Minimum Symbol Error Rate Turbo Multiuser Beamforming Aided QAM Receiver

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Abstract—This paper studies a novel iterative soft interference cancellation (SIC) aided beamforming receiver designed for high-throughput quadrature amplitude modulation systems communicating over additive white Gaussian noise channels. The proposed linear SIC aided minimum symbol error rate (MSER) multiuser detection scheme guarantees the direct and explicit minimisation of the symbol error rate at the output of the detector. Based on the extrinsic information transfer (EXIT) chart technique, we compare the EXIT characteristics of an iterative MSER multiuser detector (MUD) with those of the conventional minimum mean squared error (MMSE) detector. As expected, the proposed SIC-MSER MUD outperforms the SIC aided MMSE MUD.

I. INTRODUCTION

Iterative detection was proposed by Berrou et al. [1] in the context of turbo codes. This work has later been extended to serially concatenated codes [2] and then found its way into iterative detector designs, such as iterative equalisers [3]–[5] and iterative multiuser detectors (MUDs) [6]. Most studies consider the minimum mean square error (MMSE) soft interference cancellation (SIC) aided iterative receiver [4]–[6]. However, the MMSE algorithm does not guarantee the direct and explicit minimisation of the system’s error ratio. Hence in references [7], [8] the bit error ratio (BER), rather than the mean square error (MSE) was minimised at the MUD’s output for binary phase shift keying and quadrature phase shift keying signals. Minimum BER detectors are challenging to derive for binary phase shift keying and quadrature phase shift keying. However, the MMSE algorithm does not guarantee the direct and explicit minimisation of the system’s error ratio. Hence in references [7], [8] the bit error ratio (BER), rather than the mean square error (MSE) was minimised at the MUD’s output for binary phase shift keying and quadrature phase shift keying signals. Minimum BER detectors are challenging to derive for binary phase shift keying and quadrature phase shift keying. However, the MMSE algorithm does not guarantee the direct and explicit minimisation of the system’s error ratio. Hence in references [7], [8] the bit error ratio (BER), rather than the mean square error (MSE) was minimised at the MUD’s output for binary phase shift keying and quadrature phase shift keying signals. Minimum BER detectors are challenging to derive for binary phase shift keying and quadrature phase shift keying. However, the MMSE algorithm does not guarantee the direct and explicit minimisation of the system’s error ratio. Hence in references [7], [8] the bit error ratio (BER), rather than the mean square error (MSE) was minimised at the MUD’s output for binary phase shift keying and quadrature phase shift keying signals. Minimum BER detectors are challenging to derive for binary phase shift keying and quadrature phase shift keying.

The novel contribution of this treatise is that iterative SIC aided MSER beamforming is proposed for QAM signals and its performance is studied with the aid of multi-user EXIT charts. Note that the shifting properties and the symmetrical distribution of the output signal’s probability density function (PDF), which are used in the derivation of the original MSER beamforming solution [10], are no longer valid in our iterative system. Therefore, we derive a new a priori information assisted MSER MUD suitable for the employment in the proposed iterative SIC aided receiver. The structure of this contribution is as follows. In Section II, we outline the signal model used, followed by the portrayal of our iterative beamformer design. The focus of Section III is the novel MSER soft-input soft-output (SISO) interference canceller advocated. Section IV introduces the EXIT chart principles. Our simulation results and EXIT chart analysis are presented in Section V, followed by our conclusions in Section VI.

