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FACULTY OF BUSINESS, LAW AND ART
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The Impact of Tax Legislation on Inventory Management and Sourcing Strategies

by

Hua Jin

A thesis submitted for the degree of
Doctor of Philosophy in Management Science

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Abstract

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Operational Research (OR) provides the methods and techniques by which firms can maximise their profits by taking smart decisions. The OR literature in the area of logistics, however, pays scant attention to the cash flows that the firm needs to fulfill its legal obligations. In particular, the thesis investigates how Value Added Tax (VAT) and Corporate Tax schemes work in the United Kingdom at the current time, and how these schemes interact with the logistics aspects of the firm.

The thesis develops a methodology for constructing models that explicitly account for the impact of tax legislation on a series of classic operational research problems. It does this by expressing the future profits of the firm after tax as the Net Present Value or Annuity Stream Value of the cash-flow function associated with the activity for the firm, including these cash-flows exchanged with relevant third parties and the government that are needed in the context of ensuring compliance with tax legislation.

Using current legislation in the United Kingdom, and also drawing from European Union directives on acquisitions and removals, the thesis established how to explicitly consider Value Added Tax (VAT) scheme, Corporate Tax (CT) scheme, and import duties and tariffs rules into decision models, and how these affect optimal decisions and profitability for a firm. The focus lies on OR decision models within the area of inventory management, including decisions about supplier and product selection, and optimal promotion strategy to influence the timing of demand. In particular, we look at four different aspects for handling tax and inventory management problems: (i) traditional economic order quantity (EOQ) models; (ii) the EOQ method in a context of cross-country supply chain activities; (iii) multi-products sourcing strategy; and, (iv) dynamic lot sizing problems.

This work contributes to the body of OR theory supporting the tax-effective supply chain. Its contribution lies in proposing a method of how to account for UK/EU taxation rules into inventory and sourcing optimisation models by means of the Laplace transform of all relevant cash-flows, including those associated with taxes, and investigating the impact this has on optimal decisions. By comparison of these models with traditional OR models, it is demonstrated that not only the tax rates but also the tax schemes can be crucial determinants to operational decision making and profitability. To our knowledge, this is the first study to use this approach, and the first study to show the impact of the V.A.T.-scheme on inventory and supply decisions. The tax-adjusted inventory models in particular have a tendency to lead to operational decisions that become more synchronised with the tax points used in these taxation schemes, with savings in net profits that can amount to several percentages.

Contents

List of Symbols

Chapter	Four
p	unit sales price (excluding VAT)
y	annual demand rate
s	order cost (excluding VAT)
w	unit purchase price (excluding VAT)
T	cycle time (decision variable)
T_v	$T_v = \frac{1}{12}$, one month (years)
Q_{eoq}	lot-size (decision variable)
α	opportunity cost of capital
AS	annuity stream profit function (objective function)
t_S	the (start) time of a physical transaction (= date of supply)
t_I	the time the corresponding invoice is issued
L_I	$L_I = t_I - t_S$
t_C	the time the cash due on the invoice is paid
L_C	$L_C = t_C - t_S$
t_T	the VAT tax point
τ	VAT tax rate (20%)
τ'	VAT tax effect (= NPV-adjusted VAT rate)
τ_f	VAT flat rate (between 4% and 14.5%)
Q^*_{vat}	the VAT-adjusted order quantity (decision variable)
OVAT	total annual expected Output VAT
IVAT	total annual expected Input VAT
NVAT	net annual VAT liability to HMRC
AS_o	operational annuity stream function (with suppliers and customers)
AS_τ	tax-exchange annuity stream function (with HMRC)
AS_t	$\text{AS}_t = \text{AS}_o + \text{AS}_\tau$, tax-adjusted annuity stream function (objective function)
ϵ	the Corporate Tax (CT) rate
ϵ'	the CT effect (= NPV-adjusted tax rate)
OP	Operating Profit (in accounting period)
GP	Gross Profit (in accounting period)
NS	Net Sales (in accounting period)
COGS	Cost Of Goods Sold (in accounting period)

OE	Operating Expenses (in accounting period)
FOC	fixed overhead costs allocated to this activity
δ	fraction of FOC exempt from VAT ($0 \leq \delta \leq 1$)
T_a	$T_a = 1$, accounting year (= 1 year)
T_v	$T_v = 1/12$, one month (years)
AS_ϵ	tax-exchange annuity stream function (with HMRC)
Q^*	the CT and VAT adjusted order quantity (decision variable)
Q_s^*	the CT and VAT adjusted order quantity for small firms
Q_m^*	the CT and VAT adjusted order quantity for medium firms
Q_l^*	the CT and VAT adjusted order quantity for large firms
Chapter	Five
p	unit sales price in UK domestic market (excluding VAT)
d	unit delivery cost for sales in the UK market
p_{on}	unit sales price to non-VAT-registered customers in EU
d_{on}	unit delivery cost for sales to non-VAT-registered customers in EU
p_{or}	unit sales price to VAT-registered customers in EU
d_{or}	unit delivery cost for sales to VAT-registered customers in EU
y	annual demand rate in domestic market
y_{on}	annual demand rate to non VAT-registered customers in EU
y_{or}	annual demand rate to VAT-registered customers in EU
Y	$Y = y + y_{on} + y_{or}$ total annual demand rate
τ	domestic tax rate
τ_{on}	tax rate for non-VAT-registered customer in EU
τ_{or}	tax rate for VAT-registered customer in EU
s	set up cost (excluding VAT)
τ_s	tax rate for set-up cost (either from domestic or EU provider)
w	unit purchase price (excluding VAT)
τ_w	tax rate for purchasing cost (either from domestic or EU supplier)
T	cycle time
α	opportunity cost of capital rate ($0 < \alpha << 1$)
AS_{a0}	operational annuity stream function in acquisition
AS_a	AS function in acquisition (objective function)
AS_{a1}	AS function derived from AS_a all domestic sourcing
AS_{a2}	AS function derived from AS_a all domestic purchasing, service from EU
AS_{a3}	AS function derived from AS_a all EU purchasing, service from domestic market
AS_{a4}	AS function derived from AS_a purchasing and service from EU
Q_a^*	CT and VAT adjusted order quantity in acquisition (decision variable)
Q_{a1}	CT and VAT adjusted order quantity in domestic sourcing
Q_{a2}	CT and VAT adjusted order quantity in domestic purchasing, service from EU
Q_{a3}	CT and VAT adjusted order quantity in EU purchasing, service from domestic

Q_{a4}	CT and VAT adjusted order quantity in EU sourcing
ASN	AS profit function for import activity(without tax)
${Q_N}^*$	Optimum order quantity in NPV of import activity(without tax)
xs	freight cost to UK border or EU border
$(1-x)s$	freight cost from border to UK customer(VAT adjustment)
w	this is product cost, insurance and any other related cost
τ_1	Uk VAT rate(τ) or EU VAT rate(τ_d)
θ	duty rate for import
L_I	purchasing cost payment time in amount of wYT
L_s	proportion setup cost payment time before reach to the UK or EU border
L_N	time between clearance from border to arrives its UK destination $L_N \geq 0$
AS_{i0}	operational annuity stream function in import
AS_i	annuity stream profit function in import (objective function)
AS_{i_s}	AS profit function in import for small size firms
AS_{i_m}	AS profit function in import for medium size firms
AS_{i_l}	AS profit function in import for large size firms
Q_i	CT and VAT adjusted order quantity in import (decision variable)

Chapter	Six
Indices	
i	$1,,,n$, index of suppliers
j	$1,,,m$, index of products
Parameters	
p_j	selling price of product j
w_{ij}	purchase of unit item j from supplier i
S_i	main set up cost for supplier i
s_{ij}	minor product set up cost for supplier i by product j
y_j	annual demand rate of product j
h_j	holding cost of product h_j
k_{ij}	integer number decides the replenishment schedule of item j from supplier i
T_i	joint replenishment cycle time from supplier i (decision variable)
τ	value added tax rate
τ'	value added tax adjusted AS function
ϵ	corporation tax rate
ϵ'	corporation tax adjusted AS function
ζ'	tax adjusted rate $1-\epsilon'+\tau-\tau'$
f_j	unit capacity requirement of item j
F_i	the capacity provided by supplier i
ASP	net present value based profit function
AS_{0d}	operational annuity stream function domestic sourcing
AS_{0p}	operational annuity stream function off-shore sourcing

AS_d tax-adjusted annuity stream function domestic sourcing

AS_p tax-adjusted annuity stream function off-shore sourcing

Dependent Variable

C_i $\begin{cases} 1 & \sum_{j=1}^{m} X_{ij} > 0 \\ 0 & otherwise \end{cases}$

$Chapter$ Seven

Parameters

N dynamic lot sizing planning horizon

n number of setups to be performed, between 1...N

π profit function

R_i tax adjusted revenue in period i

C_i tax adjusted expenses in period i

H_i tax adjusted holding cost in period i

$R(i)$ tax adjusted revenue from period $i+1$ to N

$C(i)$ tax adjusted expenses from period $i+1$ to N

$H(i)$ tax adjusted holding cost from period $i+1$ to N

T_a yearly basis

$CT(i)$ corporation tax return day on ith period

$VT(i)$ value added tax return day on ith period

p_i unit selling price in period i

c_i unit purchasing price in period i

y_i demand in period i

Q_i production quantity in period i (Decision Variable)

Declaration of Authorship

I, Hua Jin , declare that this thesis titled, 'The Impact of Tax Legislation on Inventory Management and Sourcing Strategies' and the work presented in it are my own, and have been generated by me as the result of my own original research. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualication at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed :

Date :

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To no one....

Chapter 1

Introduction

1.1 Research Background

Operational Research (OR) provides the methods and techniques by which firms can maximise their profits by making smart decisions. To date, the OR literature in the area of logistics has so far not explicitly considered the cash-flows that arise in order for the firm to fulfill its legal tax obligations. As most firms wish to make decisions that maximise the Net Present Value (NPV) of future profits after tax, these additional cash-flows should in principle be accounted for. In particular, this thesis investigates how legislation in the United Kingdom of Value Added Tax (VAT), Corporate Tax (CT), and Import Duties (ID) interacts with inventory and supplier selection decisions of the firm.

There is an increased awareness in the literature of the importance of taxes to the operations of firms and the management of their supply chains (see Chapter 3). This includes studies which consider the relevance of capital structure, supply chain financing, accounting methods to value inventories, exchange rates, and location and sourcing strategies in isolation or in combination with transfer pricing strategies for multinationals. Related issues (tax havens, Brexit) have received regular attention in the press and illustrate the importance of governmental impact on the strategic, tactical, and operational decisions of the firm.

Despite advancements in the above areas, the OR literature in the area of inventory management has so far paid scant attention to tax flows. One reason for this might be that most inventory/lot-sizing models are based on average cost and profit functions. In this modelling framework, it would seem intuitive to conclude that neither CT nor VAT would impact on decisions. Indeed, VAT is a tax collected on end consumers, not on firms, and CT may seem to reduce the firm's operating profit only by a constant. Optimising the system before tax therefore seems to lead to the same result as optimising it after tax.

However, this is no longer true when looking at the processes by which most governments collect VAT and CT. In the UK, as in most EU countries, consumption tax is collected via a VAT system such that each firm collects VAT on sales and pays VAT on purchases, whereby the net difference is to be settled with the government at different times, typically much later than the moments when the VAT cash-flows exchanged with customers and suppliers initially arose. A similar delay occurs for the settlement of CT. Taking account of the differences in timing between incoming and outgoing cash-flows lies at the heart of inventory and lot-sizing theory, as exemplified by the fact that a large part of the holding costs of a typical product consists of the opportunity cost of capital. In models where the costs and revenue streams are modelled as cash-flows, it no longer seems so counterintuitive that CT and VAT might have some role to play in inventory and lot-sizing theory.

Brexit, the process of the UK leaving the EU, may affect the rules related to taxation of firms within the UK and in its interactions with suppliers and customers located in other countries inside and outside of the EU. Current rules related to VAT collection for transactions between nations within the EU may change. Regulations may also change for custom duties and for the double tax relief that multinational currently benefit from. Changes to trading rules may alleviate or reinforce current barriers, or introduce new onces. These changes may also affect the relative desirability of firms to source from firms located in certain countries. A study on the impact of current and possible future tax rules for transactions between a firm and its suppliers and customers is therefore also timely.

It is worthwhile to stress that our focus is on improving the capability of OR models for operational problems of inventories and product supply. The main quest in this work is developing a proper (mathematical) methodology as to account for operational processes associated with taxes in these optimisation models. Testing the usefulness of these tax-adjusted models is accomplished through comparing the optimal solutions with those achieved from the original models. While there is certainly a need for empirical work to establish the desire from practice for taking account of taxation in inventory and sourcing decisions, this is work that we leave for further research. More importantly, we argue that so little support has been offered from existing OR methods, that very little is known in the literature about the potential value of incorporating taxation schemes into operational decisions. This in part also explains the focus of this work on OR model development and analysis.

1.2 Problem Description

The thesis aim is to contribute to the area of research devoted to the development of OR methodologies and techniques that can explicitly account for taxes, and to increase our insight into whether, and if so, under which conditions, the consideration of profits after tax explicitly in such models produce better OR decision support models in comparison to models which do not explicitly consider taxes.

Many organisations have to deal with both inventory decisions and accounting for taxes on a day-to-day operational level. In the literature, however, mainly strategic interactions between taxation and inventories/sourcing have been researched. This includes research on location decisions, transfer price setting, and methods for the valuation of inventories. While the linkages between operations and finance have been more extensively researched, there is comparatively very little research on how OR optimisation models for inventory management and product sourcing can be adapted as to account for taxation rules.

It is not only the issue of not knowing whether it is or is not important to account for taxation in inventory optimisation models, there seems also no good methodology so far developed in the literature by which taxation can be explicitly incorporated into the inventory optimisation models. *How* to account for tax in the operational inventory decisions, such as in lot-sizing decisions (when and how to order from which supplier, or when and how much to produce where), is still a largely open research question.

In this thesis, we want to develop the optimisation methods that account for tax rules explicitly, and use these to determine whether or not the inclusion of these tax rules have much of an impact in steering the optimal decisions from the model. Other researchers may want to focus on empirical research, looking into how companies currently account for taxation in their optimisation decisions. While we cannot offer concrete evidence of this here, there certainly seems to be a rational argument to do so. Consider a builder, facing over a long period of years a problem of supplying, constructing and delivering physical buildings, while also paying suppliers and receiving partial sums for work milestones completed from customers. A rational argument can certainly be made that the builder may wish supplier invoices to arrive just prior to, and customers invoices to be issued just past any of the VAT tax points (see Chapter 2), because this will help to maximise the Net Present Value of extracted cash that the builder can invest in the next best investment opportunity. Furthermore, it also known that costs made towards investments in the company reduce its taxable profits. This thought leads to the argument that Corporation Tax has the effect that transportation costs in an inventory optimisation model are actually cheaper than the VAT exempt price, which should lower optimal lot-size decisions. This thesis specifically investigates whether there is a rationale for embedding taxation rules and processes in operational research models that help firms make decisions about their inventory policies, and the associated selection of suppliers and markets to sell products to further their promotion strategy. The rationale is deemed to be established if either or both of the following phenomena occur: (1) the optimal solutions of the tax-adjusted model differ significantly from the non-adjusted model; (2) the non-adjusted model's decisions lead to inferior profitability when evaluated through the tax-adjusted model's objective function.

Taxation rules vary across the world, and also evolve over time. We use UK and EU tax rules of the year 2015 as the basis for the understanding of relevant legislation, and from which the base case models are constructed. The taxes of particular interest in this context are Value Added Tax (VAT), Corporate Tax (CT), and Import Duties (ID). The cash-flow approach we develop here is hopefully flexible so that many similar types of legislation from other areas of the world can likewise be incorporated into optimisation models.

In our investigations, we mainly consider the situation of a firm located in the UK and which interacts with another firm that is its supplier, and sells the products on to customers. A distinction can then be drawn between different situations, depending on the country where the supplier is located, the countries where the customers are located, and whether these customers are registered companies or end consumers.

In some cases, there is also a need to consider third party logistics providers who are responsible for transporting the products from the supplier to the firm, and to which tax rules these firms are subjected to.

HMRC is the body responsible for the tax collection in the UK, and a useful repository of documents is available from the HMRC website. From the study of these rules (see Chapter 2), it is clear that not only the nominal tax rates are important, but also the processes by which these taxes are collected.

While nominal tax rates can differ depending on the type of product or firm, within each set of firms subject to the same nominal tax rates, the processes by which this tax is to be settled with HMRC can still differ greatly. It is thus also important to distinguish between these processes, and to identify to which degree firms may or may not be able to choose their preferred tax settlement process.

We will call a particular set of rules under which a firm will operate for settling its tax obligations a *tax scheme*. As such schemes will differ depending on the particular tax considered, we distinguish between VAT, CT and ID tax schemes.

An important characteristic of a tax scheme is the difference in the timing between the moments firms exchange cash-flows with suppliers and customers, and when they exchange tax-related cash-flows with the government.

The methodology applied in this thesis should be able to account for these different tax-related cash-flows. From the analysis of the developed tax models on particular scenarios, insights are to be formulated with respect to how and to what degree tax rules affect the optimal (logistics) decisions of a firm. In order to develop the theory of inventories and sourcing, the analysis is to include a comparison with traditional inventory models. A valuable question, for example, is whether or not existing models that do not consider taxes explicitly may still be used by considering the impact of taxes e.g. through a suitable choice of their parameter values.

1.3 Research Objectives

This thesis aims to develop models for inventory management supplier selection, sourcing decision and promotion strategy that explicitly account for tax schemes and aim to maximise the Net Present Value (NPV) of this activity for the firm.

It is difficult to identify the impact of tax schemes when using traditional modelling techniques which do not explicitly consider the impact of the time value of money. A technique which offers a better foundation is to use cash-flow NPV modelling. Cash-flow functions can explicitly account for both the magnitude and timing of cash that a firm exchange with exchanges with the outside world.

In particular, we aim to show how the use of the Laplace transform of all relevant cash-flows, including those associated with tax payments, can lead to improved OR models about inventory and sourcing decisions in comparison to the corresponding 'non-adjusted' OR models (i.e. models that do not account for the cash-flow effects of taxes in this manner).

A literature review (see Chapter 3) reveals that no study yet exists which examines the impact of CT and VAT tax schemes on inventory-related problems in tactical and strategical levels in either the traditional inventory modelling framework or the cash-flow based NPV framework. Following the well-used principle known as Occam's razor, We start the investigation at the roots of inventory theory.

We apply deterministic inventory models. The origin of deterministic inventory is traced back to the Economic Order Quantity (EOQ) model and the Dynamic Lot Sizing Problem (DLSP).

It is not within the aim of this dissertation to advance the theory of cash-flow NPV optimisation, but rather to use existing methods from this area in the study of the impact of taxes on inventory order quantity, ordering time and related promotion strategy. Given the state-of-the-art as sketched above in both the investigation of tax schemes and the application of cash-flow NPV modelling, the research objectives of this thesis are thus to develop models that can demonstrate:

1. the impact of tax legislation and tax schemes applicable to a firm's activity that is subject to the assumptions of the classic EOQ model;

2. the impact of relevant characteristics of suppliers, customers, or transport firms with which the firm trades in the context of EOQ-type activities;
3. the impact of taxes on the selection of suppliers for multiple products subject to EOQ-type assumptions;
4. the impact of taxes on an activity that is subject to the assumptions of the classic DLSP model and promotion strategy.

The first objective aims to arrive at an analytical model and solution of similar complexity as the classic EOQ model. This will increase our understanding of how taxes fundamentally alter the classic trade-off between holding and set-up costs, and offer insight into the impact on the overall profitability of the activity. It will also enable us to identify which other characteristics of the firm affect the optimal lot-size decision and, if a firm can choose its tax schemes, which tax schemes will help to maximise the firm's NPV.

The second objective is to help understand which characteristics of the parties involved in the firm's activities (suppliers, customers, transport firms) are possibly of relevance in this context. This will helps us understand how possible changes to legislation, for example in the context of Brexit, might affect inventory management, the relative profitability of selling certain types of products, and the relative attractiveness of sourcing from and selling to local, EU, or international markets.

The third objective will help establish the degree by which taxes may impact the decisions related to the sourcing of multiple products from multiple suppliers. This may lead to an insight into how (changes to) tax legislation at the UK, EU, or international level might impact the level of national, European, or international trade relationships.

The fourth objective is to assess whether tax schemes affect the timing and size of order quantities in a dynamic context in which demand may change over time. Models such as the DSLP may help us also to get an insight into whether it is optimal for a firm to account for the due dates at which VAT and CT settlements are made with HRMC. It seems intuitive that this might be the case. For example, it would seem to be optimal to get orders in from suppliers just prior to the next tax point in order to reclaim the firm's input VAT and promotion strategy just after the tax point to retain the output tax for a longer period.

Overall, by addressing the above five objectives, the thesis will develop our understanding about the usefulness of the Cash-flow based Net Present Value technique in its ability to incorporate tax-related cash-flows into these well-known optimisation models. The work can also be considered to contribute to development of theory about the tax-effective supply chain. This work also supports the debate about the usefulness of crossing the boundaries between the areas of operations, finance, and tax accounting.

1.4 Outline of the Thesis

The remainder of the thesis is organised as follows. Chapter 2 gives general taxation rules in UK, it includes Valued Added Tax, Corporation Tax, Trading with EU and non-EU tax and tariff regulations. Chapter 3 provides a survey of relevant academic literature on the interface between operations and finance, and taxes and inventory decisions in particular. It identifies the gap in the literature on the study of VAT and CT tax schemes in the operations research literature in general, and thus in particular also in the study of inventory management and supplier selection. The chapter also reviews the literature on the methodology of cash-flow NPV modelling of logistics systems.

Another precursor activity needed to start this work is to develop a detailed understanding of the tax regulation on CT, VAT and ID relevant in the context of inventory management and supplier selection. This is developed in Chapter 4. From this we can appreciate that the tax schemes available to a firm depend on various characteristics of the firm and how it sources and sells its products.

Research objective 1 is developed in Chapter 4. This work demonstrates that CT and VAT tax schemes are both affecting optimal economic order quantity decisions, and that the impact of taxes is dependent on the total turnover of the UK-based firm. It is shown that not accounting for taxes may lead a firm to choose lot sizes which may be up to 22% over the tax-optimal lot sizes. The importance of accounting for taxes decreases with the profit margin on the product, but is significant for products that sell at low margins. The classic EOQ formula can still be applied by either adjusting the opportunity cost of capital, or by adjusting the financial values of set-up costs and unit holding costs. The classic EOQ objective function, however, needs to be adapted in order to assess the profitability after taxes of an activity. Using the EOQ formula without this consideration may lead companies to sell products that, from the NPV perspective, are not worth selling.

Chapter 5 develops the material to meet research objective 2. The models developed in Chapter 4 are extended to cases where suppliers may be located within other countries of the EU or outside of the EU, and where the UK firm may sell a part of the demand to UK customers, and another part to customers outside of the UK. Current EU rules on VAT processing appear to affect the relative profitability of an activity, and lead UK-based firms to prefer, ceteris paribus, suppliers and transport companies that are located outside of the UK but in another EU country.

Chapter 6 focuses on developing models to develop optimal sourcing strategies so as to meet objective 3. As Chapter 5 develops models to show how the logistic costs and overall profitability depend on whether trading is done domestically or internationally, these results can be used as a starting point towards the development of supplier selection models. The general set-up is that multiple types of products can be offered by multiple suppliers at certain prices and logistics costs. The first case examined is the single-supplier situation, where one type of product can only be supplied by one preferred supplier. Each supplier has a capacity constraint on the total volume of product that can be delivered in a single order. This problem is solved with Lagrangian relaxation.

The fourth research objective is covered in Chapter 7. The Dynamic Lot Sizing Problem (DLSP) modelling framework is used to investigate whether corporations would benefit from considering tax effects in their decision about the optimal timing of sales promotions and ordering in a context of dynamically changing demand levels. We use the framework of dynamic programming to solve the corresponding cash flow models over finite planning horizons. In the further analysis we illustrate how the input tax payment structure impacts ordering time and quantity decisions, and how the output tax payment system may affect the optimal timing and duration of promotion efforts.

Chapter 8 provides a summary of contributions and findings, implication and further research direction.

Chapter 2

Tax Regulations

2.1 Introduction

The consideration of tax schemes, including accounting for the methods by which a firm and a government settle taxes, needs to start with a thorough review of relevant regulatory rules and processes. The purpose of this chapter is therefore to review and explain the tax rules that will be used in most examples of subsequent chapters. It starts with the most complex of taxes, the Value Added Tax. It continues with a description of Corporate Tax, and a review of EU-specific rules related to the transfer of goods or services across borders, and a short description of possible Brexit implications.

2.2 Value Added Tax (VAT)

Value Added Tax (VAT) schemes are adopted by most countries in the world, including Europe and China, and replace the more traditional sales tax schemes. In 2011, only 11 countries and 9 territories under two countries did not use the VAT method. This notably includes the USA where a sales tax is used.

In principle, VAT and sales tax have the same objective - only tax end consumers. In practice, they differ. A sales tax is only once collected from the end consumer and remitted to the government. With VAT, each time goods are sold wherever in the supply chain, collections of taxes occur and remittances to the government and credits of taxes already paid have to be accounted for. We illustrate with the following two examples the differences between a sales tax and VAT approach.

Sales tax can be explained with the following example. The tax rate is set at 20%.

Example 2.1.

- *A manufacturer spends £*100 *on raw materials paid out to his supplier, and certifies to her supplier that she is not a final consumer;*
- *The manufacturer charges a retailer £*140 *for the product produced, and checks that the retailer is not a final consumer; the manufacturer's profit is £*40*;*
- *The retailer sells the product for £*180 *to a final consumer, charges the consumer* $1.2(180) = £216$*, and pays the government* $216 - 180 = £36$*; the retailer's profit is £*40*.*

With a VAT, the process looks as follows:

Example 2.2.

- *A manufacturer spends £*100 *on raw materials and pays the supplier £*120*; the supplier will pay the government £*20*;*
- *The manufacturer sells to the retailer for £*140*, charges the retailer £*168*; the manufacturer pays the government* $0.2(140) - 0.2(100) = £8$*; the manufacturer's profit is £*40*;*
- *The retailer sells the product for £*180 *to a final consumer, charges the consumer* $1.2(180) = £216$*, and pays the government* $0.2(180) - 0.2(140) = £8$*; the retailer's profit is £*40*.*

The government receives £36 under both a sales tax and VAT scheme. In a sales tax scheme, a seller must check whether a buyer is a final consumer or not, but there is little incentive to do so. In a VAT scheme, every buyer is incentivised to reclaim the Input VAT back from the government. The two systems may hence differ in the total amount of tax a government actually collects in practice.

Tax rates naturally differ between countries. The operational mechanisms of how governments allow businesses to settle their VAT claims differ too. Within the UK, for example, more than a handful of different VAT schemes currently exist. This chapter gives an overview of the different VAT systems in place in the UK as was the situation in May 2015. We use this as a framework for further analysis in subsequent chapters.

Value Added Tax (VAT) is an indirect tax collected by HMRC on consumer expenditure and imports into the UK. The tax rate depends on the type of product or service consumed. The rate applicable to most products is set at 20% of the sales price. For a firm, it is important to make a distinction between its Output VAT and Input VAT.

Definition 2.1. The OUTPUT VAT is a tax collected by a firm from its buyer on a sale of goods or services to this buyer. INPUT VAT is a tax paid out by a firm to its supplier on a purchase of goods or services from this supplier.

Hence, whenever there is a sales transaction of physical goods or services between a firm and its outside world, VAT has to be exchanged in the opposite direction of the flow of goods or services. If a firm makes a sale, the Output VAT that the firm collects from its buyer is to be paid out to HMRC. If a firm purchases goods or services from a supplier, the Input VAT that has been paid out to this supplier can be claimed back from HMRC if the firm is VAT-registered at HMRC and if the goods or services were purchased for its business purposes. However if the firm is not registered, it cannot claim it back.

If the firm is a non-registered business, VAT is thus a tax on its consumption. If the firm is registered, it ends up paying to HMRC any strictly positive net difference of its Output VAT and Input VAT, or may claim back this net difference if it is negative. Businesses with a turnover (excluding VAT) for the previous 12 months exceeding a given threshold must register. The current threshold level is $\pounds 82,000$. Note that this turnover is based on the sales price before VAT from HMRC (2015a) document.

The net result of registered buyers claiming back their Input VAT and sellers paying out their Output VAT is that eventually HMRC receives a net payment of the total VAT on sales to all end consumers *and* non-registered businesses in the country.

We will now focus on the details of when firms need to exchange VAT with HMRC. In the UK, businesses are allowed to choose between different VAT schemes in an aim to let them select a scheme that best fits their business models. These schemes are discussed in subsequent sections. Before discussing this in detail, the table below provides a summary of various VAT accounting schemes and the threshold values for firms to join and leave these different VAT schemes.

VAT Accounting Scheme	Threshold to Join	Threshold to Leave
Flat Rate Scheme	£150,000 or less	More than £230,000
Cash Accounting Scheme	£1.35 million or less	More than £1.6 million
Annual Accounting Scheme	£1.35 million or less	More than £1.6 million
VAT Retail Scheme	No specification	More than £130 million
Bespoke Scheme	£130 million	No specification

2.2.1 Standard VAT Accounting Scheme

There are several options to be distinguished. All VAT schemes however are based on VAT payments with HMRC at specific points in time. In the standard scheme, four VAT returns must be completed each year, one at the end of each quarter. Both VAT due to HRMC and VAT refunds from HMRC are payable and repayable quarterly. At those times, businesses have to settle their VAT liabilities from all relevant sales transactions. The tax point determines whether a transaction is to be included in a firm's next exchange with HMRC.

Definition 2.2. The TAX POINT is the time when the VAT on a sales transaction becomes a liability to the firm in respect to HMRC. It will now have to be included in the next VAT return to HMRC.

With invoice based accounting in the standard scheme, the default tax point is the VAT invoice issue date (which usually coincides with the invoice issue date). The time an invoice is issued does not have to coincide with the time that the cash is exchanged nor the date of supply of physical goods or services.

If the VAT invoice is issued 15 or more days after the date of supply, the tax point will be the date of supply. If payment or invoice is issued before the date of supply, the tax point is the date the invoice was issued or payment was made, whichever is earlier. If payment was made in advance of supply and no VAT invoice was yet issued, the tax point is the date of payment received. The tax point will be the time that cash is exchanged if the corresponding invoice has a later issue date.

A cash-flow advantage of invoice based accounting may arise when the firm pays its supplier later than the moment the invoice is issued, since the firm can already claim back the VAT from HMRC in its next VAT return even when it has not yet paid the VAT to its supplier. A disadvantage may arise when customers pay their invoices late as the VAT will be due in the next VAT return although the firm may not yet have received the cash from HMRC (2015b).

The net payment to HMRC can in general to be expected to be positive since the price a firm must charge for its products must be larger than the total costs it makes to realise this. However, since a firm's expenses may occur earlier than its revenues, it may be that, in a submission period, the input VAT exceeds the output VAT and then the firm can claim back from HMRC.

Business with an annual turnover exceeding a certain threshold must move to standard VAT accounting. The current threshold is £1.6 million. However, they may still have the choice between adopting invoice based accounting or one of the other available options. The discussion these other options is post-poned to later sections.

2.2.2 Annual Accounting VAT Scheme

Only businesses with an annual turnover not exceeding £1.35 million(continue use as estimated turnover remain below £1.6 million) and which are not a division of a company or part of a group of companies can join this scheme. In this scheme a firm only needs to file one VAT return per year. However, in addition, it needs to make either nine monthly or three quarterly payments in between. The scheme is offered as a means to lower the administration costs and help a firm in managing its cash flow.

More specifically, in the 10% scheme the firm makes nine monthly installment payments to HMRC at the end of months 4 to 12 in its accounting year. Each payment consists of 10% of the firm's total annual VAT liability of the *previous* year. A balance payment is then to be made at the end of the second month after the end of the firm's current accounting year (i.e. month 14). This balance payment will be the difference between total annual output VAT due and total annual input VAT of the current accounting year, minus what has already been paid out in the nine installments. This balance payment is accompanied by the submission of the firm's annual VAT return form to HMRC from HMRC (2015c) document.

Alternatively, the firm can choose the 25% option, by making three quarterly installments of 25% of the firm's VAT liability of the previous year, paid at the end of months 4, 7, and 10, and then a final balance payment two months after the accounting year at the same time of its VAT return.

A firm that has not yet been registered for more than 12 months and adopts the annual accounting scheme will have to base its first-year installments on an estimate of its total VAT liabilities for that year.

The determination of VAT liability is typically based on invoice based accounting, but firms may also choose to use this annual accounting scheme based on cash accounting or a flat rate. These are explained in the next two sections, respectively.

2.2.3 Cash Accounting VAT

In the cash accounting scheme the tax point is determined by the time that cash has been exchanged rather than the time that the invoice was issued. Cash accounting can be used to replace invoice based accounting in a standard VAT scheme or annual VAT scheme.

Its main purpose is to help a firm with their cash flow if customers pay late, since the firm only pays the VAT to HMRC when the customers have paid. However, the firm then cannot claim back the VAT on its supplies if it has not yet paid its suppliers. It also means firms may want to make cash purchases just before the next VAT return date in order to reclaim VAT quickly.

This scheme can only be used for the onward supply of goods inside the UK. There are further restrictions on the use of cash accounting in an annual VAT scheme. In particular, cash accounting cannot be used with the annual VAT scheme if the firm: imports goods from other EU countries; issues VAT invoices where full payment is not due within 6 months time or in advance of providing goods and services (issue invoice in advance which can kept the business below the turnover limit to joining the scheme);

or when it buys or sells goods using lease purchase, hire purchase, conditional sale or credit sale.

The best way to use cash the accounting scheme is if you are a sole trader, or running a business as a partnership and providing services on cash-basis like electricians or painters. The scheme is not available to incorporated business such as limited companies and Limited Liability partnerships.

There is a 25% tolerance in this scheme, which means that firms with an annual turnover exceeding £1.35 million can still use it until the annual taxable supplies reaches £1.6 million(see HMRC (2015d)).

2.2.4 Flat Rate VAT

A flat rate scheme is issued by the government in an aim to simplify taxes for small businesses. Only firms with an estimated VAT taxable turnover less than £1500,000 (excluding VAT) can join the flat rate scheme, and can only make use of it until this reaches £230,000. The flat rate VAT can be used in both the standard accounting scheme or the annual accounting scheme. The flat rate scheme works on invoices, but also has its own version of cash based accounting. It can also be combined with a retail scheme, which is a special scheme for firms which sell many low-valued items, but which is not further considered in this thesis.

Instead of paying the output VAT the firm has received from its customers (at say 20%), the firm has to pay a flat tax rate on its turnover (*including* VAT charged to customers). This flat rate is a function of the sector in which the firm is classified. Currently, the flat rate ranges between 4% and 14.5%. The firm also gets a 1% discount on the first year of operation. The firm, however, cannot claim back any input VAT charged by its suppliers (seeHMRC (2015e)).

Although companies cannot claim back any VAT on purchased goods, they can reclaim VAT on capital asset purchases over £2000 on the same receipt. Like standard VAT, flat rate scheme requires to complete a quarterly VAT return.

Example 2.3. *A firm has made a sale at a VAT free price of* £1000 *and charges the customer* £1200 *(i.e. including the* 20% *standard VAT rate). If the flat rate is at* 10%*, the VAT due to HMRC will be* £120.

The flat rate scheme may not work well if the firm also incurs significant input VAT. It is in particular recommended for businesses that have very few VAT chargeable purchases and expenses.

Example 2.4. *Consider the firm of Example 2.3. If for this sale the firm had expenses of at a VAT free price of* £500 *and has to pay its supplier in total* £600 *(including the*

VAT), the firm cannot claim back the £100. Overall, the firm ends up paying £20 more to HMRC then if it would have used a standard invoice or cash accounting scheme.

2.3 Corporation Tax

Corporation tax is a tax on profits from doing business, from investment, and from capital gains. Corporation tax is paid by businesses as a limited company, by any foreign company with a UK branch or office, and also a club, cooperative or other unincorporated association. A UK company pays CT on all its profits from the UK and abroad. If the company is not based in UK but has a branch here, it only pays CT on profits from its UK activities.

Corporation tax is charged on taxable profits. This includes trading profits and most investment profits. It also includes any capital gains (where an asset is sold for more than what was initially paid for it), usually referred to as chargeable gains. UK corporation tax rules set out exactly which reliefs and capital allowances can be set against business income in calculating company taxable profits.

The accounting year end is linked to the date that a limited company chooses to begin its accounting year, and determines when CT is due. An accounting period for CT purposes cannot be longer than 12 months.

Many companies pay CT nine months and a day after their accounting year end. For example, if the accounting year ends on 30th April, the CT will be due by 1st Feb in the next year. Large companies will have to pay their CT earlier. Starting from April 2017, companies with annual taxable profits of over £20 million will be required to pay CT in instalments four months earlier than at present. Payments will be due in months 3, 6, 9 and 12 of a 12-month accounting period (pwc), but the rate of corporation tax is expected to be reduced to 19% in 2017 and to 18% in 2020.

There are common rules if CT is paid early or late. Early payments receive and an interest rate deduction from HMRC or 'credit interest'. Late payments incur a late payment interest. Any late payment interest a firm pays to HMRC is tax deductible for CT purposes. This means that the firm can include this expense in its company accounts for the accounting period.

