

Coordinating Electric Vehicle Flow Distribution and Charger Allocation by Joint Optimization

Abstract—A two-stage stochastic programming model is established to minimize EV’s expected total journey time under stochastic traffic conditions, by jointly optimizing the allocation of chargers and the distribution of EV flows. Based on sample average approximation, a feasible deterministic equivalent of the original stochastic problem is obtained. Then, a hybrid solution method, composing of a Tabu-based search and sequential quadratic programming (SQP), is proposed. The Tabu heuristic manages the charger allocation problem, where each solution candidate undergoes a second-stage EV flow optimization. SQP is applied to optimally distribute the EV flows, which is proved to be a convex problem. Extensive simulations are carried out using the eastern Massachusetts highway network. Results show that the proposed algorithm outperforms existing approaches by finding high-quality charger allocation and optimal flow distribution policies under different traffic conditions. Additionally, the two-stage model designates charging resource sufficiency by statistically estimating a lower bound for the number of chargers to allocate, which in practice helps to prevent over-investment on charging resources.

Index Terms—Electric vehicle (EV), traffic flow distribution, charger allocation, two-stage stochastic programming, convex optimization.

I. INTRODUCTION

ACCORDING to the “Global Electric Vehicle (EV) Outlook 2019” published by International Energy Agency [1], the projected EV fleet under the EV30@30 campaign [2], which aims for at least 30% new EV sales by 2030, has a global well-to-wheel (WTW) greenhouse gas (GHG) emission of less than 230 million tonnes of carbon-dioxide equivalent (Mt CO₂-eq). By contrast, the equivalent internal combustion engine (ICE) vehicle fleet would emit an enormous 770 Mt CO₂-eq. Such GHG emission reduction brought by transport electrification can potentially be even more significant, given a faster decarbonization of electricity generation using renewable energy.

Although the scale of electric mobility has been expanding rapidly (exceeding 5.1 million globally in 2018), there is still a long way to go towards the EV30@30 target. As for the prospective drivers of EV, what they care most about is the EV driving experience. Unfortunately, EV’s limited driving range and long re-charge time can result in the so-called “range anxiety”, indicating the EV drivers’ concern of running out of power before arriving at the destinations.

To alleviate the range anxiety among EV drivers and consequently promote a large-scale adoption of EVs, extensive research efforts have been made in recent years, focusing on the planning of EV’s en-route recharge activities [3], [4], [5], [6], [7], [8], [9], [10] and the design of charging infrastructure [11], [12], [13], [14], [15], [16], [17], [18]. For instance, [7] suggests efficient itineraries under time-dependent stochastic traffic condition for EVs used in logistics

industry; [8] considers a city-wide system that recommends the location and occasion of re-charge for plug-in electric taxi by coordinating with the charging infrastructure via an information center; [16] proposes to place charging stations, by capturing the competitive and strategic re-charge behaviors among EV drivers using game theory; in [17], the charging infrastructure and the coupled power distribution systems are planned collaboratively to not only minimize the joint cost but also maximize the utilization.

Despite that the aforementioned works have covered many practical conditions and constraints in their models, the planning of EV’s en-route re-charge activities and the allocation of charging resources were addressed independently. On one hand, charging resources are allocated according to some patterns of EV’s re-charge activities which are usually generated based on existing data sets [15], [16] or models [17], [18]. On the other hand, the EV’s en-route re-charge activities are planned with the support of specified charging infrastructure which may be designed by real-world data [7], [10] or random assignment [8], [9]. There is no doubt that these two problems are indeed closely related. An obvious issue of making independent decisions is that any changes in EV’s re-charging pattern or charging infrastructure can potentially lead to a different result. It might be feasible to adjust the planning strategy for EV’s en-route re-charging according to the actual condition, e.g. available charging resources, but not the other way around, because the deployment of charging infrastructure is more of an irreversible process.

Therefore, to fully empower an electrified transportation system, it is essential to coordinate the present deployment of charging infrastructure and subsequent planning of EV’s re-charge activities, which is also the focus of this work. The main contributions are summarized as follows:

- 1) A two-stage stochastic programming model is defined over a road network with stochastic traffic flow to characterize the coordination between EV flow distribution and charger allocation. The two-stage decisions can then jointly minimize EV’s expected total journey time to improve the driving experience.
- 2) A feasible deterministic equivalent of the original joint optimization problem is derived based on sample average approximation (SAA). The quality of the SAA estimator is evaluated by estimating the objective function bounds of the original problem.
- 3) The EV flow distribution problem is proved to be convex, and can thus be solved optimally and efficiently using sequential quadratic programming (SQP). Since the optimal solution becomes available, it outperforms other existing works, for example the heuristic methods, as used in [3].

4) A hybrid approach, incorporating a dedicated Tabu-based heuristic and SQP, is designed to solve the joint optimization problem. It not only outperforms existing ones by yielding a high-quality charger allocation and optimal EV flow distribution policies under different traffic conditions, but also facilitates the estimation of a lower bound for the number of chargers to allocate. In practice, such a lower bound helps to prevent the over-allocation of charging resources.

The rest of the paper is organized as follows. The two-stage stochastic programming problem is formulated in Sect. II, while the design of solution algorithms is introduced in Sect. III. In Sect. IV, simulation results for an illustrative example of the eastern Massachusetts highway network and discussions are presented. Finally, conclusions are given in Sect. V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Road Network with Stochastic Traffic Flow

A road network with charging infrastructure installed can be represented by a directed and weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. \mathcal{V} is the set of nodes (points of interest) including charging station locations denoted by $\mathcal{V}_c \subset \mathcal{V}$. $\mathcal{E} = \{(i, j) | i, j \in \mathcal{V}, i \neq j\}$ is the set of edges and $\mathcal{A} = \{(d_{ij}, \tau_{ij}^0, q_{ij}, \delta_{ij}) | (i, j) \in \mathcal{E}\}$ is the set of weights associated to the edges such that

- d_{ij} is the traveling length of the edge (i, j) (in km);
- τ_{ij}^0 denotes the free-flow traveling time of the edge (i, j) (in hours);
- q_{ij} stands for (i, j) flow capacity (in vehicles per hour);
- δ_{ij} is the proportion of q_{ij} being occupied by non-EVs (e.g. ICE vehicles) and those EVs that demand no en-route re-charging.

