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Secondary School Mathematics Learners Constructing Geometric Flow-chart Proofs with a Web-based Learning Support System

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As international research confirms, many secondary school students can find it difficult to understand and construct mathematical proofs. In this paper we report on a research project in which we are developing a web-based learning support platform (available in both Japanese and English) for students who are just starting to tackle deductive proving in geometry. In designing this learning platform we are using Flash (Adobe systems) to produce classroom tasks that involve the properties of parallel lines and congruent triangles. In using the technology, students can complete open and closed flow-chart proofs by dragging sides, angles and triangles to on-screen cells and our system automatically translates the figural elements to their symbolic form. The system identifies any errors in the students' solutions and students receive relevant feedbacks on-screen in appropriate orders. This paper explains the design of our web-based system and illustrates, from initial classroom trialling, how it can support students' proof construction processes and promote their mathematical thinking.

Introduction

The discussion document for the recent ICMI study on *Digital Technologies and Mathematics Teaching and Learning* identified a key question for mathematics education research as "how can technology-integrated environments [in mathematics education] be designed so as to capture significant moments of learning?" (IPC, 2005, p. 356). This paper reports on the design of a web-based learning support platform (available in both Japanese and English) for students who are just starting to tackle deductive proving in geometry; see: http://www.schoolmath.jp/flowchart_en/home.html

In designing this learning platform we are using Flash (Adobe systems) to produce a learning environment in which students tackle geometric problems by dragging sides, angles and triangles to on-screen cells. As this happens, our system automatically translates the figural elements to their symbolic form. The system identifies any errors in the students' solutions and students receive relevant feedback on-screen. This paper explains the design of our web-based system and illustrates, from initial classroom trialling, how it can support students' proof construction processes and promote their mathematical thinking. For more examples of the technology-based tasks that we have designed within the learning platform, see Miyazaki, Fujita, Murakami, Baba and Jones (2011).

The design of technology-based learning platforms: research and theory

In mathematics education, there exists a wide range of different types of technological support for teaching and learning. These, as Brown, Cadman, Cain, Clark-Jeavons, Fentem, Foster, Jones, Oldknow, Taylor, and Wright (2005, p. 28) illustrate, can be categorised into, for example, mathematics teaching software (such as dynamic geometry, graph-plotting, or symbolic algebra software), suitable programming languages (such as Logo), "small programs" that address very specific aspects of the mathematics curriculum (often in the form of games and simulations), "micro-worlds" (see below), and general purpose software (such as spreadsheets). Much technology-related research in mathematics education continues to be concerned with the use (or lack of use) of all these various forms of technology in teaching, including the design of teaching tasks and the professional development needs of teachers. Much less research has focussed on the design of technology-based learning platforms in mathematics education (although such concerns are wide-spread in the ICT literature in general); two examples in mathematics education research being Christou, Jones, Mousoulides and Pittalis (2006) and Confrey, Hoyles, Jones, Kahn, Maloney, Nguyen, Noss and Pratt (2009).

A "micro-world", according to Goldenberg (1982), is "a well-defined, but limited environment in which interesting things happen and in which there are important ideas to be learned" (p. 218). Edwards (1998) reviews the different ways in which the term “micro-world” has come to be used and concludes that a micro-world can be seen as “embodying” a sub-domain of mathematics (or science) because of the opportunity that such an environment provides "for students to kinaesthetically and intellectually interact with the designers’ construction of these entities, as mediated through the symbol system of a computer program" (p. 74). Yet, as Hoyles and Noss (2003) explain, it is important to take account of the potential differences in learning arising from interaction with ‘open’ micro-worlds – based on learner use of a programming language such as Logo – and those that are not.

While our web-based learning support platform (hereinafter called the *system*) is not based on learner use of a programming language, our system fits with how Goldenberg (1982) and Edwards (1998) characterise a micro-world. Hence we are interested in the potential of our system even if it is not "open" in the sense used by Hoyles and Noss (2003). In the next section we outline the design decisions used in developing our system.

The design of technology-based learning platform for proof: methodology

In characterising our learning system as a "micro-world" we note that Hoyles and Noss (1987) have identified four aspects of a micro-world as requiring consideration: the technical component, the pedagogical component, the pupil component, and the contextual component. In this paper we focus primarily on the technical component, with some attention to the other three.

We have designed our system to use flow-chart proofs (Ness, 1962) as a means of supporting students in grasping the structure of proof in mathematics. The system includes both open and closed problems (Miyazaki et al., 2011) involving the properties of parallel lines and congruent triangles. An example of an on-screen task is provided in Figure 1. For more on the design of the tasks available within our system, see Miyazaki et al. (2011) where we provide more consideration of the pedagogical, pupil, and contextual components.

