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The effect of using real world contexts in post-16 mathematics questions

Chris Little (ctl@soton.ac.uk) and Keith Jones

School of Education, University of Southampton, Southampton, UK.

This paper reports on a study into the effect of real-world contextual framing in sequence questions. Alternative versions of the same questions were presented in explicit, algebraic, word and pattern contexts, and set to a sample of 594 Year 13 students (aged 17-18) in a one-hour test. Facility levels of the questions were then compared. In addition, the paper presents results of a student questionnaire on real-world context which accompanied the test.

Keywords: Assessment, post-16 mathematics, real-world context, modelling

Introduction

A well known problem from the Rhind (or Ahmes) papyrus of 1650 BC (Boyer, 1985) may be stated in the form:

> 'Seven houses have seven cats per house. Each cat catches seven mice. Each mouse eats seven ears of spelt, each ear would have produced seven hekats of grain. What is the sum of the houses, cats, mice, ears and grain?'

'Word' problems, set in some context, have therefore been around a long time, and have been the subject of considerable research. While current mathematics curricula often frame mathematics questions in 'real-life' terms, the utility and realism of many of the contexts used in mathematics problems have been called into question (Wiliam, 1997, Boaler, 1993b). Some research has suggested that the perceived realism of contexts may be gender-dependent (Boaler 1994) Much research has focused on understanding the difficulties that many children seem to have in applying realistic constraints to 'real-world' problems (Verschaffel et al., 1994, Verschaffel et al., 1997, Cumming and Maxwell, 1999, Silver, 1993), or of transferring from one context to another (Boaler, 1993a). In terms of designing suitable examinations in mathematics, the validity of contexts deployed in summative assessment tasks has been identified as a particular issue (Ahmed and Pollitt, 2007, Pollitt and Ahmed, 2001, Pollitt et al., 2007). Some research has suggested that linguistic and cultural factors may affect the interpretation of real-world context in questions (Pollitt et al., 2000, Barwell, 2001). Cooper and Dunne (2000) have, in particular, provided a critique of the use of real-world contexts in national curriculum assessments on the grounds that they discriminate by social class.

Despite this work, little research has directly investigated the effect of real-world context on the facility of questions, or on the attitudes of learners to its usage in tasks. Moreover, most research has focussed on pre-16 mathematics. This paper describes a study conducted to investigate these issues, using 17-year old students and the AS-level topic of sequences.

The extent to which real-world contexts are used in UK public examination syllabuses varies substantially. For example, an analysis (carried out as part of the study reported in this paper) of pure mathematics papers from two current GCE A/AS specifications – OCR 'A' and 'B' (MEI) - suggests that they utilise real-world contexts in questions contributing about 5% of the marks and 30% of the marks respectively. This difference of emphasis can be traced back to 'modern' and 'traditional' A-level syllabuses of the 1970s and 1980s (Little, 2010, in press).

The research indicated above suggests a fundamental dilemma concerning the use of real-world context, or, as we prefer to call it, real-world contextual framing (RWCF) (Little, 2008a, Little and Jones, 2007) in mathematics questions. On the one hand, by making a connection between the abstract world of mathematics and everyday, or scientific, contexts, we are reinforcing the utility of mathematics as a language for explaining the patterns and symmetries of the 'real' world. On the other hand, if we manipulate and 'sanitise' real-world experiences to enable them to be modelled by a pre-ordained set of mathematical techniques, then the result can appear to be artificial and contrived, or, in the words of Wiliam (1997) a 'con'-text, providing a deception that the activity is worthwhile.

Another dilemma concerns the effect of real-world contextual framing on questions. On the one hand, it may be said to help to solve mathematical tasks by providing the solver with a 'mental scaffolding' (Vappula and Clausen-May, 2006) for solving the problem; on the other, it may be seen to complicate the task by requiring the solver to identify and match mathematical concepts with elements of the real-world context (Pollitt and Ahmed, 2001). This may assume knowledge of the context as well as the mathematics, which may actually be disadvantageous in solving problems in the manner intended, or according to the artificial rules of the mathematics classroom (Boaler, 1994).

This paper considers the effect of RWCF on post-16 examination questions. The study forms part of doctoral research which also explores the wider historical, philosophical and pedagogical roots of real-world context (Little 2010, in press).

