

On the Switching Control of the DC-DC Zeta Converter Operating in Continuous Conduction Mode

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Abstract: In this paper, we propose a switching control mechanism for the stabilization of a DC-DC Zeta converter operating in continuous conduction mode. The switching control algorithm is based on a control Lyapunov function and extends the method proposed for a two-dimensional boost converter model presented in the literature to a four-dimensional Zeta converter model. The local asymptotical stability of the operating point is established using LaSalle's invariance principle for differential inclusions. By applying spatial regularization, a modified switching control algorithm reduces the switching frequency and keeps the state-trajectory around a neighbourhood of the operating point. The method works well even if the operation point changes significantly and it is valid for both step-up and step-down operations. Furthermore, by approximating the state-trajectory near the operating point, we obtain an explicit relation between the modified switching algorithm and the switching frequency, which allows us to choose systematically the desired switching frequency for the converter to operate. The effectiveness of the proposed method is illustrated with simulation results.

1. Introduction

In energy harvesting systems, DC-DC converters are part of the power management system. Because of the uncertain nature of the ambient energy, for example, low or high irradiance of sun and fluctuation of the wind speed, the voltage generated by the energy harvester, which is connected to the input of the DC-DC converter, can be higher or lower than the output voltage. For this reason, a fourth-order DC-DC converter is a good candidate to be deployed, since it has step-up and step-down capability. There are a few topologies available, and the Zeta topology is selected for our research due to two reasons: 1) positive output voltage, and low output voltage ripple [3], 2) natural DC input-to-output voltage isolation [4].

To control a DC-DC converter, the conventional fixed-frequency, average-based system control methods are commonly deployed such as proportional integral (PI) [5–9], optimal [11–16], sliding-mode [17, 18], fuzzy [19, 20], model predictive [21, 22], adaptive [23], and fuzzy-neural [24], to name a few. The PI control produces fast output voltage regulation, however, it suffers from high control duty-ratio effort [9] that can lead to PWM circuitry problem [12]. The conventional optimal linear quadratic regulator (LQR) control produces optimal compensation with minimal control effort, but lack of robustness if the parameter is uncertain [14]. While LMI-LQR control is robust, its control duty-ratio signal has quite a large ripple [15], which may produce nonlinear effect if the ripple exceeded 20% [10]. As for the sliding-mode, fuzzy, model predictive, adaptive, and fuzzy-neural, because they use the so-called small-signal average model, when the duty ratio largely deviates from the nominal one, the small-signal average model is not a good approximation. As a result, the controller design is no longer valid, which in turn jeopardize the system performance. On the other hand, non-average-based system control, typically known as hybrid control, is a variable switching frequency type of control

where the switching frequency is initially low and it becomes arbitrarily fast at operating point. The hybrid control is more robust than average-based system control [2], due to the former's ability to execute the switching mechanism online. Hybrid control has been implemented for the stabilization of the DC-DC converter [25–32]. In [25–28], the authors propose a switching algorithm by approximating the state-trajectory and restricting the state-trajectory within a limits specified by the guard conditions. Even though the output voltage regulation is achieved and demonstrated, no theoretical work is presented to proof the stability of the system. In [29–32], the authors propose a Lyapunov-based hybrid control to stabilize the DC-DC converters. The study [29] basically proposes the switching rule that assigns the mode decreasing the value of the Lyapunov function most. When the trajectory reaches the switching boundary, it evolves as a sliding mode solution, which means that the switching interval becomes infinitesimally small. In [30, 31], the authors use sampled-data control to avoid sliding mode solutions. Though switching frequency is controlled by the sampling period, it tends to be small because the method is based on sufficient conditions. Hence the trajectory is close to the sliding solution. Though our paper uses the same switching mechanism as in [32], the stability analysis is fundamentally different. A second-order boost converter model in [32] can be analyzed by the standard Lyapunov approach by showing the decrease of the Lyapunov function along the trajectory. The derivative along the trajectory of the fourth-order Zeta converter model becomes zero even if the point is not the operating point. Hence LaSalle's invariance principle proved for this class of differential inclusion should be established. The preliminary and much shorter version of our work were presented in the conference proceedings [1,2].

The remainder of the paper is organized as follows. In Section 2, we establish a switching control mechanism for a two-mode system. The two-mode system is instrumental to model the DC-DC zeta converter operating in continuous conduction mode (CCM). In Section 3, we analyze the

stability of the switching system in Section 2, and apply it to the Zeta converter. In Section 4, the switching control mechanism discussed in Section 2 is modified to limit the switching frequency of the Zeta converter. In addition, by using linear-line-approximation of the trajectory, we show how to decide the switching frequency. Simulation results to show the effectiveness of our proposed method are presented in Section 5. Lastly, in Section 6, we conclude our work and state the plan for future work.

Notation: \mathbb{R} denote the set of real numbers. For $\rho \in \mathbb{R}$, $\mathbb{R}_{>\rho}$, $\mathbb{R}_{<\rho}$, $\mathbb{R}_{\geq\rho}$, and $\mathbb{R}_{\leq\rho}$ denote the set of real numbers larger, smaller, larger than or equal to, and smaller than or equal to ρ , respectively. The notation conv denotes the convex hull of a set. For a function $\alpha: \mathbb{R}^n \rightarrow \mathbb{R}$, α^{-1} denotes the inverse image of α . For a singleton $\{c\}$, $c \in \mathbb{R}$, we use the simplified notation, $\alpha^{-1}(c) = \alpha^{-1}(\{c\})$. A set valued map F is denoted as $F: \mathbb{R}^n \rightsquigarrow \mathbb{R}^m$, where for $x \in \mathbb{R}^n$, $F(x) \subset \mathbb{R}^m$.

2. Switching Control of Two-mode System

Because transistors and diodes exhibit on-off behaviours, many converters can be modeled as multi-mode systems, where each mode is described as a linear state-space model. A two-mode system can be used to model the continuous-conduction mode of any DC-DC converters. In this section, we introduce a Lyapunov-function-based switching control strategy and motivate its stability analysis.

Two linear systems of the same state dimension are given by

$$\frac{dx}{dt} = A_1x + B_1u, \quad (1)$$

$$\frac{dx}{dt} = A_2x + B_2u, \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state and $u(t) \in \mathbb{R}$ is the input. Fix $u_0 \in \mathbb{R}$ and $\lambda \in \mathbb{R}$. Assume that

$$\lambda A_1 + (1 - \lambda)A_2$$

is invertible. Define

$$x^* = -(\lambda A_1 + (1 - \lambda)A_2)^{-1}(\lambda B_1 + (1 - \lambda)B_2)u_0. \quad (3)$$

Note that we do not assume the stability nor the non-singularity of the matrices A_1 and A_2 . Let \mathcal{S}_i ($i = 1, 2$) denote the set of stationary points of the systems (1) and (2); namely

$$\mathcal{S}_1 = \{x: A_1x + B_1u_0 = 0\}, \mathcal{S}_2 = \{x: A_2x + B_2u_0 = 0\}. \quad (4)$$

If A_i is non-singular, \mathcal{S}_i is a singleton; otherwise it may be empty or infinite. The following proposition is easy to derive, but useful in the subsequent discussions.

Proposition 1. The point x^* is given by (3) if and only if it satisfies the following equation:

$$\lambda(A_1x^* + B_1u_0) = -(1 - \lambda)(A_2x^* + B_2u_0). \quad (5)$$

Furthermore, $A_ix^* + B_iu_0 \neq 0$ ($i = 1, 2$) if and only if $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$.

Proof. Since

$$(\lambda A_1 + (1 - \lambda)A_2)x^* = (\lambda B_1 + (1 - \lambda)B_2)u_0,$$

the equivalence of (3) and (5) is immediate. If $x^* \in \mathcal{S}_1 \cap \mathcal{S}_2$, then

$$\lambda(A_1x^* + B_1u_0) = -(1 - \lambda)(A_2x^* + B_2u_0) = 0,$$

which implies $x^* = x^*$ by the non-singularity of $\lambda A_1 + (1 - \lambda)A_2$. Conversely, if $A_1x^* + B_1u_0 = 0$, then $A_2x^* + B_2u_0 = 0$ by (5). Thus, $x^* \in \mathcal{S}_1 \cap \mathcal{S}_2$. ■

We are interested in a switching control law that drives the state of the switching system with the modes (1) and (2) to x^* under $u(t) \equiv u_0$. For this, define a candidate Lyapunov function

$$V(x) = (x - x^*)^T P (x - x^*), \quad (6)$$

where P is a positive definite matrix. Because $P > 0$, there exist $c_1 > 0$ and $c_2 > 0$ such that

$$c_1 \|x - x^*\|^2 \leq V(x) \leq c_2 \|x - x^*\|^2, \quad (7)$$

holds. The derivatives of $V(x)$ along the trajectories of (1) and (2) are

$$\begin{aligned} \alpha_1(x) &:= \frac{\partial V}{\partial x}(A_1x + B_1u_0) \\ &= (x - x^*)^T (PA_1 + A_1^T P)(x - x^*) + 2(A_1x^* + B_1u_0)^T P(x - x^*), \end{aligned} \quad (8)$$

$$\begin{aligned} \alpha_2(x) &:= \frac{\partial V}{\partial x}(A_2x + B_2u_0) \\ &= (x - x^*)^T (PA_2 + A_2^T P)(x - x^*) + 2(A_2x^* + B_2u_0)^T P(x - x^*), \end{aligned} \quad (9)$$

respectively.

