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WAVE-BASED ACTIVE CONTROL FOR NONRECIPROCAL SOUND TRANSMISSION AND ABSORPTION

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Recent interest has arisen in the realisation of nonreciprocal acoustic devices that achieve unidirectional sound transmission. In other words, these devices allow perfect transmission from waves travelling in one direction, whilst blocking waves travelling in the opposite direction. These devices have thus been described as the acoustic analogue of an electrical diode and could be useful in many applications. There are a variety of passive and active devices that have been designed based on the concept of acoustic metamaterials to achieve unidirectional transmission by introducing non-linearities or breaking spatial and temporal symmetries. This paper will investigate how unidirectional transmission can be achieved using active control within a one-dimensional duct. The proposed active control system has the capability to manipulate either the positive or negative travelling waves within the duct in order to achieve zero transmission in one direction, whilst allowing perfect transmission in the opposite direction. The advantage of this system is not only its capability to achieve nonreciprocal sound transmission, but it is also fully adaptable.

Keywords: active noise control, nonreciprocal transmission

1. Introduction

Reciprocity is a fundamental property in acoustic wave propagation that describes the symmetry in sound transmission between two points. For example, if a wave travels from a source to an observer, the sound transmission in this case will be equal to the sound transmission in the opposite direction when the observer and source are interchanged. This symmetry is used, in many practical applications in acoustics, to simplify the measurement process, for example, obtaining head-related transfer functions measurements [1], engine and tyre noise modelling in transport [2] and seismic applications [3]. However, there are a variety of applications where reciprocity is not desirable, such as full-duplex sound communication where acoustic waves can be transmitted and received from the same transducer on the same frequency channel [4]. This has led to significant interest in recent years to design nonreciprocal acoustic devices that break the time-reversal symmetry that is inherent in conventional media. This allows these devices to achieve perfect transmission in one direction, whilst the incident waves are blocked in the opposite direction.

There has been a variety of active and passive devices proposed to achieve unidirectional sound transmission by introducing nonlinearities [5, 6, 7] or breaking the symmetry by momentum-biasing in fluid motion [8] or spatial-temporal modulation [9] in resonant cavities. Although nonlinear nonreciprocal acoustic devices have been shown to achieve unidirectional transmission, these materials typically have a number of limitations since they introduce signal distortions and only operate over a limited amplitude range [8]. There are also a number of limitations with linear nonreciprocal devices, which are typically based on the design of acoustic metamaterials, these materials require a large number cavities to achieve moderate performance over a narrow frequency range. Although a number of active nonreciprocal devices have been proposed to achieve unidirectional transmission [5, 10], whilst maintaining a lightweight and compact package, they are typically combined with passive resonators and, therefore, they still only achieve this behaviour over a narrow frequency range.

An alternative approach to achieve broadband unidirectional transmission, whilst maintaining a lightweight and compact package, is active control. Active control systems have the ability to control the transmitted and reflected waves separately in a duct [11] and create unidirectional radiation using single monopole and dipole sources [12]. Building on this previous work, this paper will investigate how broadband unidirectional sound transmission and absorption can be achieved using active control. The investigated active control system has the capability to control either the positive or negative travelling waves to achieve unidirectional sound transmission or absorption in a one-dimensional duct. The advantage of using the investigated active control system is that it is fully adaptable and tuneable. Sections 2 and 3 describe the system setup and wave decomposition used in this paper. Section 4 describes the control formulations used by the active control system to achieve unidirectional sound transmission and absorption and Section 5 presents the results of simulations when implementing the proposed active control strategies. Finally, Section 6 presents the conclusions.

2. System Setup

This section presents the system setup used in this paper to investigate how unidirectional sound transmission can be achieved using active control. This study has been carried out using the system shown in Fig. 1, which consists of a plane wave simulated duct with anechoic terminations and primary disturbance sources located at both ends, an active control system placed at the centre and two pressure microphones either side of the control source. This figure also shows the positive and negative travelling waves in the duct, which are indicated by the coefficients A to D. Evanescent waves are neglected in the model, since it is assumed that the pressure microphone locations are sufficiently spaced from the sources for the evanescent wave contribution to have decayed.

