Optimal Economic Ship Speeds, the Chain Effect, and Future Profit Potential

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Abstract

In this research, we study at which speeds an oceangoing ship should *ideally* travel on each of a series of legs of a journey as to maximise the Net Present Value (NPV) of the ship. A novel class of models for the ship speed optimisation problem, which we refer to as $\mathcal{P}(n, m, G_o)$, is presented. It is based on incorporating cash-flow functions and is flexible in modelling journey structures of variable composition. By studying properties of optimal leg speeds within this NPV framework, we demonstrate two novel elements of ship speed optimisation: (a) When executing a series of identical journeys, optimal ship speeds from one execution of the journey to the next are shown to change. We refer to this as the *chain effect*. (b) The ship's optimal speed is in general highly dependent on the decision maker's views on the ship's *future profit potential*(FPP). We present two efficient algorithms to solve the models. The methodology is applied to case studies based on the literature and the results are compared with classic model formulations. Net Present Value Equivalence Analysis (NPVEA) shows how the proposed framework increases understanding of the applicability and limitations of these classic model formulations. The use of the FPP concept is recommended in speed optimisation and job selection models.

Keywords: Maritime speed optimisation, Chain effect, Future profit potential, Dynamic programming

1. Introduction

The OR-related literature in seaborne transportation is primarily concerned with (computer-assisted) decision support and optimisation towards the efficient planning and execution of operational and logistics processes. Impactful reviews such as Ronen (1983, 1993); Christiansen et al.

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(2004, 2013, 2019) track important publications of ship scheduling and routing problems decade by decade. These reviews make clear that Maritime OR is an active and growing research field.

Many OR models in the literature assume fixed and known speeds while solving routing and scheduling problems, e.g., Agarwal & Ergun (2008); Grønhaug et al. (2010); Song & Xu (2012); Li et al. (2019). This assumption removes the flexibility of decision making deemed important in many practical applications, according to Psaraftis & Kontovas (2013, 2014), who also provide comprehensive reviews on speed optimisation models in maritime OR literature. Ship speed has also been a decision variable in stochastic settings, see e.g. Magirou et al. (2015). Speed optimisation models gain in importance in the last years due to the impact of fuel consumption on the environment, see e.g. Du et al. (2019), who investigate the benefit of speed and trim optimisation for a containership over a single voyage as a means to minimise fuel consumption.

The flexibility of ship speed allows ships designed for a higher nominal speed to travel slower according to market conditions (Psaraftis & Kontovas, 2014). The report of Devanney (2010) states that with modern engines and techniques installed the speed limits of a Very Large Crude Carrier (VLCC), for example, can be extended from the traditional 2 to 3 knots range around design speed, to a range of over 6 knots. With retrofitting technologies, such flexibility is also in reach for existing ships, see Eason (2015). Allowing for ship speed flexibility rather than speed reduction can serve an important role in helping maximise the economic (and/or environmental) benefit of shipping activities for different decision makers, see also Psaraftis (2019).

This paper investigates the differences between two methodological approaches for the construction of ship speed optimisation models in maritime transportation: a novel approach based on the Net Present Value criterion versus (three flavours of) the 'conventional' approach that is not based on NPV principles. In the NPV approach, the objective function is the Laplace transform of the cash-flow function associated with the ship's future activities. This generic approach to the formulation of economic problems has been proposed in Grubbström (1967) and applied to the field of production and inventory system management in Grubbström (1980) to show how it can be used to gain insight into the applicability or limitations of classic model formulations not based on NPV principles. This kind of analysis is formalised in Beullens & Janssens (2014) and called NPV Equivalence Analysis (NPVEA). To our knowledge, this NPV approach has not yet been thoroughly applied, analysed, and compared with conventional model formulations of the ship speed optimisation problem[1].

In conventional approaches to ship speed optimisation modelling, profit or cost functions are typically expressed in one of two ways. A first 'conventional' speed optimisation method considers the criterion *dollars per journey* (or *per nautical mile*). This approach is found in e.g. Corbett et al. (2009); Norstad et al. (2011); Psaraftis & Kontovas (2014); Fagerholt et al. (2015); Wen et al. (2017). Importantly, Psaraftis & Kontovas (2014) formulate the *decomposition principle*: they

[1]Further links to related literature on NPV, NPVEA, and other disciplines of research are discussed in Section 6. We also postpone the discussion of the limited literature of NPV in speed optimisation to Section 6.2, after we have been able to present our main results, as this helps us to better clarify our contribution.

prove that an m - leg journey in their model can be *decomposed* into m one-leg sub-problems, and each solved independently for the optimal ship speed on that leg[2].

A second popular 'conventional' speed optimisation method is based on the criterion *dollars per unit of time*, as perhaps first used in the *income-generating-leg model* in Ronen (1982), and adopted in many studies for representing the basic scenario that a (tramp) ship transports cargo for a reward, see e.g. Devanney (2010); Ronen (2011); Magirou et al. (2015); Fagerholt & Psaraftis (2015); Lee & Song (2017). It is a curious fact that the decomposition principle was in fact explicitly proposed in Ronen (1982) as being 'beneficial', although not formally proven[3]. Models following this method thus divide the profit earned on a leg by total travel time of the leg to arrive at the objective function.

Ronen (1982) also presented a *positioning (empty) leg model*, minimising the total cost on the leg. We can consider this to be a third 'conventional' model. This model, however, has not received much actual application in the subsequent literature. The model crucially relies on a parameter C_a, which Ronen called the *Daily Alternative Value*, but for which he provided no further information about how to determine it. We speculate that this may have contributed to the model not being so popular.

It is a fact that these speed optimisation models produce different optimal speed values. This difference has not really been explained in the literature. Only Psaraftis (2017) points out the difference between the first two conventional approaches that we have discussed above. He argued in favour of Ronen's approach, as only here optimal speed is also a function of the revenue generated on the leg considered, which he believes captures a fundamental facet of shipping industry behaviour. Indeed, the importance of freight rates in guiding speed choice is also mentioned in e.g. Stopford (2009) and Devanney (2010).

In this paper, we support the idea that if ships have speed flexibility, that optimal economic speeds should be based on the consideration of relevant future revenues and cost[4]. For instance, it seems natural for a ship to travel at higher average speed on a leg if the company knows this may gain it access to a future profitable job (e.g., due to the competition), while other companies without such information or opportunity may instead wish to safe fuel now by travelling slower. The class $\mathcal{P}(n, m, G_o)$ of NPV models presented incorporates these kinds of trade-offs between present and future. Models in this class are not solved on an individual leg-basis (i.e. by decomposition), but need to be solved by determining optimal speeds for all legs simultaneously across all journey(s) considered. Efficient algorithms are developed and run in (pseudo-) polynomial time.

[2]The optimality of decomposition holds in that conventional model. In a NPV model, this decomposition approach is no longer optimal, see Theorem 1, and Section 4.3.

[3]Ronen (1982): "Although it is common in the shipping industry to calculate profitability on a roundtrip basis, ships are often not operating in such a manner (...)... Moreover, the most profitable speed may be different for an empty and a full leg of the same roundtrip. Thus, it is beneficial to analyze the speed for each voyage leg separately and decouple the various legs of the voyage."

[4]As this accounts best for decision makers' objective to maximise shareholder wealth, one of the principles of corporate finance, see also Brealey & Myers (2003).

The rest of paper is structured as follows. The descriptions, notations and assumptions of the modelling framework are introduced in Section 2. Model formulations and solution algorithms are presented in Section 3. Section 4 investigates and illustrates properties of our proposed models. The application of our models, including a comparison with conventional models are found in Section 5. We find it easier to present further links with other literature after the presentation of our models and their analysis. This discussion is thus presented in Section 6. Conclusions follow in Section 7.

2. Assumptions, notation and problem description

In this research, we study at which speeds an oceangoing ship should *ideally* travel on each of a series of legs of a journey as to maximise the NPV of the ship. The decision maker is the ship operator (e.g. owner, time charterer). Our primary purpose is to develop greater understanding of the degree by which the future is important in determining the optimal current usage of the vessel, and to examine to what degree this accounted for in the conventional methods introduced in Section 1. For this reason, we assume that the ship is not constrained in its choice of speeds by time windows imposed by external influences (e.g. ports, shippers). In this way, we can study optimal speeds strategies in a context that sketches the ideal position of the operator.

This is less removed from practise then perhaps initially recognised, in particular in the bulk cargo and tanker industries. Here, time windows are not an exogenous input but rather agreed between shipper and operator before signing the contract, see also Psaraftis (2017). It should be of interest to the operator, prior to engaging in these negotiations, to know how speed choices affect the NPV of the ship's activities[5].

Because of the direct applicability to the sector, we adopted modeling assumptions in Sections 2.1 to 2.5, and in most numerical examples later, based on the operations of a Suezmax oil tanker. The theoretical results of Section 3 and 4, however, do not crucially rely on the particular details of the assumptions related to how the components of the revenue and cost cash-flows and their timing are exactly dependent on the journey data. The main results are of much more general applicability to any ship not subjected to time windows and that transports cargo for a revenue, where we only need mild conditions of a concave profit structure of a leg.

2.1. Model structure: example

In this paper, we consider a single vessel, with a known sequence of port visits, and the decision maker is not restricted by e.g. time windows in ports or contractual limitations about the ship

[5]In liner services, speed optimisation is also important, but additional constraints, see e.g. Christiansen et al. (2019), may make it difficult to implement the identified optimal speeds from an unconstrained model as presented in this paper. Operators do implement the practice of slow steaming to save on fuel. Because the ships work in a group as to offer an agreed visit frequency, more ships may then be needed. For an insightful model explaining this trade-off, see Ronen (2011). We believe that the unconstrained model presented can still offer understanding of the fundamental economic forces when a containership would execute a series of (repeated) journeys, and may lead to refinements in the network optimisation of such services in future research.

speed except for a known upper and lower bound on the ship's speed. This setting is similar to Psaraftis & Kontovas (2014).

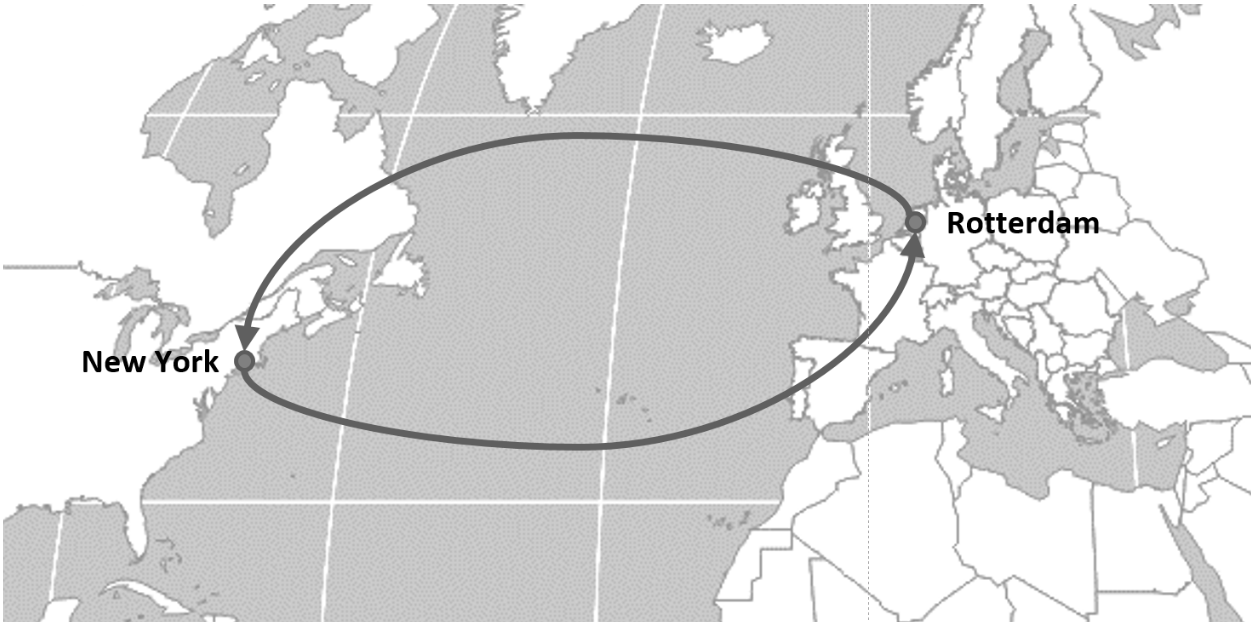


Figure 1: An example of roundtrip

The single ship will execute a *journey* n times, where n is allowed to take any value 1, 2, 3, ... We adopt the journey sequence $\{n, n-1, ..., 2, 1\}$, where n is the first and 1 is the last time that the journey is executed.

An often encountered situation is the *round-trip journey* between two ports. This is illustrated in Figure 1: the first *leg* of the journey goes from Rotterdam to New York, the second leg from New York back to Rotterdam. In general, a journey may consist of visits to a series of $m+1$ $(1 < m < \infty)$ ports in a certain order, covering m legs, where the last port not necessarily needs to be the point of origin. If the ship carries cargo on a leg, it is *laden*, otherwise it is a *repositioning* or *ballast* leg. Cases $n > 1$ cannot be realistically implemented unless the journey itself is a round-trip.

Journey repetitions of smaller roundtrips do occur as part of a time charter, a consecutive voyage charter, a contract of affreightment, and even liner shipping services, see also Stopford (2009). In the transport of oil or natural gas, the repeated ballast-laden journey is often encountered.

The organisation of repeated journeys may benefit in practical terms from adopting consistent speeds that do not change from one repetition to the next. However, how much would the company potentially gain from optimal speed choices, which may change over time? This is also of theoretical interest, since it can inform us about the accuracy of the decomposition approach used in Psaraftis & Kontovas (2014) and Ronen (1982). Allowing for journey repetitions in our models thus gives us the means to examine these questions.

It is clear that the model is very general by changing the values of n and m. In contrast to the example in Figure 1, we could also look at models where $n = 1$ and m is large. This is a situation where the journey is long, every leg may be unique, and there is no repetition, as in certain tramp shipping situations.

Returning to the example in Figure 1, let V_{AB}^{i} denote the average speed adopted by the ship on the leg from A (Rotterdam) to B (New York) on the i^{th} journey in the sequence $\{n, n-1, ..., i, ..., 2, 1\}$, and V_{BA}^{i} be the speed on the return leg. The general question addressed in this paper on how to decide on the optimal ship speeds now becomes determining optimal values for $\{(V_{AB}^{i}, V_{BA}^{i})|i = 1, ..., n\}$ that maximises the NPV of all relevant cash-flows.

2.2. Daily running costs

We assume a given fixed daily cost of value f^{TCH} for the ship, not including the voyage-related costs (fuel, cargo handling, port fees, etc). In e.g. a time charter, f^{TCH} is the Time Charter Hire (TCH) that is paid by the ship charterer to the owner. In the case of a ship owner operating in the spot market, f^{TCH} may represent the owner's fixed operating costs to cover the ship's crew, maintenance, lubrication, insurance, etc. For a detailed discussion on cost structure, see Stopford (2009)[Chapter 6].

2.3. Journey characteristics

Define the *journey* as the ship completing a route on which it will visit a sequence of ports $\mathbb{P} = \{P_0, P_1, \ldots, P_m\}$, and $\{P_{j-1}, P_j\} \in \mathbb{P}$ is *leg* j of the journey ($j = 1, ..., m$). To study situations where this journey is repeated $n > 1$ times, we need $P_m \equiv P_0$. A leg on which the ship does not carry any revenue-generating cargo is a *ballast leg*, otherwise it is a *laden leg*. For each leg j, the travel time (in days) $T_j^{s,i}$ is given by:

$$T_j^{s,i}(v_j^i) = \frac{S_j}{24 v_j^i}, \tag{1}$$

where S_j is the given distance (in nm) from P_{j-1} to P_j and v_j^i is the average chosen speed (knot per hour) on the leg j of the journey with index i in the sequence $\{n, n-1, \ldots, i+1, i, i-1, \ldots, 2, 1\}$. The total time associated with the execution of leg j is:

$$T_j^i(v_j^i) = T_j^l + T_j^{s,i}(v_j^i) + T_j^w + T_j^u, \tag{2}$$

where the loading time T_j^l at port P_{j-1} and unloading time T_j^u at P_j are linearly dependent on the tonnage to be (un)loaded (as the pumps in e.g. a tanker and at berth have limited capacity), and T_j^w is the waiting time at P_j (for queueing, towing, ...).

We define L_j^i as the total elapsed time over the first j legs of the journey with index i ($j = 1, 2, \ldots, m$; $i = 1, 2, \ldots, n$):

$$L_j^i(V^{i,j}) = \sum_{k=1}^{j} T_k^i(v_k^i), \tag{3}$$

where $V^{i,j} = \{v_1^i, v_2^i, \ldots, v_j^i\}$ and L_m^i represents the total duration of operations for completing the journey with index i. We call $\sum_{i=1}^{n} L_m^i$ the *termination time*; it is the time when the ship will become available for future work, assuming the start of operations (= loading the ship for first leg with index n) occurs at decision time 0.