Here, δ_{ij} is assumed as a realization of a random variable Δ_{ij} which follows a normal distribution, i.e., $\Delta_{ij} \sim \mathcal{N}(\mu_{ij}, \sigma_{ij}^2)$. Based on the widely used Bureau of Public Roads (BPR) volume-delay function [19], the actual traveling time from i to j can be estimated by

$$\tau_{ij} = \tau_{ij}^0 \left(1 + \alpha \left[\delta_{ij} + \frac{\lambda_{ij}}{q_{ij}} \right]^\beta \right) \quad (1)$$

where λ_{ij} (in vehicles per hour) denotes the rate of EV flow at edge (i, j) that demands en-route re-charging; α and β are tuning parameters that control how traffic congestion affects the traveling time. Due to the existence of uncertainty δ_{ij} , the traveling time τ_{ij} characterized by (1) is essentially stochastic and should be taken into consideration when planning EV's en-route re-charge activities, i.e. λ_{ij} .

B. Distribution of EV Flows

En-route re-charging is necessary for EVs traveling from a source node m to a destination node n in the road network \mathcal{G} , if their driving range $d_r < d(m, n)$ where $d(m, n)$ denotes the shortest path length from m to n . This source-destination pair, denoted by $(m \mapsto n)$, is said to be unreachable to the EVs. In this case, the EVs are to be re-distributed to a set of charging stations $\mathcal{V}_s \subseteq \mathcal{V}_c$, which provide re-charging

services. Assuming the EV flow departing from node m (to n) follows a Poisson process with an intensity λ , such re-distribution of EV flow is essentially splitting the Poisson process to multiple Poisson processes [20]. An illustrative example is given in Fig. 1. With the re-charging services available at $\mathcal{V}_s = \{s_1, s_2, \dots, s_6\}$, the EV flow at rate λ can be distributed onto the eligible paths from m to n . For example, $m \Rightarrow s_1 \Rightarrow n$ is an eligible path. All the eligible paths then form a tree structure T_{mn} , where the source m acts as the root and destination n is excluded. The charging stations s_1, s_4, s_5, s_6 are the leaf nodes from which EVs can arrive n without further re-charging. It is remarked that two successive charging stations (e.g. s_2 and s_4) may not be directly connected, in which case EVs travel the shortest path in between.

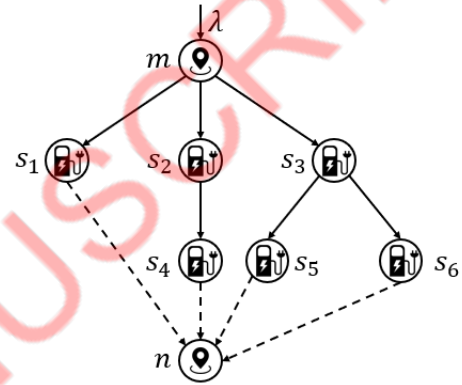


Figure 1: Illustration of the EV flow's distribution due to en-route re-charging.

For the sake of demonstration, whilst reflecting the practical situation, the following assumptions are made for the charging stations \mathcal{V}_c in this work.

- All of the chargers installed at \mathcal{V}_c are level-3 [21] and with the same specifications, e.g. 50KW DC.
- A charging station provides charging services under the first-in first-out principle.
- The charging time with a single charger is characterized by a general distribution F_c , e.g. truncated normal distribution, with mean $\frac{1}{\mu_c}$ and variance σ_c^2 .

Since the EV arrivals at charging stations are Markovian, an individual EV's average delay (including both queuing and charging times) at a charging station $s_k \in \mathcal{V}_c$ with c_k chargers can be approximated by $M/G/K$ queuing model [22] with $K = c_k$, which is given by

$$\mathbb{E} \left[W^{M/G/c_k}(s_k) \right] = \frac{CV^2 + 1}{2} \mathbb{E} \left[W^{M/M/c_k}(s_k) \right] \quad (2)$$

where CV is F_c 's coefficient of variation; $\mathbb{E} \left[W^{M/M/c_k}(s_k) \right]$ refers to the EV's average delay given exponentially distributed charging time with a parameter of μ_c such that

$$\mathbb{E} \left[W^{M/M/c_k}(s_k) \right] = \frac{B[\lambda(s_k)]}{c_k \mu_c - \lambda(s_k)} + \frac{1}{\mu_c} \quad (3)$$

with $B[\lambda(s_k)]$ being the Erlang C formula and $\lambda(s_k)$ being the aggregated rate of EV flows arriving at s_k .

C. Joint Optimization of EV Flow Distribution and Charger Allocation

Expanding the charging infrastructure is one of the most direct initiatives to alleviate EV drivers' range anxiety. Obviously, how to allocate chargers at the charging stations largely affects how well the EV flows can be distributed. Therefore, these two decisions have to be made coordinately.

Here, a joint optimization problem is considered to characterize such a coordination. A given number of chargers is firstly to be allocated to the charging stations. Then, EV flows are needed to be distributed onto the road network according to the realizations of traffic condition uncertainties, i.e. δ_{ij} . This formulation naturally results in a two-stage stochastic programming problem, which has also been considered in many other practical applications [23]. However, it should be emphasized that the formulation and solution algorithm are dedicated to the problem in concern.

In this work, the joint optimization problem aims to minimize the EV's expected total journey time. Mathematically, it is formulated as follows

$$\min_{\mathbf{c}'} \mathbb{E}[H^*(\mathbf{c}, \boldsymbol{\delta})] \quad (4)$$

$$\text{s.t.} \quad \sum_{k=1}^{|\mathcal{V}_c|} c'_k = U \quad (5)$$

$$\mathbf{c} = \mathbf{c}' + \mathbf{c}^0 \quad (6)$$

where $|\cdot|$ is the cardinality of a set; $\mathbf{c}' = \{c'_k\} \in \mathbb{Z}^{|\mathcal{V}_c|}$ is the first-stage decision vector indicating the number of chargers to be further allocated at each of charging stations and $c'_k \geq 0$; U is the total number of chargers to be installed; $\mathbf{c}^0 = \{c_k^0\} \in \mathbb{Z}^{|\mathcal{V}_c|}$ are the numbers of chargers originally installed and $c_k^0 \geq 0$; $\boldsymbol{\delta} \in \mathbb{R}^{|\mathcal{E}|}$ is a vector of δ_{ij} realization; $H^*(\mathbf{c}, \boldsymbol{\delta})$ indicates the optimal value of second-stage EV flow distribution problem, which is to be further explained in the next sub-section.

D. Formulation of EV Flow Distribution Problem

Given the charger allocation \mathbf{c} and traffic condition $\boldsymbol{\delta}$, the second-stage optimization aims to minimize EV's expected delay $H(\mathbf{c}, \boldsymbol{\delta})$, which includes not only the traveling delay, but also the en-route re-charge delay. To tackle this optimization problem, we can first identify \mathcal{V}_s and extract the tree T_{mn} for each unreachable source-destination pair ($m \mapsto n$). This can be solved by the Iterative Tree Construction Algorithm (ITCA) as described in [3], [24], which is essentially a direct heuristic algorithm, and thus details are omitted here.