Proposition 2. Suppose that $PA_1 + A_1^T P \leq 0$ and $PA_2 + A_2^T P \leq 0$. Then,

$$\alpha_2^{-1}(\mathbb{R}_{\geq 0}) \subset \alpha_1^{-1}(\mathbb{R}_{\leq 0}), \quad \alpha_1^{-1}(\mathbb{R}_{\geq 0}) \subset \alpha_2^{-1}(\mathbb{R}_{\leq 0}), \quad (10)$$

$$\alpha_1^{-1}(0) \cap \alpha_2^{-1}(0) = \{x: x - x^* \in \ker(PA_1 + A_1^T P) \cap \ker(PA_2 + A_2^T P) \cap \ker(A_1x^* + B_1u_0)^T P\}. \quad (11)$$

The proof is based on the following observation.

Lemma 1. Let $Q_1 \leq 0$ and $Q_2 \leq 0$ be $n \times n$ symmetric matrices. Let $v_1 \in \mathbb{R}^n$ and $v_2 \in \mathbb{R}^n$ satisfy $\lambda v_1 + (1 - \lambda)v_2 = 0$ for some $0 < \lambda < 1$. Define

$$p_1(x) := x^T Q_1 x + v_1^T x, \quad p_2(x) := x^T Q_2 x + v_2^T x.$$

Then

$$\begin{aligned} p_1^{-1}(\mathbb{R}_{\leq 0}) &\subset p_2^{-1}(\mathbb{R}_{\geq 0}), \quad p_2^{-1}(\mathbb{R}_{\leq 0}) \subset p_1^{-1}(\mathbb{R}_{\geq 0}), \\ p_1^{-1}(0) \cap p_2^{-1}(0) &= \ker Q_1 \cap \ker Q_2 \cap \ker v_1^T. \end{aligned}$$

Proof. If $p_1(x) > 0$, then

$$\begin{aligned} 0 &< \lambda p_1(x) = \lambda x^T Q_1 x + \lambda v_1^T x \\ &\leq \lambda v_1^T x = -(1 - \lambda)v_2^T x \\ &\leq -(1 - \lambda)x^T Q_2 x - (1 - \lambda)v_2^T x = -(1 - \lambda)p_2(x) \end{aligned}$$

holds. Thus, $p_2(x) < 0$. Because $p_1^{-1}(\mathbb{R}_{\leq 0}) = \mathbb{R}^n \setminus p_1^{-1}(\mathbb{R}_{> 0})$ and $p_2^{-1}(\mathbb{R}_{\geq 0}) = \mathbb{R}^n \setminus p_2^{-1}(\mathbb{R}_{< 0})$, $p_2^{-1}(\mathbb{R}_{\geq 0}) \subset p_1^{-1}(\mathbb{R}_{\leq 0})$. By interchanging p_1 and p_2 , in the argument, it follows that $p_1^{-1}(\mathbb{R}_{\geq 0}) \subset p_2^{-1}(\mathbb{R}_{\leq 0})$. If $p_1(x) = p_2(x) = 0$, then

$$\begin{aligned} 0 &= \lambda p_1(x) = \lambda x^T Q_1 x + \lambda v_1^T x \\ &\leq \lambda v_1^T x = -(1-\lambda)v_2^T x \\ &\leq -(1-\lambda)x^T Q_2 x - (1-\lambda)v_2^T x = -(1-\lambda)p_2(x) = 0. \end{aligned}$$

Hence all the inequalities hold as equalities. This implies $v_1^T x = 0$, $x^T Q_1 x = 0$, and $x^T Q_2 x = 0$, which means $x \in \ker Q_1 \cap \ker Q_2 \cap \ker v_1^T$. Conversely, if $x \in \ker Q_1 \cap \ker Q_2 \cap \ker v_1^T$, then $x \in \ker v_2^T$ and $p_1(x) = p_2(x) = 0$. ■

Proof of Proposition 2. Define

$$\begin{aligned} Q_1 &:= PA_1 + A_1^T P, \quad Q_2 := PA_2 + A_2^T P, \\ v_1 &:= 2P(A_1 x^* + B_1 u_0), \quad v_2 := 2P(A_2 x^* + B_2 u_0), \\ p_1(x) &:= \alpha_1(x + x^*), \quad p_2(x) := \alpha_2(x + x^*). \end{aligned}$$

Notice that the assumptions of Lemma 1 are satisfied since (5) holds. Then the proof is immediate from Lemma 1. ■

Based on Proposition 2, we propose the following switching control mechanism.

Switching Mechanism A

- If the system is operating in mode 1 and reaches $\alpha_1^{-1}(0)$, then it switches to mode 2.
- If the system is operating in mode 2 and reaches $\alpha_2^{-1}(0)$, then it switches to mode 1.

Remark 1. Switching Mechanism A is initially proposed for a boost converter model in [32]. Proposition 2 clarifies a condition that ensures that Switching Mechanism A is well defined.

To analyze the stability of the switching control law, we consider the differential inclusion

$$\frac{dx}{dt} \in F(x), \quad (12)$$

$$F(x) := \begin{cases} \{A_1 x + B_1 u_0\} & \text{if } x \in M_1, \\ \{A_2 x + B_2 u_0\} & \text{if } x \in M_2, \\ \text{conv}\{A_1 x + B_1 u_0, A_2 x + B_2 u_0\} & \text{if } x \in M_0, \end{cases}$$

where

$$\begin{aligned} M_1 &= \{x: \alpha_1^{-1}(\mathbb{R}_{< 0}) \cap \alpha_2^{-1}(\mathbb{R}_{> 0}) = \alpha_1^{-1}(\mathbb{R}_{< 0})\}, \\ M_2 &= \{x: \alpha_1^{-1}(\mathbb{R}_{> 0}) \cap \alpha_2^{-1}(\mathbb{R}_{< 0}) = \alpha_2^{-1}(\mathbb{R}_{< 0})\}, \\ M_0 &= \{x: \alpha_1^{-1}(\mathbb{R}_{\leq 0}) \cap \alpha_2^{-1}(\mathbb{R}_{\leq 0})\}. \end{aligned}$$

The set-valued map $F: \mathbb{R}^n \rightsquigarrow \mathbb{R}^n$ is upper semi-continuous, and its values are bounded closed convex sets. Solutions of (12) include solutions of (1), (2) with the switching control mechanism. We shall analyze the stability of the operating point x^* of the differential inclusion (12).

Remark 2. The switching mechanism that selects the mode defined by

$$\arg \min \{\alpha_i(x): i = 1, 2\}$$

is in line with the method used in [29], except that [29] assumes that

$$P(\lambda A_1 + (1-\lambda)A_2) + (\lambda A_1 + (1-\lambda)A_2)^T P < 0$$

for some $\lambda \in (0, 1)$. This mechanism does not stabilize the operating point asymptotically when we only assume $PA_1 + A_1^T P \leq 0$ and $PA_2 + A_2^T P \leq 0$. The following simple example

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

results in the differential inclusion

$$\frac{dx}{dt} \in F(x)$$

$$F(x) = \begin{cases} \{1\} & \text{if } \begin{bmatrix} 0 & 1 \end{bmatrix} x \neq 0, \\ \{1, 2\} & \text{if } \begin{bmatrix} 0 & 1 \end{bmatrix} x = 0. \end{cases}$$

If $\begin{bmatrix} 0 & 1 \end{bmatrix} x_0 = 0$, then the differential inclusion has the unique solution $x(t) \equiv x_0$, which shows that it is not asymptotically stable. Note that this is not a counterexample of [29, Theorem 2]. Nevertheless, the example shows that the distinction of positive definiteness and positive semi-definiteness is meaningful.

3. Stability of Switching System

In this section, we analyze the stability of the switching mechanism proposed in the previous section and apply the method to a DC-DC Zeta converter operating in the continuous conduction mode (CCM).

3.1. Stability Analysis

Consider the differential inclusion (12) and the function $V(x)$ in (6). Define $\dot{V}(x): \mathbb{R}^n \rightsquigarrow \mathbb{R}$ by

$$\dot{V}(x) = \frac{\partial V}{\partial x} F(x) := \left\{ \frac{\partial V}{\partial x} \omega : \omega \in F(x) \right\}.$$

It is easy to verify that

$$\dot{V}(x) = \begin{cases} \{\alpha_1(x)\} & \text{if } x \in M_1, \\ \{\alpha_2(x)\} & \text{if } x \in M_2, \\ \text{conv}\{\alpha_1(x), \alpha_2(x)\} & \text{if } x \in M_0. \end{cases} \quad (13)$$

The inverse image $\dot{V}^{-1}(S)$, where $S \subset \mathbb{R}$, is defined by

$$\dot{V}^{-1}(S) := \{y \in \mathbb{R}^n: \dot{V}(y) \cap S \neq \emptyset\}.$$

When $S = \{a\}$, we write $\dot{V}^{-1}(a) := \dot{V}^{-1}(\{a\})$.

