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DEVELOPING GEOMETRIC DISCOURSES USING DGS IN K-3

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This paper reports on ideas developed during an analysis of data from a project involving young children (aged 5-7 years) in a whole-class situation using dynamic geometry software. The focus is a classroom episode in which the children try to decide whether two lines that they know continue (but cannot see all of the continuation) will intersect, or not. The analysis illustrates how the children can move from an empirical, visual description of spatial relations to a more theoretical, abstract one.

BACKGROUND

Research shows that a majority of students in North America have an inadequate understanding of geometric concepts and poorly developed skills in geometric reasoning, problem solving and proof (e.g., Battista, 2009; Clements and Battista 1992). Clements and Sarama (2008) have done pioneering work in describing some learning trajectories for various strands of geometry, focusing especially on those vectored towards curricular aims of middle school (such as identifying and composing shapes, transformations, etc.). What remains a central question for research in this area is how best to develop children's explanations in a way that appropriately supports their growing understanding of the nature of proof and proving in mathematics (Stylianou, Knuth and Blanton, 2009).

This paper reports on portions of a project involving young children (aged 5-7 years old) in a whole-class situation using dynamic geometry software (specifically *Sketchpad*). The focus for this paper is a classroom episode in which the children try to decide whether two lines that they know continue (but cannot see all of the continuation) will intersect, or not. The episode relates to two important, and growing, areas of research in primary school education: first, the nature of proof and proving in the elementary grades, and second the development of understanding of spatial relations in the early years of school.

Research on young children and proof

Research has pointed to the abrupt transition that children can encounter as they move from primary school, where proof can be absent, to secondary school mathematics, where it becomes more of a central concern (Balacheff, 1988; Ball *et al*., 2002; Jones and Rodd, 2001; Sowder and Harel, 1998). In order to mitigate the effects of this abrupt transition, researchers have argued that proof should begin in the early grades (Bartolini-Bussi, 2009; Stylianides, 2007; Stylianou *et al*, 2009). Further, there is growing evidence that young children can be capable of engaging in deductive reasoning and proving (Galotti *et al*., 1997; Maher and Martino, 1996).

What it means to engage in 'proving' requires some explanation, as Jahnke (2007) notes, since a proof must depend on the concept of a theory. While some might argue that empirical activity cannot lead to proving, Bartolini-Bussi (2009: 53) argues that in the primary school, theories are "germ theories" that are "based on empirical evidence, with expansive potential to capture more and more principles." In other words, an experimental approach does not necessarily work against the production of general methods and the construction of mathematical proofs. Bartolini-Bussi argues that proving in the early years depends on the teacher being able to lead children from an experimental activity, through discussion, towards general methods and justification, in order to nurture a theoretical attitude.

In a somewhat different approach, Stylianides draws parallels between a Grade 3 child's argument and Balacheff's (1988) notion of a "thought experiment" which is the highest level of his hierarchy of arguments (and which transcends the empirical arguments used in lower levels). Here it is worth noting that Balacheff's "thought experiment" describes not only proof, but broader forms of mathematical argument:

> The thought experiment invokes action by internalizing it and detaching itself from a particular representation. It is still coloured by an anecdotal temporal development, but the operations and foundational relations of the proof are indicated in some other way than by the result of their use. (p. 219)

Research on young children and parallel lines

As Bryant (2009: 9), confirms, children's spatial understanding begins early; certainly before the start of formal schooling. By five, according to Bryant, children can take in and remember the orientation of horizontal and vertical lines very well. In contrast, at this age, children have considerable difficulty in remembering either the direction or slope of obliquely-oriented lines. Yet, the research summarised by Bryant indicates that if there are other obliquely oriented lines (in the background) that are parallel to an oblique line, the children's memory of the slope and direction for the oblique line improves dramatically. Apparently, children can use the parallel relation between the line that they have to remember and stable features in the background framework to store and recognise information about the oblique line.

Bryant concludes that younger children probably perceive and make use of parallel relations without necessarily being aware of doing so. A goal of the teaching experiment reported in this paper was to make children's implicit knowledge more explicit by inviting them to reason about the relationships between lines. Further, in keeping with the emphasis on proof and argument in the early years of school, the project followed Bartolini-Bussi in designing classroom tasks that would start experimentally but then provide an opportunity for nurturing a theoretical attitude.

THEORETICAL PERSPECTIVES

In previous research, we have found Sfard's (2008) 'commognition' approach suitable for analysing the geometric learning of students interacting with dynamic geometry software (see Sinclair & Yurita, 2008). That research showed that the use of DGS can lead users to think about geometric objects and relations in very different ways than they do using static, pencil-and-paper materials, thereby changing the fixed, linear development proposed by the van Hiele's. For Sfard, learning corresponds to a change in discourse: learning geometry thus corresponds to changing the way one communicates about geometric objects and relationships.