The EV flow at rate $\lambda(m, n)$ for ($m \mapsto n$) is then distributed to the eligible paths in T_{mn} . Let $\lambda_{ij}^{(mn)}$ denote the portion of $\lambda(m, n)$ assigned to edge (i, j) , the problem of EV flow distribution under stochastic traffic conditions can be formulated as

$$\min_{\boldsymbol{\Lambda}} H(\mathbf{c}, \boldsymbol{\delta}) = \frac{1}{N} \sum_{\substack{m, n \in \mathcal{V}, m \neq n; \\ d_r < d(m, n)}} \left\{ \frac{1}{\lambda(m, n)} \sum_{P \subseteq T_{mn}} \lambda_P^{(mn)} \tau_P \right\} \quad (7)$$

subject to:

$$\sum_{u=C(m)} \lambda_{mu}^{(mn)} = \lambda(m, n), \quad (8)$$

$$\lambda_{ij}^{(mn)} = \sum_{u=C(j)} \lambda_{ju}^{(mn)}, \quad \forall j \in T_{mn} \setminus \{m \cup L(T_{mn})\} \quad (9)$$

$$\lambda_{ij}^{(mn)} \geq 0, \quad (10)$$

$$\lambda_{ij} = \sum_{\substack{m, n \in \mathcal{V}, m \neq n; \\ d_r < d(m, n)}} \lambda_{ij}^{(mn)}, \quad \forall (i, j) \in \mathcal{E} \quad (11)$$

$$\lambda(s_k) = \sum_{(i, j) \in \mathcal{E}, j=s_k} \lambda_{ij}, \quad \forall s_k \in \mathcal{V}_s \quad (12)$$

$$\lambda(s_k) < c_k \mu_c, \quad \forall s_k \in \mathcal{V}_s \quad (13)$$

where $\boldsymbol{\Lambda} = \{\lambda_{ij}^{(mn)}\} \in \mathbb{R}^D$ and $D = \sum_{m, n} |BR(T_{mn})|$ with $BR(T_{mn})$ denoting the set of branches in T_{mn} ; N is the total number of trees which is also equal to the number of unreachable source-destination pairs; $C(j)$ indicates the set of children nodes of node j ; $P \subseteq T_{mn}$ refers to an eligible path in T_{mn} associated with an EV flow of rate $\lambda_P^{(mn)}$ ($= \lambda_{L(P), n}^{(mn)}$); τ_P is the total delay for an EV to travel from m to n via path P which is given by

$$\tau_P = \tau_{L(P), n} + \sum_{(i, j) \in P} \tau_{ij} + \sum_{s_k \in P} \mathbb{E} \left[W^{M/G/c_k}(s_k) \right] \quad (14)$$

where $L(P)$ indicates the node just before the destination n in path P ; $\tau_{L(P), n}$ and τ_{ij} evaluate the time delay to travel corresponding path segments using (1) based on λ_{ij} and δ_{ij} , respectively.

As for the constraints, (8) and (9) require that the in-flow rate equals to the summation of out-flow rates at any s_k in the tree T_{mn} ; (10) specifies the non-negativity of the decision variables; (11) indicates that the EV flow at edge (i, j) essentially contains all the EVs that travel for any unreachable pair via (i, j) ; (12) defines the aggregated EV flow rate $\lambda(s_k)$ arriving at charging station s_k and (13) is to avoid over-utilization of any charging station $s_k \in \mathcal{V}_s$.

E. A Feasible SAA for Joint Optimization Problem

Since problem (7) is defined over a network with stochastic traffic flow, the joint problem (4) cannot be solved directly. To address this issue, SAA method as suggested in [23] is adopted to approximate $\mathbb{E}[H^*(\mathbf{c}, \boldsymbol{\delta})]$ using randomly generated samples of vector $\boldsymbol{\delta}$, i.e. $[\boldsymbol{\delta}^1, \boldsymbol{\delta}^2, \dots, \boldsymbol{\delta}^I]$, by the average

$$\hat{q}_I(\mathbf{c}) = I^{-1} \sum_{l=1}^I H^*(\mathbf{c}, \boldsymbol{\delta}^l) \quad (15)$$

Then, (4) can be transformed to a deterministic equivalent, which is a collection of minimization problems given by

$$\min_{\mathbf{c}'} \hat{q}_I(\mathbf{c}) \quad (16)$$

$$\text{s.t.} \quad \sum_{k=1}^{|\mathcal{V}_c|} c'_k = U \quad (17)$$

$$\mathbf{c} = \mathbf{c}' + \mathbf{c}^0 \quad (18)$$

The constraint (13), i.e. $\lambda(s_k) < c_k \mu_c, \forall s_k \in \mathcal{V}_s$, is a strict inequality, which leads to an open feasible domain. To ensure the existence of an optimal solution, it can be relaxed by introducing an additional parameter ϵ which controls the percentage of charging resources being retained for the EVs that improves for re-charging. Therefore, (13) is modified as

$$\frac{\lambda(s_k)}{c_k \mu_c} \leq 1 - \epsilon, \quad \forall s_k \in \mathcal{V}_s \quad (19)$$

Note that the requirement of sufficient charging capacity for the routing options in ITCA needs to consider ϵ accordingly as well.

III. SOLUTION ALGORITHMS

A. Convexity of EV Flow Distribution Problem

An optimization problem can be efficiently solved if it can be recognized or transformed into a convex one [25]. Hence, before proposing the algorithm for solving the optimization problem (16), the following useful theorem is firstly established:

Theorem 1. *The EV flow distribution problem (7) is convex.*

Proof. Combining (11) and (12), the EV's aggregated arrival rate at charging station s_k , $\lambda(s_k)$, is related to the decision variables $\lambda_{ij}^{(mn)}$ as follows

$$\lambda(s_k) = \sum_{\substack{(i,j) \in \mathcal{E} \\ j=s_k}} \sum_{\substack{m,n \in \mathcal{V}, m \neq n; \\ d_r < d(m,n)}} \lambda_{ij}^{(mn)}, \quad \forall s_k \in \mathcal{V}_s \quad (20)$$

Substituting (20) into (19), one has

$$\left[\sum_{\substack{(i,j) \in \mathcal{E} \\ j=s_k}} \sum_{\substack{m,n \in \mathcal{V}, m \neq n; \\ d_r < d(m,n)}} \left(\lambda_{ij}^{(mn)} \right) \right] - c_k \mu_c (1 - \epsilon) \leq 0 \quad (21)$$

The left-hand side of inequality (21) for each s_k is essentially a linear function (thus convex) with respect to $\lambda_{ij}^{(mn)}$.

