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Task design with DGEs: The case of students' counterexamples

Keith Jones[1] and Kotaro Komatsu[2]

[1]University of Southampton, Southampton Education School, UK; d.k.jones@soton.ac.uk

[2]Shinshu University, Institute of Education, Japan; kkomatsu@shinshu-u.ac.jp

Keywords: Task design, dynamic geometry environment, counterexample, proof.

Introduction

Geometry, being one of the main areas for proof-related activity in school mathematics, is a topic in which learners can work with counterexamples. Sinclair et al. (2016) recently highlighted that more research is needed on task design with dynamic geometry environments (DGEs). We are addressing these issues by elaborating a set of theory-informed task design principles involving DGEs and using these to study students working with geometric diagrams (Jones & Komatsu, 2017; Komatsu & Jones, 2019). Given the importance of networking theoretical approaches, in this short paper we build on our studies by employing a conceptual framework on virtual manipulatives (Osana & Duponsel, 2016). Our research question is how, in using Osana and Duponsel's framework, our task design helps students to produce and address what they think are counterexamples in geometry.

Virtual manipulatives

Following Moyer-Packenham and Bolyard (2016), DGE tasks where on-screen objects are draggable can be regarded as virtual manipulatives. Osana and Duponsel (2016), based on a thorough review, proposed that task design with virtual manipulatives should take into account: the *surface features of representations*, the *pedagogical support*, and *students' perceptions and interpretations*.

Method

We undertook a task-based interview (of 35 minutes) with two prospective teachers from a national university in Japan. Each of the participants had experience of using DGEs. They worked on the tasks in Figure 1, which were based on our design principles (for details, see Komatsu & Jones, in press). For data analysis, we used video records of the task-based interview, transcripts of the undergraduates' talk, their written work, and their DGE file.

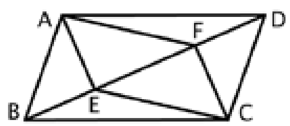
Task 1. In parallelogram ABCD, draw perpendicular lines AE and CF to diagonal BD from points A and C, respectively. Prove that quadrilateral AECF is a parallelogram.	
Task 2. Construct the diagram shown in Task 1 using a DGE. Move the vertices to change the shape of parallelogram ABCD, and examine whether quadrilateral AECF is always a parallelogram.	

Figure 1: Tasks used in the interview

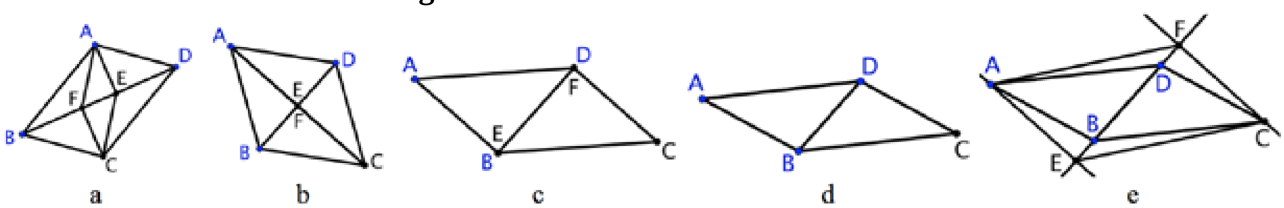


Figure 2: Types of diagrams produced by the undergraduates

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The undergraduates completed Task 1 by producing a valid proof where they showed that AE = CF and AE // CF. In our analysis, we focus on how the undergraduates worked on Task 2.

Analysis

The undergraduates produced various diagrams with the DGE, as shown in Figure 2. They considered that the cases shown in Figures 2b and 2d were counterexamples to the statement (quadrilateral AECF is a parallelogram). They successfully addressed the case of Figure 2d by extending the original statement (as in Figure 2e) and proving that quadrilateral AECF in this case is a parallelogram. Here, we summarise how the set of tasks in Figure 1 embody the three elements of the virtual manipulatives framework of Osana and Duponsel (2016):

- Task 1 contains a given diagram. This diagram includes a specific representational feature; namely a 'hidden' condition implying that perpendicular lines from points A and C *always* intersect with diagonal BD. By this *surface feature*, and the *pedagogical support* in the text in Task 2 explaining the DGE use (i.e. use of the DGE to check the truth of the statement), the undergraduates could discover what they viewed as 'counterexamples' to the statement (as in Figures 2b and 2d).
- *Pedagogical support* included in Task 2 was not excessive in the sense that this task gave the undergraduates the opportunity to explore the task on their own by not specifying what they were expected to do when discovering what they might consider to be 'counterexamples'.
- The sequence of Task 1 and Task 2, designed by taking *students' interpretations* of counterexamples into account (see Komatsu & Jones, 2019), stimulated the undergraduates' subsequent activity where they spontaneously dealt with the case of Figure 2d by extending the original statement with proving in Figure 2e.

Conclusion

This analysis illustrates how our theory-informed design principles are further supported with the conceptual framework on virtual manipulatives of Osana and Duponsel (2016) in terms of the *surface features of representations*, the *pedagogical support*, and *students' perceptions and interpretations*.

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