2.4. Economic parameters

Each visit to a port incurs costs. Without loss of generality, we split the total costs incurred at a port P_j $(j = 0, 1, \ldots, m)$ into two components.

The first component C_j^u represents costs at P_j for unloading, and typically includes the costs of waiting to enter the port and dock at berth, fixed port charges, any mooring dues, and unloading charges. The second component C_{j+1}^l represents costs at port P_j made towards undertaking the next leg $j+1$. This typically includes cargo loading costs, cost of readjusting ballast, and the costs of main and auxiliary fuel intake (Stopford, 2009)[p.236]. We assume these costs arise the moment unloading at the port has finished, and loading starts[6].

The amount of main fuel to cover leg j depends on the sailing speed v_j^i, and the deadweight tonnage (DWT) w_j carried, which includes cargo, ballast, supplies, and main and auxiliary fuels carried[7]. We adopt a daily fuel consumption function as proposed in Psaraftis & Kontovas (2014):

$$F(v_j^i, w_j) = k(p + (v_j^i)^g)(w_j + A)^h, \tag{4}$$

where A the lightweight of the ship, and typically $g \approx 3$ and $h \approx 2/3$. For simplicity, we assume the bunker intake only covers the next leg and that the cost occur at loading.[8] For reasons of stability, we impose a constraint that w_j should be at least a given percentage (30% in this paper) of the design DWT of the vessel: any shortcoming in minimum DWT arising from zero or low amounts of cargo carried is to be made up from ballast water. For further information about the ballast water function, see David (2015).

The loading cost for leg j $(j = 1, \ldots, m)$ of the journey with index i can be stated as:

$$C_j^l(v_j^i) = C_j^h + c_j^f F(v_j^i, w_j) \frac{S_j}{24 v_j^i}, \tag{5}$$

where c_j^f is the fuel cost (USD/ton) at P_{j-1} at the time of bunkering, and C_j^h represent the sum of other costs associated with the loading operations.

Revenues are based on freight rates, i.e. (market) unit prices of the cargo type and the distance

[6] The model can easily accommodate payments occurring earlier or later than the considered timing. For example, it is straightforward to implement that port costs are only paid three days after leaving the port, or that revenues are typically received some hours prior to starting the unloading operations. See also Beullens et al. (2020). These details do not fundamentally alter the findings presented.

[7] It fulfills a function similar to the "displacement" in engineering literature, see e.g. Meng et al. (2016).

[8] Bunkering decisions can be quite important for ship operators. It is relatively straightforward in the model to implement any feasible bunkering policy along the journey, as long as this is then repeated. For example, it may be feasible to take bunkers at the port where this is cheapest, and not bunker at the other port in a ballast/laden scenario. More sophisticated economic instruments of paying for bunker fuels are not considered here.

between origin and destination ports, and negotiations with shippers. For the journey described with known tonnages transported, we assume we can calculate in advance the total revenues R_j from unloading cargo at port P_j. Knowing the cargo transported on each leg j, we calculate from the stability constraint the amount of ballast water needed (if any), and find the value w_j. The resulting problem is thus a function of the speeds $\{(v_1^i, v_2^i, \ldots, v_m^i) | i = 1, \ldots, n\}$.

2.5. $\mathcal{P}(n, m, G_o)$: Problem description summary

We summarise the above problem formulation as follows. A ship is to execute a journey n times. The journey consists of m legs. The problem is to find the optimal speed $v_j^i \in (v_{min}, v_{max})$ for each leg j of each repetition i that will maximise the NPV of all future operations of the ship. We call the class of problems considered henceforth $\mathcal{P}(n, m, G_o)$.

Since the information of journey sequence is known a priori, other parameters such as w_j can be calculated in advance, and this then transform all the functions introduced to become dependent on ship speeds only. The main assumptions are summarised below:

- For each day the ship is used during these journeys, including for the time spend in ports, we pay a fixed cost f^{TCH}, see Section 2.2;
- At the start of a leg j, just prior to any loading activity at a port, the ship incurs a loading cost $C_j^l(v_j^i)$, see Section 2.3;
- The loading cost includes the cost of fuel intake to cover main and auxiliary fuel for the next leg, see Section 2.3;
- The loading costs and time in the port increase with the amount of cargo to be loaded, see Section 2.4;
- After the execution of the sea voyage, the ship spends a given waiting time prior to the start of unloading activities, see Section 2.3;
- At the end of a leg j, just after unloading is finished, the ship receives a revenue R_j and an unloading cost C_j^u, see Section 2.4;
- The unloading cost and time in the port increase with the amount of cargo to be unloaded, see Section 2.3;
- The ship must carry a minimum of 30% of its design DWT during a sea voyage, or make up the shortfall with ballast water, see Section 2.4;
- The fuel consumption per day at sea for the main bunker fuel is given by (4); w_j is the sum of cargo, ballast water, and main fuel at the port of loading and can be calculated in advance, see Section 2.4;

- As times in the formulation are expressed in days, the decision maker's opportunity cost of capital α is likewise the percentage earned in the next best available alternative per day, see e.g., Brealey & Myers (2003);
- The parameter G_o is called the *Future Profit Potential* (FPP); its function is introduced in Section 3.2.

We will also look more specifically at some noteworthy special cases, as the model then reduces to (an NPV-formulation) of a specific type of speed optimisation model from the literature (see also Section 5). Examples of special sub-classes include $\mathcal{P}(1,1,0)$, $\mathcal{P}(1,m,0)$, and $\mathcal{P}(\infty,m,-)$.

3. Model formulation and solution algorithms

Before we address the proposed model class $\mathcal{P}(n,m,G_o)$, it is instructive to first present in Section 3.1 a simpler case which serves to illustrate how to develop the model and the dynamic programming solution approach. The idea of accounting also for future plans then follows naturally as an extension in Section 3.2.

3.1. $\mathcal{P}(n,m,0)$: Dynamic Programming Formulation

The speed decision for leg j ($j = 1,2,\ldots,m$) within the roundtrip journey i ($i = n, n-1,\ldots,1$) will result in a *leg execution time* T_j^i, or *leg time* for short. Let $h_j(T_j^i)$ denote the profitability associated with this leg, discounted to the start time of execution of this leg:

$$h_j(T_j^i) = (R_j - C_j^u)e^{-\alpha T_j^i} - C_j^l - \int_0^{T_j^i} f^{TCH} e^{-\alpha t} dt. \tag{6}$$

The leg time T_j^i is a function of the chosen speed v_j^i as in (2), and as the ship's speed must remain between a given minimum and maximum value, say v_{min} and v_{max}, it is subject to the condition:

$$\frac{S_j}{24v_{max}} \leq (T_j^i - T_j^l - T_j^w - T_j^u) \leq \frac{S_j}{24v_{min}}. \tag{7}$$

Let $G_j^i(.)$ denote the accumulated profitability, associated with the sum of journeys $i-1,...,1$ up to and including of legs $m, m-1,...,j$ of journey i, discounted to the start time of the leg j of journey i. This property of a NPV discounted to a future time is also defined as the *Goodwill* in Preinreich (1940). For the special case of $i = 1$ and $j = m$, i.e. $\mathcal{P}(1,m,0)$, let:

$$G_m^1(T_m^1) = h_m(T_m^1), \tag{8}$$

and thus:

$$G_{m-1}^1(T_{m-1}^1, T_m^1) = h_{m-1}(T_{m-1}^1) + h_m(T_m^1)e^{-\alpha T_{m-1}^1}. \tag{9}$$

Let T_m^{1*} be the leg time value that maximises G_m^1, its maximum value denoted as $G_m^{1*}(T_m^{1*})$, then maximising (9) is equivalent to finding the value of T_{m-1}^1 that maximises (see also Section 4):

$$G_{m-1}^1(T_{m-1}^1) = h_{m-1}(T_{m-1}^1) + G_m^{1*}(T_m^{1*})e^{-\alpha T_{m-1}^1}. \tag{10}$$

Similarly, for all other (i, j) combinations the following recursion applies:

$$G_j^i(T_j^i) = h_j(T_J^i) + G_{j+1}^i(T_{j+1}^i)e^{-\alpha T_j^i}. \tag{11}$$

The value for the leg time T_j^i that maximises G_j^i, denoted as $G_j^{i*}(T_j^{i*})$, is not dependent on T_l^k $(k > i)$ nor on T_l^i $(l < j)$. Therefore, the *principle of optimality* holds, and we can find optimal speeds for each leg and journey with dynamic programming (Bellman, 1954) using Algorithm 1 (take $G_0 = 0$, see also Section 3.2). In the step of finding a new G value, we solve the following problem:

$$T_j^{i*} = arg \;\; max \;\; \left(h_j(T_J^i) + G_{j+1}^{i*}(T_{j+1}^{i*})e^{-\alpha T_j^i}\right). \tag{12}$$

Algorithm 1: Dynamic Programming to solve $\mathcal{P}(n, m, G_o)$.

Result: $\{T_1^{1*}, \ldots, T_m^{n*}\} = arg \;\; max \;\; G$
Initialisation: $i \leftarrow 1, \quad j \leftarrow m, \quad G = G_0$;
while $i \le n$ **do**
  $G \leftarrow \max_{T_j^i}\left(h_j(T_j^i) + Ge^{-\alpha T_j^i}\right)$;
  Store optimal value T_j^{i*};
  if $j > 1$ **then**
    $j \leftarrow j - 1$;
  else
    $j \leftarrow m$;
    $i \leftarrow i + 1$;
  end
end

This T_j^{i*} maximises G_j^i as given by (11). Note that G_{j+1}^{i*} at this time is already a known constant, and represents the optimal value of the accumulated profitability of all future journeys up to an including the leg following the currently considered leg. This optimisation problem may be implemented in linear time as a search over a finite number η of leg time values within the boundaries given by (7) to any desired degree of approximation[9]. Algorithm 1 then runs with a

[9]The discretisation of T with any grid $\epsilon = 1/\eta$ would imply an upper bound ϵ on the error between the true optimum and the optimal solution calculated. For example, with $\eta = 330$, results are within 1 hour accuracy on

polynomial time complexity $O(\eta nm)$.

To aid the further analysis of the *chain effect* in Section 4, we consider also the following reformulation at the level of a journey. The profitability associated with the journey $h(T_1^i, \ldots, T_m^i)$, discounted to the start of execution of its first leg, is given by:

$$h(T_1^i, \ldots, T_m^i) = h_1(T_1^i) + h_2(T_2^i)e^{-\alpha T_1^i} + \cdots + h_m(T_m^i)e^{-\alpha(T_1^i + \cdots + T_{m-1}^i)}. \tag{13}$$

We will later use the shorthand notation $h(\Gamma_i)$ where $\Gamma_i := \{T_1^i, \ldots, T_m^i\}$.

Let $G_i(.)$ denote the accumulated profitability across journeys $i, i-1, \ldots, 1$ discounted to the start time of the journey with index i. The recursive formula can now be stated as:

$$G_i(\Gamma_i) = h(\Gamma_i) + G_{i-1}^*(T_1^{i-1*}, \ldots, T_m^{i-1*})e^{-\alpha L_m^i}, \tag{14}$$

where $G_{i-1}^*(T_1^{i-1*}, \ldots, T_m^{i-1*})$ indicates the optimal goodwill value obtained across journeys $i-1, i-2, \ldots, 1$.

3.2. Model Extension $\mathcal{P}(n, m, G_o)$: The Future Profit Potential G_0

In the last section, we have arrived at a quite generally applicable approach to represent a speed optimisation problem in class $\mathcal{P}(n, m, 0)$, with a straightforward algorithm to solve it to any desired level of accuracy. Equation (12) demonstrates that the optimal leg speed does not depend on the speeds and profitability achieved on legs that preceded this leg, but does depend on the goodwill to be realised on future legs and journeys. Recall that the goodwill G_{j+1}^{i*} in (12) is the NPV of all cash-flows associated with the ship's execution of future legs and journeys, discounted to the time that these activities start.

Consider the following extension of (8):

$$G_m^1(T_m^1) = h_m(T_m^1) + G_0 e^{-\alpha T_m^1}, \tag{15}$$

where G_o is defined as follows:

Definition: Future Profit Potential (FPP).
The Future Profit Potential is the goodwill G_o of the ship at termination time. It is the NPV of all future cash-flows the ship is deemed to be able to generate, discounted to the moment that the ship completes its n journeys that are currently considered in the optimisation problem.

For practical purposes and in examples, we will often describe the FPP using the related αG_o or Annuity Stream (AS), the value of which is expressed in USD/day.

Application of (15) will now lead to T_j^{i*} in (12) to to be influenced by G_0 through G_{j+1}^{i*}. When

---

arrival time, and correspond to specifying the ship's optimal speed with a range ± 0.02 knots. (On a journey of $8,000$ nm, $v^- = 10$ kn and $v^+ = 17$ kn).

$G_0 = 0$, we revert back to the formulation developed in Section 3.1. This approach would only be accurate, therefore, if the future profit potential is considered to be zero. This may be the case when the decision maker has chartered the ship and will return it to the owner after completion of the n journeys. Solving this model with $G_0 = 0$ implies that the charterer does not need to be concerned about what happens with the ship afterwards. There is no constraint or consideration placed on the finishing time of the last journey. Thus, the decision maker is not concerned about the exact time the ship is returned to the owner[10] other than that it will be such as to maximise the charterer's NPV across the n journeys considered.

The situation will in general be different if the decision maker is the ship owner operating the ship in the spot market. The optimal speeds for a series of planned legs currently considered will crucially depend on the profit potential that the owner considers the ship will have at the time and at the port of finishing this set of legs. This is exactly the function that G_0 can fulfill. For any value $G_0 \neq 0$, Algorithm 1 obviously still applies, but drastically different optimal speeds across the n journeys considered may be arrived at. We will further demonstrate in Section 4.4 the impact of G_0 on optimal speeds, as well as discuss how the decision maker may arrive at estimates for G_0.

3.3. Specific case $\mathcal{P}(\infty, m, -)$: Value Iteration

Imagine the situation that the decision maker knows that the future profitability will be exactly the same as today. The G_o value will then have to take on a very specific value.

We consider the problem class of a single journey $\mathcal{P}(1, m, G_o)$. We can then reformulate the journey representation (14) to consider the FPP in a similar fashion as in (15) to:

$$G_1(\Gamma_1) = h(\Gamma_1) + G_0 e^{-\alpha L_m^1}. \tag{16}$$

For any given feasible choice of the leg speeds Γ_1 producing the journey profitability $h(\Gamma_1)$, the corresponding AS value (in USD/day) earned over the journey up to the termination time is given by:

$$\mathrm{AS}(\Gamma_1) = \frac{\alpha h(\Gamma_1)}{1 - e^{-\alpha L_m^1}}. \tag{17}$$

In the special case that the future profit potential is the same as today's journey profits, we thus need the FPP αG_0, by either chance or design, to be equal to (17) for the leg speeds that would optimise (16). We thus arrive at the specific problem class $\mathcal{P}(1, m, \mathrm{AS}(\Gamma_1)/\alpha)$. For reasons further examined in Sections 4 and 5, we will see that we can call this special case also $\mathcal{P}(\infty, m, -)$. The intuition behind this is that if the future is the same as today, it will be indistinguishable from the situation that the ship will execute the current journey indefinitely and under exactly the same

[10] Obviously, the ship owner may think differently about this. In reality the owner may require an agreement about the return time, with perhaps penalties for late delivery etc. We consider here the situation of ship charterer having the power to first decide on what would be the best return time as to maximise his or her own profits, and use this result in the further negotations about redelivery time.

economic conditions.

Algorithm 2: Value Iteration to solve $\mathcal{P}(\infty, m, -)$

Result: AS^*, Γ_1^*

Initialisation: $G_0 \leftarrow 0$, $AS \leftarrow 0$, *Convergence* $\leftarrow$ *FALSE*;

while *Convergence* = *FALSE* **do**

  Call Algorithm 1 to solve: $G \leftarrow \max_{\Gamma_1}\left(h(\Gamma_1) + G_0 e^{-\alpha L_m^1}\right)$;

  Store optimal values $h(\Gamma_1^*), {L_m^1}^*$;

  if $|\frac{\alpha h(\Gamma_1^*)}{1-e^{-\alpha {L_m^1}^*}} - \alpha G_0| \leq \epsilon$ **then**

    $AS^* \leftarrow \frac{\alpha h(\Gamma_1^*)}{1-e^{-\alpha {L_m^1}^*}}$;

    Convergence $\leftarrow$ *TRUE*;

  else

    $G_0 \leftarrow \frac{h(\Gamma_1^*)}{1-e^{-\alpha {L_m^1}^*}}$;

  end

end

Algorithm 2 describes how one can find the optimal leg speeds for this special case by repeatedly calling Algorithm 1 to solve a single journey problem, each time with an improved estimate for G_o. We found Algorithm 2 in our experiments to converge very fast, often requiring less than 4 iterations to achieve an accuracy of one USD/day. The proof of the optimality of the algorithm, and the various interpretations we could give this special case, will be given in Sections 4.2 and 5.1, respectively.

