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GEOMETRICAL PROPERTIES AS ASSUMPTIONS IN PROOFS IN JAPANESE JUNIOR HIGH MATHEMATICS TEXTBOOKS

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INTRODUCTION

Given the international evidence that students have trouble in understanding proofs (e.g., Hanna & de Villiers, 2012), the appropriateness of the content of textbooks is one of crucial factors (Fan et al., 2018). Here, we focus on the introduction of deductive proof in Japanese junior high school mathematics textbooks. In major textbooks authorized by the Japanese Ministry of Education, a deductive proof is defined as *an explanation of a statement based on properties that are already known to be true*. If these 'already known to be true statements' (i.e. assumptions) are treated ambiguously in the textbooks, then this might cause some of the difficulties in the teaching and learning of proofs. Hence our research question is: in Japanese junior high school mathematics textbooks, what is the relationship between assumptions prescribed in the definition of a proof, and the properties that appear as statements to be proved?

ANALYTIC FRAMEWORK AND METHOD

In order to examine the relationships, we define *correspondence* as the consistency between facts for students and properties, and *coherence* as the deductive consistency between properties. Using two major authorized textbooks, Tokyosyoseki (T) and Keirinkan (K), with over 70% of national share of classroom use, we examined all proofs in the 8th grade textbooks where a mathematical proof was introduced. In doing so, we analysed the relationships between assumptions prescribed in the definition of a proof and properties used in the proofs in the textbooks.

RESULTS

Through our analysis, we found that although the assumptions in these two textbooks were defined as *properties that are known to be correct*, some of this was without mentioning the methods of verifying the correctness (e.g. by demonstration, deduction etc.). The number of assumptions which come under the definition of a proof before introducing this definition (Group A) was 13(T) and 12(K) respectively. The number of properties whose correctness was verified by deductive proofs after introducing the definition of proof (Group B) was 12(T) and 6(K). By divided Group A into the two groups, Group $A \cap B$ and Group $\{A\text{-}A \cap B\}$, we found that properties of Group $A \cap B$ were proved after introducing the proof definition, although the all properties of Group A were already 'stamped' as being already known as correct. As such, by distinguishing the two criteria of purposeful correctness, 'correspondence' and 'coherency', we found that a premediated distinction was being made in the textbooks as to properties of Group A that were proved and those which were not. For instance, in both authorized textbooks while the conditions of congruent triangles is used as an assumption of proofs without being proved (Group {A-A∩B}), the property of the base angles of isosceles triangle is proved (Group A∩B), Although many properties belonged to Group {B-A∩B}, we found that 5(T) and 2(K) properties belonged to Group A∩B (e.g. the property of the base angles of isosceles triangle), and 8(T) and 12(K) properties belong to Group {A-A ∩ B}(e.g. the conditions of congruent triangles). 'correspondence' + 'coherency').

Textbook	Assumptions which come under the proof definition (Group A)	Properties whose correctness was verified by deductive proofs (Group B)	Group A∧B
T	13	12	5
K	12	6	2

Table 1. Assumptions as properties that are known to be correct

DISCUSSION AND CONCLUSION

The ambiguous treatment of assumptions that we found may inhibit students' mathematical thinking. A cause of this ambiguity is the simultaneous usage in the textbooks of 'justification' and 'systematization' as functions of proof (de Villiers, 1990). An implication is that Japanese textbook design might be improved by overcoming the distinction by providing an introductory section on proof structure (Miyazaki, Fujita, and Jones, 2017), including the definition of a proof and sections on 'justification' and 'systematization'. In a section on 'justification', all properties of Group A might be used as assumptions to prove unknown properties. Then, in a section on 'systematization', properties of Group A∩B might be proved by considering the intended local system of properties after stating that the function of proof is different.

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