Analysis of axial response of submarine pipeline to debris flow loading

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ABSTRACT

This technical note presents simplified parametric solutions for the axial response of surface-laid submarine pipelines subjected to axial drag from debris flows. In assessing the response of pipelines impacted by debris flow emanating from a submarine landslide, both normal and axial responses must be considered. Previous work has indicated that these can be decoupled, at least as a first stage analysis. The most critical aspect of axial drag is the potential for the pipeline to buckle. However, in order to make preliminary estimates of displacements and forces along the pipeline prior to buckling, simple assumptions of elastic pipeline response with elastic perfectly plastic interaction with the seabed are justified. These allow the development of parametric solutions that contain only three non-dimensional quantities. The technical note documents the solutions and illustrates their application for some typical input conditions.

KEYWORDS

Analysis, axial response, debris flow, submarine pipelines
INTRODUCTION

The offshore oil and gas industry commonly operates in deep water, beyond the continental shelf, where infrastructure is vulnerable to a number of geohazards including submarine landslides, mud and volcanoes, seismicity, shallow gas and gas hydrates (Kvalstad et al., 2001). One of the most significant geohazards on the continental slope is the threat of submarine landslides, which typically originate from the shelf-break but may run out several kilometres into development zones or across pipeline routes. It is therefore necessary to consider both normal and axial responses of pipelines impacted by debris flow, although the two modes of response can be decoupled, at least as a first stage analysis (Randolph et al., 2010). Attention here is focused on the axial response.

The most critical aspect of axial drag is the potential for the pipeline to buckle due to compressive loading. However, in order to make preliminary estimates of displacements and forces along the pipeline prior to buckling, it is sufficient to consider purely elastic response of the pipeline, together with elastic perfectly plastic interaction with the seabed. The axial drag resulting from the submarine slide may be considered as a uniform traction applied to the pipeline over a defined zone.

These simple assumptions allow the development of parametric solutions to the problem that contain only three non-dimensional quantities. The technical note documents the solutions and illustrates their application for some typical input conditions. As an aside, it may also be noted that many of the underlying relationships presented here may also be applied to related pipeline problems, such as thermal expansion and contraction.

PROBLEM DEFINITION

The submarine slide-pipeline-seabed interaction problem may be divided into three parts: active slide zone, passive plastic zone and elastic zone as shown in Figure 1. Within the slide zone,
the axial drag, $F_{\text{slide}}$, is assumed to overcome the ‘passive’ seabed resistance, resulting in a net
traction of $F_{\text{net}}$ applied over the width of the submarine impact zone. Nominaly this may be
considered as the difference between the slide loading and the passive seabed resistance,
although in practice the latter may be modified, and even eliminated, within the slide zone.
Beyond that zone, the seabed provides either an axial load-transfer stiffness (in the far-field
‘elastic’ zone) or a limiting passive resistance $F_{\text{passive}}$ within the intermediate ‘passive’ zone.
Key axial tractions within each zone, and the loads and displacements at the interface points
between zones, are indicated in the schematic. The response in each zone is solved analytically
for the relevant boundary conditions in the following sections.

**Input parameters and dimensionless groups**

The perfectly straight pipe is defined by diameter, $D$, wall thickness, $t$ submerged unit weight,
$W'$, and Young’s modulus, $E$, from which the axial rigidity $EA$ can be calculated. The slide is
defined as a block zone of length, $L_{\text{slide}}$; from symmetry of the problem, only the half slide
length, $L_{\text{AB}}$ is considered here, with the axial force $P_A$ in the pipeline at the centre of the slide
zone taken as zero. Any existing axial force distribution in the pipeline is ignored here, although
it would be relatively straightforward to extend the solutions presented to allow for that. The
length of the ‘passive plastic zone’ is $L_{\text{BC}}$, beyond which point (C onwards) the pipeline-seabed
interaction is elastic (Figure 1). The displacement at the centre of the slide, $A$, is $u_A$, at the
interface of ‘active’ and ‘passive’ zones, $B$, is $u_B$ and at the interface between ‘passive’ and
‘elastic’ zones, $C$ is $u_C$. The axial load generated within the pipeline due to the slide movement
along the length is defined as $P$. The loads take values of $P_A$, $P_B$ and $P_C$ at the points
corresponding with $u_A$, $u_B$ and $u_C$.

