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Theoretical perspectives on the design of dynamic visualisation software

Keith Jones, *University of Southampton, UK*
d.k.jones@soton.ac.uk
Constantinos Christou, *University of Cyprus*
Marios Pittalis, *University of Cyprus*
Nicholas Mousoulides, *University of Cyprus*

Designing learning environments entails drawing on theoretical perspectives on learning while, at the same time, being cognisant of the affordances and constraints of the technology. This paper reflects on the design process through utilising evidence from the design stage of the development of a dynamic visualisation software environment called 3DMath. *During the development of* 3DMath*, a dynamic three-dimensional geometry microworld aimed at enabling learners to construct, observe and manipulate geometrical figures in a 3D-like space, the key elements of visualisation – covering mental images, external representations, and the processes and abilities of visualisation- were taken into consideration. The aim of this paper is to illustrate how the design of this particular software was informed by these elements of visualisation, as well as by theories related to the philosophical basis of mathematical knowledge and by semiotics. The paper illustrates how the features of software may be designed to take account of relevant theoretical notions and to satisfy the characteristics of instructional techniques that are appropriate to theoretical perspectives on learning.*

Introduction

A prime aim of the *8th World Conference on Computers in Education*, held in July 2005, in Stellenbosch, South Africa, was to develop and produce a "global vision for ICT in education". The vision document produced at the conference, known as the Stellenbosch Declaration (IFIP, 2005), sets challenges for all stakeholders in ICT in education - teachers, practitioners, researchers, academics, managers, decision-makers and policy-makers- all with a view to increasing the access to education for everyone around the world.

While acknowledging that the development of well-designed ICT-based educational material is growing, the Stellenbosch Declaration sets out a radical agenda for research, specifying the need for the research community both to bridge the gap between technology and pedagogy, and to ensure the development of solid theoretical frameworks for software design and utilisation. This is because, the Declaration argues, "in the field of ICT-supported learning, pedagogy and technology have often been treated separately; *pedagogy often being based on what the technology appears to permit, rather than fully integrated as a basis for technological design*" (emphasis added) and that "*the possibility of relying on solid theoretical frameworks is one of the key factors that can* *enable conception of the many positive experiences already taking place* in order to reach the definition of reliable innovative reference models" (*ibid*, p4, emphasis added). The Declaration also insists that "the output of research should be made widely available, as open source, for improving practice, decision-making, and resources development".

This paper is offered as a modest contribution to meeting the challenges to research set out in the Stellenbosch Declaration. It does this by reporting on the theoretical perspectives underpinning the design of a dynamic visualisation software environment called *3DMath* (Christou *et al*, 2006) aimed at enabling learners to construct, observe and manipulate geometrical figures in a 3D-like space.

Theoretical Perspectives on Design

Traditionally, three-dimensional geometry is taught using static pictures of geometric solids presented in textbooks. Students, however, are known to have difficulty reasoning from two-dimensional representations of three-dimensional objects (Raquel, 2002; Parzysz, 1988). Moreover, developing visualization skills is difficult in a traditional lesson environment using the standard chalkboard because representing 3D objects by a 2D sketch is complicated and time consuming. Such difficulties remain even when commonly available 2D dynamic software is used. For example, Dixon (1995) showed that 2D software can be effective in improving students' two dimensional visualisation but was not effective in improving students' three dimensional visualisation.

In an attempt to overcome these difficulties, and to take account of relevant theory, this paper reports on the design of a 3D software named *3DMath*. The main objective of the software development project is to develop a dynamic three dimensional geometry microworld, which enables students to construct, observe and manipulate geometrical figures in 3D-like space on the computer screen. To meet these purposes, the design of the proposed software followed three major fields of educational theory:

(a) the constructivist perspective about learning which argues that learning is personally constructed and is achieved by designing and making artifacts that are personally meaningful (Kafai & Resnick, 1996),

(b) the semiotic perspective about mathematics as a meaning-making endeavour which argues that any single sign (e.g. icon, diagram, symbol) is an incomplete representation of the object or concept, and thus multiple representations of knowledge should be encouraged during learning (Yeh & Nason, 2004), and

(c) the fallibilist nature of mathematics which argues that mathematical knowledge is a construction of human beings and is subject to revision (Ernest, 1994).