Proposition 3. Consider the differential inclusion (12) and the Lyapunov function (6). Then,

$$\dot{V}^{-1}(0) = \alpha_1^{-1}(0) \cup \alpha_2^{-1}(0).$$

Proof. If $x \in \alpha_1^{-1}(0)$, then $\dot{V}(x) = \text{conv}\{0, \alpha_2(x)\} \ni 0$. Similarly, we have $0 \in \dot{V}(x)$ if $x \in \alpha_2^{-1}(0)$. Hence $\dot{V}^{-1}(0) \supset \alpha_1^{-1}(0) \cup \alpha_2^{-1}(0)$. Conversely, if $0 \in \dot{V}(x)$, then $x \in \alpha_1^{-1}(\mathbb{R}_{\leq 0}) \cap \alpha_2^{-1}(\mathbb{R}_{\leq 0})$ and $0 \in \text{conv}\{\alpha_1(x), \alpha_2(x)\}$. Because $\alpha_1(x) \leq 0$ and $\alpha_2(x) \leq 0$, this implies either $\alpha_1(x) = 0$ or $\alpha_2(x) = 0$. ■

Proposition 4. Let x^* , \mathcal{S}_1 , and \mathcal{S}_2 be defined by (3) and (4). Then $\{x^*\} \cup \mathcal{S}_1 \subset \alpha_2^{-1}(0)$ hold.

Proof. We have already shown that $x^* \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ in Proposition 2. If $x \in \mathcal{S}_1$, then $\alpha_1(x) = 0$ by (8). Similarly, if $x \in \mathcal{S}_2$, then $\alpha_2(x) = 0$. ■

Proposition 5. Let $x^\# \in \{x^*\} \cup \mathcal{S}_1 \cup \mathcal{S}_2$. Then, the differential inclusion (12) has a stationary solution $\phi(t, x^\#) \equiv x^\#$.

Proof. If $x^\# \in \mathcal{S}_1$, then $\alpha_1(x^\#) = 0$ by Proposition 4. Thus $F(x^\#) = \text{conv}\{0, A_2 x^\# + B_2 u_0\} \ni 0$. Similarly, $x^\# \in \mathcal{S}_2$ implies $0 \in F(x^\#)$. By Proposition 4, $\alpha_1(x^*) = \alpha_2(x^*) = 0$. Consequently $F(x^*) = \text{conv}\{A_1 x^* + B_1 u_0, A_2 x^* + B_2 u_0\} \ni \lambda(A_1 x^* + B_1 u_0) + (1 - \lambda)(A_2 x^* + B_2 u_0) = 0$. ■

By Proposition 5, the operation point x^* is not globally asymptotically stable if $\mathcal{S}_1 \cup \mathcal{S}_2 \neq \emptyset$. We shall study local asymptotic stability of x^* . The next result shows that x^* is stable in this sense.

Theorem 1. Suppose that $PA_1 + A_1^T P \leq 0$ and $PA_2 + A_2^T P \leq 0$ hold. If $\alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ contains no solution of (12) except for $x(t) \equiv x^*$, then x^* is locally asymptotically stable.

The proof of Theorem 1 hinges on a couple of Lemmas. The first one states that the operating point x^* is stable.

Lemma 2. Suppose that $PA_1 + A_1^T P \leq 0$ and $PA_2 + A_2^T P \leq 0$ hold. Then x^* is stable.

Proof. Note that the set-valued function $F(x)$ in (12) is defined for all $x \in \mathbb{R}^n$ from (10) in Proposition 2. Let $\varepsilon > 0$, and choose $\delta = \frac{\varepsilon c_1}{c_2} > 0$. If $\|x_0 - x^*\|^2 < \delta$, it follows from (7) that $V(x_0) \leq c_2 \delta = c_1 \varepsilon$. Along the trajectory $\phi(t, x_0)$ of (12), it holds that

$$\frac{d}{dt} V(\phi(t, x_0)) = \dot{V}(\phi(t, x_0)) \subset \mathbb{R}_{\leq 0},$$

by (13), and hence $V(\phi(t, x_0)) \leq V(x_0)$ holds. This implies that $\|\phi(t, x_0) - x^*\|^2 \leq \varepsilon$, and therefore x^* is stable. ■

One of the important observations is the property of the limiting set of a solution of a differential inclusion with an upper semi-continuous set-valued map. Let

$$\frac{dx}{dt} \in F(x), \quad (14)$$

be a differential inclusion where $F: \mathbb{R}^n \rightsquigarrow \mathbb{R}^n$ is upper semi-continuous and its values are bounded closed convex sets. Let $\phi(t, x_0)$ be a solution of (14). A point $\omega \in \mathbb{R}^n$ is called a

limit point of $\phi(t, x_0)$ if there is a sequence $\{t_k\}$ in $[0, \infty)$ such that $t_k \rightarrow \infty$ and

$$\lim_{k \rightarrow \infty} \phi(t_k, x_0) = \omega.$$

The set of all limit points of $\phi(t, x_0)$ is called the limit set of $\phi(t, x_0)$ and is denoted as Ω .

Lemma 3. Consider the differential inclusion (14). Suppose that a solution $\phi(t, x_0)$ is bounded. Then the limit set Ω is nonempty, closed, and bounded. Furthermore, if $\omega \in \Omega$, then there exists a solution of $\phi(t, \omega)$ of (14) with initial condition $x(0) = \omega$ satisfying $\phi(t, \omega) \in \Omega$ for all $t \geq 0$.

Proof. The first half is elementary, see for example [33, Lemma 5.30]. To prove the second half, let $\{t_k\}$ be a sequence in $[0, \infty)$ such that $t_k \rightarrow \infty$ and $\omega_k := \phi(t_k, x_0)$ tends to $\omega \in \Omega$. Let $T > 0$ be fixed, and define $\psi_k(t) := \phi(t + t_k, x_0)$ for $t \in [0, T]$. Note that $\psi_k(t)$ is a solution of (14) with the initial condition $x(0) = \omega_k$. Using a similar argument as in [34, p.13, Theorem 4] and [34, p.104, Theorem 1], one can prove there exists ψ and a convergent subsequence of $\{\psi_k\}$ where limit is ψ , and $\psi(t) = \phi(t, \omega)$ is a solution of the differential inclusion (14) with the initial condition $x(0) = \omega$. Then any point on $\psi(t)$ is a limit point of $\phi(t, x_0)$ and hence $\phi(t, \omega) \in \Omega$ for $0 \leq t \leq T$. Since this is true for any $T > 0$, this concludes the proof. ■

Lemma 4. There exists $r > 0$ such that for every ω in the set $\{(\alpha_1^{-1}(0) \setminus \alpha_2^{-1}(0)) \cup (\alpha_2^{-1}(0) \setminus \alpha_1^{-1}(0))\} \cap \{x: V(x) < r\}$ and every solution $\phi(t, \omega)$ of (12), there exists $\tau > 0$ such that $V(\phi(\tau, \omega)) < V(\omega)$.

Proof. Define

$$\dot{\alpha}_1(x) := \frac{\partial \alpha_1(x)}{\partial x} (A_1 x + B_1 u_0),$$

and recall from (9) that

$$\frac{\partial \alpha_1(x)}{\partial x} = 2((x - x^*)^T (PA_1 + A_1^T P) + (A_1 x^* + B_1 u_0)^T P).$$

It follows that $\dot{\alpha}_1(x)$ is a continuous function; moreover,

$$\dot{\alpha}_1(x^*) = (A_1 x^* + B_1 u_0)^T P (A_1 x^* + B_1 u_0) > 0,$$

and from the continuity of $\dot{\alpha}_1(x)$, $\dot{\alpha}_1(x) > 0$ in some neighborhood of x^* , say $N = \{x: V(x) < r\}$ for some $r > 0$. Let $\omega \in (\alpha_1^{-1}(0) \setminus \alpha_2^{-1}(0)) \cap N$. Since $\alpha_1(\omega) = 0$, $\alpha_2(\omega) < 0$ by (10). We can take $\tau > 0$ small enough, so $\alpha_2(\phi(\tau, \omega)) < 0$ for all $t \in [0, \tau]$. If $V(\phi(t, \omega)) = V(\omega)$ for all $t \in [0, \tau]$, then

$$\frac{d}{dt} V(\phi(t, \omega)) = \frac{\partial V}{\partial x} \frac{d}{dt} \phi(t, \omega) = 0, \quad (15)$$

for almost all t . Let $\mathcal{T} = \{t: \alpha_1(\phi(t, \omega)) > 0\}$. Note that \mathcal{T} is an open set. If $t \in \mathcal{T}$, then by (12) $F(\phi(t, \omega)) = \{A_2 \phi(t, \omega) + B_2 u_0\}$, and hence $\frac{\partial V}{\partial x} \frac{d}{dt} \phi(t, \omega) = \alpha_2(\phi(t, \omega)) < 0$. Hence $\mathcal{T} = \emptyset$. Consequently, $F(\phi(t, \omega)) = \text{conv}\{A_1 \phi(t, \omega) + B_1 u_0, A_2 \phi(t, \omega) + B_2 u_0\}$.