On the other hand, since $\lambda(m, n)$ is basically split over the branches of T_{mn} (see (8) and (9)), for any $\lambda_{ij}^{(mn)}$, it is always true that

$$\lambda_{ij}^{(mn)} + \sum_{\exists (a,b) \subseteq T_{mn}} \lambda_{ab}^{(mn)} - \lambda(m, n) = 0 \quad (22)$$

where $\lambda_{ab}^{(mn)}$ are the EV flows assigned at some branches (a, b) in T_{mn} . The left-hand side of the equality is an affine function with a translation of $-\lambda(m, n)$. Therefore, it can be concluded that the feasible region of problem (7) is convex [25].

It remains to show that the objective $H(\mathbf{c}, \boldsymbol{\delta})$ is a convex function of $\boldsymbol{\Lambda}$. It is noted that $H(\mathbf{c}, \boldsymbol{\delta})$ is inherently a weighted aggregation of a collection of τ_P as defined in (14). Therefore, proving the convexity of (14) would complete the proof, as a non-negative weighted sum of convex functions remains convex. Referring to (14), the first two terms and the third term are essentially (1) and (2), which can be proved as convex functions by the following.

- 1) It is observed that the rate of EV flow at edge (i, j) , λ_{ij} , is a linear aggregation of the entries of $\boldsymbol{\Lambda}$. (see (11)). With $\boldsymbol{\Lambda} \in \mathbb{R}^D$, (1) essentially defines a function $\tau_{ij}: \mathbb{R}^D \rightarrow \mathbb{R}$, for each $(i, j) \in \mathcal{E}$. To prove the convexity of τ_{ij} , its Hessian, $\text{Hess } \tau_{ij}$, on \mathbb{R}^D needs to be at least positive semi-definite.

An entry of $\text{Hess } \tau_{ij}$ can be computed by

$$\begin{aligned} (\text{Hess } \tau_{ij})_{st} &= \frac{\partial^2 \tau_{ij}}{\partial \lambda_s \partial \lambda_t} \\ &= \frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2} \cdot \frac{\partial \lambda_{ij}}{\lambda_s} \cdot \frac{\partial \lambda_{ij}}{\lambda_t} + \frac{\partial \tau_{ij}}{\partial \lambda_{ij}} \cdot \frac{\partial^2 \lambda_{ij}}{\partial \lambda_s \partial \lambda_t} \end{aligned} \quad (23)$$

where λ_s and λ_t are the entries of $\boldsymbol{\Lambda}$. Therefore, for the diagonal terms of $\text{Hess } \tau_{ij}$, if λ_s is aggregated by $\lambda_{ij}^{(mn)}$, one has $(\text{Hess } \tau_{ij})_{ss} = \frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2}$; otherwise, $(\text{Hess } \tau_{ij})_{ss} = 0$. For off-diagonal terms, there are three cases:

- If both λ_s and λ_t are not aggregated by $\lambda_{ij}^{(mn)}$, then $(\text{Hess } \tau_{ij})_{st} = 0$
- If only one of them is aggregated, $(\text{Hess } \tau_{ij})_{st} = 0$
- If both of λ_s and λ_t are aggregated, $(\text{Hess } \tau_{ij})_{st} = (\text{Hess } \tau_{ij})_{ts} = \frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2}$.

Since the non-zero entries in $\text{Hess } \tau_{ij}$ are essentially identical and equals to $\frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2}$, a succession of elementary operations can be performed and $\text{Hess } \tau_{ij}$ becomes

$$\text{Hess } \tau_{ij} = \frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2} \begin{bmatrix} \mathbf{1}_{\eta \times \eta} & \mathbf{0}_{\eta \times (D-\eta)} \\ \mathbf{0}_{(D-\eta) \times \eta} & \mathbf{0}_{(D-\eta) \times (D-\eta)} \end{bmatrix}. \quad (24)$$

where $\mathbf{1}$ and $\mathbf{0}$ represent the matrices of one and zero with corresponding sizes, respectively; η denotes the total number of non-zero entries in $\text{Hess } \tau_{ij}$. Moreover, $\frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2}$ can be derived as

$$\frac{\partial^2 \tau_{ij}}{\partial \lambda_{ij}^2} = \frac{\tau_{ij}^0 \alpha \beta (\beta - 1)}{q_{ij}^2} \left(\delta_{ij} + \frac{\lambda_{ij}}{q_{ij}} \right)^{\beta - 2} \quad (25)$$

By considering the typical BPR formula, i.e. $\alpha = 0.5$ and $\beta = 4$ [19], it can be easily deduced that (25) is positive.

Since the eigenvalues of $\mathbf{1}_{\eta \times \eta}$ equal to η and 0, with multiplicity of 1 and $\eta - 1$, respectively [26], $\text{Hess } \tau_{ij}$ should also have a single positive eigenvalue (others are all zero) according to the Sylvester's law of inertia [26]. As a result, the Hessian of (1) with respect to $\boldsymbol{\Lambda}$ is positive semi-finite.

- 2) As implied by (20), the entries of $\boldsymbol{\Lambda}$ are also linearly aggregated for $\lambda(s_k)$, and thus the convexity proof for (1) is also applicable here. Similarly, to show that the Hessian of $\mathbb{E} [W^{M/G/c_k}(s_k)]$ on \mathbb{R}^D is also positive semi-definite, the second derivative of $\mathbb{E} [W^{M/G/c_k}(s_k)]$ with respect to $\lambda(s_k)$ needs to be positive. In [27], (3) has been proved to be a convex function of arrival rate on \mathbb{R} . The approximation (2) is thus also convex because the scalar, i.e. $\frac{CV^2+1}{2}$, is always positive, which completes the proof. \square

B. Optimization of EV Flow Distribution

Since problem (7) is convex (according to **Theorem 1**), and also strictly feasible, i.e. Slater's constraint qualification holds, there exist dual optimal ψ^* and ω^* , together with the optimal EV flow rates Λ^* , satisfying the Karush–Kuhn–Tucker (KKT) conditions. Classical convex programming tools can be applied to obtain Λ^* efficiently [25] and SQP method [28] is adopted here. It is remarked that problem (7) can only be solved given a sample of vector δ , i.e. stochastic traffic flow. The approximation of $\mathbb{E}[H^*(c, \delta)]$ using SAA method essentially yields a collection of EV flow distribution policies with respect to different traffic conditions under charger allocation c .