Two different active control strategies have been investigated in this paper; the first strategy uses a single monopole control source that is driven to minimise the transmitted wave, C, to achieve unidirectional sound transmission and the second strategy uses a pair of monopole control sources that are driven to minimise the reflected and transmitted waves, B and C, to achieve unidirectional sound absorption.

3. Wave Decomposition

In order to control the positive and negative travelling waves independently, these wave components need to be separated from the total pressure field using a wave decomposition method. This paper uses the integration method, summarised in [11], and a pair of microphones in the upstream and downstream sections of the duct to separate the positive and negative travelling waves as shown in Fig. 1. As described in [11], the total pressure and particle velocity at the midpoint of the pair of pressure microphones need to be calculated in order to separate the positive and negative travelling waves. Since the distance between

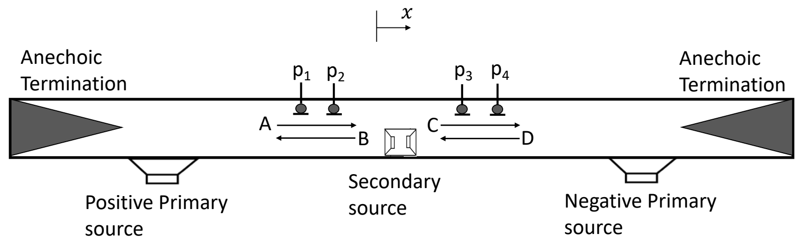


Figure 1: Diagram of the one-dimensional duct system used to investigate how unidirectional sound transmission and absorption can be achieved using active control.

the pair of microphones, Δx, is small compared to the smallest wavelength of the incident sound field, the total pressure at the midpoint, p, can be approximated as

$$p = \frac{p_1 + p_2}{2} \tag{1}$$

where p_1 and p_2 are the pressures measured at the first and second microphones respectively. Assuming only plane waves, the conservation of momentum can be expressed as

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \tag{2}$$

where ρ_0 is the ambient air density, u is the particle velocity, x is the coordinate position in the duct and t is time. In this case, the spatial derivative can be approximated by

$$\frac{\partial p}{\partial x} = \frac{p_2 - p_1}{\Delta x}. \tag{3}$$

Substituting Eq. (3) into Eq. (2), the particle velocity can be written as

$$u = \frac{1}{\rho_0 \Delta x} \int_0^t p_1 - p_2 \, dt. \tag{4}$$

The total particle velocity at the midpoint of the pair of microphones is comprised of the incident and reflected components and it can be defined as

$$u = \frac{1}{\rho_0 c_0}(A - B). \tag{5}$$

Similarly, the total pressure at the midpoint of the pair of microphones is a superposition of the incident and reflected sound fields and can be expressed as

$$p = A + B. \tag{6}$$

Combining Eqs. (5) and (6), the incident and reflected pressures can be written as

$$A = \frac{1}{2}(p + \rho_0 c_0 u) \tag{7}$$

$$B = \frac{1}{2}(p - \rho_0 c_0 u) \tag{8}$$

where c_0 is the speed of sound. Therefore, this wave decomposition method calculates the total pressure and particle velocity at the midpoint of the pair of microphones using Eqs. (1) and (5) respectively and then the positive and negative travelling waves can be calculated using Eqs. (7) and (8). The positive and negative waves in the upstream and downstream sections of the duct, which are indicated by the coefficients, A to D, in Fig. 1, have been calculated using the wave decomposition method described in this section.

4. Wave-Based Active Control Formulations

Using the wave decomposition method described in Section 3, active control can be used to manipulate the positive and negative travelling waves independently to achieve unidirectional sound transmission or absorption. This section describes the two active control strategies used in this paper: in the first, a single monopole control source is driven to minimise the positive travelling wave, C, and in the second, a pair of monopoles are driven to minimise the reflected and transmitted waves, B and C. In both cases, a leaky filtered-reference least mean squares (FxLMS) adaptive algorithm [13] has been used for feedforward control to obtain the optimal control source strengths.

