

## **Task design in a school-based professional development programme**

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In this paper we report on two principles of task design arising from our study of a school-based mathematics teacher professional development programme in Shanghai, China. The two principles are: a) developing a ‘hypothetical learning structure’ for the topic; and b) developing tasks within a web-like structure of knowledge connections. This paper provides an example of each and connects the literature review with the described research design.

Keywords: task design; school-based professional development; expert teachers; Chinese mathematics pedagogy; collective task design.

### **Introduction**

This paper focuses on the issue of how published tasks (sourced from textbooks) are appropriated by teachers for instructional purposes and hence how task design influences mathematics teaching. In particular, we focus on the central aim of the Working Group Theme D (in the ICMI Study 22 Discussion Document) by reporting on the design and implementation of tasks by a team drawn from three communities: academic researchers; the local school district professional developer, who is an expert teacher with considerable skill in school-based teacher professional development; and a group of Chinese teachers from Shanghai Soong Ching Ling School, Shanghai. In doing so, we take the stance highlighted by Kieran, Krainer and Shaughnessy (2013) that teachers are key stakeholders in research and can play a significant role in the design of rich tasks for mathematical learning. To add to this, we argue that the expert teacher/professional developer can also play a significant role in task design and its ‘implementation’.

In China, the 2011 version of National Mathematics Curriculum Standards (briefly called Standards in this paper) emphasises the shift from the traditional “Two Basics” (basic knowledge and basic skills) to the current “Four Basics” (adding basic mathematical thinking, and basic activity experiences). As a result, teachers are

expected to carefully consider and prepare the teaching of particular mathematical knowledge and skills, in addition to the development of students' individual understandings and thinking, and to building up students' learning activity experiences in mathematics (Shi, 2012). The problem now facing educators and teachers in China is which mathematical tasks, and task sequences, should be selected/considered (for particular topics), and how to work with such tasks, so as to implement effectively the range of intentions laid out by the Standards.

### **Theoretical framework and key pedagogical features**

Simon (1995) proposed the notion of Hypothetical Learning Trajectory (HLT) to help mathematics educators (i.e. teachers, researchers, curriculum developers) to think about the design and use of mathematical tasks to promote mathematical conceptual learning. The HLT comprises of three components (ibid p. 136): the learning goal; the learning activities; and the hypothetical learning process - a prediction of how the students' thinking and understanding might evolve in the context of the learning activities. Simon (1995) further points out that teachers need to develop skills to generate hypotheses about students' understandings (which go beyond soliciting and attending to students' thinking), to generate HLTs, and to engage in conceptual analysis related to the mathematics that they teach.

Ma (1999, p. 97) reports that one feature of Chinese teachers' knowledge is their well-developed "knowledge packages" of the range of ideas needed to teach a topic like arithmetic. Within such a package, there is a certain "key" piece of knowledge that is fundamental to enable students to learn other knowledge. Another kind of key piece of knowledge in the package is the "concept knot" (p. 98), which links to several different concepts of the learning topic. Ma (1999) considers that teachers' knowledge packages reveal their understandings of the longitudinal process of opening-up and developing mathematics of a particular field in students' minds.

Gu, Huang and Marton (2004) identify two types of teaching with variation in the Chinese mathematics classes. One is conceptual variation (CV), concerned with understanding concepts from multiple perspectives; the other is the procedural variation (PV), focused on developing insights into the hierarchical features of mathematical activity. Gu (2012) further interprets that the PV can be used to connect, in a dynamic way, different mathematical processes as a whole process. That is, a teacher can divide the process/es of a mathematical activity into a number of sub-activities, and then use variation as a means of "*pu dian*" (i.e. scaffolding) of knowledge between the sub-activities.

