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The topic of sequences and series in the curriculum and textbooks for schools in England: a way to link number, algebra and geometry

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The topic of sequences and series is important in mathematics and in a wide range of other disciplines. The mathematics of infinite series, in particular, has significant applications in physics, biology, economics, medicine and other disciplines. This paper provides an analysis of the topic of sequences and series in the curriculum and textbooks for schools in England. It shows that ideas about mathematical sequences begin to be taught in primary school and extent to cover sequences of natural numbers, often set in a geometrical context. Students in England who opt to continue with mathematics beyond age 16 study both arithmetic and geometric sequences, including the sum to infinity of a convergent geometric series. Further options at this age mean that some students study the summation of simple finite series and induction proofs for summation of series and for finding general terms. Overall, in the topic of sequences and series there are opportunities to link numeric, algebraic and geometric ideas.

Key words: school, curriculum, textbooks, sequences and series, England

Introduction

I begin with a joke (attributed to British comedian Bill Bailey). Understanding the joke relies on knowing some relatively sophisticated mathematics. The joke goes like something like this:

> An infinite number of mathematicians go into a bar. The first mathematician asks the barman for a beer. Before the barman can react, the second mathematician asks for half a beer. The barman surveys the infinitely large group of mathematicians and pours two beers. He then says to the assembled mathematicians: “Here are two beers; now you sort it out!”

Mathematically, this joke involves the following infinite series:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots$$

The joke, of course, replies on the following remarkable mathematical result:

$$\sum_{k=0}^{\infty} 2^{-k} = 2$$

This is related, clearly, to the following infinite sequence of numbers which sums to 1:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots$$

Such examples have been known to be a source of puzzlement and wonder since antiquity. They hark back to what have become known as *Zeno's paradoxes* (or dichotomies), a set of infinite series problems thought to have been assembled by Zeno of Elea, an ancient Greek who lived from about 490 BCE to about 430 BCE.

In mathematics, a *series* is the sum of the terms of an infinite *sequence*, where a *sequence* is an ordered list of objects such as the even positive integers (2, 4, 6, ...). In the examples above the sums of the sequences are remarkable because while each sequence of numbers is *infinite*, the sum is a *finite* number. When a sequence of numbers is *infinite* the term *infinite series* is sometimes used to emphasize the fact that series contain an infinite number of terms. Examples of such series include *arithmetic series* (ones for which the difference between successive terms is a constant) and *geometric series* (with the ratio between successive terms a constant). Particularly important types of series include the binomial series, harmonic series, power series, Fourier series, Taylor series, and so on.

The mathematics of sequences and series in general, and of infinite series in particular, has a wide span of applications, both within mathematics and science and across a remarkable range of disparate disciplines. Such applications range from calculus through to differential equations and beyond in mathematics, and to wave mechanics and many other applications in physics, biology, economics, medicine and other disciplines. Many of these have geometric components (for example, areas under curves and volumes of revolution in calculus). Yet, as Gonzalez-Martın, Nardi and Biza (2011) explain, when students encounter the topic of infinite series they can find it both complex and often counter-intuitive. As such, it might be anticipated that coverage of mathematical *series* in the school curriculum and in school textbooks would likely be limited; perhaps restricted, if the topic occurs at all, to the final year or so of high school prior to studying mathematics at University. Much more likely to occur in the school curriculum and in school textbooks are ideas about mathematical *sequences*, probably mainly sequences involving the natural numbers.

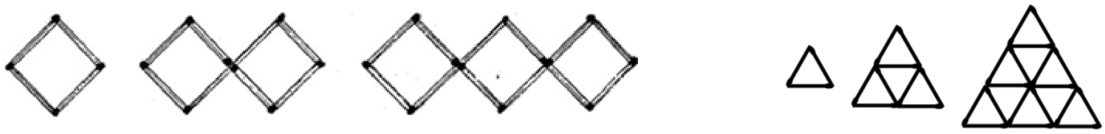
Given that the topic of sequences and series is important in mathematics and in a wide range of other disciplines, the aim of this paper is to report on the coverage given to the topic in the curriculum and textbooks for schools in England, something that has not previously been reported in any detail (except for some brief consideration in a 1997 report by a Royal Society/JMC Working Group; see Royal Society, 1997). In what follow, I begin by providing a concise overview of some of the research that has been conducted on the topic of sequences and series in the school curriculum as this is likely to have had some influence on the design of the mathematics curriculum and of school textbooks. This leads to an analysis of the topic of sequences and series in the curriculum and textbooks for schools in England, presented as Appendix A.