II. SYSTEM DESCRIPTION

A. Signal Model

The system supports $K$ number of QAM users and each user transmits on the same angular carrier frequency $\omega$. The receiver is equipped with a linear antenna array consisting of $L$ elements, which have a uniform element spacing of $\lambda/2$, where $\lambda$ is the wavelength. Assume that the channel is non-dispersive in both the angular and time domains, and hence does not induce intersymbol interference. Then the symbol-rate-sampled received signal can be expressed as $r_l(n) = \sum_{k=1}^{K} h_k s_k(n) e^{j\omega t_l(\theta_k)} + n_l(n)$ for $1 \leq l \leq L$, where $h_k$ is the non-dispersive complex-valued channel coefficient of user $k$, $s_k(n)$ is the $n$th symbol of the $k$th user, $n_l(n)$ is a complex-valued additive white Gaussian noise process with $E[|n_l(n)|^2] = 2\sigma_n^2$, and $t_l(\theta_k)$ is the relative time delay at array element $l$ for the source signal of user $k$, with $\theta_k$ being the direction of arrival for user $k$. The received signal vector $r(n) = [r_1(n)\ r_2(n)\ \cdots\ r_L(n)]^T$ is given by $r(n) = Hs(n) + n(n)$, where $n(n) = [n_1(n)\ n_2(n)\ \cdots\ n_L(n)]^T$, the transmitted symbol vector is $s(n) = [s_1(n)\ s_2(n)\ \cdots\ s_K(n)]^T$ and the system matrix is denoted by $H = [h_1\ h_2\ \cdots\ h_K]$, which is associated with the steering vectors $h_k = [h_ke^{j\omega t_1(\theta_k)}\ h_ke^{j\omega t_2(\theta_k)}\ \cdots\ h_ke^{j\omega t_L(\theta_k)}]^T$, $1 \leq k \leq K$.
B. Iterative Multiuser Beamforming Receiver Structure

The iterative multiuser beamforming receiver's structure is shown in Fig. 1, which consists of two stages, namely the SISO interference cancellation aided beamforming MUD, followed by $K$ parallel single-user SISO channel decoders. The two stages are separated by the usual bit-based deinterleavers $\Pi$ and interleavers $\Pi$. The first term in (1), denoted by $\tilde{s}_k(n)$ corresponding to the transmitted symbol $s_k(n)$ from the received signal $r(n)$ with the aid of a linear transformation. Let us now define $b_k(n, i)$ as the $i$th $(i \in \{1, \cdots, \log_2 M\})$ bit of the $M$-QAM symbol $s_k(n)$, whereas $b_k(j)$ is the same bit but in a different position of the bit-based interleaving block after the deinterleaver. $L_A(\cdot)$, $L_P(\cdot)$ and $L_E(\cdot)$ denote the a priori, a posteriori extrinsic information and extrinsic information in terms of logarithmic likelihood ratio (LLR), and the subscripts $m$ and $c$ are associated with the MUD and channel decoder, respectively. Then the SISO beamforming MUD delivers the a posteriori information of bit $b_k(n, i)$ expressed in terms of its LLR as [6]

$$L_{P,m}(b_k(n, i)) = \ln \frac{P[b_k(n, i) = 0 | \tilde{s}_k(n)]}{P[b_k(n, i) = 1 | \tilde{s}_k(n)]} + \ln \frac{P[b_k(n, i) = 0]}{P[b_k(n, i) = 1]} = L_{E,m}(b_k(n, i)) + L_{A,m}(b_k(n, i)), \quad (1)$$

where the second term, denoted by $L_{A,m}(b_k(n, i))$, represents the a priori LLR of the interleaved and encoded bits $b_k(n, i)$. The first term in (1), denoted by $L_{E,m}(b_k(n, i))$, represents the extrinsic information delivered by the SISO MUD, based on the received signal $r(n)$ and the a priori information about the encoded bits of all users, except for the $i$th bit of the desired user $k$. The extrinsic information is then deinterleaved and fed into the $k$th user’s channel decoder, which will provide the a priori information in the next iteration.

As seen in Fig. 1, between the banks of channel decoders and interleavers, we compute the extrinsic LLR based on the a priori information $L_{A,c}(b_k(j))$ provided by the SISO beamforming MUD for the SISO decoder as $L_{E,c}(b_k(j)) = L_{P,c}(b_k(j)) - L_{A,c}(b_k(j))$ [6], where the extrinsic information is gleaned from the surrounding encoded bits, excluding the specific bit considered [6]. After interleaving, the extrinsic information delivered by the channel decoders is fed back to the SISO MUD, as the a priori information concerning the encoded bits of all the users for exploitation during the next iteration.