$B_2 u_0\}$, but $\frac{\partial V}{\partial x} A_2 \phi(\tau, \omega) + B_2 u_0 = \alpha_2(\phi(\tau, \omega)) < 0$ implies $\frac{d}{dt} \phi(\tau, \omega) = A_1 \phi(\tau, \omega) + B_1 u_0$ for almost all t . Hence $\phi(\tau, \omega)$ is the solution of the differential equation

$$\frac{dx}{dt} = A_1 x + B_1 u_0, \quad x(0) = \omega, \quad 0 \leq t \leq \tau,$$

$V(\phi(t, \omega))$ is twice continuously differentiable, and

$$\frac{d^2}{dt^2} V(\phi(t, \omega)) = \dot{\alpha}_1(\phi(t, \omega)) > 0, \quad t \in [0, \tau].$$

This implies that $\alpha(\phi(t, \omega)) > \alpha(\phi(0, \omega)) = 0$ for some t . But $\mathcal{T} = \emptyset$, and this is not possible. Hence, $V(\phi(\tau, \omega)) \leq V(\phi(t, \omega)) < V(\omega)$ for some $t \in [0, \tau]$. The proof for $\omega \in (\alpha_2^{-1}(0) \setminus \alpha_1^{-1}(0)) \cap N$ is similar. ■

Proof of Theorem 1. Since $\phi(t, x_0)$ is bounded, its limit set Ω is an invariant set by Lemma 3. Since $V(x)$ is bounded from below and $V(\phi(t, x_0))$ is monotonically non-increasing for every sequence $\{t_k\}$ such that $t_k \rightarrow \infty$ as $k \rightarrow \infty$, $c := \lim_{k \rightarrow \infty} V(\phi(t_k, x_0))$ exists. If $\omega \in \Omega$, then there exists a sequence $\{t_k\}$ such that $\omega = \lim_{k \rightarrow \infty} \phi(t_k, x_0)$. This means $V(\omega) = V(\lim_{k \rightarrow \infty} \phi(t_k, x_0)) = \lim_{k \rightarrow \infty} V(\phi(t_k, x_0)) = c$. Because Ω is an invariant set, $0 \in V(\omega)$ for any $\omega \in \Omega$. From Proposition 3, $\omega \in \alpha_1^{-1}(0) \cup \alpha_2^{-1}(0)$. Take $r > 0$ and $N = \{x: V(x) < r\}$ as in Lemma 4. If $\omega \in (\alpha_1^{-1}(0) \setminus \alpha_2^{-1}(0)) \cup (\alpha_2^{-1}(0) \setminus \alpha_1^{-1}(0)) \cap N$, then ω is not a limit point by Lemma 4. Thus, $\omega \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$. Hence, if $V(x_0) < r$, then $\phi(t, x_0)$ does not have a limit point except x^* . ■

Remark 3. Theorem 1 is a consequence of LaSalle's invariance principle proved for the differential inclusion (12). This is a useful tool to prove the stability of the switching control applied to a DC-DC Zeta converter in Section 3.2.

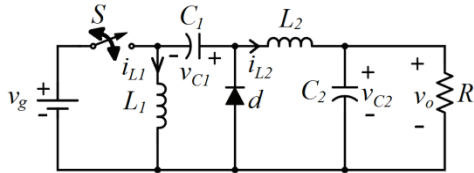


Fig. 1. A DC-DC Zeta converter circuit.

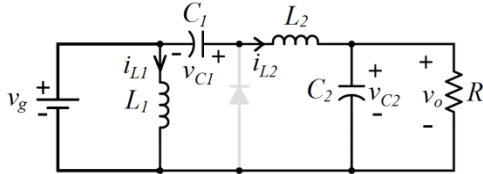


Fig. 2. Equivalent circuit when the switch is closed.

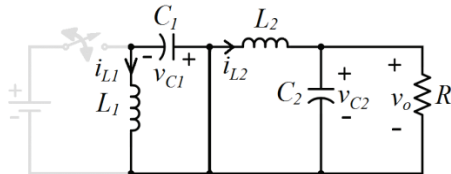


Fig. 3. Equivalent circuit when the switch is open.

3.2. DC-DC Zeta Converter Operating in Continuous Conduction Mode

Consider the DC-DC Zeta converter circuit shown in Fig. 1. The circuit consists of two inductors L_1 and L_2 , two capacitors C_1 and C_2 , an ideal diode d , a DC voltage source v_g , a resistive load R , and an ideal switch S . Denote the currents of L_1 and L_2 as i_{L1} and i_{L2} , the voltages of C_1 and C_2 as v_{C1} and v_{C2} , respectively.

The converter is in continuous conduction mode (CCM) if the diode d is open when the switch S is on and it is shorted when the switch is off. When the switch is closed (mode 1), the converter is equivalent to the circuit shown in Fig. 2, and when the switch is open (mode 2), the converter is equivalent to the circuit shown in Fig. 3. With the state vector $x = [i_{L1} \ i_{L2} \ v_{C1} \ v_{C2}]^T$ and the input $u = v_g$, the matrices for the two modes are given by

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}, \quad B_1 = \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \\ 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (16)$$

If $u_0 > 0$, then the mode 1 has no stationary solution, and the mode 2 has a unique stationary solution $x^* = [0 \ 0 \ 0 \ 0]^T$.

For $\lambda \in (0, 1)$,

$$x^* = -(\lambda A_1 + (1 - \lambda) A_2)^{-1} (\lambda B_1 + (1 - \lambda) B_2) u_0$$

$$= \begin{bmatrix} 0 & 0 & -\frac{1-\lambda}{L_1} & 0 \\ 0 & 0 & \frac{\lambda}{L_2} & -\frac{1}{L_2} \\ \frac{1-\lambda}{C_1} & -\frac{\lambda}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} \frac{\lambda}{L_1} \\ \frac{\lambda}{L_2} \\ \frac{\lambda}{C_1} \\ 0 \end{bmatrix} u_0$$

$$= \begin{bmatrix} \frac{v_g^2}{R v_g} \\ \frac{v_g}{R} \\ v_r \\ v_r \end{bmatrix} =: \begin{bmatrix} i_{L1}^* \\ i_{L2}^* \\ v_{C1}^* \\ v_{C2}^* \end{bmatrix}, \quad (17)$$

where $v_g := u_0$ and $v_r := \frac{\lambda u_0}{1 - \lambda}$. Based on the energy stored in the Zeta converter, define

$$P := \begin{bmatrix} \frac{L_1}{2} & 0 & 0 & 0 \\ 0 & \frac{L_2}{2} & 0 & 0 \\ 0 & 0 & \frac{C_1}{2} & 0 \\ 0 & 0 & 0 & \frac{C_2}{2} \end{bmatrix}. \quad (18)$$

Then

$$\begin{aligned}
 PA_1 + A_1^T P &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R} \end{bmatrix} \leq 0, \\
 PA_2 + A_2^T P &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R} \end{bmatrix} \leq 0, \\
 (A_1 x^* + B_1 u_0)^T P &= \begin{bmatrix} \frac{v_g}{2} & \frac{v_g}{2} & -\frac{v_r}{2R} & 0 \end{bmatrix}, \\
 (A_2 x^* + B_2 u_0)^T P &= \begin{bmatrix} -\frac{v_r}{2} & -\frac{v_r}{2} & -\frac{v_r^2}{2Rv_g} & 0 \end{bmatrix}. \quad (19)
 \end{aligned}$$

From (19), $\ker(PA_1 + A_1^T P) \cap \ker(PA_2 + A_2^T P) \cap \ker(A_1 x^* + B_1 u_0)^T P = \text{span}\{d_1, d_2\}$ where

$$d_1 = \begin{bmatrix} \frac{v_r}{R} \\ 0 \\ v_g \\ 0 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 0 \\ \frac{v_r}{R} \\ v_g \\ 0 \end{bmatrix}. \quad (20)$$

The function $V(x)$ decreases along the trajectory as long as $x \notin \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$. It remains to see what happens when the trajectory reaches $\alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$.

Lemma 5. Let $x \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$. Then the following properties hold:

- (a) $0 \in F(x)$ if and only if $x = x^*$.
- (b) if $x - x^* \notin \text{span}\{d_1\}$, then $F(x) \cap \ker(PA_1 + A_1^T P) \cap \ker(PA_2 + A_2^T P) \neq \emptyset$.
- (c) if $x - x^* \in \text{span}\{d_1\}$, and $x \neq x^*$, then $F(x) \cap \text{span}\{d_1\} \neq \emptyset$.

Proof. It follows from (17) that

$$A_1 x^* + B_1 v_g = \begin{bmatrix} 0 \\ 0 \\ -\frac{v_r}{c_1 R} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{v_g}{L_1} \\ \frac{v_g}{L_2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{v_g}{L_1} \\ \frac{v_g}{L_2} \\ -\frac{v_r}{c_1 R} \\ 0 \end{bmatrix}, \quad (21)$$

$$A_2 x^* + B_2 v_g = \begin{bmatrix} -\frac{v_r}{L_1} \\ -\frac{v_r}{L_2} \\ \frac{v_r^2}{c_1 R v_g} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{v_r}{L_1} \\ -\frac{v_r}{L_2} \\ \frac{v_r^2}{c_1 R v_g} \\ 0 \end{bmatrix}. \quad (22)$$

Note that $x \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ if and only if $x = x^* + \Delta x$ with

$$\Delta x = \delta_1 d_1 + \delta_2 d_2 = \begin{bmatrix} \delta_1 \frac{v_r}{R} \\ \delta_2 \frac{v_r}{R} \\ (\delta_1 + \delta_2) v_g \\ 0 \end{bmatrix}. \quad (23)$$

From this, it follows that

$$A_1 \Delta x = \begin{bmatrix} 0 \\ (\delta_1 + \delta_2) \frac{v_g}{L_2} \\ -\delta_2 \frac{v_r}{c_1 R} \\ \delta_2 \frac{v_r}{c_2 R} \end{bmatrix}, \quad A_2 \Delta x = \begin{bmatrix} -(\delta_1 + \delta_2) \frac{v_g}{L_1} \\ 0 \\ \delta_1 \frac{v_r}{c_1 R} \\ \delta_2 \frac{v_r}{c_2 R} \end{bmatrix}.$$