4. Chain effect and Future Profit Potential in shipping: Analysis

In this section we study properties of the proposed modelling framework. We start in Section 4.1 by studying problems of the type $\mathcal{P}(n, m, 0)$. We here identify the *chain effect*, meaning that the optimal speeds on identical leg of successive repetitions will gradually shift in value. A possible solution is illustrated in Figure 2: in this case the ship is depicted to gradually slow down when values of i reach single digits.

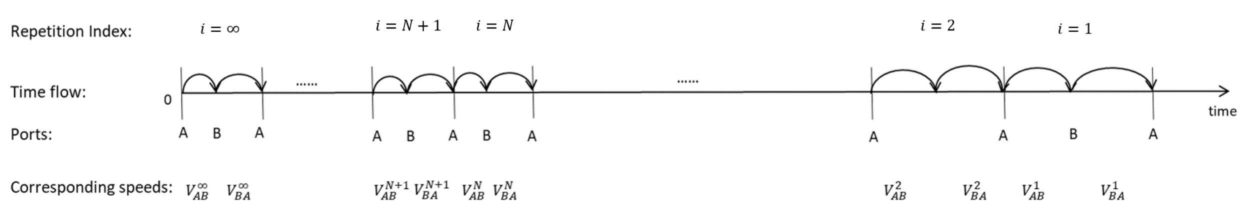


Figure 2: Optimal speeds on identical journeys

In Section 4.2 we consider taking the limit $n \rightarrow \infty$, which gives the infinite horizon case

$\mathcal{P}(\infty, m, -)$. In the final three sections, we provide more insight into the magnitude of the chain effect in $\mathcal{P}(n, m, G_o)$, the role of the FPP (G_o), and how it can be estimated.

4.1. Finite time horizon: $\mathcal{P}(n, m, 0)$ and $n < \infty$

It can be shown that if daily fixed costs f^{TCH} or future revenues R_k ($k \geq j$) in a journey are not excessively large, then the profit structure of a single leg h_j given by (6) and the profit structure of a m-leg journey h given by (13) are concave, and otherwise the optimal solution is always travelling as fast as possible (see Appendix D).

Before we present a general theorem, we provide an intuitive explanation why the decomposition principle (recall Section 1) does not apply in the NPV framework, and we instead observe a chain effect. We use for this Figure 3, which depicts the general case of a concave profit structure of a leg, and the situation that the optimal execution time T_j^{i*} of a leg (in a decomposition approach) is found within the ship's physical limits from setting the derivative $h'_j = \partial h_j(T_j^i)/\partial T_j^i$ to zero.

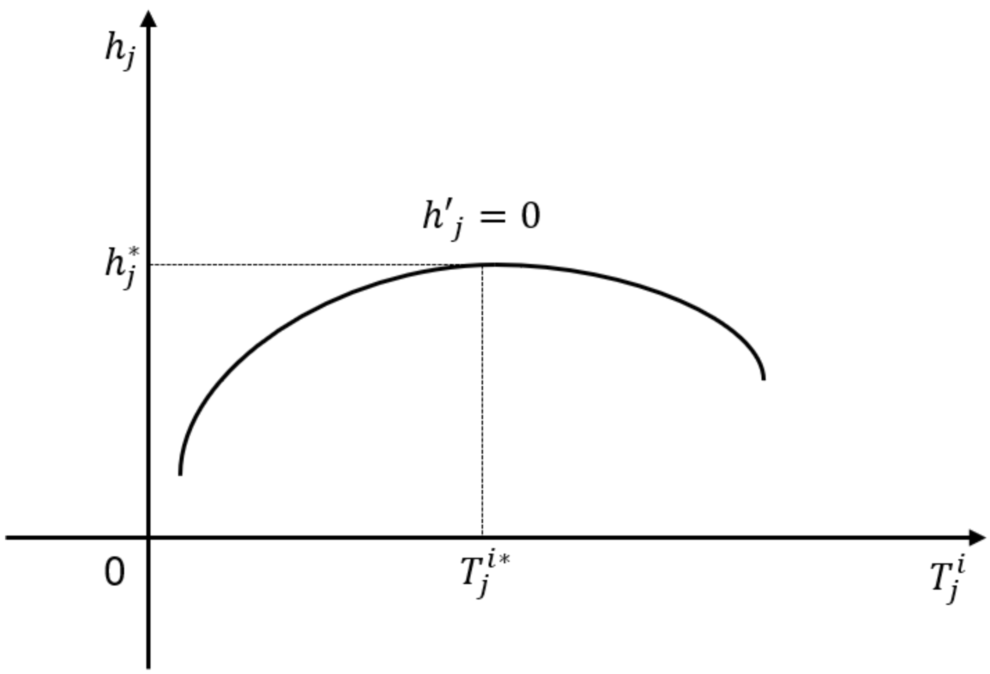


Figure 3: a general example of h_j

For $n = 2$ we can write the aggregated G_2 function:

$$G_2 = h(\Gamma_2) + G_1^* e^{-\alpha L_m^2} = h(\Gamma_2) + h(\Gamma_1^*) e^{-\alpha L_m^2}. \tag{18}$$

By taking the derivative with respect to leg $T_j^2 \in \Gamma_2$, we have:

$$\frac{\partial G_2}{\partial T_j^2} = \frac{\partial h(\Gamma_2)}{\partial T_j^2} - \alpha h(\Gamma_1^*) e^{-\alpha L_m^2}. \tag{19}$$

In the traditional models based on the *USD per journey* criterion, the decomposition principle implies that identical speeds are found for identical legs on identical roundtrips being repeated. This would imply here that $\Gamma_1^* = \Gamma_2^*$ and $\partial h(\Gamma_2)/\partial T_j^2 = \partial h(\Gamma_1)/\partial T_j^1 = 0$. This is in contradiction to (19), unless the opportunity cost of capital α is zero. With $\alpha > 0$, $h'(\Gamma_2)$ is pushed away from zero to either positive or negative values.

The decomposition principle is thus not valid. Instead, we observe a chain effect. This is intuitively understood as follows. Consider the case $m = 1$ for a journey as in Figure 1, and assume $A - B$ and $B - A$ are identical laden legs. This can then be repeated for $n > 1$. We deduce the following phenomenon: (1) if $h(.) > 0$, then $h'(\Gamma_2) > h'(\Gamma_1) = 0$ (see (19)); and thus the optimal solutions for earlier legs would be to travel faster (see Figure 3); (2) if $h(.) < 0$, then $h'(\Gamma_2) < h'(\Gamma_1) = 0$, and thus the ship would travel slower on earlier legs. The case $h > 0$ typically corresponds to a laden leg or profitable journey, while the case $h < 0$ may represent a ballast leg or repositioning journey.

In general for $\mathcal{P}(n, m, 0)$, we formulate the following theorem (proof in Appendix E.1).

Theorem 1. *(The Chain Effect) For a finite number of identical m-leg journeys indexed* $(n, n-1, ..., 2, 1)$*, when the opportunity cost of capital* $\alpha > 0$*, the following is an optimal strategy:*

- *If the journey is profitable* $(h(.) > 0)$*, the ship travels faster on earlier journeys:* $T_j^{n*} < T_j^{n-1*} < T_j^{n-2*} \cdots < T_j^{1*}$, $j = 1, ..., m$;
- *If the journey is unprofitable* $(h(.) < 0)$*, the ship travels slower on earlier journeys:* $T_j^{n*} > T_j^{n-1*} > T_j^{n-2*} \cdots > T_j^{1*}$, $j = 1, ..., m$.

Recall that we consider here the situation that optimal speeds can be found within the physical boundaries (v_{min}, v_{max}). In practice, it is of course possible to hit these boundaries. For example, if profits are very high, the optimal speed may reach v_{max} for certain j and repetitions higher than some number k: $T_j^{n*} = \ldots T_j^{k+1*} = T_j^{k*} < T_j^{k-1*} \cdots < T_j^{1*}$.

The theorem implies that we cannot let the ship travel at speeds that are optimal for the current journey (a myopic viewpoint), but have to consider the trade-off with postponing future profits. On profitable journeys, the ship travels fastest on the first journeys, and then slowly reduces speed somewhat. Repetitions of an unprofitable journey ($h < 0$) may not happen in practice unless the company would be forced to do so by e.g. contractual obligations. But if it would happen, then it is optimal to start at slow speed, and gradually increase speed. It also implies the practical consequence that one must always consider revenues, even if they are fixed, in deciding on optimal ship speeds. In traditional methods, the objective is often to minimise costs, and revenue structure is ignored, see e.g. Wang & Meng (2012).

Within each journey repetition, and when $m > 1$, legs speeds are also influenced by the profitability of later legs in the journey. For $m = 2$, for example, the optimal speed decisions for $n = 1$ are determined by:

$$\frac{\partial h(\Gamma_i)}{\partial T_2^i} = h_2'(T_2^i)e^{-\alpha T_1^i}, \tag{20}$$

$$\frac{\partial h(\Gamma_i)}{\partial T_1^i} = h_1'(T_1^i) - \alpha h_2(T_2^{i*})e^{-\alpha T_1^i}. \tag{21}$$

Optimal speeds are also pushed away from their myopic optimum by the profitability of later legs. We can thus also expect a difference in optimal speeds between the situation of a laden leg $h_1 > 0$ before a ballast leg $h_2 < 0$ and the situation where the ballast leg is first, but the difference in practice may be small (see also Example 5.10 in Section 5).

A numerical example illustrating the chain effect is given in Section 4.3.

4.2. Infinite time horizon: $\mathcal{P}(\infty, m, -)$

We consider here the case $\mathcal{P}(n, m, 0)$ for $n \to \infty$. We are thus asking what happens with the leg time sequences of Theorem 1. For example, when the journey is profitable ($h > 0$), is it possible that the leg times for higher n values reach a limit even if they are not constrained by physical boundaries (here v_{max})? This then implies that a stationary state is achieved, i.e. where the optimal speed of leg j is independent of its the repetition number. To our knowledge, no previous study has proven how this stationary (or steady) state is achieved or guaranteed.

Theorem 2. *(Convergence) If h is bounded in $\mathbb{R}$, i.e., there exists a positive number $M \in \mathbb{R}$, such that $|h| \leq M$, then there exists Γ^*, and $\Gamma^* := arg \quad max(G^* = G_n = h(\Gamma_n) + G_{n-1}e^{-\alpha L_m^n})$ with $n \to \infty$.*

Theorem 2 (proof in Appendix E.2) shows the existence of a stationary set of optimal speed decisions and a maximum *Goodwill* in the infinite time horizon. The optimal speeds obtained in finite time horizon will converge to the stationary speed decisions with the increase of journeys.

To see the link with the traditional models based on the *USD per unit of time* criterion, we extend (18) to G_{k+1}:

$$G_{k+1} = h(\Gamma_{k+1}) + G_k^* e^{-\alpha L_m^{k+1}}. \tag{22}$$

Consider a large enough number N such that for $k > N$, the set of optimal speed decisions Γ_k have converged to Γ^*, then substitute this and $G_{k+1}^* = G^* = G_k^*$ into (22) to get:

$$\alpha G^* = \alpha \frac{h(\Gamma^*)}{1 - e^{-\alpha L_m^*}}, \tag{23}$$

which proves the correctness of Algorithm 2 (Section 3.3) for solving the $n \to \infty$ case. Using the linear approximation of the Maclaurin expansion: $e^{-\alpha L_m^*} \approx 1 - \alpha L_m^*$, we further obtain:

$$\alpha G^* = \alpha \frac{h(\Gamma^*)}{1 - e^{-\alpha L_m^*}} \approx \frac{h(\Gamma^*)}{L_m^*}, \tag{24}$$

which means that a good approximation for the optimal annuity stream value αG^* (or *daily profits*) is obtained by finding the speed decisions Γ that maximise the journey profits divided by the journey time. This is an approach similar to the *income-generating leg model* of Ronen (1982)[11].

By adopting the concept of roundtrip, we know from the proof that G_i is monotone, since $h(\Gamma_i)$ is positive definite for a profitable journey and negative definite for a non-profitable journey. This property is not guaranteed if we consider the problem leg by leg, because negative T_j^i are not possible. If we consider a laden-ballast journey ($m = 2$), we get:

$$\begin{aligned} G_1^{k+1} &= h_1(T_1^{k+1}) + G_2^{k+1} e^{-\alpha T_1^{k+1}} \\ &= h_1(T_1^{k+1}) + h_2(T_2^{k+1}) e^{-\alpha T_1^{k+1}} + G_1^k e^{-\alpha (T_1^{k+1} + T_2^{k+1})}, \end{aligned} \tag{25}$$

where we are sure that for a sufficiently large N such that $k > N$, $G_1^{k+1*} = G_1^{k*} = G^*$ (but it is not guaranteed that $G_1^{k+1*} = G_2^{k+1*}$), giving as before:

$$\alpha G^* = \frac{h_1(T_1^*) + h_2(T_2^*) e^{-\alpha T_1^*}}{T_1^* + T_2^*} \neq \frac{h_1(T_1^*)}{T_1^*} + \frac{h_2(T_2^*)}{T_2^*}. \tag{26}$$

This indicates that an approach on a total journey basis is guaranteed to produce fair results, but that this is not so when optimising speed on an individual leg by leg basis, as proposed in Ronen (1982). We present a further analysis in Section 5.1.

4.3. Example of the Chain Effect in $\mathcal{P}(n, 4, 0)$

The following numerical example illustrates the chain effect in maritime shipping. The roundtrip journey of the ship consist of the sequence of port visits $A - B - C - D - A$, laden from A to B, ballast to C, laden to D, and laden to A. Further data are given in Appendix A.

Table 1: Chain effect of $\mathcal{P}(n, 4, 0)$ example; NPV (kUSD) or (*) AS (kUSD/year); and optimal speeds (knots)

n	G^*(kUSD)	V^n{AB}	V^n{BC}	V^n{CD}	V^n{DA}	V^2{AB}	V^2{BC}	V^2{CD}	V^2{DA}	V^1{AB}	V^1{BC}	V^1{CD}	V^1{DA}
1	1,645									10.9	12.6	11.9	11.5
2	3,246					11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
3	4,802	11.0	12.7	12.1	11.6	11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
4	6,316	11.1	12.8	12.1	11.7	11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
10	14,579	11.3	13.1	12.4	12.0	11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
20	25,705	11.7	13.5	12.8	12.3	11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
30	34,258	11.9	13.8	13.1	12.6	11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
40	40,864	12.1	14.1	13.3	12.8	11.0	12.7	12.0	11.6	10.9	12.6	11.9	11.5
∞	5,131*	12.7	14.8	14.0	13.5								

Table 1 reports profits and optimal leg speeds for various repetitions ($n = 1, 2, 3, ...$), obtained using Algorithm 1. The result for $n = 1$ indicates that the journey is profitable at optimal speeds ($h > 0$). In accordance to Theorem 1, we should thus observe that the ship travels faster on earlier

[11] The differences with the NPV approach is that Ronen's model considers a single leg, not a roundtrip, and does not discount the costs and revenues arising during the journey based on their relative timing of occurrence (whether they arise at the start or end of the leg, for example). The latter difference will typically produce only minor differences in speed. See also Section 5.1.

repetitions. This is confirmed by these results. For $n = 20$, for example, optimal speeds on the first few repetitions are approximately 1 knot above those on the last few repetitions.

Applying the decomposition principle of Psaraftis & Kontovas (2014) to this example, we would only need to solve[12] the case $n = 1$, and in particular we would solve the leg A to B separately to find V_{AB}^1, and the leg B to A separately to find V_{BA}^1, etc. Theorem 1 indicates that this will not produce optimal speed values and profits.

In order to illustrate the relative performance of a decomposition approach, we compare the profits, termination time (total time of all journeys), and optimal speeds for this problem in class $\mathcal{P}(n, 4, 0)$, where the optimal speeds of each repetition differ, to a corresponding problem in class $\mathcal{P}(1, 4, 0)$, where we would reuse the obtained optimal speeds for each of the other repetitions. (Note that we thus decompose the problem into separate journeys, rather than individual legs.) Table 1 illustrates that this decomposition approach provides a fairly good lower bound on the speeds when $n < 10$ (the gap is then not larger than 0.5 knots). Figure 4 plots the percentage loss in NPV and percentage increase in termination time of the decomposition method relative to the optimal solution. Here the differences remain again very small when $n < 10$. If n is large (or, by extension, if m is large), then the results in Table 1 indicate that optimal speeds should rise instead by more than 1.5 knots, and Figure 4 shows the gap in profitability increasing to over 7%.