Based on the literature (e.g. Ma 1999; Gu et al. 2004), we identify the following key features of Chinese mathematics pedagogy: 1) understanding concepts from multiple perspectives; 2) designing for individual student's actual learning processes and responses; 3) gradually deepening learning through an orderly-layered teaching procedure (a teaching/learning method called "*Xun Xu Jian Jin*" in Chinese). In our project we utilised a school-based teacher professional development program (TPDP) (as detailed in the next section) as a means to enable researchers, expert teacher and school teachers to work together. In the TPDP, we are aware of Gravemeijer's (2004) notion of a teacher's "local instruction theory" (highlighted in Choppin, 2011), which is concerned with student thinking about complex tasks and how this is situated within a broader instructional 'space'. For more on the mathematical and epistemological basis of our task design study, see Lin (2011).

## The task design study

The main study was arranged in Shanghai Soong Ching Ling School located in Qingpu district, a western suburb of the city. The school was an international laboratory school funded by China Welfare Institute with the key mission of launching innovative and experimental educational classroom studies aimed at improving the quality of compulsory education for children in the country.

The reasons for this study are twofold. First, the school planned to launch its own innovative mathematics curriculum to complement the strengths of the Shanghai (SH) mathematics curriculum. Second, an innovative school-based model of curriculum was expected to bridge the gap between curriculum developers' intentions and school teachers' classroom practice.

The methodology of this study is based on a Design-Based Research (DBR) (e.g. Design-Based Research Collective, 2003) approach to study, document and advance a model for sustainable teacher professional development. The approach involved processes such as iteration and feedback loops in such ways that development and research took place through cycles of design, enactment, analysis, and redesign (Cobb et al. 2003).

The tasks in the pilot study were sourced from published material from two countries: the SH official textbooks; and the online professional development resources from New Zealand (NZ):

<http://www.nzmaths.co.nz/professional-development>

The participant groups of the pilot study were: (1) three researchers (the three authors); (2) an expert teacher (called 'Mr Zhang' in this paper); and (3) three teachers, one of them a teacher-researcher (first author). The three researchers (authors) designed the study and provided the theoretical background (literature, research design, etc.). Next, in the school-based TPDP, the first author (a teacher-researcher) designed the first intervention based on ideas from the NZ online resources. The expert teacher, Mr Zhang, commented on it together with other teachers in the TPDP. Two Grade 4 teachers (one experienced teacher, and one less experienced teacher) then took Mr Zhang's comments into account in their re-designed lessons of the same topic in two other grade 4 classes. Mr Zhang reflected upon both lessons, together with other teachers. Subsequently, the three researchers analysed the data, which consisted of data on the context; observations and videos of the three lessons; discussions/interviews with Mr Zhang after each lesson; and documents, such as curriculum materials and textbooks; also official documents, guidelines for teaching, syllabi, etc..

In this paper we define a 'task' as a learning situation with a specific teaching goal in a single lesson. The main body of the task design is therefore to create a sequence of multi-layered learning situations within a broader instructional plan and to develop a general picture of the students' learning paths. In the pilot study the teachers were actively engaged in the work of how to select, modify, sequence, teach, observe and reflect on a single lesson and on a sequence of lessons.

As laid out in the SH Grade 4 mathematics textbook (term II), the chapter on decimals (30 pages in total) consists of the following sub-topics: decimals in real life; the meanings of decimals; comparing decimals; the properties of decimals; the movement of the decimal point; the addition and subtraction of decimals; and the application of the addition and subtraction of decimals in problem solving. In their teaching teachers were expected to appreciate the key connections of the teaching/learning goals embedded in the SH textbook as follows: (for pupils)

1. to learn/understand the relationship between decimals and fractions (to understand the base-ten place value system of decimals and the meaning of 0.1, 0.01, and 0.001);
2. to learn/understand the units of decimals (e.g., 0.1) (to understand the meanings of decimals with one, two, three digitals; and to know a whole number with a decimal);
3. to learn/understand the concept of place value of decimals (to understand the place value table of decimals).

In what follows, we focus on delineating the two major principles that emerged from the data analysis of the three teachers' lessons on 'place value of decimals' (briefly called 'decimal value') and of Mr Zhang's comments during the post-lesson TPDP.