The axial load transfer stiffness between pipeline and seabed has been considered by Guha et
al. (2016). For a partially embedded pipeline contacting the seabed over chord width $D'$
(i.e. with $0 < D' \leq D$), the elastic load transfer stiffness may be approximated as
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where $G_D'$ is the seabed shear modulus at a depth of the pipeline-seabed contact width $D'$. More detailed expressions, allowing for the pipeline embedment and exact profile of the seabed shear modulus, are provided by Guha et al. (2016) and extend over a range of ±20% relative to the above approximation. However, given the inevitable uncertainty in estimating shear modulus values at very shallow depth, Equation (1) is considered sufficient.

The output quantities may be non-dimensionalised and expressed in terms of various input properties. The maximum axial load, $P_B$, may be normalised by the axial elastic stiffness of the pipe, $EA$, and presented as compressive strain, $\varepsilon = P/EA$; the axial displacement, $u$, may be normalised by the slide length, $L_{slide}$, as $u/L_{slide}$. These normalised output parameters may then be expressed in terms of normalised input parameters, i.e. the driving force, $a_1 = F_{net}L_{slide}/EA$; passive resistance, $a_2 = F_{passive}L_{slide}/EA$; and pipe-soil stiffness, $a_3 = k_sL_{slide}^2/EA$. These three groups can be shown to be sufficient to determine the longitudinal profile of load and displacement of the pipe non-dimensionally.
Table 1 summarises the problem variables, together with relevant ranges for each that are considered later. The range for the pile-soil axial stiffness $k_x$ is quite large, reflecting conditions from a small (0.1 m) diameter pipe half embedded in a soft clay with shear modulus of perhaps 500 kPa, to a large (1 m) diameter pipe shallowly embedded in dense sand with $G_D \sim 10$ MPa.

From a practical point of view, very high combinations of the net force and length of slide impact will lead to buckling of the pipeline, which is outside the scope of the solutions presented here (see Guha, 2020), or at least localised plastic yield. Since the maximum normalised force induced in the pipeline (i.e. average axial strain in the pipe) is, by inspection, $a_i/2$, an upper limit of the normalised slide force is about 0.004 for elastic conditions to be maintained, and rather less than that once buckling is considered.
ANALYTICAL SOLUTION

Elastic zone

The axial load generated in the pipe due to the presence of frictional resistance of the seabed is

$$\frac{dP}{dx} = -F$$

(2)

The compressive strain, $\varepsilon_x$, of the pipeline (assumed elastic) is written in terms of the load, $P$, transmitted by the pipe at any length $x$,

$$\varepsilon = -\frac{du}{dx} = -\frac{P}{EA}$$

(3)

Differentiating equation (3) and using equations (1) and (2) gives

$$\frac{d^2u}{dx^2} = \frac{F}{EA} \frac{k}{EA} - u$$

(4)

The solution of this equation is

$$u(x) = C_2e^{ix} + C_2e^{-ix}$$

(5)

where $\lambda = \sqrt{k/EA}$ is the inverse of a characteristic length with dimensions m\(^{-1}\). To satisfy the boundary conditions of zero displacement at large $x$, and $u = u_C$ at $x = x_C$, the displacement variation within the elastic zone be expressed in terms of the displacement at the passive-plastic and elastic zone interface by:

$$u(x) = u_C e^{-\lambda(x-x_C)}$$

(6)

The profile of load in the pipe may then be obtained by substituting equation (6) into equation (3) and integrating to yield:
\[ P = \frac{k_x}{\lambda} u_c e^{-\lambda(x-x_c)} \]  

(7)

from which

\[ P_C = \sqrt{k_x E A u_c} \]  

(8)

In non-dimensional form, this may be written as

\[ \frac{P_C}{E A} = \sqrt{\frac{k_x L_{\text{slide}}^2}{E A}} \frac{u_c}{L_{\text{slide}}} = \sqrt{a_3} \frac{u_c}{L_{\text{slide}}} \]  

(9)