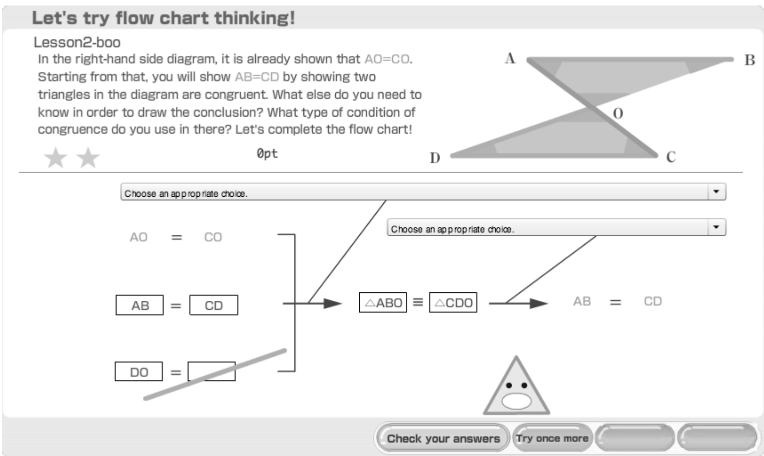


Figure 1: an example task

To make proofs accessible for as many students as possible, we utilise various technological ideas in design of our system. For example, it is constructed so as to be available via the Internet. By using Flash-based technology (Adobe system), which enables interactive actions on the web, students complete proofs by dragging sides, angles and triangles to on-screen cells and our system automatically transfers figural to symbolic elements. Using Flash-based technology also means that students and teachers do not need to install specific software as most computers (and even smart phones, apart, currently, from Apple) automatically include Flash with whatever Internet browser is being used. This means that our system can be used anytime from anywhere. By this technology, we consider that students can concentrate on their proof construction rather than pondering ‘what symbols do I have to use?’, and so on.

Given students sometimes make errors when tackling tasks, it is, of course, very important for learning that students are able to identify errors and refine their proofs to develop further their proof construction abilities (this relates to the pedagogical component of the micro-world). With our system, students receive various types of feedback in appropriate orders based on the hierarchical structure of proof to promote analytical thinking when they click the on-screen ‘Check your answer’ button. The aim of this feature, of course, is to encourage students to use the feedback to adjust their proofs for themselves.

Using a technology-based learning platform: selected classroom examples

We have been conducting studies about how the system impacts on students’ learning and understanding of proofs in geometry. In this section, we provide selected episodes to show how students (UK secondary school students and trainee teachers) are encouraged by the system to engage with analytical and deductive thinking and how they use the system feedback to engage in productive proving activities, mathematical communication, and so on.

Analytical thinking to plan their proof

The system includes some open problems which give conclusions basically, and we consider that this type of problem would be particularly important to develop students’ analytical thinking, because students learn to think backwards in various ways what they should find to show the given conclusion. For example, after completing several open problems, including one that had more than one solution, a 14 year old student K reflected on their thought processes as follows (I: Interviewer):

29	K	Well it depends how you define, no, choose, your way to show that it’s congruent. If you do that way, you’ve got to prove that those two angles are the same, as those sides, and then if you do it another way, then these two and those two, like that. You can do three; all three sides are the same.
30	I	OK I see…

We have observed this kind of thinking on many occasions during our pilot studies. Another example is a pair of undergraduate trainee teachers in their first year of University training, J1 and J2, immediately after successfully completing an open problem with two stages of reasoning, tackled the closed problem illustrated in Figure 2: if AB=AC and ∠BAD=∠ACD then ∠ABD=∠ACD. By this point, they had already completed more than five tasks, and had received several types of feedback from the system. This means that they had begun to pay more attention to specific components of their proofs, such as ‘which conditions should be chosen’, ‘what is a conclusion’, etc. The following dialogue illustrates how these students explored analytical thinking to plan their proof, i.e. starting from considering a conclusion, congruent triangles, and then which condition to be used and so on.