Sfard characterizes mathematical discourse in terms of four categories: *word use* (how are words used that are specific to mathematics: a word such as 'regular' is used in a particular way in geometry), *visual mediators* (pictures, symbols, graphs, etc.), *routines* (repetitive patterns characteristic of a given discourse: for example, how you tell whether two expressions are the same) and *narratives* (any sequence of utterances framed as a description of objects, or relations between objects that is subject to being endorsed (true) or rejected (false)). By analysing these different components of discourse, and identifying changes that are relevant to mathematical thinking, we can evaluate what students have learned through their interactions with the teacher, other students, and the software.

RESEARCH CONTEXT AND METHODS

We worked with grade 1 and kindergarten children from a University Lab pre-K-6 school in an urban middle SES district. There were 22 children per class from diverse ethnic backgrounds and with a wide range of academic abilities, with 25% being special needs learners. We worked with the students for three days on a variety of geometric concepts including identifying shapes, working with definitions, describing and creating rotational and reflectional symmetries, and identifying parallel and intersecting lines—a topic not covered in North American primary school curricula.

Each lesson lasted approximately 30 minutes and was conducted in a small group (half class at a time) with the children seated on a carpet in front of a large screen. Two researchers, and the classroom teacher, were present for each lesson. The first author conducted the lessons. Each lesson was videotaped and transcribed. The lesson presented in this paper focused on conceptualizing intersecting and parallel lines. The students had already had two previous lessons involving *Sketchpad*. The students had never received formal instruction related to extended lines, intersections, or the notion of parallel lines.

We first describe the lesson and then analyse the lesson in terms of the evolution of the geometric discourse using Sfard's four characteristic features.

EXPLORING INTERSECTING LINES

The lesson began with the children being shown several examples of pairs of points tracing out thickly-coloured linear paths, with some pairs intersecting and others not. In talking about these pairs of lines, the children described the former as "touching." After students successfully identified pairs of lines that "touch" or not, the instructor offered the more technical word "intersection" to describe the former, which the children immediately connected to road crossings—and, interestingly, cars crashes.

The instructor opened a new sketch and used the line tool to construct two lines, colouring one red and the other blue. The lines were positioned so as to be non-parallel, but so that the intersection was not visible (see Figure 1). When asked “Do you think these two lines meet?” the students all said “No” in chorus. Then one girl said “But they can if you tilt it all the way down.” The instructor began dragging the top line toward the bottom one and as the intersection became visible, one student said, “now they have an intersection” and another added “a very small one.” The instructor dragged the top line up again and asked “And here do they make an intersection?” The students chorused “no.” After a few seconds, one boy said “Oh yes they do, they do.” Several students began talking at once, and one said, “Because they go out of the screen.” So the instructor adjusted the screen (dragging the right corner of the window to enlarge it) so that the intersection was made visible, and the children talked excitedly when seeing the intersection.

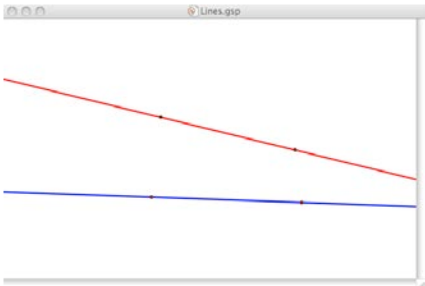

Figure 1: A non-visible intersection

The instructor then dragged the lines even further apart, so that their intersection was not visible, and asked the students to “use their imagination” to decide whether they intersect. This time most children said “yes.” Then a few said that they wouldn’t, then many others joined in. Other children hedged, “I think it might.”

Instructor: Can we make some theories about why it might intersect?

Natasha: Because it’s tilting (referring to the red (top) line).

Robert: The lines, um, can’t meet at the edge of the screen because they are too far apart and they can’t just like suddenly just have a straight line going down and meet.”

But Jamie seemed to change his mind:

Jamie: Cause they are going like this (*tracing with index finger two lines coming together*).

Instructor: But do you think they would ever meet?

Robert: Yes, because they are both slanting and the red one is slanting toward the blue one.

Natasha: It’s going to always connect somewhere because the red one is slanting so its going to connect somewhere over here (*pointing toward the outside right of the screen*).

Instructor: Even if we can't see it, it's going to connect, it's going to intersect somewhere over here?

Jamie: I think it's never going to intersect

Instructor: Why?