Research on sequences and series in the school curriculum

Through surveying the literature for this paper, the available research points to ideas of number sequences being considered important for learning counting and number operations in the early years of schooling (pupils aged 4-7), and thence for learning to be confident and competent with relevant algebraic ideas in the middle school years (pupils aged 8-16). Research on ideas relating to infinite series points to some ideas sometimes occurring in the final year or so of high school prior to studying mathematics at University but with the key ideas mainly encountered during University-level courses in mathematical analysis. The main points of some of this research are summarised in what follows, beginning with the start of formal schooling.

In the early years of schooling (when pupils are in the 4-7 age-range), available research indicates that ideas of number sequences can be introduced when children are learning counting and number operations. For example, Munn (2008, p. 25) notes that during the first two years of primary school in England (when pupils are 5-7 years old), most children begin "constructing the number word sequence beyond 30, and are beginning to work out the pattern of decade numbers [10, 20, 30, ….]". Such experience of counting strategies, says Munn, build to sequence-based (jump) strategies for early arithmetic; for example, when computing 23 + 16, a sequence-based (jump) strategy would be first to compute 23 +10 = 33, then 33 +6 = 39. Such practice is embedded in curricula advice for primary school teachers in England which states that, at this stage in their learning, children should learn to "observe number relationships and patterns … and use these to derive [number] facts" (DfES 2006, p. 70). Some of these ideas can be traced back to, amongst others, Steffe (1992). For more on how children use their early knowledge of number sequences in constructing fractional concepts and operations, see Steffe and Olive (2010).

In the middle school years (when pupils are aged 8-16) available research indicates that the topic of number sequences is considered useful when children are learning relevant algebraic ideas. A key strand in this research focuses on children understanding algebraic relations and how they are represented, in addition to learning to manipulate algebraic symbols and expression. As such, one idea suggests that pupils should study number sequences in order to express general functional rules that relate the n^{th} term in a sequence to its value; a pedagogic strategy that can be traced back to at least Kieran (1990). In this sort of learning the stages of understanding that middle school pupils go through have been identified by Bishop (2000) as: counting and verbal directions; relationships as single operations; term-to-term sequential relationships; all leading to functional relationships between sequence position and the value of the term. As Bishop confirms and illustrates, many sequence problems used in this way with students in Grades 7 and 8 are set in a geometric context where the numbers in the sequence relate to a pattern of geometrical figures in which each figure is derived from the previous figure by some well-defined procedure; see Figure 1 (see also Rivera and Becker, 2008; Rivera, 2010).



a) 4, 8, 12, …… b) 3, 9, 18, ……

Figure 1: examples of patterns of geometrical figures use to generate number sequences

At much higher levels of mathematics, at University level, ideas of the infinite series and the convergence of sequences become fundamental notions. Indeed, the idea of the limit of a sequence of real numbers (based on the Cauchy-Weierstrass definition of the continuity of a function) is central to advanced calculus and mathematical analysis (the latter being the branch of pure mathematics that deals with the calculus of differentiation, integration, limits, and analytic functions). Here the evidence to date has focused on the many difficulties that students have with the mathematics of infinite series and the convergence of sequences. Such research shows, for example, that few secondary school students [aged 16-19] “perceived a sequence as a function” (Przeniosloa, 2006, p. 821) and that difficulties with the concept of series impact on undergraduate students’ understanding of the concept of integral and, in particular, the concept of improper integral (Gonzalez-Martın et al, 2011). Interestingly, Alcock and Simpson (2004, p. 1), argue for “the centrality of visual reasoning in solving real analysis problems”, concluding that that “visual images provide students who focus on them with a strong sense that they understand the material and can correctly answer questions about it” (p. 29).

Overall, the research suggests that for the topic of sequences and series, ideas of mathematical sequences are developed from the primary school years, become a vehicle for learning about algebraic generalisation in the lower secondary years, and can develop towards ideas of infinite series for students specialising in mathematics at University. I now turn to aspects of an analysis of the curriculum and textbooks for schools in England.

