**A stochastic model for the speed of leak noise propagation in plastic water pipes**

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**Highlights**

* Stochastic model of the speed of leak noise propagation in plastic water pipes is derived.
* Uncertainties in the pipe material/geometry and soil properties are modelled and their influence on the speed of leak noise propagation is investigated.
* Properties of statistical moments are used to derive the confidence intervals for the variance of the wave speed.
* The proposed approach is applied to data from in-air and buried plastic water pipes.

**Abstract**

A good estimate of the speed of leak noise propagation is necessary to pinpoint the location of a leak using acoustic correlators. Models currently exist for this purpose, but they do not consider uncertainties in the pipe geometry and the properties of the pipe and soil. Using the fact that leak noise propagates as a predominantly fluid-borne wave, this paper develops a stochastic model of the speed of leak noise propagation in plastic water distribution pipes that can account for these uncertainties. The model provides confidence limits for the estimate of the wave speed related to the leak noise excitation. It is based on the mean and standard deviation of the pipe geometry as well as the pipe and soil material properties, which have strong influence on the speed in which the leak noise propagates in the pipe. Numerical examples, using parameters from water supply systems found in the field, in which the pipe is made from Medium-Density Polyethylene (MDPE) and Polyvinyl Chloride (PVC) are presented to validate the model. Monte Carlo simulations for both in-air and buried pipes are presented to check the 99.7% confidence interval. To verify that the predictions from the stochastic models give realistic results, they are compared with some measurements from different sites, in which nominal properties and tolerances for the pipe and soil properties are assumed.

1. Introduction

Pipelines are widely used for many purposes, such as transporting gases, oil and fluids, including potable water distribution [1]. Detection and location of leaks in such pipes has been of interest for many years, in particular for buried water pipes [2], as water has become an increasingly scarce resource, and there are regions of increasing population densities, especially in the so-called megacities.

Motivated by these factors, researchers have developed methods to detect and locate leaks in water distribution networks [3,4]. Among these, vibro-acoustic techniques have proven to be effective and are commonly used nowadays [5-6]. One of these techniques uses leak noise correlators, where vibration sensors are placed on convenient pipe fittings either side of the suspected leak position [7]. The difference in the arrival times (time delay) of the leak noise at the measurement positions is estimated using the peak in the cross-correlation function between two measured signals. This time delay is then used together with an estimate of the speed of the leak noise propagation to determine the location of the leak [7].

The speed in which the leak noise propagates in the pipe has been the study of much research, for example [8-9]. It has been shown that in plastic pipes, it propagates as a predominantly fluid-borne wave. Models that have been developed to predict this wave speed have hitherto been deterministic and do not account for uncertainties in the model parameters relating to the pipe and the surrounding soil. To account for variability in system parameters, a stochastic model is needed [10]. Modelling systems which have uncertain parameters involves methods that are generally categorized as probabilistic and non-probabilistic. Non-probabilistic methods include interval analysis [11], an anti-optimization technique [12], and fuzzy set theory [13]. These methods are used when the statistics of uncertain parameters are not known, but their limits are predefined. When the statistical properties are assumed, probabilistic methods such as Monte Carlo simulation [14], first and second order reliability methods [15], numerical integration based methods [16], spectral methods [17] and statistical moments based analytical methods [18,19] are employed. Among these methods, the Monte Carlo simulation is the most preferred technique due to its simple application. However, since it requires numerous simulations to determine the response statistics it can be time consuming.

To the authors’ knowledge a stochastic model for the speed of leak noise propagation in plastic water pipes has not been published to date. This motivates the work described in this paper, which aims to fill this gap in knowledge and to provide a useful add-on that can be incorporated into leak noise correlators. Analytical expressions are derived for the mean and standard deviation of the predominantly fluid-wave speed for in-air and buried plastic water pipes. To achieve this, several properties of statistical moments are used, which are summarised in the Appendix. Confidence intervals for the estimate of the wave speed are provided, which are a function of the pipe geometry and the pipe and soil material properties. The confidence intervals are validated for in-air and buried pipes made from Medium-Density Polyethylene (MDPE) and Polyvinyl Chloride (PVC) using Monte Carlo simulations. Further, to verify that the predictions from the stochastic models give realistic results, they are compared with some measurements from different sites, in which nominal properties and tolerances for the pipe and soil properties are assumed.