In addition, the aim of developing the *3DMath* software was to develop abilities and processes in students that are closely associated with the idea of visual imagery as a mental scheme depicting spatial information (Presmeg, 1986). It is generally accepted that learning 3D geometry is strongly associated with spatial and visual ability (Dreyfus, 1991) and that incorporating spatial visualisation and manipulation into learning activity could improve geometry learning (Tso and Liang, 2002).

Spatial ability has had many definitions in the literature. For example, Tartre (1990) defines spatial ability as the mental skills concerned with understanding, manipulating, reorganizing, or interpreting relationships visually, while Linn and Petersen (1985) defines it as the process of representing, transforming, generating, and recalling symbolic, non-linguistic information. Lohman (1988) proposes a three factor model for spatial ability, including "spatial visualization", "spatial orientation", and "spatial relations". "Spatial visualization" is the ability to comprehend imaginary movements in a three-dimensional space or the ability to manipulate objects in imagination. "Spatial orientation" is defined as a measure of one's ability to remain unconfused by the changes in the orientation of visual stimuli that requires only a mental rotation of configuration. "Spatial relation" is defined by the speed in manipulating simple visual patterns such as mental rotations and describes the ability to mentally rotate a spatial object fast and correctly.

The core visual abilities that should be taken into account in developing 3D dynamic geometry software could be said to be the following (following Gutiérrez, 1996):

(a) "Perceptual constancy", i.e., the ability to recognize that some properties of an object are independent of size, colour, texture, or position, and to remain unconfused when an object or picture is perceived in different orientations,

(b) "Mental rotation", the ability to produce dynamic mental images and to visualize a configuration in movement,

(c) "Perception of spatial positions", the ability to relate an object, picture, or mental image to oneself,

(d) "Perception of spatial relationships", the ability to relate several objects, pictures, and/or mental images to each other, or simultaneously to oneself, and

(e) "Visual discrimination", the ability to compare several objects, pictures, and/or mental images to identify similarities and differences among them.

In addition, given Yakimanskaya's (1991) claim that the creation of mental images is possible because of the accumulation of representations that serve as the starting point, 3D dynamic geometry software should aim to provide the learner with a variety and richness of spatial images.. The richer and more diverse the store of spatial representations, the easier is to use images in solving problems.

Design Principles for *3DMath*

Based on the above theoretical perspectives, and the rich concept of visualisation noted above, the following guided the design and the construction of the *3DMath* software:

(a) The software needs to allows students to see a geometric solid represented in several possible ways on the screen and to transform it, helping students to acquire and develop abilities of visualization in the context of 3D geometry.

(b) Given Gutiérrez's (1996) view that when a person handles a real three-dimensional solid and rotates it, the rotations made with the hands can be so fast, unconscious, and accurate that the person can hardly reflect on such actions, then the software could usefully place some small limits on the directions and speed of rotation, thus forcing students to devise strategies of movement and to anticipate the result of a given turn.

(c) The interface of the software needs to be intuitive and to provide an open and generative environment that enables learning to learn through making and designing personally meaningful artefacts. It also needs to employ rich semiotic resources that enable multiple perspectives and representations for mathematical meaning-making (for example, students need to be able to represent a solid in 3D, or its correspondence in 2D).

(d) The software needs to be designed to provide the means for students to focus on the mental images they create, and the processes and abilities of visualization they use to solve problems. Given that a mental image is any kind of cognitive representation of a mathematical concept by means of spatial elements, *3DMath* needs to makes it straightforward for students to construct different solids and perceive them in a concrete or pictorial form. This is because the repetition of this process helps students to formulate a "picture in their mind's eyes" (Presmeg, 1986). In addition, the software needs enables students to see solids in many positions on the screen and consequently gain a rich experience that allows them to form richer mental images than from textbooks or other static resources.

(e) In terms of external representations, a visual representation means the manipulation of visual images and the transformation of one visual image into another (Bishop, 1980). The form of software developed by the *3DMath* project aims to be rich in the ability to manipulate and transform solids - see Christou *et al* (2006) for an example of how the *3DMath* software aids the user in distinguishing between a representation of a pyramid and an octahedron.

