STUDENTS' LEVEL OF 3D GEOMETRICAL THINKING: THE INFLUENCE OF 3D REPRESENTATIONS
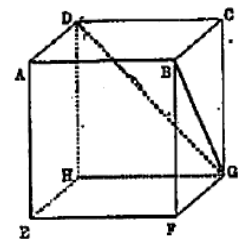
Yutaka Kondo[1], Taro Fujita[2], Susumu Kunimune[3], Keith Jones[4]

[1]Nara University of Education, Japan; [2]University of Exeter, UK; [3]Shizuoka University, Japan; [4]University of Southampton, UK

While representations of 3D shapes are in common use in the teaching of geometry in lower secondary school, it is known that the ways that 3D shapes are represented in 2D can provide challenges for students (Parzysz, 1988). In this paper, we use the levels of 3D geometrical thinking proposed by Gutiérrez (1992) to analyse students' reasoning about a 2D representation of a 3D shape.

Gutiérrez's level descriptors synthesise the two aspects of a) recognising the properties of 3D shapes and comparing 3D objects, and b) manipulating different representational models of 3D objects and obtaining correct answers. The levels can be described as follows: Level 1 No manipulations of representations; Level 2 Some manipulations; Level 3 More advanced manipulations; Level 4 Effective manipulation.

As a part of a larger study, in the research reported in this paper we analysed classroom episodes from two experimental lessons in which 28 Grade 7 students (aged 12-13) tackled a challenging problem in 3D geometry. The problem was to find the size of angle BGD in the accompanying diagram of a cube.

Through analysis of the students' responses during the lessons, we found that students at lower levels of 3D thinking could not manipulate representations effectively, e.g. they drew a net of the cube and by referring to the net concluded that angle BGD was 90 degrees. In contrast, students operating at higher levels of 3D thinking could reason correctly; for example, they could change the oblique parallel projection of the cube to an orthogonal projection in order to demonstrate that angle BGD was 60 degrees. Nevertheless, we found that even these students sometimes were unable to reason correctly for the problem. Based on our analysis, we propose that it is necessary to modify the level descriptors suggested by Gutiérrez in order to better capture students' understanding of representations of 3D shapes. Our proposal is to sub-divide levels 2 and 3 with two sub-levels each.

References

Gutiérrez, A. (1992). Exploring the links between van Hiele levels and 3-dimensional geometry. *Structural Topology*, *18*, 31–48.

Parzysz, B. (1988). 'Knowing' vs. 'seeing': problems of the plane representation of space geometry figures, *Educational Studies in Mathematics*, *19*(1), 79–92.