### Findings: two principles of the task design study

*Principle 1: Developing a 'hypothetical learning structure' for a particular topic*

The first teacher intended to use the NZ online teaching resources to complement the strengths of the SH mathematics textbook and pedagogy. For the lessons of decimal value, the teacher's "local instruction theory" (Gravemeijer, 2004) was to enable her students to accumulate learning experiences with the big idea that the decimal number system is a base-ten place value system. Thus, she used the NZ online tasks to enable students to examine two sub-ideas associated with the base-ten place value system.

1. The places (or columns) in the number system are based around groupings of ten. For example, 10 ones = 1 ten, 10 hundreds = 1 thousand.
2. The decimal point is a convention that indicates the units place.

The teacher started the lesson by presenting the place value of whole number (briefly called whole number value) on the blackboard (see the left half in Figure 1). While students observed it, they recalled previously learnt knowledge such as series, and a 10-to-1 relationship between the values of any two adjacent places (or columns). For instance, some students discussed that 10000 equals 1000 of ten. Others argued that  $10000 \div 10 = 1000$ . Still others added that the "Wans" (tens of thousands) are 3 places to the left of "Shi" (tens). This means that there is a  $10 \times 10 \times 10$  relationship between "Shi" and "Wan".

...	Qian (Thousands)	Bai (Hundreds)	Shi (Tens)	Ge (Ones)	Shi Fen Zhi Yi (Tenths)	Bai Fen Zhi Yi (Hundredths)	Qian Fen Zhi Yi (Thousandths)	...
...	1000	100	10	1	1/10 0.1	1/100 0.01	1/1000 0.001	...

Figure 1. The decimal number system

Next, the teacher drew students' attention to the decimal value (see the right half in Figure 1). For instance, according to the newly learnt knowledge of decimals, students were able to discuss in the class that 10 of  $1/100$  is  $1/10$ . Some represented such a relationship by decimals (10 of 0.01 is 0.1). The teacher then led students to pay attention not only to a 1-to-10 relationship between adjacent places to the right where the places were getting smaller by a factor of ten, but also to the relationship between any two places in the number system. In observing such relationships as  $0.1 \div 10 = 0.01$ ,  $0.1 \div 100 = 0.001$ , and  $0.1 \div 1000 = 0.0001$ , students were also led to think

about the movement of the decimal point. Moreover, the teacher posed tasks to enable students to see the connection between the movement of the decimal point and the units of a number. For instance, 6501.4 (in this case the “ones” is assumed), □650.14 tens (in this case the units are tens), □65014 tenths (in this case the units are tenths), □and 6.5014 thousands (in this case the units are thousands).

In the first post-lesson professional activity (PPA), Mr Zhang firstly commented on the teacher’s lesson from three perspectives, namely (1) the HLT of students; (2) the learning methods; and (3) the degree of learning difficulty.

I found that the (first) teacher dealt with the textbook in her lesson differently from those in our common lessons. ... There is a starter in the HLT. That is, the anchor of knowledge. For instance, what types of knowledge, experience and methods of learning the students already have? And to what degree? The starter of this lesson was at an abstract level, such as the comparison of the place values, enlarge or reduce a number, the relationship of places, etc. ... Some students in the class may be lost at this abstract level. ... Moreover, the learning method is different. In this lesson, the teacher provided a bit of context of the problems, students then developed a discussion in the class and then solved some points of problems by themselves. It’s considerably random. The logical structure of learning itself is loose. For instance, what is the first step and then the next step of learning? In a whole, what is the general goal of learning? The learning was designed in a macro way.

Next, Mr Zhang drew the teachers’ attention to the HLT from a micro perspective of the “concept knots” (the meaning and the places of decimals) (Ma 1999) in the decimal chapter of the SH textbook.