(f) Bishop (1980) identified two relevant processes of visualisation: interpreting figural information and the visual processing of abstract information; the translation of abstract relationships and non-figural data into visual terms, the manipulation and extrapolation of visual imagery, and the transformation of one visual image into another. The *3DMath* software incorporates Bishop's ideas by focusing on the processes of observation, construction and exploration (see Christou *et al*, 2006, for examples) in that

- observation allows students to see and understand the third dimension by changing the spatial system of reference (axes), choosing perspective and displaying visual feedback on objects. The *3DMath* software is being designed so that students can rotate a geometric figure in reference to the three axes and thence obtain a holistic view of it. The speed and the direction of the rotation are e controlled by the user of the software and the drawing style of the object can be in a solid colour view or in a transparent line view. Students can select, label and colour the edges and faces of the objects.

- construction entails providing users with the facilities to allows a dynamic construction of geometrical figures from elementary objects (points, lines, planes) and construction primitives (intersection, parallel, etc.). Students can also construct geometrical figures by selecting the appropriate 2D figures and then forming the solids by dynamic animations.

- exploration allows students to explore and discover geometrical properties of the figure. This is the main procedure adopted in most of the teaching scenarios being designed to accompany the *3DMath* software.

(g) The *3DMath* software is being designed in such a way as to accommodate the development of the following visualisation abilities (see Gutiérrez, 1996): (a) the figure-ground perception, (b) perceptual constancy, (c) mental rotation, (d) perception of spatial positions, (e) perception of spatial relationships, and (f) visual discrimination. The following features are that are thought to contribute to the development of the abilities are being integrates into the software:

- the dragging capability of the software is being designed to enable students to rotate, move and resize 3D objects. Rotation can be executed in all directions by controlling a rotation cursor and determining the speed of the rotation. In addition, students can resize proportionally all the dimensions of the object or resize it only in one dimension, according to the requirements of the problem.
- tracing is a particular instance of the interface where only parts of the figure are displayed. The intended purpose of this feature is to provide the learners with a way to perform a visual filtering of the main construction represented on the screen, i.e., to allow them to extract and observe parts of the construction in an independent view.
- as in 2D dynamic geometry software, students can carry out useful measurements, in the case of *3DMath* measures of the length of edges, the area of faces, and the volume of a solid. All measurements are dynamic as solids are resized by dragging. The dynamic characteristic of the measurement facility allows the exploration of properties within and among figures e.g. users can measure the volume of a cone and then double its height and see how its volume is altered.
- a textual feature, which represents the declarative description of the figure, provides the learners with a textual and chronological list of all the geometrical objects involved in the construction of the figure. Additionally, the *History* file can be used as input to the system. For example, a *History* file created by one student in one country can be used by another student in another country to reconstruct (or re-use) the same model. Using this feature, it would be possible to construct not only *Interactive* models, but also *Declarative* models (by importing *History* files) and *Interactive Programming* models.
- a diagrammatic feature provides a representation of the structural dependency graph of the figure.

(h) Other features being developed include:

- the ability to export constructions as images (BMP, JPEG, etc), or in other rendering format (PS, XML, etc). This should help teachers to create supporting educational materials, preparing reports, printed material, etc.

- the locking/unlocking of features (primitives), making them hidden from view. For example, the primitive to find the distance from a point to a plane might be initially locked (or hidden). To find that distance, students must solve the problem by making appropriate constructions. Once they do this correctly, the primitive may be unlocked (or made visible) so that it can be used freely in further constructions.

Concluding Comments

As illustrated by this paper, the design of *3DMath* is informed by theories based on philosophy of mathematical knowledge, such as constructivism, and by semiotics. The main purpose of *3DMath* is to enhance students' understanding of 3D geometry with an emphasis on visualisation. Thus, during the developmental process the key elements of visualization, as defined by Presmeg (1986), Bishop (1980), Clements (1982) and Gutiérrez (1996) (mental images, external representations, processes, and abilities of visualization), are carefully taken into consideration.