Hence if $x \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$, then

$$\begin{aligned}
 A_1 x + B_1 v_g &= A_1 x^* + B_1 v_g + A_1 \Delta x \\
 &= \begin{bmatrix} \frac{v_g}{L_1} \\ \frac{v_g}{L_2} \\ -\frac{v_r}{c_1 R} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ (\delta_1 + \delta_2) \frac{v_g}{L_2} \\ -\delta_2 \frac{v_r}{c_1 R} \\ \delta_2 \frac{v_r}{c_2 R} \end{bmatrix}, \quad (24)
 \end{aligned}$$

$$\begin{aligned}
 A_2 x + B_2 v_g &= A_2 x^* + B_2 v_g + A_2 \Delta x \\
 &= \begin{bmatrix} -\frac{v_r}{L_1} \\ -\frac{v_r}{L_2} \\ \frac{v_r^2}{c_1 R v_g} \\ 0 \end{bmatrix} + \begin{bmatrix} -(\delta_1 + \delta_2) \frac{v_g}{L_1} \\ 0 \\ \delta_1 \frac{v_r}{c_1 R} \\ \delta_2 \frac{v_r}{c_2 R} \end{bmatrix}, \quad (25)
 \end{aligned}$$

Hence if $\omega \in F(x)$ for $x \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0) \setminus \text{span}\{d_1\}$, then

$$[0 \quad 0 \quad 0 \quad 1] \omega = \frac{\delta_2 v_r}{c_2 R} \neq 0,$$

which shows $F(x) \cap \ker(PA_1 + A_1^T P) \cap \ker(PA_2 + A_2^T P) \neq \emptyset$ and $0 \notin \text{conv}\{A_1 x + B_1 v_g, A_2 x + B_2 v_g\}$. Suppose $x - x^* \in \text{span}\{d_1\}$, then,

$$\begin{aligned}
 \text{rank} \begin{bmatrix} A_1 x + B_1 v_g & A_2 x + B_2 v_g \end{bmatrix} &= \text{rank} \begin{bmatrix} \frac{v_g}{L_1} & -\frac{v_r}{L_1} - \delta_1 \frac{v_g}{L_1} \\ \frac{v_g}{L_2} + \delta_1 \frac{v_g}{L_2} & -\frac{v_r}{L_2} \\ -\frac{v_r}{c_1 R} & \frac{v_r^2}{c_1 R v_g} + \delta_1 \frac{v_r}{c_1 R} \end{bmatrix} \\
 &= \text{rank} \begin{bmatrix} \frac{v_g}{L_1} & -\delta_1 \frac{v_g}{L_1} \\ \frac{v_g}{L_2} + \delta_1 \frac{v_g}{L_2} & \delta_1 \frac{v_r}{L_2} \\ -\frac{v_r}{c_1 R} & \delta_1 \frac{v_r}{c_1 R} \end{bmatrix} \\
 &= \text{rank} \begin{bmatrix} \frac{v_g}{L_1} & -\frac{v_g}{L_1} \\ \frac{v_g}{L_2} + \delta_1 \frac{v_g}{L_2} & \frac{v_r}{L_2} \\ -\frac{v_r}{c_1 R} & \frac{v_r}{c_1 R} \end{bmatrix} \\
 &= \text{rank} \begin{bmatrix} 0 & -\frac{v_g}{L_1} \\ \frac{v_r + v_g}{L_2} + \delta_1 \frac{v_g}{L_2} & \frac{v_r}{L_2} \\ 0 & \frac{v_r}{c_1 R} \end{bmatrix}.
 \end{aligned}$$

If $\delta_1 \neq -\frac{v_r + v_g}{v_g}$, then $A_1 x + B_1 v_g$ and $A_2 x + B_2 v_g$ are linearly independent, and hence $0 \notin F(x)$. If $\delta_1 = -\frac{v_r + v_g}{v_g}$, then

$$\begin{aligned}
x &= x^* + \Delta x = \begin{bmatrix} \frac{v_r^2}{Rv_g} \\ \frac{v_r}{R} \\ \frac{v_r}{v_g} \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{v_r+v_g}{v_g} \left(\frac{v_r}{R} \right) \\ 0 \\ -\frac{v_r+v_g}{v_g} (v_g) \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{v_r}{R} \\ \frac{v_r}{R} \\ -v_g \\ v_r \end{bmatrix}, \\
A_1 x + B_1 v_g &= \begin{bmatrix} \frac{v_g}{L_1} \\ \frac{v_g}{L_2} \\ -\frac{v_r}{C_1 R} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{v_r+v_g}{v_g} \left(\frac{v_g}{L_2} \right) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{v_g}{L_1} \\ -\frac{v_r}{L_2} \\ -\frac{v_r}{C_1 R} \\ 0 \end{bmatrix}, \\
A_2 x + B_2 v_g &= \begin{bmatrix} -\frac{v_r}{L_1} \\ -\frac{v_r}{L_2} \\ \frac{v_r^2}{C_1 R v_g} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{v_r+v_g}{v_g} \left(\frac{v_g}{L_1} \right) \\ 0 \\ -\frac{v_r+v_g}{v_g} \left(\frac{v_r}{C_1 R} \right) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{v_g}{L_1} \\ -\frac{v_r}{L_2} \\ -\frac{v_r}{C_1 R} \\ 0 \end{bmatrix}.
\end{aligned}$$

Because $A_1 x + B_1 v_g = A_2 x + B_2 v_g \neq 0$, we have $0 \notin F(x)$. Finally, note that

$$\begin{aligned}
&\Delta x^T P (A_1 x + B_1 v_g) \\
&= \left[\delta_1 \frac{L_1 v_r}{2R} \quad \delta_2 \frac{L_2 v_r}{2R} \quad (\delta_1 + \delta_2) \frac{C_1 v_g}{2} \quad 0 \right] \left\{ \begin{bmatrix} \frac{v_g}{L_1} \\ \frac{v_g}{L_2} \\ -\frac{v_r}{C_1 R} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ (\delta_1 + \delta_2) \frac{v_g}{L_2} \\ -\delta_2 \frac{v_r}{C_1 R} \\ \delta_2 \frac{v_r}{C_2 R} \end{bmatrix} \right\} \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
&\Delta x^T P (A_2 x + B_2 v_g) \\
&= \left[\delta_1 \frac{L_1 v_r}{2R} \quad \delta_2 \frac{L_2 v_r}{2R} \quad (\delta_1 + \delta_2) \frac{C_1 v_g}{2} \quad 0 \right] \left\{ \begin{bmatrix} \frac{v_r}{L_1} \\ \frac{v_r}{L_2} \\ -\frac{v_r}{C_1 R v_g} \\ 0 \end{bmatrix} + \begin{bmatrix} -(\delta_1 + \delta_2) \frac{v_g}{L_1} \\ 0 \\ \delta_1 \frac{v_r}{C_1 R} \\ \delta_2 \frac{v_r}{C_2 R} \end{bmatrix} \right\} \\
&= 0.
\end{aligned}$$

Let $x - x^* \in \text{span}\{d_1\}$ and $x \neq x^*$. Then for every $\omega \in F(x)$, $d_1^T P \omega = 0$. Since $0 \notin F(x)$, it follows that $F(x) \cap \text{span}\{d_1\} = \emptyset$. ■

Theorem 2. Consider the differential inclusion (12) defined by the system matrices (16) and the operating point x^* in (17). Then, the operating point x^* is local asymptotically stable.

Proof. Assume that $\phi(t, x_0)$ is a solution of the differential inclusion satisfying $\phi(t, x_0) \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ for $t \geq 0$ and $x_0 \neq x^*$. If $x_0 - x^* \notin \text{span}\{d_1\}$, then $\frac{d}{dt} \phi(t, x_0) \notin \ker(PA_1 + A_1^T P) \cap \ker(PA_2 + A_2^T P)$ by Lemma 5, but this contradicts the assumption that $\phi(t, x_0) \in \alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ for $t \geq 0$. If $x_0 - x^* \in \text{span}\{d_1\}$, then from Lemma 5, there exists t_1 such that $x_1 := \phi(t_1, x_0)$ satisfies $x_1 - x^* \notin \text{span}\{d_1\}$. Then the trajectory $\phi(t, x_1)$ cannot stay in $\alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ just as we have proved before. This completes the proof. ■

Remark 4. The stability of switching control of a boost converter is proved in [32]. The state-space of the boost converter model is two dimensional, and the set $\alpha_1^{-1}(0) \cap \alpha_2^{-1}(0)$ is a singleton consisting of x^* . The state dimension of the Zeta converter model is four, and the set $\alpha_1^{-1}(0) \cap$

$\alpha_2^{-1}(0)$ includes the two-dimensional affine set spanned by d_1 and d_2 in (20). The stability of the operating point is a consequence of Theorem 1, which is a differential-inclusion version of LaSalle's invariance principle.

Remark 5. The switching mechanisms proposed in [29], [30], and [31] basically pick up the mode which nearly decreases the Lyapunov function most while our method retains the mode as long as it decreases the Lyapunov function. The trajectory of [29] evolves as a sliding mode solution when it approaches the sliding boundary, which means that the switching interval becomes infinitesimally small. The sampled-data control approach in [30] and [31] can reduce the switching frequency by adjusting the sampling period. However, the period depends on the feasibility of matrix inequality, which is a sufficient condition and hence incurs conservativeness. Our method approaches a sliding mode solution only when the trajectory is near the operating point. Further reduction of switching frequency to the predetermined level is possible by using the modified switching mechanism stated in the next section.

4. Limiting the Switching Frequency

The switching control proposed in Section 2 requires unbounded number of switching as a solution approaches the operating point x^* . In this section, the switching control mechanism discussed in Section 2 is modified to limit the switching frequency of a Zeta converter.