III. SISO Interference Cancellation

Given the a priori LLRs, we first define the mean and variance based on the a priori information of the $k$th user’s symbols for the QAM constellation as in [5]: $s_k = E[s_k]$ and $\nu_k = E[|s_k|^2] - |s_k|^2$, where the symbol-index $n$ was dropped for notational convenience. When using the SIC principle, the estimated symbol of user $k$ can be expressed as [5]

$$\hat{s}_k = w_k^H r - H \tilde{s}_k,$$

where $\tilde{s}_k = [\tilde{s}_1 \cdots \tilde{s}_{k-1} \ 0 \ \tilde{s}_{k+1} \cdots \tilde{s}_K]^T$. In the next two subsections we outline the differences between the MSER MUD and the MMSE MUD.

A. SISO Interference Cancellation Using the MMSE MUD

Classically, the MMSE solution for the beamformer’s weight vector $w_k$ is expressed as [6]

$$w_{k,mmse} = (H V_k H^H + E_k h_k h_k^H + 2\sigma_n^2 I_L)^{-1} E_k h_k,$$

where $E_k$ is the average symbol energy, $I_L$ denotes the $L \times L$ identity matrix and $V_k = \text{diag}[v_1 \cdots v_{K-1} 0 v_{K+1} \cdots v_K]$, in which $\text{diag}[\cdot]$ denotes a diagonal matrix.

As stated in [6], the conditional PDF $P[\tilde{s}_k = s^{(p)} | s_k]$ where $s^{(p)}$ is the $p$th $(p \in \{1, 2, \cdots, M\})$ legitimate value of the QAM constellation, may be assumed to be Gaussian distributed and the corresponding extrinsic output LLR is given by [6]

$$L_E(b_k(i)) = \sum_{b_k^{(p)}(i) = 0} \exp \left( \frac{-|s_k - b_k^{(p)}|^2}{\sigma_k^2} \right) \prod_{b_k^{(p)}(i') \neq 0} P(b_k^{(p)}(i')) \prod_{b_k^{(p)}(i') = 0} P(b_k^{(p)}(i')) \quad (4)$$

where $b_k^{(p)}(i)$ denotes the $i$th bit of $s^{(p)}$, $\mu_k^{(p)} = s^{(p)} w_k^H h_k$, $\sigma_k^2 = E_k w_k^H h_k (1 - w_k^H h_k)$, and the $a priori$ probability of the $i$th bit in symbol $s_k$ is $P(b_k^{(p)}(i')) = \frac{1}{2} (1 + \text{sgn}(\frac{1}{2} - b_k^{(p)}(i')) \cdot \text{tanh}(\frac{L_{A,c}(b_k^{(p)}(i'))}{2})).$

B. SISO Interference Cancellation Using the MSER MUD

In [10], the MSER algorithm is investigated when the MUD has access to no a priori information. Moreover, the weight vector $w_k$ is rotated to make $w_k^H h_k$ real and positive. Under these conditions, the subset PDFs conditioned on the different values of the estimated signal $\tilde{s}_k$, when all the $M^K$ possible symbol combinations are transmitted, satisfy the shifting properties and are symmetrically distributed [10], which may be used to simplify the weight vector calculation. However, when the MUD is provided with a priori information, these properties are invalid and the MSER method of [10] cannot be applied directly to our iterative system. Hence in this subsection we introduce the a priori information aided MSER MUD to resolve this problem.