The paper is organised as follows. Following this introduction, Section 2 reviews the way in which a leak is located using acoustic correlation. Stochastic models of the predominantly fluid-borne wave speed for in-air pipes and buried pipes are given in Sections 3 and 4 respectively, where the theoretical development is supported by numerical simulations and experimental work. Finally, some conclusions are then given in Section 5.

2. An overview of leak detection using acoustic correlation

This section gives an overview of the way in which leaks are detected using acoustic correlators, and shows why it is important to have an accurate estimate of the speed at which leak noise propagates in the pipe. It is further shown how the speed at which the leak noise propagates is dependent on the physical properties of the pipe and its surrounding medium.

**2.1. Pin-pointing the location of a leak**

Figure 1 shows a typical situation in which leak noise is used to locate the leak position using the correlation of measured leak noise from vibration or acoustic sensors attached to access points either side of the suspected leak position.

Figure 1. Schematic diagram of a typical leak detection problem in a buried plastic water pipe via the cross-correlation technique by using acoustic or vibration measurements.

The leak position from the right-hand sensor shown in Fig.1 is given by [7]



 (1)

where is the speed of propagation of the leak noise,  is the total distance between the sensors, and  is the time delay corresponding to the difference in arrival times of the leak noise at the sensor positions. Leak noise in plastic pipes is propagated in the form of a predominantly fluid-borne wave that is strongly coupled to the radial motion of the pipe-wall [8,9], which is discussed in the next sub-section. The cross-correlation function , between the two measured signals  and  which is used to estimate the time delay, is given by [7]



 (2)

where  is the inverse Fourier transform,  is the cross-spectral density function between the measured signals  and ,  is the circular frequency and . The time delay estimate between the measured signals is given by a distinct peak in the correlation function. A good estimate of the wave speed, which depends on the geometry and material properties of the pipe, is necessary for an accurate estimate of the leak position [20]. It is often estimated from tables which are compiled from calculations or from a historical database. If there is an error in the wave speed estimate, the resulting error in the leak location is given by [20]

 (3)

where  and , in which  and  are the estimated location of the leak, and the wave speed, respectively. It is evident that if the leak is half-way between the measurement positions, the time delay is zero, and the wave speed estimate has no effect on the estimate of the leak location. However, if the leak position is close to one of the measurement points it is important to have an accurate wave speed estimate.

**2.2. Leak noise propagation in water-filled plastic pipes**

Figure 2 shows the geometry of a typical water pipe. It has a mean radius of *a* and pipe-wall thickness *h*. The mechanism of leak noise propagation in such a pipe has been studied extensively because of its role in the detection of leaks as discussed in the previous section. For plastic pipes, the leak noise propagates in the predominantly fluid-borne wave that is well-coupled to the pipe-wall [9].

Figure 2. Schematic of the pipe showing the pipe geometry.

For frequencies well below the ring frequency of the pipe, the wavenumber for this wave is given by [9]

 (4)

where  is the free-field wavenumber of water, in which  is angular frequency and  is the wave speed in water;  is the stiffness of the water inside the pipe, in which  is the bulk modulus of water;  is the dynamic stiffness of the pipe-wall, where, , and   and  are the density, Young’s modulus and loss factor of the pipe respectively;  is the dynamic stiffness of the surrounding medium. The real part of the wavenumber is related to the pipe wave speed, in that , and the imaginary part of the wavenumber is related to the wave attenuation in the pipe, because the attenuation per unit distance along the pipe is equal to . Of interest in this paper is the wave speed, which is given by

 (5)

In the following sections, stochastic versions of the wave speed are derived to provide confidence intervals, which can be used for leak location. Two cases are then studied. The first, in Section 3, is the in-air case to study the effects of the pipe-wall and geometry when the surrounding medium is air, and the second, in Section 4, is the when the surrounding medium is soil.

