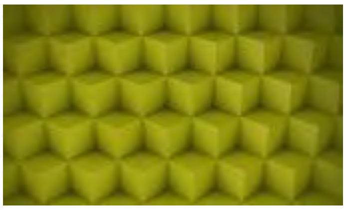
Cite as: Jones, K., Pratt, D. & Watson, A. (2014). Key ideas in teaching mathematics: Geometric and spatial reasoning. *NCETM Secondary Mathematics Magazine*, 106, 3-4.

Secondary
Magazine 106



National Centre
for Excellence in the
Teaching of Mathematics

Key Ideas in Teaching Mathematics – geometric and spatial reasoning

In this and subsequent issues, the Secondary Magazine will feature a set of six articles, written by Keith Jones, Dave Pratt and Anne Watson, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article is the second in the series and focusses on Geometric and spatial reasoning in Key Stage 3. Future articles will feature Statistical reasoning, Place Value, Algebra and Probabilistic Reasoning. The previous article focussed on similarity, ratio and trigonometry in Key Stage 3.

The geometry curriculum for Key Stage 3 makes use of the word “derive” quite often. Examples include “derive and apply formulae to calculate and solve problems involving perimeter and area”, “derive and use the standard ruler and compass constructions”, “derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures”, “derive and use the sum of angles in a triangle”, and so on. Here the word “derive” means to obtain something by some process of reasoning. This chimes with the overarching curriculum statement that students learn to “reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language”. In particular, the curriculum signals that it is at Key Stage 3 that students “begin to reason deductively in geometry” (as well as in number and algebra).

When teaching about perimeter and area, available research suggests that overly focusing on rectilinear shapes drawn on squared paper and on working on unconnected area formulae doesn’t necessarily help students to develop long-term understanding. With such an approach to teaching, students may not move much beyond the procedural counting of squares with the result that many students can end up confusing perimeter and area.

Geometric constructions have been in the curriculum for some time, but research indicates that developing mechanical competence with handing the relevant instruments (be it ruler and compass or the equivalent computer software) takes time. Throughout such teaching the focus can be on the accuracy of the construction, which, while quite proper, can mean that the students do not get to understand the link between a construction that they have been asked to perform and a related proof. In terms of properties of shapes and the use of theorems such as the angle sum in a triangle, research confirms that students can be satisfied by empirical arguments and not appreciate the need for anything beyond that.

All this points to the issue that teaching students to reason deductively in geometry is, as the available research consistently shows, not at all straightforward. The scale of the issue is reflected in what can be a lack of clarity in referring to measures. In any situation in the physical world, any measure has an error. This applies to ruler and compass constructions just as it does to the equivalent computer software. In contrast, when using geometric properties, or theorems, when we say that the circumference of a circle is πD, we do not mean approximately πD; we mean precisely πD. When we refer to a unit square, we do not mean that each side is approximately one unit; likewise, when we refer to a right-angled triangle we do not mean that the right-angle is approximately 90 degrees. In the first cases, we mean precisely one unit and, in the second, we mean precisely 90 degrees. The key point is that geometric objects such as unit squares and right-angled triangles are figural concepts in that they combine figural and conceptual aspects. As such, in teaching and learning geometry, *attention has to be paid to both*. By that I mean the spatial aspects, and the aspects that relate to reasoning with geometrical properties and theorems.

These two aspects, the spatial aspects, and the aspects that relate to reasoning, are not separate, they are entwined; they are the *yin and yang* of geometry teaching and learning. This means that when attempting to derive and use some geometric property or theorem, a secondary school student might move from making conjectures using measures taken from a geometrical drawing, to using definitions and theorems, then go back to the drawing, and so on. This moving back and forth between what might be termed 'spatio-graphic geometry' and 'theoretical geometry' is what is behind the challenge of teaching geometric ideas at the secondary school level (just as it's behind the challenge of specifying a suitable curriculum).

A further challenge is the breadth of approaches to geometry that need to be incorporated into teaching. Attention has to be paid not only to a range of key geometric ideas, including symmetry, invariance, transformation, similarity, congruence and so on, but also to various approaches to geometry, including synthetic, transformation, and analytic geometry.

With the curriculum set out as a set of bullet points, it could be tempting to treat each one in isolation. An alternative is to keep in mind the need to focus on the spatial aspects of geometry as well as the aspects that relate to reasoning. The following guidelines for successful teaching are adapted from research:

- geometrical situations selected for the classroom should, as far as possible, be chosen to be useful, interesting and/or surprising to students;
- classroom tasks should expect students to explain, justify or reason and provide opportunities for them to be critical of their own, and their classmates', explanations;
- tasks should provide opportunities for pupils to develop problem solving skills and to engage in problem posing;
- the forms of reasoning expected should be examples of local deduction, where pupils can utilise geometrical properties that they already know in order to deduce or explain other facts or results.
- in order to build on learners' prior experience, classroom tasks should involve the properties of 2D and 3D shapes, aspects of position and direction, and the use of transformation-based arguments that are about the geometrical situation being studied (rather than being about the transformations themselves solely as mathematical operations);
- while measures are important in mathematics, and can play a part in the building of conjectures, the generating of data in the form of measurements should not necessarily be an end point to learners' geometrical activity. Indeed, where sensible and in order to build geometric reasoning and counter possibly deep-seated reliance on empirical verification, it is worth considering classroom tasks where measurements (or other forms of data), or purely perceptual reasoning, are not generated.

Paying attention to the two closely entwined aspects of geometry, the spatial aspects, and the aspects that relate to reasoning with geometrical theory, across both 2D (plane) and 3D (solid) geometry, takes time; perhaps more time than is currently allocated to geometry within the typical school scheme of work. As with other key aspects of mathematics teaching at secondary school level, effective teaching of geometry needs coordination of the development of various components of geometry across school years and between teachers.

Keith Jones, Dave Pratt and Anne Watson

Image credit
Page header by Paul Cross, some rights reserved