2.4 Imports, Exports, Acquisitions and Removals

2.4.1 Transferring Goods out of an EU Country

A firm located in an EU country selling a small volume B2C (Business-To-Consumer) to customers located in another EU country, must charge Output VAT at the UK tax rate. The same applies when selling B2B (Business-To-Business) to firms in another EU country which are not VAT registered. However, each country has a distance selling threshold. If the value of business sales to that country exceeds this limit, firms must register for VAT in that country and charge the rate of VAT on sales using the VAT rate applicable in that country. Sales to another EU country are called ‘dispatches’ or ‘removals’.

If a firm is sending goods to another business which is VAT registered in the destination EU country, the firm can zero-rate for VAT purposes.

Zero-rated does not means that the goods and services are VAT exempt, but the seller will charge the buyer an Output VAT of £0, which is then claimed back by the buyer as an Input VAT at £0. In effect, it cancels out the cash-flow between the firms and between the tax authorities across different nations of the EU. It is worth highlighting the difference between zero-rated VAT transactions and goods which are VAT exempt, which are goods that are not included in the VAT system.

The sales of goods that are exported to countries outside the EU can be zero-rated for VAT purposes, but firms must provide evidence of the goods indeed having been exported within 3 months of the time of sale. The time of sale is the earlier of the days the goods are send to the customer, or the day firms receive full payment for them. Export to non-EU countries may occur through sales or transport via another EU country. Firms can also zero-rate these products destined for export, so long as the business retains proof that the goods have indeed been ultimately exported to a non-EU country within the time allowed.

2.4.2 Transferring Goods into an EU Country

Most goods can be imported B2B from another country inside the EU with a zero-rate VAT and mostly no import duty to pay. Such movements are called ‘acquisitions’ rather than ‘imports’. However, the acquiring firm will still account for this unpaid acquisition tax at the UK VAT rate on its VAT return, which will cancel out with its VAT reclaim.

Imports from outside the EU are to be declared to UK customs, and the firm will generally have to pay import duty and Import VAT (plus VAT on import duty). Typically, import duty is a percentage of customs value of the goods. The import duty percentage in general depends on the classification of the goods and where they come from. The customs value of the goods includes the price paid for the goods, the shipping costs, and the insurance cost. The VAT is then priced on the total of customs value and import duty.

Import VAT is paid directly to HMRC, whereas domestic VAT is normally paid to a supplier of goods. As a registered business, a firm can then reclaim this Import VAT on its next VAT return. It cannot reclaim import duty.

Normally the Import VAT is to be paid before customs will clear the goods for entry in the UK. However if the firm does not need the goods immediately, or intends to re-export them, goods can be stored in an authorised customs warehouse, and only pay the excise duty or VAT until it removes the goods into free circulation. If the firm is importing goods that it plans to supply to another EU member state, then claiming Onward Supply Relief (OSR) allows the firm to import the goods without paying import VAT. Instead, VAT is paid when the firm supplies the goods to its customer.

2.5 Possible Implications from Brexit

The 2016 vote of Britain to leave the EU may potentially have significant economic implications, including on the UK tax system. The official announcement to leave was made in March 2017, and in principle this implies that the UK will not actually leave the EU for at least another two years. So in the short term there will be no significant impacts on the trading across the UK and the rest of the EU.

Leaving the EU means the UK government may investigate introducing new VAT rates or changing its VAT rules. However, as the VAT is one of the major sources of tax revenue in the UK, we can safely assume that the VAT will still be applied in some form.

Exiting EU means exciting the free trade arrangements with the EU, implying the abandonement in principle of the acquisition and removal systems. This means that import of the products from another EU country into the UK may now be regarded as a taxable event for VAT purposes. In principle then, the UK tax rate will be applicable and the VAT is charged as input VAT. It is likely that EU countries will reciprocate. From a financial perspective, the implications from VAT are perhaps not that great beside the potential widening differences in VAT rates itself, but it can be expected to impost a significant additional administrative burden on the firms involved in such transactions.

Without any new agreements, the transactions with the EU countries may now be regarded as imports and exports. It is likely that the UK may consider levying import duties on the products so as to provide equal treatment as given to other third countries.

In return, however, the EU countries may then adopt similar strategies for imports from the UK. This will surely impact the amount of profitable trade between the UK and the EU.

It is also unclear how negotiations will develop around the special schemes for exports through other EU countries, such as the OSR schemes, etc.

It is possible that firms located in the UK but with a large EU footprint will reconsider their location. The UK already has one of the lowest corporation tax rates in the EU but ministers believe a further cut could help keep companies in the UK and attract new investment (HMRC (2016)).

2.6 Conclusion

Most countries in the world use a Value-Added-Tax (VAT) system. VAT is an indirect tax on consumption. In the UK, businesses collect this tax on behalf of the government, and then submit VAT returns to work out the net payments to be made to HMRC. Depending on its expected annual turnover or type of business, a firm may have several options to choose from for exchanging this tax with the government.

In this chapter we have reviewed two major schemes which will be used in the further analysis in subsequent chapters. The standard accounting scheme uses quarterly payments based on invoice-based accounting, but may also be combined with cash-based accounting or flat rate accounting. The annual accounting scheme comes in two options in terms of number of interim payments and can be based on invoice, cash, or flat rate accounting.

Corporation Tax (CT) rates and rules are an important instrument for a country to affect an organisation's strategy on logistic decisions. As from the new tax year in 2017, the Corporation Tax (CT) has been cut to 19% from the high of 20%. This gave the UK the lowest CT rate of the European countries. This rate is said to be further cut to 17% by 2020.

UK firms selling to or buying from other countries in the EU can currently still work in the system of removals and acquisitions, and enjoy mostly zero import duties. Import from outside EU is typically subject to import duty and VAT, while export businesses can be VAT zero-rated. With or without any new negotiated agreements, Brexit is likely to affect the rules of trade with the EU, and perhaps beyond.

Chapter 3

Literature Review

3.1 Introduction

This thesis is about finding out the role of taxation considerations in operational research models that help firms make decisions about the inventories and the selection of suppliers and associated inventory policies, and whether the consideration of profits after tax explicitly in such models will produce better OR decision support models. There is actually very little research published on this particular topic.

In the first part we consider literature on the links between operations, finance and taxes. This area is broad, but upon closer inspection not that relevant to the aim in this dissertation. In the second part, we proceed with literature specifically on inventories and taxes. We complete with literature on constructing cash-flow oriented Net Present Value models. While this avenue has not yet been explored in the literature, we view this method to be particularly suited for the examination of how taxes can be explicitly considered.

3.2 Tax and Operations

3.2.1 Operations and Finance

Perhaps a useful starting point is to view the topic of this dissertation as being a subject in the multi-disciplinary field of operations and finance. Yang, Birge, and Parker (2015) point out the two ways in which one can view the purpose of studying the interaction of operations with finance: either that finance related activities have the potential to make an impact on operational decisions, or that the consideration of operational behaviour provides new aspects driving further financial decisions. In the review of Zhao and Huchzermeier (2015), the authors describe these fields as matching supply and demand for material flows and monetary flows, respectively, and view these as a 'closed-loop' system. They presents a risk management framework based on the level in which they are complements or substitutes, and the level of centralisation versus decentralisation. While an insightful paper, it is limited in its discussion of taxation issues and does not mention Net Present Value. Multiple topics on this interface between operational decisions and financial decisions have gained considerable interest, such as: liquidity constraints on inventory decisions in Kouvelis and Zhao (2012) where both retailer and supplier are capital constrained and and optimal structure of the trade credit contract is designed; the impact of accounting methods to value inventories and tax implications as in Bougheas, Mateut, and Mizen (2009) and Guenther and Sansing (2012), which also indicates that the value assigned to inventory is important as it affects the balance sheet and further value of the company; impact of capital structure on the retailer's operational decisions as in Xu and Birge (2004) and Hu and Sobel (2005); working capital management in supply chains as in Protopappa-Sieke and Seifert (2010); and the capital constrained news vendor problem in Dada and Hu (2008). A particular subset of articles focus on supply chain finance (SCF). According to Gomm (2010), the purpose of SCF is to optimise finance flows across company borders to decrease the whole cost and accelerate the cash utility. For a further excellent review of this area, we refer to Zhao and Huchzermeier (2015). These studies show primarily the strategic links between operations and finance; they do not address how the tax collection process as we have described in Chapter 2 actually unfolds, and how this may affect classic OR optimisation models that we can find in OR textbooks, in particular those about inventories.

3.2.2 Operations and Corporation Tax

Corporate Tax (CT) has received attention in the operations literature through transfer pricing. Cost minimization is now replaced by after-tax profit maximization, see e.g. Perron, Hansen, Le Digabel, and Mladenović (2010), K.-K. Kim and Park (2014) and Martini (2015). The focus of these research studies is on location and allocation of production and distribution volumes, where transfer prices can be part of the decisions variables as to take advantage of different CT rates and rules between nations, and these studies focus on the pricing decision rather than looking at how tax can influence a firm's operational decisions. More recent work by Niu, Xu, Lee, and Chen (2019) show that operational decisions like ordering time, pricing and market move timing can be adjusted to generate additional profits through tax planning. These studies address the tax implications without making use of cash-flow thinking, while modelling operational aspects from a high-level perspective.

3.2.3 Operations, Value Added Tax and Trade Tariffs

Trade tariffs and tax play an important role in the global manufacturing network design, which is also connected to sourcing strategies with different duty rate or tariff concessions. Fernandes, Hvolby, Gouveia, and Pinho (2009) investigated how a multinational company should optimally allocate the inventory in geographical dispersed subsidiaries and illustrate the impact of tax elements on the distribution network design. Y. Li, Lim, and Rodrigues (2007) considered a free trade agreement (FTA) in the manufacturing management at the firm level. Kouvelis, Rosenblatt, and Munson (2004) proposed a model used to design a global facility network which incorporates trade tariffs and taxation issues. Because it is a difficult problem to obtain an optimal solution, the contribution of this work provides some insights and analysis for certain scenarios. Arntzen, Brown, Harrison, and Trafton (1995) presented mixed integer programs to optimise the global supply chain to minimise the sum of variable production costs, inventory holding costs, shipping costs, fixed set up costs minus the savings from duty drawbacks and duty relieves. However, these taxes are only considered as part of production costs. Most research regarding tax and operations focuses on corporation tax. Other research covers value added tax and tariffs like V. N. Hsu and Zhu (2011), who developed an analytic framework to study the impact of China's export-oriented tax and tariff rules on the optimisation of major supply chain structures. The analysis shows that the optimal decision depends on the purpose of a product whether it is for export or for domestic customers. Zhen (2014) and Xiao, Hsu, and Hu (2015) proposed production and outsourcing decision under China's VAT regulation and developed a solution method based on cross-entropy-based algorithm. Xu et al. (2018) studied MNF's procurement strategy on whether to buy its component procurement or rely on its contract manufacturer to purchase the requirement components, taking into consideration factors such as variance tax rules (VAT refund rate) and multi-market structures. The tactical strategy decision comes with the export-oriented tax policy in China, which means there is in flow and out flow of VAT payment differences in export products. As products are exported, the firm would be unable to collect output VAT from the buyer but can receive a partial refund of input VAT based on the Chinese tax rules. Hence, there is no impact on sourcing and production decision if the tax policy used in the domestic market as input and output tax is at the same rate. These works focus on mainly through the perspectives of different VAT refund policies. No study has yet approached modelling VAT by considering also the timing of these cash-flows in a NPV approach.

3.3 Tax and inventories

3.3.1 Economic Order Quantity Model

The Economic Order Quantity (EOQ) (Harris, 1913) captures the essential trade-off in many applications of production and inventories. In subsequent chapters we will make use of the Net Present Value-equivalent model extensively, and we refer for the detailed description of this model to Chapter 4.

The classic EOQ model balanced three main components: the ordering cost, the demand rate, and the holding cost. The ordering or set-up cost is a fixed cost for placing an order. In just-in-time (JIT) philosophy, the aim is to reduce it as to achieve almost no change-over costs and time and thus to have very small batches in production.

The holding cost is determined by a unit holding cost parameter. Silver, Pyke, Peterson, et al. (1998) for example, propose the formula:

$$h = \alpha v + f, \tag{3.1}$$

where v is the money invested per unit of product held in stock and f is the unit 'out-of-pocket' holding costs, representing the sum of real costs incurred from keeping stock (e.g. warehouse rent, electricity usage, ...) but which are variable with the amount of stock held. For a further discussion, see e.g. Azzi, Battini, Faccio, Persona, and Sgarbossa (2014). In this work we investigate whether it is possible to obtain an EOQ result by simply knowing how taxes will 'adjust' the set-up and holding cost parameters in the model. This suggestion has been made in Silver et al. (1998), but it is never really explored further to our knowledge.

The EOQ has been extended in many ways, including: planned back-orders (Huang & Wu, 2016), finite production rate as (Björk, 2012), quantity discounts (Mendoza & Ventura, 2008) and imperfect quality (J.-T. Hsu & Hsu, 2013). The constant demand rate assumption is extended to different stochastic demand cases in e.g. Presman and Sethi (2006). Majority of the contributions in the literature that are based on the EOQ model use Harris' direct costing method. One strong advantage of NPV, compared to the direct costing method, it that it is sensitive to the temporal allocation of the payments. See also Section 3.4.

Literature looking specifically towards understanding how these traditional inventory optimisation models are affected by corporation tax are very limited. Yi and Reklaitis (2007) indicate that the consideration of corporation taxes will decrease the optimal production lot and storage sizes. Michalski (2013) is the only author who adapts the EOQ formula with the explicit inclusion of the CT rate, indicating a reduction of the optimal lot-size. There is a lack of research that offers a comprehensive methodology for addressing the actual processes by which corporation tax is collected, and how this process may further affect production and inventory lot-sizing decisions. The methodology applied in this dissertation is aimed at closing this gap.

3.3.2 Accounting Methods for Valuing Inventory

When dealing with inventories and the cost of carrying inventories, we have seen in the previous section that we need to know the value invested into the products in order to determine optimal inventory levels (see also e.g. Section 3.4).

The value of items in stock is also important for tax reasons. As seen in Chapter 2, the price charged for a product determines the amount of Value Added Tax (see e.g. Section 4.3). The value of the products in stock has also implications for determining corporate tax on operational profits (see e.g. Section 4.4). On the balance sheet, inventory is reported as a current asset. The way a company value assets is thus also important as it affects how investors value the company. In short, it is essential in order to understand the role between taxes and inventories, to understand how to assign value to these stocks.

A stream of literature, see Guenther and Sansing (2012) and Bougheas et al. (2009) and references therein, examines the impact of accounting methods to value inventories and tax implications. Most of the companies use the FIFO (First-In-First-Out), the average, or the standard costing method for internal uses. However, for external reporting and tax purposes, some companies prefer to use the LIFO (Last-In-First-Out) method. In a context with cost price inflation, however, the LIFO accounting method will increase the cost of goods sold in an accounting period, and hence reduce the companies taxable profits. While acceptable within the US GAAP (Generally Accepted Accounting Principles), LIFO is now prohibited under the IFRS (International Financial Reporting Standards).

In a context where prices are constant, there is no difference between these accounting methods. In most of the models we consider in this dissertation, the valuation method will thus not matter. In the case of the DLSP, we will adopt the FIFO rule.

The cash-flow NPV approach adopted in this dissertation automatically assigns value to stocks via the process of dealing with prices explicitly, see also Section 3.4.

3.3.3 Trade Credits in the Supply Chain

From the descriptions and the definition of the tax point in the previous chapter, it is clear that one of the basic features of both the value added tax and corporation tax is the time lag. That is, there is a difference between when the input or output tax affects the cash flow of the firm, and when the cash due to the government is to be settled.

Because of this, there are similarities between the tax system and trade credits in the supply chain. Trade credit is a type of short-term financing in the supply chain in which a supplier allows a buyer a certain period of delaying payment for received goods or services. This could be a period of, for example, 30, 60, or 90 days. If this outstanding amount is paid within the permitted fixed time period, then there are no further additional (interest) charges. If retailers hence sell goods in their stores for which they yet have to pay suppliers, retailers can temporarily reinvest revenue and earn interest rate on it. This can become an important source of external financing for retailers.

The literature that looks at how trade credits affect optimal order quantities and profit is quite extensive. In a single level trade credit system, the supplier offers the delay payment time period, but the retailer would not offer the trade credit to their customers: see S.-C. Chen, Cárdenas-Barrón, and Teng (2014), Teng, Min, and Pan (2012), W.-C. Wang, Teng, and Lou (2014), J. Wu and Chan (2014). In a two level trade credit system, the retailer has a permissible delay in payment from the upstream supplier and at the same time provides a permissible delay payment time to the downstream customers, see L. Feng and Chan (2019), R. Li, Chan, Chang, and Cárdenas-Barrón (2017) S.-C. Chen and Teng (2015). For a comprehensive literature review in trade credit we refer to Xu et al. (2018) and Seifert, Seifert, and Protopappa-Sieke (2013). Despite the conceptual similarities, existing literature on (trade credits in) inventory decisions in the supply chain has ignored the impact of the time lags in VAT and CT systems. In this work, we focus on the tax system's potential influence by seeking to develop a methodology by which these tax cash flows can be explicitly incorporated in inventory management models.

The tax point, explained in Chapter 2, is an important feature in accounting and finance because it will tell us which VAT or CT period a transaction belongs to, and on which VAT or CT return to include in the transaction with the government. As there is time difference to pay back to the government, operations can accrue interest on any VAT collected prior to due date. Therefore, and as demonstrated explicitly in later chapters, the features of both the VAT and CT tax schemes currently adopted in the UK involve some kind of trade credit that is naturally embedded in the system between the tax paying firm body and the government. None of the studies in the trade credit literature have actually investigated taxes as a form of trade credit.

3.4 Cash-flow NPV Modelling

The investigation of tax regulations in Chapter 2 shows that the times when corporate taxes are paid differ from the times when profits are made, and that the settlement of value-added-taxes to the government also occurs at times that differ to those when these cash-flows are exchanged between the firm and its suppliers and customers. An approach that is most suitable to capturing these aspects of tax schemes is cash-flow NPV modelling.

3.4.1 Classic versus NPV Modelling

The breakthrough article demonstrating the relative effectiveness of the Net Present Value method in modelling systems of production and inventories was arguably Grubbström (1980). In comparison to the classic inventory method, it offered a way to get more accurate estimates for the true cost of capital invested in stocks. It also offered a way to check whether the model produces a similar results as the NPV model. This approach is described in Beullens and Janssens (2014) as NPV Equivalence Analysis (NPVEA).

Beullens and Janssens (2014) describe the main difference between classic inventory modelling and cash-flow NPV based modelling. Traditional inventory theory is based on the concept of unit holding costs, and develops objective functions that represent average costs (or profits) per unit of time. The holding cost components in these objective functions are typically the most difficult to develop. These terms are typically found from the integration over a relevant time period T:

$$\frac{1}{T}\int_0^T h(t)I(t)dt,$$

where $I(t)$ is the inventory level at time t, and $h(t) = h$ is the unit holding cost, typically taken to be a constant. Cost are not discounted according to their time of occurrence, but the time value of money is implicitly incorporated by the inclusion into h of the financial opportunity cost from investments in inventory. Silver et al. (1998) for example, propose the formula:

$$h = \alpha v + f, \tag{3.2}$$

where v is the money invested per unit of product held in stock and f is the unit 'out-of-pocket' holding costs, representing the sum of real costs incurred from keeping stock (e.g. warehouse rent, electricity usage, ...) but which are variable with the amount of stock held.

The parameter α is the firm's continuous capital rate also known as the opportunity cost of capital, representing the return per monetary unit of investing in the next best available alternative for the firm. Typical values used in inventory models range but are typically $\alpha \in (0,05; 0.25)$.

It is clear that classic (inventory) modelling, by not considering the explicit timing of when costs or revenues are incurred, will make it difficult to assess how tax cash-flows could be accurately incorporated.

The opportunity cost of capital α is also used in calculating the Net Present Value (NPV) of an activity. Grubbström (1967) defines the NPV as the Laplace transform of a cash-flow function $a(t)$ of an activity:

$$\text{NPV}(\alpha) = \int_0^\infty a(t)e^{-\alpha t}dt.$$

The cash-flow function $a(t)$ represents the function that captures the rate by which revenues minus costs cash-flows enter the firm. More formally, let A be a function where the domain represents time and $A(t)$ is the cumulative amount of money received by the firm before or at time t as a result of engaging in an activity. If $A(t)$ is differentiable over $[0, \infty)$, then $a(t)$ is defined as follows:

$$\frac{dA(t)}{dt} = a(t).$$

As the time value of money is now explicitly accounted for through the Laplace formula, as pointed out in (Beullens & Janssens, 2014), $a(t)$ can no longer obtain financial holding costs as used in the traditional modelling framework. Only real revenues, costs, and other cash-flows can be included into $a(t)$.

As explained in Grubbström (1980), the cash-flow function $a(t)$ may not only be a flow, but may contain multiple Dirac delta functions a_i at points t_i, representing finite payments a_i. The cumulative function $A(t)$ is no longer differentiable everywhere, and will contain "jumps" at times t_i. The calculation of the NPV of a discrete cash-flow a_i occuring at time $t_i \geq 0$ is then simply given by:

$$\text{NPV}(\alpha) = a_i e^{-\alpha t_i}.$$

We will use this modelling approach to allow for the consideration of tax flows as part of the cash-flow function $a(t)$ relevant to this activity.

3.4.2 NPV Equivalence Analysis

Hadley (1964) was one of the first to present a method by which one can compare the objective functions of classic inventory models with NPV-derived objective functions. This approach typically involves deriving a linear approximation of the NPV objective function, by Maclaurin expansion of (exponential) terms in the NPV function, or thus a

Taylor series around 0, and such that the resulting approximation of the NPV function can be compared to the classic inventory model's objective function. Applied to the EOQ, this led them to the insight that the unit holding cost h has indeed the form as presented in Eq.(3.2). Their underlying assumption about the cash-flows was that set-up cost as well as purchasing costs to acquire the products are incurred the moment that the lot size arrives at the firm.

The general applicability to systems of production and inventories of this approach was first clearly demonstrated in Grubbström (1980), and also used in Van der Laan (2003), and formalised in Beullens and Janssens (2014), who called it NPV Equivalence Analysis or NPVEA. These works showed for example, that under different assumptions about when set-up costs and investment costs occur, the formula of how to set h in the classic EOQ model may differ from Eq. (3.2).

In this dissertation we use NPVEA to help determine the benefit of using a tax-adjusted inventory OR model.

3.5 Conclusions

3.5.1 Summary of Findings

The literature on the interface between logistics decisions and financial decisions is relatively small but growing in importance in recent years. The consideration of corporate taxes is fairly established in works that consider aspects of what to produce or source across borders, where the focus is primarily on the differences in tax rates. Few of these studies consider also issue related to differences in consumption or VAT tax rates.

None of the studies in the literature appears to account for what we defined in Chapter 2 as *tax schemes*, in which one would not only consider the nominal tax rates but also the timing when taxes due are to be settled with these governments of these countries. This thesis thus appears to be first in examining the impact of tax schemes on inventory and supplier selection decisions.

Although the classic average cost (profit) production and inventory models aim to implicitly account for the time value of money through accounting for this in the unit holding costs, they do not offer a method by which one can estimate the impact of tax schemes on the values of the model parameters.

Cash-flow NPV modelling seems to offer a better foundation by using cash-flow functions as the starting point for constructing models. Cash-flow functions can explicitly account for both the magnitude and timing of cash that the firm exchanges in the context of an activity with the outside world, i.e. with suppliers, customers, third parties, and the government. The objective function of the firm can then be expressed as the Net Present

Value (NPV) of all cash-flow functions relevant to the execution of this activity. The aim of solving the model is then to find the decisions which maximise this NPV value. The inclusion of tax cash-flows in this approach has not yet been undertaken.

The technique of NPVEA applied may furthermore lead to an increased understanding of whether traditional models still apply or need adaptation. Such comparisons can thus lead to increased understanding of how the theory of inventory models needs to be interpreted or adapted so as to account for taxes.

3.5.2 Implications for this work

The literature review reveals that no study yet exists which examines the impact of CT and VAT tax schemes on inventory and supplier selection decisions in either the traditional inventory modelling framework or the cash-flow based NPV framework.

A well-known problem-solving principle, known as Occam's razor, states that when presented with a set of different hypothetical solutions to a problem, one should select the one that makes the fewest assumptions. It is often applied in science as a guide in the construction of theoretical models.

Applied to our investigation of the impact of tax schemes on inventory and sourcing strategies, it suggests starting from the most basic models first. If tax schemes show their impact in these models, a good foundation may be developed for deriving fundamental insight into the links between taxes and logistics decisions.

We can distinguish between deterministic and stochastic inventory models. The origin of deterministic inventory is traced back to (1) the Economic Order Quantity (EOQ) model of Harris (1913), and (2) the Dynamic Lot Sizing Problem (DLSP) of Wagner and Whitin (1958).

The application of cash-flow NPV modelling to deterministic inventory theory can be traced back to at least Hadley (1964) and Grubbström (1980). This is by now a well-established technique in this area. The application of NPV Equivalence Analysis (NPVEA) as formalised in Beullens and Janssens (2014) furthermore enables us to assess the relative benefits of cash-flow NPV optimisation over the classic models.

As the main aim of this thesis is not to develop the theory of cash-flow NPV optimisation to stochastic models, the thesis will mostly consider deterministic models which are commonly used by small medium sized firms, and there is high proportion of business running in small and medium-sized enterprises (SMEs), leading to the particular research objectives as outlined earlier in Section 1.3 to achieve tax schemes applicable to a firm's activity with the assumptions of EOQ and it can be further investigated with multi products and relevant supply chain parties.

Tax already attracts considerable attention in operations decisions. How the inflows and outflows of tax related payments may impact the inventory modelling theory has not yet been investigated. The NPV framework is applied to construct the models and then the components of these models and their behaviour are compared with the models based on the classical inventory theory. The results are likely to be identified in two areas: (1) how the explicit consideration of tax rules and schemes may alter inventory modelling; (2) when it is advisable for a firm to consider tax more explicitly in inventory management decisions compared to the classic inventory theory.

Chapter 4

Value Added Tax, Corporation Tax and Economic Order Quantity models

4.1 Introduction

In business situations, many logistics activities involve dealing with direct or indirect taxes. One important area, known as deterministic inventory theory, is concerned with optimal decisions on order size and frequency. In Chapter 2 we have looked at the VAT and Corporation tax rules in the UK. In this chapter, we will examine how deterministic inventory models can account for these regulations and from this, derive some insight into how taxes impact classic inventory theory. We will find that the consideration of both types of taxes simultaneously is needed in arriving at the optimal order quantity and frequency that maximises a firm's profit after tax.

Some textbooks on inventory theory do mention taxes, see e.g. Silver et al. (1998), p.45: 'The cost of carrying items in inventory includes the opportunity cost of money invested, expenses incurred in running the warehouse, deterioration of stock, damage, theft, obsolescence, insurance, *and taxes.*' (Italics added.) An almost identical description can be found in Axsäter (2006), p.44. To our knowledge, no explicit treatment of taxes in analytical models exist which provide insights into which types of taxes would effect economic lot-size decisions and to which degree.

A principle from finance theory is that firms wish to engage in activities which maximise the Net Present Value (NPV) of profits after tax. In this study we focus on different types of VAT schemes and corporation tax which are typically relevant to a firm and its operations. How should the financial parameters in the model be specified so that its application ensures compatibility with the NPV optimisation of profits after tax?

These questions are important to help bridge the gap between inventory theory and its applicability to practice. This chapter fills part of this gap by presenting adaptations to the Economic Order Quantity (EOQ) model of Harris (1913). The EOQ model lies at the root of inventory theory and captures the essence of the economics of optimal lot-sizing decisions, and is still of high relevance to current research on supply chains, see e.g. Beullens (2014) and the other articles in the special issue 155 of the *IJPE*. Furthermore, the technique used in this paper is quite generally applicable to all kinds of inventory systems, in particular deterministic models.

To our knowledge, no prior work has been conducted on deriving an explicit analytical result that shows the impact of taxes on economic order quantity decisions, with the exception of Michalski (2013), who derives an EOQ model which considers corporate tax. Based on the news vendor problem, J. Liu, Fu, Lu, and Shang (2015) investigate tax-effective supply chain decisions for Chinese enterprises under China's valued added tax policies and export-oriented tariff policies, and find that the optimal order quantity and the allocation of profit are both affected by the export tax rebate policy. Yi and Reklaitis (2007) investigate the influence of the macroscopic economic factors such as taxes and exchange rate on the operational decision and showed that the optimal production lot sizes are typically smaller when tax is taken into consideration than the without scenario. Although the above cited studies added tax factors in their model, more focus is on how refund tax and tariff work in the case of export oriented policy. No study in the operations literature has considered the possible implications of how a country collects *consumption tax* or Value-Added-Tax (VAT) on inventory decisions. Perhaps more importantly, as concluded in Chapter 3, none of the prior studies has accurately considered the process by which taxes are settled with governments.

Our work is also different from prior studies on taxes and operations in its methodology. In line with corporate finance theory principles, we develop NPV-based profit after tax models by the explicit consideration of all relevant tax cash-flows associated with this activity for the firm. An important distinguishing factor from all existing literature on taxes in OR studies is the use of the Laplace transform. As shown in Grubbström (1967, 2007), the Laplace Transform of the cash-flow function produces the NPV of the activity considered when interpreting the Laplace frequency α as the (continuous) cost of capital rate of the firm. Grubbström (1980) was arguably first to convincingly show how this approach can lead to an accurate insight into the financial implications of production and inventory decisions, and that it is quite generally applicable to all kinds of operational systems. In addition, he shows how the linear approximation of the equivalent Annuity Stream (AS) function can provide insight into the relative performance of average cost models using classic inventory theory principles and unit cost parameters. This approach is formalised as NPV Equivalence Analysis (NPVEA) in Beullens and Janssens (2014), who illustrate how this also can lead to the identification of correction factors to classic models so as to give these models the ability to maximise the NPV of profits for the firm for various different contracts or payment structures that the firm adopts with its suppliers and customers. Further illustrations of the benefits of NPVEA towards the advancement of inventory and supply chain management theory are given in Beullens (2014) and Ghiami and Beullens (2016).

The cash-flow based NPV method lends itself to the examination of taxes in an operational context by adding the tax authorities as an additional player with which the firm exchanges cash-flows. We can then specify on the one hand these cash-flows as functions of the firm's decision variables and other operational parameters, and express on the other hand the firm's operational cash-flows with suppliers and customers, where needed, also as functions of the relevant tax regime. Since the timing of various cash-flows exchanged between the firm and its outside world are affected by the tax regime that is (to be) adopted by the firm, we can expect this approach to produce more accurate insights compared to classic average cost methods in which the relative timing of when costs and revenues occur is not explicitly modelled.

4.2 Standard EOQ Model

The standard EOQ model is well-known and derived from Harris (1913). A firm satisfies a constant demand rate y without shortage and purchases, or produces at infinite rate, in batches of size $Q = yT$, where T is the cycle time. With each lot, a set-up cost s is incurred, and h is the unit holding cost. The question is to determine the optimal value of Q.

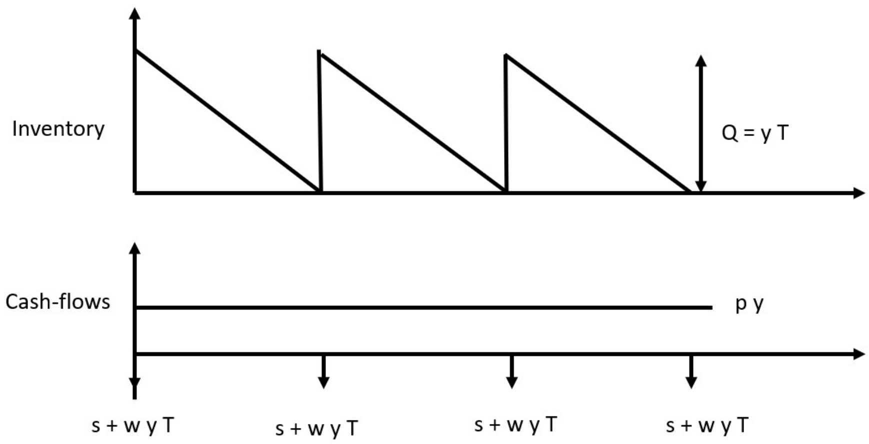

Figure 4.1: EOQ and Cash Flow

Figure 4.1 sketches inventory level and corresponding cash flows as functions of time. The derivation of Harris' model from the Net Present Value was first presented in Hadley and Whitin (1963), see also Beullens (2014). The Annuity Stream (AS):

$$\text{AS} = py - (s + wyT)\sum_{i=0}^{\infty} \alpha e^{-i\alpha T} = py - \frac{\alpha(s + wyT)}{1 - e^{-\alpha T}}, \tag{4.1}$$

can be re-written by Maclaurin expansion of the exponential term in αT, and then the linear approximation is given by:

$$\overline{\text{AS}} = (p - w)y - \frac{s}{T} - \alpha\frac{s}{2} - \alpha w\frac{yT}{2}. \tag{4.2}$$

Since $Q = yT$, the optimal order quantity is therefore:

$$Q^*_{eoq} = \sqrt{\frac{2sy}{\alpha w}}, \tag{4.3}$$

and substitution gives:

$$\overline{\text{AS}}^* = (p - w)y - \sqrt{2sy\alpha w} - \alpha\frac{s}{2}. \tag{4.4}$$

.

The first term is called the marginal profit term, while the second term denotes the logistics costs. The third term is often absent from the traditional EOQ model if it is not derived from the NPV criterion; this term accounts for the financial opportunity cost of set-ups. It is often a constant and also relatively small.

4.3 VAT Accounting Schemes and the EOQ

4.3.1 Calculating the Tax Point

The standard EOQ model does not make clear some of the underlying assumptions we need to make explicit when applying VAT accounting schemes. The NPV derivation in Section 4.2 assumes that at the time when a batch arrives in the EOQ model, the firm has an outgoing cash-flow of $s + wQ$. Likewise, the revenues are received as an annuity stream py at the rate of sales, such that customers receiving a product immediately pay for it.

In reality, there may be a time difference between the moment a physical transaction takes place and the time that the invoice is issued and the time that cash towards meeting this invoice is paid.

In an invoice based accounting scheme, the tax point t_T is specified as follows. If $L_I \leq 15$ days, then

$$t_T = min\{t_I, t_C\}, \tag{4.5}$$

else

$$t_T = t_S. \tag{4.6}$$

In a cash accounting scheme, however, this would be:

$$t_T = t_C. \tag{4.7}$$

The default assumption henceforth will be that $L_I = L_C = 0$, and thus that $t_T \equiv t_S$, unless otherwise specified.

4.3.2 Annual VAT Accounting Scheme in Nine Interim Payments

In its interaction with customers and suppliers, the firm will have to account for the VAT being charged. It is easy to see that the annuity stream of these interactions is given by Eq.(4.1) in which $(1 + \tau)$ is added to p, s and w. The operational AS profit function, representing the cash-flows exchanged between the firm and its outside world with the government, is then:

$$\text{AS}_o = p(1+\tau)y - \frac{\alpha(s(1+\tau) + w(1+\tau)yT)}{1 - e^{-\alpha T}}, \tag{4.8}$$

and therefore:

$$\overline{\text{AS}}_0 = (p-w)(1+\tau)y - \frac{s(1+\tau)}{T} - \alpha\frac{s(1+\tau)}{2} - \alpha w(1+\tau)\frac{yT}{2}. \tag{4.9}$$

Hence, AS_0 is equal to Eq.(4.1) times a constant $(1 + \tau)$.

The firm's annual expected VAT liabilities are:

$$\text{NVAT} = \text{OVAT} - \text{IVAT} = p\tau y - (w\tau y + \frac{s\tau}{T}) \tag{4.10}$$

and 10% of this has to be exchanged with HMRC at the end of month four, five, six, ..., and twelve of every accounting year. The final instalment then has to occur at the end of the second month after the accounting year. Because of the deterministic assumptions of the EOQ, the VAT liabilities of each year are constant, and hence this final instalment will be the remaining 10%. Therefore:

$$\text{AS}_\tau = -\text{NVAT}\frac{0.1\alpha}{1 - e^{-\alpha T_a}}(\sum_{i=1}^{9} e^{-(i+3)\alpha T_v} + e^{-14\alpha T_v}), \tag{4.11}$$

and its linear approximation is:

$$\overline{\mathrm{AS}}_\tau = -\mathrm{NVAT}(\frac{1}{T_a} + \frac{\alpha}{2})(1 - 8.6\alpha T_v) \tag{4.12}$$

$$= -\mathrm{NVAT}(1 + \frac{\alpha}{2} - \frac{8.6\alpha}{12})$$

$$= -\mathrm{NVAT}(1 - \frac{2.6\alpha}{12})$$

Definition 4.1. The VAT TAX EFFECT τ' is an adjustment to an adopted VAT tax rate τ and which is to account for the time-dependent VAT scheme adopted by the firm to pay the government the net annual VAT liability at the VAT tax rate τ.

