Cite as: Ding, L., Jones, K., Pepin, B. & Sikko, S.A. (2014). An expert teacher's local instruction theory underlying a lesson design study through school-based professional development. In C. Nicol, P. Liljedahl, S. Oesterle & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (PME38 & PME-NA36) (Vol 2, pp.401-408). Vancouver, Canada: PME.

AN EXPERT TEACHER'S LOCAL INSTRUCTION THEORY UNDERLYING A LESSON DESIGN STUDY THROUGH SCHOOL-BASED PROFESSIONAL DEVELOPMENT

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Using data from a research project in Shanghai, China, this paper reports on an expert teacher's implicit 'Local Instruction Theories' (LIT) (Gravemeijer, 2004) that underpin his guidance of a junior teacher in lesson design and implementation. Our analysis focuses on the expert teacher's input to the junior teacher to help her understand how and why to redesign a lesson as part of a school-based teacher professional development project. We identified three key points of the expert's implicit LIT: mathematics has its own form of exploration; each student should have their own thinking path at each key point of the learning process; and each student should not only be able to experience use of their own representation, but also learn about other students' representations and the excellence of representations.

INTRODUCTION

At a PME36 Research Forum, Li and Kaiser (2012) examined "the concept and nature of teacher expertise in mathematics instruction valued in selected education systems" (p121). In doing so, they highlighted different approaches, practices and cultural resources that are used to develop teacher expertise in mathematics instruction in different countries. In similar vein to Jaworski (2004), who sees teachers and educators working together in an *inquiry community* and in a "reciprocal relationship of a reflexive nature" (Jaworski 2001, p. 315), the analysis of five nation-wide teacher professional programs (Canada, China, Japan, Norway, and USA) by Kieran, Krainer and Shaughnessay (2013) concludes that teachers should be viewed as key stakeholders in research – "stakeholders who co-produce professional and scientific knowledge" (p. 387).

In Shanghai (SH), China, Gu and Wang (2003) have proposed the 'Action Education' (AE) model ('Xingdong Jiaoyu' in Chinese) to tackle the challenge of improving teaching through inservice teacher professional development (TPD). Three key features are emphasized in the AE model: the use of *Keli* ('exemplary lesson development' in English) (see Huang & Bao, 2006), the collaborative work of teachers with expert teachers and university researchers (mostly local but sometimes foreigners in the case of SH), and teacher follow-up reflection and action in their own class. Paine and Fang (2006, p286) consider that this SH AE as a hybrid model – a means of connecting Chinese educators to foreign ones – that characterizes reform in Chinese TPD. Such a teacher/expert collaboration attempts to develop and promote the teacher's expertise by absorbing and building on a combination of Chinese experts' accumulated "wisdom of practice" (Shulman, 1986, p. 9) and international expertise. Given the long tradition of China's own cultures of teaching and learning (Paine, Fang, & Wilson, 2003), it remains under-researched how this combination works out in practice. It is this that is a focus of our research.

In a previous paper, Ding, Jones and Pepin (2013) report how an expert teacher guided a junior teacher to develop what we called a 'hypothetical learning structure' (HLS) in her lesson design. We carefully distinguished this HLS from Simon's (1995) 'hypothetical learning trajectory' (HLT), as the HLS in our study was not based on constructivist theory but rather on the Chinese expert teacher's 'wisdom of practice' in the form of their expertise and experiences with local classroom practice. In this paper, we seek a deeper understanding of the pedagogical principles of this local expert teacher through studying his coaching of a junior teacher during our lesson design study.

In this we refer to Gravemeijer's (2004) '*local instruction theories*' (LIT) of the expert teacher. As pointed out by Gravemeijer (2004), local instruction theories go "beyond the level of an instructional sequence in terms of a series of instructional activities" (p. 108); rather, LIT are a "description of, and *rationale for*, the envisioned learning route" (p. 107; emphasis added). Our research question in this paper is: "what are the expert teacher's implicit LIT that underpin his guidance of a junior teacher in lesson design and implementation, with the particular teaching objective of developing individual children's mathematical reasoning in the class?"