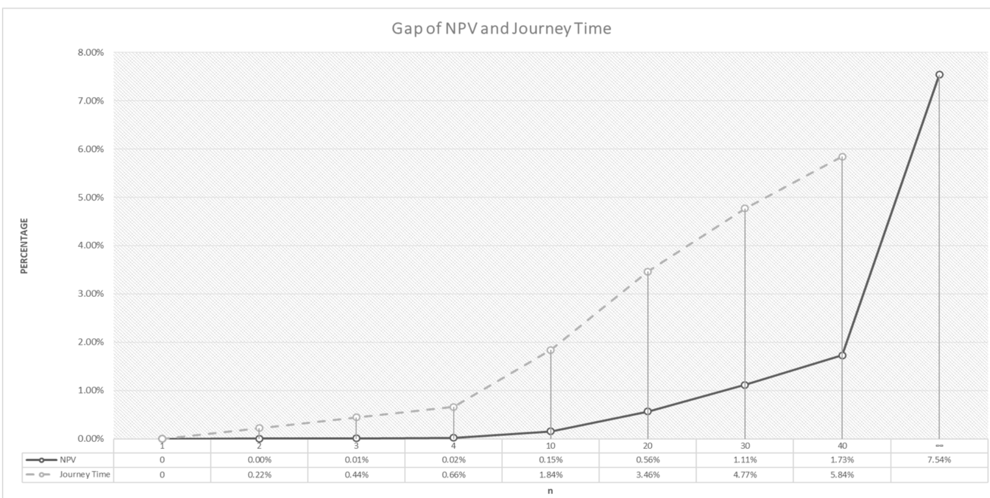


Figure 4: Optimility gaps of NPV and termination time if adopting decomposition (x-axis plots n)

We have also seen in Theorem 2 that when n values increase to large numbers, optimal speeds converge. The results in Table 1 confirm this, and also show that in the limit for $n \to \infty$ (solved with Algorithm 2), these speeds are about 2 knots above the optimal speeds when $n = 1$, yet the convergence is slow as can be observed from comparison with the results for $n = 10, 20, 30$ and 40.

When Ronen (1982)'s *income-generating-leg model* (see Introduction, and also Section 5.1) is applied to the example, we would for $n = 1$ thus proceed exactly as before: solve the leg A to B

[12] When $n > 1$, if the decomposition principle would apply, it would lead us to conclude that $V_{AB}^n = \cdots = V_{AB}^1$ and $V_{BA}^n = \cdots = V_{BA}^1$, and thus that we could restrict our attention to cases where $n = 1$.

separately to find V_{AB}^1, and the leg B to A separately to find V_{BA}^1, etc. Applied to any repetition $n > 1$, we would find that $V_{AB}^N = \cdots = V_{AB}^1$ and $V_{BA}^N = \cdots = V_{BA}^1$, etc. Because in Ronen's model the criterion is *USD per unit time*, optimal speeds in this approach, however, correspond more closely to the speed values obtained in Table 1 for the case $n \to \infty$ (see Section 4.2 for an intuitive explanation, and Section 5.1 for a deeper explanation).

For intermediate n values in the range $10, ..., 100$, neither of the traditional methods adopting the decomposition principle will produce good approximations of the NPV results in this example. Thus it is not valid to apply this principle in the NPV models. This is a consequence of the chain effect.

4.4. Example of the impact of G_0 in $\mathcal{P}(1, 4, G_o)$

In the problem class $\mathcal{P}(1, m, G_o)$, the profit function (16):

$$G_1(\Gamma_1) = h(\Gamma_1) + G_0 e^{-\alpha L_m^1}, \tag{27}$$

requires an estimate for G_o.

If we solve this problem with Algorithm 2, however, we don't need to provide a value for G_o as it will be the value that maximises the expression (17). This, as seen in Section 4.2, corresponds to the solution in the steady state, having the G_o value, see (24):

$$\hat{G}^* = \frac{h(\Gamma_1^*)}{1 - e^{-\alpha L_m^1{}^*}}. \tag{28}$$

In the steady state, the daily profits that can be made in the future equal those made today, at optimal speeds to maximise the NPV of all future profits.

Notice that G_o in (16) fulfills a similar role as $h(\Gamma_1^*)$ in (18); positive G_o values will thus according to the same mechanism that led to Theorem 1 push speeds on the journey to higher values, while negative values will reduce these speeds. We can analyse the effects of different estimates by taking:

$$G_0 = \beta \hat{G}^*, \tag{29}$$

and varying the value of β.

Table 2 summarises the optimal speed decisions and the corresponding journey time with different G_0 values. We use the same settings as for the example presented in Section 4.3. For $\beta = 1$, we find the steady state solution, which corresponds to the result for the $n = \infty$ entry in Table 1. At $\beta = 0$, the solution is the same as for the $n = 1$ case in Table 1. These two solutions correspond to the traditional models based on the criteria *USD per unit time* models and *USD per trip*, respectively. At all different values for β, the ship's optimal speeds will be different.

Figure 5 illustrates the trade-off that is considered in the optimisation of the NPV criterion between the FPP and the current journey's profits. When the FPP is zero ($\beta = 0$), the decision

Table 2: The impact of $G_0 = \beta\hat{G}^*$ on optimal speeds for $\mathcal{P}(1, 4, G_o)$

β	**Duration (Days)**	V^1**{AB}**	V^1**{BC}**	V^1**{CD}**	V^1**{DA}**
-1	147.1	10.0	10.0	10.0	10.0
-0.5	141.2	10.0	11.1	10.6	10.2
0[(1)]	127.6	10.9	12.6	11.9	11.5
0.5	118.2	11.9	13.8	13.0	12.6
1[(2)]	111.2	12.7	14.8	14.0	13.5
1.5	105.9	13.4	15.7	14.8	14.2

(1) Equivalent to classic approach using criterion *USD per trip.*

(2) Appromixating classic approach using criterion *USD per unit of time.*

maker does not consider the ship to have any value after the completion of the current journey. This corresponds to choosing ship speeds that will maximise the current journey's NPV. With positive or negative FPP values, the ship speeds on the current journeys adjust accordingly, which will reduce the NPV of the current journey. In particular for positive FPP values, we observe a dramatic decrease in NPV. In steady state ($\beta = 1$), for example, the current journey's profits are reduced by nearly 7%. Profit-seeking decision makers are thus willing to sacrifice current profits for longer-term benefit. At negative FPP values, the decrease is less dramatic, since the ship will soon hit the v_{min} boundary (see also Table 2), at which point the NPV will remain constant.

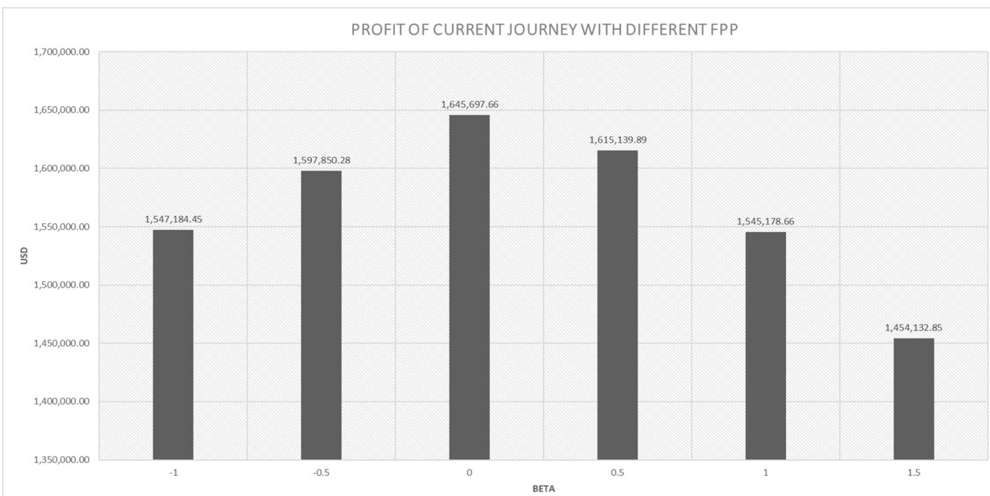


Figure 5: Profitability of the current journey (NPV value, in USD) for different $G_0 = \beta\hat{G}^*$ levels

4.5. Estimating G_o

Industry reports estimates of current profitability for various ship types, routes, and contract types. These are based on averages obtained by brokers or other agencies. The *Time Charter Equivalent* (TCE) earnings, in particular, represent the average daily revenue performance that companies can use to establish insight into the evolution of a particular market.

The calculation of the TCE for one particular journey is based on taking the freight revenue received on the journey (e.g. a ballast-laden trip), subtracting the variable journey costs (mainly port and fuel costs), and dividing by the total number of days of the journey. Note that this value does not account for the fixed daily running costs of the ship.

According to the definition of FPP in Section 3.2, αG_0 represents the daily net profit (=daily revenues - daily operational costs - daily fixed costs) expected in the future. A simple approach that the ship operator as decision maker could use therefore consists of using this industry index value TCE as presented in (29):

$$G_o = \beta \hat{G}^* = \frac{\beta}{\alpha}\big(\mathrm{TCE} - f_F^{TCH}\big), \tag{30}$$

where f_F^{TCH} is the future value of the daily fixed costs, a value which is typically private information to the ship operator. The α value corresponds to the opportunity cost of capital of the decision maker, while β represents the decision maker's level of optimism or pessimism with respect to the future evolution of this market.

Shipping companies would use these TCE values cautiously since it can highly depend on underlying assumptions such as the journey structure considered. (See also e.g. Example 5.1. in Section 5.1.) Alternative forecasts provided by either brokers or based on in-house calculations of past performance can be adapted in the same manner.

5. Comparison with models from the literature

In this section we show how the models and algorithms presented in Sections 2 and 3 can help in assessing the applicability of a few important ship speed optimisation models presented in the literature. To this end, we use the technique of NPV Equivalence Analysis (NPVEA) to identify under which assumptions the classic method can produce results that are also (close to) optimal from the NPV perspective. For more information about NPVEA, please see Section 6.3.

5.1. Model I of Ronen (1982): Income generating leg

Model I in Ronen (1982) is an analytical model for finding the speed of the ship when executing a laden leg which will maximise the net profit associated with this leg. While readers are encouraged to consult the original article for details, a representation of the model is given in (31), but using our notation for a leg j, and replacing Ronen's propeller law by a fuel consumption function $F(v_j, w_j)$.

$$\max\left\{ Z_j = \frac{R_j - C_j^h - C_j^f F(v_j, w_j)\frac{S_j}{24 v_j} - C_j^u - f^{TCH} T_j^i}{T_j^i} = \frac{R_j - C_j^l - C_j^u - f^{TCH} T_j^i}{T_j^i} \right\}. \tag{31}$$

NPVEA's main question is the following: When Ronen's income generating leg model (31) is used to solve a ship speed decision problem, what kind of assumptions are needed if this approach is to produce results in line with the optimisation of the NPV of the ship's activities?

The profits of one arbitrary leg j in $\mathcal{P}(n, m, G_o)$ are given by (6). This expresses the NPV of net earnings discounted to the start of loading operations for this leg. We assumed that unloading operations and revenues were received at the time of completion of this leg. We can consider arbitrary modifications to the timing of payments, for example as follows:

$$h(T_j, \Delta^u, \Delta^l) = (R_j - C_j^u)e^{-\alpha(T_j+\Delta^u)} - C_j^l e^{-\alpha\Delta^l} - \int_0^{T_j} f^{TCH} e^{-\alpha T} dt, \tag{32}$$

where values $\Delta^u > 0$ and $\Delta^l > 0$ would indicate further delays in time of cash-flows associated with the unloading and loading operations, respectively.

We now consider an infinite repetition of this profit generating leg as a problem in $\mathcal{P}(\infty, 1, -)$. An infinite repetition produces an infinite chain. We have seen in Section 4.2 that in $\mathcal{P}(\infty, m, -)$ the chain effect is not observable; optimal leg durations of each leg repetition are equal. This therefore certainly applies in $\mathcal{P}(\infty, 1, -)$. The corresponding Annuity Stream (AS) value achieved (dropping the index j) is then given by the equation:

$$\text{AS} = h(T, \Delta^u, \Delta^l) \sum_{i=0}^{\infty} \alpha e^{-i\alpha T}. \tag{33}$$

The leg duration T that maximises this expression is optimal from the NPV perspective. Ronen's Model I, instead, considers as objective function the profits of the single leg divided by its duration.

Working out (33), substituting (32):

$$\text{AS} = \Big[(R - C^u)e^{-\alpha(T+\Delta^u)} - C^l e^{-\alpha\Delta^l} - \frac{f^{TCH}(1 - e^{-\alpha T})}{\alpha}\Big] \frac{\alpha}{1 - e^{-\alpha T}}. \tag{34}$$

Maclaurin expansion of the exponential factors, and retaining only constant and linear order terms in α gives:

$$\text{AS} \approx (R - C^u)\Big(\frac{1}{T} - \frac{\alpha}{2} - \frac{\alpha\Delta^u}{T}\Big) - C^l\Big(\frac{1}{T} + \frac{\alpha}{2} - \frac{\alpha\Delta^l}{T}\Big) - f^{TCH},$$

which reduces to:

$$= \frac{1}{T}\Big[R - C^u - C^l - f^{TCH}T\Big], \tag{35}$$

when $\Delta^u = -T/2$ and $\Delta^l = +T/2$. Recalling (2) and (5), it is easily verified that (35) is equivalent to (31), which gives the objective function of Ronen's Model I (equation (6) in that paper.) We thus get:

Lemma 3. *(Equivalence of Model I) (a) The objective function of Model I is a linear approximation of (33) when $\Delta^u = -T/2$ and $\Delta^l = +T/2$. (b) Solving Model I is equivalent (in approximation) to solving the infinite repetition on the leg in $\mathcal{P}(\infty, 1, -)$, i.e. maximising the AS of an infinite repetition of this leg under identical economic conditions; and such that all cash-flows associated with a leg are exchanged when the ship is halfway in leg duration.*

Because of the infinite repetition of the leg in this interpretation of Model I, it is difficult to assume other conditions but that the leg, somehow, would be a *roundtrip* journey all by itself. The single leg assumption explicitly made in Ronen is not compatible with this result. The result is, however, in alignment with Ronen's own comment that 'it is common in the shipping industry to calculate profitability on a roundtrip basis'. To make Ronen's model more practical can thus be done by replacing the single leg by a roundtrip. We can rewrite (33) using the journey profit expression (13):

$$h(T_1^i, \ldots, T_m^i) \sum_{i=0}^{\infty} \alpha e^{-\alpha i L_m}, \tag{36}$$

with the understanding that we now need to seek m optimal leg durations (or leg speeds), which are in general all different. The implications of this are profound, as illustrated by the next example.

Example 5.1: *Model I: Single leg vs Roundtrip.*

We use the example of a Suezmax with a speed range of 10-17 kn (see Appendix B for full data). The scenarios #1 and #2 examined in this example are listed in Table 3. Scenario #1 considers a single profitable leg, as Ronen intended. In #2 the ship must return in ballast in order to make it a roundtrip; an assumption in line with Lemma 3 (b). To calculate the optimal speeds in both scenarios, we use the NPV method and Algorithm 2 (infinite repetition of the journey), since the equivalence results in Lemma 3 (b) indicate this to be a fair representation of the solutions obtained with Ronen's Model I.

The optimal solution in #1 is to travel at full speed of 17 knots. In #2, the ship travels at 13.61 knots on the laden leg, and at 15.91 knots on the ballast leg. In #1 the ship makes much larger profits per day, because no days have to be accounted for that the ship travels in ballast. The question is not in which scenario the profits per day are the highest, of course, but how plausible each scenario is. In scenario #1, the ship would have to travel back to the origin port at infinite speed and zero costs in order to start the next repetition. In #2, we do not face such an impossible requirement. □

Table 3: Evaluating Ronen (1982) (I^1 and II^1: modified to multiple legs; (*) i.e. $C_a = 42,968$, see (44))

#	Model	Journey	αG_o (USD/day)	**Journey (USD/day)**	**Days at sea**	**Speed (knots)**
1	I	Laden	77,340	77,340	20.32	17.0
2	I^1	Laden; Ballast	12,968	12,968	25.40; 21.72	13.61; 15.91
3	II	Ballast	12,968 (*)	-65,022	21.72	15.91
4	II	Ballast	2,000	-58,976	23.81	14.52
5	II^1	Laden, Ballast	2,000	12,529	27.71; 23.81	12.47; 14.52
6	II^1	Laden, Ballast	20,000	12,826	24.26; 20.71	14.25; 16.68
7	II	Ballast	20,000	-68,787	20.71	16.68

Ships do not necessarily make roundtrips, however. This indeed was part of Ronen's motivation to contruct separate single leg models. He states: 'ships do (often) not return to the same origin to begin the next trip'. Is there an alternative interpretation of Model I that does not need to rely on the infinite repetition of a roundtrip journey? Let us thus return to the single leg interpretation, and rewrite (33) as follows:

$$\text{AS} = \alpha h(T, \Delta^u, \Delta^l) + h(T, \Delta^u, \Delta^l) \sum_{i=1}^{\infty} \alpha e^{-\alpha i T}$$

$$\text{AS} = \alpha h(T, \Delta^u, \Delta^l) + h(T, \Delta^u, \Delta^l) e^{-\alpha T} \sum_{i=0}^{\infty} \alpha e^{-\alpha i T}$$

$$\text{AS} = \alpha h(T, \Delta^u, \Delta^l) + \frac{\alpha h(T, \Delta^u, \Delta^l)}{1 - e^{-\alpha T}} e^{-\alpha T} \tag{37}$$

$$\text{NPV} = \frac{\text{AS}}{\alpha} = h(T, \Delta^u, \Delta^l) + G_o e^{-\alpha T} \tag{38}$$

Maximising (38) will maximise (33), and thus the equivalence with Model I, as given in Lemma 3 (a), still holds. Equation (38) can be recognised as the dynamic programming approach developed in Section 2, where G_0 represents the future profit potential. Because in this case it must be, for the equivalence to hold, equal to the specific value as seen in (37), we can use Algorithm 2 to solve situations as considered in Ronen's Model I. (Alternatively, we can intuitively deduce this from Ronen's (implicitly adopted) steady state assumption and results of Section 4.2.)