228	J2	So we need to say these triangles [in Figure 2] are congruent.

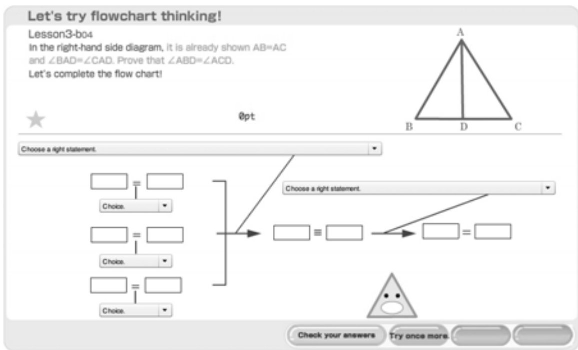


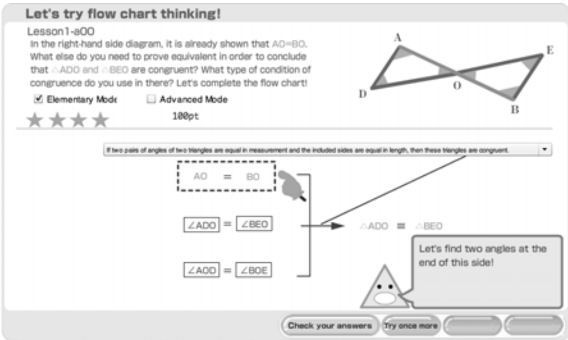
Figure 2: an example task

229	J1	And then we need to choose ‘angles’
230	J2	Yes, we are trying to find angles, wait, yes.
231	J1	And then you can put the angle ABD [into the conclusion box]
232	J2	Right [reading the question again] We have got AB=AC, BAD=CAD.
233	J1	That, we’ve got side angle and side?

Feedback effect on the proof construction process

Students sometimes do not consider which angles and sides should be chosen when trying to solve a proof problem; rather, they might simply select, for example, just one side and two angles which are not related to the chosen side. As an example, we observed two 14 year olds, PF and PM, make such a mistake. Yet, as illustrated by the dialogue below, they were able to utilise the on-screen feedback provided by our system to carry out a modification to their proof which resulted in a correct one. As captured in line 111 below, the students started seeking analytically which sides and angles should be used to draw the conclusion by using ASA condition of congruency. What is more, although the system responded "Let's find two angles at the end of this side," students found the sides and changed the condition of the congruent triangles.

109 PF Yes, I am going to try once more [completing a proof and checking the answer]



I think the middle one is wrong? Then we need to choose the sides…

110 PM These two? [AD and BE]

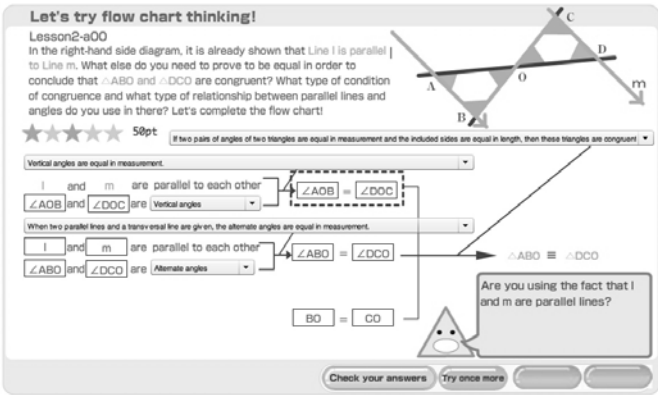
111 PF Which angles have we chosen? Then DO and EO [instead of ∠ADO=∠EBO]? And we need to change the condition [from ASA to SAS]?

112 PM Middle one.

113 PF Middle one. Yes!

For some tasks in our system, students are required to use the properties of parallel lines to deduce that two angles are the same, and our system provides a means of supporting them to reason correctly why. In the example dialogue below, undergraduate trainee teachers J1 and J2 illustrate the proof construction process:

99 J1&J2 [both reading and thinking]



100 J2 We can use different angles? Those two [∠OAB & ∠BAO] or those two [∠AOB & ∠ABO], we haven't done?

101 J1 OK.

102 J2 Start with vertically opposite angles…

103 J1 Don't forget extra boxes are appearing!

104 J2 OK. And then these angles [ABO&DCO]… extra boxes… [completing why these angles

		are the same]
105	J1	That's the … Z one again so corresponding, no alternate! And we need to this box to fill into.
106	J2	Alternate angles are equal?
107	J1	Yes. And then [condition]
108	J2	Yes… [checking the answer]
109	J1	Oh no?! [reading the hint] 'Are you using the fact l and m are parallel…'? [pause 10 sec].
110	I	… I think your reasoning is actually correct, but because l and m are parallel to each other.
111	J1	Yes.
112	I	You need to use either alternate angles or corresponding angles.
113	J1	Ah! This one and this one [indicating l and m]. Ah, this [l and m are parallel to each other] does not tell you anything about vertically opposite angles!

As can be seen in the above dialogue, both J1 and J2 concentrated just on choosing angles and sides for the ASA conditions (lines 102-107). They used $\angle AOB = \angle DOC$ (vertically opposite angles) despite the system asked them to use the properties of parallel lines. The system recognised the approach being used by the students as an error, and with the interviewer's clarifications (lines 110 and 112), J1 finally noticed that they had to use the angles deduced from the properties of parallel lines (113).

These two episodes illustrate that feedback provided by our system gives students an opportunity not only to get correct their answers but also to think more productively and flexibly in the appropriate open problems in which conclusions and parts of premises are given.