We thus find the *VAT tax effect*:

$$\tau' = \tau(1 + \frac{\alpha}{2})(1 - \frac{8.6\alpha}{12}) \approx \tau(1 - \frac{2.6\alpha}{12}) \tag{4.13}$$

Hence, simplified $\mathrm{AS}_\tau = (p - w)y\tau' - \frac{s}{T}\tau'$

Therefore, since $\mathrm{AS}_t = \mathrm{AS}_o + \mathrm{AS}_\tau$:

$$\mathrm{AS}_t = p(1+\tau)y - \frac{\alpha(s(1+\tau) + w(1+\tau)yT)}{1 - e^{-\alpha T}} + (p - w)y\tau' - \frac{s}{T}\tau' \tag{4.14}$$

Linear approximation of the Maclaurin expansion

$$\overline{\mathrm{AS}}_t = (p - w)y(1 + \tau - \tau') - \frac{s}{T}(1 + \tau - \tau')$$

$$- \alpha\frac{s(1+\tau)}{2} - \alpha w(1+\tau)\frac{yT}{2} \tag{4.15}$$

The optimal order quantity:

$$Q^*_{vat} = \sqrt{\frac{2s(1 + \tau - \tau')y}{\alpha w(1+\tau)}} \tag{4.16}$$

or:

$$Q^*_{vat} = Q^*\sqrt{\frac{1 + \tau - \tau'}{1 + \tau}} \tag{4.17}$$

Replacing τ' into Q^*_t leads to the following approximation:

$$Q^*_{vat} \approx Q^*\sqrt{\frac{1 + \frac{2.6\alpha\tau}{12}}{1 + \tau}}. \tag{4.18}$$

Since $\frac{2.6\alpha}{12} << 1$, the correction factor is also smaller than 1, and the optimal lot-size under this VAT scheme will be smaller than the optimal lot size of the standard EOQ model. By substitution:

$$\overline{\text{AS}}_t^* = (p-w)y(1+\frac{2.6\alpha\tau}{12}) - \sqrt{2sy\alpha w}\sqrt{(1+\frac{2.6\alpha\tau}{12})(1+\tau)} - \alpha\frac{s(1+\tau)}{2}. \tag{4.19}$$

The logistics cost, as given by the second term, increases approximately by $1+\tau/2$ compared to the standard EOQ model, and this will be largely independent of the value of α. The marginal profits, as given by the first term, will slightly increase as well, but this small increase, if marginal profits are much larger than the logistics costs, may more than offset the increase in logistics costs.

Example 4.1. *For $\tau = 0.2$, the logistics costs increase by approximately 10%, and for $\alpha = 0.2$, $1+\frac{2.6\alpha\tau}{12} = 1.00866...$, and therefore the marginal profits increase approximately by 0.9%.*

Example 4.2. *Continuing the previous example, let $p = 30$, $w = 15$, $y = 3000$, and $s = 100$. Then:*

$$Q^*_{eoq} = \sqrt{\frac{2(100)(3000)}{(0.2)(15)}} = 447.2$$

$$\overline{AS}^* = (30-15)3000 - \sqrt{2(100)(3000)(0.2)(15)} - 0.2(100/2) =$$

$$= 45,000 - 1,341.6 - 10 = 43,648.4$$

$$Q^*_{vat} = \sqrt{\frac{2(100)(3000)(1.00866..}{(0.2)(15)(1.2)}} = 410.0$$

$$\overline{AS}^*_t = 45,000(1.00866..) - 1,341.6\sqrt{1.00866..(1.2)} - 10(1.2) =$$

$$= 45,390 - 1,476.0 - 12 = 43,902.$$

4.3.3 Second Order Approximation

We return to Eq.(4.11). Since the terms involved are multiples of T_v, a linear approximation of these terms may perhaps not sufficiently account for the impact of the VAT scheme on the firm's profit function. We will therefore derive a second order approximation using:

$$\frac{\alpha}{1-e^{-\alpha T_a}} \approx \frac{1}{T_a} + \frac{\alpha}{2} + \frac{\alpha^2 T_a}{12}, \tag{4.20}$$

$$e^{-\alpha k T_v} \approx 1 - \alpha k T_v + \frac{\alpha^2 k^2 T_v^2}{2}. \tag{4.21}$$

This produces:

$$\overline{\overline{\text{AS}}}_\tau = -\text{NVAT}(1+\frac{\alpha}{2}+\frac{\alpha^2}{12})(1-8.6\alpha T_v + 416\alpha^2 T_v^2) \tag{4.22}$$

$$= -\text{NVAT}(1 - \frac{2.6\alpha}{12} - \frac{4\alpha^2}{288}),$$

and therefore, the correction from the second order approximation is quite small.

$$\text{AS}_\tau = -\text{NVAT}\frac{0.25\alpha}{1 - e^{-\alpha T_a}} \sum_{i=1}^{4} e^{-(3i+1.25)\alpha T_v} \tag{4.23}$$

$$\tau' = \tau \frac{0.1\alpha}{1 - e^{-\alpha T_a}} (\sum_{i=1}^{9} e^{-(i+3)\alpha T_v} + e^{-14\alpha T_v})$$

$$\tau' = \tau \frac{0.25\alpha}{1 - e^{-\alpha T_a}} (\sum_{i=1}^{3} e^{-(3i+1)\alpha T_v} + e^{-14\alpha T_v})$$

$$\tau' = \tau \frac{0.25\alpha}{1 - e^{-\alpha T_a}} \sum_{i=1}^{4} e^{-(3i+1.25)\alpha T_v}$$

4.3.4 Annual VAT Accounting Scheme in Three Interim Payments

The approach is similar to the previous one, but we now have three interim payments at the end of month four, seven, and ten, respectively:

$$\text{AS}_\tau = -\text{NVAT}\frac{0.25\alpha}{1 - e^{-\alpha T_a}} (\sum_{i=1}^{3} e^{-(3i+1)\alpha T_v} + e^{-14\alpha T_v}), \tag{4.24}$$

and its linear approximation:

$$\overline{\text{AS}}_\tau = -\text{NVAT}(1 + \frac{\alpha}{2})(1 - \frac{8.75\alpha}{12}), \tag{4.25}$$

$$\tau' = \tau(1 + \frac{\alpha}{2})(1 - \frac{8.75\alpha}{12}) \approx \tau(1 - \frac{11\alpha}{48}) \tag{4.26}$$

According to $AS_t = AS_0 + AS_\tau$ which can lead to the equation of 4.15, replace $\tau' = \tau(1 - \frac{11\alpha}{48})$,annual VAT accounting with three interim payment AS can be:

$$\overline{\text{AS}}_t = (p - w)y(1 + \frac{11\alpha\tau}{48}) - \frac{s(1 + \frac{11\alpha\tau}{48})}{T} - \alpha\frac{s(1 + \tau)}{2} - \alpha w(1 + \tau)\frac{yT}{2}. \tag{4.27}$$

This therefore produces corrections to the optimal lot-size of similar magnitude to the nine month interim payments scheme. The logistics costs will increase approximately with $1 + \tau/2$, but since $11/48 > 2.6/12$, the marginal profits will increase slightly more. In cases where the marginal profits are much larger than the logistics costs, the three interim payments scheme is therefore slighlty preferable.

Example 4.3. *For* $\tau = 0.2$*, the logistics costs increase by approximately* 10%*, and for* $\alpha = 0.2$*,* $1+\frac{11\alpha\tau}{48} = 1.009166...$*, and therefore the marginal profits increase approximately by* 0.9%*.*

Example 4.4. *Continuing the previous example, let* $p = 30$*,* $w = 15$*,* $y = 3000$*, and* $s = 100$*. Then:*

$$Q^*_{vat} = \sqrt{\frac{2(100)(3000)(1.009166..}{(0.2)(15)(1.2)}} = 410.1$$

$$\overline{AS}^*_t = 45,000(1.009166..) - 1,341.6\sqrt{1.009166..(1.2)} - 10(1.2) =$$

$$= 45,412.4 - 1,476.4 - 12 = 43,924.$$

4.3.5 Standard VAT Accounting Scheme

In this scheme the firm pays at the end of months three, six, nine, and twelve, each time the actual difference between output VAT and input VAT. In EOQ situations every year is equal, and therefore the approach is as before, leading to:

$$\overline{\text{AS}}_t = (p - w)y(1 + \frac{6\alpha\tau}{48}) - \frac{s(1 + \frac{6\alpha\tau}{48})}{T} - \alpha\frac{s(1 + \tau)}{2} - \alpha w(1 + \tau)\frac{yT}{2}. \tag{4.28}$$

Because $6/48 < 2.6/48 < 11/48$, the increase in marginal profits is less while the impact on logistics costs remains at the level of $1+\tau/2$. An annual accounting scheme with three interim payments method therefore seems the preferred accounting scheme compared to the nine-interim or standard VAT account schemes in EOQ-type conditions.

4.3.6 Flat Rate VAT Accounting

In its interaction with customers and suppliers, the firm will account for the VAT being charged as before, and AS_o is given by Eq.(4.8).

The firm's annual expected VAT liabilities are:

$$\text{NVAT} = \tau_f p(1 + \tau)y. \tag{4.29}$$

Flat rate scheme requires the completion a quarterly VAT return like standard VAT.

$$\text{AS}_e = -\text{NVAT}(1 - \frac{6\alpha}{48}) \tag{4.30}$$

Substituting Eq.(4.29), we find:

$$\overline{\text{AS}}_t = (p(1 - \tau_f(1 - \frac{6\alpha}{48})) - w)y(1 + \tau) - \frac{s(1 + \tau)}{T} - \alpha\frac{s(1 + \tau)}{2} - \alpha w(1 + \tau)\frac{yT}{2}. \tag{4.31}$$

Under a flat rate accounting scheme, the optimal lot-size is Q^* of the standard model. However, the logistics costs now increase significantly with $1+\tau$, i.e. much higher than the increase of $1+\tau/2$ for invoice based VAT accounting.

The logistics manager hence has to know whether the company uses flat rate VAT accounting or invoice based VAT accounting, since the optimal order quantity will be different, as well as the logistics costs.

If we focus on the marginal cost term, we see that it is now very different. Let:

$$p' = p(1-\tau_f(1-\frac{6\alpha}{48})),$$

then the profit margin now becomes:

$$(1+\tau)(p'-w)y,$$

whereas under invoice based accounting, this is:

$$(1+\frac{2.6\alpha}{12})(p-w)y$$

A flat rate would be beneficial if:

$$(1+\tau)(p-w)y - \tau_f(1+\tau)py(1-\frac{6\alpha}{48}) > (p-w)y(1+\frac{2.6\alpha}{12})$$

$$\Leftrightarrow (p-w)(1+\tau-(1+\frac{2.6\alpha}{12})) > \tau_f(1+\tau)p(1-\frac{6\alpha}{48})$$

$$\Leftrightarrow (p-w)(\tau-\frac{2.6\alpha}{12}) > \tau_f(1+\tau)p(1-\frac{6\alpha}{48})$$

$$\Leftrightarrow \frac{p-w}{p} > \tau_f\frac{(1+\tau)(1-\frac{6\alpha}{48})}{\tau-\frac{2.6\alpha}{12}} \tag{4.32}$$

Example 4.5. *For $\tau = 0.2$, $\tau_f = 0.1$ and $\alpha = 0.2$, the right-hand side of Eq.(4.32) is $7.4665\tau_f = 0.74665$.*

If we would ignore the NPV value of the particular invoice based annual scheme, the flat rate scheme would be beneficial if:

$$(1+\tau)(p-w)y - \tau_f(1+\tau)py > (p-w)y$$

$$\Leftrightarrow \frac{p-w}{p} > \tau_f\frac{(1+\tau)}{\tau} \tag{4.33}$$

Example 4.6. *For $\tau = 0.2$, $\tau_f = 0.1$, the right-hand side of Eq.(4.33) is $6\tau_f = 0.6$.*

Not accounting for the NPV effect would hence lead to a bound on the relative profit margin $(p-w)/p$ that is too optimistic towards adopting a flat rate.

If we include the logistics costs, hence using the complete $\overline{\text{AS}}$ objective functions, the criterion becomes less elegant and it will need to take into account the logistics costs. Then we then also observe it is a function of the demand rate as well:

$$(1+\tau)(p-w)y-\tau_f(1+\tau)py(1-\frac{6\alpha}{48})-\sqrt{2sy\alpha w}(1+\tau)-\alpha\frac{s(1+\tau)}{2}$$

$$> (p-w)y(1+\frac{2.6\alpha}{12})-\sqrt{2sy\alpha w}\sqrt{(1+\tau)(1+\frac{2.6\alpha}{12})}-\alpha\frac{s(1+\tau)}{2}$$

$$\Leftrightarrow \frac{p-w}{p} > \tau_f\frac{(1+\tau)(1-\frac{6\alpha}{48})}{\tau-\frac{2.6\alpha}{12}}+\sqrt{\frac{2s\alpha w}{py}}\frac{1+\tau-\sqrt{(1+\tau)(1+\frac{2.6\alpha}{12})}}{\tau-\frac{2.6\alpha}{12}}$$

$$\Leftrightarrow \frac{p-w}{p} > \tau_f\frac{(1+\tau)(1-\frac{6\alpha}{48})}{\tau-\frac{2.6\alpha}{12}}+\sqrt{\frac{w}{p}}\sqrt{\frac{2s\alpha}{y}}\frac{1+\tau-\sqrt{(1+\tau)(1+\frac{2.6\alpha}{12})}}{\tau-\frac{2.6\alpha}{12}} \tag{4.34}$$

Example 4.7. *For $\tau = 0.2$, $\tau_f = 0.1$, $\alpha = 0.2$, $y = 3000$, and $s = 100$ the right-hand side of Eq.(4.34) is $7.4665\tau_f + 0.07\sqrt{w/p}$.*

Example 4.8. *Continuing from the previous example, let $w = 7.5$ and $p = 30$. Observe that therefore $(p-w)/p = 0.75$, and therefore meets the NPV-based profit margin criterion as given by Eq.(4.32), see Example 4.5. However, the right-hand side of Eq.(4.34) now gives a lower bound of $7.4665\tau_f + 0.07\sqrt{7.5/30} = 0.74665 + 0.07(0.5) = 0.78165$.*

It is hence important to consider the impact of the logistics costs to determine whether the flat rate is beneficial.

To summarise, we have found that in order to establish that an annual VAT scheme with nine interim payments should use invoice VAT accounting or flat rate VAT accounting, it is important to consider the impact of the NPV of the scheme as well as the impact of the logistics costs. Both effects tend to significantly increase the lower bound on the relative profit margin $(p-w)/p$ we should have in order for flat rate accounting to be financially beneficial.

The use of the other schemes will give comparable results. For the annual scheme with three interim payments, adjust the term $2.6\alpha/12$ to $11\alpha/48$, and for the standard accounting scheme adjust it to $6\alpha/48$.

4.4 Corporation Tax Schemes and the EOQ

We assume the accounting period for tax purposes is one year. For numerical examples, we typically use the CT rate $\epsilon = 0.20$, which was the standard rate for company profit in the UK in May 2015.

The CT is charged as a percentage of Operating Profit (OP):

$$\text{OP} = \text{GP} - \text{OE}, \tag{4.35}$$

where OE are the Operating Expenses, and GP is the Gross Profit, equal to the Net Sales (NS) minus the Cost Of Goods Sold (COGS). In the EOQ model, a volume of y products per year is sold at a constant price p, so NS amounts to py. COGS is in general given by the formula:

$$\text{COGS} = C(I_0) + C(Q_{01}) - C(I_1), \tag{4.36}$$

where I_0 is the inventory at the start of the accounting period, I_1 is the inventory at the end of the accounting period, Q_{01} accounts for the amount of inventory purchased during the accounting period, and C(.) is a function which returns a cost value. In the EOQ model, the amount of products purchased may differ from y due to the lot-size decision and the times when the accounting year starts and ends relative to the inventory cycle. For a constant purchase price w, however, these effects cancel out:

$$\text{COGS} = wI_0 + w(y - I_0 + I_1) - wI_1 = wy. \tag{4.37}$$

The Operating Expenses (OE) in the EOQ model are the set-up costs of ordering from the supplier, the fixed out-of-pocket holding costs, and other fixed overhead costs (FOC). FOC are those costs not affected in size or timing by the lot-size decision or the pricing of the goods, but which nevertheless are associated with performing this activity. These costs are also important when assessing the overall profitability of selling the product although they do not affect the lot-size decision itself. Without loss of generality, we take FOC to be the annuity stream value of all involved overhead expenses[1]. The firm's operating profit (OP) realised through this activity therefore equals:

$$\text{OP} = (p - w)y - \frac{s}{T} - \text{FOC}. \tag{4.38}$$

Firms subject to corporate tax in the UK with a taxable profit of £1.5 million or less pay CT nine months (and one day) after the end of the accounting year. The contribution of CT to the annuity stream profit function of the firm can thus be expressed as:

$$\begin{aligned} \text{AS}_\epsilon &= -\epsilon \text{ OP } e^{-(12+9)\alpha T_v} \sum_{i=1}^{\infty} \alpha e^{-i\alpha T_a} \\ &= -\epsilon \text{ OP } \frac{\alpha e^{-(12+9)\alpha T_v}}{1 - e^{-\alpha T_a}}. \end{aligned} \tag{4.39}$$

[1] A large part of a firm's expenses are salaries and wages of its personnel. Making the reasonable assumption that this does not change with the lot-size decision, a fraction of these costs for the work of the employees on this activity form then part of the FOC. This typically includes also the firm's contribution to National Insurance, which are also tax deductible.

where $T_a = 1$ and $T_v = 1/12$.

Definition 4.2. The CORPORATION TAX EFFECT ϵ' is an adjustment to an adopted CT tax rate ϵ and which is to account for the time-dependent CT scheme adopted by the firm to pay the government the net annual CT liability at the CT tax rate ϵ.

We find the corporation tax effect of this CT payment scheme as:

$$\epsilon' = \epsilon \frac{\alpha e^{-\alpha(12+9)(1/12)}}{1 - e^{-\alpha}}, \tag{4.40}$$

and thus find:

$$\text{AS}_\epsilon = -\epsilon' \text{ OP}. \tag{4.41}$$

After linearisation of the Maclaurin expansion of Eq. (4.39), we get the following approximations:

$$\overline{\epsilon}' = \epsilon(1 + \frac{\alpha}{2})(1 - \frac{21}{12}\alpha), \tag{4.42}$$

$$\overline{\text{AS}}_\epsilon = -\overline{\epsilon}' \text{ OP} . \tag{4.43}$$

Firms with an annual taxable profit above £1.5 million in the UK, however, pay CT in quarterly instalments based on an estimation of operating profits made in that quarter. Currently, payment is due at times (in months from start of accounting year) 6.5; 9.5; 12.5; and 15.5. The contribution of CT to the firm's AS profit function is:

$$\text{AS}_\epsilon = -\epsilon \frac{\text{OP}}{4} \frac{\alpha}{1 - e^{-\alpha T_a}} (\sum_{i=1}^{4} e^{-(3.3+3i)\alpha T_v}). \tag{4.44}$$

The CT effect of this CT payment scheme is now defined as follows:

$$\epsilon' = \epsilon \frac{1}{4} \frac{\alpha}{1 - e^{-\alpha}} (\sum_{i=1}^{4} e^{-(3.3+3i)\alpha(1/12)}), \tag{4.45}$$

so that AS_ϵ can still be expressed by Eq.(4.41), but in which ϵ' is given by Eq.(4.45).

After linearisation of the equation we get the following approximation:

$$\overline{\epsilon}' = \epsilon(1 + \frac{\alpha}{2})(1 - \frac{11}{12}\alpha), \tag{4.46}$$

and then $\overline{\text{AS}}_\epsilon$ is still given by Eq.(4.43).

As of April 2017, however, firms with an annual taxable profit over £20 million will be required to make payments earlier, and will be due at the end of months three, six, nine and twelve. Each time it is expected that CT is charged on a quarter of the total annual

OP. This gives Eq.(4.41) and Eq.(4.43) where:

$$\epsilon' = \epsilon \frac{1}{4} \frac{\alpha}{1-e^{-\alpha}} (\sum_{i=1}^{4} e^{-3i\alpha(1/12)}), \tag{4.47}$$

$$\overline{\epsilon}' = \epsilon(1+\frac{\alpha}{2})(1-\frac{7.5}{12}\alpha). \tag{4.48}$$

It is worth pointing out that the current main CT rate in the UK is set at 20% ($\epsilon = 0.20$), but this will be reduced as of April 2017 to 19%, and in April 2020, to 18%.

4.5 The EOQ with CT and VAT Effects

We assume that a fraction δ of expenses in the activity-related FOC is composed of expenses to which VAT does not apply, while the remainder of expenses in FOC are liable to VAT charges.

Examples of fixed overhead cost can be found in the areas of rent, insurance, salaries and office expenses. VAT does not apply to salaries and wages of personnel, but is to be charged for services received from external domestic parties.

$$\begin{aligned} \mathrm{AS}_o = p(1+\tau)y - \frac{\alpha(s(1+\tau)+w(1+\tau)yT)}{1-e^{-\alpha T}} \\ - (1-\delta)FOC(1+\tau) - \delta FOC \end{aligned} \tag{4.49}$$

The firm's annual expected VAT liabilities to the government are:

$$\mathrm{NVAT} = \mathrm{OVAT} - \mathrm{IVAT} = p\tau y - (w\tau y + \frac{s\tau}{T}) - \tau(1-\delta)FOC \tag{4.50}$$

$$AS_\tau = (py - wy - \frac{s}{T} - (1-\delta)FOC)\tau'$$
$$AS_\tau = (OP + \delta FOC)\tau'$$

Having looked at the impact of CT and VAT on the AS profit function of the firm, we arrive at the tax adjusted profit function of the firm's EOQ problem:

$$\mathrm{AS} \equiv \mathrm{AS}_o + \mathrm{AS}_\epsilon + \mathrm{AS}_\tau = \mathrm{AS}_o - \epsilon'\mathrm{OP} - \tau'(\mathrm{OP} + \delta\mathrm{FOC}), \tag{4.51}$$

which, after some algebraic manipulation, can be written as:

$$\mathrm{AS} = \Big[py - (1-\delta)\mathrm{FOC}\Big](1 - \epsilon' + \tau - \tau') - \delta\mathrm{FOC}(1-\epsilon')$$

$$- (s + wyT)\Big[\frac{\alpha(1+\tau)}{1-e^{-\alpha T}} - \frac{\epsilon' + \tau'}{T}\Big]. \tag{4.52}$$

Maclaurin expansion of the exponential term in decision variable T, and ingnoring second and higher order terms in αT, gives after some algebraic manipulation the following approximate AS function:

$$\begin{aligned}\overline{\text{AS}} &= (p-w)y(1-\epsilon'+\tau-\tau') - \alpha\frac{s(1+\tau)}{2}\\ &-\text{FOC}(1-\epsilon') - (1-\delta)\text{FOC}(\tau-\tau')\\ &- s\frac{(1-\epsilon'+\tau-\tau')}{T} - \alpha w(1+\tau)\frac{yT}{2}.\end{aligned} \tag{4.53}$$

The optimal lot-size is therefore:

$$Q^* = \sqrt{\frac{2s(1-\epsilon'+\tau-\tau')y}{\alpha w(1+\tau)}}. \tag{4.54}$$

4.6 Numerical Examples

We summarise what kinds of VAT and CT can be combined according to the UK tax rules. Based on turnover, different VAT and CT schemes can be notated as follow:

- ϵ'_s : CT scheme when taxable profits $<$ £1.5 million;
- ϵ'_m : CT scheme when taxable profits exceed $>$ £1.5 million (2016);
- ϵ'_l : CT scheme when taxable profits $>$ £20 million (2017);
- τ'_9 : Annual VAT scheme nine interim payments;
- τ'_3 : Annual VAT scheme three interim payments;
- τ'_{st} : Standard VAT scheme.

We have made a distinction between three situations, based on the notation introduced:

- The 'small'-sized firm having less than £1.5 million taxable profits, using the CT scheme corresponding to ϵ'_s and the VAT scheme corresponding to τ'_3;
- The 'medium'-sized firm, having less than £20 million taxable profits, using the CT scheme corresponding to ϵ'_m and the VAT scheme corresponding to τ'_{st};
- The 'large'-sized firm, using the CT scheme corresponding to ϵ'_l and the VAT scheme corresponding to τ'_{st}.

Table 4.1 demonstrates how the cash flow of the tax rates change according to the fluctuation of opportunity cost of capital and government tax rate.

Table 4.1: Typical values for CT and VAT effects

Scenario	ϵ	τ	α	ϵ_s'	ϵ_m'	ϵ_l'	τ_9'	τ_3'	τ_{st}'
UK 2016	0.20	0.20	0.20	0.1555	0.1840	0.1950	0.1914	0.1911	0.1910
	0.20	0.20	0.10	0.1764	0.1918	0.1975	0.1957	0.1955	0.1955
	0.20	0.20	0.05	0.1879	0.1956	0.1988	0.1978	0.1977	0.1977
Alternatives	0.18	0.175	0.20	0.1400	0.1660	0.1755	0.1675	0.1672	0.1671
	0.18	0.175	0.10	0.1588	0.1727	0.1778	0.1712	0.1711	0.1710
	0.18	0.175	0.05	0.1681	0.1763	0.1789	0.1731	0.1730	0.1730

Note: UK tax rate for 2020 is targeted to become $\epsilon = 0.18$; Prior to 2011 the UK VAT rate was $\tau = 0.175$.

Table 4.2 illustrates how these gaps are a function of firm size and opportunity cost of capital rate at current UK and future UK CT tax levels.

For example, in the first row scenario, if firms use the classic pre-tax EOQ formula then the small firm will choose lot-sizes that are 18.56% too high and arrive at a logistics costs that are 1.455% above optimal, while a large firm's lot-size will be 21.423% too high and its logistics costs are 1.889% above optimal. Comparing the first three rows, it can be observed that when opportunity costs of the firm decrease, these gaps increase. In other words, when out-of-pocket costs are zero, firms with smaller opportunity costs derive more benefit from the adoption of the tax-adjusted EOQ. The reduction in the CT tax rate by 2020 will decrease these gaps, but otherwise the above insights remain valid. The profit different follows the same rules. From Figure 4.2 we can further see that the higher the opportunity cost of capital rate, the larger the difference in profitability between the model with taxes considered and the model without taxes incorporated.

Table 4.2: Gaps between using pre-tax and tax-adjusted lot-sizes for small, medium and large firms

Scenario	ϵ	α	$\frac{Q^*_{eoq}}{Q^*_s}$	$\frac{Q^*_{eoq}}{Q^*_m}$	$\frac{Q^*_{eoq}}{Q^*_l}$	$\frac{TC_s(Q^*_{eoq})}{TC_s(Q^*_s)}$	$\frac{TC_m(Q^*_{eoq})}{TC_m(Q^*_m)}$	$\frac{TC_l(Q^*_{eoq})}{TC_l(Q^*_l)}$
UK 2016	0.20	0.20	1.1856	1.2061	1.2142	1.01455	1.01761	1.01889
	0.20	0.10	1.2038	1.2152	1.2194	1.01725	1.01905	1.01974
	0.20	0.05	1.2139	1.2199	1.2221	1.01884	1.01982	1.02017
UK 2020	0.18	0.20	1.1752	1.1929	1.1999	1.01305	1.01559	1.01665
	0.18	0.10	1.1912	1.2010	1.2047	1.01534	1.01683	1.01740
	0.18	0.05	1.2001	1.2053	1.2072	1.01668	1.01749	1.01779

Subscripts s, m and l refer to the small, medium, and large firm situations, respectively, as defined earlier, and set $\tau = 0.2$.

Table 4.3 illustrates the sensitivity of the optimal lot-size decisions as a function of the opportunity cost of capital, the fixed out-of-pocket costs, and firm size, and other parameters kept fixed and given values as reported below the table. Overall, pre-tax EOQ lot-sizes are too high in the range of $10\% - 22\%$ and lead to excess logistics costs in the range of $0.5\% - 2\%$.

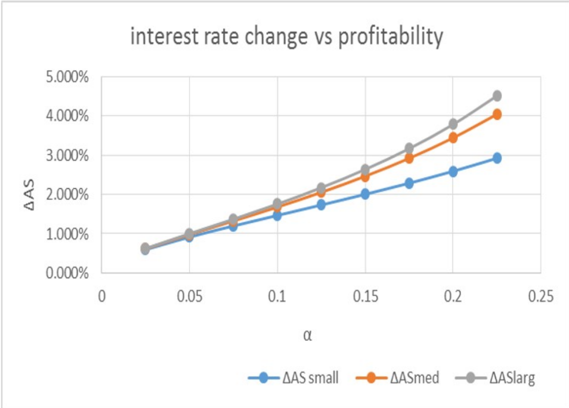

Figure 4.2: Capital Cost versus Profitability

Table 4.3: Tax-adjusted optimal lot-sizes and logistics costs, and excess when using pre-tax lot-sizes

α	Q^*_{eoq}	Q^*_s	Q^*_m	Q^*_l	TC^*_s	TC^*_m	TC^*_l
0.20	2553.8	2153.6	2117.5	2103.2	11888	11688	11609
		+18.58%	+20.60%	+21.40%	+1.45%	+1.76%	+1.89%
0.08	4037.9	3343.4	3317.8	3308.5	7382.1	7325.8	7305.1
		+20.77%	+21.70%	+22.05%	+1.79%	+1.93%	+1.99%

For $p = 30$, $w = 23$, $y = 30,000$, $s = 500$, $\epsilon = 0.20$, $\tau = 0.20$. Q^*_{eoq} is the EOQ lot-size using pre-tax parameter values; subscripts s, m and l refer to the small, medium, and large firm situations, respectively, as defined earlier. Even rows report excess when using Q^*_{eoq} instead.

4.7 Impact on Classic Inventory Theory

The following propositions are valid under the assumption of the firm adopting the annual or standard VAT schemes investigated in this chapter (but excluding the flat rate scheme).

We use Q^*_{eoq} when referring to the standard optimal lot-size formula, i.e. $Q^*_{eoq} = \sqrt{2sy/h}$, where s is the set-up cost and h is the unit holding cost.

Proposition 4.3. *A logistics manager wishing to use the standard EOQ model will arrive at the EOQ lot-size that helps to maximise the AS of profits after tax by using the following CT- and VAT-adjusted parameter values:*

$$\textit{Set-up cost} = s(1 - \epsilon' + \tau - \tau') \tag{4.55}$$

$$\textit{Unit holding cost} = \alpha w(1 + \tau) \tag{4.56}$$

Proof. This can be easily observed from Eq.(4.54) and comparing this with the standard EOQ model. ⋄

Proposition 4.4. *For any given values of $\epsilon > 0$ and $\alpha > 0$ it holds that $\epsilon'_s < \epsilon'_m < \epsilon'_l < \epsilon$.*

Proof. This can be easily observed to hold for the linear approximations of these effects by inspection and comparison of Eqs.(4.42), (4.46), and (4.48). Analytic comparison of Eqs. (4.40), (4.45), and (4.48), and since $e^{-x} > e^{-y}$ when $x < y$, proves this also holds for the unapproximated CT effects. ⋄

Proposition 4.5. *For any given values of $\tau > 0$ and $\alpha > 0$ it holds that $\tau'_9 < \tau'_3 \leq \tau'_{st} < \tau$.*

Proposition 4.6. *For any given values of $\epsilon > 0$, $\tau > 0$ and $\alpha > 0$, it holds that the optimal tax-adjusted lot-size is strictly smaller than the optimal EOQ lot-size based on prices before tax.*

Proof. We can rewrite Eq.(4.54) to:

$$Q^* = \sqrt{\frac{2sy}{\alpha w \gamma}}, \tag{4.57}$$

where:

$$\gamma = \frac{1+\tau}{1+\tau-\epsilon'-\tau'}. \tag{4.58}$$

Since $1 > \epsilon' > 0$ when $\epsilon > 0$ and $0 < \tau' < \tau$ due to Proposition 4.5, it follows that:

$$\frac{(1+\tau)-\epsilon'-\tau'}{(1+\tau)} < 1,$$

and thus that $\gamma > 1$. When $\gamma > 1$, it holds that

$$Q^* < \sqrt{\frac{2sy}{\alpha w}}, \tag{4.59}$$

and hence the proposition holds. ⋄

Definition 4.7. The TAX ADJUSTED OPPORTUNITY COST OF CAPITAL α' is an adjustment to an adopted opportunity cost of capital rate α that can be used in classic inventory theory but accounts for the time-dependent VAT and CT schemes adopted by the firm to pay the government its VAT and CT liabilities at adopted CT and VAT tax rates ϵ and τ.

Proposition 4.8. *A logistics manager wishing to use the standard optimal EOQ lot-size formula by using pre-tax prices, will arrive at the EOQ lot-size that helps to maximise the AS of profits after tax by using an adjusted and increased opportunity cost of capital rate set to $\alpha' = \alpha\gamma$.*

Proof. This follows easily from Eq.(4.57) and that $\gamma > 1$, as proven previously. ⋄

It can be observed from Table 4.4 that the effect of taxes significantly increases the opportunity cost of capital to be used in the EOQ formula. Comparing the γ values in the table illustrates that the larger tax adjustment needed, the larger the firm and the smaller its opportunity cost of capital. In the UK 2016 scenario, the upwards adjustment is in the range of 40%−49%, while in the UK 2020 scenario, the reduction of the corporate tax rate to 18% reduces this somewhat, but the range is still a considerable 38% − 46%.

Table 4.4: CT- and VAT-adjusted opportunity costs of capital $\alpha\gamma$

				UK 2016				
ϵ	τ	α	$\gamma(\epsilon'_s, \tau'_3)$	$\gamma(\epsilon'_m, \tau'_{st})$	$\gamma(\epsilon'_l, \tau'_{st})$	$\alpha\gamma(\epsilon'_s, \tau'_3)$	$\alpha\gamma(\epsilon'_m, \tau'_{st})$	$\alpha\gamma(\epsilon'_l, \tau'_{st})$
0.20	0.20	0.20	1.4061	1.4547	1.4743	0.2812	0.2909	0.2949
0.20	0.20	0.10	1.4491	1.4766	1.4869	0.1449	0.1477	0.1487
0.20	0.20	0.05	1.4735	1.4881	1.4934	0.07368	0.07441	0.07467
				UK 2020				
ϵ	τ	α	$\gamma(\epsilon'_s, \tau'_3)$	$\gamma(\epsilon'_m, \tau'_{st})$	$\gamma(\epsilon'_l, \tau'_{st})$	$\alpha\gamma(\epsilon'_s, \tau'_3)$	$\alpha\gamma(\epsilon'_m, \tau'_{st})$	$\alpha\gamma(\epsilon'_l, \tau'_{st})$
0.18	0.20	0.20	1.3810	1.4229	1.4398	0.2762	0.2846	0.2880
0.18	0.20	0.10	1.4189	1.4425	1.4514	0.1419	0.1443	0.1451
0.18	0.20	0.05	1.4402	1.4528	1.4574	0.07201	0.07264	0.07287

Proposition 4.9. *For any given values of $\epsilon > 0$ and $\tau > 0$, a larger tax-adjustment γ to the opportunity cost of capital in the classic EOQ formula is needed the larger the firm and the smaller its opportunity cost of capital.*

Proof. This follows from the definition $\diamond$

Proposition 4.10. *It holds that:*

$$\frac{Q^*_{eoq}}{Q*} = \sqrt{\gamma}, \tag{4.60}$$

and

$$\frac{TC(Q^*_{eoq})}{TC(Q^*)} = \frac{1}{2}\Big[\sqrt{\gamma} + \frac{1}{\sqrt{\gamma}}\Big], \tag{4.61}$$

where Q^*_{eoq} is the EOQ lot-size at pre-tax level, Q^* is the tax-adjusted EOQ lot-size given by (4.54), and $TC(Q)$ is the sum of lot-size relevant logistics costs:

$$TC(Q) = s(1 - \epsilon' + \tau - \tau')\frac{y}{Q} + \Big[\alpha w(1 + \tau)\Big]\frac{Q}{2}. \tag{4.62}$$

Proof. This follows from simple analytical manipulation. $\diamond$

Note that this sensitivity result Eq.(4.61) has exactly the same shape as the sensitivity of the standard EOQ model to using wrong estimates for either the set-up cost or the unit holding cost. Our analysis leads to a more refined insight into why CT and VAT consideration is needed to improve the accuracy of these parameters. In particular, the CT is mostly important as a tax relief on the set-up costs while the VAT is primarily a tax penalty on keeping inventory.

4.8 Conclusions

This chapter looked into the classic inventory model and explains how to account for the effects of Value-Added-Tax (VAT) and Corporation Tax (CT) according to the rules of the UK government valid in 2015 and 2016.

The tax-adjusted EOQ model, derived from NPV principles, demonstrates that not only the tax rates themselves are important, but also the tax scheme, which describes the method of when the firms pays which amount of the taxes due to the government.

If the VAT scheme was not considered, one would find there to be no impact from VAT, since the both the Output VAT and Input VAT of a firm only serve to ensure that it collects any tax on final consumption, which it pays out to the government in full. When accounting for the method in which VAT is collected through either an annual standard accounting scheme, the impact of VAT is revealed on the firm's Annuity Stream profit function mainly as a correction factor that increases the cost of holding inventories. The impact on the set-up cost is non-zero and positive but quite small.

The net benefit from VAT on marginal profits of a firm is non-zero and positive and also quite small. However, in cases where the marginal profit is much larger than the sum of logistics costs (of set-ups and holding inventories), the small positive increase of profits may well outweight by far the increase in logistics costs. In those cases, VAT overall produces a net benefit to the firm. In those cases, an annual VAT scheme is also preferable to the standard VAT accounting scheme, and the three interim payments method fares better than the nine interim payments method.

While VAT increases the marginal profits of the firm with a modest factor, the CT greatly reduces marginal profits. However, accounting for the typical delays in which firms pay the government the taxes due reveals that the CT effect on marginal profits is typically several percentages below the CT tax rate. In particular, small firms which can enjoy paying taxes nine months after the accounting year benefit in that the CT effect is significantly smaller than the CT tax rate.

The CT scheme does not affect the holding costs in the inventory model but reduces the set-up costs, although by a smaller amount than the CT tax rate itself. The combined effect of CT and VAT, reducing set-up costs and increasing holding costs, means that they both reinforce each other in that they decrease the optimal lot size and increase the order frequency. At current tax rates, optimal order quantities are typically in the order of 20% smaller. This may mean in practical terms several more orders to be placed per year in comparison to using a standard EOQ model, but less warehouse space needed.