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Let us define a symbol in the $M$-QAM constellation as $s^{(m,n)} = \gamma(2m - \sqrt{M} - 1) + j \cdot \gamma(2n - \sqrt{M} - 1)$, where $\gamma = \sqrt{\text{Eb}/\text{N0}}$. We assume that the $q$th ($q \in \{1, 2, \cdots, M^K\}$) symbol combination $s^{(q)}$ is transmitted, in which the desired user $k$ transmits symbol $s_k^{(q)} = s^{(m,n)}$. Fig. 2 shows the estimated signal $\hat{s}_k^{(q)}$ and its marginal PDFs under this condition. The PDF of $\hat{s}_k^{(q)}$ is a Gaussian distribution with a mean value $\bar{\hat{s}}_k^{(q)} = w_k^H (\mathbf{H}s^{(q)} - \mathbf{H}s_k^{q})$, as seen in Fig. 2. When the $k$th user transmits symbol $s_k^{(q)}$, the conditional PDF of $\hat{s}_k$ is a scaled mixture of all the Gaussian PDFs in the subset $\{p(s_k^{(q)} | s_k^{(m,n)} = s^{(m,n)})\}$, defined by
\[
p(\hat{s}_k | s_k^{(m,n)}) = M \sum_{q \in \{1, 2, \cdots, M^K\}} \sum_{s_k^{(q)} = s^{(m,n)}} P_k(s^{(q)}) \cdot p(\hat{s}_k^{(q)} | s_k^{(q)} = s^{(m,n)}) = \frac{M}{2\pi \sigma_k^2 w_k^H w_k} \sum_{q \in \{1, 2, \cdots, M^K\}} P_k(s^{(q)}) \cdot \exp\left(-\frac{|\hat{s}_k^{(q)} - \bar{\hat{s}}_k^{(q)}|^2}{2\sigma_k^2 w_k^H w_k}\right),
\]
where $P_k(s^{(q)}) = \frac{1}{M} \prod_{k' \neq k} P(s_k^{(q)} = s^{(q)})$ is the probability of transmitting the $q$th possible QAM symbol combination $s^{(q)}$, given the $a$ priori information of the other $(K-1)$ users, except for user $k$. By defining $b_i = \gamma(2i - \sqrt{M})$ for $1 \leq i \leq \sqrt{M}-1$, the decision boundaries of $\hat{s}_k$ are determined by $b_i |w_k^H h_k|$ for the in-phase component and by $jb_i |w_k^H h_k|$ for the quadrature-phase component, as seen in the diagram.

Fig. 2. Then the error rate is the integral of the conditional PDF outside the corresponding boundaries. Fig. 2 only portrays the scenario of the inner constellation boundaries, which is enclosed by boundaries. The points at the edge of the constellation may have open boundaries in one or two directions, which should be considered for the error rate calculation.

Let us now assume that the $k$th user transmits symbol $s_k^{(m,n)}$. Then the in-phase component’s conditional error probability of $\mathbb{R}[\hat{s}_k] \neq \mathbb{R}[s_k^{(m,n)}]$ can be shown to be
\[
P_{eI}(s_k^{(m,n)}) = \begin{cases} \int_{b_{m-1}^{\infty}} p(\hat{s}_k | s_k^{(m,n)}) d\hat{s}_k, & m = 1, \\ \int_{b_{m-1}^{\infty}} p(\hat{s}_k | s_k^{(m,n)}) d\hat{s}_k + \int_{b_{m-1}^{\infty}} p(\hat{s}_k | s_k^{(m,n)}) d\hat{s}_k, & 2 \leq m \leq \sqrt{M} - 1, \\ \int_{b_{m-1}^{\infty}} p(\hat{s}_k | s_k^{(m,n)}) d\hat{s}_k, & m = \sqrt{M}, \end{cases}
\]

