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Geometry in the A-level Mathematics Curriculum

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Currently there is little opportunity in England to study geometry after the age of 16. The aim of this short paper is to contribute to discussions of what geometry might be suitable for inclusion in the specification for A-level mathematics.

The recent Royal Society report on the teaching and learning of geometry 11-19 (Royal Society, 2001) identifies progression in geometry within the curriculum as a major issue. The Chair of the working group that produced the report, in his preface to the main report, comments:

"The 11-16 geometry curriculum in England continues to concentrate on techniques for working in 2 dimensions, such as the plane geometry derived from Euclid, together with elements of transformation, vector and coordinate geometry. Yet little of this finds its way into current AS/A-level specifications in mathematics, whose geometrical content has been drastically reduced over time. Similarly, the kind of geometry studied by mathematics undergraduates bears little resemblance to that studied either pre- or post-16.

While the working group is optimistic about the possibility for significant improvement in teaching geometry 11-16, (which is not to underestimate the challenges to be addressed), it is far less sanguine about the state of geometry in 16-19 education. The geometrical content of the current AS/A-level specifications in pure mathematics is very small and offers little by way of progression from what has come before. But there is little point in advising content changes at this level when the whole basis of 16-19 qualifications in mathematics and all other subjects has just undergone a series of changes, the consequences of which have yet to be fully felt. Our view is that the general position of mathematics in 16-19 education needs a fundamental review before geometry can be accorded an acceptable place".

Royal Society 2001, p viii

One of the recommendations of the working group is that:

"a fundamental review be made of all 16-19 mathematics provision. This should include considering how:

a) the structure and content of the current AS/A-level Mathematics and Further Mathematics specifications can better meet the needs of students and include a greater emphasis on geometry; and
b) other post-16 mathematics qualifications, such as Free Standing Mathematics Units (FSMUs) and AS-level Use of Mathematics, can enable students to have the opportunity to continue their study of geometry".

As the Royal Society report makes clear, specifying the geometry content for post-16 mathematics is a complex matter which requires considerable thought and discussion. This paper represents one small contribution to the debate about what might be appropriate content for the geometry component of A-level Mathematics.

Below is a list of topics which may be suitable for consideration for inclusion in A-level Mathematics. The intention is to focus on *geometrical* topics, rather than those that, while geometric in origin, are usually treated in a predominantly *algebraic* way or rapidly become entirely algebraic (eg: conic sections; polar coordinates; trigonometric functions; etc).

Plane Geometry

There are several ways of approaching plane geometry, all equally valid from a mathematical perspective. The approach adopted at any particular stage of the curriculum depend on the aims and choices of theorems. Suitable approaches include:

- via congruent triangles and parallels;
- through the use of transformations;
- through co-ordinate geometry;
- through vector geometry

Suitable theorems might be selected from ones involving circles and triangles. Examples of problems might include Apollonius, the nine point centre of a triangle, and so on.

Spherical geometry

Suitable topics might include:

- Spherical similarity
- Spherical triangles
- Girard's Theorem

Projective geometry

Suitable topics might include:

- Observation points and vanishing points
- Maps

Polyhedra

Suitable topics might include:

- The Platonic Solids
- The Archimedean solids
- Euler's Theorem (and the link with Girard's Theorem)
- Circumscribing and inscribing spheres of polyhedra

Networks

Suitable topics might include:

- Traveling-salesperson problems
- Map colouring problems
- Eulerian tour (for example, the highway inspector's problem)

Knot theory

Suitable topics might include:

- Classification
- Trefoil knots
- Topological equivalence
- Reidemeister moves

Contemporary applications of geometry

Suitable topics might include:

- Robotics
- Crystallography
- Art and Architecture
- Fractals

Final Comment

In an overcrowded curriculum, it may be tempting to select only one or two of the above topics. This would be regrettable as it is only by experiencing a range of geometry that learners gain insight into any one particular geometry (for further discussion on this, see, Jones, 2001).

References

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