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$$a+b-a=b$$

National Centre for Excellence in the Teaching of Mathematics

Key Ideas in Teaching Mathematics – Algebraic reasoning

In this and other issues, the Secondary Magazine will feature a set of six articles, written by Anne Watson, Keith Jones and Dave Pratt, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the fourth in the series and focusses on Reasoning with decimals in Key Stage 3. Future articles will feature Place Value, Algebra and Probabilistic Reasoning. Previous articles focussed on similarity, ratio and trigonometry in Key Stage 3, Geometric and spatial reasoning in Key Stage 3, statistical reasoning in Key Stage 3, and reasoning with decimals in Key Stage 3.

Algebraic manipulation without any meaning or purpose is a source of mystery, confusion and disaffection for adolescents. 'Meaning' in school algebra comes from the way relations between quantities and variables are expressed. 'Relations between quantities' and 'algebraic reasoning' – used in the title of the chapter of the corresponding book – pervade mathematics. Manipulating algebraic expressions enables us to express mathematical relations in different ways, and know more about them, when it is associated with some underlying meaning or purpose.

Algebraic reasoning involves:

- formulating, transforming and understanding generalisations of numerical and spatial situations and relations;
- using symbolic models to predict and explain mathematical and other situations;
- controlling, using, understanding and adapting spreadsheet, graphing, programming and database software

Expressing school algebra in these three ways links it to what young students know about relations between quantities. The simplest uses of algebra express relations between numbers when students already understand the relations. For example, finding x when $2 + x = 5$ involves the additive relation $2 + 3 = 5$; similarly, $a + b - a = b$ expresses the relation that students can spot when asked to calculate 37 + 49 - 37.

There is wide availability of symbolic manipulators which can transform expressions, solve equations, and carry out other algebraic techniques. Their use will help students answer questions which reflect all three statements above, rather than the more limited use of traditional algebra questions focusing on manipulation.

Students' problems with early algebra mainly stem from incorrect interpretation of notation. As with any notation, it is more effective to know what you are expressing before having to use standard notation to do so. If a student writes that $a+b = ab$ it does not mean that they do not understand addition, but that they do not understand what '$a+b$' and 'ab' and '=' are telling them. Saying that 'letters stand for numbers' is not sufficient as there are several different uses of letters in mathematics. They need to have a purpose for writing the sum of two unknown numbers or variables, and know that '$a+b$' is how we write that. For example, writing $a + b = b + a$ is the way we represent a fact about addition of numbers that they will already know. There are too many sources of particular confusion to list them here, but many are in the book *Key Ideas in Teaching Mathematics*.

We could talk endlessly about problems, so what about successful learning in algebra? There is no guaranteed way to teach and learn algebra. All approaches have limitations and many have significant strengths. We do know that traditional ways of teaching leave many students confused and switched off mathematics, so diving in with rules and practice and mnemonics is probably not a good way. In this table, we summarise eight main approaches and some of their strengths and weaknesses.

Approach	Example	Strengths	Weakness
Create a need to express equivalent relations	$4a+4 \equiv 4(a+1) \equiv 2(a+2)+2a \equiv 4(a+2)-4$ can arise from counting paving slabs round an a by a square	Supports learning about notation and manipulations; has meaning	May not lead to fluent use
Express relations among quantities	$p+q=10$, so if I know p I can use $10-p$ to find q.	expresses what is already understood; a foundation for functions and equations	It is hard to get beyond squares and maintain understanding
Create a need for algebra to model phenomena	How high am I after turning through x degrees on a fairground wheel?	Algebra is needed to describe and predict, so students appreciate its power	Tendency to think additively or use ad hoc reasoning. Graphs help overcome this.
Learn about expressions by substituting numbers	What is 60 degrees Celsius in Fahrenheit? What about -40 degrees?	Can explore equivalence, equality, inequality, and differences between notations	Substitution without purpose can lead to a belief that algebra is like code-breaking. It emphasises arithmetic rather than relations and structures.
Learn about expressions by 'reading' them	I think of a number, divide it by 5, subtract it from 2, square what I get, add one to it and square root everything and the outcome is equal to 4	Connects notation meaning and the relations being expressed	Gets complex with higher order functions and equations, and multiple variables.

Keith Jones, Dave Pratt and Anne Watson

In keeping this series of articles brief, there is no space for full references; these can be found in the book Key Ideas in Teaching Mathematics