Jamie: Because I just do.

Instructor: What do you think about the theory though that this (*pointing to the red line*) is slanting more and more toward the blue?

Jamie: (*Standing up*) But the blue is also going like this (*using hands and arms to show that both lines are slanting*).

Instructor: Oh I see. Interesting, so the blue is slanting as well.

Jamie: As long as both, the red's going down the blue's going down beside it so the line can't just go like that (*bringing his hands together, curving the top one down to touch the bottom one*) and then intersect.

Instruction: That's interesting. Let's look at a situation where we can definitely see an intersection (*dragging the two lines so that their intersection is visible on the screen*). So now they're both slanting just like Robert said.

Natasha: But it's always going to slant because right there (*pointing to the left on the screen*) that's how thick it was so it's always going to slant.

Intructor: It's always going to slant.

Saskia: It's going to intersect.

Robert: It's going to intersect at one point but it might, it might intersect somewhere far, far away.

Instructor: We need to figure out how we're going to know when the lines are going to intersect even when we can't see it. So Jamie, no Natasha said they're going to intersect between the red one is slanting toward the blue one.

Natasha: No because that right there (*hand positioned so that index and thumb at a certain distance away*) isn't the same thickness and it's going to intersect because it always gets smaller.

When asked what gets smaller, Natasha came to the screen and put her index finger on the red line and her thumb on the blue and moved toward the intersection while decreasing the gap between her index and thumb.

The instructor then announced they would look at another situation in which the intersection is not visible, at which point Jamie asked "can we *see* if it is going to intersect or not?" No one expressed any surprise when the window was enlarged in order to make the intersection visible. Jamie then got up and traced his fingers along the intersection. The instructor invited him to explain what he'd done.

Jamie: Because the red one is slanting enough (*gets up to trace to lines off the screen and create their intersection with his fingers*).

Finally, the instructor dragged the red line so that the two lines were parallel to each other and asked the students whether they would intersect. All students said "nooo." One student used Natasha's gesture of measuring the thickness. Jamie used both arms and said, "because they are going away from each other." And Charlotte said "Because they are both going the same way. One of them, they're not slanted, so, they're kind of slanted but they're not going to meet since one of them is not really slanted because they're just going like (*gesturing with one straight arm the direction of a line*) they're both going (*now bringing the other arm to move parallel with the first*) like that so they're never going to meet (*using her right hand to curve down towards the left one*). The instructor then offered the word "parallel" to describe two lines that are never going to intersect.

In terms of their word use, the children's initial discourse is about shapes immediately visible to their visual field. So, for example, "line" is a linear segment drawn on the screen. This evolves into an unbounded process that leaves a linear trace, as can be seen in the way the children begin to talk about "they are going like this" and "the red one going down." This change may seem marginal at first, but it signifies a huge leap from the geometric discourse of being captive of one's visual field and speaking about static visible objects, to the discourse of talking about possibilities (hypothetical things: "it's going to connect somewhere other here") and abstract objects (an invisible point of intersection).

The role of the instructor is crucial in bringing about the change in discourse, not only in terms of the manipulation of the lines—which go from having visible to invisible intersections, and which move as entire objects all at once—but in terms of modeling the new discourse. The questioning begins with "do the two lines meet" and then turns into a more hypothetical formulation about "why it might intersect"—the former concerning the static, visible lines and the latter going beyond the here and now, implying that the "line" is not just what is contained in the children's visual field. This discursive shift is evident in Natasha's statement "It's going to always connect […] so it's going to connect somewhere over there," which involves a hypothetical, dynamic way of talking. The instructor reinforces this way of talking when she asks "Even if we can't see it, it's going to connect, intersect somewhere over there?" and when she re-voices the dynamic description "the red is slanting more and more toward the blue." While the word intersection was initially reserved for a visible place where two lines meet, toward the end of the episode, the children use it to describe any place where two lines meet, be they visible or not. Indeed, besides Jamie, who insists on actually seeing the intersection, even though he has argued that the two lines must intersect, the other students express no such empirical need, and their lack of response to the empirical evidence suggests they are neither surprised nor relieved.

In terms of routines (repetitive patterns found in the discourse of the students as a class), we see a shift from the routine that depends on the visual identification of the intersection, to one that involves working with the properties of the lines. In one routine, the children assess whether one line is slanting toward another and in the other, they determine whether the "thickness" of the lines varies. Both routines are more sophisticated in terms of their geometric discourse in that they rely on assertions about the relationships between the lines—in which the lines are conceived as objects that can be transformed. The ensuing narratives are expressed in the statements about intersecting lines, namely, that two lines intersect if one slants more than the other or if the thickness between the two gets smaller.