More importantly in this context of how to interpret Model I, we see from the form of (37) that G_o can be interpreted as the NPV of an Annuity Stream (AS) value *in size* equal to the optimal value of the AS function of the single leg under consideration. We are thus solving a problem in class $\mathcal{P}(1, 1, G_o')$, where G_o' refers to being a very specific value. Solving Ronen's Model I can thus be alternatively achieved by maximising (38) if we can correctly verify that the value of the future profit potential would be:

$$G_o' = G_o(T^*) = \frac{h(T^*, \Delta^u, \Delta^l)}{1 - e^{-\alpha T^*}}. \tag{39}$$

This means that Ronen's Model I would also be applicable when the future profit potential (expressed in USD per day) is in size equal to the profits per day of the current single leg, *while these future profits however may be reached undertaking other types of (non-)roundtrip journeys and under different economic circumstances*:

Lemma 4. *(Equivalence of Model I) (c) The objective function of Model I is a linear approximation of (38) when* $G_0 = \frac{h(T, \Delta^u, \Delta^l)}{1 - e^{-\alpha T}}$*, which can be solved by Algorithm 2; (d) Solving Model I is equivalent (in approximation) to solving a problem in* $\mathcal{P}(1, 1, G_o(T^*))$*, i.e. maximising the Goodwill of the current leg when the future profit potential equals the specific value as given in (39).*

This would thus seem to allow for Ronen's argument in support of the single leg model for situations when the ship executes e.g. tramp operations. The situation, however, is very unlikely corresponding to reality. Indeed, adopting the assumption of a future producing *exactly the same* daily value as in the optimal solution of the current single laden leg means that the future would have to consist of profit generating legs that are *more* profitable per day than the current leg, given that it is not unlikely that the ship may have to incorporate ballast or repositioning legs in future operations, which are not profitable by themselves.

In conclusion, through NPVEA we have offered two possible interpretations of Ronen's Model I. As explained in the above paragraph, it seems fair to conclude that the first interpretation (Lemma 3) is much more plausible and acceptable. This means that we should consider replacing the "leg" with a roundtrip journey of two or more legs, and assume this roundtrip is infinitely repeated under the same economic and operational conditions.

5.2. Model II of Ronen (1982): Positioning (empty) leg

We check under which conditions there is equivalence of Ronen's Model II with our approach. This also provides an interpretation for a parameter in Ronen's model termed the *Alternative Daily Value* of the ship, named C_a in that paper. For ease of reference, Model II in our own notation for the other parameters and variables, and for such a repositioning leg j, is given by (40):

$$\min\left\{Z_j = C_a\frac{S_j}{24v_j} + c_j^f F(v_j, w_j)\frac{S_j}{24v_j} = C_a T_j^s + c_j^f F(v_j, w_j) T_j^s\right\}. \tag{40}$$

Model II considers a single leg that repositions the ship, i.e. it carries no cargo. In the NPV framework, we can consider a problem in $\mathcal{P}(1, 1, G_o)$ with profits of the leg in general given by (32). Since the ship carries no cargo, we set the revenues R and cargo unloading costs C^u to zero. In addition, after the execution of this journey, the future profit potential can be realised. This gives the objective function:

$$G = -C^l e^{-\alpha\Delta^l} - \frac{f^{TCH}}{\alpha}(1 - e^{-\alpha T}) + G_o e^{-\alpha T}, \tag{41}$$

of which the linear approximation is equal to:

$$G \approx -C^l(1 - \alpha\Delta^l) - (\alpha G_o + f^{TCH})T + G_0. \tag{42}$$

Constant terms can be dropped for the purpose of determining optimal ship speed v. Only T (through T^s, the time at sea) and C^l (through the bunker fuel consumption function) are functions of the ship speed. Using (2) and (5), maximising (42) thus equals minimising:

$$c^f F(v, w) T^s(1 - \alpha\Delta^l) + (\alpha G_o + f^{TCH})T^s. \tag{43}$$

It can be verified that (43) is equivalent to the objective function (40) of Ronen's Model II (equation

(8) in that paper) for $\Delta^l = 0$ and when Ronen's *Daily Alternative Value* is set to:

$$C_a = \alpha G_o + f^{TCH}. \tag{44}$$

Ronen's C_a parameter equals the annuity stream value of the goodwill of *future* profits after finishing this leg, plus the *current* daily charter hire value (or the daily fixed costs) that is due for every day the ship executes this leg.

Lemma 5. *(Equivalence of Model II) (a) The objective function of Model II is a linear approximation of the speed-dependent terms in (41) when $\Delta^l = 0$ and Ronen's Alternative Daily Value of the ship is set as in (44). (b) Solving Model II is equivalent (in approximation) to solving a problem in $\mathcal{P}(1, 1, G_o)$, i.e. maximising Goodwill of the current leg using (41) when $G_o = (C_a - f^{TCH})/\alpha$.*

Example 5.2: *Model II and C_a.*

In this example we focus on scenarios #3 and #4 of Table 3. In #3 we consider a single ballast leg for repositioning the ship and assume a value $C_a = 42,968$ USD/day. With a current hire cost $f^{TCH} = 30,000$ USD/day, this corresponds to a FPP of $\alpha G_o = 12,968$ USD/day, see (44). The optimal repositioning speed in #3 is 15.91 knots. This is the same as the optimal ballast speed in #2 because the ship faces at the start of either the ballast leg in Model I, or the repositioning leg in Model II, the same future profit potential. If the future looks less promising, as in #4, then optimal repositioning speed is also lower at 14.52 knots and the ship spends about 2 extra days at sea, saving on the other hand also about 6,000 USD/day in costs during the repositioning journey. □

In the development of Model II, Ronen did not include the daily costs into the objective function, unlike what he did for Model I. This may seem to find support from intuition, since Model II investigates the marginal cost of an extra day sailing on a repositioning (non-earning) leg. In other words, it examines the trade-off about how slower steaming saves fuel on the one hand, but also means loosing out on realising the alternative value on the other hand, and during this time the ship's daily costs are obviously the same. At first sight, it may thus seem counterintuitive to have to add the daily fixed cost f^{TCH} of the current leg into the alternative daily value C_a, as indicated by (44), in order for this method to be leading to optimal decision making in the NPV framework. It is nevertheless correct, given the above mathematical logic leading up to (44). The reason is that the ship's daily costs are not necessarily remaining the same in the above trade-off as we need to compare *current* with *future* operations. A further clarification may help, and goes as follows. Goodwill G_o is based on net future profits, and thus the daily future profits are based on:

$$\alpha G_o = \textit{Future Revenues - Future Operational Costs - Future Fixed Cost}$$

The operational costs refer to fuel and port costs mainly, while the fixed costs represent either the time charter hire or the costs to cover crew, maintenance, lubricants, insurance, etc. Equation (44) can then be written as:

$$C_a = \textit{Future Revenues - Future Operational Costs - (Future Fixed Cost - Current Fixed Cost)}.$$

Let $\triangle = \textit{Future Fixed Costs} - \textit{Current Fixed Costs}$. If the future and current fixed costs (per day) are equal, which may be the case if the ship owner operates in the spot market, C_a would be a measure for the operational profits that can be realised (before fixed costs). In that case, C_a matches the *Time Charter Equivalent* (TCE) (see also Section 4.5). If future and current fixed costs per day are *not equal*, however, one should not forget to correct for this, i.e. $C_a = \text{TCE} - \triangle$. This situation may arise, for example, if the current (charter) contract ends after completing this leg, and the daily hire in the future changes.

In conclusion, Ronen's Model II did not include the daily hire cost; and we have given also an intuitive explanation why this may seem sufficient. However, the NPVEA method has given us a proof that this is incorrect in the case of future hire cost being different, unless if we correctly account for this difference. The practical formula (44) derived from NPVEA can ensure that Ronen's Model II will be applicable in all cases, however, as it captures both eventualities automatically.

5.3. Ronen's switching strategy between Models I and II

Ronen (1982) recommends to switch between Model I and II to make the most profitable decision. In particular, if the maximum daily profit achieved on the profit generating leg using Model I is less than the daily alternative value C_a used in Model II, then it is advised to switch to Model II. We interpret this that in that case, the ship's next leg will not be a profit generating leg, but becomes a repositioning leg instead. Let Z^* indicate the optimal solution from Model I, then Ronen's advice is thus to use Model I when $Z^* > C_a$, and switch to Model II otherwise.

Given Lemma's 3 and 5, and (44), however, we must make the following adaptation to this rule if we want to make the NPV-optimal decision: Use Model I when:

$$Z^* \geq C_a - f^{TCH}, \tag{45}$$

where f^{TCH} reflects the *current* daily costs on the current leg, and check with Model II otherwise, *and take the scenario with best overall NPV.* Example 5.3 illustrates.

Example 5.3 *Switching Model I to Model II: Ronen's rule vs updated rule.*

Consider that the ship currently executes operations at profits of $12,968$ USD/day as in scenario #2 (Table 3), but it could potentially make the switch to #3. Current daily hire is $30,000$ USD/day, and the future daily hire in #3 will be $35,000$ USD/day, but with a Time Charter Equivalent value of $47,968$ USD/day. We then have: $C_a = 47,968 - 35,000 + 30,000 = 42,968$ USD/day. Note that $Z^* << C_a$, so according to Ronen's rule we should switch to undertake the repositioning leg instead, i.e. adopt #3. The updated rule given by (45), however, gives an equality. In addition, while the future in #3 promises equal daily value as in #2 at $12,968$ USD/day, the fact that we have a repositioning journey to make first makes its NPV lower (at only $12,580$ USD/day). We should thus definitely stay in scenario #2. □

Ronen's models do not capture all possible 'switching' situations. In fact, the following example is not uncommon: It may be that in order to reach the area of the world in which the ship could realise C_a, the ship could travel along a profit generating route instead of a revenue-less repositioning leg. We would thus need to extend Model II (see also the example below) to a situation in which as part of the repositioning strategy, the ship may first make a laden leg, and then a ballast leg as to arrive at the area in which C_a can be realised.

Note that using Model I for this (once used) laden leg journey, in combination with the switching rule, as Ronen suggested, would not lead us to the correct decisions. Indeed, Lemma 3 and 4 show that Model I would work under the assumption that the economic conditions experienced during this laden, ballast journey would also be there in the future. Not only would then optimal speeds obtained from Model I be wrong, one would not be able to estimate the actual future profits (or losses) the ship could make. The following example illustrates.

Example 5.4 *Extending Model II with profit generating legs.*

Consider scenarios #4 and #5 in Table 3, and assume the ship is currently idle. In #4 the ship will face a loss of $-58,976$ USD/day for almost 24 days during a single repositioning leg, before arriving at a situation where it can make $2,000$ USD/day. In #5, the ship first executes a laden leg and then travels in ballast to the area of future profits. It takes the ship almost 28 days longer to arrive there, but it will make a profit of $12,529$ USD/day on the way. Here, #5 is clearly superior to #4, because the current daily earnings are higher than in the future. Note that the speeds obtained in #5 are optimal as to maximise the NPV of future profits of the ship. If we would follow Ronen and have used #1 for deciding on the laden leg as in Model I, it would travel too fast (as it implicitly assumes a much more optimistic future in this case). □

The above examples make clear that the class of models $\mathcal{P}(n, m, G_o)$ offers increased flexibility and allows for the consideration of multiple possible scenarios for the ship to balance current operations with future profit potential. When using Ronen's models, instead, a decision maker needs to think about which model to use, how to correctly apply which switching rule. In addition, when using USD per unit of time as a criterion, it becomes potentially problematic comparing different scenarios, since the profits per unit of time earned today can be different from those earned tomorrow. The next example illustrates this issue. When adopting the NPV modelling approach instead, all scenarios deemed possible can be modelled similarly and then compared not based on switching rules, but simply based on which scenario has the highest NPV[13].

[13]While in investment analysis, care should be given when comparing investments of different duration, see e.g. Brealey & Myers (2003) (p.132), models in class $\mathcal{P}(n, m, G_o)$ do not suffer from such complications because they, per definition, should account for all relevant future plans. If $G_o = 0$, for example, you do account for the fact that future profitability is not there, and thus the termination time can be freely chosen to help optimise the NPV of the

Example 5.5 *NPV as the ultimate arbiter.*

Consider scenarios #6 and #7 in Table 3. These are equal to #5 and #4, respectively, save for the fact that the future profit potential is 20,000 USD/day. In #6 we make profits per day during the laden, ballast repositioning journey, but it takes us about 25 days longer to get to a future where we could earn even much more per day. In #7, we make a loss during the repositioning leg but we'll be 25 days earlier to the promising future of high earnings. It is now not immediately clear from these numbers reported in the table which of these scenarios is the best. However, as our method is based on maximising the NPV (or AS) of the ship's future activities, it can be easily decided which scenario is best by looking at the NPV (or AS) values and retaining the scenario with the highest value. The AS of #6 is 7,270,608 USD/year and of #7 is 7,146,125 USD/year, and thus the laden journey is worth doing. □

We can formulate the following conclusions. First, we demonstrated that the application of Ronen's models may lead to erroneous decisions if the underlying assumptions are not well understood. With NPVEA we have been able to shed light on these assumptions, as well as provided a correct formula to help calculate Ronen's Daily Alternative Value (C_a) parameter. In particular, we derived that it will have to account for any differences in daily hire values between the current repositioning journey and the future. We have also demonstrated with examples that in order to make better decisions, the shipping company should extend the models of Ronen to include multiple legs. Because of the equivalence results we have obtained with our modelling framework, Ronen's Models I and II can both be solved with our model (and Algorithms 2 and 1, respectively), guaranteeing the optimality according to the NPV criterion, while at the same time it easily allows for extending these models to multi-leg journeys and journey repetition. Finally, our modelling approach can deal with a wide range of future profit potential values, which allows it to consider many more realistic situations. Not accounting for the future profit potential leads to erroneous speed decisions and profit values that will not optimise the ship's net present value.

5.4. Fagerholt and Psaraftis (2015): optimal speeds in Emission Control Areas

In this section, we consider the problem presented in Fagerholt & Psaraftis (2015). The study concerns the situation in which a ship has to travel through an Emission Control Area (ECA), introduced by the International Convention for the Prevention of Pollution from Ships (MARPOL). As a consequence, vessels have to burn cleaner but more expensive marine gas oil (MGO) when travelling within an ECA, while they can use heavy fuel oil (HFO) outside of such areas. Fagerholt & Psaraftis (2015) have built their model based on Ronen (1982), Model I, assuming a ship travelling a single journey starting outside and crossing into an ECA zone to end in the destination port, see

current journeys.

Table 4: Instance characteristics: Cases 2 to 10 are alternative plausible scenarios in the setting considered in Fagerholt & Psaraftis (2015)

#	Repetitions (n)	Journey Structure	FPP (USD/day)
1	∞	Laden	-
2	∞	Laden-Ballast	-
3	1	Laden	0
4	1	Laden-Ballast	0
5	1	Laden	-10,000
6	1	Laden-Ballast	-10,000
7	1	Laden	10,000
8	1	Laden-Ballast	10,000
9	∞	Ballast-Laden	-
10	1	Ballast-Laden	0

model (46).

$$\max\left\{Z = \frac{R - c^f_{HFO}F(v_{outside}, w_{outside})T_{outside} - c^f_{MGO}F(v_{inside}, w_{inside})T_{inside}}{T_{outside} + T_{inside}}\right\}. \tag{46}$$

In (46), we have used similar notation as in our paper, but have added subscripts to indicate two different fuel consumption costs in the numerator, associated with the travel outside and inside the ECA, respectively. The time spend on these two parts of the journey is $T_{outside}$ and T_{inside}, respectively. We illustrate how one could arrive at very different conclusions about optimal speeds, and thus also environmental impact, depending on the choice of model.

Within our modelling framework of Section 3, this type of problem can be viewed as a journey between three ports A, B and C, where the leg $A - B$ is outside of the ECA, and the leg $B - C$ is inside of the ECA, and B is a virtual port with zero residence time and costs, but where the fuel is switched. The impact of ECAs can be denoted by assuming a different fuel price for each leg, where HFO is normally much cheaper than MGO.