The investigation has also shown that the classic EOQ formula can still be applied if one substitutes the firm's opportunity cost of capital with an adjusted capital rate which accounts for the CT and VAT schemes applicable to the firm and this activity. This may be good news to firms in that the logistics manager can keep using the classic inventory method and does not need to be concerned with the impact of taxes explicitly. Financial accountants of the firm may instead provide the logistics manager with adjusted capital rate values to be used for the inventory planning of each type of product.

It is worthwhile to point out that this adjusted capital rate is significantly different from the firm's opportunity cost of capital. At current CT and VAT rates, it has been shown to be close to 50% higher than the firm's opportunity cost of capital, and that the exact value depends on firm size through the CT and VAT schemes it can select. The tax-adjusted EOQ model is as user-friendly in practical applications as the original model. Corporations can apply the model once they have identified their CT and VAT tax schemes. The firm's accountants may work out the NPV adjusted parameter values. This refined model is especially recommended for items with lower profit margins.

This research uses the current UK tax regulations, but we envisage that the approach would be relatively easily transferable to other countries which use similar VAT and CT schemes. Nations making use of the sales tax systems have not been addressed here, but the principles developed in the chapter of looking at the cash-flows and their impact on the AS profit function of the firm are likely applicable there too.

In this chapter, only activities that happen inside a domestic market of suppliers and customers has been investigated. The next chapter will expand this research by looking at trade of firms with suppliers and customers located in other nations.

Chapter 5

Economic Order Quantities Across Borders - Impact of Taxes and Tariffs

5.1 Introduction

Recent trading data show that the UK has significant trade flows with other nations, and this is in particular import-oriented. About 44% of UK exports in goods and services went to other countries in the EU in 2016, and 53% of their imports into the UK came from other countries in the EU in 2016(Kent (2016)). More recently, in April 2017 the value of exports (EU and Non-EU) was £26.5 billion, and total imports were £38.3 billion. The UK was a net importer, with imports exceeding exports by £11.8 billion according to UK trade office for National Statistics. This is contributed to by globalization, rapidly increasing number of regional trade agreements and free trade agreements between the nations.

Domestic as well as cross-country tax events are found in internationally operating businesses in global trading. It is a given that the operations and corporations are influenced by the legislative measures, as well as the trade policy imposed by the government. This policy is expressed in terms of import tariffs (duty), consumption tax (Value Added Tax), corporation tax and other factors. Despite the complexity of regulatory elements imposed by government authorities, it is surprising that much of the existing literature that has addressed the inventory and sourcing problem fails to account for the effect of theses policies. A few models have accounted for the issue of tax payments such as duty drawback can be found in Oh and Karimi (2006), consider duty in total cost function in Degraeve, Labro, and Roodhooft (2005), export oriented VAT policy in production decision in V. N. Hsu and Zhu (2011), but none of them considers this regulatory tax event combined with inflow and outflow of the timing of the tax payment.

In this chapter we particularly look at a UK based corporation, and cross-country trading regulation adopted by the firms for their trading activities. We investigate how domestic UK firm's inventory decision changed by cross-country supplier selection, separately look at supplier in EU and non-EU. First we introduce basic policy set by the UK authority to cross-border activities in both EU and non-EU trading regulations before Brexit. Second, we present a new deterministic EOQ model that can precisely account for three main tax factors - value added tax, import duty, corporation tax. Specific attention is paid to the particular schemes by which firms pay the government these taxes due. We use these models to examine how this would affect logistics decision making in the firm, in particular with respect to classic inventory optimisation. It is concerned with finding optimal order quantities and frequencies which represent trade-off factors on inventory theory. In more detail, however, it is concerned with finding out what the percentage difference is from the original inventory problem. Furthermore, the tax added method can give more accurate reference price in scouring decisions.

During import and export activities, VAT thus arises in two different formats. One is the domestic VAT, the other is the import VAT levied on the physical transaction of goods that move between nations. Both these chargeable VAT events can result in VAT cash-flows for the firm. Tariff is added as a cost of purchasing products and it ends up with different amounts of corporation tax payment.

Tariff included in this problem, which is naturally connect to the sourcing strategy for the operations. Operations management literature reports a wide range of factors that trigger international sourcing decisions. Sourcing mainly from lower purchasing price - see Nassimbeni (2006); inventory costs - see Callioni, de Montgros, Slagmulder, Van Wassenhove, and Wright (2005); and financial costs in terms of hedge against exchange risk see - B. Kim, Park, Jung, and Park (2017). Those papers do not consider the impact of outsourcing on equilibrium cross-border taxes and duty while other research considered taxes, tariffs and duties in the outsourcing decision. Holweg, Reichhart, and Hong (2011) considered comprehensive total cost which covers all aspects of outsourcing cost. Import duty is included overall purchasing price, and the same is considered in Kumar and Wilson (2009), Y. Liu and Tyagi (2011), Q. Feng and Lu (2012). In this sourcing decision considered about tax and tariffs, which is embedded within the model, but it may not be present with sufficient detail in the collection of consumption tax and cash flow of corporation tax in inventory decision process.

In the next sections, we first model the case of a UK based firm with domestic operations and which interacts with EU countries only. We assume that this situation would fall under 'Acquisition and Removal' rules (the case as before Brexit). We then proceed with the case that the firm interacts with other countries in an 'Import and Export' scenario.

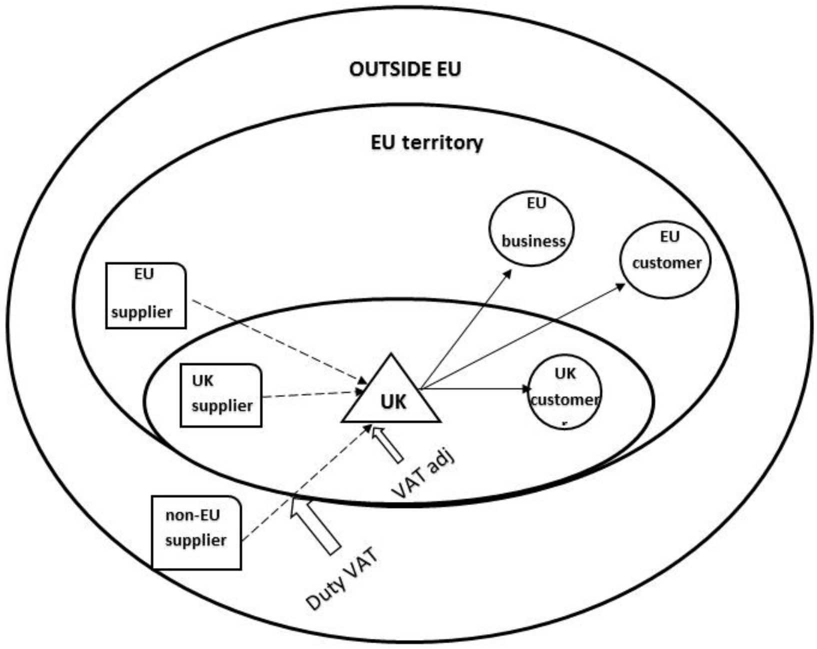

Figure 5.1: Supply Chain

5.2 EOQ with Acquisitions and Removals

We consider the case of a UK domestic buyer trading with an EU supplier and where the mechanism of 'reverse charge' applies. The VAT reverse charge is at its simplest a mechanism where the liability to account for and pay VAT on cross-border services is transferred from the supplier to the receiver of certain services. It is important to note that reverse charge only applies to Business to Business (B2B) transactions and when services are supplied.

The seller of the service accounts the VAT as zero, and the service receiver or buyer in their VAT return form will write an equal amount of VAT output and input, so it is cancelled out. The importance of this reverse charge is that there is no cash flow for input VAT. So, if a UK business uses a delivery service from an EU-registered business, the EU seller sets a zero rate for VAT in the invoice, so the UK business only pays for the pricing without tax. Reverse charge works in our problem when the domestic supplier chooses the delivery service from EU VAT-registered operations.

Yao, Huang, Song, and Mishra (2018) indicate that service outsourcing is very common in the commercial supply chain, like the transportation services. In this modelling approach we assume that the buyer outsources the transportation service and can choose either domestic supplier or EU supplier.

Goods purchased from other EU countries are referred to as 'acquisitions', while selling to customers located in other EU countries are called 'dispatches'. There is a VAT charge difference for VAT-registered and non-VAT-registered businesses. For the non-VAT-registered businesses in the UK, it would be the same whether it acquires goods in the domestic market or from other EU markets as long as they have to pay domestic VAT or the other EU country's VAT rate for their acquired goods. They cannot claim back this VAT paid from the UK government. In our further treatment, we consider a UK firm that is VAT-registered and will refer to the UK market as the domestic market.

In the basic EOQ model of an activity of such firm, it has a choice of buying from a supplier in the domestic market, a supplier from another EU nation, or from a non-EU supplier. We use dash lines in Figure 5.1 to demonstrate that only one possible supplier is chosen. Furthermore, in this section we will only consider suppliers located inside the EU.

If the firm acquires goods from its own country, it pays Input VAT on their purchases, and this situation is the same as the domestic transaction model which was treated in Chapter 3. If the firm, however, acquires goods from a supplier located in another EU nation, they can zero-rate for Input VAT. See also Chapter 2.

The firm can also choose to sell to customers in the domestic marker, to dispatch to customers in another EU nation, or to export the goods. In the models being developed in this chapter, we will consider that these three possible sales options can be simultaneously deployed if desired. For this reason, we have used solid lines in Figure 1. In this section, however, we exclude exports.

For domestic customers, the firm charges the Output VAT and the UK VAT rate. For dispatches to customers which are not VAT-registered, the UK firm either charges at the UK VAT rate or the VAT rate that applies in the EU country where the customers are located, depending on whether the firm exceeds a given threshold volume of trade to that nation. For dispatches to VAT-registered customers, the UK firm can zero-rate for VAT purposes. See also Chapter 2. We assume that the sales prices the UK firm charges for the same good are allowed to differ from nation to nation (one reason to justify this may be, for example, differences in transportation costs).

The consideration of various possibilities arising from the location of the supplier, the location and type of the customers, and whether or not the threshold of sales is or is not exceeded, will be examined and compared in the models developed in the remainder of this section, and in the following section.

Adopting the methodology developed in Chapter 4, and accounting for the detailed explanations below, we can develop the AS profit function for either purchasing from a domestic or EU market as follows:

$$\begin{aligned}\mathrm{AS}_{a0} = & \; py(1+\tau) + p_{on}y_{on}(1+\tau_{on}) + p_{or}y_{or}(1+\tau_{or}) \\ & -dy(1+\tau) - d_{on}y_{on}(1+\tau_{on}) - d_{or}y_{or}(1+\tau_{or})\end{aligned}$$

$$- s\frac{\alpha(1+\tau_s)}{1-e^{-\alpha T}} - wYT\frac{\alpha(1+\tau_w)}{1-e^{-\alpha T}} - (1-\delta)\text{FOC}(1+\tau) - \delta\text{FOC}. \tag{5.1}$$

The VAT payment is the difference of input and output VAT. Considering acquisition and removal, the net VAT payment can be summarised as below:

$$\begin{aligned}\text{NVAT} &= \text{OVAT}_{uk} + \text{OVAT}_{on} + \text{OVAT}_{or} - \text{IVAT} \\ &= py\tau + p_{on}y_{on}\tau_{on} + p_{or}y_{or}\tau_{or} - dy\tau - d_{on}y_{on}\tau_{on} - d_{or}y_{or}\tau_{or} \\ &\quad - \frac{s}{T}\tau_s - wY\tau_w - (1-\delta)\text{FOC}\tau. \end{aligned} \tag{5.2}$$

The value added tax AS function depends on which scheme is used. Hence:

$$AS_\tau = -NVAT\tau'. \tag{5.3}$$

The detailed explanation of the various terms in the above equations is described below:

- $py\tau$, VAT charge on sales to domestic customers;
- $p_{on}y_{on}\tau_{on}$, VAT charge on sales to EU non-VAT-registered customers. The domestic supplier will charge $\tau_{on} = \tau$, if it sells below the distance selling threshold, but takes the value of the VAT rate of the EU country of sales otherwise;
- $p_{or}y_{or}\tau_{or}$, VAT charges on sales to VAT-registered EU customers. In accounting terms, the VAT rate could either be τ or τ_{on} as above, but in cash-flow terms this rate can currently under EU regulations of dispatches be zero-rated, and thus $\tau_{or} = 0$;
- $dy\tau$, VAT charge on cost of delivery to the domestic market;
- $d_{on}y_{on}\tau_{on}$, VAT charge on cost of delivery to EU non-VAT-registered customer market. The basic rule for supplies to non-business customers is that the supplier will account for their own domestic VAT rate as τ;
- $d_{or}y_{or}\tau_{or}$, VAT charge on cost of delivery to EU VAT-registered customer market. For business to business supply, the customer will typically handle the VAT, and under the EU rules of dispatches, τ_{or} can be zero-rated;
- $\frac{s}{T}\tau_s$, VAT charge on supply or order costs. Businesses have the option to either purchase delivery services from a domestic supplier, then $\tau_s = \tau$, or delivery services from the EU market, which can then be zero-rated at rate or $\tau_s = \tau_{or} = 0$;
- $wY\tau_w$, VAT charge of purchasing costs on each order from the supplier. If the supplier is domestic, then $\tau_w = \tau$, otherwise if the supplier is from the EU, this can be zero-rated, and thus $\tau = \tau_{or} = 0$.

The operating profit function is arrived at following the logic similar to that leading to Eq.(4.38) of Chapter 4:

$$OP = py + p_{on}y_{on} + p_{or}y_{or} - dy - d_{on}y_{on} - d_{or}y_{or} \\ - \frac{s}{T} - wY - \text{FOC}. \tag{5.4}$$

From the above equation, and following the notation introduced in Chapter 3, the total AS function can be written as $AS_a = AS_{ao} + AS_\tau + AS_\epsilon$. In explicit form, this gives:

$$AS_a = py(1+\tau) + p_{on}y_{on}(1+\tau_{on}) + p_{or}y_{or}(1+\tau_{or}) \\ -dy(1+\tau) - d_{on}y_{on}(1+\tau_{on}) - d_{or}y_{or}(1+\tau_{or}) \\ -s\frac{\alpha(1+\tau_s)}{1-e^{-\alpha T}} - wYT\frac{\alpha(1+\tau_w)}{1-e^{-\alpha T}} - (1-\delta)\text{FOC}(1+\tau) - \delta\text{FOC}. \\ -(py\tau' + p_{on}y_{on}\tau'_{on} - dy\tau' - d_{on}y_{on}\tau'_{on} - \frac{s}{T}\tau'_s - wY\tau'_w - (1-\delta)\text{FOC}\tau') \\ - \epsilon'(py + p_{on}y_{on} + p_{or}y_{or} - dy - d_{on}y_{on} - d_{or}y_{or} - \frac{s}{T} - wY - \text{FOC}). \tag{5.5}$$

After rearranging, this gives;

$$AS_a = (p-d)y(1+\tau-\tau'-\epsilon') + (p_{on} - d_{on}y_{on}(1+\tau-\tau'-\epsilon') + (p_{or}-d_{or})y_{or}(1-\epsilon') \\ -FOC(1-\epsilon') - FOC(1-\delta)(\tau-\tau'). \\ - s\frac{\alpha(1+\tau_s)}{1-e^{-\alpha T}} + \frac{s}{T}\tau'_s + \frac{s}{T}\epsilon' - wYT\frac{\alpha(1+\tau_w)}{1-e^{-\alpha T}} + wY\tau'_w + wY\epsilon' \tag{5.6}$$

Linearisation of this acquisition and removal of AS function produces:

$$\overline{AS_a} = (p-d)y(1+\tau-\tau'-\epsilon') + (p_{on}-d_{on})y_{on}(1+\tau_{on}-\tau_{on}{}'-\epsilon') \\ +(p_{or}-d_{or})y_{or}(1+\tau_{or}-\tau_{or}{}'-\epsilon') - wY(1+\tau_w-\tau_w{}'-\epsilon') \\ -\frac{s\alpha}{2}(1+\tau_s) - \text{FOC}(1-\epsilon') - \text{FOC}(1-\delta)(\tau-\tau') \\ - s\frac{(1+\tau_s-\epsilon'-\tau_s{}')}{T} - \Big[\alpha w(1+\tau_w)\Big]\frac{yT}{2}. \tag{5.7}$$

From this formula we can derive optimal order quantities in different situations.

$$Q_a^* = \sqrt{\frac{2s(1+\tau_s-\epsilon'-\tau'_s)Y}{\alpha w(1+\tau_w)}}. \tag{5.8}$$

5.3 EOQ Formula under Different Particular Assumptions

5.3.1 Domestic Purchasing and Supply Delivery

In this case, set-up costs and purchasing costs are subject to the domestic VAT rate, and the VAT scheme and CT schems developed in Chapter 4 are applicable.

This leads to the EOQ formula that is the same as Eq.(4.54) in Chapter 4, but where y is replaced by Y:

$$\begin{aligned}\overline{AS_{a1}} &= (p-d)y(1+\tau-\tau'-\epsilon') + (p_{on}-d_{on})y_{on}(1+\tau_{on}-\tau_{on}'-\epsilon') \\ &+(p_{or}-d_{or})y_{or}(1+\tau_{or}-\tau_{or}'-\epsilon') - wY(1+\tau-\tau'-\epsilon') \\ &-\frac{s\alpha}{2}(1+\tau) - \text{FOC}(1-\epsilon') - \text{FOC}(1-\delta)(\tau-\tau') \\ &- sY\frac{(1+\tau-\epsilon'-\tau')}{Q} - \alpha w(1+\tau)\frac{Q}{2}. \end{aligned} \tag{5.9}$$

This gives:

$$Q^*_{a1} = \sqrt{\frac{2s(1+\tau-\epsilon'-\tau')Y}{\alpha w(1+\tau)}}. \tag{5.10}$$

5.3.2 Domestic Purchasing and Supply Delivery by EU

Purchasing happens in the domestic market but the set-up cost is charged by an EU tax- registered company, which can be zero-rated. The AS profit function simplifies to:

$$\begin{aligned}\overline{AS_{a2}} &= (p-d)y(1+\tau-\tau'-\epsilon') + (p_{on}-d_{on})y_{on}(1+\tau-\tau'-\epsilon') \\ &+(p_{or}-d_{or})y_{or}(1-\epsilon') - wY(1+\tau-\tau'-\epsilon') - \frac{sY(1-\epsilon')}{Q} - \alpha w(1+\tau)\frac{Q}{2} \\ &- \frac{s\alpha}{2} - FOC(1-\epsilon') - FOC(1-\delta)(\tau-\tau'). \end{aligned} \tag{5.11}$$

This gives:

$$Q^*_{a2} = \sqrt{\frac{2s(1-\epsilon')Y}{\alpha w(1+\tau)}}. \tag{5.12}$$

5.3.3 EU Purchasing and Supply Delivery by Domestic Firms

Goods acquired from a business in the EU can be zero-rated for VAT purposes, but the set-up cost is to be rated at the domestic VAT rate:

$$\overline{AS_{a3}} = (p-d)y(1+\tau-\tau'-\epsilon') + (p_{on}-d_{on})y_{on}(1+\tau-\tau'-\epsilon')$$
$$+(p_{or}-d_{or})y_{or}(1-\epsilon') - wY(1-\epsilon') - \frac{sY(1+\tau-\tau'-\epsilon')}{Q} - \alpha w\frac{Q}{2}$$
$$-\frac{s\alpha}{2} - FOC(1-\epsilon') - FOC(1-\delta)(\tau-\tau'). \tag{5.13}$$

Therefore:

$$Q^*_{a3} = \sqrt{\frac{2s(1-\epsilon'+\tau-\tau')Y}{\alpha w}}. \tag{5.14}$$

5.3.4 EU Purchasing and Supply Delivery

This means that VAT on purchases and set-ups costs can be zero-rated;

$$\overline{AS_{a4}} = (p-d)y(1+\tau-\tau'-\epsilon') + (p_{on}-d_{on})y_{on}(1+\tau-\tau'-\epsilon')$$
$$+(p_{or}-d_{or})y_{or}(1-\epsilon') - wY(1-\epsilon') - \frac{sY(1-\epsilon')}{Q} - \alpha w\frac{Q}{2}$$
$$-\frac{s\alpha}{2} - FOC(1-\epsilon') - FOC(1-\delta)(\tau-\tau'). \tag{5.15}$$

This produces:

$$Q^*_{a4} = \sqrt{\frac{2s(1-\epsilon')Y}{\alpha w}}. \tag{5.16}$$

5.3.5 Impact of Sales Mix on the VAT in the AS Profit Function

From Eq.(5.9) we derive that the impact of the sales mix on the AS profit function is captured in the following profit terms of this function:

$$(p-d)y(1+\tau-\tau'-\epsilon') + (p_{on}-d_{on})y_{on}(1+\tau_{on}-{\tau_{on}}'-\epsilon')$$
$$+ (p_{or}-d_{or})y_{or}(1+\tau_{or}-{\tau_{or}}'-\epsilon') \tag{5.17}$$

As mentioned before, criteria that influence the values of the VAT impacts in the above include the amount sold into the domestic market versus the amount sold to EU customers.

The term $(p_{or}-d_{or})y_{or}(1+\tau_{or}-{\tau_{or}}'-\epsilon')$ is the impact of selling to business customers (B2B) who are VAT-registered in the EU country. Under the EU regulation of dispatches, there is not a VAT cash flow and we can rewrite it to $(p_{or}-d_{or})y_{or}(1-\epsilon')$. Selling to VAT-registered customers in the UK is better with respect to the firm's AS function in comparison to selling to VAT-registered customers in other EU nations, because the latter can be zero-rated.

The term of $(p_{on}-d_{on})y_{on}(1+\tau_{on}-\tau_{on}'-\epsilon')$ is the result of selling to consumers (B2C) who are non-VAT-registered in the EU, and VAT payments occur. Here, we further clarify the Distance Selling Threshold (DST). DST applies when a VAT-registered business in one of the EU countries sells to another EU country's non-VAT-registered customers by Watson (2014). The selling firm will have to register for VAT in the EU country it sells to if the following condition is satisfied:

$$p_{on}y_{on} > \mathrm{DST}_o, \tag{5.18}$$

where DST_o is the Distance Selling Threshold value of the destination country. If Eq.(5.18) is not satisfied, then the selling firm charges the VAT rate of its own country, and the impact can be rewritten as $(p_{on}-d_{on})y_{on}(1+\tau-\tau'-\epsilon')$; otherwise, the selling firm must register at the destination country and charges the VAT rate of the destination country, and the impact can be rewritten as $(p_{on}-d_{on})y_{on}(1+\tau_d-\tau_d'-\epsilon')$, where τ_d is the VAT rate for the destination country, and VAT payment depends on that country's VAT rules.

5.3.6 Double Tax Relief (DTR)

If DSTs are exceeded, or firms sell B2B in other countries, the impact on CT has also has to be considered. As in Law (2016): 'The general principle is that a UK resident company is subject to UK corporation tax on its worldwide profits and gains.' However, Double Tax Relief (DTR) is the mechanism that if adopted can reduce the impact of overseas income being taxed twice, so in the term of $(1+\tau_d-\tau_d'-\epsilon')$, the ϵ' can be relief or exempted.

5.4 Examples of Acquisitions and Removals

We presented three types of firms in Chapter 4, Table 1. In the following examples we consider the case of a medium-sized firm using the CT scheme corresponding to ϵ'_m and the VAT scheme corresponding to τ'_{st}. We examine the difference between using the basic EOQ formula versus the tax-adjusted EOQ models developed in this chapter.

Table 5.1 reports the difference of using the classic EOQ formula on the order size, the logistics costs TC, and the profitability AS. All sales are VAT chargeable. We observe that the smallest differences occur when the firm purchases from another EU country. This is a consequence of the zero-rating of the VAT due to the EU regulations on acquisitions.

While the impact on profits in Table 5.1 remains modest, the change in profitability is higher in a second series of experiments reported in Table 5.2. In these, we have used

Table 5.1: Medium sized firm EOQ, logistic cost, Profit change of pre-tax versus tax-adjusted model

ϵ	τ	α	$\frac{Q^*_{eoq}}{Q^*_{a1}}$	$\frac{Q^*_{eoq}}{Q^*_{a2}}$	$\frac{Q^*_{eoq}}{Q^*_{a3}}$	$\frac{Q^*_{eoq}}{Q^*_{a4}}$
0.2	0.2	0.2	1.2060	1.2127	1.1009	1.1070
0.2	0.2	0.10	1.2151	1.2185	1.1093	1.1124
0.2	0.2	0.05	1.2199	1.2216	1.1136	1.1152
0.18	0.2	0.2	1.1929	1.1992	1.0889	1.0945
0.18	0.2	0.10	1.2011	1.2043	1.0964	1.0994
0.18	0.2	0.05	1.2053	1.2070	1.1003	1.1018
ϵ	τ	α	$\frac{TC_{a1}(Q^*_{eoq})}{TC_{a1}(Q^*_{a1})}$	$\frac{TC_{a2}(Q^*_{eoq})}{TC_{a2}(Q^*_{a2})}$	$\frac{TC_{a3}(Q^*_{eoq})}{TC_{a3}(Q^*_{a3})}$	$\frac{TC_{a4}(Q^*_{eoq})}{TC_{a4}(Q^*_{a4})}$
0.2	0.2	0.2	1.0176	1.0186	1.0046	1.0052
0.2	0.2	0.10	1.0190	1.0196	1.0054	1.0057
0.2	0.2	0.05	1.0198	1.0201	1.0058	1.0059
0.18	0.2	0.2	1.0155	1.0165	1.0036	1.0041
0.18	0.2	0.10	1.0168	1.0173	1.0042	1.0044
0.18	0.2	0.05	1.0175	1.0177	1.0046	1.0047
ϵ	τ	α	$\frac{AS_{a1}(Q^*_{a1})}{AS_{a1}(Q^*_{eoq})}$	$\frac{AS_{a2}(Q^*_{a2})}{AS_{a2}(Q^*_{eoq})}$	$\frac{AS_{a3}(Q^*_{a3})}{AS_{a3}(Q^*_{eoq})}$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a4}(Q^*_{eoq})}$
0.2	0.2	0.2	1.0135	1.0141	1.0027	1.0029
0.2	0.2	0.1	1.0085	1.0087	1.0020	1.0021
0.2	0.2	0.05	1.0055	1.0056	1.0014	1.0014
0.18	0.2	0.2	1.0117	1.0123	1.0021	1.0023
0.18	0.2	0.1	1.0073	1.0075	1.0015	1.0016
0.18	0.2	0.05	1.0048	1.0049	1.0011	1.0011

In the example we use the value for $p = 35$,$w = 28$,$y = 3000$,$s = 500$,$FOC = 9\%py$,$y_{on} = 0, y_{or} = 0$ and Q^*_{eoq} in Eq. 4.3; Q^*_{a1} in Eq.4.54; Q^*_{a2} in Eq. 5.12; Q^*_{a3} in Eq. 5.14; Q^*_{a4} in Eq. 5.16. In TC only think logistic cost which can be expressed as follow: $TC_{a1} = \frac{sy(1+\tau-\tau'-\epsilon')}{Q} + \alpha w(1+\tau)\frac{Q}{2}$; $TC_{a2} = \frac{sy(1-\epsilon')}{Q} + \alpha w(1+\tau)\frac{Q}{2}$; $TC_{a3} = \frac{sy(1+\tau-\tau'-\epsilon')}{Q} + \alpha w\frac{Q}{2}$; $TC_{a4} = \frac{sy(1-\epsilon')}{Q} + \alpha w\frac{Q}{2}$.

Table 5.2: Medium sized firm profitability change when higher $\frac{w}{p}ratio$

ϵ	τ	α	$\frac{AS_{a1}(Q^*_{a1})}{AS_{a1}(Q^*_{eoq})}$	$\frac{AS_{a2}(Q^*_{a2})}{AS_{a2}(Q^*_{eoq})}$	$\frac{AS_{a3}(Q^*_{a3})}{AS_{a3}(Q^*_{eoq})}$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a4}(Q^*_{eoq})}$
0.2	0.2	0.2	1.3331	1.3106	1.0123	1.0134
0.2	0.2	0.1	1.0386	1.0393	1.0067	1.0070
0.2	0.2	0.05	1.0177	1.0179	1.0040	1.0041
0.18	0.2	0.2	1.2325	1.2229	1.0093	1.0103
0.18	0.2	0.1	1.0328	1.0334	1.0052	1.0054
0.18	0.2	0.05	1.0153	1.0154	1.0031	1.0032

In the example we use the value for $p = 35$,$w = 30$,$y = 3000$,$s = 500$,$FOC = 9\%py$ and $y_{on} = 0, y_{or} = 0$.

equal parameter values as in the first table, except for the purchasing price w, which was now increased from 28 to 30. It demonstrates that the small change in costs can have important impacts on profit when the products are sold at small marginal profit.

Table 5.3 compares the difference for all UK versus EU sourcing strategies. In purchasing cost $w = 28$, AS_4 function which point the EU sourcing and AS_1 all UK purchasing. There is a 21% difference in the high capital rate (first line), while in the higher w value of 30, the difference is over four times as high.

Table 5.3: UK sourcing versus EU sourcing strategy

ϵ	τ	α	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a1}(Q^*_{a1})}, (w = 28)$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a1}(Q^*_{a1})}, (w = 30)$
0.2	0.2	0.2	1.2114	5.059
0.2	0.2	0.1	1.099	1.4473
0.2	0.2	0.05	1.0511	1.1652
0.18	0.2	0.2	1.2057	4.4335
0.18	0.2	0.1	1.0966	1.4289
0.18	0.2	0.05	1.0500	1.16048

In the example we use the value for $p = 35$,$y = 3000$,$s = 500$,$FOC = 9\%py$ and $y_{on} = 0, y_{or} = 0$.

In the above two examples all sales have output VAT payments. In the case of zero-rated VAT outputs, the economic order quantities and logistic costs are not affected as they are not valued by selling price values. Table 5.4 reports the impact on profitability, using the same parameters and the experiments of Table 5.2, except for the fact that only 90% of sales occur with VAT charged, while the other 10% have no VAT charge on them. (Table 5.5 85%, Table 5.6 is 80% for VAT charged sales).

Table 5.4: Profitability change participial selling does not have positive output VAT payment in Medium sized firm $y_{uk} = 0.9$

ϵ	τ	α	$\frac{AS_{a1}(Q^*_{a1})}{AS_{a1}(Q^*_{eoq})}$	$\frac{AS_{a2}(Q^*_{a2})}{AS_{a2}(Q^*_{eoq})}$	$\frac{AS_{a3}(Q^*_{a3})}{AS_{a3}(Q^*_{eoq})}$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a4}(Q^*_{eoq})}$
0.2	0.2	0.2	1.5771	1.4961	1.0131	1.0143
0.2	0.2	0.1	1.0399	1.0406	1.0069	1.0072
0.2	0.2	0.05	1.0179	1.0181	1.0040	1.0041
0.18	0.2	0.2	1.3468	1.3181	1.0099	1.0109
0.18	0.2	0.1	1.0338	1.0345	1.0053	1.0055
0.18	0.2	0.05	1.0154	1.0156	1.0031	1.0032

$p = 35$,$w = 30$,$y = 3000$,$s = 500$,$FOC = 9\%py$, and 90% of total demand sell in domestic market which has positive output VAT while another 10% selling cannot benefit of output VAT, $y = 90\%3000, y_{on} = 0, y_{or} = 10\%3000$.

Table 5.5: Profitability change participial selling does not have positive output VAT payment in Medium sized firm $y_{uk} = 0.85$

ϵ	τ	α	$\frac{AS_{a1}(Q^*_{a1})}{AS_{a1}(Q^*_{eoq})}$	$\frac{AS_{a2}(Q^*_{a2})}{AS_{a2}(Q^*_{eoq})}$	$\frac{AS_{a3}(Q^*_{a3})}{AS_{a3}(Q^*_{eoq})}$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a4}(Q^*_{eoq})}$
0.2	0.2	0.2	1.9104	1.7074	1.0135	1.0147
0.2	0.2	0.1	1.0405	1.0413	1.0069	1.0072
0.2	0.2	0.05	1.0180	1.0182	1.0041	1.0042
0.18	0.2	0.2	1.4597	1.4044	1.0103	1.0113
0.18	0.2	0.1	1.0344	1.0350	1.0053	1.0056
0.18	0.2	0.05	1.0155	1.0157	1.0031	1.0032

Table 5.4 shows that this now further increases the differences between the classic EOQ formula and the refined EOQ formula developed in this chapter. In conclusion, products sold at low profit margins to VAT-registered businesses in other EU countries requires the company to be more careful about economic order quantity decisions, and these firms should consider the impact of the tax regulations carefully. This is particularly

Table 5.6: Profitability change participial selling does not have positive output VAT payment in Medium sized firm $y_{uk} = 0.8$

ϵ	τ	α	$\frac{AS_{a1}(Q^*_{a1})}{AS_{a1}(Q^*_{eoq})}$	$\frac{AS_{a2}(Q^*_{a2})}{AS_{a2}(Q^*_{eoq})}$	$\frac{AS_{a3}(Q^*_{a3})}{AS_{a3}(Q^*_{eoq})}$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a4}(Q^*_{eoq})}$
0.2	0.2	0.2	3.1556	2.2326	1.0141	1.0152
0.2	0.2	0.1	1.0412	1.0420	1.0070	1.0073
0.2	0.2	0.05	1.0181	1.0183	1.0041	1.0042
0.18	0.2	0.2	1.6818	1.5549	1.0106	1.0116
0.18	0.2	0.1	1.0349	1.0355	1.0053	1.0056
0.18	0.2	0.05	1.0155	1.0157	1.0031	1.0032

$p = 35$,$w = 30$,$y = 3000$,$s = 500$,$FOC = 9\%py$, and 90% of total demand sell in domestic market which has positive output VAT while another 10% selling cannot benefit of output VAT, $y = 80\%3000, y_{on} = 0, y_{or} = 20\%3000$.

the case when the supply comes from the UK. Such firms incur Input VAT but cannot benefit as much from collecting Output VAT.

5.5 Impact on Classic Inventory Theory

Based on the above models and derived insights, some general theoretical conclusions as well as potential areas for further research can be formulated.

5.5.1 Impact on Order Quantity Decisions

This section on acquisitions and removals in a context of trading within the EU shows that the optimal order quantity depends on the VAT and CT schemes implemented by different governments.

A UK firm that purchases from a UK supplier will order, ceteris paribus, in different order quantities than a UK firm where the supplier is located in another EU country.

The main factor here is the fact that firms can zero-rate the VAT on transactions across borders, while they must charge the VAT for transactions within the UK.

5.5.2 Impact on supplier selection

In terms of profitability, ordering from an EU supplier is, ceteris paribus, preferable to ordering from a UK supplier. This is again due to the VAT rules on acquisitions.

5.5.3 Impact on Sales Strategy

Ceteris paribus, it is preferable to sell B2C rather than B2B to customers in other EU countries, because of the difference between collecting Output VAT or no VAT.

It is, ceteris paribus, most attractive to sell over the distance selling threshold (DST) to countries where the VAT rate is the highest, provided that the VAT scheme by which the Output VAT has to be paid to the respective government offers a net positive benefit on the AS function for the firm.

It is noteworthy to point out that the DST is different from country to country. The EU allows countries to set this at either 35,000 or 100,000 euros, while the UK sets it at 70,000 pounds.

5.5.4 Impact on Location Decisions

It was shown that small- to medium-sized UK businesses using local suppliers trying to broaden their B2B customer base to other EU countries have an extra hurdle to overcome due to experiencing the negative effect of Input VAT while not enjoying the positive effect of Output VAT (see e.g.Table 5.6).

This provides some incentive for the firm to have a base or satellite in the EU country it tries to sell to, as ordering from the UK supplier from the firm's branch in the destination country means it can now zero-rate the Input VAT. If it keeps using the local UK suppliers, it can now cancel the negative effect of Input VAT

Furthermore, having a branch in the destination country allows the firm to enjoy the financial rewards of collecting the Output VAT of the destination country (assuming that VAT collection in that country occurs according to similar schemes as those investigated in Chapter 4).

A similar incentive occurs for firms broadening their B2C customer base to other EU countries, but the incentive will be smaller since it can only alleviate the Input VAT effect from establishing a satellite firm, while the Output VAT effect is already there without having the satellite firm. VAT may hence have some influence on whether an expanding firm will want to establish a satellite office in the country it wants to sell to. Whether this benefit is significant can be investigated from comparing the profits the firm could make in both situations.

Taking this one step further, we can consider the possible trade-off between the firm purchasing from the local UK supplier from its original UK base where because of volume it may enjoy quantity discounts, or let the satellite make its own purchases, saving on the Input VAT effect and transportation cost for big products but then possibly not enjoying benefits of quantity discounts. There are possible ways around problem, if the firm can agree with the supplier discounts not based on invididual order but based on a promised annual total volume that would be the sum of the volumes needed by the firm in the UK plus its satellite firm in the other country.

5.6 EOQ with Imports and Exports

We now focus our attention on the case of a UK based firm trading with (non-EU) countries. In particular, we have to introduce new cash flows associated with the collection of import duties.

UK firms exporting to outside the EU can zero-rate most goods to non-EU countries. Evidence that the goods have left the EU and a record of the exported amount must be reported in VAT accounts.

Figure 5.1 shows the three possible ways for firms to purchase supplies. The situation of supplies from other EU countries has been addressed in previous sections of this chapter, and of goods supplied from inside the UK in Chapter 4. Goods that are being imported from outside EU countries need to be declared to HMRC, as well as the duty and VAT charged by the government authorities. A tariff is a tax applied to goods that are traded on international markets.