**3. Stochastic model for wave speed estimation for in-air water-filled plastic pipes**

**3.1 Theory**

For an in-airpipe the surrounding medium is considered to be a vacuum, so that . Further, as the pipe loss factor only has a negligibly small effect on the wave speed, the normalised wave speed in this case can be approximated by

 (6)

Although Eq. (6) is useful in that it gives an estimate of the wave speed, it does not facilitate a prediction of how the wave speed varies if there is tolerance in the pipe parameters. A stochastic model is required, in which the system parameters are random variables, each described by a mean and a standard deviation. As it is unlikely that the bulk modulus of water changes appreciably from place to place it is not considered to be a random variable. Accordingly, the structural parameters *E*, *a*, *h* and  are considered to be Gaussian random variables. The mean of the normalised wave speed is given by

 (7)

where the over bar denotes the mean. The variance of the wave speed, which is the square of the standard deviation , is given by

 (8)

Setting  where , and applying the rules for the manipulation of variances given in the Appendix, results in . This can be written in terms of the means and variances of *E*, *a*, *h* and , and after straightforward but tedious algebra results in

 (9a)

where  and . If the analysis is restricted to small variations in the system parameters, such that , Eq. (9a) can be approximated to

 (9b)

If the frequency range is restricted such that , then Eq. (9b) can be further simplified to

 (9c)

The standard deviation of the wave speed can be determined by taking the square root of Eq. (9c), rearranging, and noting that  is the mean stiffness of the pipe-wall and  is the mean stiffness of the water, to give

 (10)

One further approximation is possible for some plastic pipes in which , then Eq. (10) simplifies to .

**3.2 Simulations**

To check the accuracy of Eq. (9a) and the validity of the approximations leading to Eq. (10), predictions using these equations are compared with Monte Carlo simulations for two materials commonly used in water distribution networks, which are Medium-Density Polyethylene (MDPE) and Polyvinyl Chloride (PVC). The pipes used in the experimental work described in the next sub-section are also made from these materials, and their nominal properties and dimensions are given in Tab. 1 [21,22].

Table 1. Pipe, water and soil properties used in the numerical simulations and the experimental work. The overbar denotes the mean value.

Monte Carlo simulations are shown in Figs. 3(ai) and (bi) for the MDPE pipe and the PVC pipe respectively together with the model predictions. The Monte Carlo simulations were performed for 500 realisations of the model described by Eq. (6) in which *E*, *a*, *h* and  were considered to have the mean values given in Tab. 1. It was assumed that *a* and  had standard deviations of 1% of the mean values, and *E* and *h* had standard deviations of 5% and 10% of their mean values respectively. The results for the wave-speed were calculated over the frequency range 0-1000 Hz with step of 1 Hz. The values were chosen conforming standard specifications reported in ASTM standard, ASTM D1785-12 [23]. The mean  and the 99.7% confidence interval  are shown. The grey area in Figs. 3(ai) and (bi) are from the Monte Carlo simulations. Note that the reduction of the wave speed with frequency, due to the inertial effects of the pipe, is more evident in Fig. 3(ai). For the MDPE pipe it ranges from 351 m/s to 329 m/s (a reduction of approximately 6.3%) whereas for the PVC pipe it ranges from 402 m/s to 400 m/s (a reduction of 0.5%) over the frequency range 0-1000 Hz. This smaller deviation is because the PVC pipe has a much smaller pipe-wall and higher density compared to the MDPE pipe. Furthermore, the standard deviations given by Eq. (10) are 18.6 m/s and 21 m/s for the MDPE and PVC pipes respectively, which are about 5.3% and 5.2% of the mean values at zero frequency respectively. It can be seen from the simulations that Eq. (10), which neglects the inertial effects of the pipe, gives a good approximation to the standard deviation for the pipe parameters used.

Figure 3. Predicted mean wave speed and ranges of values with confidence limits of 68%, 95% and 99,7% for the in-air pipe. (i) wave speed, (ii) normalised distribution function at low frequency; (a) MDPE pipe and (b) PVC pipe. Dotted-dashed blue line () is the mean wave speed from Eq. (7), dotted red line () denotes the confidence levels, continuous black line () and dashed yellow line () are the standard deviations calculated using Eq. (9a) and the approximation given in Eq. (10) respectively; Grey area: Monte Carlo simulations.