In principle, point C represents the interface between passive plastic and elastic zones (see Figure 1), although if the active slide force is small the passive plastic zone may disappear. An upper limit for the displacement at C is

\[ u_{c-\text{slip}} = \frac{F_{\text{passive}}}{k_x} \frac{u_{c-\text{slip}}}{L_{\text{slide}}} = \frac{F_{\text{passive}} L_{\text{slide}}}{E A} \frac{E A}{k_x L_{\text{slide}}} = \frac{a_3}{a_3} \]  

(10)

Substituting this into equation (9) gives the maximum load at the boundary of the elastic zone, for the long pipe considered here, as

\[ P_{c,\text{max}} = F_{\text{passive}} \sqrt{\frac{E A}{k_x}} \]  

hence

\[ P_{c,\text{max}} = \frac{a_3}{E A} \sqrt{a_3} \]  

(11)

**Passive plastic zone**

In general, there will be a passive plastic zone between the active slide zone and the elastic zone, where slip occurs between the seabed and the pipe and the resistance force per unit length is \( F_{\text{passive}} \). The governing equations of the plastic zone are similar to those for the elastic zone, but with \( F = F_{\text{passive}} \) in equation (2). This results in a linear increase in force in the pipe between points C and B, with
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\[ P_B = P_C + F_{\text{passive}}L_{BC} \]

hence

\[ \frac{P_B}{EA} = \frac{P_C}{EA} + a_2 \frac{L_{BC}}{L_{\text{slide}}} \]  \hspace{1cm} (12)

This may be used to determine the length of the passive zone, \( L_{BC} \) as

\[ \frac{L_{BC}}{L_{\text{slide}}} = \frac{1}{a_2} \left( \frac{P_B}{EA} - \frac{P_C}{EA} \right) \]  \hspace{1cm} (13)

When the passive zone \( L_{BC} = 0 \), point \( B \) coincides with point \( C \) leading to \( P_B = P_C \). In general, though, we may write \( P_B \geq P_C \) and \( L_{BC} \geq 0 \).

Integration of equation (3), allowing for the linear variation of \( P \) between \( B \) and \( C \), yields

\[ \frac{u_B}{L_{\text{slide}}} - \frac{u_C}{L_{\text{slide}}} = \frac{(P_B + P_C) L_{BC}}{2EA} \frac{L_{BC}}{L_{\text{slide}}} = \frac{1}{2} \left[ \left( \frac{P_B}{EA} \right)^2 - \left( \frac{P_C}{EA} \right)^2 \right] \]  \hspace{1cm} (14)

For a small active slide load (or strong passive resistance), \( u_C \) may not reach the elastic limit of \( u_{C\text{-slip}} \), in which case \( L_{BC} = 0 \), \( P_B = P_C \) and \( u_B = u_C \).

Active zone

In the active zone the interaction between the pipe and the soil is assumed to be plastic. The displacement is taken as \( u_A \) at the centre of the slide (\( x = 0 \)) from symmetry. Similarly, the axial force \( P_A \) in the pipe is zero at \( x = 0 \), and increases linearly to \( P_B \) at the edge of the slide material, where

\[ P_B = \frac{F_{\text{net}L_{\text{slide}}}}{2} \]

hence

\[ \frac{P_B}{EA} = \frac{F_{\text{net}L_{\text{slide}}}}{2EA} = \frac{a_2}{2} \]  \hspace{1cm} (15)

Note that \( P_B \) represents the largest axial force generated in the pipeline, and hence the maximum compressive strain in the pipe is \( \varepsilon_{\text{max}} = a_{1/2} \).

Integrating equation (3), for the linear variation of \( P \) from zero at \( A \) to \( a_{1/2} \) at \( B \), yields
\[ u_A - u_B = \frac{P_B L_{slide}}{4EA} = \frac{F_{net} L_{slide}^2}{8EA} \]

hence
\[ \frac{u_A}{L_{slide}} = \frac{u_B}{L_{slide}} + \frac{a_1}{8} \quad (16) \]

**Summary of solution**

For convenience the main expressions are summarized here in non-dimensional form. The key loads may be expressed as

\[ \frac{P_A}{EA} = 0; \quad \frac{P_B}{EA} = \frac{a_1}{2}; \quad \frac{P_C}{EA} = \text{Min}\left(\frac{a_1}{2}, \frac{a_2}{\sqrt{a_3}}\right) \quad (17) \]

