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A WEB-BASED LEARNING SYSTEM FOR CONGRUENCY-BASED PROOFS IN GEOMETRY IN LOWER SECONDARY SCHOOL

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International research confirms that many secondary school students can find it difficult to understand and construct mathematical proofs. In this paper we report on a research project in which we are developing a web-based learning support platform (available in Japanese, English and Chinese) for students who are just starting to tackle congruency-based in geometry in lower secondary school. In using the technology students can complete the congruency-based proofs by dragging sides, angles and triangles to on-screen cells and our system automatically translates the figural elements to their symbolic form. Using the notion of 'conceptions of congruency' as our framework, we compare the tasks provided in our web-based learning system with similar tasks in a typical textbook from Japan. Our analysis shows that the tasks provided in the web-based platform aim to help learners to develop a correspondence conception of triangle congruency.

INTRODUCTION

The discussion document for the ICMI study on *Digital Technologies and Mathematics Teaching and Learning* identified a key question for mathematics education research: "how can technology-integrated environments [in mathematics education] be designed so as to capture significant moments of learning?" (IPC, 2005, p. 356). This paper reports on aspects of the design of a web-based learning support platform (available in Japanese, English and Chinese) for students in lower secondary school who are just starting to tackle congruency-based proofs in geometry; see: http://www.schoolmath.jp/flowchart_en/home.html

When using this learning platform, students can tackle geometric problems by dragging sides, angles and triangles to on-screen cells. As this happens, our system automatically translates the figural elements to their symbolic form. When students complete their proof, the system identifies any errors and provides relevant feedback on-screen. Using the theoretical notion of 'conceptions of congruency' as our framework (see below), we set out in this paper to compare the tasks provided in our web-based learning system with similar tasks in a typical textbook from Japan. Our research question is 'How do the tasks provided in our web-based learning system compare with similar tasks in a typical textbook from Japan?' For more examples of the technology-based tasks that we have designed within the learning platform, see Miyazaki, Fujita, Murakami, Baba and Jones (2011).

WEB-BASED PROOF LEARNING SYSTEM IN GEOMETRY

Building on the description of our proof learning support system in an earlier paper (Miyazaki, et al, 2011), we focus here in this section on how and why we designed the tasks that are available within our learning platform.

At this stage of our project, we have designed 20 tasks. The mathematical content is based on the Japanese geometry curriculum for 13-14 year-olds (Grade 8 in Japan). As such, our system is aimed at students who are starting to learn deductive proving through the use of properties of basic 2D objects (lines, angles, parallel lines, triangles and quadrilaterals). Our motivation for developing this system is the need to improve geometry teaching as, from our classroom observations, we are aware that many Grade 8 students can find proofs with congruent triangles difficult (see Fujita et al, 2011).

To make proofs accessible to as many students as possible, we utilise a range of technological capabilities in the design of our system. For example, it is constructed so as to be available via the Internet. By using Flash-based technology (Adobe system), which enables interactive actions on the web, students can complete proofs by dragging sides, angles and triangles to on-screen cells and our system automatically transfers figural to symbolic elements illustrated in Figure 1 (left-hand illustration). Students also choose appropriate conditions for triangles by using drop-down menus. By this automatic translation, students can concentrate on making a proof without being distracted by how the conventions of how to 'write' their proof. In addition, to help learners construct a proof step by step, answers within the system are data-based so that if a learner constructs an incorrect proof, then the system gives relevant feedback by indicating where the proof needs to be corrected. This latter capability of the system is illustrated by the right-hand part of Figure 1. Decisions for giving what feedback would be provided are based on our theoretical ideas for learners' structural understanding of proof (Miyazaki & Fujita, 2010).