The probability density functions (PDFs) given by are plotted in Figs. 3(aii) and (bii) for the MDPE and PVC pipes respectively, using Eq. (7) with the inertial effects neglected (i.e., at zero frequency) together with Eqs. 9(a) and (10). Also plotted are the Monte Carlo simulations as grey areas. Again, it can be seen that Eq. (10) is a good approximation for the standard deviation of the wave speed for the frequency range shown. For reference, on these graphs, lines for  which correspond to confidence intervals in which 68%, 95% and 99.7% of all wave speed estimates are likely to occur, are also shown.

Thus, Eq. (7) can be used together with Eq. (10) to give approximate confidence intervals for wave speed estimate. In [7] it was shown that the variance of the time delay estimate is very small in practice, therefore approximate confidence intervals for the position of the leak in an in-airwater pipe can be determined by combining Eqs. (1), (7) and (10).

**3.3 Experimental work**

In this sub-section, the approach described above is examined in relation to two experimental test rigs, one in the UK which has an MDPE pipe, and the other one in Brazil which has a PVC pipe. The aim is to see if the measured wave speed is within the confidence interval predicted using the stochastic model in the previous sub-section using the nominal values given by Tab. 1. Photographs of both test rigs, the wave excitation mechanism and the schematic diagrams are given in Fig. 4.

Figure 4. Experimental arrangements for measurements made on in-air plastic water pipes: (a) Photograph illustrating each test rig, (b) Leak noise excitation mechanism and (c) Schematic diagram (rotated 90 degrees) for the UK (i) and Brazilian (ii) test rigs (not to scale).

The UK test rig has been described in [8]. It consists of a 2 m long water-filled MDPE pipe arranged vertically in the laboratory as shown in Fig. 4(ai). The water was excited at the upper end by an electrodynamic shaker attached to a light, rigid piston depicted in Fig. 4(bi). The mean radius of the pipe is 84.5 mm and the wall thickness is 11 mm. The centre section of the pipe was instrumented with four calibrated PVDF wire ring transducers, which are shown in Fig. 4(bi) and the schematic diagram depicted in Fig. 4(ci). The transducers indirectly measure the acoustic pressure inside the pipe by measuring the circumferential strain of the pipe-wall. Three of the transducers, spaced 0.5 m apart, were used to extract the wavenumber from measured data for the wave propagating directly from the piston, which was excited with a swept sine input from 30 Hz to 1 kHz, using the method described in [8]. More details on the acquisition system and parameters can be found in the reference [8].

The test rig in Brazil, consists of a 30 m long PVC pipe with mean radius and wall thickness of 28.4 mm and 3.3 mm, respectively. The pipe is laid on a bed of sand, and is otherwise open to the air as shown in Fig. 4(aii). The pipe pressurized by a water main, and the wave is generated by a leak from a 0.05 mm diameter hole as shown in the figure. Acceleration of the pipe-wall was measured using two Bruel & Kjaer type 4506B003 accelerometers and a LMS SCADAS acquisition system from SIEMENS, where time histories were recorded at a sampling frequency of 12.8 kHz for 60 s. One of the accelerometers was positioned next to the leak as shown in Fig. 4(bii) and the other was placed 3 metres away as shown in the schematic diagram shown in Fig. 4(cii). The wave speed was determined by calculating the time of flight from the cross-correlation between the two signals [9].

Figures 5(a) and 5(b) show the results for the UK pipe system and for the Brazilian pipe systems, respectively. Note that the frequency range in which leak noise was measured for the UK pipe system is from 0 to 1 kHz compared to 0 to 2 kHz for the Brazilian pipe system. The frequency range is much higher for the Brazilian pipe system because the pipe has a smaller diameter and has different material properties. The oscillations in the measured curves at frequencies below approximately 370 Hz for the UK system and 450 Hz for the Brazilian system are due to a variety of reasons. For the UK test rig, in which the PVDF sensors measure circumferential strain in the pipe-wall, which indirectly measures acoustic pressure in the water-filled pipe, the fluctuations in the estimated wave speed are thought to be associated with small errors in decomposing the outgoing and reflected waves in the pipe, as well as in the phase unwrapping. This is most probably due to the small distances between the sensors compared to a wavelength at low frequencies. The larger fluctuations at lower frequencies could possibly have been reduced with a greater distance between the sensors. For the Brazilian test rig, in which accelerometers measure radial vibration of the pipe-wall, which indirectly measures acoustic pressure in the water-filled pipe, there is low frequency attenuation of the signals related to the pressure. This means that there is poor signal to noise ratio and hence the fluctuations in the estimated wave speed are thought to be due to issues in phase unwrapping. Note that in the Brazilian test rig, there was no attempt to decompose the signals in terms of the outgoing and reflected waves in the pipe, so there may have been a component of the reflected waves in the measured signals, which could also have caused fluctuations in the estimate of the wave speed.