THEORETICAL FRAMEWORK

Simon (1995) suggested the HLT as a way to consider the reflexive relationship between a teacher's design of activities and considerations of students' thinking as the students engage and participate in particular classroom tasks. As pointed out by Simon (1995), the term HLT underscores the importance of having a goal for teaching, some ideas for learning activities, and a sense of the direction of students' learning. The HLT consists of three components: the learning goal; learning activity/ies; and the hypothetical learning process.

Gravemeijer (2004) points out that it is not easy for teachers to design the HLT for reform mathematics in which the aim is to transform of students' current ways of reasoning to more sophisticated ways of mathematical reasoning. The central problem that teachers face involves the tension between the openness toward the students' own constructions and the obligation to work toward certain given endpoints. As Gravemeijer (2004) clarifies:

> I reserve the term *hypothetical learning trajectories* for the planning of instructional activities in a given classroom on a day-to-day basis, and I use the term *local instruction theories* to refer to the description of, and rationale for, the envisioned learning route, as it relates to a set of instructional activities for a specific topic. (p. 107)

That is, the term local instruction theory is coined to "convey the intention of offering more than a description of a learning route, or the corresponding instructional activities. In addition to these two, a local instruction theory also includes a rationale" (Gravemeijer, 2004, p. 100). As such, and akin to Simon's HLT with the addition of a rationale, the conjectured LIT consists of three components: (a) learning goals for students; (b) planned instructional activities and the tools that will be used; and (c) a conjectured learning process in which one anticipates how students' thinking and understanding could evolve when the instructional activities are used in the classroom.

In our study, we use the three components of Gravemeijer's (2004) conjectured LIT (noted above) to analyse both the junior teacher's and the expert teacher's pedagogical thinking and decision-making during the lesson design and implementation, as well as during the lesson redesign.

METHOD

Our school-based TPD study is being conducted in a local laboratory school located in Qingpu district, a western suburb of SH (see also Ding et al., 2013). The overall methodological approach of our TPD study is in the form of the AE model by Gu and Wang (2003) that aims at developing the teacher's professional knowledge – in the nature of absorbing and building on the accumulated "wisdom of practice" (Shulman, 1986) – through the teacher's lesson planning, lesson delivery, post-lesson reflection and lesson re-delivery. Two features highlighted by Huang and Bao (2006) distinguish the SH AE model from other types of TPD used in other countries – such as 'Japanese Lesson Study', case inquiry (Shulman, 1986), and course-based training and workshops: (1) the expert's input to upgrade teacher ideas in the context of peer support; and (2) the whole process of teacher action follow-up and reflection is included. At the present stage of our data analysis, we particularly focus on the expert's input to the junior teacher to help her understand *how* and *why* to redesign the lesson.

The participant groups of the study were: (1) four researchers (the four authors); (2) an expert teacher (Mr Zhang); and (3) three teachers (two in Grade 3 (G3) and one in G4; 4) twelve mathematics teachers from the mathematics teacher group of the school (from G1 to G6, ranging from newly-appointed teachers to teachers with about ten years teaching experience). In this paper we focus on one of the G3 teachers, who we call Peipei (a pseudonym), who, at the time of the research, had four years teaching experience in primary school mathematics.

Our data sources include: Peipei's initial lesson plan and accompanying classroom tasks; the transcript of her video-recorded lesson; the transcript of the video-recorded comments of the expert teacher and his work/documents to redesign the lesson and tasks; and the transcript of the video-recorded re-taught lesson.

The analysis of the development/design research approach (Gravemeijer, 2004) was used to analyse the cumulative interactions between the junior teacher's initial lesson design and implementation, and the expert teacher's comments and lesson re-design.

In so doing, we aim to make the expert teacher's implicit LIT explicit, and explain *how* and *why* the teacher reflected and revised her mathematics teaching across an interactive series of teaching cycles.

FINDINGS

Understanding the learning goal of the lesson

In Peipei's initial lesson plan and implementation, we found that the teacher tried to guide students to achieve the learning goal given in the SH official teacher's textbook reference (TTR). The TTR suggested the teacher to make one point of mathematical knowledge clearly to students in the lesson inquiry activity. In this case, the core of the inquiry was the relationship of the area, length and width of rectangles (including squares) with the constant perimeter as a stepping stone to understanding the relationship of the constant sum of two numbers and the maximum product of them.

