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OPEN TASKS IN JAPANESE TEXTBOOKS: THE CASE OF GEOMETRY FOR LOWER SECONDARY SCHOOL

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From the early 1970s Japanese mathematics teaching has put particular emphasis on designing and implementing lessons in which students can explore different approaches and ways to solve given problems. This is generally known as the open-ended approach because the tasks tackled by students are 'open' to different solution strategies and approaches. The purpose of this paper is to report on the extent to which such an open approach is realised in current mathematics textbooks in Japan. Our focus is geometrical reasoning in lower secondary school, as this is one of the important topics in mathematics. In analysing the topic of angles in polygons, we found that open problems were utilised by Japanese textbook authors as worthy approaches which all teachers could take in everyday lessons on this topic. We further found that while each of the seven textbook series had undergone the same official authorisation process, the textbooks showed different approaches for the same geometry topic. This illustrates the variety of ways in which the open-ended approach can be enacted in the teaching of mathematics.

Keywords: open-ended approach, geometry, secondary school, Japan

INTRODUCTION

Beginning in the early 1970s Japanese mathematics teaching began putting particular emphasis on designing and implementing lessons in which students can explore different approaches and ways to get given problems. This has become generally known as the open-ended approach because the tasks tackled by students in such lessons are 'open' to different solution strategies and approaches (Shimada, 1977; Becker & Shimada, 1997). The pedagogical value of such an open approach is widely recognised in mathematics education research (e.g. Silver, 1997).

School mathematics textbooks are important objects for analysis as they represent the 'potentially implemented curriculum' (Valverde, et al, 2002) that influences the ways of teaching and learning of mathematics in everyday lessons. In Japan, textbooks may be published by private publishers but the textbooks need to reflect the official 'Course of Study' and the accompanying 'Teaching Guide', both published by the Ministry of Education and Science. What is more, all textbooks must pass through a textbook authorization process overseen by the Ministry of Education and Science, a process that can take about three years from initial development to classroom use (Shimizu & Watanabe, 2010). In practice, there are usually around seven different textbook series on offer from different publishers. The use of textbooks by teachers can vary, but textbooks are one of the most influential resources for planning and implementations in daily lessons in Japan (Sekiguchi, 2006), and it is therefore important to study textbooks in order to understand the complexities of mathematics lessons.

The purpose of this paper is to address the following research questions: To what extent are open approaches realised in current school mathematics textbooks in Japan?; Can we observe any different approaches among the textbooks published by the seven publishers? In this paper, we particularly focus on open approaches in angles in polygons, because a) it is one of the common geometrical topics in lower secondary schools internationally, and b) our preliminary analysis suggests that this is one of the topics in which open approaches are evident compared to topics such as proving.

OPEN APPROACHES IN GEOMETRY REASONING-AND-PROVING

By elaborating the original ideas proposed by Shimada and his colleagues between 1971-6 (Shimada, 1977), Becker and Shimada (1997) described "incomplete" or "open-ended" problems as those in which "students are asked to focus on and develop different method, ways, or approaches to getting an answer to a given problem" (p. 1). In this situation, methods for arriving at answers are seen as just as important as the actual answer. The approach has been found to be effective in not only raising the general level of students' performance, but also in cultivating students' mathematical thinking and creativity (e.g. Kwon, Park & Park, 2006). Our focus is geometrical reasoning in lower secondary schools, as this is one of the important topics in mathematics. In particular, in Japan geometry is used to introduce ideas of formal proving (Jones & Fujita, 2013; Fujita & Jones, 2014).

In order to conceptualise activities involved in proving in geometry, we refer to "reasoning-and-proving" (Stylianides, 2009, p. 259); that is, the classroom activities of "identifying patterns, making conjectures, providing non-proof arguments, and providing proofs". We have already obtained an overview of G8 geometry content within this framework (see Fujita & Jones, 2014). Through our analysis, we reported that in G8 geometry lessons start from a problem solving situation, with the geometrical facts to be proved and learnt often coming later. A sequence from conjecturing to proving is prominent in the process of reasoning-and-proving in the textbook.

In this paper, we take a step further and consider how "open situations" are intended in the textbook in geometrical reasoning. By considering the activities identified by the reasoning-and-proving framework, 'open' approaches in geometry can be conceptualised as follows: (a) devising different ways to identify patterns, (b) devising different conjectures, (c) devising different methods of proving, (d) devising different methods of non-proof argument, (e) devising different new statements after proving a statement. We used this as an analytic framework and have conducted an analysis of the two chapters related to geometry in Tokyo Shoseki's Mathematics G8 (textbook A), one of the most popular textbooks in among seven publishers. Table 1 summarises our analysis of 34 (+4 flexible) lesson chapters related to geometry (Ch. 4 and 5).