When importing to the UK, or via the UK into other EU countries, VAT is charged at the same rate as if business purchases goods from a supplier inside the UK. VAT-registered businesses in the UK can reclaim the import VAT employing the same process used to reclaim Input VAT on purchases of supplies within the UK. Firms registered for VAT elsewhere in the EU and importing via the UK can reclaim VAT paid in the UK.

In most cases, import duty is calculated as a percentage of the customs value of the import. The customs value varies by country while duty rate depends on the product classification. The customs value can be based on FOB value or CIF value. All EU countries use the CIF value for calculating the duty on an import. The CIF value is the sum of the price paid for the goods C, the insurance cost I, and the shipping cost F.

HMRC considers six methods to calculate the import valuations. We illustrate here one method which applies to over 90% of import consignments. The first example illustrates a straightforward approach that is, however, not adopted by HMRC:

Example 5.1. *Goods are bought by a company located in Aberdeen from China for £*5000 *and are subject to £*250 *UK Duty. The shipping quote to the location of the firm is £*500*. Then, the VAT due would be £*1150*:* $VAT = 20\% of (\$5000 + \$250 + \$500) = £1150$.

HMRC adopts the principle that two companies importing identical products purchased for the same amount should pay the same duty and VAT. If the goods arrive in Southampton, for example, a firm located in Southampton would receive a cheaper shipping quote than the firm located in the Highlands. So, in the above example, the company would be located in Southampton might have a shipping quote of only £200, while arrival of Highland cost £500 which implying a £60 difference in the VAT that the firm would have to pay.

The principle adopted by HMRC, therefore, is that the CIF value for the shipping cost is only the cost to the EU border, excluding the delivery cost of additional transport within the EU. This would make the VAT payment for both companies in the above example equal.

The process adopted by HMRC is not to account for the full door-to-door shipping cost being used for the VAT calculation, just the costs to the border. VAT Value Adjustment (VAT-VA) is introduced to calculate the shipping charges from the UK border to the first point of delivery in the UK. The cost of delivery from the point of entry into the EU to the location of the firm in the UK can be done by either a domestic or an EU carrier. When using a domestic carrier, the Input VAT rules developed in Chapter 4 apply. When using the EU carrier, the reverse-charge mechanism works and it can be zero-rated for VAT purposes. Hence, the whole VAT payment consists of the shipping cost to get the goods to the UK (EU) border and VAT-VA figure that depends on the size of shipment and distance from the port.The following example illustrates VAT-VA. The firm in the UK may also wish to import via another EU country, and similar principles apply. The timing of cash flows are shown in Figure 5.2.

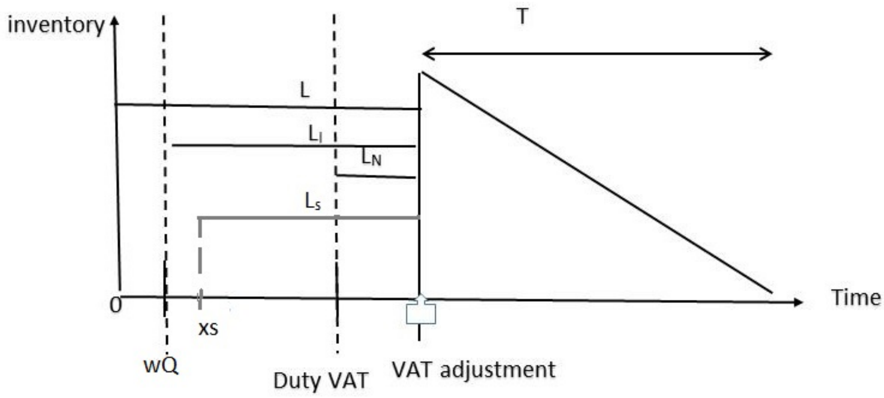

Figure 5.2: import cash flow

Example 5.2. *CIF Value of Goods 12500.00 GBP, Duty Rate 6%, VAT Adjustment 450.00 GBP, VAT Rate 17.5%. Duty Payable = 12500.00 GBP x 6% = 750.00 GBP. VAT Payable = (12500.00 GBP + 750.00 GBP + 450.00) GBP x 17.5% = 2397.50 GBP.*

There are different payment structures for CIF value. In logistic terms, CIF value refers to $wYT + xs$. The first term is purchasing cost, and the second term refers to shipping and insurance costs. In the most common case, the buyer pays wYT and xs at the same time. In general, they can occur at different times. Thus, we assume the case of paying the amount wYT at a time L_I prior to the expected arrival time of the goods to compensate the supplier for the goods, while the amount of xs is payed at a time that is L_s prior to the arrival of the goods at the UK buyer to compensate the logistics' party of the shipping and insurance cost.

As illustrated in Figure 5.2, we can model these relative times by placing an *anchor point* at some arbitrary time in future L from current decision time. This methodology is based on Beullens and Janssens (2011). The anchor point corresponds to the time the first batch of products are delivered at the location of the UK firm. Placing an anchor point means that this moment is assumed to be dictated by the customer and is not affected by a change in lead times, or different choice of the decision variables. In other words, if the lead times would be longer, the order needs to be placed sooner. Note that alternative placements of the anchor point are also possible, e.g. at the time that the order would be placed at the supplier. The placement of the anchor point may affect the inventory decision model.

5.6.1 Net Present Value Based Import Method

As illustrated in Figure 5.2, the purchasing payment happens at time L_I before the anchor point, and an instalment payment of the set-up cost occurs at a time L_s prior to the anchor point. If we assume that there is no instalment payment of set-up cost happens, then $x = 0$, and this is the the same as in Example 5.1, while in Example 5.2 the set-up cost is split into two parts. Depending on the circumstances, maybe the first part set-up cost xs pays out at time L_s and $L_s > L_I$ or the purchasing cost wYT comes at time L_I and $L_s < L_I$, otherwise they might be at the same time as $L_I = L_s$. There is no duty or VAT handling time in NPV based import AS function, hence $L_N = 0$. It is easy to see that the net present value of import without tax consideration is now given as below:

$$ASN = \Big[py - (wYTe^{\alpha L_I} + xse^{\alpha L_s} + (1-x)s)\sum_{i=0}^{\infty} \alpha e^{-i\alpha T} - FOC\Big]e^{-\alpha L}.$$

The first term is revenue from sales, the second term is unit purchasing cost paid at time L_I, the third term is set-up cost payment at time L_s and the fourth term is remaining set-up payment at the anchor point.

After linearisation, this becomes:

$$[py - (wYe^{\alpha L_I} + \frac{\alpha wYTe^{\alpha L_I}}{2} + \frac{xs}{T}e^{\alpha L_s} + \frac{\alpha xse^{\alpha L_s}}{2} + \frac{(1-x)s}{T} + \frac{\alpha(1-x)s}{2})$$

$$- FOC\big]e^{-\alpha L}. \tag{5.19}$$

Hence, the NPV based import EOQ without tax is,

$$Q_N{}^* = \sqrt{\frac{2sY[xe^{\alpha L_s} + (1-x)]}{\alpha w e^{\alpha L_I}}}. \tag{5.20}$$

We can see that the holding cost is affected by the time of the procurement payments relative to the arrival time of the goods in the UK firm's warehouse. Also note that if there are no instalment payments and thus $x = 0$ and $L_s = 0$, we would find the classic EOQ result.

Putting the optimum order quantity in the linear AS, we can find the optimum NPV based profit:

$$ASN^* = \big[py - wye^{\alpha L_I} - \frac{\alpha w Q e^{\alpha L_I}}{2}$$
$$- \frac{xsy}{Q}e^{\alpha L_s} - \frac{sy(1-x)}{Q} - \frac{s\alpha(xe^{\alpha L_s + (1-x)})}{2}\big]e^{-\alpha L}. \tag{5.21}$$

5.6.2 Net Present Value with Tax-adjusted Import Method

When we do consider the tax of import activity, we should add the cash flows of duty, VAT and CT payments. Duty and VAT payments occurs at the point of entry into the UK(EU) region at the time of L_N relative to the anchor point, while the tax adjustement happens at the anchor point. We further assume that the purchasing cost at CIF value incurred at relative time of L_I is always a positive value as there is a longer transition time between placing the order and receiving the final stocks.

Following the previously developed process leading to Eq.(4.8), we can develop the AS function for importing from outside EU countries as follows:

$$\mathrm{AS}_{i0} = \Big\{py(1+\tau) + p_{on}y_{on}(1+\tau_{on}) + p_{or}y_{or}(1+\tau_{or})$$
$$-dy(1+\tau) - d_{on}y_{on}(1+\tau_{on}) - d_{or}y_{or}(1+\tau_{or})$$
$$-\big[wYTe^{\alpha L_I} + xse^{\alpha L_s} + (wYT + xs)\theta e^{\alpha L_N} + (wYT + xs)(1+\theta)\tau_1 e^{\alpha L_N}$$
$$+ (1-x)s(1+\tau_s)\big]\sum_{i=0}^{\infty}\alpha e^{-i\alpha T} - (1-\delta)\mathrm{FOC}(1+\tau) - \delta\mathrm{FOC}\Big\}e^{-\alpha L}. \tag{5.22}$$

The logistic cost that occurs in the seventh term is the CIF value at time L_I, the eighth term is the partial setup cost payment at time L_s which is the same as the NPV import model, the ninth term is the duty payment in time L_N with the amount of CIF value in time L_I and partial set-up cost in time L_s, and the tenth term is VAT payment in time

L_N which is the amount of duty added CIF and partial setup cost. The eleventh term indicates the final VAT-adjustment payment.

The net VAT payment is the difference between Output and Input VAT which is imported from outside EU countries:

$$\text{NVAT} = \text{OVAT}_{uk} + \text{OVAT}_{on} + \text{OVAT}_{or} - \text{IVAT}$$

$$= py\tau + p_{on}y_{on}\tau_{on} + p_{or}y_{or}\tau_{or} - dy\tau - d_{on}y_{on}\tau_{on} - d_{or}y_{or}\tau_{or}$$
$$- (1+\theta)\frac{xs}{T}\tau_1 - (1+\theta)wY\tau_1 - \frac{(1-x)s}{T}\tau_s - (1-\delta)\text{FOC}\tau. \quad (5.23)$$

The Input VAT for set-up s and purchasing cost w are as follows: $-(1+\theta)\frac{xs}{T}\tau_1$ is the duty added VAT payment for set-up cost up to the UK/EU border. If it is the UK can, this can be denoted as τ, and if it is other EU country, it can be denoted as τ_d ; $(1+\theta)wY\tau_1$ is the duty added VAT payment for the purchasing cost; and $-\frac{(1-x)s}{T}\tau_s$ is the VAT charge on the remaining set up cost for the transport from the UK/EU border to the location of the firm, $\tau_s = \tau$ or $\tau_s = 0$ depends on the delivery company they can use, and it is τ if a UK company or zero if it is an EU VAT-registered company.

The AS function for value added tax payment is $AS_\tau = NVAT\tau'$:

$$AS_\tau = -[py\tau' + p_{on}y_{on}\tau'_{on} + p_{or}y_{or}\tau'_{or} - dy\tau - d_{on}y_{on}\tau'_{on} - d_{or}y_{or}\tau'_{or}$$
$$- (1+\theta)\frac{xs}{T}\tau'_1 e^{\alpha L_N} - (1+\theta)wY\tau'_1 e^{\alpha L_N} - \frac{(1-x)s}{T}\tau'_s - (1-\delta)\text{FOC}\tau']e^{-\alpha L}. \quad (5.24)$$

The operating profit function should add the cost of duty in the model.

$$OP = py + p_{on}y_{on} + p_{or}y_{or} - dy - d_{on}y_{on} - d_{or}y_{or}$$
$$- wY(1+\theta) - \frac{xs}{T}(1+\theta) - \frac{(1-x)s}{T} - \text{FOC}. \quad (5.25)$$

The AS function for corporation tax is shown as below:

$$AS_\epsilon = -OP\epsilon'. \quad (5.26)$$

The total annunity stream function considering the CT and VAT schemes is then:

$$AS_i = \Big\{py(1+\tau) + p_{on}y_{on}(1+\tau_{on}) + p_{or}y_{or}(1+\tau_{or})$$
$$-dy(1+\tau) - d_{on}y_{on}(1+\tau_{on}) - d_{or}y_{or}(1+\tau_{or})$$
$$-\big[wYTe^{\alpha L_I} + xse^{\alpha L_s} + (wYT+xs)\theta e^{\alpha L_N} + (wYT+xs)(1+\theta)\tau_1 e^{\alpha L_N}$$

$$+(1-x)s(1+\tau_s)]\sum_{i=0}^{\infty}\alpha e^{-i\alpha T}-(1-\delta)\mathrm{FOC}(1+\tau)-\delta\mathrm{FOC}\Big\}e^{-\alpha L}.$$

$$-[py\tau'+p_{on}y_{on}\tau'_{on}+p_{or}y_{or}\tau'_{or}-dy\tau-d_{on}y_{on}\tau'_{on}-d_{or}y_{or}\tau'_{or}$$

$$-(1+\theta)\frac{xs}{T}\tau'_1e^{\alpha L_N}-(1+\theta)wY\tau'_1e^{\alpha L_N}-\frac{(1-x)s}{T}\tau'_s-(1-\delta)\mathrm{FOC}\tau']e^{-\alpha L}$$

$$-\epsilon'[py+p_{on}y_{on}+p_{or}y_{or}-dy-d_{on}y_{on}-d_{or}y_{or}$$

$$-wY(1+\theta)-\frac{xs}{T}(1+\theta)-\frac{(1-x)s}{T}-\mathrm{FOC}]e^{-\alpha L}. \tag{5.27}$$

Linearisation through Maclaurin expansion gives:

$$\overline{AS_i}=\Big\{py(1+\tau)+p_{on}y_{on}(1+\tau_{on})+p_{or}y_{or}(1+\tau_{or})$$

$$-dy(1+\tau)-d_{on}y_{on}(1+\tau_{on})-d_{or}y_{or}(1+\tau_{or})$$

$$-wYe^{\alpha L_I}-\frac{xs}{T}e^{\alpha L_s}-\frac{\alpha}{2}(wYTe^{\alpha L_I}+xse^{\alpha L_s})$$

$$-wY\theta e^{\alpha L_N}-\frac{xs}{T}\theta e^{\alpha L_N}-\frac{\alpha}{2}(wYT+xs)\theta e^{\alpha L_N}$$

$$-wY(1+\theta)\tau_1e^{\alpha L_N}-\frac{xs}{T}(1+\theta)\tau_1e^{\alpha L_N}-\frac{\alpha}{2}(wYT+xs)(1+\theta)\tau_1e^{\alpha L_N}$$

$$-\frac{(1-x)s(1+\tau_s)}{T}-\frac{\alpha}{2}(1-x)s(1+\tau_s)-(1-\delta)\mathrm{FOC}(1+\tau)-\delta\mathrm{FOC}$$

$$-py\tau'-p_{on}y_{on}\tau'_{on}-p_{or}y_{or}\tau'_{or}+dy\tau+d_{on}y_{on}\tau'_{on}+d_{or}y_{or}\tau'_{or}$$

$$+(1+\theta)\frac{xs}{T}\tau_1'e^{\alpha L_N}+(1+\theta)wY\tau_1'e^{\alpha L_N}+\frac{(1-x)s}{T}\tau_s'+(1-\delta)\mathrm{FOC}\tau'$$

$$-\epsilon'py-p_{on}y_{on}\epsilon'-p_{or}y_{or}\epsilon'+dy\epsilon'+d_{on}y_{on}\epsilon'+d_{or}y_{or}\epsilon'$$

$$+wY(1+\theta)\epsilon'+\frac{xs}{T}(1+\theta)\epsilon'+\frac{(1-x)s}{T}\epsilon'+\mathrm{FOC}\epsilon'\Big\}e^{-\alpha L}. \tag{5.28}$$

Rearranging the above gives us:

$$\overline{AS_i}=\Big\{(p-d)y(1+\tau-\tau'-\epsilon')+(p_{on}-d_{on})y_{on}(1+\tau_{on}-\tau_{on}'-\epsilon')$$

$$+(p_{or}-d_{or})y_{or}(1+\tau_{or}-\tau_{or}'-\epsilon')$$

$$-wY[e^{\alpha L_I}+e^{\alpha L_N}\theta+e^{\alpha L_N}\tau_1(1+\theta)-e^{\alpha L_N}(1+\theta)\tau'_1-(1+\theta)\epsilon']$$

$$-\frac{s\alpha}{2}[xe^{\alpha L_s}+x\theta e^{\alpha L_N}+xe^{\alpha L_N}(1+\theta)\tau_1+(1-x)(1+\tau_s)]$$

$$-FOC(1-\epsilon')-FOC(1-\theta)(\tau-\tau')$$

$$-\frac{xs}{T}[(e^{\alpha L_s}+e^{\alpha L_N}\theta+e^{\alpha L_N}\tau_1(1+\theta))-(1+\theta)(\epsilon'+\tau_1'e^{\alpha L_N})]$$

$$-\frac{s}{T}(1-x)(1+\tau_s-\epsilon'-\tau_s')$$

$$- \frac{YT}{2}[w\alpha(e^{\alpha L_I} + e^{\alpha L_N}\theta + \tau_1(1+\theta)e^{\alpha L_N})]\Big\}e^{-\alpha L}. \tag{5.29}$$

We introduce the following additional notation: $\psi = e^{\alpha L_I} + e^{\alpha L_N}\theta + \tau_1(1+\theta)e^{\alpha L_N}$, and $\omega = e^{\alpha L_s} + e^{\alpha L_N}\theta + \tau_1(1+\theta)e^{\alpha L_N}$.

We assume $\tau_s = \tau$, and $\tau_1 = \tau$. The relevant terms that determine the optimal lot-size are then given by:

$$\Big\{ - \frac{s}{T}[x\omega - x(1+\theta)(\epsilon' + \tau' e^{\alpha L_N}) + (1-x)(1+\tau-\epsilon'-\tau')] - \frac{YT}{2}\alpha w\psi\Big\}e^{-\alpha L}. \tag{5.30}$$

Since L is constant, this delay does not affect optimal policy in the model so it can be further ignored.

The economic order quantity including import situation is then given by:

$$Q_i^* = \sqrt{\frac{2sY[x\omega - x(1+\theta)(\epsilon' + \tau' e^{\alpha L_N}) + (1-x)(1+\tau-\tau'-\epsilon')]}{\alpha w\psi}} \tag{5.31}$$

5.6.3 Comparison of NPV versus Tax-adjusted NPV

We compare the linear AS function in Eq.(5.19) and Eq.(5.29).

First, purchasing cost term, $wYe^{\alpha L_I}$ versus $wY[e^{\alpha L_I} + e^{\alpha L_N}\theta + e^{\alpha L_N}\tau_1(1+\theta) - e^{\alpha L_N}(1+\theta)\tau_1' - (1+\theta)\epsilon']$. Second, set-up cost term before the border of the UK, $\frac{xs}{T}e^{\alpha L_s}$ versus $\frac{xs}{T}[(e^{\alpha L_s} + e^{\alpha L_N}\theta + e^{\alpha L_N}\tau_1(1+\theta)) - (1+\theta)(\epsilon' + \tau_1' e^{\alpha L_N})]$. In the first and second cases, on the basis of $L_I = L_s$, the tax effect terms have the same impact. Without tax consideration it includes $e^{\alpha L_I}, e^{\alpha L_s}$, in the tax added model it is included these basic terms, and duty added VAT payment term $e^{\alpha L_N}\tau_1(1+\theta)$, VAT tax adjusted term $e^{\alpha L_N}\tau'(1+\theta)$ and corporation tax adjusted payment $(1+\theta)\epsilon'$.

Third, setup cost term from the border to the destination of customers, classical model $\frac{s}{T}(1-x)$ versus tax adjusted method $\frac{s}{T}(1-x)(1+\tau_s-\epsilon'-\tau_s')$, tax effect term is the same with UK tax adjusted model in Chapter 4 except the remaining balance payment of x value.

Fourth, holding cost term $\frac{YT}{2}\alpha we^{\alpha L_I}$ versus $\frac{YT}{2}w\alpha(e^{\alpha L_I} + e^{\alpha L_N}\theta + \tau_1(1+\theta)e^{\alpha L_N})$. Without tax consideration, holding cost is only measured by the L_I time, in the tax added model it is includes basic time value of L_I, duty payment of $e^{\alpha L_N}\theta$ and duty added VAT payment term of $\tau_1(1+\theta)e^{\alpha L_N}$.

5.7 Imports and Exports Parameter Analysis

5.7.1 Holding Cost

Table 5.7: Holding cost change depends on lead time L_I

days	L_I	$e^{\alpha L_I}$	ψ
0/365	0	0	1.2
30/365	0.08219	1.0166	1.3986
60/365	0.1643	1.0334	1.4155
90/365	0.2465	1.0506	1.4326
180/365	0.4931	1.1037	1.4857

$\tau = 0.2$ $\epsilon = 0.2, \alpha = 0.2$, $L_N = 0.027$, $\theta = 0.15$, except for the first line $\theta = 0$.

Table 5.8: Profitability change in import case for different firm sizes

L_I	ϵ	α	ψ	$\frac{AS_{io_s}(Q^*_{io})}{AS_{io_s}(Q^*_{eoq})}$	$\frac{AS_{io_m}(Q^*_{io})}{AS_{io_m}(Q^*_{eoq})}$	$\frac{AS_{io_l}(Q^*_{io})}{AS_{io_l}(Q^*_{eoq})}$
$\frac{60}{365}$	0.2	0.2	1.4155	1.1123	1.1911	1.246
	0.2	0.1	1.3976	1.0189	1.0214	1.0224
	0.2	0.05	1.3887	1.0099	1.0105	1.0107
	0.18	0.2	1.4155	1.0876	1.1336	1.1609
	0.18	0.1	1.3976	1.0165	1.0184	1.0192
	0.18	0.05	1.3887	1.0086	1.0091	1.0093
$\frac{30}{365}$	0.2	0.2	1.3986	1.0377	1.0493	1.0547
	0.2	0.1	1.3892	1.0159	1.0178	1.0186
	0.2	0.05	1.3846	1.0093	1.0098	1.0100
	0.18	0.2	1.3986	1.0325	1.0414	1.0455
	0.18	0.1	1.3892	1.0139	1.0155	1.0161
	0.18	0.05	1.3846	1.0082	1.0086	1.0087
	$L_I = \frac{60}{365}$			$L_I = \frac{30}{365}$		
	AS_{io_s}	AS_{io_m}	AS_{io_l}	AS_{io_s}	AS_{io_m}	AS_{io_l}
	702.7040	519.0348	447.9604	2015.6392	1829.0014	1756.7835
	3198.5509	3086.1741	3044.7873	3864.3838	3751.0966	3709.3750
	4696.1212	4632.8708	4610.1565	5031.5196	4968.0121	4945.2055
	803.6575	637.7639	573.5148	2118.2321	1949.6473	1884.3600
	3327.6845	3226.2786	3188.9199	3994.5659	3892.3364	3854.6747
	4844.7042	4787.6718	4767.1877	5180.7071	5123.4425	5102.8750

$p = 35$,$w = 25$,$y = 3000$,$s = 500$,$FOC = 9\%py$, $x = 0.8$, $\theta = 15\%$, $L_N = 10/365$, $L_s = L_I$, $L = L_I$, $\tau = 0.2$.

The first line in Table 5.7 is the same as the UK sourcing case due to the fact that there is not no duty payment and the lead time $L_I = 0$. The $e^{\alpha L_I}$ explains the NPV holding cost without tax consideration, and $\psi = e^{\alpha L_I} + e^{\alpha L_N}\theta + \tau_1(1+\theta)e^{\alpha L_N}$ is import of tax considered holding cost. From the experiment without tax consideration the value $e^{\alpha L_I}$ is the same as nearly 1, but when duty and VAT payment are included, the value approximates to 1.4, and the difference comes from duty payment $e^{\alpha L_N}\theta$ and VAT payment term $\tau_1(1+\theta)e^{\alpha L_N}$.

Compared to the sourcing from domestic(UK) market, in the tax added import case, holding cost is increased and this is significantly dependent on the value of L_I. The reason for this is the comparably longer transition period, so placed order in time L_I, until the product arrived at the anchor point it calculated holding cost which is transition time period. The longer the lead time for the first payment of wy, the higher the holding cost. Table 5.8 further shows how L_I can affect holding cost and pricing. Even if the cost is the same, $w = 25$, depending on the value of L_I, the difference varies. In the case of $L_I = 30/365$, the difference will be smaller than $L_I = 60/365$. With the cost of $w = 25$, if the $L_I = 90/360$ will have a negative profit in high $\alpha = 0.2$. In our NPV tax added analysis for import case the lead time for the first payment of CIF value is important because the end result is different profit.

5.7.2 Set-up Cost Percentage Change

Table 5.9: Import case Set-up cost change based on the advance percentage payment $L_I = 60$

ϵ	α	ψ	$\iota_{s(x=0.2)}$	$\iota_{m(x=0.2)}$	$\iota_{l(x=0.2)}$	$\iota_{s(x=0.8)}$	$\iota_{m(x=0.8)}$	$\iota_{l(x=0.8)}$
0.2	0.2	1.4155	0.8858	0.8566	0.8452	0.9832	0.9514	0.9391
0.2	0.1	1.3976	0.8563	0.8404	0.8346	0.9411	0.9238	0.9174
0.2	0.05	1.3887	0.8405	0.8322	0.8293	0.9189	0.9099	0.9067
0.18	0.2	1.4155	0.9091	0.8828	0.8725	1.0297	1.0011	0.9899
0.18	0.1	1.3976	0.8780	0.8637	0.8585	0.9749	0.9594	0.9537
0.18	0.05	1.3887	0.8616	0.8542	0.8515	0.9469	0.9388	0.9359

$\iota = x\psi - x(1+\theta)(\epsilon' + \tau' e^{\alpha L_N}) + (1-x)(1+\tau-\tau'-\epsilon')$, and $\iota_s, \iota_m, \iota_l$ separately is for small, medium and large firms' tax schemes. The other parameters are $p = 35, w = 25, y = 3000, s = 500, FOC = 9450, \theta = 0.15, y_u k = 3000, L_I = L = L_s = 0.1643, L_N = 0.0273, \tau = 0.2, \omega = \psi$.

Table 5.10: Profitability difference compared with classical method followed by example in Table 5.9

$x = 0.2$			$x = 0.8$		
$\frac{AS_{i_s}(Q^*_{i_s})}{AS_{i_s}(Q^*_{eoq})}$	$\frac{AS_{i_m}(Q^*_{i_m})}{AS_{i_m}(Q^*_{eoq})}$	$\frac{AS_{i_l}(Q^*_{i_l})}{AS_{i_l}(Q^*_{eoq})}$	$\frac{AS_{i_s}(Q^*_{i_s})}{AS_{i_s}(Q^*_{eoq})}$	$\frac{AS_{i_m}(Q^*_{i_m})}{AS_{i_m}(Q^*_{eoq})}$	$\frac{AS_{i_l}(Q^*_{i_l})}{AS_{i_l}(Q^*_{eoq})}$
1.1242	1.1864	1.2233	1.1123	1.1911	1.2459
1.0258	1.0287	1.0298	1.0190	1.0214	1.0224
1.0135	1.0142	1.0145	1.0099	1.0105	1.0107
1.1019	1.1422	1.1639	1.0876	1.1336	1.1609
1.0229	1.0251	1.0260	1.0165	1.0184	1.0192
1.0121	1.0126	1.0128	1.0087	1.0091	1.0093

Table 5.9 shows the payment of set-up cost change, particularly look at the change of x value in $2sY[x\omega - x(1+\theta)(\epsilon' + \tau' e^{\alpha L_N}) + (1-x)(1+\tau-\tau'-\epsilon')]$ term. The classical model EOQ is based on $\frac{2sy}{\alpha w}$ which treats $2sy * 1$, in import tax adjusted model when $x = 0.8$ treated nearly the same as 1 (see $\iota_{s(x=0.8)}, \iota_{m(x=0.8)}, \iota_{l(x=0.8)}$) and for $x = 0.2$ is is smaller than 1. Hence, if the holding cost is evaluated the same, compared to the classical model versus for tax-adjusted model with smaller x (x=0.2), there is higher profitability differences in the import scenario compared with the basic NPV model (see Table 5.10, first line of the table for small firms is 1.1242 and 1.1123).

Table 5.11: Profitability difference in tax adjusted model in different x value payment following the example in Table 5.9

$x=0.2$			$x=0.8$					
AS_{i_s}	AS_{i_m}	AS_{i_l}	AS_{i_s}	AS_{i_m}	AS_{i_l}	$S_{\%}$	$M_{\%}$	$L_{\%}$
933	747	675	702	519	447	0.3286	0.4406	0.5086
3343	3230	3188	3198	3086	3044	0.0454	0.0467	0.0473
4792	4728	4705	4696	4632	4610	0.0204	0.0206	0.0207
1035	867	802	803	637	573	0.2889	0.3609	0.3999
3474	3371	3334	3327	3226	3188	0.0440	0.0451	0.0455
4941	4884	4863	4844	4787	4767	0.0200	0.0202	0.0202

$S\% = \frac{AS_{i_s}(x=0.2)}{AS_{i_s}(x=0.8)}$, based on x value of the profitability difference for small firms, medium $M\%$ and large size of firms $L\%$.

Table 5.12: Zero duty rate follows the example in Table 5.9

$x=0.2$			$x=0.8$					
AS_{i_s}	AS_{i_m}	AS_{i_l}	AS_{i_s}	AS_{i_m}	AS_{i_l}	$S_{\%}$	$M_{\%}$	$L_{\%}$
10621	10120	9926	10575	10074	9879	0.0043	0.0046	0.0047
12777	12491	12385	12760	12474	12369	0.0013	0.0013	0.0013
14063	13909	13854	14057	13903	13848	0.0004	0.0004	0.0004
10896	10444	10269	10850	10398	10223	0.0042	0.0044	0.0045
13105	12847	12752	13088	12831	12736	0.0012	0.0013	0.0013
14424	14285	14236	14418	14279	14230	0.0004	0.0004	0.0004

By comparing the set-up cost in this section, we can find out when all conditions are the same, less payment for the set-up cost before the goods come to UK or EU boundaries, in which case the buyer will benefit. This is mainly because of the duty payment. For set-up cost payments due before the goods come to the border (the x value), it is written as $\frac{xs}{T}[(e^{\alpha L_s} + e^{\alpha L_N}\theta + e^{\alpha L_N}\tau_1(1+\theta)) - (1+\theta)(\epsilon' + \tau_1{}' e^{\alpha L_N})]$, compared to the remaining set-up cost payment in the UK, which is $\frac{s}{T}(1-x)(1+\tau_s-\epsilon'-\tau_s{}')$, not hard to see the set-up cost payment before the border should add duty payment on it. Hence, we do another experiment to see whether all conditions are the same as those reported in Table 5.9, and we assume that the duty payment is zero, as in Table 5.12, there is not as high profit difference compare to Table 5.11. Obviously the profit difference will vary depending on the tariff rate applied. For the set-up costs(whether this involves delivery cost paid to a private agency or paid to a supplier) - the lower the payment of x value, the more benefit for the buyer in high tariff scenarios.

Following the analysis of late payment of the percentage of set-up costs, we consider payment structure of unit purchasing cost which happens in time L_I. We think about whether the buyer and vendor have a good relationship and pay the purchasing costs when they receive the item at the anchor point. In this scenario, unit purchasing cost is still charged by duty and VAT when it comes to the border, the only difference is give the buyer more credit time to pay at the anchor point when the item arrives.

5.7.3 Purchasing Cost Paid at Anchor Point

Table 5.13: Purchasing cost paid at anchor point with $x = 0.2$ and $x = 0.8$ and $L = 60$ $L_s = 60$

$x = 0.2$			$x = 0.8$		
$\frac{AS_{i_s}(Q^*_{i_s})}{AS_{i_s}(Q^*_{eoq})}$	$\frac{AS_{i_m}(Q^*_{i_m})}{AS_{i_m}(Q^*_{eoq})}$	$\frac{AS_{i_l}(Q^*_{i_l})}{AS_{i_l}(Q^*_{eoq})}$	$\frac{AS_{i_s}(Q^*_{i_s})}{AS_{i_s}(Q^*_{eoq})}$	$\frac{AS_{i_m}(Q^*_{i_m})}{AS_{i_m}(Q^*_{eoq})}$	$\frac{AS_{i_l}(Q^*_{i_l})}{AS_{i_l}(Q^*_{eoq})}$
1.0321	1.0389	1.0418	1.0237	1.0296	1.0322
1.0189	1.0208	1.0215	1.0138	1.0154	1.0161
1.0120	1.0126	1.0128	1.0088	1.0093	1.0095
1.0289	1.0344	1.0367	1.0210	1.0257	1.0277
1.0170	1.0185	1.0191	1.0122	1.0135	1.0140
1.0108	1.0113	1.0114	71.0077	1.0081	1.0083

$\epsilon, \tau, \alpha, \psi, \omega$ is the same value in Table 5.9. For $L_I = 0, L_N = 0.02739, L = 0.1643, L_s = 0.1643$, $\omega = \delta$, $p = 35, w = 25, y = 3000, s = 500, FOC = 9450, \theta = 0.15, y_u k = 3000$ x separately 0.2 and 0.8.

Table 5.14: Profit difference pay py at different L_I time in tax adjusted model

	$L_I = 60$			$L_I = 0$					
x	AS_{i_s}	AS_{i_m}	AS_{i_l}	AS_{i_s}	AS_{i_m}	AS_{i_l}	$S_\%$	$M_\%$	$L_\%$
0.2	934	748	676	3409	3222	3150	2.6515	3.3096	3.6615
	3344	3230	3189	4584	4471	4429	0.3709	0.3839	0.3889
	4792	4728	4706	5412	5349	5326	0.1294	0.1311	0.1318
	1036	868	803	3512	3343	3278	2.3904	2.8519	3.0826
	3474	3372	3334	4715	4612	4574	0.3571	0.3679	0.3720
	4942	4884	4864	5562	5504	5484	0.1255	0.1270	0.1275
	$L_I = 60$			$L_I = 0$					
x	AS_{i_s}	AS_{i_m}	AS_{i_l}	AS_{i_s}	AS_{i_m}	AS_{i_l}	$S_\%$	$M_\%$	$L_\%$
0.8	703	519	448	3181	2996	2925	3.5266	4.7729	5.5295
	3199	3086	3045	4440	4327	4286	0.3880	0.4021	0.4076
	4696	4633	4610	5316	5253	5230	0.1321	0.1339	0.1346
	804	638	574	3282	3116	3051	3.0842	3.8852	4.3200
	3328	3226	3189	4569	4467	4430	0.3730	0.3847	0.3892
	4845	4788	4767	5465	5408	5388	0.1281	0.1296	0.1301

In Table 5.13 the profitability difference is not as high as in Table 5.10 because the purchasing cost with input VAT was not paid before, which means it was only paid at the anchor point. We further look at how the same set-up cost payment condition the profit difference for the purchasing cost paid before anchor point versus at the anchor point. This is shown in Table 5.14. Tax adjusted model give as high over 3 times difference if all normal tax rate and interest rate high case for the small firms. There is no doubt that if set-up cost paid in advance, the higher the difference.

5.7.4 Guide Price for Import Activity

Before an example is given, some general information on import activity is needed. The reason that buyers import is that there must be an advantage in terms of price, policy or currency difference. The buyer has no incentive to import if the purchasing price is the same as that in the domestic market. Therefore, in the next experiment, we assume that the selling price is the same but that, due to currency differences, the purchasing price or set-up costs are lower. This then raises the following questions: if the buyer wants to enjoy the same profitability as they would make from the domestic market, what are the purchasing price? Do they differ compared with purchasing from the domestic market? Table 5.15 shows the results.

In the example $AS_{a1}(Q^*_{eoq})$ is the annuity stream profit when purchasing from UK domestic market. Hence, from the buyer's perspective, we can see what the overseas purchasing price is if the buyer is to acquire at least the same profit, where w indicate the maximum acceptable price. As we want to see the impact of EOQ in different situations, we use the same profitability to compare whether, in the case of import purchasing, there is a greater gap than when purchasing in the domestic market. The following example shows that there is indeed a higher percentage gap than in the other situations under different α, ϵ value in Table 5.15.

Table 5.15: Profitability change in import case for Medium sized firm with lower bound of purchasing cost

L_I	ϵ	τ	α	$AS_{a1}(Q^*_{a1})$	$w_{0.1643}$	$\frac{AS_i(Q^*_i)}{AS_i(Q^*_{eoq})}$	$\frac{AS_{a1}(Q^*_{a1})}{AS_{a1}(Q^*_{eoq})}$
0.1643	0.2	0.2	0.2	297.3458	25.07539	1.40	1.3331
0.1643	0.2	0.2	0.1	1517.8705	25.5466	1.046	1.0386
0.1643	0.2	0.2	0.05	2373.7818	25.79902	1.021	1.0177
L_I	ϵ	τ	α	$AS_{a4}(Q^*_{4q})$	$w_{a0.1643}$	$\frac{AS_i(Q^*_i)}{AS_i(Q^*_{eoq})}$	$\frac{AS_{a4}(Q^*_{a4})}{AS_{a4}(Q^*_{eoq})}$
0.1643	0.2	0.2	0.2	1504.3476	24.66497	1.0603	1.0132
0.1643	0.2	0.2	0.1	2196.8279	25.30995	1.0311	1.0070
0.1643	0.2	0.2	0.05	2765.9831	25.6603	1.0181	1.0041

$p = 35$,$w = 30$,$y = 3000$,$s = 500$,$FOC = 9\%py$, $x = 0.8$, $\theta = 15\%$, $L_N = 0.0274$, $L = 0.1643$, $L_s = 0.1643$. In Table 5.2 when $\epsilon = 0.2, \alpha = 0.2$ which got the $AS_{a1}(Q^*_{eoq}) = 297.3458$. In order to at least achieve the same profit,w in import case should be $w = 25.07$. The gap is 1.40 in import activities while it is 1.33 in domestic purchasing.