Analytic framework and method

The analysis reported in this paper comes from a larger comparative study being conducted at the University of Southampton, UK. In this paper, the data is restricted to the sources listed in Table 1. The analytic procedure was first to scour each source for explicit coverage of the topic of sequences and series. This was followed up by a page-by-page detailed inventory in which everything related to the topic of sequences and series was recorded.

Findings

The analysis of the curriculum and textbooks for schools in England, for ease of presentation, is laid out in Appendix A.

The analysis shows the following:

- Children are taught to “describe and extend number sequences” from the age of five, but the word “sequence” is probably not used with children until they are 7 or 8.
- By the time children are 10 or 11 they can continue simple number sequences and find numbers that have been omitted from a simple sequence.

Primary mathematics education (pupils aged 5-11)	
National Curriculum	DfEE (Department for Education and Employment) (1999) *Mathematics: The National Curriculum for England*. London: DfEE.
Curriculum guidance	DfES (Department for Education and Skills) (2006), *Primary Framework for Literacy and Mathematics*. London: DfES.
Textbooks	*Abacus Evolve* series, published by Pearson
Lower secondary mathematics education (pupils aged 11-14)	
National Curriculum	DfEE (Department for Education and Employment) (1999) *Mathematics: The National Curriculum for England*. London: DfEE.
Curriculum guidance	DfEE (Department for Education and Employment) (2001), *Key Stage 3 National Strategy: Framework for Mathematics*. London: DfEE.
Textbooks	*Exploring Maths* series, published by Pearson
Upper secondary mathematics education (pupils aged 14-16)	
National Curriculum	DfEE (Department for Education and Employment) (1999) *Mathematics: The National Curriculum for England*. London: DfEE. QCA (Qualifications and Curriculum Authority) (2006), *GCSE Mathematics Subject Criteria* [QCA/06/2901]. London: QCA.
Curriculum guidance	Edexcel (2010), *Edexcel GCSE Content Exemplification*. London: Edexcel.
Textbooks	*Edexcel GCSE Mathematics* textbook, published by Pearson
Mathematics education for students aged 16-18	
National Curriculum	QCA (Qualifications and Curriculum Authority) (2002), *GCE A-level Subject Criteria for Mathematics*. London: QCA.
Curriculum guidance	Edexcel (2010), *GCE A-level Mathematics Specification*. London: Edexcel.
Textbooks	*Edexcel Core Mathematics 1 & 2* textbooks, published by Pearson

Table 1: sources used for the analysis

- The process of finding a term in a sequence from its position is introduced when children are 11 or 12; at 12 or 13 they are taught to use position-to-term rules, including with a spreadsheet on a computer.
- In the age-range 14-16, the higher-attaining students are taught term-to-term and position-to-term definitions of sequences, including finding the n^{th} term of an arithmetic sequence.
- Students aged 16-18 who opt to specialise in mathematics cover arithmetic and geometric series, including the use of $\sum$ notation. They also are taught how to find the sum to infinity of a convergent geometric series and are introduced to the binomial series.

From age 7 or 8, when children are finding the terms of a number sequence, the context is often spatial (of the form illustrated in Figure 1).

Discussion

Both Sutherland (2002), in a comparative survey of the algebra curriculum across a range of countries, and Mason and Sutherland (2002), in review of the key aspects of teaching algebra in schools, found that 'expressing generality' (as exemplified by finding the terms of a number sequence) was an approach to algebra evident mainly in mathematics curricula in English-speaking countries (eg: Australia, Canada, England). In such countries, Mason and Sutherland (2002, p. 25) go on to suggest, expressing generality "has tended to be confined to finding formulae for patterns, whether from a geometric source… or in the structure of number". Despite the prevalence in English-speaking countries, in a national survey in Chinese Taipei, Lin and Yang (2004, p. 464) concluded that "reasoning on geometric number patterns is a proper initial activity for learning algebraic thinking in Grade 7".

Concluding comments

The evidence presented in this paper, from existing research and from the analysis of the curriculum and textbooks for schools in England, shows that the topic of sequences and series can appear at all levels of mathematics education, from early primary school (where the focus is on learning the number sequence as a starting point for arithmetic calculation) through middle school (where the topic can be a vehicle for learning basic algebra) through to the upper years of secondary school (where the focus becomes preparation for the study of infinite sequences). Despite this structure, Gonzalez-Martın et al (2011) highlight that University-level students continue to exhibit confusion between sequences and series as well as resisting seeing sequences as a type of function. This shows that more efforts are needed in mathematics education research if more students are going to be successful in mathematics. Such efforts may benefit from more attention being paid in the curriculum and in textbooks to the links between number, algebra and geometry (Jones, 2010).