Theoretical Framework

Some theories of teaching and learning, such as Realistic Mathematics Education (see, for example, Freudenthal, 1991, Treffers, 1987, Dickinson and Eade, 2005) espouse the view that 'realistic' contexts, or contexts that are 'real' to learners, play a coordinating role in the development of mathematical concepts. When used in public examinations, however, assessment theory requires that questions are evaluated according to their construct validity (Wiliam, 2007). Ahmed and Pollitt (2007) use the term *construct fidelity* to assess the extent to which a real-world context faithfully tests what it intends to assess. They see context as a threat to validity, as it introduces variability to the question–setting and answering process. Additional demands of comprehension on candidates may be considered to introduce a *construct irrelevant variance* (Wiliam, 2007) to the assessment.

On the other hand, the subject criteria for GCE A/AS Mathematics (Qualifications and Curriculum Authority, 2002) require schemes of assessment to:

> 'Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinements of such models.'
>
> 'Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.'

These assessment objectives would appear to require a degree of real-world contextual framing in test items.

How do questions involving real-world context differ from those deploying language from the mathematics register (Pimm, 1987)? They require the solver to match elements of the real-world context with mathematical concepts and variables. The extent to which this matching process is explicit or implicit in the question, or overt or camouflaged, would appear to be relevant to the facility of the question.

Methodology

In order to investigate the effect of RWCF in post-16 mathematics, the study selected a topic from A/AS level which appeared to be amenable to a variety of approaches, both with and without RWCF. An analysis of past paper suggested that sequences and series was such a topic, and that questions on this topic may be categorised as follows:

- Explicit (e) questions, which contained no real-world context, and explicitly define the sequence to be used in the question;
- Algebraic (a) questions, which used mathematical notation, such as u_n or sigma notation;
- Word (w) questions, which used a real-world context defined in words;
- Pattern (p) questions, in which a sequence was defined using a pattern context.

The study therefore constructs e, a, w and p versions questions of with similar or identical solutions, in order to compare their facility.

The study comprised a one-hour test and a short student questionnaire. The test consisted of four questions on arithmetic sequences (AI to AIV) and four on geometric sequences (GI to GIV), and was given to a sample of 594 year 13 students (aged 17-18) from four centres. Students were randomly allocated to one of four tests (A, B, C or D), each of which contained one of four versions (e, a, w and p) of the questions. The solutions to each version were, except in a few small details, the same.

The total mean scores for the 'e', 'a', 'w' and 'p' versions were calculated and compared, to establish an overall measure of difficulty for each type of question. Each part question was then considered, using four difference of two means test to compare the 'e' version with the 'a', 'w' and 'p' versions, and the 'w' version with the 'p' version. The question versions, and their solutions, were then analysed in greater detail, in order to conjecture reasons for significant differences in response.

The questionnaire invited students to consider six statements on pure and applied mathematics and real-world context in questions, and register their level of agreement, from 'strongly disagree' to 'strongly agree'. These questions are listed in Fig. 2 in the 'results' section. They were then given space to make further comments. The results of the questionnaire were compared by gender and by whether students declared English as their first language. Open comments were classified into broad similarities of opinion.

Results

The mean total number of marks out of 40 for each version were as follow:

Explicit: 28.17, Algebraic: 20.17, Word: 22.31, Pattern: 23.37

Thus, the 'e' versions (explicit), as might be expected, gained higher marks overall, followed by the 'w' and 'p' versions, with the 'a' questions proving to be the most difficult. However, this pattern of results did not hold for all of the questions

Fig. 1 gives the versions of question AI, together with the mean scores for each part question, and is followed by a sample analysis.

AIe (A1)
An arithmetic progression has first term 7 and common difference 3.

(i) Which term of the progression equals 73? [3] [2.60]

(ii) Find the sum of the first 30 terms of the progression. [2] [1.76]

AIa (D8)
The nth term of an arithmetic progression is denoted by u_n. $u_1 = 7$, $u_2 = 10$ and $u_3 = 13$.
(i) If $u_n = 73$, find n. [3] [2.35]

(ii) Find $\sum_{r=1}^{30} u_r$. [2] [0.99]

AIw (C5)
Chris saves money regularly each week. In the first week, he saves £7. Each week after that, he saves £3 more than the previous week.

(i) In which week does he save £73? [3] [2.19]

(ii) Find his total savings after 30 weeks. [2] [1.18]

AIp (B4)
A spiral is formed with sides of lengths 7 cm, 10 cm, 13 cm, … which are in arithmetic progression.