In Chapter 4 Section 1, students are particularly encouraged to devise different methods to identify the pattern (code (a)) or devise different ways of proof or non-proof arguments (codes (c) and (d)). For example, after the problem was introduced on p. 89, on p. 90, the following two different methods to find the sum are shown as examples (Fig. 1a & b). Also on p. 91 as an extension activity (Fig. 1c) is suggested.

Table 1: Suggested open activities in G8 textbook

	(a)	(b)	(c)	(d)	(e)
Ch. 4* Sec. 1	3		2	1	
Ch. 4* Sec. 2					
Ch. 5** Sec. 1		1			1
Ch. 5** Sec. 2		1	1		1

*Chapter 4. Parallelism and congruence (Section 1: Parallel lines and angles; Section 2: Congruent figures)

**Chapter 5. Triangles and quadrilaterals (Section 1: Triangles; Section 2: Parallelograms)

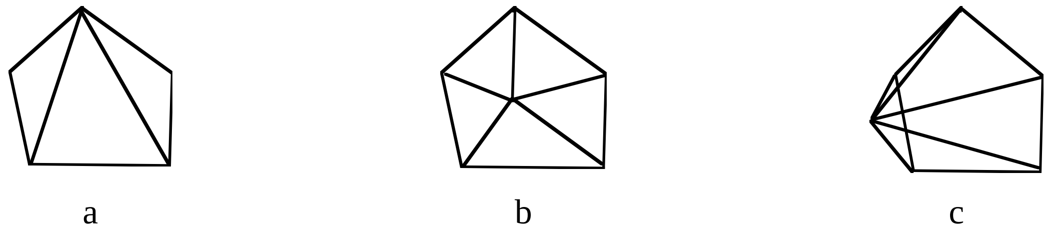

Figure 1: Different methods to find the sum of inner angles of polygons

For (c), different methods of proving the sum of inner angles of a triangle is 180 are shown on p. 99 and p. 100, and as a non-proof argument an alternative way of finding the sum of exterior angles of a polygon is shown.

OPEN APPROACHES FOR THE SUM OF POLYGON INNER ANGLES

We conducted further examinations of how the angles in polygons are taught across the seven textbooks. We found at least four different methods: (1) drawing lines from one vertex to the other (e.g. fig. 1a), (2) drawing lines from a point inside polygons (fig. 1b), (3) drawing lines from a point on one of the sides and (4) drawing lines from a point outside polygons (e.g. Fig 1c, although this method is unlikely to be considered by the majority of Grade 8 students).

Method (1) is used as an introductory example in the all seven textbooks. In these problem situations, cases for triangles, quadrilaterals, pentagons etc. are summarised in a table, and encourage students to inductively identify a pattern of the number of triangles in polygons (‘n-2’). Then ‘n-2’ is used as a premises to generalise the formula ‘180 x (n-2)’. This approach is used in the all seven textbooks. Also, this is an expected progression from primary school where students have already experienced investigating the angles in polygons. Their learning experience in primary schools is generalised through the case of the n-polygon, which is one of the aims of this lesson.

In contrast, treatments for (2)-(4) are quite diverse among the seven textbooks. For example, the method (2) appears as a main content except textbook D which treats the method (2) as an optional contents. This means a teacher using textbook D is not, on the one hand, expected to teach this method during the lesson. On the other hand, this does not necessarily mean that a learning opportunity with method (2) is missed (regardless the textbook design) because students can devise method (2) by themselves if the teacher encourage their students to think openly (Haneda, et al, 2001). The other treatments imply the method should be taught within a lesson. Also, textbook A uses a table so that students can inductively identify a pattern, but others do not use table but ask students to find the sum as an ‘exercise’. The method (3) appears as a main contents in textbooks E, F and G, and as optional contents in A, C and D, but does not appear in textbook B. Also, textbook D asks students to examine the method (3) for only the case of hexagons, whereas A, C, E, F and G ask the case of n-polygon. Finally, the method (4) appears in only textbook A. When we refer to teacher’s guide, this textbook tries to encourage students to see geometrical figures from a dynamic point of view. Also, the guide for textbook C suggests using dynamic geometry software as well.