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References

Bishop, J. (2000). Linear geometric number patterns: Middle school students' strategies. *Mathematics Education Research Journal*, 12(2), 107–126.

DfES (Department for Education and Skills) (2006), *Primary Framework for Literacy and Mathematics*. London: DfES.

Gonzalez-Martın, A. S., Nardi, E. & Biza, I. (2011), Conceptually-driven and visually-rich tasks in texts and teaching practice: the case of infinite series. *International Journal of Mathematical Education in Science and Technology*, 42(5), 565–589.

Jones, K. (2010) Linking geometry and algebra in the school mathematics curriculum. In, Usiskin, Z., Andersen, K. & Zotto, N. (Ed.) *Future Curricular Trends in School Algebra and Geometry* (pp. 203-215). Charlotte, NC: Information Age Publishing.

Kieran, C. (1990). Cognitive processes involved in learning school algebra. In E. P. Nesher, & J. Kilpatrick (Eds.), *Mathematics and Cognition* (pp. 96-112). New York, NY: CUP.

Lin, F.-L. & Yang, K.-L. (2004). Differentiation of students' reasoning on linear and quadratic geometric number patterns. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Psychology of Mathematics Education Conference* (Vol. 4, pp. 457–464). Bergen: PME.

Mason, J. & Sutherland, R. (2002). *Key Aspects of Teaching Algebra in Schools*. London: QCA.

Munn, P. (2008), Children's beliefs about counting. In I. Thompson (Ed.) *Teaching and Learning Early Number* (2nd edition) (pp.1-36). Maidenhead: Open University Press.

Przeniosloa, M. (2006), Conceptions of a sequence formed in secondary schools, *International Journal of Mathematical Education in Science and Technology*, 37(7), 805-823.

Rivera, F.D. (2010), Visual templates in pattern generalization activity, *Educational Studies in Mathematics Education*, 73(3), 297–328.

Rivera, F. D. & Becker, J. R. (2008), Middle school children's cognitive perceptions of constructive and deconstructive generalizations involving linear figural patterns, *ZDM : The International Journal on Mathematics Education*, 40(1), 65–82.

Royal Society (1997), *Teaching and Learning Algebra pre–19: Report of a Royal Society/JMC Working Group*. London: Royal Society.

Steffe, L.P. (1992), Learning stages in the construction of the number sequence. In J. Bideau, C. Meljac, & J-P. Fischer (Eds.), *Pathways to Number: Children's developing numerical abilities* (pp. 83–98). Hillsdale, NJ: Lawrence Erlbaum Associates.

Sutherland, R. (2002), *A Comparative Study of Algebra Curricula*. London: QCA.

Steffe, L. P. & Olive, J. (2010), *Children's Fractional Knowledge*. New York: Springer.