After observing Peipei's initial lesson, the first point that Mr Zhang suggested to Peipei was carefully to consider about the learning goal suggested by the TTR. Mr Zhang explained to Peipei the learning goal as follows:

> In primary mathematics, this content is considered as a typical topic to learn how to establish a mathematical proposition. Strictly speaking, it is not about concept learning, but about proposition learning [learning how to find laws and relations in mathematics].

Redesigning the instructional activities and the tools

In the initial lesson, Peipei directly used the task given in the textbook (using 20 matches to form rectangles and to find the largest area). To achieve the learning goal explained in the TTR, Peipei organized three main instructional activities in her lesson plan and implementation: (1) The starting activity: Peipei asked students to use four numbers 1, 3, 4, 5 to combine two two-digital numbers, and then to guess which of the two to multiply to get the largest result. (2) The main activity: Peipei asked students to cooperate in a group of four students and to respectively use 20 and 18 matches to form rectangles and to record the possible length, width and area of rectangles with the constant perimeter on the worksheet. Students were also asked to use mathematical language to represent their findings on the worksheet. (3) The exercise activity: One of the tasks in this activity was to ask students to find the larger product of 94×83 and 93×84.

Mr Zhang considered that Peipei constructed the learning process not from the perspective of students, but from the perspective of the textbook. Mr Zhang said the following:

> From the teaching perspective, the logic of the lesson structure [the three instructional activities] is clear. If the teacher added one more activity to ask students to talk about the conclusion of the lesson, I guess most students could make it. Such a way of teaching is very traditional as it merely concerns on students' learning product, not on their learning process. However, students would gain benefits from the learning process, not merely from the learning product. The application of the learning product is based on students' learning

> experience, method and thinking path. To support individual learning, the teacher should address questions [pertaining to] students' starting points in their own learning and experience and what they can achieve in the lesson.

Accordingly, Mr Zhang suggested to Peipei not to use the activity of four numbers 1, 3, 4, 5 to start the lesson. Instead, Mr Zhang suggested Peipei to start the lesson by using a smaller number of matches so as to enable students with various levels of skills to handle the task within the available lesson time. The instructional activities were redesigned to enable students to experience the whole reasoning process of rediscovering the mathematical proposition (e.g., observation/operation – guesses – plausible reasoning / proving – using proper representations and language to represent the mathematical proposition) as follows: (1) Starting activity: students were asked to use matches to form rectangles and then to record the length, width, perimeter and area of the rectangles in a table; (2) Follow-up activity: students could make guesses and reasoning about their findings and then confirm their own guess. (3) Conclusion of the activity: students should learn to use different representations (e.g., drawing, symbols, their natural language and mathematical language) to characterize and to simplify the mathematical proposition of the relationship of perimeter and area of rectangles.

The expert teacher's implicit LIT

We analysed the complexity of Mr Zhang's implicit LIT according to his perspectives on students' learning methods, students as active learners, and students' mathematical reasoning development.

Students' learning methods: Mr Zhang highlighted the role of the worksheet as an effective tool to develop individual students' independent learning method. For instance, in the redesigned starting activity, students were given opportunities to independently decide the length and width of rectangles and the number of matches. As the worksheet was A4 size, the space was limited for students to draw and put matches on the worksheet. A maximum of 10 matches could be used. In using the worksheet, students would have opportunities to experience the process of reviewing their own previously learned knowledge of perimeter and area of rectangles and squares, drawing and forming rectangles and gradually to develop their reasoning of their observations and guesses.

Students as active learners: Mr Zhang explained to Peipei the complex relationship between the cognitive processes of an individual student and the classroom learning community. Mr Zhang's view is evident in his discussion with Peipei about students' group discussion, as follows:

> Students' group [or class] discussion is based on each individual's own learning experience and the related learning results. It would be too abstract for students if the teacher asked students to discuss their observation during the starting activity. Because students had not yet experienced the cognitive processes such as from sample [of matches] to operations [form rectangles by matches], and from the diagram to language, the group [or class] discussion encouraged by the teacher was from one student's language to another

> student's language. The individual student's cognitive process was interrupted by others' discussion. Sometimes, other students' talk is positive to develop the individual student's thinking development. Yet, other times, it may prevent the individual student's independent thinking. The teacher should reflect on her role of how to enable each student to develop their own learning outcome and then how to help students to correct and revise their learning experience.