(i) How many sides does the spiral have if its longest side is 73 cm? [3] [2.52]

(ii) Find the total length of the spiral with 30 sides. [2] [1.52]

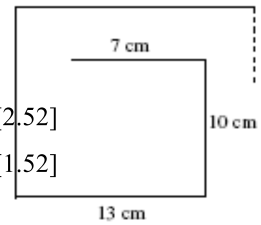

Fig. 1 Versions of question AI with mean scores.

For this question, both parts of the 'a' version were significantly more demanding than the 'e' version, and the 'w' version proved more difficult than the 'e' version, with a more significant difference in part (ii). Comparing the 'e' with the 'p' versions, there was no significant difference in scores for part (i), but part (ii) was significantly more difficult in the 'p' version. Finally, comparison of the 'w' and 'p' versions shows the 'w' version to be significantly more difficult than the 'p' version.

The 'e' v 'a' results are easily explained in terms of additional demand of algebraic notation, in particular sigma notation. In the 'e' version, the explicit reference to 'arithmetic progression', 'term' and 'sum' leads the solver directly to the appropriate formulae. These cues were not present in the 'w' version, which requires solvers to translate elements from the real-world context to the algebraic model ('first week £7' = a, '£3 more' = d). This would account for the extra difficulty of the 'w' version compared to the 'e' version.

On the other hand, the 'p' version of part (i) proved to be no harder than the 'e' version. This might be because the first three terms of the sequence are stated explicitly (7 cm, 10 cm, 13 cm, ..), thus making the match to an arithmetic model easier than in the 'w' version. Indeed, it is possible to think within the context to derive the number of terms (length increased by 67 = 3 × 29, so 30 sides). This type of 'first principles' thinking is not available in part (ii), which perhaps explains why this proved harder than the 'e' version.

The explicit statement of the first three terms in the 'p' version, thus hinting at an arithmetic sequence model, might also explain why this version proved to be easier than the 'w' version, where this cue was not given

Similar analyses of all eight questions suggest a number of factors which affect the facility of questions, as follows.

- The use of algebraic language such as sigma notation and iteration formulae added substantially to the difficulty of questions.
- The requirement to identify the type of sequences and to use the appropriate term or sum formula from data given in a real-world context also adds to the demand.
- Some real-world contextual framing requires solvers to interpret text carefully in order to select the appropriate match between context and model. Semantic ambiguities may also cause unintended errors.
- In contrast, questions which can be solved by 'thinking within the context' can be as easy as explicit formulations.

The results of the questionnaire, together with the questions asked, are shown in Figure 2. Two thirds of the students believed that questions set in real-world context are harder than those without context. 33% agreed and 30% disagreed that RWCF made questions more interesting, whereas 55% agreed and 30% disagreed, with the statement that real-world context shows how mathematics is useful. Over half of the students preferred pure mathematics to applied mathematics, and felt that pure mathematics is interesting in its own right.

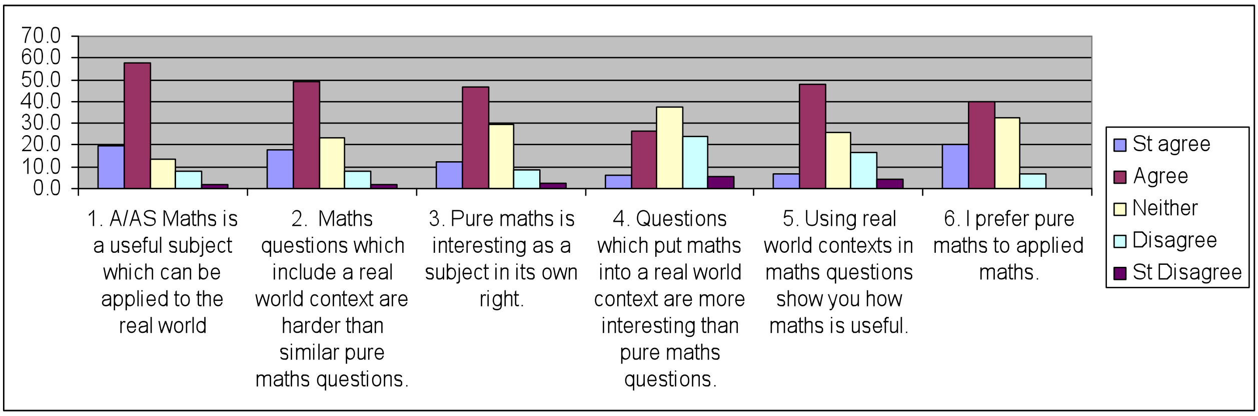

Fig. 2: questionnaire results

The results suggested that more girls prefer pure to applied mathematics. Girls agreed more with the statements that real-world contexts make questions harder and that pure maths is interesting as a subject in its own right, whereas boys agreed more with the statement that questions with real-world context are more interesting. Overall, these results show a consistent pattern of girls preferring pure mathematics questions without real world contextual framing.