The length of the (plastic) passive zone is given by

\[ \frac{L_{BC}}{L_{slide}} = \text{Max}\left(0, \frac{a_1}{2a_2} - \frac{1}{\sqrt{a_3}}\right) \quad (18) \]

The displacements at key points are

\[ \frac{u_A}{L_{slide}} = \frac{a_1}{8} + \text{Max}\left(\frac{a_1}{2\sqrt{a_3}}, \frac{1}{8} \frac{a_1^2}{a_2}, \frac{1}{2} \frac{a_2}{a_3}\right) \]

\[ \frac{u_B}{L_{slide}} = \text{Max}\left(\frac{a_1}{2\sqrt{a_3}}, \frac{1}{8} \frac{a_1^2}{a_2}, \frac{1}{2} \frac{a_2}{a_3}\right) \quad (19) \]

\[ \frac{u_C}{L_{slide}} = \text{Min}\left(\frac{1}{2} \frac{a_1}{\sqrt{a_3}}, \frac{a_2}{a_3}\right) \]

These relationships are illustrated in Figure 2, which shows the length of the plastic zone \( L_{BC} \) as a function of the normalised net slide force \( a_1 = F_{net} L_{slide}/EA \), and Figure 3, which shows corresponding key displacement ratios. As might be expected intuitively, the length of the plastic zone grows proportionally with the ratio of driving to resisting force \( (F_{net}/F_{passive}) \), with almost no influence of the elastic stiffness ratio \( a_3 = kL_{slide}^2/EA \) apart from at very low ratios of \( F_{net}/F_{passive} \).
In a similar vein, the magnitude of displacements $u_A$ and $u_B$, both normalised by $L_{slide}$, grow proportionally with the ratio of driving to resisting force, except where that ratio falls below unity. Once $F_{net}/F_{passive}$ reduces below unity, the maximum displacement at the mid-point of the slide ($u_A$) asymptotes to a plateau that corresponds to the pipe compression within the slide zone, essentially half the ratio $0.5a_2/a_3$, as the displacement at $B$ reduces towards zero. In most cases the displacement at interface between passive and elastic zones ($u_C$) is negligible.

**EXAMPLE NUMERICAL SOLUTION**

As a check on the analytical solution, and to explore the effect of different slide loading on a given pipeline, three example cases are considered here, with results compared with those obtained from finite element analysis (Guha, 2020). The three cases were for a 1 m diameter pipeline with $D/t$ of 25, subjected to slide loading of 11.9 kN/m over slide lengths of 100, 300 and 500 m, beyond the seabed passive resistance is 3.8 kN/m. The input data and corresponding normalised parameters are summarised in Table 2.

Figure 4 shows the resulting profiles of (a) axial force, and (b) axial displacement along the pipeline for the three cases. Note the axial displacements have been factored up by 1000. Corresponding displacements from finite element analyses (Guha, 2020) are shown for comparison. The zones of slide loading, plastic passive resistance and elastic resistance are colour coded, respectively blue, red and green. The finite element data confirm the accuracy of the analytical solution. Overall, the results also show that, provided the pipe does not fail through plasticity or buckling, the axial displacements remain rather small, varying quadratically with the magnitude of total slide load ($F_{net}L_{slide}$) and, for these cases, ranging between 2.7 mm and 62 mm.
CONCLUDING REMARKS

This technical note has documented a simple analytical solution to the distribution of axial force, strain and displacements in a pipeline loaded axially by a debris flow. The solutions facilitate simple calculation of the potential for failure of a pipe due to plastic strains or (in a broader context not considered here) by lateral buckling.

DATA AVAILABILITY STATEMENT

All data, models, and code generated or used during the study appear in the submitted article.