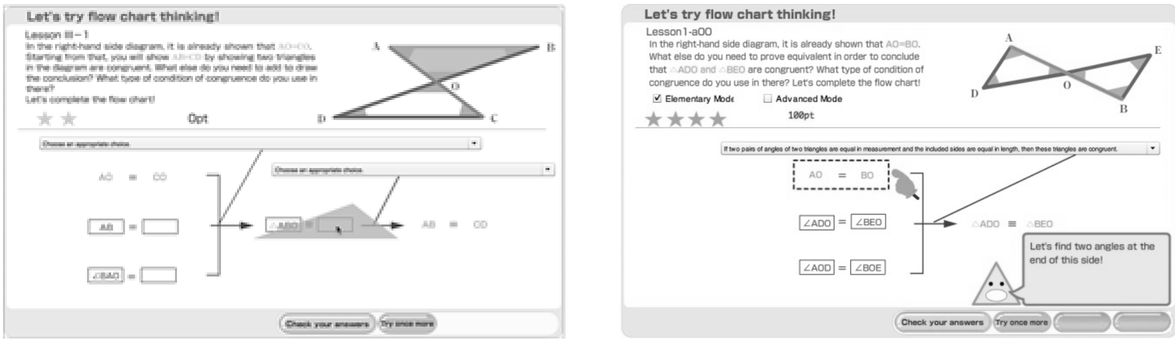


Figure 1: proof tasks within the web-based learning support platform

Overall, our interest is to investigate how and why our system can be an effective tool to promote students' proof learning experience. So far, evidence from our pilot studies (e.g. Miyazaki, et al, 2011; Fujita et al, 2011) suggests that learners' proving processes can be enriched when learners used our proof system. In this paper, we explore further the features of our system by characterising the tasks it contains.

ANALYTIC FRAMEWORK AND METHOD

In the analysis we present in this paper we follow the approach of González and Herbst (2009, p. 154) in taking a 'conception' as being "the interaction between the cognizant subject and the milieu – those features of the environment that relate to the knowledge at stake". In this approach, a conception comprises the following quadruplet (P, R, L, ∑): P: a set of problems or tasks in which the conception is operational; R: a set of operations that the agent could use to solve problems in that set; L: a representation system within which those problems are posed and their solution expressed; ∑: a control structure (for example, a set of statements accepted as true). In their paper, González and Herbst (2009, pp. 155-156) propose the following four conceptions of congruency:

- The perceptual conception of congruency (PERC) "relies on visual perception to control the correctness of a solution to the problem of determining if two objects (or more) are congruent".
- The measure-preserving conception of congruency (MeaP) "describes the sphere of practice in which a student establishes that two objects (e.g. segments or angles) are congruent by way of checking that they have the same measure (as attested by a measurement instrument)".
- The correspondence conception of congruency (CORR) is such that "two objects (segments or angles) are congruent if they are corresponding parts in two triangles that are known to be congruent".

- The transformation conception of congruency (TRANS) “establishes that two objects are congruent if there is a geometric transformation, mapping one to the other, which preserves metric invariants”.

By using the above ideas as our analytic framework, we have analysed tasks which can be found in a commonly-used Grade 8 textbook in Japan; see Jones and Fujita (2013). What we found, in brief, is that the Japanese textbook contained a lesson progression from PERC or MeaP to CORR. Nevertheless, National Survey data from Japan has indicated that Japanese Grade 8 students struggle to solve geometrical problems. For example, a recent national survey in Japan reported that the proportion of Grade 9 students who could identify the pair of equal angles known to be equal by the SAS condition in a given proof was 48.8% (National Institute for Educational Policy Research, 2010). This indicates that many students in Japan have not fully developed their CORR conception of congruency despite studying congruent triangles and related proofs during Grade 8.

With our proof learning system, learners can select and drag the sides and angles of various shapes, and also select from a choice of congruency conditions. From each set of actions, feedback is provided from the system. This is likely to influence learners’ subsequent actions. Thus the system offers opportunities for students to learn proofs in a way that is different from traditional textbook-based learning. As such, we are interested in how the tasks in our system can be characterised in terms of the conception of congruency, and whether we might be able to identify similarities and differences between tasks in the textbook and our system.