To explain the first line in Table 5.15 , in import case, the purchasing cost should be $w = 25.0754$ which is maximum acceptable price for the business sourcing from outside the EU country, and in this case the profitability difference in the import case versus classical case can show a 40% difference, while in the UK sourcing case versus classical case can end up about 33% difference, there is no doubt the import strategy has more difference as the duty payment and time delay factors in the problem. If the business originally sourced from the whole EU without tariff and borders, in order to at least achieve $AS_{a4}(Q^*_{a4})$, the guide price is lower than the UK sourcing case and this is mainly because of the zero input VAT payment.

Different tax and tariff policies have an impact on classical theory. First, comparable higher holding cost:in the UK sourcing case, the holding cost is evaluated as $1.2\alpha w$ ($\tau = 0.2$) while in the import case it is assumed to be $1.4\alpha w$ (Table 5.7, $\psi = 1.4155$). Second, price guidance for businesses in the domestic market is given. This tax added import EOQ model can lead the buyer to have right decision on their logistic decision and also give the chance to compare in which price they can have import activities always benefit than purchasing in domestic market. Third, Table 5.14 shows how, if the buyer decides to import, the timing of the payment can affect the order quantity and profitability. Paying the same amount of setup cost in the whole sourcing strategy, but the timing of the payment to the supplier whether it is inside the border or not can have different profit.

5.8 Analysis of Tax Payment Difference

The main focus of this research is on whether businesses should still use the traditional EOQ to make their decisions, or is it better to have more accurate details on tax contained in their logistic decisions? The following example in Table 5.16 gives a clue as to what the tax payment difference is when traditional EOQ ignore the VAT and CT consideration.

- In VAT payment $[(p-w)y-\frac{sy}{Q}]\tau($ small firms $\tau=\tau_3$, medium and large firms $\tau=\tau_{stand})$.
- CT payment $[(p-w)y-\frac{sy}{Q}]\epsilon($ small firms $\epsilon=\epsilon_{small}$, medium firms $\epsilon=\epsilon_{med}$, large firms $\epsilon=\epsilon_{large})$.
- $Q^*_{eoq}=\sqrt{\frac{2sy}{\alpha w}}$, $Q^*_{a1}=\sqrt{\frac{2s(1-\epsilon'+\tau-\tau')y}{\alpha w(1+\tau)}}$
- $Q^*_i=\sqrt{\frac{2sY[x\omega-x(1+\theta)(\epsilon'+\tau' e^{\alpha L_N})+(1-x)(1+\tau-\tau'-\epsilon')]}{\alpha w\psi}}$
- VAT percentage difference can be used as $\frac{Q^*_{eoq}\text{VAT}-Q^*_{a1}\text{VAT}}{Q^*_{a1}VAT}$, for the corporation tax percentage difference is $\frac{Q^*_{eoq}CT-Q^*_{i}CT}{Q^*_{i}CT}$.

Table 5.16: Tax payment difference with traditional EOQ and tax-added EOQ

Classical	Q^*_{eoq}	Q^*_{eoq} VAT	Q^*_{eoq} CT		
small	707.11	2460.906	2002.64		
medium	707.11	2460.09	2369.54		
large	707.11	2460.09	2511.88		
Domestic	Q^*_{a1}	Q^*_{a1} VAT	Q^*_{a1} CT	$VAT\%$	$CT\%$
small	596.31	2385.59	1941.35	3.16	3.16
medium	586.29	2376.59	2289.12	3.51	3.51
large	582.35	2373.28	2423.24	3.66	3.66
Import	Q^*_i	Q^*_i VAT	Q^*_i CT	$VAT\%$	$CT\%$
small	589.33	2379.90	1936.72	3.40	3.40
medium	579.71	2371.04	2283.77	3.76	3.76
large	575.93	2367.80	2417.64	3.90	3.90

Comparing the traditional model with the tax-adjusted EOQ, a tax payment difference of up to 3% can be seen, when $p = 35, w = 30, y = 3000, s = 500$.This is a big difference and corporations should consider their tax payment when they base their logistic decision on this effect.

In general, businesses that have very high profitability do not need to consider too much as they have enough cash to run, but in most cases corporations run by profits with small margins have to seriously consider how tax affects their logistic cost as they do not have enough liquidity to cover small changes. From the above analysis it is clear that products with small profit margins should pay more attention to tax regimes and which can help to improve overall profit.

5.9 Brexit on Tax

The UK has already voted to leave the EU, but until there is complete negotiation with the EU, the UK and the EU still need to operate under the original rules which are based on HMRC documentation.

As we model in this Chapter for acquisition and dispatches, UK businesses get some benefit for apply acquisition tax on their imports from the EU ,while after Brexit, the UK will lose intra-community trading status with the EU.Recent comments made by the UK government that the UK will adopt exciting models like The Norwegian model, The Swiss model, The Turkey Model and WTO model.

The Norwegian model means Britain will have access to the single market in exchange for a financial contribution but without the additional burdens of being a member of the European Union. The Swiss model, with its tailor-made solutions, covers some but not all areas of trading, and makes a financial contribution which is smaller than Norway's. To adopt this model, however, means that some EU regulations have to be implemented for trading and it depends on what kind of agreement can be reached. The Turkish model means not being part of the EEA or EFTA, but those that adopt this model face no tariffs (taxes or duties) or quotas on industrial goods sent to the EU. In all these circumstances, the UK might have free circulation in the EU which have similarities to the acquisition and removal. Hence no duties apply except in cases where there is some specific trading agreement with the EU.

The WTO model means no free movement and not obligation to apply EU laws and some tariffs would be in place on trade with the EU. which means that goods brought from the EU will be treated as imports' and 'exports'. Thus, all the duty and VAT rules that apply to imports (or exports) will apply to goods moving in and out of the EU, hence acquisitions and dispatches will no longer be relevant at some point. The most tangible consequence of Brexit would likely be the imposition of "import" VAT and duty payment when trading between the UK and the EU. The VAT would often be recoverable – but there may be an unwelcome cash flow cost for the period between import and recovery for many businesses. It addressed the important part of net present value analysis which is necessary to take logistic decisions. For duty part, Currently, the UK applies the EU duty rate; however, when the UK leaves the EU, it can set its own duty rates. This will be one of the major areas of negotiation between the EU and the UK. Options include, no duties being applied like Norway, otherwise using the current duty rates that the EU applies to third party countries, or some other variation. The import model in this chapter can use even trading with EU businesses.

5.10 Conclusions

The main question considered in this chapter has been how government tax and import duty regimes affect operations decisions as well as profitability of these operational activities when products cross borders. This chapter in particular presents NPV and AS approaches to building inventory decision models which explicitly recognise the timing of the cash flows for the collection of VAT, corporations tax, and import duties (or tariffs). We integrate UK trading rules in the context of sourcing from or selling to EU and non-EU countries to demonstrate how tax and tariffs can be implemented in the supply chain inventory decision model in different trading situations.

We find that the tax adjusted EOQ models can more accurately show how profitability depends on these taxes for businesses with small marginal profits. Trading tariffs have an impact on sourcing decisions. Under current EU rules, acquisition from another EU country offers more benefit than import from outside the EU due to duty rate and time lag differences. Furthermore, acquisition would, ceteris paribus, be more beneficial to the firm than using a local supplier, due to the avoidance of the input VAT payment. In addition, the tax payment and tariffs (duty) also affects optimal Economic order quantities, and may also influence the profitability of sales markets. Although this work is only related to UK trading with other countries, it can be easily adopted to firms with their operations located in other EU countries.

Chapter 6

Impact of taxation on supplier selection in single-source multi-product EOQ settings

6.1 Introduction

Retailers do not always buy only one type of product from a supplier. It is more common to buy groups of products from a supplier especially when there is a physical long distance between them, and in this situation the retailer tries to replenish different types of products in the same order to reduce fixed ordering costs. This problem is referred to as the joint replenishment problem (JRP). JRP is the multi-item inventory problem to coordinate the cycles of items that may be jointly ordered from a supplier (S. K. Goyal, 1974; L. Wang, He, Wu, & Zeng, 2012).

Commonly suppliers can offer different prices and quality for the same group of products. The retailer then has to decide which products should be sourced from which supplier in how many quantities, and how often the order cycle times should be. The criteria of supplier selection can vary; a range of decision approaches have been proposed in the literature to achieve optimal supply chain design like sustainable supplier selection by Park, Kremer, and Ma (2018), supplier selection and carrier selection on the lot-sizing problem by Choudhary and Shankar (2014) and multi-criteria supplier selection by Setak, Sharifi, and Alimohammadian (2012). However in the off-shore supply chain other important factors are tax and tariffs. Tax and tariffs become more and more important as criteria in the context of increased levels of globalisation and outsourcing within the supply chain structure. We can find some research for tax applied on operational decisions like Xiao et al. (2015).Hamad and Gualda (2014) show that tax planning is the most significant element to control the operational cash outflow.

L. Wang, Shi, and Liu (2015) state that JRP problem from can be solved in both direct grouping or indirect grouping methods. The direct grouping method means that products are grouped in sets and in each group the products follow the same cycle time. In the indirect grouping method each product has its own cycle time, which is a multiple of a basic cycle time. In the indirect grouping method we need to find out how to decide this integer multiplier. From literature we can see it can be solved by enumeration methods S. K. Goyal (1974) , simple heuristics Silver (1976), the RAND method Kaspi and Rosenblatt (1991) or simple spreadsheet searches Nilsson, Segerstedt, and Van Der Sluis (2007), among others. We follow Nilsson et al. (2007) extensive work, and adopt Nilsson and Silver (2008) method to figure out this multiplier.

For a multi-item inventory system, a realistic assumption that may need to be made is that a capacity limitation. This may correspond to a number of practical limitations. For example, it may be that transport is of limited capacity (e.g. one full container maximum), or that there is a limitation on either the total capital investment (per order), or on the storage capacity constraint (in total, or for each product).

This chapter extends the JRP with tax consideration and includes the issues of storage capacity constraint for each item in the order. This constraint has already been widely studied by many researchers including Hoque (2006), Porras and Dekker (2006), Khouja and Goyal (2008) and Amaya, Carvajal, and Castaño (2013). For this constraint, Lagrange multiplier are usually adopted to solve the relative optimization problem in S. Goyal (1975), Moon and Cha (2006).

The main interest in this chapter is to look into the following questions. First, how is supplier selection changed based on the tax and tariffs consideration? How is the joint replenishment cycle time re-explained in the NPV analysis under tax implications? Second, with the capacity constraint, the research investigates whether there are different decisions on classical JRP versus the tax-adjusted JRP problem, if capacity is applied in off-shore sourcing strategies in which the lower bound of price for the tax model can still benefit from the off-shore sourcing strategy.

This research is related to four streams of literature. The first one is on procurement strategy. Tax planning in sourcing strategy can be found in Balaji and Viswanadham (2008) who developed a tax-adjusted optimal decisions model in different stages of the global supply chain design by taking into account the export and import tax liabilities in the model to meet the demand for its products in different countries. This is to decide whether to set up a subside or outsource some main compartments. Niu, Liu, Luo, and Feng (2019) studied on sourcing strategies for a nonferrous metal product for an original equipment manufacturer(OME) who can purchases from a domestic supplier or from abroad, and they find that quality of metal and government tariff policy are important.

This work is also related to EOQ on supplier selection due to the fact that the classical assumptions of the JRP are similar to the EOQ, and briefly look at EOQ supplier selection. Single product supplier selection is based on the EOQ model with some capacity and quality constraint : Rosenblatt, Herer, and Hefter (1998), Chang (2006), Ghodsypour and O'brien (2001), Kheljani, Ghodsypour, and O'Brien (2009),Mendoza and Ventura (2012) and Kamali, Ghomi, and Jolai (2011); The multi products supplier selection problem is presented in Narasimhan, Talluri, and Mahapatra (2006),Rezaei and Davoodi (2008) Ozkok and Tiryaki (2011), Shahroudi and Rouydel (2012). Jain, Kundu, Chan, and Patel (2015) modelled the system of one buyer with multiple suppliers inventory problem under all units discount to select one supplier and allocating optimal order quantities under mixed integer nonlinear programming.

The stream of the literature on supply chain NPV on supplier selction is also related to this work. Mousavi, Hajipour, Niaki, and Alikar (2013) used meta-heuristic algorithms to solve the multi-item and multi-period inventory systems with the discounted cash flow approach. The NPV method also found in multi products under inflation and value of money invested with space and budget constraints in Jana, Das, and Maiti (2014), who also proposed a genetic algorithm method to obtain optimal solution.

The JRP is developed on the single supplier case, while some of them extended to the JRP with multi suppliers. Benton (1991) developed single objective to minimise the inventory purchasing, holding and ordering cost and used Lagrangian relaxation to solve this nonlinear program under the condition of multi products and multi suppliers with some resource limitation. Yoo and Gen (2007) and Moon, Goyal, and Cha (2008) proposed JRP for items are sourced from multi suppliers and used genetic algorithm and simulated annealing. In this works, they considered a single sourcing method and both used IDS to decide supplier selection and replenishment cycle schedule of each item. The contribution of Moon et al. (2008) was to address the gap in the JRP literature with discount. In supplier selection and JRP in direct grouping method Mohammaditabar and Ghodsypour (2016) presented simulated annealing algorithm with multi sourcing case. They proposed DGS to decide suppliers and order allocation, joint replenishment of the items, and finally decide the optimal number of groups and allocation each item in the groups. Items ordered from the same supplier are not necessarily in the same group as it depends on demand. The main difference from our research is that we think every single supplier has their independent basic cycle time, but both Moon et al. (2008) and Mohammaditabar and Ghodsypour (2016) developed the basic cycle time for n number of different suppliers.

6.2 JRP from Literature

The JRP problem in indirect grouping method was formulated from Olsen (2005). One method for replenishing multiple items from a single supplier is to submit an order on a regular cycle. There are a total of m products available, with say basic cycle time T

. Thus, for each item $j(j = 1, 2, ...m)$, major set-up cost S and minor set-up cost s_j are incurred if products are ordered. Hence, the optimum order quantity is ordered in every k_jT units of time. The total relevant cost includes the ordering cost for the first term and holding cost for the second term. The following explains how the deterministic joint replenishment problem is formulated as shown the literature.

$$TC(T, k_1, ...k_m) = \frac{S + \sum_{j=1}^{m} \frac{s_j}{k_j}}{T} + \sum_{i=1}^{m} \frac{y_j k_j h_j T}{2}. \tag{6.1}$$

For a fixed set of k_j, take partial derivatives of T. Based on $\frac{\partial TC(T,k_j)}{\partial T} = 0$, the optimal value T^*, is given by the equation.

$$T^*(k_1, ..., k_m) = \sqrt{\frac{2(S + \sum_{j=1}^{m} \frac{s_j}{k_j})}{\sum_{j=1}^{m} k_j y_j h_j}}. \tag{6.2}$$

Substitution of value T^* into Eq. 6.1 gives the optimum total cost as TC^* for the JRP as a function of the $k_j's$.

$$TC^*(k_1, ..., k_m) = \sqrt{2(S + \sum_{j=1}^{m} \frac{s_j}{k_j})(\sum_{j=1}^{m} k_j y_j h_j)}. \tag{6.3}$$

Then, Silver (1976) method is followed to find the solution of integer multiplier k_j. In Silver (1976), the products are renumber in the order of increasing $\frac{s_j}{y_j h_j}$, then the first product is the lowest value of this ratio should have its $k_1 = 1$. Hence, this product j is to be included in every time of replenishment. Moreover it follows

$$k_j \leq k_l, for j \leq l. \tag{6.4}$$

Next, start with the item m, and work downwards $(z = m - 1, m - 2...)$, the procedure does not usually have to proceed through all of the products, based on the Eq.6.4, as long as we can find the value $k_j = 1$ for all $j \leq l$.

$$k_j(k_j + 1) > \frac{s_j}{y_j h_j} \frac{\sum_{i=1, i\neq j}^{m} k_i y_i h_i}{S + \sum_{i=1, i\neq j}^{m} \frac{s_i}{k_i}}. \tag{6.5}$$

6.3 Assumptions and Notations

There are more than one suppliers can produce the same groups of products m. The buyer or retailer replenishes the stocks jointly from an exclusive source specifically divide domestic and off-shore based on EOQ policy. The ordering cost includes two parts, one of which is independent from the number of items in the order S_i. This major set-up cost happens when purchasing from supplier i. In addition, a minor set-up cost s_{ij} is charged for each particular item which is included in the replenishment. Due to the existence of the major set-up cost, grouping inventory items and using joint replenishment can lead to substantial cost savings. We use exhaustive search approach to examine all of the possibilities to choose profit maximisation combination.

- There are multi products with independent demand.
- There are multi suppliers can provide identical products.
- The demand rate of each product is deterministic and constant.
- The unit holding cost of each product is known and constant.
- The lead time is known and constant.
- The main set-up cost is fixed and incurred if the supplier is selected.
- The minor set-up cost is incurred by the product.
- Shortage and backorder are not allowed.
- Although several suppliers can be considered while purchasing each item, it can be purchased from only one supplier.

The profit maximsation model for the joint replenishment problem can be found in J.-M. Chen and Chen (2007) based on the single supplier. Under the joint replenishment policy, the buyer or retailer determines a common replenishment cycle T for the simultaneous replenishment of all items. The profits considered by the buyers consist of revenue from selling products, purchasing cost, minor set-up cost for each products, holding cost, and the number of major replenishment set-ups that are reduced to one over the cycle.

$$AS = \sum_{j=1}^{m} \{(p_j - c_j)y_j - \frac{s_j}{T} - \frac{h_j y_j T}{2}\} - \frac{S}{T}.$$

6.4 Net Present Value on Multi-item EOQ

The buyer has revenue for selling all m products which is $\sum_{j=1}^{m} p_j y_j$, and these products can sourced from n different suppliers. The second term which is purchasing cost of w_{ij} by products depend on each product demand y_j for the common cycle time of T_i from supplier i, and this purchasing cost X_{ij} only happens if supplier i selected products j. The third term which is major set-up cost only happens when at least one product is selected by the supplier. The last term, minor set-up cost, also happens only when X_{ij} is non-zero value. We write buyer's profit in terms of annuity stream function.

$$ASP = \sum_{j=1}^{m} p_j y_j -$$
$$\sum_{i=1}^{n} \Big(\Big(\sum_{j=1}^{m} w_{ij} y_j T_i X_{ij} + (S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}) \Big) \sum_{b=0}^{\infty} e^{-b\alpha T_i} \Big)$$

$$\begin{array}{lll} \text{subject to} & \sum_{i=1}^{n} X_{ij} = 1 & \forall i = 1...n \\ & Q_{ij} = y_j T_i X_{ij} \geq 0 & \forall j = 1, ..., m \\ & C_i \geq \sum_{j=1}^{m} X_{ij} & \forall j = 1...m \\ & Q_{ij}, X_{ij} \geq 0 & \forall j = 1, ..., m \\ & C_i \in \{0, 1\} & \forall i = 1...n \end{array} \tag{6.6}$$

The first constraint means only one supplier satisfies every product, while the second constraint indicates every item has to ordered. Based on Eq. 6.6 if take partial derivatives on every cycle time T_i by supplier i is denoted as follow.

$$T_i = \sqrt{\frac{2(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{\alpha \sum_{j=1}^{m} w_{ij} y_j X_{ij}}}. \tag{6.7}$$

As buyer source from total of n different suppliers, the optimum profit function for the buyer purchasing from n suppliers is displays as:

$$\overline{ASP}^* = \sum_{j=1}^{m} p_j y_j -$$
$$\sum_{i=1}^{n} \Big(\sum_{j=1}^{m} w_{ij} y_j X_{ij} + \sqrt{2(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}) \sum_{j=1}^{m} w_{ij} y_j X_{ij} \alpha}$$
$$+ \frac{\alpha(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{2} \Big) \tag{6.8}$$

The first terms is revenue for selling m products , the second term is purchasing cost of item j sourced from supplier i , and the third term is the logistic cost of selected products from ith supplier. The fourth term accounts for the financial opportunity cost of set-ups which both includes major and minor set-ups expenses. We further explore new models with tax interactions based on the JRP Problem proposed before.

6.5 Import Tax on Multi-item EOQ

In order to model sourcing cost are set according to geographically dispersed places, we need to look at how import duty affect to logistic decision. Here we found the following example on HMRC website to see how the set up cost re-considered by tax and duty payment.

Example 6.1. *An electric guitar and amplifier from the USA to UK. The electric guitar costs* \$600 *and the amplifier* \$400 *(including sales tax). The total shipping cost was* \$200 *and the total insurance cost* \$10*. The exchange rate was* 1.56 *USD to* 1 *GBP. The duty rate for the electric guitar is* 3.7% *and for the amplifier* 2.7%*. The VAT rate in the UK is* 20%*.*
The customs value in the UK is then $600 + 400 + 200 + 10 = \$1210$*. The guitar is* 60% *of the total value of the goods, therefore the shipping and insurance cost allocated to the guitar is* $0.6(210) = \$126$*, and to the amplifier* $0.4(210) = \$84$*. The customs value for the guitar is therefore* $600{+}126 = \$726$*, and for the amplifier* $400{+}84 = \$484$*. Converted in GBP, these values are* £465.38 *and* £310.25*, respectively. The total value inclusive import duty is therefore* $(1.037)465.38 =$ £482.59 *for the guitar, and* $(1.027)310.25 =$ £318.62*.*
The VAT is therefore $0.2(482.59) =$ £96.51 *on the guitar and* $0.2(318.62) =$ £63.72 *on the amp. The total landed costs are* $482.59 + 96.51 + 318.62 + 63.72 =$ £961.44*.*

We can formalise this as follows. First, we can simplify the exchange rate issue by assuming costs are already expressed in GBP. Let there be a set of m products in an imported order with a common set-up cost S_i which is major set up cost (including shipping and insurance costs) from supplier i. Supplier i can provide different items, assume the lot-sizes in the order for the different products from supplier i are q_{ij}, then the allocation of set-up cost to product j from supplier i :

$$S_i \frac{w_{ij} y_j T_i X_{ij}}{\sum_{j=1}^{m} w_{ij} y_j T_i X_{ij}}$$

and this fraction is charged a duty γ_j. The multiplication of X_{ij} means only one supplier can satisfies every product j . Since $q_{ij} = y_j T_i X_{ij}$ is an economic order quantity model to purchase multi products under a common cycle time from the same supplier i , hence the common cycle time T_i will deleted. The total landed cost of the set-up will be shown as below :

$$S_i(1+\tau)\frac{\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) X_{ij}}{\sum_{j=1}^{m} w_{ij} y_j X_{ij}} \tag{6.9}$$

We assume here $Z_i = C_i \frac{\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) X_{ij}}{\sum_{j=1}^{m} w_{ij} y_j X_{ij}}$ which means in multi products case the set-up cost split by every products and charged by individual duty rate. The multiplication of C_i means,if there is no orders happen, then set up cost from supplier i is zero.

6.6 Tax-adjusted JRP Sourcing Strategies

6.6.1 Modelling Approach

Sourcing from domestic market only affected by VAT and CT payment, the formula is the same as in Eq. 6.6, and every term is multiplied by tax rate which can be re-write as:

$$AS_{0d} = \sum_{j=1}^{m} p_j y_j (1+\tau) -$$

$$\sum_{i=1}^{n} \Bigg(\Big(\sum_{j=1}^{m} w_{ij} y_j T_i X_{ij} + (S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}) \Big) \sum_{b=0}^{\infty} e^{-b\alpha T_i} \Bigg) (1+\tau). \tag{6.10}$$

Sourcing from outside the country is affected by both VAT and duty payment, and duty payment is only affected by cost terms. The second term of the purchasing value is evaluated duty added value so it is multiplied by $(1+\gamma_j)$ for every product if X_{ij} is positive. The main set-up cost is delivery cost paid by buyer if sourced from supplier i and the duty payment splitted by each product which is Z_i, and the minor set up cost is the same multiplies by each product with duty payment.So the AS function can be written as follow:

$$\begin{aligned} AS_{0p} = \sum_{j=1}^{m} p_j y_j (1+\tau) - (1+\tau) \sum_{i=1}^{n} \Bigg(\Big(\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) T_i X_{ij} \\ + (S_i Z_i + \sum_{j=1}^{m} s_{ij} X_{ij} (1+\gamma_j)) \Big) \sum_{b=0}^{\infty} e^{-b\alpha T_i} \Bigg). \end{aligned} \tag{6.11}$$

Based on AS function, we can easily formulate NVAT payment, which is the difference of output VAT and input VAT.It also separates the two situations of domestic and off-shore sourcing. There is no doubt that the output VAT is the same for both case which is revenue from selling products, but the difference comes for unit purchasing price and setup cost whether tariffs added or not. Hence, the first case is non-duty value of domestic sourcing and the second one is duty added cost of NVAT off-shore sourcing.

$$
\text{NVAT} = \begin{cases} \sum_{j=1}^{m} p_j y_j - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j X_{ij} \\ -\sum_{i=1}^{n} \frac{(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{T_i} - (1-\delta)FOC & \gamma = 0 \\ \sum_{j=1}^{m} p_j y_j - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) X_{ij} \\ -\sum_{i=1}^{n} \frac{(S_i Z_i + \sum_{j=1}^{m} s_{ij} X_{ij}(1+\gamma_j))}{T_i} - (1-\delta)FOC & \gamma > 0 \end{cases} \tag{6.12}
$$

The evaluation of VAT effect is highly depends on which VAT schemes the company allowed to choose. The annuity stream for VAT payment to government is $AS_\tau = -\tau' NVAT$.

Operating profit is the difference between revenue and cost which is not included in the tax payment. It is not hard to see that the OP is nearly the same as the NPV if ignore the FOC term in both NVAT and OP function. The OP function can easily be written in two different cases depending on whether γ is charged or not.

$$
\text{OP} = \begin{cases} \sum_{j=1}^{m} p_j y_j - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j X_{ij} \\ -\sum_{i=1}^{n} \frac{(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{T_i} - FOC & \text{Domestic, } \gamma = 0 \\ \sum_{j=1}^{m} p_j y_j - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) X_{ij} \\ -\sum_{i=1}^{n} \frac{(S_i Z_i + \sum_{j=1}^{m} s_{ij} X_{ij}(1+\gamma_j))}{T_i} - FOC & \text{Off-shore, } \gamma > 0 \end{cases}
$$

The evaluation of CT effect also depends on in which turnover level is allocated. The annuity stream for the OP payment to the government is $AS_\epsilon = -\epsilon' OP$.

tax-adjusted multi-item single sourcing profit model can be obtained from Eq.6.13. AS_0 can either be $AS_0 d$ domestic sourcing which comes with NVAT and OP for $\gamma = 0$, or be $AS_0 P$ off-shore sourcing comes with NVAT and OP for $\gamma > 0$.

$$
AS = AS_0 + AS_\tau + AS_\epsilon. \tag{6.13}
$$

The main objective function depend on sourcing strategies can be written as following:

$$AS_d = \sum_{j=1}^{m} p_j y_j \zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon')$$
$$-\sum_{i=1}^{n}\Big(\sum_{j=1}^{m} w_{ij} y_j T_j X_{ij} + ((S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}) \sum_{i=1}^{n}\Big(\frac{\alpha(1+\tau)}{(1-e^{-\alpha T_i})} - \frac{\epsilon' + \tau'}{T_i}\Big)$$
$$\begin{aligned} \text{subject to} \quad & \textstyle\sum_{i=1}^{n} X_{ij} = 1 && \forall i = 1...n \\ & Q_{ij} = y_j T_i X_{ij} \geq 0 && \forall j = 1,...,m \end{aligned} \tag{6.14}$$

The linear of the objective function of Eq.6.14 is below:

$$\begin{aligned} \overline{AS_d} = & \sum_{j=1}^{m} p_j y_j \zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon') - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j X_{ij} \zeta \\ & - \sum_{i=1}^{n} \frac{S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}}{T_i}\zeta - \frac{\alpha(1+\tau)\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j T_i X_{ij}}{2} \\ & - \sum_{i=1}^{n} \frac{\alpha(1+\tau)(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{2} \end{aligned} \tag{6.15}$$

$$AS_p = \sum_{j=1}^{m} p_j y_j \zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon')$$
$$-\sum_{i=1}^{n}\Big(\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) T_j X_{ij} + ((S_i Z_i + \sum_{j=1}^{m} s_{ij} X_{ij}(1+\gamma_j)) \sum_{i=1}^{n}\Big(\frac{\alpha(1+\tau)}{(1-e^{-\alpha T_i})} - \frac{\epsilon' + \tau'}{T_i}\Big)$$
$$\begin{aligned} \text{subject to} \quad & \textstyle\sum_{i=1}^{n} X_{ij} = 1 && \forall i = 1...n \\ & Q_{ij} = y_j T_i X_{ij} \geq 0 && \forall j = 1,...,m \end{aligned} \tag{6.16}$$

For the linear approximation of the Eq.6.16 would be:

$$\begin{aligned} \overline{AS_p} = & \sum_{j=1}^{m} p_j y_j \zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon') - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j (1+\gamma_j) X_{ij} \zeta \\ & - \sum_{i=1}^{n} \frac{S_i Z_i + \sum_{j=1}^{m} s_{ij}(1+\gamma_j) X_{ij}}{T_i}\zeta - \frac{\alpha(1+\tau)\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j T_i X_{ij}}{2} \\ & - \sum_{i=1}^{n} \frac{\alpha(1+\tau)(S_i Z_i + \sum_{j=1}^{m} s_{ij}(1+\gamma_j) X_{ij})}{2} \end{aligned} \tag{6.17}$$

The optimum cycle time will be:

$$\sum_{i=1}^{n} T_i = \begin{cases} \sqrt{\sum_{i=1}^{n}\left(\frac{2\zeta(S_iC_i+\sum_{j=1}^{m} s_{ij}X_{ij})}{\alpha(1+\tau)\sum_{j=1}^{m} w_{ij}y_jX_{ij}}\right)}, & \text{if } \gamma = 0. \\ \sqrt{\sum_{i=1}^{n}\left(\frac{2\zeta(S_iZ_i+\sum_{j=1}^{m} s_{ij}X_{ij}(1+\gamma_j))}{\alpha(1+\tau)\sum_{j=1}^{m} w_{ij}y_j(1+\gamma_j)X_{ij}}\right)}, & \text{if } \gamma > 0. \end{cases} \tag{6.18}$$

In the first case of domestic purchasing, both major and minor set-up cost are re-evaluated by the cash flow of ζ payment, and holding cost is the same as the one we have in the single product tax-adjusted EOQ model,but holding cost happens only when X_{ij} is non-zero value. In the off-shore sourcing case, the duty payment is considered both in set-up and holding costs.

The optimum profit for sourcing different suppliers is:

$$AS_d = \sum_{j=1}^{m} p_jy_j\zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon') - TC_d. \tag{6.19}$$

The cost function for the domestic purchasing define TC_d as

$$TC_d = \sum_{i=1}^{n}\Big(\sum_{j=1}^{m} w_{ij}y_jX_{ij}\zeta + \sqrt{2\zeta(S_iC_i + \sum_{j=1}^{m} s_{ij}X_{ij})\sum_{j=1}^{m} w_{ij}y_jX_{ij}\alpha(1+\tau)}$$

$$\frac{\alpha(S_iC_i + \sum_{j=1}^{m} s_{ij}X_{ij})}{2}(1+\tau)\Big). \tag{6.20}$$

Sourcing from off-source supplier:

$$AS_p = \sum_{j=1}^{m} p_jy_j\zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon') - TC_p. \tag{6.21}$$

Redefine the cost function and define as TC_p:

$$TC_p = \sum_{i=1}^{n}\Big(\sum_{j=1}^{m} w_{ij}y_j(1+\gamma_j)X_{ij}\zeta$$

$$+\sqrt{2\zeta(S_iZ_i + \sum_{j=1}^{m} s_{ij}X_{ij}(1+\gamma_j))\sum_{j=1}^{m} w_{ij}y_jX_{ij}(1+\gamma_j)\alpha(1+\tau)}$$

$$\frac{\alpha(S_iZ_i + \sum_{j=1}^{m} s_{ij}X_{ij})(1+\gamma_j)}{2}(1+\tau)\Big) \tag{6.22}$$

6.6.2 Numerical Experiments

For easy understanding, examples are illustrated to see how the tax-adjusted multi products EOQ can change the supplier selection among the order quantities and total cost.These numerical results show the proposed model is optimal by its solution method used in this chapter.

Example 6.2. *There are ten different products ($m = 10$), supplied by two suppliers($n = 2$) with different set up cost of supplier 1 with S_1, supplier 2 with S_2. Demand rate is given in the table.It has $2^{10} = 1024$ possibilities. Take any three different scenarios to create an example.*

Table 6.1: Information of 10 products and 2 suppliers

item	1	2	3	4	5	6	7	8	9	10
p_j	35	110	61	33	632	18	69	92	97	65
y_j	200	3000	550	750	50	5000	2500	855	90	7000
γ_j	0.07	0.08	0.1	0.09	0.09	0.03	0.12	0.19	0.11	0.18
s_{1j}	2	2	2	2	2	2	2	2	2	2
s_{2j}	4	4	4	4	4	4	4	4	4	4
w_{1j}	18	102	45	22	550	5	55	78	84	53
w_{2j}	23	98	49	21	620	6	57	80	85	53

We consider supplier selection and order allocation with and without tax consideration. For classical method find the total cost and profit value under $\tau = 0, \epsilon = 0, \gamma = 0$. The tax-adjusted supplier selection for supplier1 assume from off-shore applies Eq.(6.21), make $\tau = 0, \epsilon = 0, \gamma = 0$, for the supplier 2 assume from domestic sourcing in Eq.(6.19) and set $\tau = 0.2, \epsilon = 0.2$. In total we have 1024 scenarios and use exhaustive search method to evaluate with and without tax supplier selection and order allocation to find optimum cost.

Table 6.2: Result Analysis

	S_1	S_2	Selected Supplier	T_1	T_2	TC	AS
Classical	1000	500	1212111112	0.18	0.09	886479	165469
	2000	500	1212111122	0.26	0.08	890933	161014
Tax	1000	500	1222212222	0.5	0.06	860959	190989
	2000	500	2222222222	0	0.06	861922	190026

Based on the information given in Table 6.1, we can see how supplier selection changes in different major set-ups.

6.6.3 Analysing the Influence of Parameters

First, we examine the influence of the in- and out- cash flow of tax payment and the tariff rate on the decisions. In the experiment, tax rate (i.e., τ, ϵ) and tariff rate (i.e.,γ) in each product determines whether or not to import from overseas, otherwise it should be purchased domestically.

Without tax consideration in Table 6.2 with set-up cost of $S_1 = 1000$, it shows seven products allocated to off-shore supplier 1, but when the set-up cost goes up to $S_1 = 2000$ (in second line), six products are given to supplier1. It shows that, in the low set-up cost, more products are allocated to supplier 1 as their unit selling price of products is lower than that of supplier 2. Here we can see both suppliers are selected in the decision. With the tax-adjusted model, under the lower unit selling price in supplier 1, in the same main set-up cost with $S_1 = 1000$, the tax-adjusted model shows only two products allocated to supplier 1 or even no products in the higher set-up cost case, $S_1 = 2000$. The tax-adjusted decision process gives totally different results for the supplier selection. The tariff increases the purchasing cost in cross-border sourcing, and most of the products are allocated to the domestic supplier.

Second, we examine the influence of the logistic cost and purchasing price. We perform several experiments by changing the set-up and purchasing price, to investigate whether the decisions regarding the sourcing channels are influenced by these changes. We look at how high the set-up cost of the classical model is when only choosing domestic sourcing strategies. In our test for $S_1 = 4600$, the classical model chooses domestic sourcing; it shows the tax-adjusted decision process imposes more restricted decisions on the cost change. It works the same for the unit purchasing price; in the tax-adjusted method only the off-shore supplier can give the discount price of 15%, then the buyer benefits with the same total cost and profit as 861922 and 190026. As the buyer considers the set-up cost is the delivery cost provided by the third party and cannot have bargaining power over it, operations may still be sourced from the offshore supplier if they provide a very good discount on the unit price. In the classical decision process, if other costs remain the same, the discount value given by the supplier may be good enough for the buyer to choose the supplier from off-shore, but when it applied in the tax-adjusted decision model it still is not low enough to source from off-source supplier. It is a trade-off between the set-up cost and unit purchasing price. Operational decision should go along with the off-shore sourcing strategy if both set-up and unit purchasing costs are comparably low.

Third, the research investigates the influence of the product demand levels on the sourcing decisions.In the experiment with set-up costs of 1000 and 500, products 1 and 6 is still need to be out-sourced in the tax integrated model. This indicates comparable low procurement cost and high demand product, can be benefit to outsource as the most cost-effective decision. This result is consistent with Balaji and Viswanadham (2008).

6.7 Capacity Constraint on Sourcing Decision

Considering the capacity constraint problem; assume f_j is the amount of space needed by one unit of product j, and F_i is the total maximum capacity of transportation vehicle chooses to delivery from ith supplier to buyer. The main objective function can be Eq.(6.14) and Eq.(6.16) with the constraint of

$$\begin{array}{lll} \text{subject to} & \sum_{j=1}^{m} f_j y_j X_{ij} T_i \leq F_i & \\ & \sum_{i=1}^{n} X_{ij} = 1 & \forall i = 1...n \\ & Q_{ij} = y_j T_i X_{ij} \geq 0 & \forall j = 1,...,m \end{array} \tag{6.23}$$

As different suppliers are located in different areas, we solve the problem by each supplier case.