Students for whom English was their first language agreed more strongly with the statements that A/AS mathematics is a useful subject which can be applied to the real world and that maths questions set in real-world context are harder.

The first result might be interpreted as showing cultural differences concerning the nature of mathematics. The second result is perhaps surprising, as one might have expected non-native speakers to find contextualised questions harder to comprehend. However, as the number of students in the first category was relatively small (49), the sample may not be large enough to be representative.

The variety of opinions which students hold on the use of real-world context is indicated by the following selection of comments:

> "Applying maths to real situations is certainly more difficult but a lot more interesting and satisfying to complete rather than straight pure maths questions."

"I strongly dislike real world context questions as they turn maths that I can do into something I can barely understand."

"I prefer the questions which are worded (without context). I believe this is due to dyslexia, which means I find (contextualised questions) harder to understand."

"Sometimes (contexts) help if you don't know terms or to get an idea of what the question requires, but otherwise they are just plain patronising!"

"Real world questions don't show you how maths is useful, because questions in context, such as question 2, are not useful. It is about a beetle, not useful in normal everyday life."

Conclusions

How do these results inform research on real-world contextual framing? It should be acknowledged first that these results apply to post-16 students of mathematics only, and test results are based on one only one topic. Further research using different topics would be needed to test the generality of the conclusions. However, they provide strong evidence that setting sequence questions in real-world contexts does indeed add to the overall demand, though a context can on occasions provide 'mental scaffolding' to help the solver to use context-specific heuristic strategies. For example, questions involving the 'term' formula from an arithmetic progression [$u_n = a + (n - 1)d$] are amenable to calculations using first principles (e.g. nth term = 1st term + (n − 1) × the 'step'). On the other hand, using the 'sum' formulae for both APs and GPs is a pre-requisite to the efficient solutions of problems involving summation.

One could argue that such solutions effectively side-track the application of standard algebraic formulae to model realistic situations. The potency of algebraic formulae lies in their universality and blindness to individual contexts (Little, 2008b), and, in resorting to context-specific thinking to solve these questions, students are avoiding the necessity to transfer and abstract from context to mathematical model, which is, arguably, the heuristic strategy intended by the questions.

However, questions with RWCF need to be carefully constructed to avoid unwanted distracters and ambiguities. They can require greater interpretative acuity from the solver, in order to correctly match the context with the mathematical model. They may therefore disadvantage students with dyslexia, or non-native language speakers. Contextualised questions therefore require careful revision to ensure that the language used is clear and unequivocal. It is also important to consider the overall length of questions in relation to the time allowed to answer them: asking students to read and comprehend complex, novel contexts in a timed written examination clearly adds to the stress of the experience, and, by placing too much emphasis on comprehension skills, compromise the mathematical goals of the assessment.

Students in this survey generally see real-world context as reinforcing the perception that mathematics is useful, although contexts can be perceived as artificial. Some comments from students resonate with researchers (e.g. Boaler, 1994, Wiliam, 1997) who have criticised real world contexts on the grounds of artificiality. The gender differences identified above are also of interest, with girls preferring pure mathematics and non-contextualised questions, but boys finding such questions more interesting.

What is the function of real-world context in these sequence questions? An earlier paper on linear equation questions argued that, notwithstanding that students link real-world context to applicability of mathematics, most contextualised questions have little or no practical utilitarian value (Little, 2008a). They are, rather, embryonic, albeit contrived, exercises in modelling, which force the student to make connections between the world of algebra and mathematics and real-world concepts such as finance, percentages, and physical patterns and shapes.

