

## **THE VALUE OF GEOMETRICAL CONSTRUCTIONS: DISCOVERING, REASONING AND PROVING IN GEOMETRY**

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The pedagogic value of geometrical constructions has been recognised for many decades and beyond (e.g. Polya, 1962). However, Tam and Chen (2012) have also identified that students often find it difficult to make a link between the construction procedures and their underlying geometric properties. Such issues arise whether constructions are performed with physical instruments or with computer software. The aim of this paper is to provide a guiding principle that can be used to inform the use of geometrical constructions in ways that foster students' proving processes (be this with or without technology).

In our research, we approach this through our central research question: how does reasoning and proving emerge during the process of geometrical construction? In doing so, we focus on issues around 'discovering' and proving. We refer to the word 'discovering' in line with De Villiers (1990). When students solve challenging geometrical construction problems such as 'Construct a square which goes through the two given points A and B', it is expected that students might 'discover' facts, reasons and relationship, for example (a) what 'conditions' are necessary to construct a shape; (b) their methods of construction; (c) reasons why their methods are correct; (d) relationship between the conjecture and conclusion.

We report two cases from Grade 7 (12-13 year old) which undertook the construction of a square described above to consider our research question. As we expected the students 'discovered' various methods to construct a square ((a) and (b)), but at the same time, they had difficulties to use properties of shapes to reason why their construction would be correct ((c) and (d)). However, after they experienced more constructions and reasoning, they gradually became aware how they could utilise properties of shapes, and a shift of their reasoning took place, i.e. a shift from relying on visual appearances or measurement to reasoning with properties of shapes. Detailed analysis of the students' construction processes and their reasoning will be presented in the actual presentation.

### **References**

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