For the reasons stated above the wave speed cannot be estimated from the measured data at low frequencies. To emphasise that wave speed estimation is only possible above a certain frequency, Figures 5(a) and 5(b) each have shaded region identifying the low frequency range where wave speed estimation is not possible. It can be seen that at frequencies above this range, the measured wave speed falls into the 99.7% confidence intervals predicted by the stochastic model in the previous sub-section for both test sites.

A better fit can be achieved between the estimated mean wave speed and the measured one by varying the Young’s modulus of the pipe. In order to achieve this, some animations are provided as supplementary material illustrating the effects of the pipe material on the predicted stochastic wave speed. The range of values adopted here are GN/m² and GN/m² for the MDPE and PVC pipes respectively [21,22]. By using GN/m² and GN/m² for the UK and Brazilian pipe systems respectively, the analytical mean wave speed behaviour captures very well the overall trend of the measured curve.

Figure 5. Actual and estimated wave speed for the in-air pipes shown in Fig. 4, and the 99.7% confidence interval calculated using the proposed model (a) UK test rig and (b) Brazilian test rig. Continuous green thick line (): measured wave speed, dashed central blue line (): mean wave speed from Eq. (7); continuous black line (): 99.7% confidence interval calculated using the approximation given in Eq. (10). The grey shaded area denotes the frequency range where wave speed estimation is not possible. Animations illustrating the results for different values of the Young`s modulus of the pipe are provided as supplementary material.

**4. Stochastic model for wave speed estimation in buried water-filled plastic pipes**

**4.1 Theory**

In this sub-section, an analytical expression for the standard deviation of wave speed in buried pipes is derived. As before the loss factor of the pipe material is neglected. Further, because only the wave speed is of interest the dynamic stiffness of the soil can be approximated to a simple expression only involving the shear modulus of the soil *G*, such that  [24]. Thus, the mean of the normalised wave speed is given by

 (11)

In the previous section, it was shown that the inertial effects of the pipe could be neglected without significantly affecting the accuracy of standard deviation of the wave speed estimate. Following the procedure given in the previous section and by neglecting inertial term, the variance of the wave speed is given by

 (12)

By applying the properties given in the Appendix, as done in the previous section, Eq. (12) becomes

 (13a)

where . Making the assumption that  results in

 (13b)

Note, that if  then Eq. (13b) reduces to the in-air case given in Eq. (10).

**4.2 Simulations**

To check the accuracy of Eq. (13a) and the validity of the approximations leading to Eq. (13b), predictions using these equations are compared with Monte Carlo simulations for two pipe systems. One system consists of a MDPE pipe buried in sandy soil, and the other system consists of a PVC pipe buried in clay soil. Note, from Tab. 1, that the material properties of these pipes are the same as those considered for the in-air pipes, but the geometry of the PVC pipe is different. The shear moduli of the soils are also given in Tab. 1 extracted from [25].