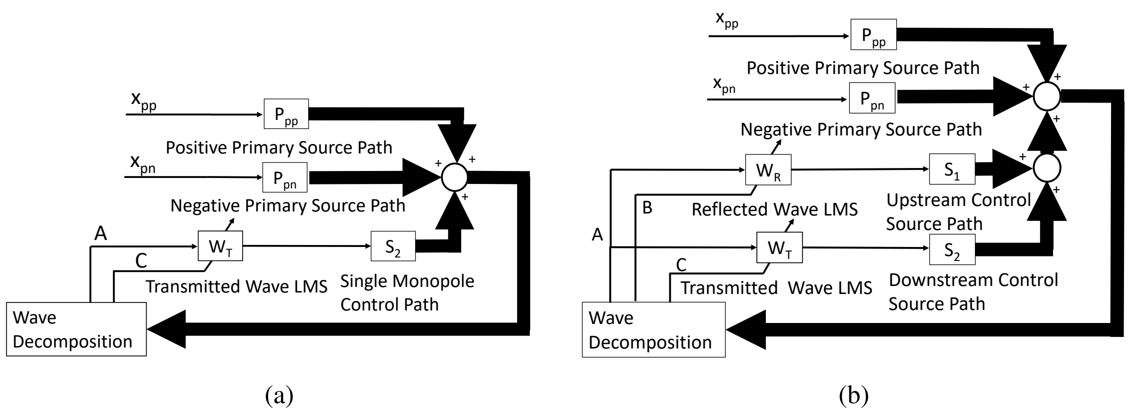


Figure 2: The feedforward control block diagrams for the unidirectional (a) transmission and (b) absorption control systems.

4.1 Nonreciprocal Sound Transmission

To realise non-reciprocal sound transmission, a single monopole control source is driven to minimise the transmitted wave, C, whilst using the positive travelling incident wave, A, as the reference signal for the FxLMS algorithm. The block diagram for this feedforward control system is shown in Fig. 2(a). The cost function for this active control strategy is the mean squared value of the transmitted wave, C, which can be defined as

$$J = |C(n)|^2 \\ \text{where } C(n) = \frac{1}{2}\left(p(n) + \rho_0 c_0 u(n)\right) \tag{9}$$

and n is the discrete time step. Substituting Eqs. (1) and (4) into Eq. (9), the transmitted wave, C, can also be expressed as

$$C(n) = d_T(n) + r_T(n)\mathbf{w}_T(n) \tag{10}$$

where

$$d_T(n) = \frac{d_3(n) + d_4(n)}{4} + \frac{c_0}{\Delta x}\int_0^t d_3(n) - d_4(n)dt, \tag{11}$$

$$\mathbf{r}_T(n) = \big[r_T(n), r_T(n-1), \ldots, r_T(n-I-1)\big] \tag{12}$$

$$r_T(n) = \frac{r_3(n) + r_4(n)}{4} + \frac{c_0}{\Delta x}\int_0^t r_3(n) - r_4(n)dt, \tag{13}$$

I is the number of tapped delays for controller, $\mathbf{w}_T(n)$ is the vector of control filter coefficients corresponding to the transmitted wave controller, $d_3(n)$ and $d_4(n)$ are the acoustic pressures due to the primary disturbance source at the third and fourth pressure microphones respectively and $r_3(n)$ and $r_4(n)$ are the filtered reference signals between the control source and the third and fourth pressure microphones respectively. The derivative of the mean squared error with respect to the control filter coefficients can then be calculated and it can be expressed as [13]

$$\frac{\partial J}{\partial \mathbf{w}_T(n)} = C(n)\frac{\partial C(n)}{\partial \mathbf{w}_T(n)} = 2\mathrm{r}_T(n)C(n) \tag{14}$$

Using the gradient given by Eq. (14), the FxLMS algorithm adapts the filter coefficients in the negative gradient direction and, thus, the FxLMS algorithm in this case can be expressed as

$$\mathbf{w}_T(n+1) = (1-\beta)\mathbf{w}_T(n) - \mu\mathbf{r}_T(n)C(n) \tag{15}$$

where μ is the convergence coefficient and β is a regularisation coefficient that has been introduced to enable limit to be imposed on the control effort.