To view the learning from a micro perspective, students are expected to master a number of factors from this topic. For instance, can a number be read on the decimal value (dv) table? Can a decimal be put on the dv table? Can the form of the decimal be explained on the dv table? In the last lesson [according to the SH textbook], it is about the structure of the meaning of decimals. In this lesson, a shift is to be made to the structure of the places of decimals. In the end of this lesson, the two structures should be connected.

From such a micro perspective, Mr Zhang also addressed the importance for teachers to recognize the connection of the “key” piece of knowledge (the whole number value) (Ma 1999) and new knowledge (the decimal value) in the HLT.

When pupils learn the decimal value, their cognitive anchor is on what they have previously learnt of the place value of whole numbers [learned in Grade 4 -Term I]. Teachers thus should create cognitive conflicts of the new topic for their students, to go beyond their previous knowledge of whole numbers. It is because ... to ‘fill’ a decimal is something new for students. In so doing, their intellect is challenged and they can be engaged in thinking about how to create a decimal value which is the topic of this new lesson.

In our study, we use the term “hypothetical learning structure” (HLS) to distinguish the Chinese expert teacher’s concept of HLT from Simon’s (1995) HLT, in terms of two considerations: (1) The HLS in our study is not based on the western constructivist theory, but largely on the Chinese expert teacher’s expertise in predicting students’ learning processes and responses through observing the same topic many years during authentic classroom practice. (2) The use-aim of the HLS is not only to address the teacher’s well-developed “knowledge packages” (Ma, 1999) in mathematics, but also to distinguish the learning methods from those of the HLT (Simon, 1995). That is, in Mr Zhang’s view, pupil learning could be more ‘efficient’ (in the sense of ‘whole class learning’) if students do not randomly use their previous knowledge to respond to the teacher’s questions in a set of points, but are engaged in

the teacher's well-designed mathematical tasks in a set of 'blocks'. Mr Zhang offered the following 'blocks of tasks' for the decimal value lesson:

1. Write decimals and then write decimals with whole number on the number line;
2. Write units of decimals on the number line (e.g., 0.1, 0.01, and 0.001);
3. Write whole number in the whole number table and then write decimals with whole numbers in the whole number table;
4. Create the decimal table, design the place value in the decimal table and recognize the role of the decimal point;
5. Develop an understanding of the units of decimals in the decimal table;
6. Write the form of decimals in mathematical notation (e.g.,  $0.23 = () \times 0.1 + () \times 0.01$ );
7. Write the form of decimals in word language;
8. Define two types of decimals (decimals and decimals with whole numbers).

*Principle 2: Developing tasks within a web-like structure of knowledge connections*

Two teachers followed up the conversations with Mr Zhang by carefully planning the second and the third lessons for each of their classes on the same topic of decimal values according to the HLS (outlined above). Their lessons were quite different due to the teachers' different instructional intentions and the different students in each class. The second teacher (with two years teaching experiences in primary mathematics) sought to develop students' understanding of the form of decimals on the concrete geometrical model (the number line), and then their abstract thinking in the decimal table. For instance, in the starter of his lesson, the teacher used a considerable amount of time for students to write decimals on the different sections of the number line (e.g. those between 0 and 1, 0 and 1/10, 0 and 1/100, 0, 1 and 2, and 61, 62 and 63).

The third teacher (with ten years teaching experiences in primary mathematics) did not use the number line, but developed her students' abstract thinking of decimals directly from the form of decimals to the decimal table. For instance, in the starter of her lesson, the teacher posed a number of decimals and a whole number on the blackboard (e.g. 0.23, 0.63, 1.08, 61.52, 88.888, 1045). She then asked students to discuss the form of these numbers. Some of the examples are given below:

$$0.23 = () \times 0.1 + () \times 0.01$$

$$1.08 = () \times 1 + () \times 0.1 + () \times ()$$

$$61.52 = () \times 10 + 1 \times () + () \times () + () \times ()$$

In the second PPA, Mr Zhang mainly commented on the two approaches applied by the two teachers, in order to help teachers to develop an insight into selected didactical ideas in mathematics teaching which are advocated in the latest national curriculum reforms. The new curriculum advocates developing students' complex thinking in a web-like knowledge structure, so as to enable students to see the most important feature of how mathematical knowledge is constituted. Mr Zhang said the following:

Different from the second teacher, the third teacher's lesson was from number to table. In fact, there are a considerable number of concepts in the (decimal) table. For instance, 10-to-1 relationship between the values of any two adjacent places, the name of the units of decimal, the name of the place of decimals, the unit of

the place of decimal, the number on the unit of decimal, the unit number of decimals, etc. The (third) teacher's instruction was clearly concerned about the first two concepts, but did not talk about the other concepts in the lesson. The main 'missing point' of the lesson is the lack of concrete diagrams to support students to understand the connection of the abstract concepts, such as the unit of the decimal place and its meaning (as the second teacher did). The advantages of the combination of decimal table and the number line are as follows: firstly, the infinity in dividing a small segment on the number line can be connected to the concept of number place in the table. At the same time, it is an opportunity for students to develop the concept of infinity. In such a way the teacher helps students to see the connection between the representation of the geometrical model and the mental representation of the decimal table. Consequently, the teacher makes the scaffolding for students to understand the abstract concepts and ideas such as infinity, set, 'approaching'. A lesson like this is likely to be a deep learning lesson. ... In the latest pedagogy reforms, 'gathering thinking' is highlighted in teaching. That is, thinking is developed as a web, rather than the linear line/progression of knowledge connections. The 'gathering thinking' is based on a rich web of knowledge and a web of experiences. New knowledge is the result of the richer/more connections. The point is to understand what kinds of web new knowledge is based on. Then teachers could see how to help students to develop multiple connections of knowledge and experiences within the web. The connection of a new point of knowledge to the multiple points of knowledge on the web would enable students to enrich knowledge representations, make knowledge transformation and application, and develop complex thinking, namely the 'gathering thinking' in mathematics.

Mr Zhang then drew two web-like structures of knowledge connections of the decimal table. Figure 2 is a macro-level of the knowledge structure of the decimal table. Figure 3 is a micro-level of the knowledge structure of the decimal table.