In developing the 3*DMath* software we are seeking to bridge the gap between technology and pedagogy, and develop solid theoretical frameworks that inform the software design. This is so that the pedagogy is fully integrated as a basis for technological design, rather than the pedagogy, as is often the case, being based on what the technology appears to permit a The output of this research, as the IFIP (2005) declaration recommends, is to be made widely available, as open source, for improving practice, decision-making, and resources development in mathematics education.

References

Bishop, A. J. (1980). Spatial abilities and mathematics education: A review. *Educational Studies in Mathematics,* **11 No 3**, 257-269.

Booth, R. D. and Thomas, M. O. (2000). Visualization in mathematics learning: arithmetic problem-solving and student difficulties. *Journal of Mathematical Behavior,* **18 No 2**, 169-190.

Christou, C., Pittalis, M., Mousoulides, N., & Jones, K. (2006), Developing 3D Dynamic Geometry Software: theoretical perspectives on design. *International Journal of Technology in Mathematics Education*, **13**(1),

Clements, M. A. (1982). Visual imagery and school mathematics. *For the Learning of Mathematics,* **2 No 3**, 33-38.

Dixon, J. K. (1995). English language proficiency and spatial visualization in middle school students' construction of the concepts of reflection and rotation using the Geometer's Sketchpad. *Dissertation Abstracts International*, **56 No 11**, 4307A.

Dreyfus, T. (1991). On the status of visual reasoning in mathematics and mathematics education. In ed. F. Furinghetti*; Proceedings of the 15 th conference of the international group for the psychology of mathematics education*, vol 1, pp33-48. Genova: University de Genova.

Ernest, P. (1994). Varieties of Constructivism: their metaphors, epistemologies and pedagogical implications. *Hiroshima Journal of Mathematics Education,* **2**, 1-14.

Gutiérrez, A. (1996) Vizualization in 3-dimensional geometry: In search of a framework. In eds. L. Puig and A. Guttierez; *Proceedings of the 20 th conference of the international group for the psychology of mathematics education*, vol. 1, pp3-19. Valencia: Universidad de Valencia.

Gutiérrez, A., & Jaime, A. (1998). On the assessment of the Van Hiele levels of reasoning, *Focus on Learning Problems in Mathematics,* **20 No 2&3**, 27-46.

International Federation for Information Processing (2005), *The Stellenbosch Declaration - ICT In Education: Make It Work.* Laxenburg, Austria: IFIP.

Kafai, Y. B. and Resnick, M. (1996). *Constructionism in practice: Designing, thinking, and learning in a digital world.* Mawhaw, NJ: Lawrence Erlbaum Associates.

Laborde, C. (1998). Visual phenomena in the teaching/learning of geometry in a computer-based environment. In eds. C. Mammana and V. Villani; *Perspectives on the teaching of geometry for the 21st century,* pp113-121. The Netherlands: Kluwer Academic Publishers.

Linn, M. C., & Peterson, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development,* **56***,* 1479–1498.

Lohman, D. (1988). Spatial abilities as traits, processes and knowledge. In ed. R. J. Sternberg; *Advances in the Psychology of Human Iintelligence*, Vol. 4. Hillsdale, NJ: LEA.

Parzysz, B. (1988). Problems of the plane representation of space geometry figures. *Educational Studies in Mathematics,* **19 No 1**, 79-92.

Presmeg, N. (1986). Visualization in high school mathematics. *For the Learning of Mathematics*, **6 No 3**, 42-46.

Raquel, J. (2002). *Thinking in three dimensions: Exploring students' geometric thinking and spatial ability with the geometer's sketchpad.* Unpublished Doctoral Dissertation: Florida International University.

Tartre, L. A. (1990). Spatial orientation skill and mathematical problem solving. *Journal for Research in Mathematics Education,* **21 No 3**, 216-229.

Tso, T. and Liang, Y.N. (2002). The study of interrelationship between spatial abilities and van hiele levels of thinking in geometry of eight-grade students. *Journal of Taiwan Normal University,* **46 No 2**, 1-20.

Yakimanskaya, I.S. (1991): *The development of spatial thinking in schoolchildren* ("Soviet Studies in Mathematics Education" vol. 3). Reston: Va, NCTM.

Yeh, A. and Nason, R. (2004). *Towards a semiotic framework for using technology in mathematics education: The case of learning 3D geometry*. Paper presented at the International Conference on Computers in Education.