4.2 Nonreciprocal Absorption

In the case of realising nonreciprocal absorption control, a pair of monopole control sources are used to control both the transmitted and reflected waves. The control source closer to the downstream end of the duct within the pair of monopoles is driven to minimise the transmitted wave, C, using the same procedure described in Section 4.1, whilst the control source closer to the upstream end of the duct is driven to minimise the reflected wave, B, simultaneously using two single channel FxLMS algorithms as shown in Fig. 2(b). In both LMS algorithms, the incident wave, A, is the reference signal. The control source closer to the downstream end of the duct is driven to minimise the transmitted wave, C, with the control filter coefficients being adapted according to Eq. (15). For the FxLMS algorithm that optimises the control source closer to the upstream end of the duct, the reflected wave, B, is the error signal and, therefore, the cost function in this case can be expressed as

$$\begin{aligned} J &= |B(n)|^2 \\ \text{where }\ B(n) &= \frac{1}{2}(p(n) - \rho_0 c_0 u(n)). \end{aligned} \tag{16}$$

Using the wave decomposition method described in Section 3, the reflected wave, B, can also be written as

$$B(n) = d_R(n) + r_R(n)\mathbf{w}_R(n) \tag{17}$$

where

$$d_R(n) = \frac{d_1(n) + d_2(n)}{4} + \frac{c_0}{\Delta x}\int_0^t d_1(n) - d_2(n)dt, \tag{18}$$

$$\mathbf{r}_R(n) = \left[r_R(n), r_R(n-1), \ldots, r_R(n-I-1)\right] \tag{19}$$

$$r_R(n) = \frac{r_1(n) + r_2(n)}{4} + \frac{c_0}{\Delta x}\int_0^t r_1(n) - r_2(n)dt, \tag{20}$$

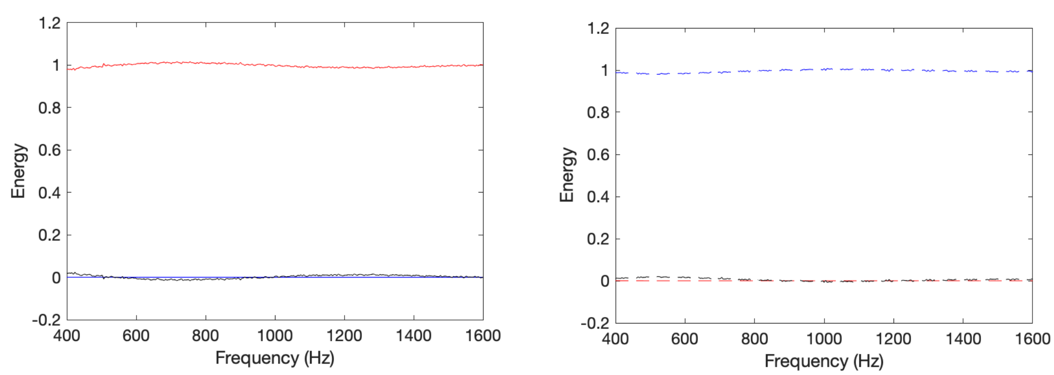
$\mathbf{w}_R(n)$ is the vector of control filter coefficients corresponding to the reflected wave controller, $d_1(n)$ and $d_2(n)$ are the acoustic pressures due to the primary disturbance source at the first and second pressure microphones respectively and $r_1(n)$ and $r_2(n)$ are the filtered reference signals between the control source and the first and second pressure microphones respectively.

The derivative of the mean squared error with respect to the filter coefficients, $\mathbf{w}_R(n)$, has then been calculated for this case and this result is used to update the FxLMS algorithm. The FxLMS algorithm in this case is

$$\mathbf{w}_R(n+1) = (1-\beta)\mathbf{w}_R(n) - \mu\mathbf{r}_R(n)B(n). \tag{21}$$