C. A Hybrid Approach for Feasible SAA

A hybrid approach, which incorporates SQP into a Tabu-based [29] charger allocation algorithm (CAA), is designed to solve problem (16). The mechanisms of CAA are detailed as follows.

Algorithm 1 Charger Allocation Algorithm (CAA)

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1: procedure CHARGERALLOCATION
2:   Initialize  $e \leftarrow 0$ ,  $c_*' \leftarrow c_s'$ , and  $\mathcal{L} \leftarrow \{c_s'\}$ 
3:   while  $e \leq E$  do
4:     Perform Pairing & Shifting to generate  $R(c_s')$ .
5:     for all  $c' \in R(c_s')$  do
6:       if  $c' \notin \mathcal{L}$  and  $\hat{q}_I(c' + c^0) < \hat{q}_I(c_s' + c^0)$  then
7:          $c_s' \leftarrow c'$ 
8:        $e \leftarrow e + 1$ 
9:       if  $\hat{q}_I(c_s' + c^0) < \hat{q}_I(c_*' + c^0)$  then
10:         $c_*' \leftarrow c_s'$ 
11:       $\mathcal{L} \leftarrow \mathcal{L} \cup \{c_s'\}$ 
12:      if  $|\mathcal{L}| > L$  then
13:         $\mathcal{L} \leftarrow \mathcal{L} \setminus \{\mathcal{L}^{(1)}\}$   $\triangleright$  Expire the 1st “Tabu”
14:    return  $c^* \leftarrow c_*' + c^0$  and  $c_*'$ 

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Note: The optimal EV flow distribution solution $\hat{q}_I(\cdot)$ for each candidate of charger allocation is obtained by SQP; c^0 are the numbers of chargers originally installed.

- 1) **Encoding Method:** The solution candidate is simply $c' = \{c_k'\} \in \mathbb{Z}^{|\mathcal{V}_c|}$, where c_k' implies the number of chargers to be assigned at the charging station s_k .
- 2) **Neighborhood Structure:** The basic idea of Tabu search is to explore the solution space by seeking improved solution among the neighbors of c' . Therefore, a two-step neighborhood structure that generates feasible neighbors of c' is devised.
 - a) *Pairing*: all the elements of c' are randomly paired;
 - b) *Shifting*: for each element pair c_i' and c_j' , if $c_i', c_j' \neq 0$, then one of them, say c_i' , will shift $c_i^t \in [1, c_i']$ chargers to c_j' . Consequently, one has $c_i' \leftarrow c_i' - c_i^t$, and $c_j' \leftarrow c_j' + c_i^t$. If one of the elements, say $c_i' = 0$, then c_j' has a probability of 0.5 to carry out the above shifting operation. If both elements are zero, the operation of this element pair is aborted.
- 3) **Operational Procedures:** The operational procedures of CAA are given in **Algorithm 1**. \mathcal{L} is the Tabu list

that contains the solutions visited in recent past and its maximum length is L . Initially, \mathcal{L} only contains one solution candidate, c_s' , which is randomly generated with equality constraint (17) preserved. c_s' is also referred as the current local best and considered as global best (denoted by c_*') (line 2). In each iteration, a set of neighbors of c_s' , $R(c_s')$, is obtained (line 4) via *Pairing* and *Shifting* operations. If the best non-Tabu neighbor in $R(c_s')$ outperforms c_s' in terms of the corresponding optimal EV flow distribution solution \hat{q}_I , c_s' is updated (line 6-7). Similarly, c_*' is also updated if c_s' is better (line 9-10). Lastly, this solution is added into the Tabu list \mathcal{L} (line 11-13). Such a searching cycle repeats until the number of iterations reaches E .

The outputs of the algorithm, i.e. c^* and c_*' , are the resultant charger allocation and the corresponding placement strategy that benefit the optimal EV flow distribution under stochastic traffic flow δ the most.

Remark 1. The most time-consuming procedure of CAA is the evaluation of non-Tabu neighbors, where the corresponding problem (7) instances are solved by SQP. As a result, the computational cost of CAA would depend on that of SQP. In each CAA iteration, there is at most $|R(c_s')|$ non-Tabu neighbors generated. Thus, the total number of SQP execution in CAA is at most $E \times |R(c_s')|$. In Sect. IV-B2, the running time of SQP will be further studied.

IV. SIMULATION RESULTS AND DISCUSSIONS

A. Dataset and Parameter Settings

To demonstrate the effectiveness of the proposed problem framework and solution algorithm, the eastern Massachusetts highway network [30] shown in Fig. 2 is employed.

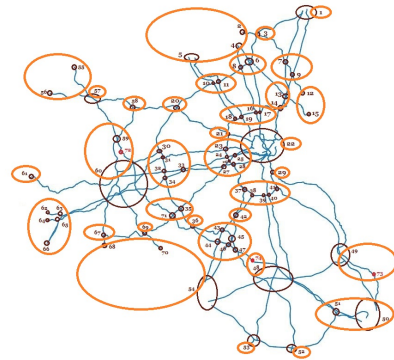


Figure 2: The eastern Massachusetts highway subnetwork. Each node of the network corresponds to a “zone”, which is an area integrating several locations.

The highway network is modeled as a directed and weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with $|\mathcal{V}| = 74$ and $|\mathcal{E}| = 258$. The distances (in km), free-flow travel time (in hours) and flow capacity of each edge are respectively employed for d_{ij} , τ_{ij}^0 and q_{ij} in \mathcal{A} . The charging stations \mathcal{V}_c are sited at nodes with

the highest betweenness centrality [31], and for any unreachable pairs ($m \mapsto n$), $\lambda(m, n) = \lambda$ is assumed. Table I lists the general settings of other parameters for the simulations. It is remarked that these settings are for illustrative purpose and they can be fully adjustable according to practical situations. All the simulations are implemented with MATLAB on a PC platform with Intel Core i7-6700H CPU @ 2.60GHz and 8 GB of RAM. In particular, the SQP method is implemented in “fmincon” function of optimization toolbox [32].