The constraint of capacity in this model is achieved when the Equation $\sum_{j=1}^{m} f_j y_j X_{ij} T_i$ does not satisfy the right hand side of equation F_i; this means greater than the right hand size, so the constraints is active. In this situation we need to the new value of Q_{ij} to satisfy the constraints and this can be done by using Lagrange method. As the buyer may choose a total n suppliers and if we rewrite the buyer's total cost under Lagrange multiplier, this can be formulated as below:

$$\begin{aligned} L(\sum_{i=1}^{n} T_i, \sum_{i=1}^{n} \lambda_i) = & \sum_{j=1}^{m} p_j y_j \zeta - (1-\delta) FOC\zeta - \delta FOC(1-\epsilon') \\ & - \sum_{i=1}^{n} \frac{S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}}{T_i} \zeta - \frac{\alpha(1+\tau) \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} y_j T_i X_{ij}}{2} \\ & - \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} y_j X_{ij} \zeta - \sum_{i=1}^{n} \frac{\alpha(1+\tau)(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{2} \\ & - \sum_{i=1}^{n} (\lambda_i \sum_{j=1}^{m} f_j y_j T_i X_{ij} - F_i). \end{aligned} \tag{6.24}$$

The optimum cycle time will be re-written as:

$$\sum_{i=1}^{n} T_{\lambda_i} = \sqrt{\sum_{i=1}^{n} \Big(\frac{2\zeta(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{\alpha(1+\tau) \sum_{j=1}^{m} w_{ij} y_j X_{ij} + 2\lambda_i \sum_{j=1}^{m} f_j y_j X_{ij}} \Big)}. \tag{6.25}$$

The λ can be obtained by putting the value T_i in $\sum_{j=1}^{m} f_j y_j X_{ij} = F_i$, as easy calculation denote $\sum_{i=1}^{n} B_i = \sum_{i=1}^{n} \frac{F_i}{\sum_{j=1}^{m} f_j y_j X_{ij}}$, hence

$$\sum_{i=1}^{n} \lambda_i = \frac{2 \frac{S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}}{(\sum_{i=1}^{n} B_i)^2} \zeta - \alpha(1+\tau) \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} y_j X_{ij}}{2 \sum_{i=1}^{n} \sum_{j=1}^{m} f_j y_j X_{ij}}. \tag{6.26}$$

Then optimum cycle time is B_i, so the profit function can be rewritten as:

$$\begin{aligned}&\sum_{j=1}^{m} p_j y_j \zeta - (1-\delta)FOC\zeta - \delta FOC(1-\epsilon') - \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j X_{ij} \zeta \\ &- \sum_{i=1}^{n} \frac{S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij}}{B_i}\zeta - \frac{\alpha(1+\tau)\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_j B_i X_{ij}}{2} \\ &- \sum_{i=1}^{n} \frac{\alpha(1+\tau)(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{2}\end{aligned} \tag{6.27}$$

In the same way we can obtain off-shore sourcing case with duty payment consideration.

The problem is if we want to know how the λ is re calculated in different sourcing strategies , then Eq.(6.26) will applies, otherwise we can decide out cycle time T_i:

$$T_i^* = min\Big[\sqrt{\sum_{i=1}^{n}\Big(\frac{2\zeta(S_i C_i + \sum_{j=1}^{m} s_{ij} X_{ij})}{\alpha(1+\tau)\sum_{j=1}^{m} w_{ij} y_j X_{ij}}}, \frac{F_i}{\sum_{j=1}^{m} f_j y_j X_{ij}}\Big] \tag{6.28}$$

Table 6.3: Capacity Constraint Analysis

	cap_1	cap_2	$SelectedSupplier$	T_1	T_2	TC	AS
Classical	NA	NA	1212111112	0.2	0.083	889294	162654
Tax	NA	NA	1222212222	0.6	0.06	861816.9	190132
Classical	2600	900	1212111112	0.2	0.083	889403	162545
Tax	2600	900	1222212222	0.5	0.0608	861913.6	190035

Table 6.3 shows the result of capacity-constrained supplier selection problem of cycle time, total cost and profit. The first two lines are without capacity constraint, the third and fourth lines adopt capacity with 2600 for supplier 1, and 900 for supplier 2. The same number is used as in Example 6.2, and we put set-up cost for each supplier is $S_1 = 1500$,

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Total
C-Sup1	45	0	125	0	11	1137	568	194	20	0	2103
C-Sup2	0	260	0	65	0	0	0	0	0	607	932
T-Sup1	122	0	0	0	0	3058	0	0	0	0	3181
T-Sup2	0	188	34	47	3	0	157	53	5	440	930
Capacity	2600	900									
C-Sup1	45	0	125	0	11	1137	568	194	20	0	2103
C-Sup2	0	251	0	63	0	0	0	0	0	586	900
T-Sup1	100	0	0	0	0	2500	0	0	0	0	2600
T-Sup2	0	182	34	45	3	0	152	52	5	425	900

Table 6.4: Capacitated Order Quantities

$S_2 = 500$. Table 6.4 shows the result of ordering quantities. The first two lines are the ordering quantities for the classical inventory model without constraint, and the third and fourth lines are the tax-adjusted ordering quantities without constraint. For the capacity constraint in the classical model in the fifth and sixth lines, from supplier 1, there is no effect on classical model, but for the supplier 2 the ordering quantities meet the constraint of 900. For the tax-adjusted method in lines seventh and eighth, only two items allocated in supplier 1 and remaining go to the supplier 2. However, capacity is down to 2500 from supplier 1 in tax-adjusted method, suggesting sourcing all products from domestic market in supplier 2. In Capacity constrained, if the payment of the rent another container or truck is larger than the after tax profit is generated from the selling of these items,it is better to source from domestic market.

6.8 Conclusions

With the tax-adjusted method considered, the sourcing strategy gives the lower bound compare to the classical method. This tax-adjusted model gives very precise decisions based on the main set-up costs and purchasing price changes. The main findings this chapter is that when businesses source from off-shore, most operational decisions did not take into account mandatory payment of tax and tariffs, and under these consideration we find except buyer have very good bargain power to have right price (set-up cost) for products and services, otherwise it should not always benefit to offshore sourcing. This work further demonstrates the re-shoring phenomenon in supply chain sourcing strategy, and from the tax point of view it is also of benefit for the operations. This chapter just give more strong evidence for re-shoring. Moreover, if all the products are sourced from indoor market, then businesses need to use the joint replenishment problem to re-decide k_j value under the common cycle time from the domestic supplier. We tried Nilsson and Silver (2008) method to find $k_j$value under tax consideration, but as JRP for decision of k_j solves with approximated value such as if ($k(k+1) > 3.46$, for tax-adjusted $k(k+1) > 5.3$, $k = 2$ still works in tax-adjusted case), the result is the same for without tax consideration result.

Chapter 7

Impact of VAT on Deterministic Dynamic Lot-Sizing models

7.1 Introduction

Tax works in pricing decision. Earlier this year government introduced a sugar tax on soft drinks to reduce the sugar content in products, followed by another argument on whether there should be a tax on red meat. The true meaning for this is to reduce the consumption of sugary products and red meat, but if we look at the relationship between tax and price, from the study it suggested a tax of 14% on red meat would mean the price of red meat increasing from £3.80 to £4.33, and for the processed meat, 79% tax would drive up the price from £1.50 to £2.69. The higher the base price the more impact on the tax-increased price. We want to point out that the tax has an impact on price and further for the demand of the products which induces the different ordering policies.

This operation related activities from ordering policy to pricing decision which links to the demand of products involves financial aspects of tax flows(payment). The timing of tax payment is fixed depending on the policy the company can choose, but the timing of operational policy is changed by the circumstance; thus, in this chapter we investigate whether the productions timing and quantities are changed by the in and out cash flow of tax within the planning horizon, in which situation, and when the tax adjusted model for the operations should be used for benefit. Based on the findings, we further investigate promotion price strategy because the ordering quantities and timing are changed by the tax consideration, so the promotion price should also be affected by the tax model.

In order to find out how the tax works in operational related activities, we look at the features of value added tax and corporation tax. The retailer receives a payment from the customer at time t by selling price p and demand y, VAT payment τ, with the total of $py(1+\tau)$, and the retailer pays back $py\tau$ to the government but not on the same day when it occurs. This happens at some point in the future L, and denote $py\tau$ as output VAT . The same applies to purchasing cost, as the retailer sources from the supplier at cost $cy(1+\tau)$, and also can reclaim back $cy\tau$ in future time L, and denote $cy\tau$ as input VAT. Depending on the regimes it applies, this can be yearly basis, pay by installment, or quarterly basis. Same description of corporation tax payment on the profit; at the same time profit occurs at time t with the product cost c being the difference for the selling and cost. Hence, profit after tax is $(p-c)y(1-\epsilon)$. The timing of the amount of tax payment $(p-c)y\epsilon$ happens is not specified, and also based on the regulation it can be paid a yearly basis or a quarterly basis. Traditional operational models take into account tax, and easily model profit after tax as $((1+\tau)py-(1+\tau)cy-\tau py+\tau cy)(1+\epsilon)$. The first terms is selling, the second terms is cost, the third term is VAT needed to payback, and the last term is the VAT that be can reclaimed, as the τ value is cancelled out without considering the time factor; the ϵ is fixed so there is only a need to maximise the $(p-c)y$ value. As we can see it ignores the processing of these tax payment happens and timing of payback. This research looks at the flows of these payments from business to government.

In operation literature we also can find the same payment method which is described as trade credit or delay payment. L. Feng and Chan (2019) describe that most frequently applied form is when the supplier offers a fixed length of delay payment time , and the manufacturer or retailer is allowed to pay back within that time period. This only happens when the supplier is willing, but in the tax payment scheme we can easily find out that, in all operational activities when dealing with tax with government terms, embedded trade credit happens especially for certain tax schemes(Standard VAT scheme, Medium and Large Corporation Tax Scheme). We find a similar payment structure in Beullens and Janssens (2014) who specifically describes the payment structure and timing between supply chain buyer and supplier with defined payment symmetry and asymmetric. The asymmetric is very appropriate way to describe tax payment structure.

For the inventory theory, we adopt dynamic lot-sizing model which is suitable to design time varying demand and clearly see whether the decision policy is changed by the tax payment date. We further investigate pricing policy based on the relationship between price and tax to find out whether there is a different price decision. In most case, the marketing department first determines a retail price and the implied quantities demanded in every period without inventory related cost, and using these demand figures, the purchasing division orders from upstream suppliers. In fact, the pricing and ordering decisions are interrelated as shown by Kunreuther and Schrage (1973). Hence, another stream of dynamic lot-sizing problem is considering the pricing in the inventory theory as most firms follow the relationship between pricing and demand and the production line with the price decision.

The main contributions of this chapter can be summarised as follows: (1) we consider tax in the dynamic lot-sizing inventory decision problem where in particular the ordering decision depends on the input tax payments and payment timing; (2) we integrate in the dynamic lot sizing and promotion pricing decision models the relationship of prices with taxes, by connecting output VAT and CT tax flows relative to the timing of operations.

The literature in this chapter addresses net present value of the dynamic lot-sizing problem. Regarding the pricing decision on DLSP, from literature we can see it is divided in two parts – the first addresses dynamic pricing decision in every period and the second part looks at constant static pricing for the whole planning horizon. Also; promotion with DLSP and trade credit with DLSP are reviewed herein.

NPV of dynamic lot-sizing problem. In order to build a cash flow analysis model the time value of money is considered and this financial consideration method called Discounted Cash Flow (DCF) model. The DCF method can be found since the work of Helber (1998) who developed a cash flow based lot-sizing model in manufacturing resource planning system, and more recently Grubbström and Kingsman (2004) who use the DCF approach present a general model for determining the optimal ordering quantities of an item when there are step changes in price. Bian et al. (2018) proposed a dynamic lot-sizing based profit maximisation discounted cash flow model. This paper first considered dynamic lot-sizing and financial aspects of working capital requirement with discounted cash flow. These studies highlight the fact that operational decision is evaluated by the finance terms of investment decisions.

The solution approaches for pricing in the dynamic lot-sizing problem, particularly in the dynamic price decision for every horizon, can be found in following literature. Wagner and Whitin (1958) and Thomas (1970) studied simultaneous dynamic pricing and lot-sizing model in a new setting by regarding prices of each planning horizon as decision variables. Demand is a linear function of the price and the problem was solved efficiently. Bhattacharjee and Ramesh (2000) proposed two heuristic algorithms and an exact method to solve the pricing and inventory problem through maximising profit considering revenue and relevant cost. Brahimi, Absi, Dauzère-Pérès, and Nordli (2017) argued against Bhattacharjee and Ramesh (2000) method; instead of solving the problem either inefficiently or heuristically, they show in this note that the problem can be solved optimally to an efficient way of polynomial time. They do this by applying a method already proposed by Thomas (1970) for a similar problem. In the dynamic promotion decision in this problem, we still use Thomas (1970)'s method which come from the root of algorithm in this field.

Another stream of pricing in the dynamic lot-sizing is constant price for the whole planning horizon; this assumption means that price must be set for the whole period. In order to simplify the pricing decision in every period, some approaches assume a single static price for the whole horizon. Kunreuther and Schrage (1973) used heuristic approaches and proposed alternating coordinates minimisation algorithm , based on the given price with the price depend demand find a optimal ordering plan, and based on this inventory ordering plan further found an optimal price. They prove that their approach does not skip any optimal solution. Gilbert (1999) proposed exact method to solve the problem, with the assumption that costs are time independent and demand in each period depend on demand intensity function. They find the profit maximisation price within the T (planning horizon) possible candidates for the number of set-ups. Van den Heuvel and Wagelmans (2006) proposed to restart the approach, and imply that the static pricing and inventory problem can be solved by a more efficient method. X. Wu, Xu, Chu, Zhang, et al. (2017) proposed deterministic dynamic lot-sizing models with pricing for a new product without tax consideration.

Promotion strategy is established in tax adjusted DLSP. Besides the assumption on pricing decision, we can look at the differences between marketing-production models. For example, demand may depend on other marketing instruments than price such as promotion. Sogomonian and Tang (1993) considered a T-period discrete model where the promotion periods and promotion levels have to be determined. A solution of the model was found by solving a number of nested longest path problems. For an overview of literature on marketing-production decision making models we refer to Eliashberg and Steinberg (1993). Our research is inspired by this work, and looks at the promotion decision.

Trade credit often considered in the inventory management literature with dynamic lot-sizing problem with payment structure in operational upstream delay payment;for example Z. Chen and Zhang (2018) investigated delay payment from manufacture due to capital flow constraint and it decrease production quantities while Tsao and Sheen (2008) studied replenishment fro a deteriorating products based on supplier's trade credit.

7.2 Payment Structure for the VAT and CT Schemes

Tax occurs in a operational transaction between the businesses and governments by an event occurring at some point t. We can find very specific definition for payment structure, payment symmetry and timing from Beullens (2014).

The payment structure describe is between business and government, is particular for the business in an decision model specifies at what future times, for the relative to the event time t, which amount of $py\tau$ is paid out by the business ,and at what future times, relative to t, which amount of $py\tau$ arrives at the government. This payment structure is depend on the government tax schemes.

There are different VAT and CT schemes in the UK including Standard VAT accounting Scheme, Annual accounting VAT scheme, Flate rate scheme and Cash accounting scheme. We specifically adopt Standard schemes which pays the VAT liabilities at the end of months three, six, nine and twelve.

For the corporation tax depend on the turnover, can be paid nine months after the end of the accounting year or on a quarterly basis. For the quarterly basis corporation tax payment (taxable profit over 20million pound) make payments on months three, six, nine and twelve.

Regarding payment symmetry, if the business pays out an amount $py\tau$ at time t' and government gets the amount $py\tau$ at time t', this is called symmetric; the same applies for the purchasing cost. In the traditional inventory model, this is the way the tax effect on inventory decision is calculated. Asymmetric means the opposite of symmetric, such as when a business receives VAT payment from the customer and payback to the government with some delay $L > 0$ at time $t' + L$.

With payment timing, the business pays out $py\tau$ at t'. The payment is conventional if the business pays out $py\tau$ at t'; the payment is credited if $t' > t$; the payment is advanced if $t' < t$. Conventional payments have zero delay which is always used in classical inventory theory, credit payments have positive delay, and advance payments have a negative delay.

Under the conventional payment structure, business pays out the event payment at the same event time. Theoretically, in the standard VAT and CT schemes, the transaction happens on tax return day 90,180,270 and 360, business pays and claims back at the same time for related operational activities. Under the credit payment structure, business pays the output VAT some time after the event time. If the buyer receives output VAT from customer on day t, and pays to the government on day $t + L$, its NPV is $-py\tau e^{-\alpha(t+L)}$. It can be seen as trade credit, and found trade credit between supply chain parties literature like Protopappa-Sieke and Seifert (2010).

7.3 The Standard Dynamic Lot-Sizing Problem (DLSP)

7.3.1 Formulation of Basic Problem

In terms of the single item lot-sizing problem, there is time-varying demand over a planning horizon of periods. The main problem is to decide when production takes place and how much to order to minimise the sum of the both production and inventory holding costs. Let N be the total length of the production planning horizon and y_i be the deterministic demand pattern in period $i(i = 1....N)$. The main costs are the unit production cost c ; fixed set up cost s incurred once in a period if the production occurs, hence $x_i = 1$; and the unit product holding cost h .Without loss of generality, inventory at the beginning are zero($I_0 = 0$, $y_1 > 0$), and thus that production in first period will be necessary, the same applies at the end of the planning horizon which is $I_N = 0$. In the Wagner-Whitin literature referred to this as not having speculative motives to hold inventory, which indicate always optimal to produce as late as possible.

A Mixed-Integer Linear Programming (MILP) formulation of the 'standard' Dynamic Lot-Sizing Problem (DLSP) is as follows:

Problem 1.

$$\begin{array}{lll} \min C(D) = & \sum_{i=1}^{N} [sx_i + hI_i + cQ_i] & \\ \textit{subject to} & I_i = I_{i-1} + Q_i - y_i & \forall i = 1, ..., n \\ & Q_i \leq Mx_i & \forall i = 1, ..., n \\ & x_i \in \{0, 1\} & \forall i = 1, ..., n \\ & I_i, Q_i \geq 0 & \forall i = 1, ..., n \end{array}$$

The objective function in this problem to minimise the total set-up, production and holding costs. The first constraint is the inventory balance equation; the expression shows current inventory level is the sum of previous period remaining inventory added to the current period production which is used to satisfy the current demand. The second constraint states that production can only occur in a period if this production is activated, and M is a big number. Constraint three is a binary variable. Constraint four means order quantity and ending inventory are both non-negative in each period.

Finally, the variable production costs in the objective function can be left out of the formulation since this total cost is constant and independent of when these costs are incurred.

7.3.2 Wagner and Whitin Theorem Solution Method

A theorem by Wagner and Whitin Wagner and Whitin (1958) states that an optimal solution must have $I_{i-1}Q_i = 0$, $\forall i \geq 2$. Hence, either $Q_i = 0$ (if $I_{i-1} > 0$), or $Q_i = \sum_{k=i}^{j} y_k$ for some $j \geq i$ (if $I_{i-1} = 0$).

This implies that the problem can be solved by dynamic programming or, equivalently, by finding a shortest path in a network formulation of the problem.

There are $N + 1$ nodes in the network named $0, 1, ..., N$. From node i to node j for $0 \leq i < j \leq N$, arc $i - j$ corresponds to the decision that $Q_{i+1} = y_{i+1} + y_{i+2} + ... + y_j$. The costs c_{ij} of arc $i - j$ is then:

$$c_{ij} = s + h(y_{i+2} + 2y_{i+3} + ... + (j - i - 1)y_j), \tag{7.1}$$

unless $y_{i+1} + 2y_{i+2} + ... + (j - i - 1)y_j = 0$ in which case $c_{ij} = 0$.

7.3.3 Net Present Value Formulation of the Standard DLSP

In a Net Present Value formulation, the times when costs are incurred become important. Assume that set-up and production costs are incurred at the start of a period, and assume holding cost happens at the end of periods. Here i indicate period.

Problem 2.

$$\begin{array}{lll} \min & \sum_{i=1}^{N}\left[se^{-(i-1)\alpha T}x_i + fe^{-i\alpha T}I_i + ce^{-(i-1)\alpha T}Q_i\right] & \\ \textit{subject to} & I_i = I_{i-1} + Q_i - y_i & \forall i = 1, ..., n \\ & Q_i \leq Mx_i & \forall i = 1, ..., n \\ & x_i \in \{0,1\} & \forall i = 1, ..., n \\ & I_i, Q_i \geq 0 & \forall i = 1, ..., n \end{array}$$

We can assume that $I_0 = 0$, $y_1 > 0$, and thus that production in period 1 will be necessary. We can further see that we can take $I_N = 0$. The variable production costs in the objective function can now no longer be left out of the formulation since this total NPV contribution of this cost is not constant and depends on when these costs are incurred.

Note that f in Problem 2 only considers the out-of-pocket holding costs. The opportunity cost of capital invested is excluded. In contrast, the standard DLSP formulation in Problem 1 uses a holding cost h in which this opportunity cost is to be included, i.e. $h = \alpha c + f$.

In particular we look at how the holding cost changes based on the time its happens as final value in NPV affected by the time discount factor.Because there is no benefit in producing earlier than necessary, the Wagner and Whitin (1958) theorem still holds. In the network formulation of the problem, we proceed as before. The cost c_{ij} of arc $i - j$ is, for non-negative demand in the periods covered, given by:

$$\begin{array}{l} c_{ij} = se^{-i\alpha T} + ce^{-i\alpha T}\sum_{k=i+1}^{j} y_k \\ +f(y_{i+2} + y_{i+3} + ... + y_j)e^{-(i+1)\alpha T} \\ +f(y_{i+3} + ... + y_j)e^{-(i+2)\alpha T} \\ +... \\ +fy_je^{-(j-1)\alpha T} \end{array}$$

but if $y_{i+1} + 2y_{i+2} + ... + (j-i-1)y_j = 0$ then $c_{ij} = 0$.

This can be re-written:

$$c_{ij} = \left[s + c\sum_{k=i+1}^{j} y_k + f\sum_{l=2}^{j-i}\left(y_{l+i}\sum_{k=1}^{l-1} e^{-k\alpha T}\right)\right]e^{-i\alpha T}. \tag{7.2}$$

7.4 Dynamic Lot-Sizing with Tax implication

7.4.1 Mathematical Formulation Tax-based DLSP

- The planning horizon is a given number of days.
- Demand is a given function of time over the planning horizon and always nonnegative.
- Sales price is a given function of time over the planning horizon, always larger than unit cost, not decision variable
- Unite cost is fixed cost charged by supplier for the unit product.
- Set-up cost is a fixed cost charge by supplier for every order delivered.
- Number of set up should be equal or smaller than planning horizon.
- Demand is measuring by daily basis
- VAT and CT payment treated as daily basis (demand/day)

The objective of this tax-adjusted model is to maximise the NPV of the profit after tax by satisfying demand. To simplify the problem, the NPV after tax profit is defined as the difference between the NPV of revenue after tax and the NPV of expenses after tax. In this problem, the revenue is a function of units sales price and by demand with consider the cash flow payment of both VAT and CT payments. The same applies to the expenses which covers unit purchasing , set-up and holding costs. As the time value of money is accounted for in this problem, all of the cash inflow(i.e., output VAT) and outflow (i.e., input VAT) are presented.

- NPV of After Tax Revenue in Period i: receive payment for demand of y_i with output VAT payment from the customer, $p_i y_i e^{-\alpha(i-1)}(1+\tau)\frac{(1-e^{-\frac{\alpha}{T_a}})}{\alpha}$; this output VAT should be paid back to the government based on the VAT scheme adopted $-p_i y_i \tau e^{-\alpha\frac{VT(i)-(i-1)}{T_a}}$; corporation tax payment for this transaction $(p_i - c_i)y_i\epsilon e^{-\alpha\frac{CT(i)-(i-1)}{T_a}}$. Add all these three terms displays as function (7.3).

$$R_i = p_i y_i e^{-\alpha(i-1)}(1+\tau)\frac{(1-e^{-\frac{\alpha}{T_a}})}{\alpha} - p_i y_i \tau e^{-\alpha\frac{VT(i)-(i-1)}{T_a}} - (p_i - c_i)y_i\epsilon e^{-\alpha\frac{CT(i)-(i-1)}{T_a}} \tag{7.3}$$

- NPV of After Tax Expenses in Period i : operational related cost is mainly set-up cost and purchasing cost with VAT paid to supplier $[s(1+\tau)+c(1+\tau)y_i]e^{-\alpha(i-1)}$; VAT reclaim back for both set up and purchasing cost $(s+cy_i)\tau e^{-\alpha\frac{VT(i)-(i-1)}{T_a}}$;

corporation tax payment for set up and purchasing cost(already include in Revenue)$s\epsilon e^{-\alpha\frac{CT(i)-(i-1)}{T_a}}$. Combine these operational expenses function (7.4).

$$C_i = s(1+\tau)e^{-\alpha(i-1)} + c(1+\tau)y_i e^{-\alpha(i-1)} - (s+cy_i)\tau e^{-\alpha\frac{VT(i)-(i-1)}{T_a}} - s\epsilon e^{-\alpha\frac{CT(i)-(i-1)}{T_a}} \tag{7.4}$$

- NPV of After Tax holding cost in period i.

$$H_i = fI_i e^{-\alpha i} - f\tau I_i e^{-\alpha\frac{VT(i)-(i-1)}{T_a}} - f\epsilon I_i e^{-\alpha\frac{CT(i)-(i-1)}{T_a}} \tag{7.5}$$

Problem 3.

$$\begin{array}{lll} \max & \sum_{i=1}^{N}[R_i - E_i - H_i] & \\ \textit{subject to} & I_i = I_{i-1} + Q_i - y_i & \forall i = 1, ..., n \\ & Q_i \le Mx_i & \forall i = 1, ..., n \\ & x_i \in \{0,1\} & \forall i = 1, ..., n \\ & I_i, Q_i \ge 0 & \forall i = 1, ..., n \end{array}$$

7.4.2 Literature Solution Method

Before we start profit maximised NPV model look at the solution method we adopted in this problem. In Gilbert (1999), the problem is jointly determining the price and production plan to maximise the total profit. The demand is not only timing varying, but also dependent on the price and expressed as $d_t(p) = \beta_t D(p)$ in which β_t is seasonal factor, and $D(p)$ is intensity of demand. There is one to one correspondence between prices and demand $D(p)$, so it can be written in reverse as $p(D)$. Demand in period t, as a function of demand in-density D. Demand in period t can be expressed as $d_t(D) = \beta_t D$, so only need to decide demand D and $p(D)$ is decided by the density of D from the market.

$R_t(p) = P(D)D\beta_t$ revenue generated from price $P(D)$and demand $\beta_t D$, and $c(D)$is operational related cost. The objective function is as follow:

$$\Pi = \max(\sum_{t=1}^{N} R_t(D) - c(D)) \tag{7.6}$$

followed by,

$$\begin{aligned} \underset{Y,I,X}{\text{minimize}} \quad & c(D) = \sum_{i=1}^{N} \left[sx_i + hI_i + cQ_i \right] \\ \text{subject to} \quad & I_i = I_{i-1} + Q_i - D\beta_i \; \forall i = 1, ..., n \\ & Q_i \leq Mx_i \; \forall i = 1, ..., n \\ & x_i \in \{0, 1\} \; \forall i = 1, ..., n \\ & I_i, Q_i \geq 0 \; \forall i = 1, ..., n \end{aligned}$$

Gilbert (1999) solved this problem as cost minimisation by allowing only solutions with exactly n setups, where n is the number between 1 and N planning horizon. They identified this minimising value involves only determination of the best n times of set-up. As the set-up is fixed it is only concerned with minimising the cost of holding inventory. Hence, $c(D) = sx_i + hI_i + cQ_i$ changed to Eq.(7.9). It is not hard to see as the set-up is fixed, so both unit purchasing cost and setup value do not change. Therefore, in every processor, we only need to decide holding cost , and holding cost calculated by Eq.(7.10) and Eq.(7.11).

$$\pi = \max_{n \in (1, ...N)} \{\pi_n\} \tag{7.7}$$

$$\Pi = \max(\sum_{t=1}^{N} R_t(D) - c(D, n)) \tag{7.8}$$

$$C(D, n) = nS + g_n(D, 1) + c \sum_{t=1}^{N} d_t(p) \tag{7.9}$$

for $n = 1, ..., N$,

$$\underset{t>j, t\leq N}{g_n(D, j)} = \min\{h \sum_{k=j}^{t-1} (k - j) d_k(p) + g_{n-1}(D, t)\} \tag{7.10}$$

for n=2,...., N and j=1,....,N-n.

$$g_1(D, j) = h \sum_{k=j}^{N} (k - j) d_k(p), forj = 1..., N. \tag{7.11}$$

7.4.3 DLSP Tax-adjusted Solution

This is the solution method what we will adopt in our tax adjusted model in the following problems. In Gilbert (1999) decided both demand D and set-up frequency n. In this tax adjusted NPV model we look at set-up decision only, hence, price and demand is independent. Wagner and Whitin theorem still holds. There is zero inventory property and non-zero demand in first period, hence, setup is required in first time period. $R(i)$ is the after tax revenue from node i to N covers period from $i+1$ to N.

$$R(i) = \sum_{k=i+1}^{N} \Big[p_k y_k e^{-\alpha(k-i-1)}(1+\tau)\frac{(1-e^{-\frac{\alpha}{T_a}})}{\alpha} \tag{7.12}$$

$$-p_k y_k \tau e^{-\alpha\frac{VT(k)-i}{T_a}} - (p_k - c_k) y_k \epsilon e^{-\alpha\frac{CT(k)-i}{T_a}} \Big]$$

The same applies to cost function, $C(i)$ is the expenses from node i to N covers period from $i+1$ to N.

$$C(i) = s(1+\tau)e^{-i\alpha} + c(1+\tau)\sum_{k=i+1}^{N} y_k e^{-i\alpha} \tag{7.13}$$

$$-(s + c\sum_{k=i+1}^{N} y_k)\tau e^{-\alpha\frac{VT(i+1)-i}{T_a}} - s\epsilon e^{-\alpha\frac{CT(i+1)-i}{T_a}} + H(i)$$

Next we Separately formulated the holding cost terms which can be indicated thus; $H(i)$ is the holding cost from node i to N covers period from $i+1$ to N:

$$H(i) = f\sum_{l=2}^{N-i}(y_{l+i}\sum_{k=1}^{l-1} e^{-k\alpha T}) \tag{7.14}$$

$$-f\tau\sum_{l=2}^{N-i}(y_{l+i}\sum_{k=i+1}^{l+i-1} e^{-\alpha\frac{VT(k)-i}{T_a}}) - f\epsilon\sum_{l=2}^{N-i}(y_{l+i}\sum_{k=i+1}^{l+i-1} e^{-\alpha\frac{CT(k)-i}{T_a}})$$

The solution method of Problem 3 tax considered profit maximised problem is re-formulated below. We find maximum profit value within the n times set-up displayed in Eq.(7.15), and in order to find this value need to find maximum j value when set-up n is fixed use Eq.(7.16). Further, to achieve this result, we use the recursion method in Eq.(7.17) and Eq. (7.18).

$$\pi = \max_{n\in(1,\ldots N)}\{\pi_n\} \tag{7.15}$$

$$\pi_n = \max_{i\in(0,\ldots N-1)}\{g_n(i)\} \tag{7.16}$$

$$\underset{t>i,t\leq(N-(n-1))}{g_n(i)} = \max\{\sum_{k=i}^{t-1}(R_n(i) - C_n(i)) + g_{n-1}(t)e^{-\alpha\frac{t-i}{T_a}}\} \tag{7.17}$$

for n=2,...., N and i=0,....,N-n.

$$g_1(i) = \sum_{k=i}^{N-1}(R_1(i) - C_1(i)) \tag{7.18}$$

for i=0...,N-1.

Example 7.1. *This example shows how the algorithm works. Consider a six-period problem for each* $N = 6, s = 50, c = 8, p = 15, y = 4475$*. Tax payment for VAT and CT set as constant of* $VT(i) = 90, CT(i) = 195$*. Optimum result shows in Table 7.3 with one set-up and profit is 386.32.*

Hence from table 7.1 we can see the detailed explanation. In $n = 1$, we use Eq.(7.18), $k = 0$ denotes node which covers period 1 up to planning horizon; the same explanation applies until $k = 5$ only covers sixth period. For set-up that exceeds 1,we apply Eq.(7.17) and find maximum value when node k is fixed. In $n = 2, k = 0$ set-up separately happens in starting period, and fifth period will give the maximum value 344.36, as shown in Table 7.2.

Table 7.1: Optimal period for different setup

n	k=0	k=1	k=2	k=3	k=4	k=5
1	1	2	3	4	5	6
2	1,5	2,5	3,5	4,6	5	
3	1,5,6	2,4,6	3,5,6	4,5,6		
4	1,4,5,6	2,4,5,6	3.4.5.6			
5	1,3,4,5,6	2,3,4,5,6				
6	1,2,3,4,5,6					

Table 7.2: Profit value for different setup

n	j=0	j=1	j=2	j=3	j=4	j=5
1	386.32	311.61	241.17	170.64	100.04	29.34
2	344.36	270.60	200.01	129.37	58.68	
3	302.32	229.34	158.71	88.01		
4	260.17	188.02	117.33			
5	217.93	146.93				
6	175.59					

Table 7.3: Algorithm results

n	periods with setups	π_n
1	1	386.32
2	1,5	344.36
3	1,5,6	302.32
4	1,4,5,6	260.17
5	1,3,4,5,6	217.93
6	1,2,3,4,5,6	175.59

7.4.4 Experiment Analysis

Example 7.2. *This example numerically assessed how the value of tax involved dynamic lot-sizing model decide the optimum ordering decision. Daily demand from day 1 to 30, (515,505,495....225) which is reduced by 10, and repeat this pattern in every month with a year planning horizon. VAT return comes along with standard scheme, set VT(1)=0, follows VT(2,3,4..91)=90 from period 2 to 91, VT(92,93...181)=180 from period 93 to 182, VT(182,183...271)=270 from period 183 to 272, until planning horizon. Corporation tax payment comes the same pattern.*

First, in the Table 7.4 numerical test shows how the inventory order policy changes with the profitability, and shows that overall tax-adjusted model is good for company's cash flow. In scenario A without tax consideration, equal cycle time and same order quantities obtained (in every 30 days, 30 items are ordered 12 times), while the tax-adjusted algorithm shows more order place before VAT return day and follows with one more small order. This is because comparable low set-up cost case (scenario A), just place large amount order before next VAT return day, then all the input VAT can be claimed back and can hold output VAT from the sellings for more longer times. But if the set-up cost comparable high case in scenario B it just follows VAT return cycles. In the case of comparable low demand case in D-ch, $p = 12, c = 10, s = 4$ demand follows (350,340...60) this pattern, and we can see that there is different policies are presented and also have around 1.5% difference. As businesses can take advantage of the holding on to output VAT longer, and onto input VAT for a shorter period, in both selling and purchasing price, the higher case should bring more benefit to the tax adjusted model like in scenario D. Comparing cases C and D, the only difference is the cash flow of selling price and purchasing cost. This indicates that even in the low margin case, selling price and purchasing cost difference can end up with different ordering policy. The reason is in tax model consider the VAT and CT payment of the cash flow, so if p is bigger, then the value of $p\tau$ also become bigger, so it gives the benefit to cash flow especially output VAT from customer. At the same time purchasing cost, as input VAT can be claim back on the VAT return day(cost from 10 to 20, hence reclaim back 20 x 0.2), boost more benefit of the operations cash flow.

Table 7.4: Constant demand of price decision

$p = 12$	$c = 10$	$s = 4$	$\pi^t = 492.6$	$\pi = 491.9$	A		
TPeriods	0	60	90	150	180	240	270
	330	365					
TQuantity	60	30	60	30	60	30	60
	37						
Periods	0	30	60	90	120	150	180
	210	240	270	300	330	365	
Quantity	30	30	30	30	30	30	30
	30	30	30	30	37		
$p = 12$	$c = 10$	$s = 12$	$\pi^t = 446.89$	$\pi = 445.26$	B		
TPeriods	0	90	180	270	330	365	
TQuantity	91	91	91	60	37		
Periods	0	60	120	180	240	300	365
Quantity	60	60	60	60	60	67	
$p = 12$	$c = 10$	$s = 4$	$\pi^t = 265.63$	$\pi = 261.625$	$a_y 200$	D-ch	
TPeriods	0	60	90	150	180	240	270
	330	365					
TQuantity	33	16	33	16	33	16	33
	21						
Periods	0	60	120	180	240	300	365
Quantity	33	33	33	33	33	38	
$p = 12$	$c = 10$	$s = 10$	$\pi^t = 457$	$\pi = 454$	C		
TPeriods	0	60	90	150	180	240	270
	330	365					
TQuantity	60	30	60	30	60	30	60
	37						
Periods	0	60	120	180	240	300	365
Quantity	60	60	60	60	60	67	
$p = 23$	$c = 20$	$s = 10$	$\pi^t = 700.37$	$\pi = 687.60$	1.85%	D	
TPeriods	0	60	90	150	180	240	270
	330	365					
TQuantity	60	30	60	30	60	30	60
	30	37					
Periods	0	42	90	124	166	210	244
	286	330	365				
Quantity	0	45	45	35	44	41	35
	44	41	37				

π^t, tax adjusted DLSP profit function. π, classical DLSP profit function.