Similarly, the quadrature-phase component’s conditional error probability of $\mathbb{I}[\hat{s}_k] \neq \mathbb{I}[s_k^{(m,n)}]$ can be shown to be
\[
P_{eQ}(s_k^{(m,n)}) = \begin{cases} M \sum_{q \in \{1, 2, \cdots, M^K\}} P_k(s^{(q)}) \cdot Q\left(\frac{|b_{m-1}^{\infty} w_k^H h_k - \mathbb{I}[s_k^{(q)}]|}{\sigma_n \sqrt{w_k^H w_k}}\right), & n = 1, \\ M \sum_{q \in \{1, 2, \cdots, M^K\}} P_k(s^{(q)}) \cdot Q\left(\frac{|b_{m-1}^{\infty} w_k^H h_k - \mathbb{I}[s_k^{(q)}]|}{\sigma_n \sqrt{w_k^H w_k}}\right), & 2 \leq n \leq \sqrt{M} - 1, \\ M \sum_{q \in \{1, 2, \cdots, M^K\}} P_k(s^{(q)}) \cdot Q\left(\frac{|s_k^{(q)} - b_{m-1}^{\infty} w_k^H h_k|}{\sigma_n \sqrt{w_k^H w_k}}\right), & n = \sqrt{M}. \end{cases}
\]

Then the average error probability of the in-phase and quadrature-phase components are given by
\[
P_{eI} = \frac{1}{M} \sum_{m=1}^{\sqrt{M}} \sum_{n=1}^{\sqrt{M}} P_{eI}(s_k^{(m,n)})
\]
and
\[
P_{eQ} = \frac{1}{M} \sum_{m=1}^{\sqrt{M}} \sum_{n=1}^{\sqrt{M}} P_{eQ}(s_k^{(m,n)}).
\]
respectively. The resultant SER can be formulated as
\[ P_{e_k} = P_{e_I} + P_{e_Q} - P_{e_I} \cdot P_{e_Q}. \] (10)

Finally, the MSER solution is defined as the one that minimises the upper bound of the SER given by
\[ w_{k,mser} = \arg \min_{w_k} (P_{e_I} + P_{e_Q}). \] (11)
The upper bound \((P_{e_I} + P_{e_Q})\) is very close to the true SER \(P_{e_k}\) because the term \(P_{e_I} \cdot P_{e_Q}\) is typically negligible. In order to arrive at the optimum weights for the MSER solution, we need the gradients of \(P_{e_I}\) and \(P_{e_Q}\) in the context of the simplified conjugate gradient algorithm [7], which can be derived from the gradients of the Q-functions in Equations (6) and (7), leading to
\[ \nabla_{w_k} Q \left( \begin{array}{c} \Re[s_k] \\ \Im[s_k] \\ b_i w_h^H h_k \end{array} \right) = \frac{1}{\sigma_n \sqrt{w_h^H w_h}} \cdot \exp \left( -\frac{(\Re[s_k] - b_i w_h^H h_k)^2}{2\sigma_n^2} \right) \cdot \begin{array}{c} w_h (\Re[s_k] - b_i w_h^H h_k) \\ w_h (\Im[s_k] - b_i w_h^H h_k) \\ w_h (b_i w_h^H h_k) \end{array} \cdot \begin{array}{c} 1 \\ j \end{array} \right). \] (12)

The output extrinsic information delivered by the MSER MUD can be expressed as
\[ L_E(b_k(i)) = \frac{1}{\ln \left( \sum_{b_{(m,n)}^{(i)} = 0} \exp \left( \frac{-(\Re[s_k] - b_{(m,n)}^{(i)} w_h^H h_k)^2}{2\sigma_n^2} \right) \right)} \] (18)
when \(b_k(i)\) is mapped to the real part of \(s_k\), and
\[ L_E(b_k(i)) = \frac{1}{\ln \left( \sum_{b_{(m,n)}^{(i)} = 0} \exp \left( \frac{-(\Im[s_k] - b_{(m,n)}^{(i)} w_h^H h_k)^2}{2\sigma_n^2} \right) \right)} \] (19)
when \(b_k(i)\) is mapped to the imaginary part of \(s_k\), where \(b_{(m,n)}^{(i)}\) denotes the \(i\)th bit of \(s_{(m,n)}^k\) and \(P(b_{(m,n)}^{(i)}) = \frac{1}{2} \left( 1 + \text{sgn} \left( \frac{1}{2} b_{(m,n)}^{(i)} \right) \cdot \tanh \left( \frac{i}{2} \right) \right) \).