Two new visual mediators are also introduced, that of prolonging lines using one's arms, and that of using one's fingers to measure the "thickness" between the lines, also indicated through gesture. These gestures, which are introduced by Jamie and Natasha, are also used by the other children in the class as they determine whether two given lines intersect.

REFLECTIONS AND CONCLUSION

In this classroom episode, the children were being asked to come up with a method whereby they could predict whether two lines might intersect. Although not explicitly about parallel lines (though the word was eventually introduced to describe lines that the children argued would not intersect), their task involved analysing the relation between lines, and characterising the difference between lines that intersect and lines that do not—a characterisation that forms the basis for the definition of parallelism. Natasha and Jamie both offered arguments that qualify as thought experiments, in Balacheff's sense.

In addition to setting up a context in which lines could be easily and precisely moved, and extended as much as desired (unlike lines on a blackboard), *Sketchpad* offered an opportunity for the teacher and the students to develop of discourse of dynamism and potentiality. This way of talking enabled the thought experiment that required the children to attend to the relationships between the lines and to devise routines for using these properties to make inferences. We note the pivotal use of gestures by the children in developing their routines; these gestures are certainly a component of their commognition, and may deserve greater articulation—beyond being classified as visual mediators—in Sfard's characterisation of discourse. Our analysis of the student episode demonstrates a substantive increase in mathematical discourse along each of Sfard's four characteristic features. We thus provide further evidence of Bartolini-Bussi's claim that young children can be capable of transcending empirical arguments and engaging in aspects of deductive argumentation.

References

Balacheff, N. 1988. Aspects of proof in pupils' practice of school mathematics. In D. Pimm (ed.), *Mathematics, Teachers and Children*. London: Hodder and Stoughton (pp. 216–235).

Ball, D.L., C. Hoyles, H.N., Jahnke, and N. Movshovitz-Hadar. 2002. The teaching of proof. In L.I. Tatsien (ed.), *Proceedings of the International Congress of Mathematicians*. Beijing: Higher Education Press, Vol. III, pp. 907–920.

Bartolini-Bussi, M. (2009). Proof and proving in primary school: an experimental approach. In Fou-Lai Lin et al (eds) *Proceedings of the ICMI Study 19 Conference: Proof and Proving in Mathematics Education*. Taipei, Taiwan: National Taiwan Normal University. Volume 1, pp 53- 58.

Battista, M.T. (2009). Highlights of Research on Learning School Geometry. In (Eds.) T.V Craine and R. Rubenstein. *Understanding geometry for a changing world: Seventy-first yearbook* (pp 91 -108). Reston, VA: NCTM.

Bryant, P. 2009. *Understanding Space and its Representation in Mathematics*. London: Nuffield Foundation. Carpenter, T.P., M.L. Franke, and L. Levi. 2003. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann.

Clements, D. H., & Battista, M.T. (1992). Geometry and spatial reasoning. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-464). New York, NY: MacMillan.

Clements, D. H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal, 45*, 443-494.

Galotti, K.M., Komatsu, L.K., and Voelz, S. (1997). Children's differential performance on deductive and inductive syllogisms. *Developmental Psychology*. 33(1): 70-78.

Jahnke, H. N. (2007). Proof and hypotheses. *ZDM - The International Journal on Mathematics Education*. 39: 79–86.

Jones, K. and M. M. Rodd. 2001. Geometry and proof. *Proceedings of the British Society for Research into Learning Mathematics*. 21(1): 95-100.

Light, P., A. Blaye, M. Gilly,. and V. Girotto. 1989. Pragmatic schemas and logical reasoning in 6- to 8-year-old children. *Cognitive Development*. 4: 49-64.

Maher, C.A. and A.M. Martino. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*. 27: 194–214.

Sinclair, N. and Crespo, S. (2006). Learning mathematics in dynamic computer environment. *Teaching Children Mathematics*. 12: 437-444.

Sinclair, N. and Yurita, V. (2008). To be or to become: How dynamic geometry changes discourse. *Research in Mathematics Education, 10*(2), 135-150.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge, England: Cambridge University Press.

Sowder, L. and Harel, G. (1998). Types of students' justifications. *The Mathematics Teacher*. 91(8): 670–675.

Stylianides, A.J. (2007). The notion of proof in the context of elementary school mathematics. *Educational Studies in Mathematics*. 65: 1–20.

Stylianou, D., E. Knuth, & M. Blanton (Eds.). (2009). *Teaching and Learning Proof across the Grades: a K-16 perspective*. New York, NY: Routledge.