Mr Zhang further used an example to explain to Peipei the teaching strategy of how to tackle such complexity of the relationship between individual, group and class during the follow-up activity in the lesson: (1) individual students should be selected by the teacher to report their worksheet data to the class; (2) a group of students would share the similar data (due to the same size of worksheet); (3) the whole class could share all reported data listed on the blackboard.

Students' mathematical reasoning development: Mr Zhang referred to two theoretical ideas to address the teacher's role in students' mathematical reasoning development: (1) the teacher can use *variation* as a means of "Pu dian" (scaffolding in Chinese) (Gu, 2012) to enable different students' reasoning and representations to be shared in the whole dynamic mathematical activity; (2) the teacher should ensure that at each key point of the learning process, each student should have their own thinking path. He said:

> Students should first develop their independent representation of their findings. After that, they can present their representations in the class. They would then learn from their peers in the class which representation is correct or incorrect, which one is a suitable, rigorous or scientific form of representation. The representation of mathematical proposition is complex as it can be represented by multiple languages and reasoning path. The teacher should ensure that each student not only has learning opportunities to demonstrate their representations and to compare with others, but also to learn to appreciate the excellence of the multiple forms of representation.

DISCUSSION AND CONCLUSION

By analysing the cumulative interaction between the junior teacher's initial lesson design and implementation and the expert teacher's comments and lesson redesign, we can identify three key points of Mr Zhang's implicit LIT as follows:

1. Mathematics has its own form of exploration. The teacher should think about how to develop students' ways of mathematical reasoning during their exploration process. The lesson should be designed in such a way that students are able to experience on their own the whole process of plausible reasoning in mathematics.
2. To experience the whole process of mathematical reasoning (plausible reason in this study), the construction of the learning process should focus on each individual student. That is, at each key point of the learning process, each student should have their own thinking path. Each student should enjoy a whole process of their own independent thinking in the learning process.

3. Mathematical proposition is complicated for it can be represented by multiple kinds of languages and various types of thinking. Each student should not only be able to experience to use their own representation, but also to learn others' representations and the excellence of representations.

In our previous studies (Ding et al., 2013; Ding, Jones, Pepin & Sikko, 2014), we focused on the expert teacher's voice. For instance, in Ding et al. (2014) we reported that while guiding the teacher to understand the new teaching norms from the overseas textbooks (e.g., Pepin & Haggarty 2001), the expert teacher simultaneously encouraged our case study teacher to use the traditional Chinese 'two basic' (basic knowledge and skills, briefly named as TB) teaching (e.g., Shen Tou) method carefully to develop students' TB in mathematics. In this paper, the expert teacher highlighted an alternative teaching method (Pu Dian) in the redesigned activities to develop students' mathematical reasoning. The expert teacher's voice on the empirically-grounded teaching approaches echoes Shulman' (1986) influential work on the nature of teachers' professional knowledge development – absorbing and building on the accumulated "wisdom of practice". In our case, it is as a *key stakeholder* (Kieran, et al, 2013) in our *inquiry community* (Jaworski, 2004).

Li, Huang and Yang (2011) show the complexity of the Chinese expert teachers' teaching expertise valued in China. In our study, we showed the complexity of the expert teacher's implicit LIT. As Mr Zhang pointed out, 'at each key point of the learning process, each student should have their own thinking path'. That is, while individual students participate into the group and class-shared thinking process, they should not stop their own thinking path and passively listen and take others' thinking path. Others' thinking path should be considered as an alternative means for individuals to develop and complete their independent thinking path. If we borrow Simon's (1995) metaphor of travel plan, the teachers ought to have a sophisticated 'travel plan' not only for one individual, but for the class of pupils. Our next step in our project is towards understanding the expert teacher's sophisticated 'travel plan' that makes the connection to each individual student's thinking in their mathematics learning journey within the class.

Acknowledgement

We thank China Welfare Institute, Shanghai Soong Ching Ling School and Shanghai Education Science Research Council who funded this school-based teacher professional development project.

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