From our analysis of a commonly-used Japanese Grade 8 textbook (Jones and Fujita, 2013), we know that the Japanese textbook includes many tasks which are related to congruent triangles. Some of the tasks entail identifying congruent figures, while others focus on proving properties of geometrical figures using congruency-based arguments. Because our web-based learning support system especially focuses on proof-related task, we chose the tasks shown in Table 1 as our sample for analysis. These tasks are similar to each other at a first glance. Our intention is to see if different intended conceptions might be observed in our system because of the technology that underpins it.

Following the approach of González and Herbst, we undertook an *a priori* analysis of the tasks in following way:

- we used the quadruplet (Problems; Operations; Representation system; Control structure) to characterise the sample tasks selected from the Grade 8 textbook and from our geometry proof system;
- we used the information from our analysis to characterise the approach to triangle congruency utilised in the sampled tasks.

FINDINGS AND DISCUSSION

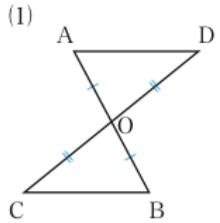
Table 2 summarises the result of our analysis of Lesson 10 (textbook) and task II-1 (proof system). In terms of the four conceptions of congruency, both tasks can be characterised as being the correspondence conception of congruency (CORR) as both tasks require learners to identify corresponding parts to deduce congruent triangles. Similar characteristics were identified for other tasks we analysed.

Despite both tasks in Table 2 being characterised as CORR, the table suggests striking differences between the way in which the same intended conception is realised in the tasks in the textbook and in our proof system. In particular, whereas both tasks provide similar problems (P), learners would face quite different learning experience in terms of operation (R), representations (L), and control structure (∑) thanks to the technology in our system.

Tasks from the Grade 8 textbook	Tasks from the geometry proof system

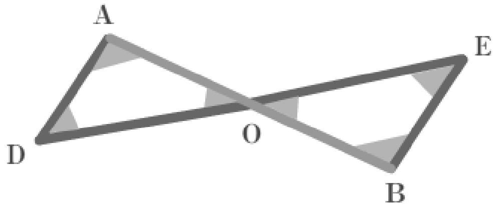
A task taken from lesson 10

In the diagram below, identify a pair of congruent triangles and name them using the / sign. Also, name the congruence condition used. The sides and angles labelled with the same marks in each diagram may be considered equal.



Lesson II-1

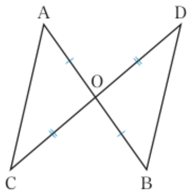
In the diagram below, prove triangles ADO and BEO are congruent by assuming what is needed (AO=BO assumed)



(Students can construct more than one proof in this problem situation.)

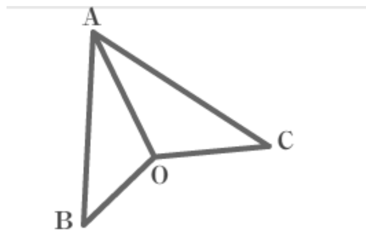
A task from lesson 13

In the diagram below, if O is the mid-point of line segments AB and CD, then angles OAC=OBD (prove this).



Lesson III-2

In the diagram below, prove that angles ABO=ACO by using triangle congruence and by assuming what is needed.



(Students can construct more than one proof in this problem situation.)

A task from Lesson 17

In triangle ABC, we need to show if AB = AC, then angle B (ABD) = angle C (ACD).

Lesson V-1

In the diagram below, if AB=AC and angle BAD = angle CAD, then angles ABD=ACD (prove this).

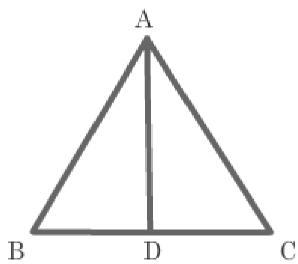


Table 1: tasks selected from the textbook and from the web-based learning support platform

In the textbook task, learners have to correspond figural elements to symbolic ones by themselves, but this can be quite hard for many learners who are just developing their CORR conceptions. The system supports this process by dragging and dropping figural elements to cells connected with the equal sign (=) or congruent sign (≡), and as a result learners can concentrate on formulating logical relationships in their proof. Also, the system does not have any measurement or superposition tools and these restrictions might help make learners aware that it is possible to study geometry theoretically as well as practically.