Appendix A

Age range	National Curriculum	Curriculum guidance	Textbook
Age 5-6	Recognise sequences, including odd and even numbers to 30 then beyond	Describe and extend number sequences: count on or back in steps of 1, 10 or 100 from any number; count on or back in twos; count on or back in steps of any size	Textbook title: *Abacus Evolve* 1. Three “workbooks”, each of 48 pages. None of the workbooks has a contents page; there is no use of the word “sequence” in any of the three books. *Workbook 1*: p3: counting to 20; p5: next number [single digit] *Workbook 2*: p3: counting next number [two digit]; p5: counting on in 10s; p37-39: counting on in 10sp40: one more, one less; p43: count back in 1s *Workbook 3*: p9: find the in-between numbers [two digit]; p37-38: counting in 5s
Age 6-7	Recognise sequences, including odd and even numbers to 30 then beyond	Describe and extend number sequences: count on or back in steps of 1, 10 or 100 from any number; count on or back in twos; count on or back in steps of any size	Textbook title*: Abacus Evolve* 2. Three “workbooks”, each of 48 pages; plus one “textbook” of 32 pages. None of the workbooks has a contents page; there is no use of the word “sequence” in the workbooks or the textbook. *Workbook 1*: p2: one more, one less [two digit]; p3: one more, one less [two digit]; p4: one more, one less [two digit]; p12: counting [two digit]; p13: counting [two digit *Workbook 2*: p2: one more, one less [two and three digit]; p3: ten more, ten less; p4: counting in 100s; p5: counting on in odd numbers; counting on in even numbers p38: counting on in 10s, 2s and 5s *Workbook 3*: Nothing related to sequences *Textbook*: p3: counting on in 1s, 10s and 100s; p13: counting back in 10s
Age 7-8	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Describe and extend number sequences: count on or back in steps of 1, 10 or 100 from any number: count on or back in twos, and recognise odd and even numbers: count on or back in steps of any size	Textbook title*: Abacus Evolve* 3. Three “textbooks” each of 80 pages. Each of the textbooks has a contents page; the word “sequence” in used in Textbook 1 only. *Textbook 1*: number sequences: p14: one more, one less; ten more, ten less [two digit]; p15: one more, one less; ten more, ten less [three digit]; p16: ten more, ten less; 100 more, 100 less [three digit]; p17: one more, one less; ten more, ten less [four digit] In additional number pattern practice occurs in *Textbook 1*: p3-4, counting in 1s; p6, counting in 10s; p10, counting in 100s; *Textbook 2*: p11, counting in 10s; p12, counting in 100s; p13, counting in 50s; p14, counting in 25s; p15-18, counting in 2s even and odd; *Textbook 3*: p19, counting in 2s, even and odd; p20-22, counting in 5s and 50s
Age 8-9	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Textbook title*: Abacus Evolve* 4; Three “textbooks” each of 80 pages. Each of the textbooks has a contents page; the word “sequence” does not appear No work on number sequences
Age 9-10	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Textbook title*: Abacus Evolve* 5. Three “textbooks” each of 80 pages. Each of the textbooks has a contents page; the word “sequence” in used in Textbook 1 only. T*extbook 1*: number sequences; p23: counting in 25s [in a sequence]; p24: finding the smallest positive starting number [in a sequence]; p25: continuing to 10 steps [of a sequence]; p26: finding missing numbers [in a sequence]

Age 10-11	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back	Textbook title*:* *Abacus Evolve* 6. Three "textbooks" each of 80 pages. Each of the textbooks has a contents page; the word "sequence" in used in Textbook 2 only. *Textbook 2*: number sequences; p62: continuing next four steps; Fibonacci sequence; p63: decimal and fraction sequences; p64: sequence of triangle numbers
Age 11-12	Generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers)	Generate and describe sequences	Textbook title: *Exploring Maths* 3 [327 pages] (one of a series of seven books covering levels 1 – 7 of the National Curriculum. Each book aims to cover one NC level. Books 3, 4 and 5 form the Core for key stage 3) p13-14 sequences and rules; "*using simple rules to extend number sequences*"
	Find the first terms of a sequence given a rule arising naturally from a context [eg: a regularly increasing spatial pattern]; find the rule (and express it in words) for the nth term of a sequence	Find the nth term, justifying its form by referring to the context in which it was generated	p15-16 finding missing terms; "*this lesson will help you to find missing terms in sequences*" p16-19 sequences from patterns; "*sequences from patterns and finding missing terms*" p19-21 using a letter symbol; "*this lesson will help you to use letters to stand for numbers*"
	Generate terms of a sequence using term to-term and position to-term definitions of the sequence; use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated	Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT	p21-22 finding a term from its position; "*find a term from its position in the sequence*" p157-159 sequences; "*find a formula for the nth term of a sequence*" pp305 – 309 Revision section on Sequences, functions and graphs; "*a reminder of how to generate sequences and plot graphs of simple functions*". Additional number pattern practice: p35 completing decimal sequences; p131 patterns in squares; p239 number of squares on a chessboard & forming patterns from pyramids of squares
Age 12-13	Generate common integer sequences Find the first terms of a sequence given a rule for the nth term of a sequence Generate terms of a sequence using term to-term and position to-term definitions; use linear expressions to describe the nth term of an arithmetic sequence, justifying its form	Generate and describe sequences; find the nth term, justifying its form by referring to the context in which it was Generated; generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT	Textbook title: *Exploring Maths* 4 [346 pages] p30-32 term-to-term rules; "*this lesson will help you to use rules to generate sequences*" p32-34 position-to-term rules; "*this lesson will help you to use position to term rules to generate sequences and find a formula for the nth term of a sequence*" p35-36 using a spreadsheet to generate sequences; "*this lesson will help you to use a spreadsheet on a computer to explore sequences*" p37-38 exploring patterns; "*solving problems involving patterns and sequences*" p299 – 302 Revision section on Expressions and sequences; "*this lesson will help you to generate sequences, simplify expressions and substitute numbers into expressions*"
Age 13-14	Generate common integer sequences Find the first terms of a sequence given a rule for the nth term of a	Generate and describe sequences; find the nth term, justifying its form by referring to the context in which it was	Textbook title: *Exploring Maths* 5 [385 pages] p11-14 generating sequences; "*this lesson will help you to use rules to generate sequences*" p15-17 making generalisations; "*this lesson will help you to find a formula for the nth term*