4.1. Modified Switching Mechanism

The switching mechanism considered in Section 2 is based on the signs of the derivatives along the trajectories (8) and (9). Let $\rho_1, \rho_2 > 0$ and $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be modified switching functions which satisfy the following:

$$\tilde{\alpha}_1(x) \geq \alpha_1(x), \quad \tilde{\alpha}_2(x) \geq \alpha_2(x), \quad (26)$$

$$\alpha_2^{-1}(\mathbb{R}_{\geq 0}) \subset \tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq \rho_1}), \quad \alpha_1^{-1}(\mathbb{R}_{\geq 0}) \subset \tilde{\alpha}_2^{-1}(\mathbb{R}_{\leq \rho_2}). \quad (27)$$

Notice that (27) is equivalent to

$$\tilde{\alpha}_1^{-1}(\mathbb{R}_{> \rho_1}) \subset \alpha_2^{-1}(\mathbb{R}_{< 0}), \quad \tilde{\alpha}_2^{-1}(\mathbb{R}_{> \rho_2}) \subset \alpha_1^{-1}(\mathbb{R}_{< 0}).$$

Based on (26) and (27), we propose the following modified switching control mechanism:

Switching Mechanism B

- If the system is operating at mode 1 and reaches $\tilde{\alpha}_1^{-1}(\rho_1)$, then it switches to mode 2.
- If the system is operating at mode 2 and reaches $\tilde{\alpha}_2^{-1}(\rho_2)$, then it switches to mode 1.

The differential inclusion (12) is modified accordingly.

$$\begin{aligned}
&\frac{dx}{dt} \in \tilde{F}(x), \quad (28) \\
&\tilde{F}(x) := \begin{cases} \{A_1 x + B_1 u_0\} & \text{if } x \in \tilde{M}_1, \\ \{A_2 x + B_2 u_0\} & \text{if } x \in \tilde{M}_2, \\ \text{conv}\{A_1 x + B_1 u_0, A_2 x + B_2 u_0\} & \text{if } x \in \tilde{M}_0, \end{cases}
\end{aligned}$$

where

$$\begin{aligned}\tilde{M}_1 &= \{x: \alpha_1^{-1}(\mathbb{R}_{<0}) \cap \tilde{\alpha}_2^{-1}(\mathbb{R}_{>\rho_2}) = \tilde{\alpha}_2^{-1}(\mathbb{R}_{>\rho_2})\}, \\ \tilde{M}_2 &= \{x: \tilde{\alpha}_1^{-1}(\mathbb{R}_{>\rho_1}) \cap \alpha_2^{-1}(\mathbb{R}_{<0}) = \tilde{\alpha}_1^{-1}(\mathbb{R}_{>\rho_1})\}, \\ \tilde{M}_0 &= \{x: \tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq\rho_1}) \cap \tilde{\alpha}_2^{-1}(\mathbb{R}_{\leq\rho_2})\}.\end{aligned}$$

Assumption 1. The sets $\tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq\rho_1}) \cap \alpha_1^{-1}(\mathbb{R}_{>0})$ and $\tilde{\alpha}_2^{-1}(\mathbb{R}_{\leq\rho_2}) \cap \alpha_2^{-1}(\mathbb{R}_{>0})$ are bounded.

Proposition 6. Suppose Assumption 1 holds. Let $c > 0$ satisfy

$$c > \sup \left\{ V(x): x \in \left(\tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq\rho_1}) \cap \alpha_1^{-1}(\mathbb{R}_{>0}) \right) \cup \left(\tilde{\alpha}_2^{-1}(\mathbb{R}_{\leq\rho_2}) \cap \alpha_2^{-1}(\mathbb{R}_{>0}) \right) \right\}.$$

Then for any solution $\tilde{\phi}(t, x_0)$ of (28), there exists $T > 0$ such that $\tilde{\phi}(t, x_0) \in \{x: V(x) < c\}$ for $t > T$.

Proof. First, we shall prove that $\tilde{F}(x) \subset F(x)$ if $x \notin \Xi_\rho := \left(\tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq\rho_1}) \cap \alpha_1^{-1}(\mathbb{R}_{>0}) \right) \cup \left(\tilde{\alpha}_2^{-1}(\mathbb{R}_{\leq\rho_2}) \cap \alpha_2^{-1}(\mathbb{R}_{>0}) \right)$. From (27), $\tilde{\alpha}_1^{-1}(\mathbb{R}_{>\rho_1}) \subset \alpha_2^{-1}(\mathbb{R}_{<0})$. So, if $\tilde{\alpha}_2(x) > \rho_2$, then

$$\begin{aligned}\tilde{F}(x) &= \{A_1x + B_1u_0\} = F(x), \quad \alpha_2(x) > 0, \\ \{A_2x + B_2u_0\} &\subset \text{conv}\{A_1x + B_1u_0, A_2x + B_2u_0\} = F(x), \quad \alpha_2(x) \leq 0.\end{aligned}$$

From Assumption 1, the number $c > 0$ exists. If $V(x_0) > c$, then a solution $\tilde{\phi}(t, x_0)$ of (28) satisfies $\tilde{\phi}(t, x_0) = \phi(t, x_0)$ as long as $\tilde{\phi}(t, x_0) \notin \Xi_\rho$ where $\phi(t, x_0)$ is a solution of (12). There exists $T > 0$ such that $V(\phi(t, x_0)) \geq c$ if $t > T$ because $V(\phi(t, x_0))$ is monotonically decreasing and $V(\phi(t, x_0)) \rightarrow 0$ as $t \rightarrow \infty$ from Theorem 1. Note that $\Xi_\rho \cap \{x: V(x) \geq c\} = \emptyset$. This implies that $\tilde{\phi}(t, x_0) = \phi(t, x_0)$ and $V(\tilde{\phi}(t, x_0)) \geq c$ for $0 \leq t \leq T$. Furthermore, $V(\tilde{\phi}(t, x_0))$ is non-increasing when $\tilde{\phi}(t, x_0) \in \Xi_\rho$. Therefore, $V(\tilde{\phi}(t, x_0)) < c$ for $t > T$. ■

4.2. Example for the DC-DC Zeta Converter

From (19),

$$\alpha_1(x) = (x - x^*)^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R} \end{bmatrix} (x - x^*) + v_g(i_{L1} - i_{L1}^*) + v_g(i_{L2} - i_{L2}^*) - \frac{v_r}{R}(v_{C1} - v_{C1}^*), \quad (29)$$

$$\alpha_2(x) = (x - x^*)^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R} \end{bmatrix} (x - x^*) - v_r(i_{L1} - i_{L1}^*) - v_r(i_{L2} - i_{L2}^*) + \frac{v_r^2}{R}(v_{C1} - v_{C1}^*). \quad (30)$$

Define

$$d_3 := \begin{bmatrix} v_g R C_1 \\ L_1 \\ v_g R C_1 \\ L_2 \\ -v_r \\ 0 \end{bmatrix}, \quad d_4 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_r \end{bmatrix}. \quad (31)$$

Then $\{d_1, d_2, d_3, d_4\}$ with d_1 and d_2 in (20) is a basis of \mathbb{R}^4 , and thus any $x \in \mathbb{R}^4$ can be written as

$$x - x^* = \Delta x = \delta_1 d_1 + \delta_2 d_2 + \delta_3 d_3 + \delta_4 d_4. \quad (32)$$

The modified functions $\tilde{\alpha}_1(x)$ and $\tilde{\alpha}_2(x)$ can be defined as

$$\tilde{\alpha}_1(x) := \alpha_1(x) + k_1 \delta_4^2 + \beta(c_1 \|\delta_1, \delta_2\|), \quad (33)$$

$$\tilde{\alpha}_2(x) := \alpha_2(x) + k_2 \delta_4^2 + \beta(c_2 \|\delta_1, \delta_2\|), \quad (34)$$

where $\|\delta_1, \delta_2\|$ is any norm in \mathbb{R}^2 , and $\beta: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is a monotone nondecreasing function satisfying

$$\beta(0) = 0, \quad \beta(z) = \rho, \quad \text{if } z \geq \rho,$$

$$0 < k_1 < \frac{v_r^2}{R}, \quad 0 < k_2 < \frac{v_r^2}{R}, \quad c_1 > 0, \quad c_2 > 0, \quad \rho > 0.$$

Proposition 7. The functions $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ defined by (33) and (34) satisfy (26), (27), and Assumption 1.

Proof. It is obvious that (26) holds. Suppose $\alpha_2(x) \geq 0$. Define $p_1(x - x^*) := \alpha_1(x) + k_1 \delta_4^2$ and $p_2(x - x^*) := \alpha_2(x) + k_2 \delta_4^2$. Then the quadratic terms of p_1 and p_2 are nonpositive, and hence by Lemma 1, we assert that $\alpha_1(x) + k_1 \delta_4^2 \leq 0$. Because $\beta(c_1 \|\delta_1, \delta_2\|) \leq \rho_1$, we obtain $\tilde{\alpha}_1(x) \leq \rho_1$. Similarly, $\alpha_1(x) \geq 0$ implies $\tilde{\alpha}_2(x) \leq \rho_2$. To show that $\alpha_1^{-1}(\mathbb{R}_{>0}) \cap \tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq\rho_1})$ is bounded, we use the representation (30) and show that the set $\{(d_1, d_2, d_3, d_4): x \in \alpha_1^{-1}(\mathbb{R}_{>0}) \cap \tilde{\alpha}_1^{-1}(\mathbb{R}_{\leq\rho_1})\}$ is bounded. If $\alpha_1(x) > 0$ and $\tilde{\alpha}_1(x) \leq \rho_1$, then

$$\rho_1 > \tilde{\alpha}_1(x) - \alpha_1(x) = k_1 \delta_4^2 + \beta(c_1 \|\delta_1, \delta_2\|) \geq \begin{cases} \beta(c_1 \|\delta_1, \delta_2\|), \\ k_1 \delta_4^2. \end{cases} \quad (35)$$

From (35), it follows that $\|\delta_1, \delta_2\| < \frac{\rho_1}{c_1}$ and $|\delta_4| < \sqrt{\frac{\rho_1}{k_1}}$.