ACKNOWLEDGEMENTS

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REFERENCES


Table 1: Summary of range of input and output parameters

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipeline diameter, $D$</td>
<td>0.1 - 1</td>
<td>m</td>
</tr>
<tr>
<td>Pipeline diameter to wall thickness ratio, $D/t$</td>
<td>13 - 20</td>
<td></td>
</tr>
<tr>
<td>Elastic modulus of pipeline, $E$</td>
<td>210</td>
<td>GPa</td>
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<tr>
<td>Length of slide loading on pipeline, $L_{slide}$</td>
<td>50 – 1000</td>
<td>m</td>
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<tr>
<td>Net slide force on pipeline, $F_{net}$</td>
<td>0.1 – 10</td>
<td>kN/m</td>
</tr>
<tr>
<td>Passive seabed frictional resistance force, $F_{passive}$</td>
<td>0.02 – 10</td>
<td>kN/m</td>
</tr>
<tr>
<td>Pipe-soil elastic axial stiffness, $k_x$</td>
<td>50 – 10,000</td>
<td>kPa</td>
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</table>

Adopted range of non-dimensional input parameters

<table>
<thead>
<tr>
<th>Normalised slide loading, $a_1$</th>
<th>$F_{net}L_{slide}/EA$</th>
<th>0.000001 – 0.01</th>
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</thead>
<tbody>
<tr>
<td>Normalised passive resistance, $a_2$</td>
<td>$F_{passive}L_{slide}/EA$</td>
<td>0.000001 – 0.01</td>
</tr>
<tr>
<td>Normalised pipe-soil elastic stiffness, $a_3$</td>
<td>$k_xL_{slide}^2/EA$</td>
<td>0.01 - 10000</td>
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</table>

Non-dimensional output quantities

<table>
<thead>
<tr>
<th>Axial loads</th>
<th>$P_B/EA, P_C/EA$</th>
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<tbody>
<tr>
<td>Length of passive zone</td>
<td>$L_{BC}/L_{slide}$</td>
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<tr>
<td>Displacements</td>
<td>$u_A/L_{slide}, u_B/L_{slide}, u_C/L_{slide}$</td>
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Table 2: Input parameters for numerical examples

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 2</th>
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<tr>
<td>Length of slide, $L_{slide}$ (m)</td>
<td>100</td>
<td>300</td>
<td>500</td>
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<td>11.9</td>
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<td>Pipe-soil elastic axial stiffness, $k_x$ (kPa)</td>
<td>6400</td>
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<td>0.000047</td>
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<td>Normalised passive resistance, $a_2$</td>
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<td>Normalised pipe-soil elastic stiffness, $a_3$</td>
<td>2.5</td>
<td>24.9</td>
<td>78.9</td>
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</tbody>
</table>
198 **Figure Captions**

199 Figure 1 Idealisation of axial slide pipeline interaction

200 Figure 2 Length of passive zone, $L_{BC}$

201 Figure 3 Normalised axial displacements of pipe

202 Figure 4 Results of three example cases
Figure 1 Idealisation of axial slide pipeline interaction
Figure 2 Length of passive zone, $L_{BC}$

solid lines: $a_3 = k_L \frac{L_{slide}^2}{EA} = 10$

Dashed lines: $a_3 = k_L \frac{L_{slide}^2}{EA} = 1000$

$a_2 = F_{passive} \frac{L_{slide}}{EA}$:

0.000001

0.00001

0.0001

0.001

0.01

0.1

1

10

100

1000

10000

Non-dimensional slide force, $F_{net} \frac{L_{slide}}{EA}$

Length of passive region, $L_{BC}/L_{slide}$
Figure 3 Normalised axial displacements of pipe

\[ a_3 = k_x L_{\text{slide}}^2 / EA = 10 \]

solid lines: \( u_A \)
dashed lines: \( u_B \)
chain dotted lines: \( u_C \)

\[ a_2 = F_{\text{passive}} L_{\text{slide}} / EA : \]

\[ a_2 = F_{\text{net}} L_{\text{slide}} / EA : \]
Figure 4 Results of three example cases

(a) Distribution of axial strain along the pipeline

Axial compressive strain, \( e = \frac{P}{EA} \) (%)

Distance along pipeline from slide centre, \( x/D \)

Case 1: \( L_{slide}/D = 100 \)
Case 2: \( L_{slide}/D = 300 \)
Case 3: \( L_{slide}/D = 500 \)

Colours (left to right): Blue - slide
Red (dashed) - passive
Green - elastic

(b) Distribution of axial displacements along the pipeline

Normalised axial displacement, \( \frac{1000 \times u}{L_{slide}} \)

Distance along pipeline from slide centre, \( x/D \)

Finite element results (Guha, 2020)