To complete a proof, learners have to exercise which condition should be applied, but our classroom observation suggest that often learners who have just learnt the conditions cannot use them effectively. The system supports learners as the known facts to be used are shown in the tabs. The system also gives various forms of feedback in accordance with learners' actions. For example, a learner might make mistakes when choosing an appropriate condition of the condition. Without our system, a learner might not know whether their proof is correct or not until the proof is shared with their peers or until a teacher points out their mistake; with our system the learner is supported within the system and this should help to activate their conceptual control structure.

	Tasks from G8 textbook	Tasks from the proof system
P	10Pa: To identify two congruent triangles. 10Pb: To identify the conditions of congruent triangles. 10Pc: To use symbols correctly.	II-1 Pa: To prove triangles ADO and BEO are congruent.
R	10Ra: To find pairs of congruent sides and angles. 10Rb: To identify equal sides/angles including not symbolised ones. 10Rc: To apply the conditions of congruent triangles. 10Rd: To apply already known facts	II-1 Ra: To identify what assumptions and conclusions are. II-1 Rb: To drag and drop sides, and angles. II-1 Rc: To choose statements (conditions of congruent) II-1 Rd: To check answers by clicking a button II-1 Re: To review already completed answers by clicking stars
L	10La: The diagram is the medium for the presentation of the problem. 10Lb The symbols are the registers of equal sides and angles. 10Lc: Already known facts such as vertically opposite angles or the conditions of congruent triangles mediate for the solution and reasoning.	II-1 La: The diagram on the computer screen is the medium for the presentation of the problem. II-1 Lb: Dragged sides/angles/triangles are the registers of equal sides/angles/triangles. II-1 Lc: The structure of the proof is visualised by the flow-chart format. II-1 Ld: Tabs are the medium of right statements to be chosen.
∑	10∑a: If we can find three components of triangles (SSS, ASA, SAS). 10∑b: If one of the conditions of congruent triangles is applied to two triangles.	II-1 ∑a: If one of the conditions of congruent triangles is applied to two triangles. II-1 ∑b: If the system gives feedback 'your proof is correct'.

Table 2: analysis of tasks selected from the textbook and from the web-based platform

CONCLUDING COMMENT

Given the sparse research on the topic of congruency, as a starting point for our research we have analysed various congruency-related tasks in textbooks and in our web-based learning system through an analysis utilising the four congruency conceptions proposed by González and Herbst (2009). In our analysis of tasks in a Japanese textbook (see Jones & Fujita, 2013) we show that the textbook is based on a learning progression from PERC or MeaP to CORR, i.e. from a practical conception of congruency to a correspondence conception. Our analysis in this paper shows that our system can be used during the introductory stage of proof learning because the tasks provided in the web-based platform are similarly designed to help learners to bridge between PERC or MeaP and CORR. One reason for developing our web-based platform is that national survey data from Japan shows this progression might not be as straightforward as we might expect and that it might be necessary to support many more learners to develop CORR in their learning of proofs in geometry.

In addition to aiming to support the development of students' CORR, with our web-based system we aims to support students' learning in various other ways, including mediating figural and symbolic elements of geometrical proofs, scaffolding the students' use of known facts, and supporting their control structure by providing relevant and timely feedback. We argue that such learning experience should be useful as students proceed to more complex and formal learning in geometry and proving, and that is why the learning with our system can be located in the introductory stage of proof learning.

Our next task is to characterise actual students' conceptions when they interact with various congruent triangle problems. In this way we aim to examine more systematically how our web-based learning system would contribute to supporting the development of students' correspondence conception of congruency.

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