Using the properties in Tab. 1, the mean wave speed and the corresponding stochastic bounds were calculated using Eqs. (11) and Eq. (13a,b) over the frequency range 0-1000 Hz. The Monte Carlo simulations were performed in a similar way to that described in Section 3.2 for the in-air case. The same parameters were used, but with the inclusion of the shear modulus of the soil given in Tab. 1. The pipe systems correspond to a MPDE pipe buried in sandy soil and a PVC pipe buried in clay soil. These were chosen as they represent the cases considered experimentally, which are described in the next Sub-section. The results are shown in Figs. 6(ai) and (bi). The grey areas correspond to the Monte Carlo simulations. It can be seen in Fig. 6(ai) that the mean of the wave speed reduces with frequency, and this is due to the inertial effects of the pipe as in the in-air case. For the sandy soil case it ranges from 375 m/s to 356 m/s (a reduction of approximately 5.1%) and for the clay soil case it ranges from 545 m/s to 543 m/s (a reduction of 0.4%) over the frequency range 0-1000 Hz. The standard deviations given by Eq. (13b) are 17.2 m/s and 17.7 m/s for the sandy soil and clay soil respectively, which are about 4.6% and 3.3% of the mean values at zero frequency respectively. It can be seen from the simulations that Eq. (13b), which neglects the inertial effects of the pipe, gives a good approximation to the standard deviation for the pipe parameters and soil parameter used.

Figure 6. Predicted mean wave speed and ranges of values with confidence limits of 68%, 95% and 99,7% for the buried pipes. (i) Stochastic wave speed, (ii) normalised distribution function at low frequency; (a) MDPE pipe buried in sandy soil and (b) PVC pipe buried in clay soil. Dotted-dashed blue line () is the mean wave speed from Eq. (11), dotted red line () denotes the confidence levels, continuous black line () and dashed yellow line () are the standard deviations estimated using Eq. 13(a) and the approximation given in Eq. 13(b) respectively; Grey area: Monte Carlo simulations.

The PDFs are plotted in Figs. 6(aii) and 6(bii) for the pipes buried in sandy and clay soil respectively, using Eq. (11) with the inertial effects neglected (i.e., at zero frequency) and Eqs. (13a,b). Also plotted are the Monte Carlo simulations as grey areas. Again, it can be seen that Eq. (13b) is a good approximation for the standard deviation of the wave speed. Thus, Eq. (11) can be used together with Eq. (13b) to give approximate confidence intervals for the wave speed estimate for buried water pipes. For reference, lines corresponding to 68%, 95% and 99.7% confidence intervals are also shown in Figs. 6(aii) and 6(bii).

**4.3 Experimental work**

The approach described above is examined in this sub-section for two test sites with very different pipe geometry and soil properties, one is in the UK where a MDPE pipe is buried in sandy soil, and one is Brazil where a PVC pipe is buried in clay soil. The aim is to verify if the measured wave speed is within the confidence interval predicted using the stochastic model with the theoretical predictions given in the previous sub-section. Photographs of both test rigs and the schematic diagrams are given in Fig. 7.

Figure 7: Experimental arrangements for measurements made on buried plastic water pipes. (a) Photograph illustrating each test rig, (b) Schematic diagram (not to scale) for the UK system (i) and Brazilian system (ii).

The UK system, located in East Anglia, has been described in detail in [8]. The test rig consists of a 32 metres long pipe, pressurised by 1.5 m head of water in the termination tanks which are located at each end of the pipe. A photograph and a schematic diagram are shown in Figs. 7(ai) and 7(bi) respectively. Rather than generating a predominantly fluid-borne wave with a leak, an underwater loudspeaker fitted at one end of the pipe was used. It was supplied with a stepped sine signal, via a power amplifier, increasing from 30 Hz to 1 kHz in 1 Hz increments. The dynamic pressure was measured using two hydrophones positioned 2 metres apart as shown in Fig. 7(bi). More details about the measurements were carried out and how the wave speed was calculated are given in [8].

The Brazilian system, located in São Paulo city, has been described in detail [9,25]. It consists of a closed-circuit system in which the pipe is much smaller than UK system. A photograph and a schematic diagram of part of the test rig are shown in Figs. 7(aii) and 7(bii) respectively.

In this test rig, the pipe was pressurised with a centrifugal pump (3.4 bar), and the predominantly fluid-borne wave was excited by opening a valve and the signals related to this leak noise wave were measured by using two PCB 333B30 piezoelectric accelerometers at two access points 7 m apart. The acquisition system used for data collection was the LMS SCADAS from SIEMENS and the time histories were recorded a sampling frequency of 12.8 kHz for 60 s. The wave speed was determined by calculating the time of flight from the cross-correlation function between the two signals [9, 26].