It is perhaps unrealistic, however, to expect short, closed examination questions to do more than this, since genuine mathematical modelling requires strategic thinking that is not possible to test in a timed written examination. Such questions might, as the current subject criteria for A/AS Mathematics require, test students' ability to 'recall, select and use their knowledge of standard mathematical models to represent situations in the real world and present and interpret results from such models in terms of the original situation'. However, they manifestly do not 'include discussion of the assumptions made and refinements of such models'. Neither do they require students to 'read critically and comprehend longer mathematical arguments', which a dedicated comprehension paper might do. It is therefore difficult to see how A/AS level syllabuses that rely wholly on short written examination papers can be said to satisfy such requirements.

References

Ahmed, A., and A. Pollitt, 2007. Improving the quality of contextualised questions: an experimental investigation of focus. *Assessment in Education: Principles, Policy and Practice* 4: 201-232.

Barwell, R. 2001. Difficulties with mathematical word problems. *Proceedings of the British Society for Research in Learning Mathematics* 21(2): 63–74.

Boaler J. 1993a. Encouraging the transfer of "school" mathematics to the "real world" through the integration of pocess and content, context and culture. *Educational Studies in Mathematics* 25: 341-73.

Boaler, J. 1993b. The role of contexts in the mathematics classroom: do they make mathematics more real? *For the Learning of Mathematics* 13: 12-17.

Boaler, J. 1994. When do girls prefer football to fashion? An analysis of female under-achievement in relation to realistic mathematics contexts. *British Educational Research Journal* 20: 551-564.

Boyer, C. 1985. *A history of mathematics*.Princeton. Princeton University Press.

Cooper, B., and M. Dunne. 2000. *Assessing children's mathematical knowledge*, Buckingham, Open University Press.

Cumming, J., and G.Maxwell. 1999 Contextualising authentic assessment. *Assessment in Education* 6: 177-194.

Dickinson, P., and F. Eade. 2005. Trialling realistic mathematics education. *Proceedings of the British Society for Research in Learning Mathematics* 25(3), 1–13.

Freudenthal, H. 1991. *Revisiting mathematics education*. Dordrecht, Kluwer.

Little, C. 2008a. The functions and effects of real-world contextual framing in A/AS mathematics questions: developing an evaluative framework. *Proceedings of the British Society for Research in Learning Mathematics* 28(3), 72-77.

Little, C. 2008b. The role of context in linear equation questions. *Proceedings of the British Society for Research in Learning Mathematics* 28(2), 55–60.

Little, C. 2010. (in press). *The Use of Real-World Contextual Framing in UK University Entrance Level Mathematics Examinations,* PhD Thesis, University of Southampton.

Little, C., and K. Jones. 2007. Contexts for pure mathematics: an analysis of A-level mathematics papers. *Proceedings of the British Society for Research in Learning Mathematics* 27(1), 48–53.

Pimm, D. 1987. *Speaking mathematically*. London, Routledge.

Pollitt, A., and A. Ahmed. 2001. Understanding students' minds: the key to writing more valid questions. *National Center for University Entrance Examinations*, Japan.

Pollitt, A., A. Ahmed, and V. Crisp. 2007. The demands of examination syllabuses and examination papers. In *Techniques for monitoring the comparability of*

examination standards.ed. P. Newton, 112-130. London, Qualifications and Curriculum Authority.
Pollitt, A., C. Marriott and A. Ahmed. 2000. Language, contextual and cultural constraints on examination performance. *International Association for Educational Assessment*.
Qualifications and Curriculum Authority. 2002. *GCE AS/A Level Subject Criteria for Mathematics*.London, QCDA.
Quinlan, M. 1995. A comparability study in Advanced level mathematics. *University of London Examinations and Assessment Council*. London.
Silver, E. A. 1993. Sense making and the solution of division problems involving remainders: an examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education* 24: 117-35.
Treffers, A. 1987, *Three dimensions*. Dordrecht, Reidel.
Vappula, H., and T. Clausen-May. 2006. Context in maths test questions: does it make a difference? *Research in Mathematics Education* 8: 99-115.
Verschaffel, L., E. De Corte, and I. Borghart 1997. Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction* 7: 339-359.
Verschaffel, L., E. De Corte, and S. Lasure. 1994. Realistic considerations in mathematical modelling of school arithmetic word problems. *Learning and Instruction* 4: 273-94.
Wiliam, D. 1997. Relevance as MacGuffin in mathematics education. *British Educational Research Association*. York.
Wiliam, D. 2007. Quality in assessment. In *Unlocking Assessment*, ed.S Swaffield,123-137. London: David Fulton.