In the following examples, we follow a ship with similar characteristics as in Fagerholt & Psaraftis (2015), including a speed range of 15-21 knots. Due to confidentiality, the full data used in the original study could not be obtained. We thus have used our own data for undisclosed parameters, see Appendix C. Table 4 provides key data of ten different journey situations. In the first eight cases, the ship travels from A outside an ECA zone into an ECA zone laden to deliver in port C. In odd case numbers 1, 3, 5, and 7 there is no subsequent route out of C explicitly modelled. This corresponds in our model to a journey A-B-C ($m = 2$). In the even cases 2, 4, 6 and 8, a ballast return journey is added so as to get back out of the zone. This is modelled as the journey A-B-C-B-A ($m = 4$).

Example 5.6 *Using Ronen's Model I with two different journey structures.*

We compare the first two cases of Table 4. Case 1 replicates the model used in Fagerholt &

Psaraftis (2015): a single income generating model without the consideration of a return journey. In Case 2, a ballast return journey is added. Both cases, as in all examples in this paper, are modelled using our framework. In the spirit of Model I, they are solved using Algorithm 2 (see Lemma 4).

Figure 6 illustrates how optimal leg speeds inside and outside the ECA vary with the fuel price (taxation) ratio MGO/HFO, where we keep the HFO price fixed. An increase in this ratio may, for example, be induced through taxation. In Case 1 of a single laden journey, the ship's speed inside the ECA is reduced from the maximum 21 knots down to 18.4 knots with increasing MGO/HFO, but outside the ECA the ship maintains maximum speed. At MGO/HFO = 2, the ship would make 44,319 USD/day, and it would be worthwhile for the ship to not slow down at all.

Case 2, however, shows much greater sensitivity to the (taxation) ratio. Most importantly, the speed inside the ECA on the laden leg is drastically reduced: at a ratio of 2.5 the speed is about 4.5 knots below the optimal speed in Case 1. At ratios above 3, also the laden leg speed outside the ECA would start to decrease. At MGO/HFO = 2, the ship makes only 8,278 USD/day, much lower than the profits in Case 1. The reason is that Case 2 also considers the ballast leg from C back to A, which reduces the average daily profits, and also greatly tempers optimal ship speeds.

The question here is not in which case the ship is most profitable but, as in Example 5.1, how plausible each scenario is. In both cases, since we use Ronen Model I's criterion of maximising USD/day, we must implicitly assume that the ship continues these activities with the same daily profits (see Lemma 3 and 4). In Case 1, this is difficult to imagine: even if the ship would have also a laden leg from C back to A, the fact that the ship first needs to go through the ECA zone makes this not identical to the A-B-C laden leg, and thus the laden leg C-B-A would need to have a very specific revenue value as to counteract this (see also Lemma 4). In Case 2, the journey is a roundtrip so we don't need to ensure specific relationships between revenue and fuel cost parameters. □

Example 5.6 also illustrates (see Figure 6) how difficult it may become to determine quantitatively how taxation on bunker fuel prices may affect emissions, since optimal speeds both inside and outside the ECA highly depend on the case considered. We think that, at best, only a qualitative effect may be predictable. This point will be further strengthened by considering the next cases.

In Case 1 and 2, the ship is assumed to continue doing activities in the future producing the same USD/day value (or thus as if the journey is infinitely repeated in the same economic conditions.) An alternative situation arises if we imagine the journey A-B-C considered to be part of a tramp shipping scenario, where the ship aims to arrive in port C to realise a future profit potential. Cases 3 to 8 consider several scenarios in which different FPP values apply.

Example 5.7 *Using a model similar to Psaraftis & Kontovas (2014).*

Cases 3 and 4 from Table 4 describe the situation of a single laden journey A-B-C, and a two-leg laden ballast roundtrip A-B-C-B-A, respectively. We solve both cases with Algorithm 1. In approach, these NPV models with $n = 1$ and FPP = 0 are comparable to the classic multi-leg journey model described in Psaraftis & Kontovas (2014) where the criterion is to maximises total

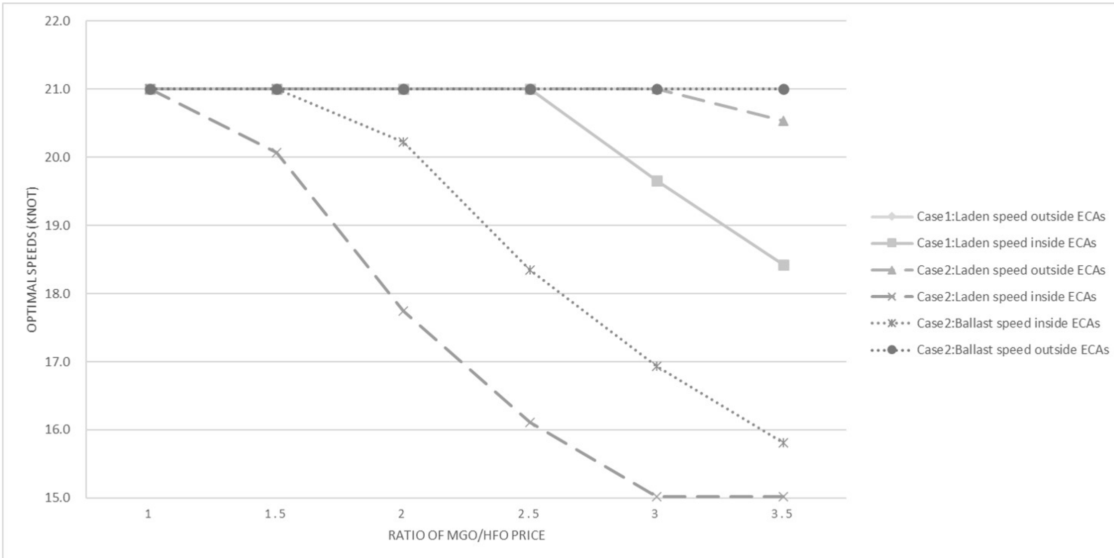


Figure 6: Optimal speeds within and outside the ECA for Cases 1 and 2

profits in USD per journey. Optimal speeds inside the ECA are depicted in Figure 7; the optimal speed outside the ECA is 21 knots at any ratio MGO/HFO. (This is a result of the fact that the unconstrained optimal speed would be above the speed limit of 21 kn, here mainly because the HFO price is low.)

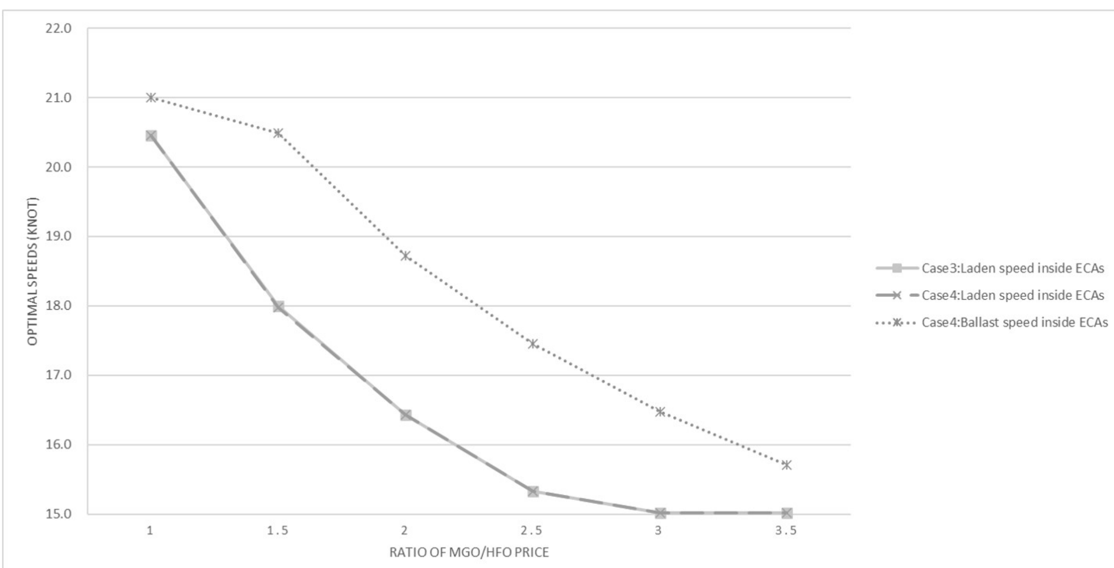


Figure 7: Optimal speeds within the ECA for Cases 3 and 4

Though the speeds outside of ECAs are affected by the fuel price inside ECAs for large n values (see e.g. Case 2), this is not observable here for $n = 1$: by changing the travel speeds, one cannot postpone or advance the future payments much. As a consequence, adding fuel tax will not change the optimal speeds outside of ECAs significantly, and thus we observe that the ship always travels at maximum speed on $A - B$. Moreover, including a ballast return leg will not urge the ship to travel slower on the laden leg, unlike its dramatic effect it had in Case 2. Because the FPP is not as attractive as in Case 1 and 2 (see Example 5.6), the ship travels slower on laden legs in Case 3 and 4, while however still showing a similar sensitivity to MGO/HFO, see Figure 7.

We can imagine a scenario in which the roundtrip journey of Case 4 would be repeated $n > 1$ times. The chain effect analysis has shown us that the future now becomes more and more attractive. This will have the effect of gradually moving the optimal speed curves observed as in Case 4 given in Figure 7 to the corresponding curves of Case 2 given in Figure 6. We would again have to conclude that the impact of taxation on optimal speeds inside and outside an ECA depend considerably on the scenario considered. □

In Case 3 and 4 we have considered FPP = 0. In the following example we compare this with the situation in which the decision maker has a more pessimistic expectation about the future. This could correspond to, for example, expecting a major global economic downturn (from e.g. the outbreak of a pandemic, such as was COVID-19 in 2020), or from the ship upon completion of the journey having to undergo a major overhaul which is expected to ground the ship for several months.

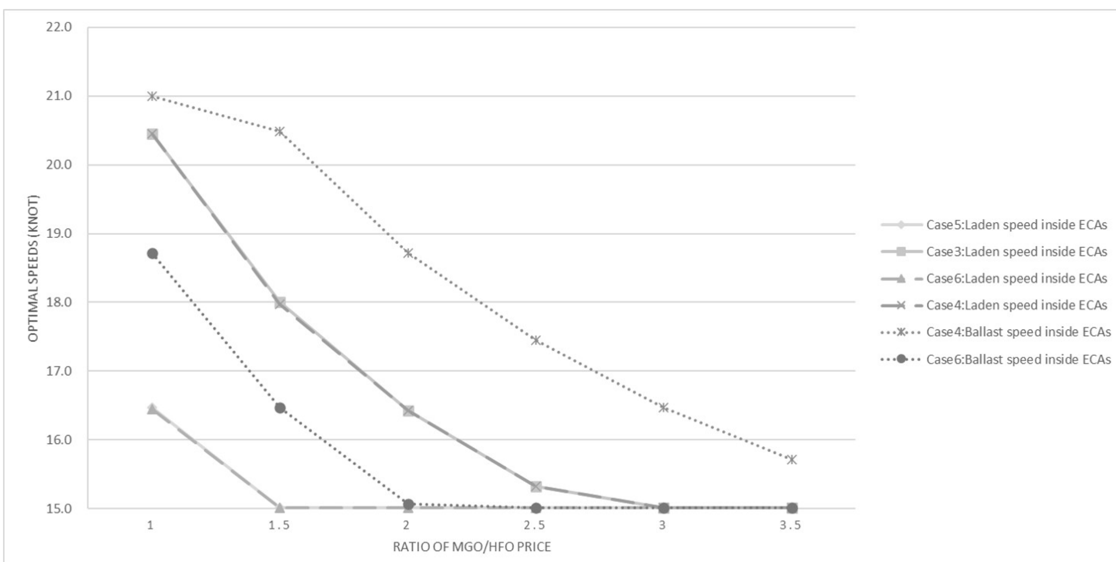


Figure 8: Optimal speeds within the ECA for Cases 5 and 6

Example 5.8 *Facing a negative FPP: future losses ahead.*

Cases 5 and 6 describe the situation when the ship, upon completion of the laden leg to port C, or the laden-ballast journey back into port A, respectively, will run into the FPP of $-10,000$ USD/day. As examined in Section 4.4, we should expect the ship to travel much slower now in order to save fuel and to postpone the onset of the negative FPP. The optimal speeds shown in Figure 8 indeed confirm this (to ease the comparison, the Case 3 and 4 results are reproduced.) As the ship is already travelling at low speeds, increased taxation has only a small impact on additional speed reductions. □

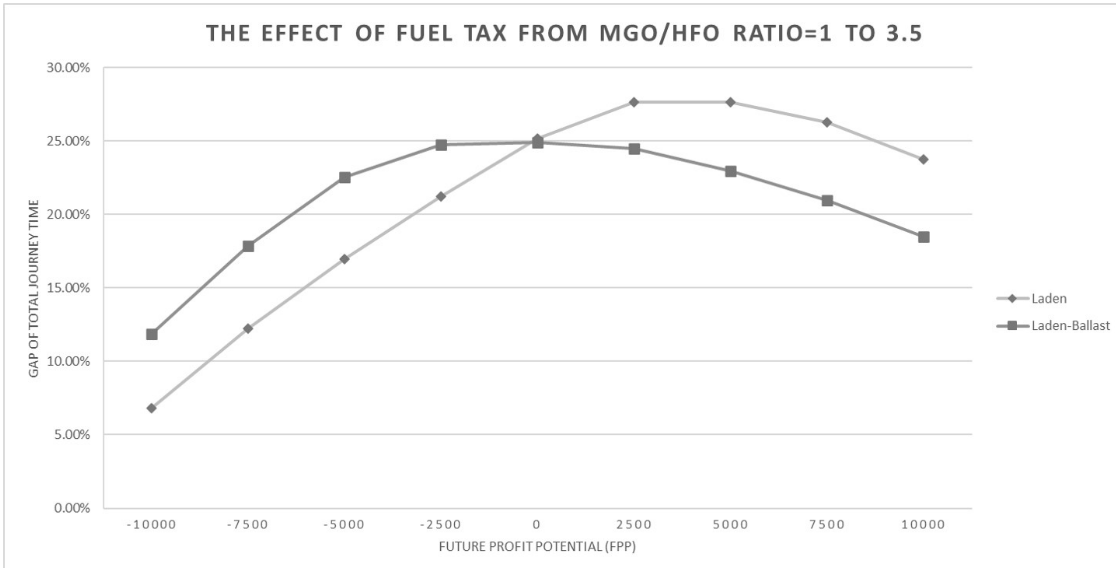


Figure 9: The impact of FPP on the power of fuel tax

Example 5.9 *Speeding towards a positive FPP.*

When the decision maker has a positive FPP, it is more likely that he will decide to let the ship travel faster in order to receive the future revenues earlier. Adding fuel tax, however, can obviously urge it to slow down.

In this last example about FPP, we summarise the impact of a taxation incentive in Figure 9, using the scenarios as in Cases 3 (laden) and 4 (laden-ballast), but for a range of different FPP values. The figure plots the gap when the MGO/HFO ratio is changed from 1 to 3.5. The percentage gap (y-axis) here is defined based on the difference of total journey time at a ratio of 3.5 and 1, divided by the total time at ratio 1.

It may seem intuitive that when the market is in prosperity, that adding a fuel tax will enlarge the gap of total time and thus contribute more to the emission reduction, as confirmed by initial increase observed. However, this is not true when the expectation of FPP is too high. With a promising economic future, the shipper expects to earn so much that he is willing to bear the cost of fuel taxes at high speed. Consequently, the power of fuel tax increases on emission reduction is then reduced. □

Example 5.10 *How ECA may affect competitiveness*

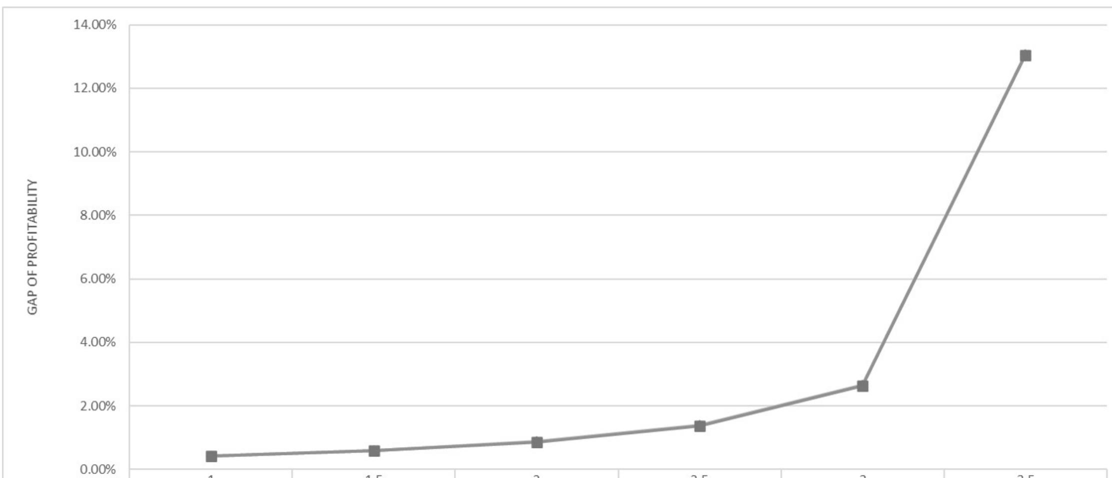
In all the above cases, the ship executes the profitable leg first, starting in port A. However, there could be another identical ship on the same route, but starting from the port C within the ECA. This alternative ship will do the journey C-B-A-B-C; the ballast leg first, then the laden leg. The latter situation is described by the Cases 9 and 10; the tasks in this set-up is to compare Case

9 to 2, and 10 to 4, respectively.