In this problem, the number of triangles ‘n-2’ plays a key role to generalise the pattern, but the treatment of ‘n-2’ again differs in the textbooks. For example, some textbooks ask students to relate the numbers of vertices and triangles and to deduce the number of triangles when there are n vertices (textbook C). Textbooks B, E and F provide more detailed diagrams and ask students to explain why it is possible to divide an n-polygon into ‘n-2’ triangles by considering the number of diagonals drawn from a point. In contrast, textbooks A, D and G just show tables and ask students inductively to identify ‘n-2’ solely by looking at numerical values.

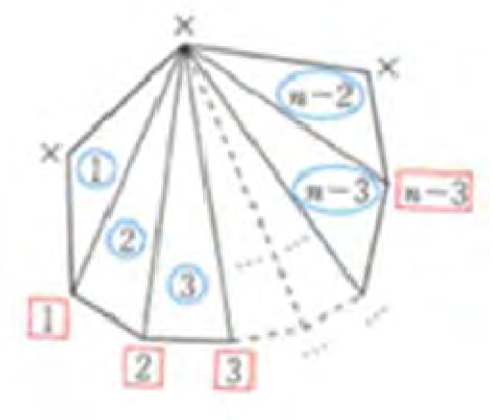

Figure 2: Deducing the number of triangles

Table 2: Open approaches for angles in polygons across the seven textbooks

	A	B	C	D	E	F	G
Method (1)	○	○	○	○	○	○	○
Method (2)	○	○	○	△	○	○	○
Method (3)	△	×	△	△	○	○	○
Method (4)	△	×	×	×	×	×	×
Deducing ‘n-2’ from diagrams	×	○	○	×	○	○	×

Table 2 summarises our findings in terms of approaches for the sum of the inner angles of triangles. ○, △ and × indicate 'appears in the main text', 'appears as optional', and 'does not appear', respectively. Note that we do not consider for example textbook D does not provide enough opportunities for open approaches for angles in polygons, but simply this table suggests that even the textbooks which have undergone the authorisation process show different approaches for the same topic in geometry. We discuss the implications of our findings in the next section.

DISCUSSION

Our analysis suggests that open approaches are particularly evident in identifying patterns. We then examined angles in polygons as an example of the open approach across the seven textbooks authorised by the Japanese Ministry of Education and Science. While all textbooks take the method to identify the formula for the sum of inner angles of polygons (method (1)), other methods are not always considered as the main topic for the lessons. However, at least the two methods (1) and (2) are 'visible' in the all textbooks. This shows how open approaches are recognised as worthy approaches which Japanese textbook authors consider that all teachers to take in everyday lessons.

One of the important purposes of G8 geometry is to introduce ideas of formal proving. Although in lessons which deal with the sum of inner angles of polygons students are not required to undertake formal proofs, it is important to provide learning opportunities to explain reasons why. Thus, some textbooks (B, E and F) explicitly ask students to explain why the number of triangles created by diagonals will be 'n-2' by relating to geometrical diagrams. Other textbooks just use tables and numerical values to find a pattern inductively. Which approach would be more appropriate for the foundation of geometrical reasoning? Again teachers who are aware of the importance of geometrical reasoning might relate the geometrical meaning of 'n-2' regardless the textbook design. In contrast, an explicit link between 'n-2' and geometrical diagrams in textbooks B, E and F might encourage teachers to teach this topic more 'conceptually' rather than 'procedurally' to find the formula.

For teachers it is always difficult to determine to what extent we should 'tell' to students. Chazan and Ball (1999, p. 10) argue the need for classroom-based research to identify and understand "what kind of 'telling' it was, what motivated this 'telling', and what the teacher thought the telling would do", together with ways of "probing the sense that different students make of different teacher moves".

CONCLUDING COMMENT

The main purpose of this topic is to find the sum of inner angles of polygons, and we want all students actively engage tasks and find different methods by themselves as much as possible. We, as educators, expect that textbooks give some insights for 'what kind of telling'. However, our analysis reveals even the authorised textbooks have varied views of 'what kind of telling' in open approaches in geometry. For textbook writers it is very difficult to decide to what extent textbooks should express their views how mathematical topics ought to be taught. Also, whereas some studies (e.g. Kwon, Park and Park, 2006) suggest that open approach might cultivate students' creative thinking, there is no guarantee teachers would follow the approaches suggested by the textbooks. For future research it might be particularly interesting to examine how this topic is actually taught in classrooms, and to what extent the ways in which teachers teach this topic are influenced by the textbooks that they use.

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