Table I: Parameter Settings

| Parameters | Settings |
|--|--------------------------|
| Number of charging stations ($ \mathcal{V}_c $) | 10 |
| Initial number of chargers ($c_k^0, \forall s_k \in \mathcal{V}_c$) | 3 |
| Utilization of road capacity ($\Delta_{ij}, \forall (i, j) \in \mathcal{E}$) | $\mathcal{N}(0.5, 0.1)$ |
| EV's re-charge time ($F_c \sim \mathcal{N}(\mu_c, \sigma_c^2)$) | $\mathcal{N}(0.5, 0.13)$ |
| F_c 's truncation interval | [0.3, 0.7] |
| Retained charging resources (ϵ) | 10% |
| Number of samples in SAA (I) | 20 |
| Maximum size of Tabu list (L) | 5 |
| Maximum iteration of CAA (E) | 100 |
| Number of neighbors generated for c'_s ($ R(c'_s) $) | 10 |

B. Algorithmic Performance Analysis

1) *Quality of SAA Solution*: The quality of SAA solution with respect to the solution of original two-stage problem (4) is firstly evaluated. Assuming the optimal objective value for problem (4) is q^* , then $\hat{q}_I(c^*) \geq q^*$ must hold, where c^* is obtained by solving the corresponding SAA problem (16). The quality of c^* can thus be measured by the optimality gap, which is defined as

$$P(c^*) := \hat{q}_I(c^*) - q^* \quad (26)$$

In [23], a statistical procedure is outlined to estimate $P(c^*)$ which is given by

$$\hat{P}(c^*) := UB(c^*) - LB \quad (27)$$

where $UB(c^*)$ and LB are the confidence upper-bound and lower-bound of $\hat{q}_I(c^*)$, respectively.

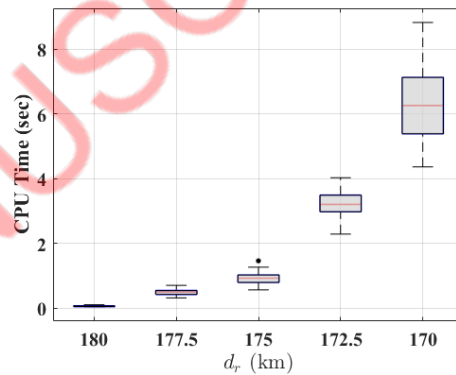
Let the EV flow rate $\lambda = 2$ (EVs/hour) at each unreachable source-destination pair and the total number of chargers for allocation $U = 30$, the quality of c^* with respect to EV's driving range d_r is presented in Table II. Note that three ranges, i.e. 180, 175, 170 (km) are examined because such variations with respect to d_r lead to different sets of unreachable source-destination pairs, covering various scenarios for the second-stage optimization problem (7).

Table II shows that the magnitude of the estimated optimality gaps $\hat{P}(c^*)$ are less than 0.02, which means the number of samples in SAA ($I = 20$) is good enough to handle the uncertainty induced by stochastic traffic flow δ .

Table II: Estimation of quality of c^* for different d_r

| Ranges (d_r) | $UB(c^*)$ | LB | $\hat{P}(c^*)$ |
|------------------|-----------|--------|----------------|
| 180 (km) | 2.8419 | 2.8304 | 0.0115 |
| 175 (km) | 2.8374 | 2.8339 | 0.0036 |
| 170 (km) | 2.9985 | 2.9815 | 0.0170 |

2) *Running Time of SQP*: Fig. 3 depicts the running times for SQP in solving problem (7) based on five different values of d_r . Since the running time may vary, 30 problem (7) instances are solved for each d_r . The results show that the smaller d_r is, the more time SQP takes to solve the problem. It is because, given a smaller d_r , the number of decision variables $\lambda_{ij}^{(mn)}$, i.e. rates of EV flows assigned at “tree branches”, would increase together with the number of unreachable source-destination pairs. However, one SQP execution only takes a few seconds in all considered cases, and thus is acceptable for practical applications.

Figure 3: Running time of SQP under different d_r .

3) *Effectiveness of SQP*: In [3], a flow distribution algorithm (FDA) is suggested to assign EV flows to feasible paths with deterministic traffic conditions. FDA is based on genetic algorithm [33] that is composed of binary tournament selection, simulated binary crossover and polynomial mutation. It can also be applied to solve problem (7). Let $d_r = 175$ (km), $U = 30$, and $\mu_{ij} = 0.5$, then for each $\lambda \in \{\lambda \in \mathbb{N} : 1 \leq \lambda \leq 6\}$, 50 sets of problem (7) are formulated under randomly generated charger allocations. These problem sets are respectively solved by both SQP and FDA for comparison. Since the uncertainty induced by δ would also affect the decision-making in (7), the same randomly generated I samples of δ are adopted for each charger allocation. The quality of a flow distribution solution under any charger allocation c can thus be measured by averaging $H^*(c, \delta)$ over I traffic samples. Therefore, $\hat{q}_I(c)$ essentially becomes the algorithmic performance metric instead of $H(c, \delta)$.

The performance improvements of SQP over FDA in terms $\hat{q}_I(c)$ and CPU time consumption, are presented in Fig. 4 (a) and (b), respectively. It is clearly observed that SQP not only obtains solutions with more than 15% less $\hat{q}_I(c)$ (except for $\lambda = 1$), but also saves at least 70% of the CPU time. As proved in Theorem 1, problem (7) is convex and applying SQP as a

solver would guarantee to find the optimal EV flow rates Λ^* . By contrast, heuristic algorithms like FDA do not have any performance guarantee. Therefore, SQP outperforms FDA in all cases.

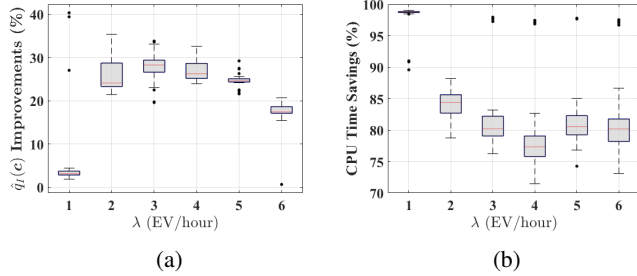


Figure 4: Performance improvement of SQP against FDA (a) solution quality (b) CPU time consumption.

4) *Effectiveness of CAA*: [3] also suggests two charger allocation schemes, namely proportional allocation (PA) and uniform allocation (UA). PA assigns more chargers to charging nodes with higher betweenness, while UA assigns equal number of chargers to each of charging stations. Let SQP-Pro/SQP-Uni denote the algorithms that solve problem (16) with PA/UA allocation and SQP. For comparison purpose, CAA, SQP-Pro and SQP-Uni are applied to solve the same sets of problem (16). Fig. 5 shows the specific parameter settings and simulation results.