For Cases 9 and 10, the decision maker incurs the costs of the ballast leg first and receives the revenues of the laden leg later, and profitability is lower compared to Cases 2 and 4.

The ship also travels faster on the ballast leg to secure the job, and then slows down on the laden leg. Consequently, if the whole journey is still profitable, the ship is likely to spend less total travel time when doing the ballast leg first, and thus create more emissions. However, this time (and speed) difference is typically rather small, especially when the distance is short or the logistics task is not that attractive.

The difference in profitability is given in Figure 10. The percentage gap is based on the difference of net profits between Case 9 (or 10) and Case 2 (or 4), and divided by net profits of Case 2 (or 4). The gap in profitability in this example grows rapidly with the ratio MGO/HFO. It should be noted, however, that with growing ratio, the whole journey also reduces in profits for both ships, which tends to boost the gap measure. □



	1	1.5	2	2.5	3	3.5
n=1	0.43%	0.59%	0.88%	1.40%	2.65%	13.07%
n=infinity	0.42%	0.58%	0.86%	1.37%	2.63%	13.05%

Figure 10: The gap of profitability between Laden-Ballast and Ballast-Laden

This actually leads to an interesting problem for competitors using identical ships on the same route with different directions. Consider the example from the introduction given in Figure 1, and assume that the ECA is around Rotterdam. Whoever has a ship in New York can not only serve the laden journey much sooner, but will also be more profitable compared to those who have a ship in Rotterdam. Figure 10 suggests that the fuel tax will enlarge this profitability gap even further.

In conclusion, the examination of the Cases 1 to 10 illustrates how differently the same fuel taxation may impact a ship's execution of a journey into (and out from) an ECA. While all odd and even cases may seem very similar at least in the logistics of the journey and associated costs and revenues considered, the optimal speeds are yet very different, and thus the environmental impact as well. To a large degree the difference in these cases results from how the NPV model, through the value of n and the FPP value, is able to capture how the decision maker views this journey as being in a trade-off with the kind of activities the ship will be able to do upon its completion: the more optimistic/pessimistic about the future relative to the current economic conditions, the more willing one may forego making myopic optimal decisions about the current journeys. The ability to capture this effect in a model as we have developed, and show its importance by conducting comparisons with classic speed optimisation models has, to our knowledge, not previously been effectively demonstrated.

6. Further comments

6.1. Chain Effect

The NPV framework has been widely used in the study of replacement (Bethuyne, 2002). Preinreich (1940) is one of the earliest studies of the single machine replacement problem from the NPV perspective. Amongst his interests was the relationship between the optimal life-time of successive usages of a single machine. He derived the insight that "the latter machine would have a longer (re-)cycle life", and referred to it as a "chain rule". The chain effect was also identified in the area of financial investment in problems concerning optimal durations of a sequence of identical projects, see Götze et al. (2015, Chapter 5.3).

We study the relationship between the optimal lengths of successive repetitions of a journey by a ship, established through decisions on leg speeds. Where the single machine replacement problem has a fixed original cost (B) in the beginning of each replacement, we instead face a daily charter cost ($\int_0^{t_i} f^{TCH} e^{-\alpha t} dt$). The chain effect has, to our knowledge, not been reported as to also apply in ship speed optimisation problems, even though ships are often used for repeated logistics services in practice. We have thus investigated the chain effect in this paper to determine its potential importance. We would conclude, see in particular Sections 4.1 and 4.2, that it has great value to help understand the transition from the finite journeys situation to the infinite horizon case.

The material presented Section 4.3 also helps us to understand that the two conventional modelling approaches first discussed in Section 1 each present solutions only valid in an extreme situation. One modeling approach considers the finite single journey situation. By using the optimisation criterion of USD over the journey, it implicitly ignores all of the potential relevant future. The other approach uses the optimisation criterion of USD per time, thereby implicitly assuming the infinite repetition of the journey over an infinite horizon, with constant economic conditions over time. The example in Table 1 illustrates that there are many plausible situations that do not fit either of these extremes.

6.2. Future Profit Potential

As a ship is an expensive investment, Net Present Value methods and discounting cash-flow projections over the life-time of the vessel form part of common business investment analysis, see e.g. Alizadeh & Nomikos (2007). It is in this application also part of the theory about maritime economics, see Stopford (2009). Yet, it has rarely been actually adopted at the operational level of ship journey and speed optimisation.

A notable exception is Magirou et al. (2015), who present a discounted profit model as an extension to their average profit model in the infinite horizon situation. They use the model to identify the optimal traversal cycle between ports that maximises the net profitability, although state that "the equations [*of the discounted model*] are more complicated and the optimal speed expressions are not as easy to interpret." They further identify a 'paradox' in which solutions violate "the principle that profitable voyages should be traversed at high speeds."

We are first to present the finite horizon case ($n < \infty$) and analyse in this context the chain effect and the impact of the future profit potential (FPP). This offers also a robust understanding about the paradox identified in the infinite horizon model of Magirou et al. (2015) by viewing the infinite horizon model as the limit of the n journey situation. It is by now hopefully clear to the reader that optimal speed on a current journey or leg is not determined by its profitability, but by the trade-off with future profit potential. We would thus argue that there is no paradox, but rather that the stated principle is simply not valid. It is worthwhile to point out that both the chain effect and the FPP concept are no longer observable in the infinite horizon case.

A pre-cursor idea of the FPP in the maritime shipping literature can be found in Brown et al. (1987), who address a scheduling problem of crude oil transportation in shipping. They made the interesting suggestion at the end of their paper that, because of uncertainty in scheduling events near the end of the planning horizon, it may be better to discount the costs of future events to a present value to ease the comparison of alternative shipping plans. We have not actually seen the development of this idea in further literature on ship scheduling.

In corporate finance theory, a similar concept exists called *horizon value* and is defined as "the forecasted value of the business at the valuation horizon, also discounted back to present value" (Brealey & Myers, 2003) (p. 77). The horizon value, also known as *terminal value*, forms an important component in the method to value a business by the discounted cash-flow method, see e.g. Chapters 4 and 19 in Brealey & Myers (2003).

The *Future Profit Potential* (FPP) in this paper can be viewed as a translation of this concept from corporate finance to the ship speed optimisation problem. The FPP then receives a very natural interpretation as the forecasted value of the ship at the time and location where the ship completes the journey. (But unlike the horizon value, it is not discounted back to the present, but to the completion time of the journeys.) We find in our numerical experiments (see Section 5) that the ship speeds on the legs of a currently planned set of journeys can be quite sensitive to the FPP.

6.3. Net Present Value Equivalence Analysis

In Section 5 we make use of the technique of *NPV Equivalence Analysis* (Beullens & Janssens, 2014). This approach has been pioneered in the field of production and inventory system management by Grubbström (1980). The NPV objective function in this theory is the Laplace transform of a cash-flow function of an activity. NPVEA is used to compare the outcomes from this NPV model to a “classic” optimisation model (not based on the Laplace transform of a cash-flow function) about the same activity.

Assuming that the cash-flow structure in the NPV model is a fair representation of reality, the NPVEA method asks the question whether the classic model can find its own optimal solutions to be also (close to) optimal from the NPV perspective. This equivalence may be subject to making specific assumptions or interpretations about the classic model’s “structure” (e.g. its parameters), which helps us to better understand the classic model’s applicability.

One of the techniques that can be used is based on Maclaurin expansion of exponential terms in α of the NPV objective function, and then linearisation of the resulting expression. This is often a fairly accurate approximation. More importantly, this linearised objective function is often close to the objective function developed in classic modelling approaches (not based on the NPV technique). This allows us to state with more confidence *under which conditions* there is “equivalence”. Grubbström (1980), for example, was able to show that in some specific production systems, equivalence only exists if the holding cost of products in stock is based on the sales price rather than the cost price. In this paper, for example, we identified that equivalence of Ronen’s Model II with the NPV framework only exists when Ronen’s parameter C_a receives a specific interpretation as given by (45).

Equivalence results not only tell us more about the applicability of classic models, but also allows us to use either the classic or the NPV model interchangeably. We make extensive use in Section 5 of equivalence results. For example, as we have established equivalence between our NPV model and Model I of Ronen (1982), we can obtain solutions of Ronen’s model by solving our NPV model instead.

7. Conclusions

Several modeling approaches exist for determining optimal ship speeds. The NPV modelling framework $\mathcal{P}(n, m, G_o)$ developed in this paper seems the first effort combining the chain effect from replacement theory with the horizon value from corporate finance. We see this marriage of the two into one NPV model having great value in the shipping context: it gives a simple yet powerful model to determine optimal ship speeds, helps build understanding of some of the fundamental forces that drive ship speed optimisation, and shows how ship speed optimisation impacts the NPV of the ship’s future operations.

The main methodological contributions consist of identifying the existence of the *Chain Effect* and the importance of accounting in speed optimisation models for the *Future Profit Potential*. The chain effect is the phenomenon that is visible for finite $n > 1$: optimal leg speeds change with each repetition of the journey. Next to theorems and proofs, we include an intuitive explanation (Section 4.1), as well as a numerical example to illustrate the effect (Section 4.3). From the study of the chain effect, we build up an appreciation of the importance of accounting for future profits. Increasing the value of n is simply one possible mechanism by which the future profits can be made more and more important to the determination of optimal speeds on the current journey. The

Future Profit Potential G_o is we believe an elegant general way to account for future plans. Only for certain decision makers (contract types) would its value be zero (see Section 3.2). We provide numerical illustrations (e.g. Section 4.4) and a way to estimate its value (Section 4.5).

We can summarise more specific contributions attributable to the framework $\mathcal{P}(n, m, G_o)$ as follows. First, we demonstrate that due to the chain effect, the *decomposition principle* no longer holds in the NPV framework. We will in general benefit from optimising the ship speeds jointly across all the (nm) legs of all journey repetitions. This will become more important the larger the number (nm), see e.g. Section 4.3. Second, the algorithm to solve for optimal ship speeds on (nm) legs simultaneously in the NPV framework runs in (pseudo-) polynomial time to any degree of accuracy. Not adopting decomposition is therefore not a computational burden. Third, the NPV Equivalence Analysis establishes that we can replace the three conventional methods investigated in Sections 4.3, 5.1 and 5.2 by the proposed unifying NPV modelling approach $\mathcal{P}(n, m, G_o)$. This has the benefit that one does not need to choose which of the conventional models would be best; or the need to apply switching rules. We can instead always use one single approach. Fourth, and the most important, we have demonstrated that models from class $\mathcal{P}(n, m, G_o)$ allow for much greater flexibility. Multiple models considering different journeys of arbitrary composition and cash-flow structure can be constructed in the same framework, and compared based on the NPV criterion (illustrated in e.g. Examples 5.3, 5.4 and 5.5). Because the approach optimises in each model the trade-off between the maximisation of the currently planned journeys' profits and the decision maker's expectation about future profits, the model that produces the maximum positive NPV value should be the most desirable from a financial point of view.

We have identified and studied the phenomenon that the corresponding ship speeds should differ with each repetition, even in the static case of time-invariant economic conditions. This generalises the *Chain Effect* first observed in Preinreich (1940) in a study about the optimal renewal period of industrial equipment. In practical terms and for a limited number of repetitions, the chain effect may not manifest into big losses if ship speeds are kept constant instead, see also Section 4.3. This implies that companies can still use the convenience of consistent speeds for short term plans. However, for long term plans such differences may become more important, and adapting speeds across different journey repetitions may be worthwhile to consider. The most important conclusion that can be made from the study, however, is the dramatic impact that increasing n has on the optimal speeds for *the first few* journey executions. This thus points towards the importance of accounting for all of the relevant future for the ship, even if the model only considers a single journey. We note that while we use *round-trip* journeys in the study of the chain effect, the model and algorithms work equally well without imposing this requirement.

The easiest way to account for that relevant future, we believe, is through the adoption of the *Future Profit Potential* (FPP) concept. This paper seems first to formally define and recognise the importance of the FPP in ship speed optimisation models, and show how it is key in driving optimal decisions. The FPP is the present value, at the time and location of completion of the n journeys, of all future cash-flows associated with the ship. The FPP shows similarities with approaches used in Corporate Finance, in particular with the concept of the *horizon value*, or *terminal value*, which forms an important component in the method to value a business by the discounted cash-flow method. Changing the FPP value can have significant impact on optimal speeds and NPV of currently planned journeys. In the example of Section 4.4, a reduction in profits from the current journey of more than 11% can be part of the overall optimal strategy if the future is promising, with speeds that would be more than 3 knots higher on average (compared to myopic optimisation of the current journey).

The consideration of the repeated journey structure in $\mathcal{P}(n, m, G_o)$ has allowed us to identify and examine the chain effect and discover and show the value of the FPP. However, the flexibility in this framework allows also the consideration of journeys of a different structure. In tramp shipping scenarios, the decision maker may not always have sufficient information about the future legs the ship will undertake, and there may be no repetition of journeys. It then seems necessary for the decision maker to take a 'myopic' viewpoint and optimise only to the next potential job, and update regularly when new information arrives. The framework developed can be of use in such scenarios, as indicated by the examples in Section 5, and in particular examples 5.4 and 5.5. The use of NPV modelling not only offers a fair criterion when comparing different potential scenarios, the inclusion of the FPP value in the analysis of each potential scenario can offer a more robust decision making process. For example, it may be that the ship would arrive in a port that offers less good prospects, which should lower the FPP value, and making this job less preferable than another job that leads to a port with a higher FPP.

In the following three paragraphs, we summarise more specific results about the applicability of the investigated conventional modelling approaches. Conventional speed optimisation models that minimise costs over a given (series of) journeys, as in Psaraftis & Kontovas (2014), are not so much inaccurate because of not accounting for the associated fixed revenues, but rather because the models fail to account for the future profit potential. See again Section 4.4. Only in the special case that $G_o = 0$ would a classic approach like this actually work well. This is the case for example when the ship is to be hired on a short time charter, where the charterer can decide on the optimal termination time, because then $G_o = 0$ is indeed a correct value (see also Section 3.2).

The *income generating leg model* of Ronen (1982) finds support in the literature because it is able to account for also revenues. However, by optimising profits per unit of time, the model is equivalent to considering an infinite repetition of the considered leg under identical economic conditions. If applied, the NPVEA analysis from Section 5.1 indicates that the model should be based on journeys that are roundtrips, contrary to what Ronen actually argued for in his paper. The modelling framework and our analysis can explain why the optimal speeds are different from the first conventional method above. In simple terms, it is due to the difference in their implicit assumption about the future profit potential. Models in class $\mathcal{P}(n, m, G_o)$ can assume many other different values of the FPP, as well as consider non-roundtrip journey structures.

We also address Ronen's *positioning (empty) leg model*. We provide a practical interpretation of *Daily Alternative Value* C_a parameter in that model, and illustrate how to correctly apply the switching rule between Ronen's two models. We demonstrate that single leg re-positioning scenarios are too restrictive for real world situations, but that models in class $\mathcal{P}(n, m, G_o)$ can extend Ronen's approach to consider multiple legs, including laden legs, in the re-positioning scenarios.

In the final demonstration of the value of $\mathcal{P}(n, m, G_o)$, we shows how important the journey structure is in the problem studied in Fagerholt & Psaraftis (2015) concerning the effects of ECAs and fuel taxes on emission reduction. We demonstrated how our modelling framework can capture the behaviour that the more optimistic/pessimistic about the future relative to the current economic conditions, the more willing profit-seeking decision makers should be to forego making myopic optimal decisions about the current journeys. The example illustrates that the effect of fuel tax on speed reduction may thus depend on the expectation of the FPP, and that either a very high or low FPP would reduce the power of taxation.

Restricting ourselves to static conditions during the journey makes it easier to observe the chain effect, as otherwise changes in speed might be attributed to the dynamic data. This setting also makes it more clear how the FPP affects the optimal ship speeds on the legs, and leads us to present two easy to apply algorithms that can solve instances for any set of (n, m) values in polynomial time. It also allows us to make direct analytical and numerical comparisons with models from the literature based on the classic approaches, which also assume such static conditions. We leave the extension of the models to dynamic (stochastic) conditions for further research, but point out that both the chain effect as well as the impact of the FPP will still be present in these models, albeit likely in a more disguised manner. The impact of the future in determining the optimal current usage of a vessel is clearly demonstrated in this paper, and the use of the FPP concept is recommended in the further development of speed optimisation and job selection models.