Table 7.5: setup changes one or two jumps

$p = 23$	$c = 20$	$s = 12$	$s = 10$	$\pi^t = 698.05$	$\pi = 681.74$	2.4%	E
TPeriods	0	60	88	148	180	240	268
	328	365					
TQuantity	60	29	60	31	60	29	60
	38						
Periods	0	35	76	120	155	196	240
	328	365					
Quantity	37	42	41	37	42	41	43
	46	38					

Second, in the case of set-up or purchasing cost changes one or two times during the periods, the optimal order planning try to escape order in high cost periods. In Table

7.5 case E, the set-up cost changes in every two periods, so on s(89)=12 and s(90)=12, the ordering time set on 88 to will escape rather than VAT return time; but this cost is continuously high during VAT return periods, like on day (88,89,90) is 12 which means changes in every 3 jumps, it still need to go with VAT return cycle. The reason is as compare with original set-up 10 not as high enough to violate tax return policy. But if set-up changes three jumps as high as 20, then the optimal order planning synchronize to the times when these cost changes, hence in this case ordering 88 become 87, the same with 268 to 267.

Table 7.6: High margin High setup

$p = 25$	$c = 5$	$s = 80$	$\pi_t = 9561.41$	$\pi = 9550.07$	0.12%
TPeriods	0	180	360	540	730
TQuantity	182	182	182	195	
Periods	0	240	480	730	
Quantity	243	243	256		

Third, in the case of set-up cost is very large, as in Table 7.6, we can find it is not synchronized to every subsequent VAT return day, but it escape one period and comes to next VAT return day.

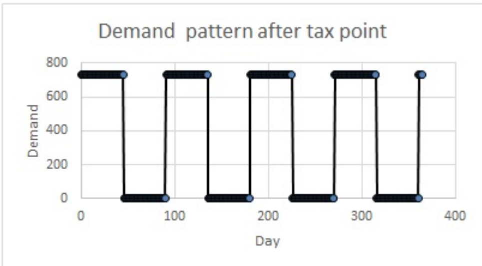

Figure 7.1: Demand Pattern

Fourth, different demand patterns have different impact. Assume, for example, that demand follows (by accident) a quarterly seasonal pattern, high in the first half of the quarter just after the tax point, while demand is low in the second half of the quarter. Demand happens as shown in Fig.7.1 just after tax point with $s = 4, c = 10, p = 12, y = 730$. Company A has demand profile with high demand season just after tax point will benefit from using Tax inventory model in Table 7.7 (2.7% in profits difference): ordering will then be occurring just before tax point as to reclaim input VAT, and output VAT can be kept to next tax point. Using the classic model, the synchronization with demand

Table 7.7: Different Demand Pattern

$p = 12$	$c = 10$	$y = 730$	$s = 4$	$\pi^t = 519.22$	$\pi = 505.28$	2.75%	a
TPeriods	0	90	180	270	302	365	
TQuantity	92	90	90	62	36		
Periods	0	23	91	114	181	204	271
	296	365					
Quantity	46	46	46	44	46	44	50
	48						
$p = 12$	$c = 10$	$y = 500$	$s = 4$	$\pi^t = 351.36$	$\pi = 337.56$	4.1%	b
TPeriods	0	90	180	270	365		
TQuantity	63	61	61	67			
Periods	0	91	181	271	296	365	
Quantity	63	61	61	34	32		
$p = 12$	$c = 10$	$y = 730$	$s = 8$	$\pi^t = 506.54$	$\pi = 483.57$	4.1%	c
TPeriods	0	90	180	270	365		
TQuantity	92	90	90	98			
Periods	0	91	181	271	365		
Quantity	92	90	90	90	98		
$p = 23$	$c = 20$	$y = 730$	$s = 4$	$\pi^t = 788.54$	$\pi = 760.39$	3.7%	d
TPeriods	0	30	90	120	180	210	270
	300	360	365				
TQuantity	60	32	58	32	58	32	58
	32	8					
Periods	0	23	91	113	181	203	271
	293	361	365				
Quantity	46	46	44	46	44	46	44
	46	8					

occurs but you will order too late so that the input VAT has to be reclaimed only at next tax point. Within this demand style, demand and cost further more have impact on its profitability and ordering policies. In case b, as the demand goes down the profitability changes up to 4% difference. The same in case c setup cost change up to 8, although the operational police comes the same result but for the profit changes goes up to more than 4%, hence, in this kind of demand pattern, it is more sensitive to any small change in price or cost. As we analyze in Table 7.4 even the same margin, higher selling price has more impact on cash flow of tax payment, and this is the same in this demand pattern (example d). Company B has demand profile with high demand season prior to tax point will not have much benefit using tax inventory model (percentage in profits difference,ordering pattern is same), but in 500 demand case it can have 1.5% difference in profit. However, other parameter changes such as setup goes 8 does not effect on profitability, and the same with $p = 23, c = 20, s = 4$ case.

Hence, demand patterns come where company A above sells on average the same amount of products per year than the company B. However, the company with second kind demand pattern will make 6,4% less profits (£486 versus £519) on the product. This indicates that companies may benefit from aligning high demand seasons to the VAT tax cycles. In A type demand pattern, it influenced more by tax and also very sensitive to the changes of cost or selling price or even demand.

Fifth, the same result can be obtained with and without tax model if the margin very high and comparable low set up such as $p = 25, c = 5, s = 10$ with 2 years planning horizon and demand pattern is the same in scenario A (Table 7.4). The main reason as high profit case tax is too small to consider because the output tax payment is far more bigger than the unit input VAT and also it can be possible selling for the one products can be cover all the VAT payment of the purchasing in that periods, hence input VAT compare the selling far too small to worth claim back. Another as we mentioned low demand comes with high set up and low margin case. In Table 7.4 scenario A, use the same parameter except demand vector decrease to (350,340...60) and $p = 12, c = 10, s = 12$, then both model provides the same result with ordering policy (90,180,270,365) with profit of 233.92. The main reason is that because demand comparable low even the tax considered cash flow does not benefit for the operational decision. However, the question still remains over how set-up cost can be say high.

From the tax-adjusted dynamic lot-sizing model, we already find that there is operational benefit to claiming input tax in short term rather than keeping input tax as cost in the operations. Based on this result, we further look at how the output tax has impact; hence we further look at promotion price strategy in dynamic lot-sizing problem.

7.5 Pricing in Dynamic Lot-Sizing with Tax-adjusted Model

7.5.1 Tax-adjusted Model with Constant Pricing in DLSP

In the classical dynamic lot sizing problem set $p_{min} = 0$ and $p_{max} = \infty$ which denotes there is no restriction on price, and it can be solved with Wagner and Whitin (1958) theorem. In this section, we set the price range and find out the optimum price in tax model versus classical model.

In pricing and ordering policy model assume demand is a function of the products price and need to figure out this single price for the all planning horizon. Demand function can vary in Kunreuther and Schrage (1973) adopted $d_t(p) = \alpha_t + \beta_t p$, with $\alpha \geq 0$, $\beta \geq 0$, which indicates that demand does not increase even if the price goes up. Pricing and ordering quantities are made at the beginning of a period, and the product demand is deterministic with price. In this problem, we adopt economic theory that provides basic demand models which are derived from the classical rational theory of consumer choice. We assume that the demand of the products is a decreasing function of its price and revenue is concave as a function of price. The same linear demand function can be adopted $y_t(p) = \alpha - \beta p$ with $\alpha \geq 0$, $\beta \geq 0$.

Hence, demand is a derived variable that is based on the price p. $y(p)$ and p denote demand and price, respectively, and where α, β are given constant value. The algorithm by which this can be done is based on a labelling technique: find the longest path from node 0 to N. Hence, for node i to node j for $0 \leq i < j \leq N$, arc ij corresponds to profit that can be formulated as R_{ij} represents revenue from selling, C_{ij} represents operational cost accounting for set-up, unit purchasing cost and holding cost.

$$R_{ij} = \sum_{k=i+1}^{j} \Big[p_k y(p)_k e^{-\alpha(k-i-1)} (1+\tau) \frac{(1 - e^{-\frac{\alpha}{T_a}})}{\alpha} \tag{7.19}$$

$$-p_k y(p)_k \tau e^{-\alpha \frac{VT(k)-i}{T_a}} - (p_k - c_k) y(p)_k \epsilon e^{-\alpha \frac{CT(k)-i}{T_a}} \Big]$$

$$C_{ij} = s(1+\tau) + c(1+\tau) \sum_{k=i+1}^{j} y(p)_k \tag{7.20}$$

$$-(s + c \sum_{k=i+1}^{j} y(p)_k) \tau e^{-\alpha \frac{VT(i+1)-i}{T_a}} - s\epsilon e^{-\alpha \frac{CT(i+1)-i}{T_a}} - H_{ij}$$

$$H_{ij} = f \sum_{l=2}^{j-i} (y_{l+i} \sum_{k=1}^{l-1} e^{-k\alpha T}) \tag{7.21}$$

$$-f \sum_{l=2}^{j-i} (y_{l+i} \sum_{k=i+1}^{l+i-1} e^{-\alpha \frac{VT(k)-i}{365}}) - f \sum_{l=2}^{j-i} (y_{l+i} \sum_{k=i+1}^{l+i-1} e^{-\alpha \frac{CT(k)-i}{365}})$$

$$\pi_{ij} = (R_{ij}^{tx} - C_{ij}^{tx}) e^{-\alpha i T} \tag{7.22}$$

NPV problem and is discounted by the delay time of i. Further use forward recursion method to solve the problem.

Solution method can be explained as following.

- 1, Decide price bound. This would be between $p_{min} < p(1 - 0.01\omega) < p_{max}$ which is based on $\omega = (1 - c/p)100\%$.
- 2, Set $i = 0$, then $\pi_i = \infty$.
- 3, $i = 1 + i$, based on p_i derives from demand vector $y_i(p_i)$.
- 4, Apply dynamic forward recursion method find optimum production plan and related profit π_{i+1}.
- 5, If $\pi_{i+1} - \pi_i > 0$, then goes back to third.
- 6, Otherwise, p_{i+1} is the optimum price.

Example 7.3. *Consider a 365 period problem with s=100, p=15, c=12,* $\alpha = 32808.34$, $\beta = 1888.889$, *hence the demand function is* $y(p) = 32808.34 - 1888.88p$. $\omega = 20$, *we* *start with* $p_{min} = 12.15$ *to* $p_{max} = 15$*. From Table 5 we can see, without tax implication of NPV model determine the price* $p^* = 15$ *with the total profit of* $NPV^* = 8671.587$*, and tax-adjusted model gives the optimum price* $p^* = 14.7$ *with the profit of* $NPV = 8863.207$*, there is* 2.16% *difference.*

Table 7.8: price decision

price	demand	profit(tax)	profit
$p = 15.00$	y(p)=4475.000	8785.779	8671.58*
$p = 14.85$	y(p)=4758.338	8855.685	8658.775
$p = 14.7$	y(p)=5041.672	8863.207*	8654.906
$p = 14.55$	y(p)=5325.005	8808.351	8650.283
$p = 14.4$	y(p)=5608.338	8691.121	8524.891

Table 7.9: Experiment

$n = 7$	π=8671.58	period	0	52	104	156	208	260	312	365	
		quantity	638	638	638	638	638	638	638	638	
$n = 8$	π=8863.207	period	0	52	90	142	180	232	270	325	365
		quantity	718	524	718	524	718	524	718	759	552

This example further proves that the consideration of cash flow of tax can change the optimum price and ordering policies. The main reason as in the operational decision takes into account the additional cash flow of the output VAT and profit, and takes advantage of this delay payment on output VAT and profit at the same time by claiming back for the purchasing. Operations can give reasonable good price to customer with comparable lower price for higher demand and this brings benefits for the operation itself, as well as for the supplier and government.

7.5.2 Tax-adjusted Model with Dynamic Pricing in DLSP

For dynamic pricing Thomas (1970) considers that demand function and cost parameters may vary over the time. The method is explained below with profit maximisation from node i to j.

$$\pi_{ij}(p_{ij}) = \sum_{k=i}^{j-1}(p_{k+1} - c_{i+1} - (k-i)h)d_{k+1}(p_{k+1}) - K_i \tag{7.23}$$

For $0 \leqslant i \leqslant j \leqslant N$ which means node i to j, and the price vector $p_{ij} = [p_i, ..., p_j]$, π_{ij} is the total profit if the production takes place in node i to satisfy demands in periods $i+1, i+2, ..., j$. We then define sub-plan which consists of periods $i, ..., j$.

$$\pi_{ij} = \max_{p_{ij}}\{\pi_{ij}(p_{ij})\} \tag{7.24}$$

In this method, Thomas (1970) shows that if a set-up takes place in period i and the next set-up in period j, then the optimal price for period $k = i, ..., j-1$ must be set at the value which maximises profit.

As the optimal solution consists of a series of consecutive subplans, using forward dynamic programming algorithm and the Zero Inventory Ordering (ZIO) property. The forward recursion for the optimal profit for the whole model:

$$F(t) = \max_{i=1,\dots,t} \{F(i-1) + \pi^*_{ij}\} \tag{7.25}$$

The label $F(t)$ represents the longest path or maximum profit path until period t. In a forward pass, we start labelling node 0, node 1, ..., up to node n. Then $F(n)$ provides us with the total maximum profit solution.

In this section, we use the same forward dynamic programming method to decide the promotion strategies and ordering quantities in our problem. The original selling price po and related demand can be $y(po)$, promotion price use pp and relatives demand function is can be $y(pp)$.The profit function can be use Eq.(7.19) and Eq.(7.20) to evaluate both selling and operational related costs and replace p and $y(p)$ change to either original price or promotion price to separately find the profit function in both po and pp cases.

Based on the profit they are given, we need to decide in which price decision the profit function from node i to j gives the maximum profit, hence

$$\pi_{ij} = \max_{p_{ij} \in (po, pp)} \{\pi_{ij}(p_{ij})\} \tag{7.26}$$

Recursion method we are using is

$$F(t) = \max_{i=1,\dots,t} \{F(i-1) + \pi^*_{ij}\} \tag{7.27}$$

Example 7.4. *We use constant demand and cost to look at the promotion price changes.* $y = 700, po = 12, c = 10, s = 10, pp = 11.5$*, promotion percentage=0.36, standard VAT payment method is combined with quarterly corporation tax payment.*

First, from the first line of Table 7.10 in case a, for the tax model the promotion price is decided with the same pattern of tax payment time while the classical model give the general result of constant price of promotion. This different ordering and pricing policy can end up with a 2.08% profit difference. In common understanding, as the demand goes up dramatically compared with promotion price, in the classical model do all promotion, with in the tax considered model result shows rather than choose all promotions, it follows the due date of tax payment. The same reason as $py(1+\tau)e^{-\alpha i}$ is the output VAT for the selling, and pay back at some point $py\tau e^{-\alpha VT(i)}$, the longer

Table 7.10: Constant demand of price decision

c=10	s=10	36%	$\pi_t = 908.68$	$\pi = 890.15$	2.08%		a
TPeriods	0	49	90	139	180	229	270
	310	340					
TPrice	2	1	2	1	2	1	2
	1	2					
TQuantity	127	78	127	78	127	78	104
	57	65					
Periods	0	37	74	111	148	185	221
	257	293	229				
Price	2	2	2	2	2	2	2
	2	2	2				
Quantity	96	96	96	96	96	93	93
	93	93	93				
c=10	s=11	36%	$\pi_t = 902.69$	$\pi = 876.37$	3%		b
TPeriods	0	49	90	139	180	229	270
	321						
TPrice	2	1	2	1	2	1	2
	1						
TQuantity	127	78	127	78	127	78	133
	84						
Periods	0	46	92	138	184	230	275
	320						
Price	1	1	1	1	1	1	1
	1						
Quantity	88	88	88	88	88	86	86
	86						
c=9	s=10	21%	$\pi_t = 1428.41$	$\pi = 1416.29$	0.8%		c
TPeriods	0	50	90	140	180	230	270
	323						
TPrice	2	1	2	1	2	1	2
	1						
TQuantity	116	76	116	76	116	76	122
	80						
Periods	0	41	82	123	164	205	245
	285	325					
Price	2	2	2	2	2	2	2
	2	2					
Quantity	95	95	95	95	95	92	92
	92	92					

Table 7.11: Demand decrease of price decision

		y=500	$\pi_t = 632.1103$	$\pi = 620.36$	1.9%		
TPeriods	0	49	90	139	180	229	270
	321	365					
TPrice	2	1	2	1	2	1	2
	1						
TQuantity	91	56	91	56	91	56	95
	60						
Periods	0	52	104	156	208	260	312
	365						
Price	1	1	1	1	1	1	1
Quantity	71	71	71	71	71	71	72

output VAT is kept, the better for the cash flow for the operation. This is the main reason to do promotion on VAT and CT cycles (if VAT and CT are paid together on a quarterly basis), which can boost demand. Also if purchasing happens before VAT return day or on VAT return day, the buyer can be reclaimed back immediately for the input VAT. That is the reason why individual items even have the same profit margin, so higher turnover scenario products should take much more advantage of these features of tax payment.

Second, in case $s = 11$ (Table 7.10), for the tax-adjusted model, the ordering policy remains the same as $s = 10$, but the classical model chooses all original prices, and it can end up with 3% difference of profitability. This is the same explanation, as the classical model cannot consider that the extra cash flow happens in both VAT and CT, with comparable high set-up cost, so it not worth doing promotion. However, in the tax-adjusted case, it benefits from the lower price with higher demand because it specifically considers the additional cash flow that is related to these operational activities. Hence, the benefit of cash flow created from tax can cover this extra expenses, and the tax-adjusted model gives the same ordering policy and promotion strategy.

Third, in case c with lower cost of c=9 (Table 7.10), which means it has a higher profit margin compared to the cost of 10. In this situation as the profit margin already higher and has less impact on promotion percentage and the classical model can choose all promotion price, but tax adjusted model leads to different decisions and complies with VAT return cycle when doing promotion. As it has comparable higher margin, the profitability difference only around 0.8%.

Fourth, for the same price and cost (Table 7.11), but the demand is decreases case, as the demand become smaller, it may not seem worth doing promotions if the price and demand are the same. In this case, however, classical model choose no promotion, but the tax consideration method chooses the VAT return day to do promotion. The same explanation applies to case b, where the cash flow created from tax is still beneficial to choosing a promo- tion strategy. In general, referring to the example in Table 7.10, with case a, if we use the classical method to decide optimal decision, it chooses the promotion for the entire horizon. In the low demand case shown in Table 7.11, the classical model gives no promotion, but the tax-adjusted model gives promotion strategies that comply with the tax return day. The benefit of tax model precisely consider selling price $py(1+\tau)e^{-\alpha t} - py\tau e^{-\alpha \frac{VT(t)-t}{T_a}}$ as well as cost happens with purchasing with both VAT and CT payments.

Fifth, Looking at another scenario for same margin with high turnover in Table 7.12 with $p = 23, c = 21, s = 10$ with $pp = 22.04$, none of the model choose the promotion price. From here we can see, even the same margin, and the same percentage of promotion versus original price($\frac{11.5}{12} = \frac{22.04}{23}$) with demand, the selling price affects for the promotion decisions.This is the same as Zhen (2014) state the larger of unit purchasing cost, the higher of the value of the tax refund rate. Regardless of set-up cost, the high selling price/low margin scenario with parameter in Table 7.12 does not choose promotion price strategies. The main reason is that with high purchasing, even claim back the amount of input VAT and the output VAT can hold until next tax return time is not benefit. Hence by either increasing promotion price or increasing demand, the tax model can be benefit to choosing the promotion price on the tax return cycle. Increase promotion price case $pp = 22.5$, as compared to the selling price 12, with selling 23 having higher turnover, so within the VAT return cycle the ordering pattern happens like (2,1,1). Alighment with in low set up cost case big order and comes with one or small order in (Table 7.12). Hence, even the same margin, with different selling price has an effect on promotion price decisions. With the promotion price changes from 22.04 to 22.5, and with other values unchanged, extra increased output VAT is beneficial for the tax considered promotion price strategy.

Table 7.12: High Turnover

		pp=22.5	$\pi_t = 858.27$	$\pi = 838.80$	2.3%		
TPeriods	0	36	63	90	126	153	180
	216	243	270	307	336	365	
TPrice	2	1	1	2	1	1	2
	1	1	2	1	1		
TQuantity	93	51	51	93	51	51	93
	51	51	96	55	55		
Periods	0	30	60	90	120	150	180
	210	241	272	303	334	365	
Price	1	1	1	1	1	1	1
	1	1	1	1	1		
Quantity	57	57	57	57	57	57	57
	59	59	59	59	59		

Example 7.5. *Price insensitive demand case.* $p = 14, s = 30, c = 9$. *For the demand, we create dynamic demand pattern based on* $y_k = y - \frac{Height}{2} + (Height - k\frac{Height}{Duration})$ *(Height=-90,Duration=30), with* $y = 900$ *and* $k = 0$ *to 29. Hence demand changes from 450 to 1320. Price is changed based on demand,* $y_p = a_p + b_p\frac{k}{365}$*, and it derives p is changes from 14 to 10.821.*

First,this example shows price changes based on the market demand. Price changes from 14 to 10 and cost are constant with 9. From table 7.13 can easily find two different policies for tax and classical models and this change remains nearly the same until set-up cost goes up to 34. Second, we further look at high marginal profit case with cost of c is 8, as shown in the constant demand and price case which requires lower promotion percentage, and it works the same in price dependent demand case. Use the promotion percentage of 15% which start choose the promotion price. Three, if demand increased to 1100, which becomes the price sensitive demand case, tax model choose the promotion strategies which are compiled on tax payment day, and profitability difference is comparable higher than insensitive demand case. This is the same with constant

Table 7.13: dynamic price insensitive demand of price decision

	c=9	y=900	21%	s=30	$\pi_t = 1798.4$	$\pi = 1780.43$	1%	
TPeriods	0	75	136	180	255	317		
TPrice	2	1	1	2	1	1		
TQuantity	208	147	115	208	150	120		
Periods	0	74	140	216	286			
Price	2	1	1	2	1			
Quantity	205	161	186	201	195			
	c=8	y=900	15%	s=30	$\pi_t = 2454.33$	$\pi = 2440.73$	.5%	
TPeriods	0	75	136	180	255	316		
TPrice	2	2	1	2	2	1		
TQuantity	198	170	115	198	170	122		
Periods	0	72	138	217	287			
Price	2	2	1	2	1			
Quantity	190	184	193	192	193			
	c=9	y=1100	20%	s=30	$\pi_t = 2278.49$	$\pi = 2256.48$	.97%	
TPeriods	0	48	90	138	180	228	270	319
TPrice	2	1	2	1	2	1	2	1
TQuantity	160	133	160	133	160	133	160	133
Periods	0	47	106	166	227	291		
Price	2	1	1	1	1	1		
Quantity	156	175	178	181	191	222		
	c=9	y=1100	20%	s=27	$\pi_t = 2296.24$	$\pi = 2270.02$	1.15%	
TPeriods	0	48	90	138	180	228	270	319
TPrice	2	1	2	1	2	1	2	1
TQuantity	160	133	160	133	160	133	160	133
Periods	0	47	106	166	227	291		
Price	2	1	1	1	1	1		
Quantity	156	175	178	181	191	222		

demand and price case discussed above. Four, Selling price difference. The selling price change from 24 to 20.82192, and price discount goes the same with 0.043, and other parameter remain the same, apart from demand goes up to 0.43. Then the tax adjusted model choose the promotion price and classical model still no promotion, and profitability difference goes to 1.9%.

From these two experiments we can summarize following things. First need to find out what is the changing point. This means in which case the tax model and traditional model have different price decisions. It is depend on the selling price and cost further for the set-up cost and demand pattern.

Second it is important to decide the scope of the promotion plan change in tax-adjusted model because consideration of tax gives the extra benefit of the cash. As long as cost change does not go over this extra benefit of cash, tax model retains its promotion price strategies. As tested in the example in Table 7.13 in first scenario, set-up between 30 to 35 have the same promotion plan result in tax-adjusted method, so within this range there is not too much difference. It is the same with constant price case as examples show the set-up cost change 10 to 21. So the the tax adjusted model chooses the promotion price strategies which is comply with VAT return cycles.

Third, the tax point is important. Except for the tax-adjusted model that chooses all promotion prices, when it chooses a mixed plan of two different prices, the promotion price always decides on the tax payment periods which come with larger ordering quantities. In the insensitive price demand case, the change of set-up cost is not decide do all promotion in the tax-adjusted model, which means the tax-adjusted model changes decisions gradually; that is, it is more stable.

The overall result shows that operational-related decisions such as ordering quantity or promotion strategy are impacted by VAT and CT payments. The main reason is that there is a time difference between when the real payment happens and pay-back time. By purchasing products on the VAT return cycle, businesses can reclaim the input VAT on that purchasing, and selling for planning period and keep the output VAT until the next VAT return cycle. In operational research literature we can see the trade credit that the buyer obtained from the seller, and the buyer or retailer can accumulate revenue by selling items and earning interests. In our problem the trade credit is obtained from government tax policy. This trade credit is important as it increases a firm's purchasing power. Businesses can benefit as long as selling price is higher than the cost, and even in the same margin case higher selling prices have more impact. It seems like people borrow money from bank at a zero-rated risk-free rate, and pay back on next CT and VAT return day. This is the reason that in the dynamic promotion price strategy, the tax adjusted model has a cost range. It does not change the ordering policy and promotion policy if the change is inside this range (like change $s = 11$, classical model does not do promotion, but tax model yield s the same result as $s = 10$), and it is mainly because tax-created cash flow is bigger than any increase of expenses can cover up, the ordering policy and promotion price is the same.

7.6 Conclusions

The research adopted the tax payment structure in the classical dynamic lot-sizing problem. Deriving profit functions from the NPV framework instead of the traditional average profit method has several advantages. The largest advantage of the NPV accounting is that the incoming and outgoing cash flows do not have to coincide with the physical transactions of the products as they move through the system and this advantage comes along with the features of inflow and outflow of tax payment. This approach is more accurate in that it can easily model the systems where operational activity happens in the moment and actual tax payment happens at some point in the future (output VAT and CT), and is considered further pay-back of the input tax. To the best of the author's knowledge, tax with production and inventory systems are always modelled in average cost function, so this thesis particularly contributes to showing how the process of tax collection for the government has an impact on the logistic decisions and related activities. This thesis introduces the concept of the tax point and shows how NPV profit function can easily construct the cash flows of actual tax in supply chain.

In the dynamic lot-sizing problem, price, ordering time and quantities changed based on the government tax payment due date, which gives us higher profitability compared to the literature models. As the time of tax payment is fixed according to the policy, the date of payment cannot be changed, but the ordering time and quantities can be changed. Hence, the benefit for the tax-adjusted inventory decision process through changing the ordering pattern and price offers greater advantages for the whole cash flow and profit.

First, it is of benefit for the businesses to take advantage of the VAT payment structure in operational strategies. These benefits can be seen in dynamic lot-sizing ordering decisions and promotion strategy which come along with tax payment policy. Second, tax has an influence on the new product decision. If it has the same margin, high selling price products are more vulnerable to the tax consideration. Third, concerning sensitivity of the product to the price, the tax-adjusted decision process is helpful for the business cash flow. Fourth, in the circumstances of (it does not always ask you to do a promotion; it only does so when the benefits of tax delivery cash flow are large) the tax model choosing promotion on tax return day and the classical model not doing so, operations do promotion, ordering more from supplier, and the government can tax more, so it is of overall benefit for all entities in the operational who are directly or indirectly involved.

During the last few years, and to the present time, customers have become increasingly sensitive to the price range, which indicates that most retailers are facing low profit situations. There is no doubt that tax should be considered if the retailer generated very high profit from the selling and the case is the same when there is comparable low demand(low margin, high set-up). Regarding how we can measure the high profit and low demand scenario, the comparison of the tax-adjusted model versus the classical model can give this kind of boundary. Overall, supply chain operational decisions regarding inventory should have this boundary, which means that, in the case of changes to any prices or demand, the two models give the same result, while in other cases it gives different policy and affects the profitability. The tax considered model can analyse all of the different situations and accurate decisions.

Chapter 8

Conclusions

8.1 Overview

Classical inventory models often do not account for many practical considerations that a company's management faces. This thesis investigates how to adapt economic order quantity and dynamic lot-sizing problem formulations as to explicitly account for the cash flows related to tax payment structures. The main purpose of this research is to find out how taxation rules change the decision models in order to improve our decision modelling theory, while also investigating in which practical situations the use of these refined models may produce more economic benefits to the firm. The particular areas investigated in this work are related to inventory ordering decisions, supplier selection in domestic, European and international context, and promotion strategies.

We have considered the 2015 UK tax schemes as the framework for modelling tax flows, with a focus on Value Added Tax (VAT), Corporation Tax (CT), and import tariffs. An overview of these regulations is presented in Chapter 2. The cash flow based NPV maximisation framework is adopted as it can account for the timing of payments.

Cash flow functions of tax payment schemes are integrated into the classical EOQ model in Chapter 4. It is shown that inventory ordering quantity and profitability are impacted by the flows of tax payments. In Chapter 5, taxation rules for UK companies trading with other EU and non-EU countries are investigated, and how these can change the decisions of supplier selection and sales strategy is also addressed. How tax may be an important element in the sourcing strategy is investigated in Chapter 6, where the joint replenishment problem considers suppliers located in both offshore and inshore markets. Chapter 7 uses the modelling framework of the DLSP and the interface between tax regulations and promotion strategies is further investigated.

8.2 Contributions and Findings

In most OR research about inventory optimisation of the past, the system by which governments collect corporate tax (CT) consumption tax (VAT) has not been explicitly accounted for. The main reason for this we have attributed to the fact that the tax does not seem to influence operational decisions when looking at averages over time. Indeed, the CT appears as a percentage deduction on profits, so maximising profits before tax seems sufficient if the CT rate is given. Likewise, the VAT cash-flows into the firm will also go out the firm, and thus overall results in zero result for the firm, and only to a net outflow of VAT for products bought for consumption.

Only a few paper have more recently appeared in the literature that discuss the VAT tax system in a supply chain context (V. N. Hsu & Zhu, 2011; Niu, Liu, et al., 2019; Niu, Xu, et al., 2019; Xu et al., 2018; Zhen, 2014). These authors consider the specific partial VAT refund policy with tariffs of China, or the combined consideration of corporation tax with tariffs.

Our works differs in the following aspects from the all previous research. First, the VAT policy included in above studies consider strategical operational decisions considering taking advantage of the partial VAT refund policy when exporting products. This can lead to an imbalance in the inflow and outflow of tax payments, which results in different operational strategies. The work developed in this thesis is based on the balanced VAT policy applied in the EU/UK and most of the rest of the world. Second, we also include all three tax elements (CT, VAT, tariffs) in the modelling approach to explain inventory related cost terms with tax implications. Third, transfer pricing is an important element for the multinational firms to save on corporation tax and to keep most of the the profit in low tax countries. Our research focuses instead on the payment structure in both value added tax and corporation tax, which is consistent with the results of Niu, Xu, et al. (2019) who show that ordering time and pricing need to be adjusted according to tax policies.

Fourth, and perhaps most importantly, we have presented a new mathematical technique to incorporate tax *schemes* into production and inventory systems. In all previous research on taxes in a logistics or operational context, the models use average cost functions. The use of the Laplace transform of relevant cash-flow functions used in this dissertation leads to a more accurate representation of the importance of not only the magnitude of tax cash-flows, but also their timing relative to the occurrence of operational events. This approach leads to models that, when optimised, aim to maximise the NPV of profits after tax, and is an approach bringing the model closer to reality.

Because we can with this method accurately account for the timing of cash-flows, we now also understand that VAT, while on average balances out and has a net zero impact on the firm, it has not got a zero impact on the NPV of the firm's activities. A similar reasoning applies to Corporate Tax (CT); because the delay in payments of CT, firms are less severly impacted by CT in their NPV that solely looking at the accounting average balance. For example, revenues made in the beginning of cycle are thus more worth that those made later into the cycle.

This thesis particularly contributes to the literature on OR optimisation models in areas of inventory, supplier selection, and production product pricing. It leads to new tax-adjusted models that can show how the process of tax collection for the government impacts the NPV of the operational activities of the firm, and that optimal operational planning may benefit from accounting for this effect.

In terms of classic EOQ-like inventory situations in supply chains, we find that the optimal order quantity is smaller and that a higher profitability is obtained when tax is considered in the supply chain decision. This result is qualitatively in agreement with the findings of Yi and Reklaitis (2007) and J. Liu et al. (2015). Our work is differs these previous papers in a quantitative sense, as we use a different mathematical process to construct the model that is unique in that it can account for the *tax schemes* of CT and VAT (and tariffs). We conclude that tax planning is one of the important factors in minimising the operational cash outflow and inventory carrying costs. Tax effective inventory decisions can also result in making different sourcing decisions and ranking selling markets according to profitability differently.

The structure of tax rules applied good purchased and sold in different markets has the potential to affect the corporations profit under different choices of procurement sourcing strategies which is shown in both Acquisition Import and JRP modelling approach in Chapter 5 and 6. Qualitatively, similar findings are reported in Niu, Liu, et al. (2019) who particularly look at China's import-export tax policies for multinational firm's procurement strategy under partial VAT refund policy.

Further, we find a fairly robust insight that optimal ordering decisions often become synchronised with the tax return points. This is shown in the dynamic lot-sizing problem in Chapter 7. With respect to sales promotions, we conclude that the *timing* sales promotions relative to the tax cycle may impact the net present values of profits for firms. Niu, Xu, et al. (2019) also find that product ordering timing and tax planning are important to sell in low-tax country, however, they conclude this because of corporation tax differences between countries. Our work looks at VAT and CT effects within a single country setting, where the impact is the result mainly from accounting accurately for the associated cash-flows.

With respect to the literature on trade credits in the supply chain, our work is first to showing that the taxation system introduces a 'trade credit' from the tax authorities to firms in the supply chain. In general, smaller firms in the UK get more trade credit from HMRC in the form of later submission of both their CT and VAT liabilities, when compared to larger firms. Seifert et al. (2013), for example, demonstrates how trade credits given by suppliers affects the holding cost of inventories and increases order quantities. The trade credit offered by the government in the EOQ, however, leads to smaller optimal order quantities. Furthermore, in the DLSP setting, the dynamics introduced by the government's trade credit system is yet different and may lead to the synchronisation of optimal ordering decisions and sales promotions to the moments when these taxes need to be submitted.

8.3 Implications

8.3.1 Theoretical Implications

It is over 100 years since the introduction of the EOQ formula by Harris (1913), and it is widely used for its robustness, and extended in different formats. This work first presented how the flows of tax payment have been reformulated in classical EOQ and DLSP problem, and it is directly influenced by the tax policy and schemes it uses. This work also contributes to the emerging area of tax-effective supply chain management and supply chain finance by investigating all these basic tax elements with payment structures in the problem.

This thesis particularly contributes to the literature on OR optimisation models in areas of inventory, supplier selection, and production product pricing. It leads to new tax-adjusted models that can show how the process of tax collection for the government impacts the NPV of the operational activities of the firm, and that optimal operational planning may benefit from accounting for this effect.

8.3.2 Practical Implications

This research has several practical implications in the context of helping corporations make more effective decisions with ordering quantity, sourcing strategy, timing of the ordering and promotion planning to maximise the firm's value. As a mandatory event of tax for operation related activities, this research gives detailed explanation how to add tax in inventory planning and provides more accurate decision compared to the classical models. Business can fully take advantage of extra cash inflow from the consumption tax payment to utilise its capital.

This thesis clarified that the price for inventories needs to include the VAT paid on it, as until the tax can be claimed back it remain a cost, and that the value of a sales also should include the VAT charged, until it has to be paid out to the government. The price of transport or other fixed costs of ordering may need to consider the regime of VAT that applies, in particular in relation to the EU legislation on acquisitions and removals. The introduction of Brexit may make these rules very different when applied to UK firms sourcing from EU countries either supplies or transport services.

The tax-adjusted models presented in this work are the more recommended, compared to their non-adjusted classic models, the lower the profit-margin on the product while the value of the product is high. As tax-adjusted inventory models more precisely consider the timing of extra cash in and out related to taxes, it can help firms make better decisions related to order and production lot-sizes, selecting between the location and prices of different suppliers, or plan for sales promotions in relation to the tax cycles. For products with very high profits, the absolute gains of accounting for taxes remains small. But in this context, optimisation of logistics related costs also become less important. For low profit margin situations, however, relative profitability may increase more than by 10% when using the tax-adjusted modeling framework developed.

The models we developed are perhaps more difficult to construct as it requires some expertise in the application of the NPV cash-flow principles, as well as expertise in the identification and translation of the tax principles that apply to the firm. However, the resulting models can typically be solved with optimisation routines (algorithms) of similar computational complexity as the classic counterpart models. As most companies are profit oriented, we see little disincentive not to use tax-adjusted models in this sense. When the firm has access to employees or consultants with an understanding of taxation and accounting practises that apply to them, and with an understanding of the techniques used in this dissertation, the models may help them find a few percentages of additional profitability on their bottom line.

8.4 Research Limitations and Future Research Directions

Like with any work, this thesis also exhibits many limitations. The main ones can be summarised as follows. First, we don't explicitly consider exchange rates, or assume that they are constant. In practise, this is another very important factor for multinational firms' procurement decisions. Second, with improved accuracy comes also the potential to optimise to the wrong taxation rules. The research mainly looked at UK tax rules of 2015. Since then, changes in the UK have been introduced in both magnitude and also the frequency and timing by which large firms need to report their VAT and CT liabilities. Other countries may still work differently. The modelling technique requires taking these differences into account, and adjusting the models accordingly. It is in this sense not as simple as applying an EOQ formula! Third, the inventory and production planning models by which companies plan may of course deviate significantly from the models we have considered. Future research could continue to look at different deterministic as well as stochastic OR problems in which the possible consideration of tax regulations may lead to further improvements. In time, we will also be able perhaps to assess the impact of Brexit on supply chain models, and the possible role taxes and tariffs may play in them.

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