IV. EXIT CHART ANALYSIS

The EXIT chart analysis computes the mutual information (MI) between the LLRs and the corresponding bits, as detailed in [11]. An example of EXIT chart is shown in Fig. 3 for the simulated system investigated in Section V. Let \(L_A\) denote the MI between the \(a\) priori values \(L_A\) and the corresponding bit-sequence, while \(L_E\) denote the MI between the extrinsic values \(L_E\) and the corresponding bit-sequence. Then the EXIT function of the channel decoder is defined by \(I_{E,E}=f_c(I_{A,c})\), which maps the input variable \(I_{A,c}\) to the output variable \(I_{E,E}\), and the specific value of \(I_{E,E}\) in the range [0, 1] characterises the quality of the output LLRs of the decoder components. Unlike in single-user turbo coding or turbo equalisation, in the multiuser detection scenario the MUD’s EXIT curve recorded for the desired user depends on all the other \((K-1)\) users’ channel decoder output MI. In our simulations all the users’ SNRs are identical. Additionally, their angular locations are selected so that the relative time delay of all the users with respect to the angularly closest neighbors is the same. Hence the turbo MUD can average all the users’ MI’s in order to simplify the EXIT chart function to \(I_{E,m} = f_m(I_{A,m}, E_b/N_0)\).

The output of one of the two constituent components is the input of the other, hence both transfer functions are shown in the same EXIT plane having coordinate axes of \((I_{A,m} = I_{E,c}, I_{E,m} = I_{A,c})\). The stair-case-shaped lines in Fig. 3, connecting the MI points evaluated during each iteration, are referred to as the detection or decoding trajectory. The substantial advantage of EXIT charts accrues from the fact that the detection trajectory points recorded for both constituent components exchanging information fall on the continuous EXIT functions obtained independently in a separate process. An infinitesimally low BER may be attained, when there
is a so-called open tunnel between the EXIT curves of the decoder and the MUD. This graphical representation gives us an immediate insight into the number of detection iterations required for attaining the best possible BER performance.

V. SIMULATION RESULTS

The system employs a two-element antenna array to support $K=3$ 16QAM users. All the users have the same transmit power. Each user employs a different randomly generated interleaver. The interleaver length of each user is $2 \times 10^4$ bits. All the users have the same channel coefficients of $h_k = 1.0+j0.0$, $1 \leq k \leq 3$, and employ the same rate-1/2 and constraint-length 4 non-systematic convolutional code using the octally represented generators (15, 17). The arrival angles of the users’ signals are $68^\circ$, $15^\circ$ and $-24^\circ$, respectively.

Fig. 3 shows the EXIT curves and the simulated trajectories of the iterative MMSE and MSER 16QAM beamforming receivers supporting $K=3$ users at $E_b/N_0=7.5$ dB. In this 16QAM system, the MMSE and MSER MUDs have almost the same output $I_{E,m}$ value at both the axes at $I_{A,m}=0$ and $I_{A,m}=1$. Between these two points of intersection, the MMSE MUD has the lower EXIT curve, and the MSER’s EXIT curve reaches a higher $I_{E,m}$ value.

Fig. 4 shows the EXIT curves and the simulated trajectories of the iterative MMSE and MSER 16QAM beamforming receivers supporting $K=3$ users at $E_b/N_0=7.5$ dB. In this 16QAM system, the MMSE and MSER MUDs have almost the same output $I_{E,m}$ value at both the axes at $I_{A,m}=0$ and $I_{A,m}=1$. Between these two points of intersection, the MMSE MUD has the lower EXIT curve, and the MSER’s EXIT curve reaches a higher $I_{E,m}$ value.

VI. CONCLUSION

This paper has introduced a novel SIC-MSER MUD employed in an iterative beamforming receiver designed for high-throughput QAM systems. This scheme directly minimises the SER instead of the MSE at the MUD’s output, which leads to a better performance than that of the conventional MMSE-based systems at a cost of higher complexity. EXIT charts have also been used for analysing the convergence behaviour of the proposed system.

REFERENCES