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Appendix A. Data of the example of base case

In the base case example of Section 4 (Table 1) used to examine the models of Ronen (1982), the following data has been used:

- **Ship characteristics**: Suezmax of 157, 800 tonne dwt capacity (scantling) and 145, 900 tonne dwt (design); $A = 49,000$ tonne lightweight of ship; $k = 3.910^{-6}$, $p = 381$, $g = 3.1$ and $a = 2/3$. Auxiliary fuel consumption (in ports) at 5 tonne/day. Maximum speed $v^{+} = 17$ kn and minimum speed $v^{-} = 10$ kn. Cargo discharge rate at 3000 m^3/hour. Ballast stability condition: minimum fill-rate of vessel at 30% of design dwt (ballast tank capacity 54, 500 m^3).
- **Journey characteristics**: Distance for any of the legs is 8, 000 nm. Loading rates 3000 m^3 per hour in any port. The waiting time needed in each port is 24 hours.
- **Demand characteristics**: parcel of 1,200,000 barrels of light crude oil at 1.07 m^3/tonne for $A - B$, 600,000 barrels of light crude oil for $C - D$ and 800,000 barrels of light crude oil for $D - A$. And 1 barrel equals 0.136 m^3.
- **Revenue structure**: Freight rate at 25 USD/tonne for $A - B$ and $D - A$, 20 USD/tonne for $C - D$.
- **Cost structure**: Total fixed port costs of a journey towards the each port equals 300, 000 USD. Unloading and loading charges are at a rate of 4, 000 USD/hour in each port. Main bunker fuel at 498 USD/tonne and auxiliary fuel at 590 USD/tonne in each port.
- **Charter party data**: Zero forward start, and TCH at 20, 000 USD/day.
- **Opportunity cost of capital**: $\alpha = 0.08$ per year.

Appendix B. Data of the example of NPVEA

In the NPVEA example of Section 5 (Table 3) that a simple laden (or laden-ballast) scenario is used to examine the equivalence of models from Ronen (1982) in NPV framework, the data are as in Appendix A, except for:

- **Journey characteristics**: Distance for any of the legs is 8, 293 nm.
- **Demand characteristics**: parcel of 1,200,000 barrels of light crude oil at 1.07 m^3/tonne and 1 barrel equals 0.136 m^3.
- **Revenue structure**: Freight rate at 29.4 USD/tonne.
- **Charter party data**: Zero forward start, and TCH at 30, 000 USD/day.

Appendix C. Data of the example of Fagerhold and Psaraftis 2015

In the numerical example of Section 5.4 (Table 4) obtained from Fagerholt & Psaraftis (2015), the data are as in Appendix A, except for:

- **Ship characteristics**: 10,000 tonne dwt capacity; $A = 5,000$ tonne lightweight of ship; $k = 3.910^{-6}$, $p = 381$, $g = 3.1$ and $a = 2/3$. Maximum speed $v^+ = 21$ kn and minimum speed $v^- = 15$ kn.
- **Journey characteristics**: The voyage used in the example is between Antwerp and Halifax with a total distance of 2873 nm (symmetric), while the distance outside the ECA zone is 773 nm and the distance inside ECA zone is 2100 nm. No waiting time at both ports. (This corresponds to data reported in Fagerholt & Psaraftis (2015).)
- **Demand characteristics**: The ship is fully loaded to transport the cargo from the port outside of the ECA zone to the port inside of the ECA zone.
- **Revenue structure**: Freight rate is 45 USD/tonne.
- **Cost structure**: No fixed port cost at both port. Main bunker fuel price of HFO, which is used outside the ECA zone, is 294.5 USD/tonne, This price is used as the baseline in the example. The auxiliary fuel is 500 USD/tonne in both ports.

Appendix D. Necessary conditions for concavity

Appendix D.1. For a single leg h_j

To begin with, we first look at the net profitability of a random leg with index i: $h_j(T_j^i)$. Now in order to find out how to can find its maximum value, we take its first and second order derivative with respect to T_j^i, viz:

$$\frac{\partial h_j(T_j^i)}{\partial T_j^i} = -\alpha(f^{TCH} + R_j - C_j^u)e^{-\alpha T_j^i} - (-2c_j^f k(W_j + A)^h S_j^3 / T_j^i + c_j^f kp(W_j + A)^h), \quad \text{(D.1)}$$

$$\frac{\partial^2 h_j(T_j^i)}{\partial T_j^{i2}} = \alpha^2(f^{TCH} + R_j - C_j^u)e^{-\alpha T_j^i} - 2c_j^f k(W_j + A)^h S_j^3. \quad \text{(D.2)}$$

By observing (D.2), $\partial^2 h_j(T_j^i)/\partial T_j^{i2}$ is decreasing with T_j^i as the travel time can not be negative. In addition, one shall notice that the discount coefficient α^2 is an extremely small multiplier. Thus $f^{TCH} + R_j - C_j^u$ is relatively small for any trip (with $T_j^i > 0$) as otherwise it would be unrealistic in the sense of practice (e.g., the daily charter hire f^{TCH} is normally around 20,000 to 30,000 USD per day, and the revenue R_j cannot be at least hundreds of millions for a single leg). This implies that, generally speaking, $\partial^2 h_j(T_j^i)/\partial T_j^{i2} < 0$ holds, and thus by solving $\partial h_j(T_j^i)/\partial T_j^i = 0$, the optimal travel time that maximises the net revenue of a single leg is found as depicted in Figure 3.

On the other hand, if, the revenue of a single leg R_j or the daily charter hire cost f^{TCH} is a very large number, e.g., the logistics task is either super profitable or the cost of chartering a ship is too high, we might encounter an extreme case of which $\partial^2 h_j(T^i_j)/\partial T^{i2}_j \geq 0$. Equivalently, we have $\alpha^2(f^{TCH} + R_j - C^u_j)e^{-\alpha T^i_j} \geq 2c^f_j k(W_j + A)^h S^3_j > 0$. By substituting it into (D.1), the equation can be re-written as:

$$\frac{\partial h_j(T^i_j)}{\partial T^i_j} \leq -2c^f_j k(W_j + A)^h S^3_j/\alpha + 2c^f_j k(W_j + A)^h S^3_j/T^i_j - c^f_j kp(W_j + A)^h. \tag{D.3}$$

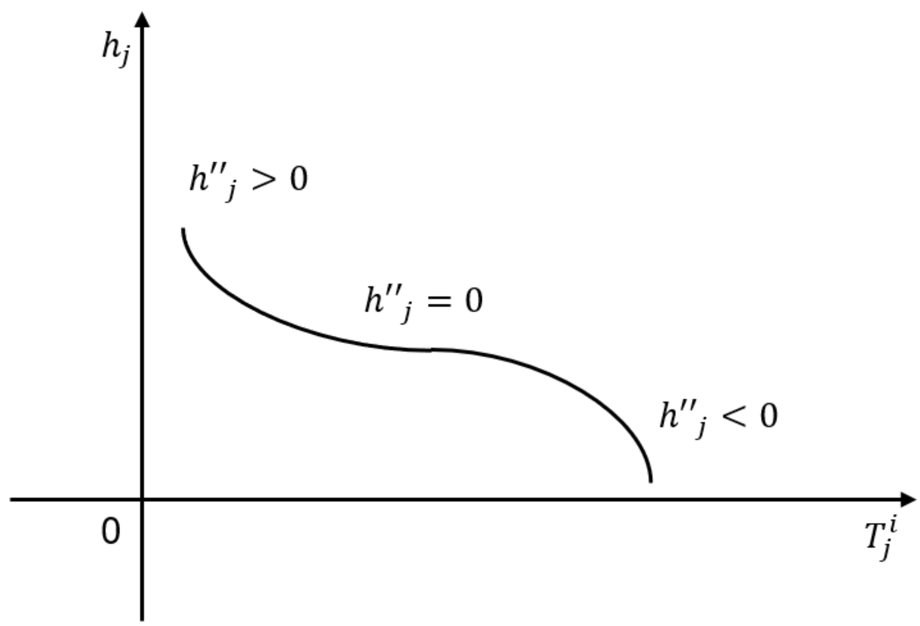

Figure D.11: an extreme example of h_j

If $\alpha < T^i_j$, which is true in practice, we have $\partial h_j(T^i_j)/\partial T^i_j < 0$. Thus we can draw the graph of h_j as given in Figure D.11. It suggests that now the optimal plan is to travel as fast as possible when the revenue is too attractive or the daily charter hire is too high.

Appendix D.2. For a journey h with m legs

Back to the discussion of roundtrip, without loss of generality, let the roundtrip journey contains two legs, i.e., $m = 2$. Thus the net profitability of any roundtrip is given:

$$h(\Gamma_i) = h_1(T^i_1) + h_2(T^i_2)e^{-\alpha T^i_1}. \tag{D.4}$$

Reminding Algorithm 1, the optimal travel time of latter leg, e.g., T^i_2 here, is solved first and it is independent of the choices of T^i_1. For simplicity, we denote T^{i*}_2 as the optimal travel time for the second leg, and $h_2(T^{i*}_2)$ is the corresponding maximum net profitability. Now take the derivatives of $h(\Gamma_i)$ with respect to T_1^i and we have:

$$\frac{\partial h(\Gamma_i)}{\partial T_1^i} = \frac{\partial h_1(T_1^i)}{\partial T_1^i} - \alpha h_2(T_2^{i*})e^{-\alpha T_1^i}, \tag{D.5}$$

$$\frac{\partial^2 h(\Gamma_i)}{\partial T_1^{i2}} = \frac{\partial^2 h_1(T_1^i)}{\partial T_1^{i2}} + \alpha^2 h_2(T_2^{i*})e^{-\alpha T_1^i}. \tag{D.6}$$

If the second leg is a ballast leg, then (D.6) is negative for sure, and thus the profit structure for the journey h is concave. However, when the second leg is laden, there are two possible scenarios.

The first one is, by noticing that α^2 is extremely small, as long as $h_2(T^{i*}_2)$ is not too large, the term $\alpha^2 h_2(T_2^{i*})e^{-\alpha T_1^i} \approx 0$, and thus the concavity of h is still guaranteed. This case is more likely to be observed in practice.

The other possibility is, when $h_2(T^{i*}_2)$ is a very large positive number, i.e., the future profitability is too attractive. Note the profit structure for the journey h is now convex, and thus to maximising h, we would like to have (D.5) to be either a large positive number or small negative number. Given the fact that $h_2(T^{i*}_2) > 0$, now the optimal T^{i*}_1 that maximises h is to force $\partial h_1(T^{i*}_1)/\partial T^{i*}_1 > 0$. According to Figure 3, this is to say that the ship will travel as fast as possible in the earlier leg, if the net profitability of later journey is too attractive. Using the same induction, we can extend the number of legs from 2 to m and arrive at the similar conclusion.

Appendix E. Proof of theorems

Appendix E.1. Theorem 1

Proof of Theorem 1. For a random leg j of the journey with index i, the optimal travel time T_j^{i*} is found by solving the following equation:

$$\frac{\partial G_i}{\partial T_j^i} = \frac{\partial h(\Gamma_i)}{\partial T_j^i} - \alpha G^*_{i-1} e^{-\alpha L^i_m} = 0.$$

If $h > 0$, we have $G^*_{i-1} > 0$ (by definition), and thus $\partial h(\Gamma_i)/\partial T^i_j > 0$ (or $\partial h(\Gamma_i)/\partial T^i_j < 0$, if $h < 0$). By comparing T_j^{i*} and T_j^{i-1*} we have:

$$\begin{aligned}
\frac{\partial h(\Gamma_i)}{\partial T_j^i} - \frac{\partial h(\Gamma_{i-1})}{\partial T_j^{i-1}} &= \alpha(G^*_{i-1}e^{-\alpha L^{i}_m{}^*} - G^*_{i-2}e^{-\alpha L^{i-1*}_m}) \\
&= \alpha e^{-\alpha L^{i}_m{}^*}(G^*_{i-1} - G^*_{i-2}e^{-\alpha(L^{i-1*}_m - L^{i}_m{}^*)}) \\
&= \alpha(1-\varepsilon_1)(G^*_{i-1} - G^*_{i-2}(1-\varepsilon_2)) \\
&= \alpha(G^*_{i-1} - G^*_{i-2}) - \varepsilon.
\end{aligned}$$

In practice, the completion time of a roundtrip ${L^i_m}^*$ cannot be super large, e.g., a few years. As a consequence, numerically the term $\varepsilon \to 0$ can be ignored, unless we assume the profitability of a single journey is also such a small number (smaller than 1). Thus, if $h > 0$, term $G^*_{i-1} - G^*_{i-2} > 0$.

Now we give the expression of $\partial h(\Gamma_i)/\partial T^i_j$ as follows:

$$\frac{\partial h(\Gamma_i)}{\partial T^i_j} = h'_j(T^i_j) - \alpha(h^*_{j+1}({T^i}^*_{j+1})e^{-\alpha T^i_j} + \cdots + h^*_m({T^i}^*_m)e^{-\alpha \sum_{k=j}^m T^i_k})$$

Since $h''_j(T^i_j) < 0$, it can be shown that $\partial h(\Gamma_i)/\partial T^i_j$ is decreasing with T^i_j. As a consequence, we have $T^i_j < T^{i-1}_j$, if $h > 0$. Similarly, the others can be proved in the same way. **Q.E.D.** □

Appendix E.2. Theorem 2

Proof of Theorem 2. Denote a mapping F:

$$F(\Phi(n)) = h(\Gamma_n) + e^{-\alpha L^n_m}\Phi(n-1).$$

Such that given any function Φ of n, it is obvious that for any $\Phi > \Phi'$, we always have $F\Phi > F\Phi'$, namely the monotone property. Another property is, for any real number $p \in \mathbb{R}$, we have $F(\Phi + pI)(n) = F\Phi(n) + pe^{-\alpha L^n_m}$ for any n, where $I(n) \equiv 1$ is a unit function. They are easily proved by the definition of F.

Now, if ϕ and φ are two bounded functions, then we can denote a constant $b = \max_n |\phi(n) - \varphi(n)|$. This gives the following inequality:

$$\phi(n) - b \le \varphi(n) \le \phi(n) + b,$$

By applying the mapping F to above inequality, we have

$$F(\phi(n) - b) = F\phi(n) - be^{-\alpha L^n_m} \le F\varphi(n) \le F(\phi(n) + b) = F\phi(n) + be^{-\alpha L^n_m},$$

which is equivalent to say

$$\max_n |F\phi(n) - F\varphi(n)| \le be^{-\alpha L^n_m} = e^{-\alpha L^n_m} \max_n |\phi(n) - \varphi(n)|,$$

where $e^{-\alpha L^n_m} \in [0, 1)$ for all $L^n_m > 0$.

Now consider the scalar function $H(n) = \sum_{i=1}^n h(\Gamma_i)e^{-\alpha \sum L^{i-1}_m}$ with $L^0_m \equiv 0$, which is the total discounted net profit across all journeys. Because $T^i_j \in [T_{min}, T_{max}]$ with $0 < T_{min} < T_{max} < +\infty$ (by definition, which is the result of the physical limits of speeds and positive travel distance which cannot be extended to ∞), we have $0 < L^i_m < +\infty$ (because $m < +\infty$). h is bounded by M, then we show $H(n)$ is also bounded in $\mathbb{R}$:

$$\lim_{n\to\infty} |H(n)| = \sum_{i=1}^{\infty} |h(\Gamma_i)|e^{-\alpha \sum L^{i-1}_m} \le \sum_{i=1}^{\infty} Me^{-i\alpha m T_{min}} = \frac{M}{1 - e^{-\alpha m T_{min}}}.$$

Denote the space $\Omega = [-\frac{M}{1-e^{-\alpha m T_{min}}}, \frac{M}{1-e^{-\alpha m T_{min}}}]$ in $\mathbb{R}$. According to *Heine–Borel theorem*, Ω is compact, since it is a closed and bounded subset of $\mathbb{R}$. By replacing ϕ by $H(n)$, φ by $\hat{H}(n)$ (we can understand it as an approximation of $H(n)$ in the same space Ω), the inequality $\max_n |FH(n) - F\hat{H}(n)| \leq e^{-\alpha T_n} \max_n |H(n) - \hat{H}(n)|$ shows that the mapping $F : \Omega \to \Omega$ is a contraction mapping. According to the *Banach fixed-point theorem*, there exists a unique point H^*, such that $\lim_{n\to\infty} FH(n) = H^*$.

Notice that $FH(n) = h(\Gamma_n) + e^{-\alpha L_m^n} H(n-1)$ is the discounted net profit at stage n, which is equivalent to $G_n = h(\Gamma_n) + e^{-\alpha L_m^n} G_{n-1}$. Thus we show that $\lim_{n\to\infty} G_n = G^*$, which is equivalent to say there exists Γ^*, such that $\Gamma^* := arg \;\; maxG^* = arg \;\; max(h(\Gamma_n) + G_{n-1} e^{-\alpha L_m^n})$ with $n \to \infty$. **Q.E.D.** $\square$