	sequence Generate terms of a sequence using term to-term and position to-term definitions; use linear expressions to describe the nth term of an arithmetic sequence, justifying its form	Generated; generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT	*of a linear sequence*" p18-20 using computers; "*this lesson will help you use a computer to explore linear sequences*" p336 – 338 Revision section on Sequences, equations and graphs; "*this lesson will remind you how to find the nth term of a linear sequence, solve linear equations and plot the graphs of simple linear functions*"
Age 14-16	Generate common integer sequences Find the first terms of a sequence given a rule for the nth term of a sequence Generate terms of a sequence using term to-term and position to-term definitions of the sequence; use linear expressions to describe the nth term of an arithmetic sequence, justifying its form	Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence	Textbook title: *Edexcel GCSE Mathematics A Linear* ***Foundation*** student book (one book covering the entire specification 584 pages; designed for ***average students***) p246-247 continuing patterns by adding; p247-248 continuing patterns by subtracting; p249 continuing number patterns by multiplying; p250 continuing number patterns by dividing; p251-253 continuing patterns in pictures; p253-254 one stage input and output machines; p255-258 two stage input and output machines Textbook title: *Edexcel GCSE Mathematics A Linear* ***Higher*** student book (one book covering the entire specification 715 pages; designed for ***higher-attaining students***) p22-23 term-to-term and position-to-term definitions
		Use linear expressions to describe the nth term of an arithmetic sequence	Textbook title: *Edexcel GCSE Mathematics A Linear* ***Foundation*** student book (one book covering the entire specification 584 pages; designed for ***average students***) p258-260 finding the nth term of a number pattern p260-261 deciding whether or not a number is in a number pattern Textbook title: *Edexcel GCSE Mathematics A Linear* ***Higher*** student book (one book covering the entire specification 715 pages; designed for ***higher-attaining students***) p24-25 the nth term of an arithmetic sequence
Age 16-18	Sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$. Arithmetic series, including the formula for the sum of the first n natural numbers. The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of r < 1 . Binomial expansion of (1+ x)n for positive integer n. The notations n! and $\binom{n}{r}$	Sequences, including arithmetic and geometric sequences, those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$ Arithmetic series, including the use of $\sum$ notation Geometric series; the sum to infinity of a convergent geometric series, including use of $\|r\| < 1$ and use of $\sum$ notation Binomial expansion of $(1+x)^n$ for positive integer n.	Textbook title*: Edexcel Core Mathematics 1*(185 pages plus CD) p92 definition and example of a sequence; p93-95 formula for the nth term of a sequence (e.g.U_n = 3n-1) ; p95-98 recurrence relationships (or recurrence formulae); p98-100 definition of arithmetic sequence; p100-103 arithmetic series as $U_1 + U_2 + U_3 + \ldots + U_n$; p103-106 sum of an arithmetic series; p107-110 $\sum$ notation Textbook title: Edexcel Core Mathematics 2 (227 pages plus CD) p103 definition of geometric sequences; p104-106 geometric sequence using first term and common ratio r; p107-109 geometric sequences for problems involving growth and decay; p109-112 sum of a geometric series; p112-117 sum to infinity of a convergent geometric series; p77-79 Pascal's Triangle to expand expressions such as $(x+2y)^3$; p79-80 combinations and factorial notation to expand binomial expressions; p80-82 $\binom{n}{r}$ to work out the coefficients in the binomial expansion. p82-85 expand $(1+x)^n$ and $(a+bx)^n$ using the binomial expansion

Students aged 16-18 studying *Further Mathematics* study the summation of simple finite series and induction proofs for summation of series and for finding general terms.