From the definition of α_1 , $\tilde{\alpha}_1$, and d_3 , we have

$$\alpha_1(x) = k \delta_3 + \gamma(\delta_1, \delta_2, \delta_4), \quad \tilde{\alpha}_1(x) = k \delta_3 + \tilde{\gamma}(\delta_1, \delta_2, \delta_4),$$

where γ and $\tilde{\gamma}$ are continuous functions and

$$k = \frac{v_r^2 R C_1}{L_1} + \frac{v_r^2 R C_1}{L_2} + \frac{v_r^2}{R} > 0.$$

Let

$$\begin{aligned}M &:= \sup \left\{ \gamma(\delta_1, \delta_2, \delta_4): \|\delta_1, \delta_2\| < \frac{\rho_1}{c_1}, |\delta_4| < \sqrt{\frac{\rho_1}{k_1}} \right\}, \\ m &:= \inf \left\{ \tilde{\gamma}(\delta_1, \delta_2, \delta_4): \|\delta_1, \delta_2\| < \frac{\rho_1}{c_1}, |\delta_4| < \sqrt{\frac{\rho_1}{k_1}} \right\}.\end{aligned}$$

Then,

$$\begin{aligned}0 < \alpha_1(x) &= k \delta_3 + \gamma(\delta_1, \delta_2, \delta_4) \leq k \delta_3 + M, \\ \rho_1 \geq \tilde{\alpha}_1(x) &= k \delta_3 + \tilde{\gamma}(\delta_1, \delta_2, \delta_4) \geq k \delta_3 + m,\end{aligned}$$

and it follows that $-\frac{M}{k} < \delta_3 \leq \frac{\rho_1 - m}{k}$. The boundedness of the set $\alpha_2^{-1}(\mathbb{R}_{>0}) \cap \tilde{\alpha}_2^{-1}(\mathbb{R}_{\leq \rho_2})$ can be proved similarly. ■

Remark 6. From Proposition 6 and 7, we conclude that any solution of (28) converges to the set $\{x: V(x) < c\}$. The spatial regularization was studied for the boost converter in [29] to reduce the high rate of switching. The method discussed in this section extends the idea to the Zeta converter by adding extra terms in (33) and (34) to cope with the four dimensional state-space.

4.3. Estimating the Switching Frequency

Although the modified switching mechanism is able to limit the switching frequency, the value of the switching frequency itself, however, is controlled by the parameters in Switching Mechanism B. In this subsection, we will show how to decide such parameters based on a linear-line approximation of the trajectory.

From Section 4.2, the switching occurs when

$$\begin{aligned}\rho_1 &:= \tilde{\alpha}_1(x^* + \Delta x_1), \\ \rho_2 &:= \tilde{\alpha}_2(x^* + \Delta x_2),\end{aligned}$$

where $\Delta x_1 := [\Delta i_{L1a} \ \Delta i_{L2a} \ \Delta v_{C1a} \ 0]^T$ and $\Delta x_2 := [\Delta i_{L1b} \ \Delta i_{L2b} \ \Delta v_{C1b} \ 0]^T$ are the difference of the approximated state-trajectory from the operating point at their respective switching instants as shown in Fig. 4.

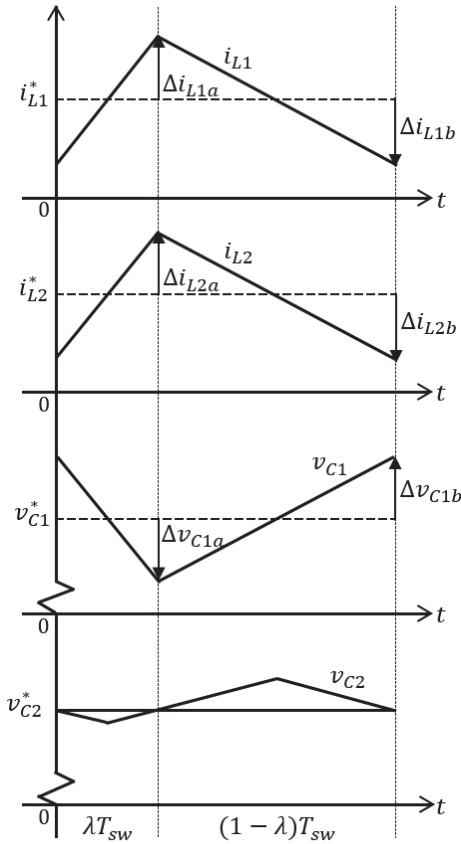


Fig. 4. Approximate state-trajectory.

Table 1 The DC-DC Zeta converter parameters.

Parameter	Value
v_g	18 V
$v_o(v_{ref})$	5 V
R	2.5 Ω
L_1	100 μ H
L_2	100 μ H
C_1	100 μ F
C_2	220 μ F
f	100 kHz

Observing Fig. 4 and from (21) and (22), the gradient of the state-trajectory at the operating point is given by

$$\frac{2\Delta x_1}{\lambda T_{sw}} = \begin{bmatrix} \frac{v_g}{L_1} \\ \frac{v_g}{L_2} \\ -\frac{v_r}{C_1 R} \\ 0 \end{bmatrix}, \quad \frac{2\Delta x_2}{(1-\lambda)T_{sw}} = \begin{bmatrix} -\frac{v_r}{L_1} \\ -\frac{v_r}{L_2} \\ \frac{v_r^2}{C_1 R v_g} \\ 0 \end{bmatrix}.$$

where $T_{sw} = \frac{1}{f}$ is the period of the switching frequency f . With $\lambda = \frac{v_r}{v_r + v_g}$ (from (17)) the above expressions can be rewritten as

$$\Delta x_1 = \begin{bmatrix} \frac{v_r v_g}{2f L_1 (v_r + v_g)} \\ \frac{v_r v_g}{2f L_2 (v_r + v_g)} \\ \frac{v_r^2}{2f C_1 R (v_r + v_g)} \\ 0 \end{bmatrix}, \quad \Delta x_2 = \begin{bmatrix} -\frac{v_r v_g}{2f L_1 (v_r + v_g)} \\ -\frac{v_r v_g}{2f L_2 (v_r + v_g)} \\ \frac{v_r^2}{2f C_1 R (v_r + v_g)} \\ 0 \end{bmatrix}. \quad (36)$$

Define penalty functions $\sigma_1 := k_1 \delta_4^2 + \beta(c_1 \|\delta_1, \delta_2\|)$ and $\sigma_2 := k_2 \delta_4^2 + \beta(c_2 \|\delta_1, \delta_2\|)$ and assume the state-trajectory near the operating point. Therefore, the penalty functions are close to 0 such that $\sigma_1 \approx 0$ and $\sigma_2 \approx 0$, consequently, $\rho_1 \approx \alpha_1(x^* + \Delta x_1)$ and $\rho_2 \approx \alpha_2(x^* + \Delta x_2)$. Nevertheless, the effect of $\sigma_1 > 0$ and $\sigma_2 > 0$ will be investigated and illustrated graphically later in Section 5. Therefore, with (17) and (36), and from (29) and (30), we have

$$\rho_1 \approx \frac{v_r(L_1 L_2 v_r^2 + C_1 L_1 R^2 v_g^2 + C_1 L_2 R^2 v_g^2)}{2f C_1 L_1 L_2 R^2 (v_r + v_g)}, \quad (37)$$

$$\rho_2 \approx \frac{v_r^2(L_1 L_2 v_r^2 + C_1 L_1 R^2 v_g^2 + C_1 L_2 R^2 v_g^2)}{2f C_1 L_1 L_2 R^2 v_g (v_r + v_g)}. \quad (38)$$

From (37) and (38), we observe how the desired switching frequency f is related to the thresholds ρ_1 and ρ_2 . Therefore, the DC-DC Zeta converter will operate at the prescribed switching frequency under the modified switching rule. Though the expressions of ρ_1 and ρ_2 look complex, they are straightforwardly processed beforehand (offline). Nowadays, considering the capability of the high-speed processors like in the DSP, FPGA, or even (maybe) microcontroller, there should be no performance issue in executing the switching mechanism.