Fig. 5 (a) clearly demonstrates the out-performance of CAA over SQP-Pro and SQP-Uni by achieving the smallest expected total journey time of EV \hat{q}_I . However, it is noticed that the improvement is less significant when the arrival rate of EV flow at each source node is small, e.g. $\lambda = 1$. To better explore the reason, the composition of \hat{q}_I is obtained. In Fig. 6 (a), EV's (expected) traveling delay on the road $\hat{q}_I^{(T)}$ and the total delay at the charging stations $\hat{q}_I^{(W)}$ (including both queuing and charging time), as given in (2), are obtained against λ using SQP. It shows that $\hat{q}_I^{(T)}$ remains similar but $\hat{q}_I^{(W)}$ decreases noticeably with fewer EVs demanding en-route re-charging, i.e. smaller λ . With a small λ , the charging resource would be relatively "sufficient" and EVs experience very little (or even no) queuing delay at the charging stations. As a result, the allocation of additional chargers becomes less critical and applying CAA can no longer achieve a significant reduction in $\hat{q}_I^{(W)}$, as shown in Fig. 5 (a). Also, from Fig. 5 (a), SQP-Pro performs better than SQP-Uni, which is consistent with the results given in [3].

Fig. 5 (b) presents the cases where the number of chargers for allocation (U) is increased from 30 to 50. As compared to Fig. 5 (a), \hat{q}_I of all the algorithms drops. Although CAA is still the best, the induced improvement becomes less. Charging resource sufficiency, as a direct implication of increasing U , is again the main reason. This is also confirmed by comparing Fig. 6 (a) and (b), showing that decreasing λ and increasing U have similar impacts on \hat{q}_I . Therefore, CAA's solutions hardly benefit from the more-than-sufficient charging resources for $1 \leq \lambda \leq 4$, as shown in Fig. 5 (b). However, when the charging resource becomes relatively "insufficient" by increasing λ to

5 or 6 (EV/hour), \hat{q}_I improvements provided by CAA become noticeable.

In Fig. 5 (c), EV's driving range d_r is set to 180 (km) rather than 175 (km), and again, the CAA's out-performance becomes less significant. Extending EV's range would reduce the number of unreachable source-destination pairs. Consequently, service load at charging stations drops, which turns out to be the same case as discussed following Fig. 5 (a).

In Fig. 5 (d), the mean utilization of edge (i, j) capacity μ_{ij} is decreased to 0.3. Based on (1), less traffic volume shall lead to less travel delay along the edges and thus smaller \hat{q}_I . The decrease in \hat{q}_I (as compared to Fig. 5 (a)) is marked in red in Fig. 5 (d). As shown, such a decrease is not substantial for all the tested cases, and CAA remains dominant.

In summary, CAA can solve problem (16) effectively under all the testing scenarios. Its out-performance over the other approaches is especially significant when the charging resource is tight.

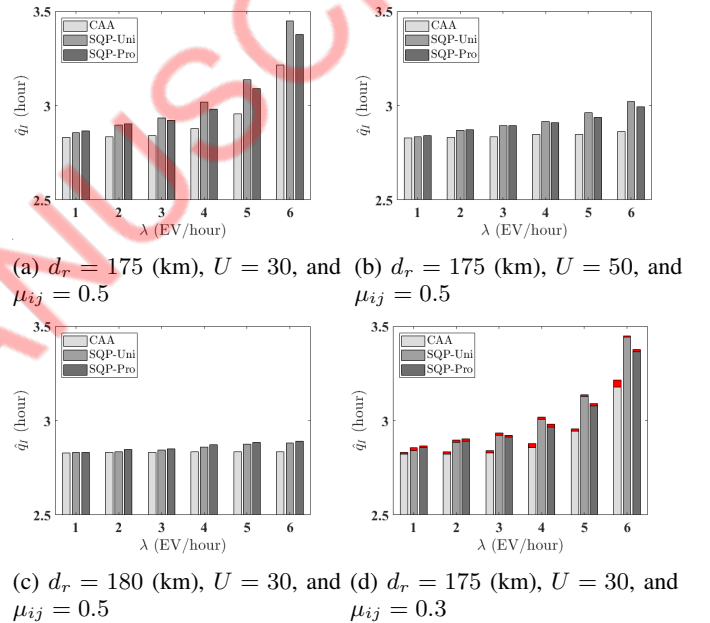


Figure 5: Performance evaluation of CAA.

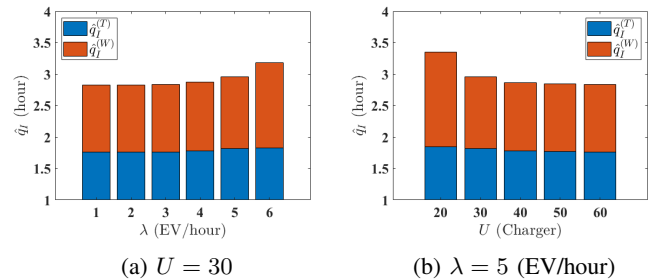


Figure 6: Composition of \hat{q}_I , where $d_r = 175$ (km).

C. Impacts of Parameter Variations

The results presented in Fig. 5 not only verify the effectiveness of CAA, but also suggest that the parameter setting

would affect the solutions of problem (16). In this section, the impacts of major parameters, including charging resource sufficiency (U), driving range (d_r) and stochastic traffic flow (Δ_{ij}), are extensively studied.

1) *Impact of Charging Resources Sufficiency*: Fig. 5 (a)–(c) suggest that the expected total journey time of the EV can be greatly reduced when the charging resource is sufficient for EVs to re-charge without queuing. A natural question arose here is how to define “sufficient” quantitatively. In other words, is there a lower bound for the number of chargers to allocate (U), such that the EV flow with bounded arrival rate can always re-charge without queuing?

