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Key Ideas in Teaching Mathematics – Probabilistic reasoning

In this and other issues, the Secondary Magazine features a set of six articles, written by Anne Watson, Keith Jones and Dave Pratt, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the fourth in the series and focusses on Reasoning with decimals in Key Stage 3. Future articles will feature Place Value, Algebra and Probabilistic Reasoning. Previous articles focussed on similarity, ratio and trigonometry in Key Stage 3, Geometric and spatial reasoning in Key Stage 3, statistical reasoning in Key Stage 3, reasoning with decimals in Key Stage 3, and Algebraic reasoning. The opinions expressed in this article are those of the authors, and not of the NCETM.

In science, pupils are taught how to formalise their experiences of cause and effect. In mathematics, they are taught deductive systems of logic that draw on precisely defined representations, which often turn out to be helpful in science. However, determinism and logic are insufficient for the management of uncertainty. When we shake two dice, we might predict a total of 7 knowing that this is the most likely, but we would not be surprised if we were wrong. When we hang an object on a spring, there will be uncertainty about exactly how far the spring might stretch because of measurement error but, by focusing on the physics of the situation, we might predict the stretch with some confidence. The level of confidence we feel in making predictions about most situations in professional and everyday life falls between these two examples. Consider the following examples, involving both causality and uncertainty.

Professionals such as actuaries use probabilistic reasoning to compute appropriate premiums for life assurance based on life expectancies; meteorologists offer probabilistic weather forecasts; doctors provide an assessment of the risks in an operation; media give publicity to research on the dangers of eating particular foodstuffs. Every day, citizens are receivers of such wisdom and need to be empowered to evaluate that information.

Probabilistic reasoning is therefore an essential tool for citizens and professionals alike, and one that is becoming ever more significant as scientific communication becomes more available and society more technological. Looked at from this perspective, the newly released National Curriculum for Mathematics in England appears somewhat impoverished. At Key Stages 1 and 2, learners are now offered no opportunity to consider chance. As a result, while their thinking about cause and effect will have developed considerably by the time they enter secondary school, opportunities for them to account for non-causal variability in observed data will have been lost, and early intuitions of relative frequency (which research has shown to exist even before children start school) will have been allowed to wither. The task therefore for Key Stage 3 teachers to introduce pupils to probabilistic reasoning presents an immense challenge.

Research has shown how peoples' intuitions for chance are often misguided. For example, there is a tendency for people to judge chance by trying to evoke occurrences of similar events from memory. Unfortunately, the memories evoked are likely to be those most salient rather than those most frequent. This could for example explain why some people find it difficult to accept the relative safety of travelling by air when any aeroplane crash is always headline news. Or it might explain why people believe that a 6 on a die is less likely to occur than other numbers, perhaps based on painful memories of trying to obtain a 6 to get started in a board game. There is also a tendency to expect a random situation to self-correct so that outcomes that are so far underrepresented in a sequence of results are now more likely to occur. For example, a gambler might expect the roulette wheel that has just created a run of red numbers to be more likely to throw up a black number. There is also a strong tendency for people not to recognise situations as amenable to probabilistic reasoning and instead regard the situation as simply a matter of luck. Nevertheless, a strategic approach based on probabilities will be more successful in the long term than one that relies on good luck.

The lack of any reference to probabilistic reasoning in the new Key Stage 1 or 2 curriculum will mean that misguided intuitions such as those above will have been firmly established by the time pupils enter secondary school. It is possible to go about everyday life without recognising the missed opportunities for applying probabilistic reasoning. Whereas young pupils might develop some basic sense of cause and effect from everyday perception, the stochastic cannot be perceived in such an immediate way since the effects are based on proportions and only noticeable in the longer term. Probabilistic reasoning is more likely to emerge through systematically organised experience in school.

Mathematics teachers at Key Stage 3 need to introduce pupils to the use of probabilistic reasoning by helping them to recognise their naïve or misguided intuitions. Situations that involve uncertainty can be analysed by asking pupils to make predictions and to account for differences between their expectations and what actually happens. Pupils' attention will need to be drawn not only to short-term variation but also to the sense in which the longer-term aggregated view is in fact predictable – at least in a probabilistic sense. Typically, pupils will not appreciate that the Laws of Probability can be used to explain the behaviour of a wide range of phenomena. The knowledge they hold about such matters will develop within particular situations. For example, the pupils might eventually realise that 'the more times you toss the coin, the more even are the numbers of heads and tails' (in a proportionate rather than absolute numerical sense). However, it does not follow that the children will realise that 'the more times you throw the die, the more even are all six outcomes'. Some effort has to be made to widen the domain of knowledge to both coins and dice and subsequently to a general Law of Large Numbers. The Key Stage 3 teacher should also expect that such knowledge when it first emerges will be overgeneralised; after only a small number of throws of a die, a pupil might be surprised that the frequencies of the scores are not at all equal. Simulations offer opportunities for pupils to generate large sets of data quickly and so facilitate systematic observation beyond what is possible in practical experiments.

It would be easy to interpret the four bullet points under probability in the new National Curriculum in a narrow literal way that did not address any of the above issues. The previous paragraph represents pedagogic knowledge about teaching probability that is not set out in the curriculum but which research would suggest is essential.

Most tasks that have developed for the teaching and learning of probabilistic reasoning have in the past focused on coins, spinners and dice. However, these situations do not embrace common partially-determined situations such as the examples given at the start of this article. The actuaries, meteorologists, doctors and researchers draw on models where uncertainty and determinism co-exist. Key Stage 3 and 4 teachers might ask their pupils to build their own probability-based models by exploiting new modelling tools (for example, TinkerPlots).

For example, pupils might enjoy trying to create their own weather forecaster that uses factors such as today's weather but also recognises that to some extent weather is unpredictable. Or they might like to model sporting outcomes such as future football results based on the form of the competitors but again recognising the uncertainty in such situations. By modelling with probabilistic reasoning in such software, pupils might not only engage with meaningful everyday decision-making, they might also see connections between probability and statistics that are perhaps less evident in the behaviour of coins, spinners and dice.

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