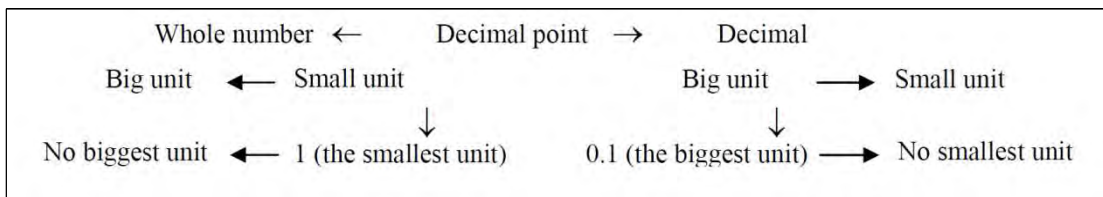


Figure 2. A macro-level of the knowledge structure of the decimal table

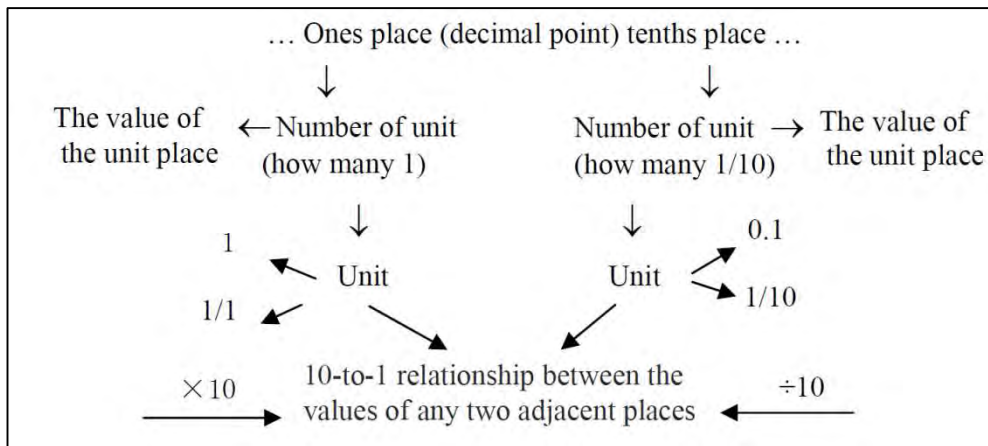


Figure 3. A micro-level of the knowledge structure of the value place of decimal table

The two figures indicate two key theoretical ideas of this expert teacher in helping teachers to develop knowledge and ideas to make the multi-layered task design: first, the conceptual variation (Gu et al., 2004) for understanding concepts from multiple perspectives; second, the "Xun Xu Jian Jin" method to develop the complex mathematical thinking within the whole structure of knowledge connections.

## Implications

In this paper, we have delineated two principles which had emerged in our pilot study of the school-based tasks and instruction design project in Shanghai. However, there are challenges for the main study that concern the difficulties of the connection between theoretical frameworks and principles for our task design across communities:

(a) It is challenging to situate knowledge from different design communities into the ‘living context’ – the classroom in a broader culture (in our study the Chinese culture). In particular, if we wish to introduce innovation in teaching, we need to understand more comprehensively the alternative “local instruction theories” (Gravemeijer, 2004) that different teachers (researchers, professional developer and school teachers in our study) appear to hold.

(b) The HLT (Simon, 1995) features a teacher’s design decisions based on her/his best guess of how learning might proceed. In our study the HLS addresses the importance for teachers to develop their “knowledge packages” (Ma, 1999) and to try things out in practice. It would be necessary to understand how the expert teacher’s HLS may help teachers to make the connection between the unpredictable nature of individual learning and the pedagogic practice/ mathematical-didactic structures (macro and micro) suggested by the expert. Further, it would be necessary to develop new insights into the different teaching/learning methods underlying Simon’s (1995) HLT in the US classroom and the HLS in the Chinese classroom. For instance, the distinction of the western construction (i.e., scaffolding) and the Chinese “*Xun Xu Jian Jin*” and “*pu dian*” (i.e., procedural variation, in Gu et al., 2004, p. 340), the theoretical ideas of the ‘proper potential distance’ (Gu et al., 2004, p. 343), and the micro and macro perspectives of learning viewed by the expert teacher in our study.

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## References

- Choppin, J. (2011). The impact of professional noticing on teachers’ adaptations of challenging tasks. *Mathematical Thinking and Learning*, 13(3), 175-197.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003) Design experiments in educational research. *Educational Researcher*, 31(1), 9-13.
- Design-Based Research Collective (2003) Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, 31(1), 5-8.
- Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning*, 6(2), 105–128.
- Gu, L. (2012). *Lecture on research of teaching with variation in mathematics teaching and learning*. Qingpu district, Shanghai, China. [In Chinese]
- Gu, L., Huang, R. and Marton, F. (2004) Teaching with variation: a Chinese way of promoting effective mathematics learning. In Fan, L., Wong, N., Cai, J. and Li, S. (eds.) *How Chinese Learn Mathematics: Perspectives from Insiders*. New Jersey: World Scientific.
- Kieran, C., Krainer, K., & Shaughnessy, J. M. (2013). Linking research to practice: Teachers as key stakeholders in mathematics education research. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick & F. K. S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 361-392). Dordrecht, The Netherlands: Springer.
- Lin, Ch. (2011). *Intellectual Development and Mathematics Learning*. Beijing: China Light Industry Press. [in Chinese]



- Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: LEA.
- Shi, N. (2012). *An interpretation of the Compulsory Education Mathematics Curriculum Standard (Version 2011)*. Beijing: Beijing Normal University Press.
- Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26, 114-145.