5. Simulation Results

The simulations are carried out using the circuit simulation software PSIM® with the parameters shown in

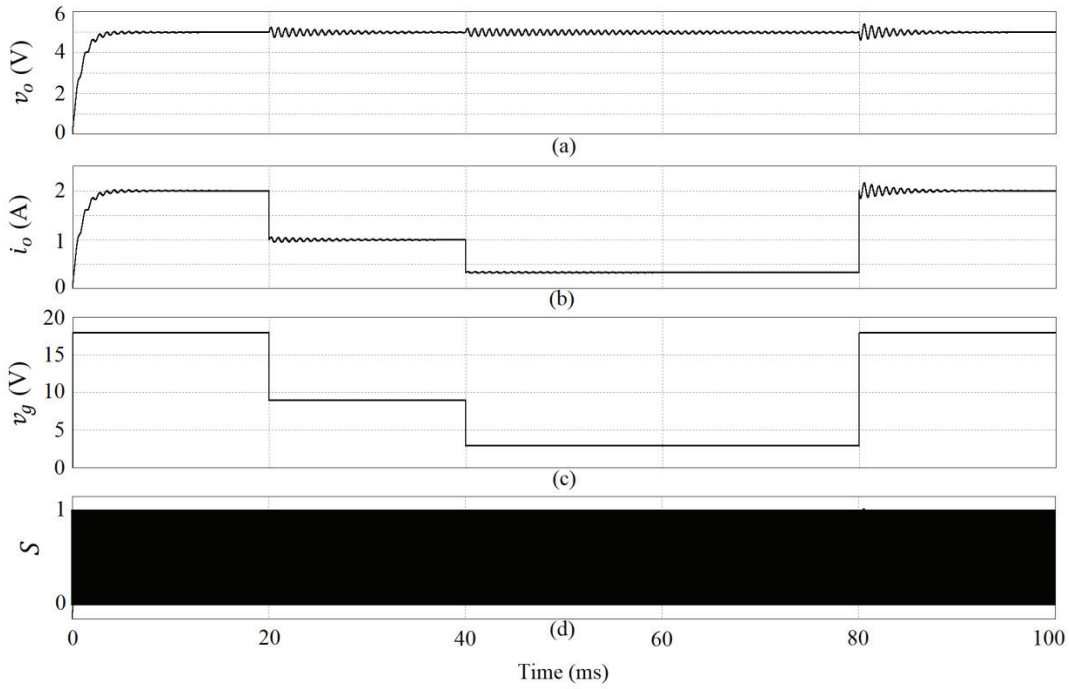


Fig. 5. Simulation results under perturbations with $\sigma_1 \approx 0$ and $\sigma_2 \approx 0$. Variations in (a) the output voltage v_o , (b) the output current i_o , (c) the input voltage v_g , and (d) the switching waveform S .

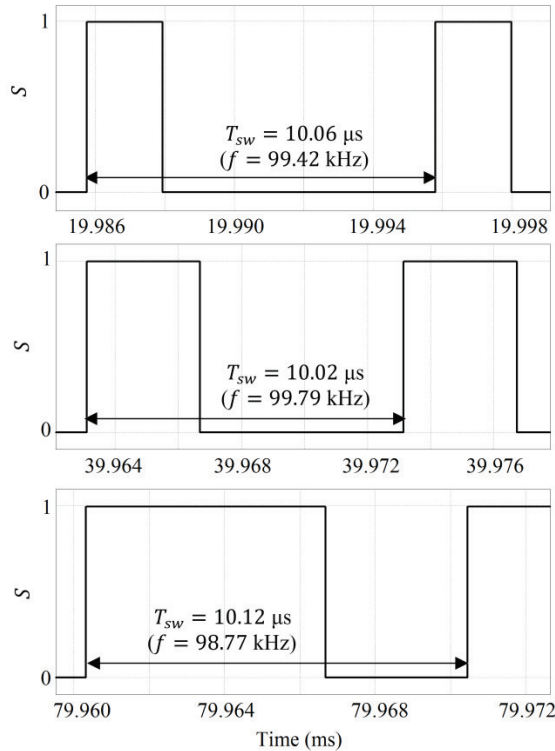


Fig. 6. Close view of the switching waveform S when $v_g = 18$ V (top), $v_g = 9$ V (middle), and $v_g = 3$ V (bottom).

Table 1. With the input voltage v_g , the capacitor voltage v_{C2} , and the load current i_o are the variables that are sensed in the circuit in Fig. 1, and fixed number computation instead of

floating-point computation is used practically to reduce computational burden; ρ_1 and ρ_2 in (37) and (38), respectively, can be rewritten as

$$\rho_1 \approx \frac{25}{100(v_g+5)} \left(25 \left(\frac{i_o}{v_{C2}} \right)^2 + (v_g^2 + 1) \right),$$

$$\rho_2 \approx \frac{125}{100v_g(v_g+5)} \left(25 \left(\frac{i_o}{v_{C2}} \right)^2 + (v_g^2 + 1) \right).$$

In Fig. 5, the simulation results for $\sigma_1 \approx 0$ and $\sigma_2 \approx 0$ are shown. As can be seen, with the nominal $v_g = 18$ V and $i_o = 2$ A, no overshoot for the output voltage v_o is observed at the start-up, and the settling time is approximately 10 milliseconds (ms). At $t = 20$ ms, v_g drops to 9 V and i_o reduces to 1 A. Despite the large input voltage drops, the overshoot at the output voltage is considerably small with some oscillations can be seen before it settles down at approximately $t = 30$ ms. Afterward, at $t = 40$ ms, the input voltage drops further to 3 V and $i_o = 0.33$ A. Similarly, although more oscillation and longer setting time are observed, nonetheless the output voltage is able to return to its operating point. Moreover, the converter is now operating in step-up mode (instead of step-down mode for the first two perturbations), thus proves the effectiveness of the switching control in regulating the output voltage at both operation modes. Finally, at $t = 80$ ms, the input voltage return to its nominal value of 18 V. Although the increment is very significant (+500 %), the switching control can regulate the output voltage well with minimum overshoot (approx. 10 %) and considerably fast settling time (approx. 8 ms). On the other hand, the steady-state switching waveforms in close view for the three different input voltage perturbations are illustrated in Fig. 6. As shown, the switching control algorithm

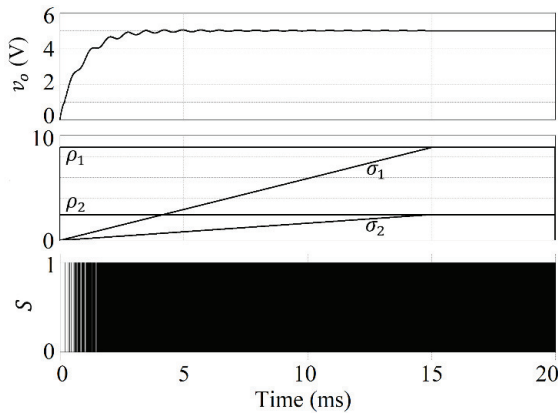


Fig. 7. Simulation results for $\sigma_1 > 0$ and $\sigma_2 > 0$.

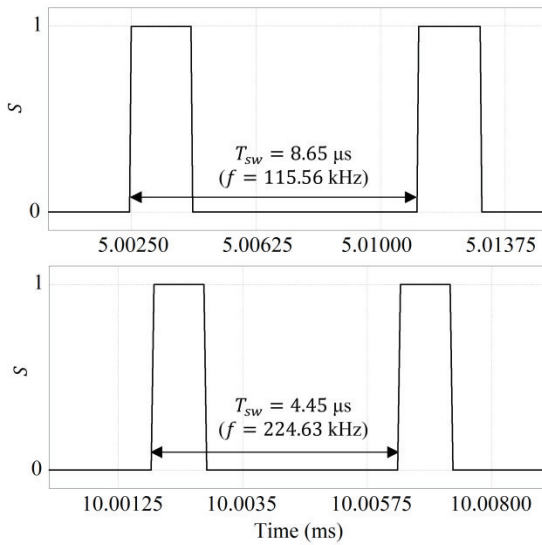


Fig. 8. Close view of the switching waveform S correspond to $\sigma_1 = 2.95$, $\sigma_2 = 0.82$ (top) and $\sigma_1 = 5.90$, $\sigma_2 = 1.64$ (bottom).

is able to produce the desired switching frequency of approximately 100 kHz for all three instances.

In the next simulation, the effect of introducing the penalty functions $\sigma_1 > 0$ and $\sigma_2 > 0$, defined in Section 4, are shown in Fig. 7. As can be observed, the introduction of σ_1 and σ_2 does not have much effect on the response of the output voltage. Increasing σ_1 and σ_2 , however, increases the switching frequency as shown in Fig. 8. These observations are expected: (37) and (38) are no longer valid, since σ_1 and σ_2 are not approximately zero. As σ_1 and σ_2 reaches ρ_1 and ρ_2 , respectively, the number of switching becomes unbounded, which is identical for the case of the switching control mechanism in Section 2.

6. Conclusion and Future Work

In this paper, we have presented a switching control mechanism which induces closed-loop stability of the DC-DC Zeta converter. A two-mode system is used to model the DC-DC Zeta converter operating in continuous conduction mode. A switching control algorithm is derived from a Lyapunov functional candidate which is basically the energy

storage function of the Zeta converter. Instrumental in our work is the establishment of an analysis of the fourth-order Zeta converter that is simple but sufficient to prove the stability of the control. Moreover, our switching control mechanism is not only able to reduce the switching frequency, but most essentially one can systematically choose the desired switching frequency for the converter to operate. Important to highlight here is how we use two different thresholds ρ_1 and ρ_2 (for mode 1 and mode 2, respectively) to define the spatial regularization, as opposed to a single common threshold for the two modes as adapted in [32]. As a result, we were able to solve the switching control law in [32] by eliminating the output voltage steady-state error. Although the approximate state-waveforms are used to find ρ_1 and ρ_2 , the close agreement between the theoretical and simulation results of the desired switching frequency shows that the approximation is indeed justified. In future, we plan to add the internal resistances in the Zeta converter model, consider the effect of interference, and most importantly validate the findings with experimental results.

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