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Key Ideas in Teaching Mathematics - Similarity, ratio and trigonometry in KS3

In this and subsequent issues, the Secondary Magazine will feature a set of six articles, written by Keith Jones, Dave Pratt and Anne Watson, the authors of the recent publication Key Ideas in Teaching Mathematics. While not replicating the text of this publication, the articles will follow the themes of the chapters and are intended to stimulate thought and discussion, as mathematics teachers begin to consider the implications of the changes to the National Curriculum. This article, which refers to the new curriculum, intended for first teaching in autumn 2014, is the first in the series and focuses on Similarity, ratio and trigonometry in Key Stage 3. Future articles will feature Geometric Reasoning, Statistical Reasoning, Place Value, Algebra, and Probabilistic Reasoning.

The geometry curriculum for Key Stage 3 requires that all children should be able to 'use ... trigonometric ratios in similar triangles to solve problems involving right angled triangles'. As with most statements in the programme of study this masks a number of interesting issues. At the very least, we need to know what is meant by 'solve problems'. The relevant overarching statement reads: 'solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.' Teaching students to solve routine problems with trigonometry is often approached in a procedural manner, sometimes using mnemonics. Typically, many students misapply these procedures by misidentifying right angled triangles, misidentifying the appropriate sides, and misapplying the inverse function to find angles. However, the inclusion of trigonometric ratios for all students by the end of KS3 may well lead to procedural teaching because of time and assessment pressures.

The research in this area focuses on the efficiency of particular methods in enabling students to solve problems without the usual errors, but most of these studies depend on particular tests of short-term outcomes rather than focusing on longer term development of understanding. Rather than focus on particular teaching methods, therefore, it seems more useful to analyse the conceptual threads which could support understanding trigonometry ratios at KS3 for a wide range of students, and which also contribute to strengthening understanding in other areas of mathematics.

In a survey of 29 teachers' preferred approaches to teaching trigonometry, most teachers started with collections of triangles with which students would make measurements, tabulate and compare results. Four different conceptual pathways underpinned these approaches:

1. similarity approach: the angle is fixed so that similar right angled triangles are being explored. The ratios of corresponding sides are found to be constant – some teachers avoid using the word 'ratio' and talk about division instead. An advantage of this approach is that it relates to work on similarity and enlargement. The trigonometric ratios can be seen to be specially related to right-angled triangles, which has given them historical and practical importance. A disadvantage of this approach is that it is limited to angles less than 90˚.
2. functional approach: the angle varies and the ratio of sides also varies with a fixed hypotenuse (for sine and cosine) or a fixed base for tangent. This approach can be related to the use of graphing software, and contributes to students' experience in analysing and interpreting graphs. The tangent approach relates to gradient. A disadvantage is that the use of ratios to solve right-angled triangles appears to be an unrelated procedure.
3. multiplier approach: if a unit radius circle is used in the functional approach, sine and cosine are multipliers. This approach relates to students' understanding of scaling. The language associated with right-angled triangles then emphasises the multiplicative relation between sides, and can be helpful later when resolving vectors.

4. combination: for a range of right-angled triangles, angles and sides are measured and ratios calculated. Results are compared and conjectures can be made. This approach can trigger discussions about accuracy with measurements, and the status of conjectures from data, but a large number of results and needed because of the number of variables involved and some students may remain unconvinced.

A purely procedural approach is not included in these descriptions, and yet in one study such an approach was found to be more efficient in helping students to solve routine problems (finding missing sides and angles in given right-angled triangles), when compared to the unit circle approach. The decision that faces teachers of the new curriculum is whether to 'cut to the chase' and use a procedural approach in order to answer routine assessment questions, or whether to build up a conceptual understanding that is connected across mathematics and of lasting value. In all the above conceptual approaches, there has to be a stage of encapsulating the ideas into definitions of sine, cosine and tangent and some practice of use. This stage of formalising processes after conceptual exploration is typical of school mathematics at this stage.

To solve non-routine problems requires students to recognise when and how the use of trigonometric ratios is relevant. This requires experience of analysing a problem and coordinating their resources to tackle the problem. At KS3 only right-angled triangles are considered, so these have to be identified in the problem. In some problems they might have to introduce a right angled triangle in order to set up the situation they can then solve. They need experience in doing this. For example, many survey problems are solved by constructing a perpendicular to a given straight line in order to create right angled triangles. This process offers a meaningful context for some classic geometrical work.

Students then have to be familiar with how the sides relate to the angles so that they can use the appropriate ratios - this is also true in routine problems. They then have to decide how to use the ratios to find the unknown measures - this is also true in routine problems. This step may not be obvious, as a problem might include several right angled triangles in which, for example, the hypotenuse of one might be the opposite side to a particular angle in another. Students therefore have to make decisions about the sequence of work they have to do to solve a particular problem.

Trigonometry, therefore, can be seen as a context for coordinating a range of KS3 mathematics as well as developing problem-solving skills more generally. As with many hard-to-learn ideas, the alternative to a procedural approach is to coordinate the development of components between years and teachers. Measurement, division, ratio, similarity, construction, graph plotting and scaling all have a part to play in this development.

Keith Jones, Dave Pratt and Anne Watson