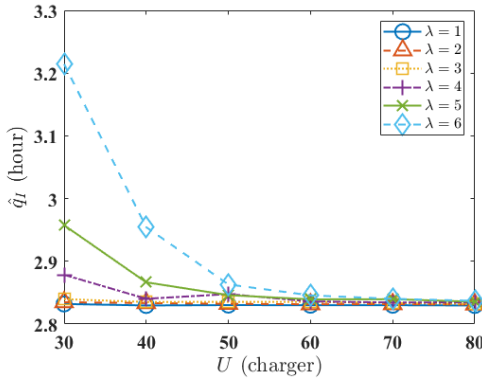


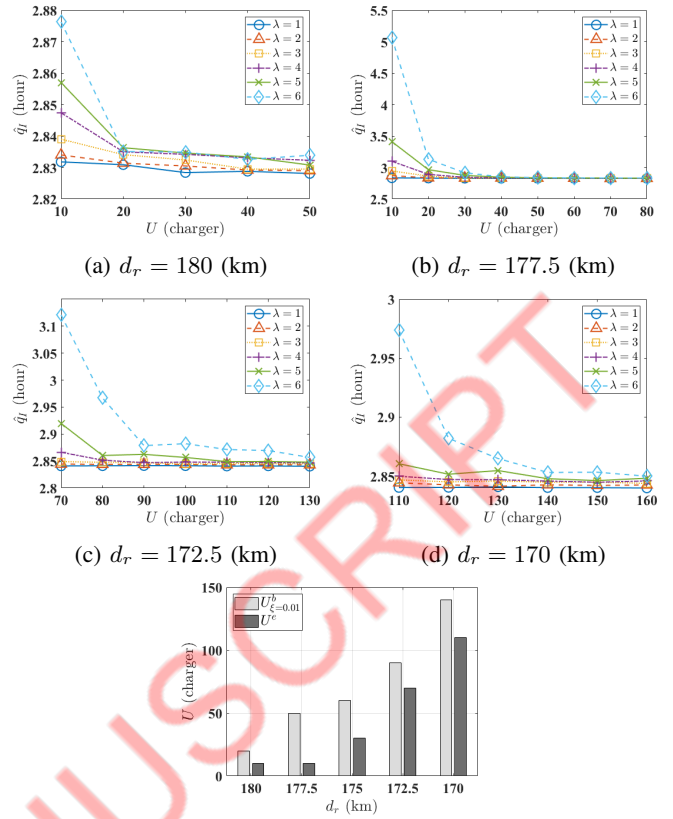
Figure 7: The expected total journey time of EV (\hat{q}_I) versus the number of chargers for allocation (U), with $d_r = 175$ (km) and $\mu_{ij} = 0.5$.

To answer this question, the expected total journey time of EV (\hat{q}_I) is derived with respect to the number of chargers for allocation (U), under various arrival rate of EV flows. The specific settings and results are presented in Fig. 7. U starts with 30 which is the least number of chargers that satisfies the inequality (19) under the current settings, and it is denoted by U^e hereafter. Fig. 7 shows that, given any λ , \hat{q}_I decreases when U increases. In particular, when $U \geq 60$, the drop of \hat{q}_I is less than a threshold $\xi = 0.01$. It implies the elimination of queuing and \hat{q}_I is only further improved with a shortened (expected) re-charging time. 60 (chargers) can thus be considered as an estimator of the lower bound of U with the EV flow rate $\lambda \leq 6$ (EV/hour), and symbolized by $U_{\xi=0.01}^b$. It's also noted that a more precise lower bound of U is possible by increasing U with a smaller step size.

In practice, the rate of EV flows in a road network is highly dependent to the market share of EV. With the market share of EV at a future point being statistically predictable, this lower bound for chargers would be a very useful guideline, to prevent over-investment of charging resources.

2) *Impact of Driving Range*: Fig. 5 (c) demonstrate that the extension of EV's driving range d_r helps to reduce the charging load, and the number of chargers that is required for a quality re-charge service can be reduced accordingly.

In Fig. 8 (a)–(d), EV's expected total journey times (\hat{q}_I) are derived with respect to the number of chargers for allocation (U), under different d_r . Let $\xi = 0.01$, the least number of chargers U^e and the lower bound $U_{\xi=0.01}^b$ for different d_r can



(e) $U_{\xi=0.01}^b$ and U^e against d_r .

Figure 8: Charging resource sufficiency under different driving range of EV, with $\mu_{ij} = 0.5$.

thus be obtained as shown in Fig. 8 (e). The results indicate that both U^e and $U_{\xi=0.01}^b$ decrease with the increase of d_r . Therefore, the allocation planning of charging resources would also be benefited from the advance of EV's battery technology.

3) *Impact of Stochastic Traffic Flow*: Fig. 5 (d) has implied that the traffic flow on edge (i, j) characterized by Δ_{ij} may not have substantial impact on the solutions of problem (16) (obtained by CAA). For further investigation, EV's expected total journey times (\hat{q}_I) are acquired with respect to EV flow rate at each source (λ) under different Δ_{ij} . The settings and the results are given in Fig. 9. In Fig. 9 (b), the number of chargers for allocation (U) is increased to 50 so as to study cases where the queuing delay is largely obviated with given sufficient charging resources.

The comparison between Fig. 9 (a) and Fig. 9 (b) clearly shows that the queuing delay caused by excessive re-charge activities (for $\lambda = 5, 6$ in Fig. 9 (a)) leads to a serious degradation in \hat{q}_I , while the travel delay induced by the extra vehicle traffic is little (see Fig. 9 (b)). In conclusion, comparing to the variations in traffic conditions, the sufficiency of charging resources plays a more critical role in the distribution of EV flows.

V. CONCLUSION

In this paper, a two-stage stochastic programming model is considered to coordinate the allocation of chargers and the

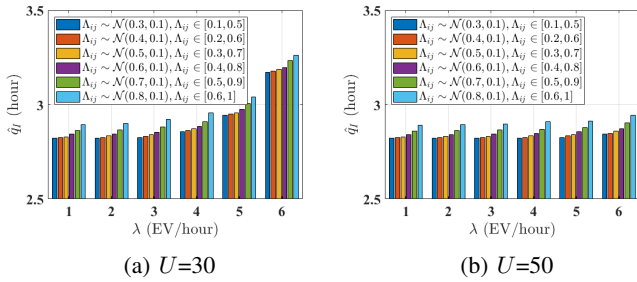


Figure 9: Impact of stochastic traffic flow, with $d_r = 175$ (km).

distribution of EV flows under stochastic traffic conditions. The EV's expected total journey time is minimized jointly by the two-stage decisions. Based on the SAA method, a feasible deterministic equivalent of the original problem is derived. Furthermore, the EV flow distribution problem is proved to be convex. Therefore, the two-stage optimization problem can be effectively solved by a hybrid approach, which incorporates SQP and a Tabu-based heuristic, so that a high-quality charger allocation and optimal EV flow distribution policy are obtained. Numerical results confirm the quality of SAA estimator as well as the out-performance of the hybrid approach over existing ones. Moreover, the proposed formulation designates the sufficiency in terms of charging resources by statistically estimating a lower bound for the number of chargers to allocate, which in practice helps to prevent over-allocation of charging resources. The simulation results also reveal that the sufficiency of charging resources is critical for EV's driving experience, because it has greater impact on traveling delay comparing to the variation in traffic conditions. In the future, this work would be extended to consider bidirectional chargers for the implementation of vehicle-to-grid (V2G) services [34], [35].

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