5. Simulation Results

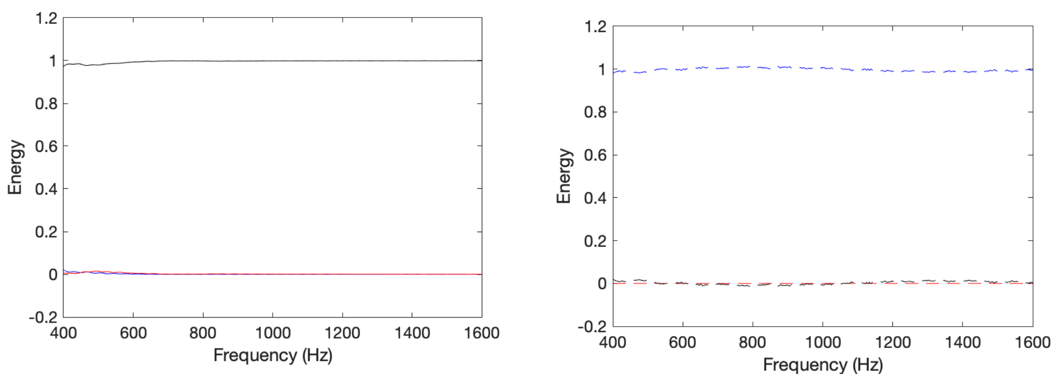
Implementing the two active control strategies described in Section 4, the performance of each strategy has been evaluated in terms of the transmitted, reflected and dissipated energy, when the incident field propagates in either the positive or negative directions. In the case of the nonreciprocal transmission controller defined in Section 4.1, which uses a single monopole control source, these results are presented in Figure 3. From Figure 3(a), which shows the results for the case of a positive travelling incident primary wave, it can be seen that the single monopole achieves zero transmission (blue), which is the objective of the controller, but the dissipated energy (black) is zero and, thus, the wave is perfectly reflected. However, when the incident field is travelling in the opposite direction, as shown by the results presented in Figure 3(b), the wave is perfectly transmitted and there is zero reflection or dissipation by the active control unit. Therefore, these results show that this active control strategy achieves broadband unidirectional sound transmission. It is worth noting that the directivity of the nonreciprocal behaviour in the proposed wave active control system can be reversed by changing the reference signal from the positive propagating incident wave A to the negative propagating incident wave D and the transmitted wave in this case is B, which shows that the proposed wave-based active control systems are fully adaptable.



(a) (b)

Figure 3: The system response when a single monopole control source is driven to minimise the transmitted wave, C. (a) and (b) show the performance of this active control strategy in terms of the transmitted (blue), reflected (red) and dissipated (black) energy when the incident field propagates in the positive (a) and negative directions (b).

Building on this concept, the results for the pair of monopole control sources that are driven to not only minimise the transmitted wave, C, but also the reflected wave, B, using the FxLMS algorithm defined by Eq. 21 are shown in Fig. 4. From Fig. 4(a), it can be seen that when the wave is incident from the positive direction, the pair of monopole control sources achieves zero transmission and reflection and, therefore, perfect dissipation of the energy related to the incident wave. In the case of a negative travelling incident wave, Fig. 4(b) shows that the wave is perfectly transmitted with zero energy reflected or dissipated. These results thus show that broadband unidirectional sound absorption can be achieved using active control, which would be desirable in sensing [14] and noise control applications. Similarly to the single monopole case, the directivity of the nonreciprocal behaviour can be reversed by changing the reference signal from A to D, and, thus, this active control system is also fully adaptable.



(a) (b)

Figure 4: The system response when a pair of monopole control sources are driven to minimise the reflected and transmitted wave, B and C. (a) and (b) show the performance of this active control strategy in terms of the transmitted (blue), reflected (red) and dissipated (black) energy when the primary source travels in the positive (a) and negative directions (b).

6. Conclusions

There has been significant recent interest in the design of nonreciprocal acoustic devices that achieve unidirectional sound transmission, however, these devices are typically based on the design of acoustic metamaterials and, thus, generally only exhibit this behaviour over a narrow frequency range. Therefore, this paper has investigated how broadband unidirectional sound transmission and absorption can be achieved using active control in a one-dimensional duct. Two different optimal feedforward control strategies have been investigated in this paper; a single monopole control source that is driven to minimise the transmitted wave and a pair of monopole control sources that are driven to minimise the reflected and transmitted waves and, thus, maximise absorption. The two proposed active control strategies have been shown to enable the realisation of broadband unidirectional control of sound transmission using a single monopole control source and sound absorption using a pair of monopole control sources. Future work should implement experimental validation of the presented methods.

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