Discussion and concluding comments

Given, as international research (such as that summarised in Mariotti, 2007) confirms, that many secondary school students find it difficult to understand and construct mathematical proofs, in this paper we report on a research project in which we are developing a web-based learning support platform (available in both Japanese and English) for students who are just starting to tackle deductive proving in geometry. In designing this learning platform we are using Flash (Adobe systems) to produce classroom tasks that involve the properties of parallel lines and congruent triangles. In using the technology, students can complete flow-chart proofs by dragging sides, angles and triangles to on-screen cells and our system automatically translates the figural elements to their symbolic form. The system identifies any errors in the students' solutions and students receive relevant feedback on-screen. This paper explains the design of our web-based system and illustrates, from initial classroom trialling, how it can support students' proof construction processes and promote their mathematical thinking

The classroom episodes we provide in this paper illustrate how, with our system, students can be encouraged to develop analytical thinking for planning their proofs. This is especially so in the early stage of proof learning, for which our system is particularly designed. The provision of appropriate open problems is designed to encourage students to consider the strategy of thinking backwards from the conclusion to the premises. After this initial stage, students are expected to be able to use analytical thinking effectively to plan proofs in closed problems.

We find that the feedback provided within our system gives students not only encouragement to get correct answers but also to engage in more productive and flexible thinking. In most of the cases we have observed, learner makes many mistakes in their proof construction process. It is important to organise the order which errors should be prioritised to stimulate their modification process most effectively. The system decides the order of feedback based on the hierarchical structure of proof, and shows the message that students can think both analytically and synthetically. This recovering from systematic error leads students to cultivate a capability not only to construct proofs but also to evaluate and improve proofs after they have constructed them. A detailed principle of the design of our feedback system will be addressed in another paper.

At the time of writing this paper, our system has fifteen open problems graded into four levels, and five closed problems. When using these problems in a sequence of lessons, we need to consider a teaching sequence to clarify questions such as ‘How many the lessons should we use to each type of problems?’, and ‘What kinds of concerns or troubles will be identified in transitions from open to closed tasks in students’ proof learning?’. This hypothesising about the process of the students’ learning to prove, coupled with our consideration of the design of appropriate mathematical tasks that can be used to promote student learning (reported in Miyazaki et al., 2011) is leading us to begin considering how we might utilise the idea of hypothetical learning trajectory (HLT), as proposed by Simon (1995) and developed by Simon & Tzur (2004). This is something we may consider as we develop our system.

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References

Brown, G., Cadman, K., Cain, D., Clark-Jeavons, A., Fentem, R., Foster, A., Jones, K., Oldknow, A., Taylor, R. and Wright, D. (2005). *ICT and mathematics: a guide to learning and teaching mathematics 11-19*. Leicester, UK: Mathematical Association (revised edition).

Christou, C., Jones, K., Mousoulides, N. & Pittalis, M. (2006). Developing the 3DMath dynamic geometry software: theoretical perspectives on design, *International Journal of Technology in Mathematics Education, 13(4)*, 168-174.

Confrey, J., Hoyles, C., Jones, D., Kahn, K., Maloney, A. P., Nguyen, K., Noss, R. & Pratt, D. (2009). Designing software for mathematical engagement through modeling. In C. Hoyles & J. B. Lagrange (Eds.), *Digital Technologies and Mathematics Education: Rethinking the terrain* (the 17th ICMI Study) (pp. 19–46). Berlin: Springer.

Edwards, L. D. (1998). Embodying mathematics and science: microworlds as representations. *Journal of Mathematical Behaviour, 17(1)*, 53-78.

Goldenberg, P. (1982). LOGO: a cultural glossary. *Byte*, 2(8), 210–229.

Hoyles, C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In Bishop, A.J., Clements, M.A.., Keitel, C., Kilpatrick, J., & Leung, F.K.S. (Eds.), *Second International Handbook of Mathematics Education* (pp. 323-349). Dordrecht: Kluwer.

IPC [International Program Committee ICMI Study 17] (2005). Digital technologies and mathematics teaching and learning: rethinking the terrain – discussion document for the seventeenth ICMI Study. *L’Enseignement Mathématique, 51*, 351–363.

Mariotti, M. A. (2007). Proof and proving in mathematics education. In A. Guitierrez and P. Boero (Eds), *Handbook of Research on the Psychology of Mathematics Education*. Rotterdam: Sense Publishers.

Miyazaki, M., Fujita, T., Murakami, Y., Baba, N., & Jones, K. (2011). A web-based learning support system to help secondary school mathematics students construct geometric flow-chart proofs. *Proceedings of the 10th International Conference on Technology in Mathematics Teaching* (ICTMT10). Portsmouth, UK, July 2011.

Ness, H. (1962). A method of proof for high school geometry, *Mathematics Teacher, 55*, 567–569.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal *for Research in Mathematics Education, 26*, 114–145.

Simon, M. & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning, 6(2)*, 91-104