Figures. 8(a) and 8(b) show the results for the UK test site and for the Brazilian test site respectively. Also plotted are the predictions made using the model described in Section 4.1 and the system properties given in Tab. 1. As illustrated in Figs. 8(a) and (b), the wave speed in the UK system marginally decreases within the frequency range 0-600 Hz, whereas the wave speed in the Brazilian system does not vary as much. This is mainly due to the inertial effect of the pipe-wall as discussed in the previous Sub-section. The measured wave speed at zero frequency is approximately 375 m/s for the UK system and 540 for the Brazilian system. This difference is mainly due to the larger shear modulus of the clay soil compared to the sandy soil. As with the in-air case it is not possible to estimate the wave speed from the measured data at low frequencies for reasons similar to those given for the in-air case discussed in Section 3.3. However, in addition these pipes are buried, so there is the additional effect of the surrounding soil. In the model it is assumed that the soil is homogeneous and of infinite extent. However, this is a poor approximation when the wavelength in the soil is much larger than the depth of the buried pipe, because the wave that is radiated from the pipe and then reflects from the surface, is small when it returns to the pipe [9,25,26]. As the wavelength decreases with frequency, the model becomes a better representation of the physical system at higher frequencies. To emphasise that wave speed estimation is only possible above a certain frequency, Figures 8(a) and 8(b) each have a shaded region identifying the low frequency range where wave speed estimation is not possible. It can be seen that at frequencies above approximately 170 Hz for the UK system and 150 Hz for the Brazilian system, the measured wave speed falls into the 99.7% confidence intervals predicted by the stochastic model in the previous sub-section for both test sites.

Figure 8. Actual and estimated wave speed for the buried pipes shown in Fig. 7, and the 99.7% confidence interval calculated using the proposed model. (a) Test site at UEA in the UK and (b) Test site at São Paulo city in Brazil. Continuous green thick line (): measured wave speed, dashed central blue lines (): mean wave speed from Eq. (11); continuous black line (): confidence interval (99,7%) using the approximation given in Eq. (13b). The grey shaded area denotes the frequency range where wave speed estimation is not possible. Animations illustrating the results for different values of the shear modulus of soil are provided as supplementary material.

Note that a better fit can be achieved between the estimated mean wave speed and the measured one by varying the shear modulus of the soil. Some animations are provided in the supplementary material to illustrate this effect.

5. Conclusions

In this paper, a stochastic model for the speed in which leak noise wave propagates in plastic water pipes has been developed. In these pipes, leak noise propagates in a predominantly fluid-borne wave, and analytical expressions for the variance of the speed of this wave have been derived for both in-air and buried plastic water pipes. This was accomplished by applying the properties of statistical moments to an analytical model of the wave speed. The approach allows uncertainties in the pipe geometry as well as in the pipe and soil properties to be incorporated into the model to give confidence intervals for the wave speed estimate. The analytical expressions for the standard deviation of the wave speed for both in-air and buried plastic water pipes were validated by comparing the 99.7% confidence intervals with Monte Carlo simulations.

To check the veracity of the predictions from the stochastic models using nominal properties and tolerances for the pipe and soil properties, they were compared with some measurements from different sites. The results show that the model is able to give reasonably good confidence intervals on the wave speed estimates, and suggest that the model can be used to predict confidence intervals for pipe systems where there are different combinations of uncertainties in both geometry and material properties.

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**Appendix: Properties of statistical moments**

In the derivation of the stochastic model for the predominantly fluid-borne wave speeds on in-air and buried wave pipes, several properties of statistical moments are used. These can be found in [18,19] and are summarised in this appendix for ease of reference.

The variance of summation of two statistical variables  with two deterministic coefficients  is given by [27]

 (A.1)

where  denotes the variance, which is the square of the standard deviation , and  is the covariance between the statistical variables. The overbar denotes the expected (or mean) value of the variable. The variance of the product of two statistical variables is given as

 (A.2)

Two other useful properties are used. One is with respect to the variance of the inverse of a statistical variable given by [28]

, (A.3)

and the other is related to the square of a statistical variable, and is given by

 (A.4)

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