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Analysing Newsvendor Problems: A Cash-flow Net Present Value Approach

by

Hafizah binti Zulkipli

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ABSTRACT

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Cash-flow based Net Present Value (NPV) modelling in supply chain and inventory management has not received much attention in the context of stochastic demand. It offers, however, an accurate approach to study the interactions between logistics decisions, demand pattern characteristics, and payment contract details with suppliers, and their combined effect on expected future profits. In this thesis, we examine the usefulness of this technique in the context of two-echelon supply chains in which the downstream firm (retailer) faces a newsvendor problem for each product to be sold during a selling season. In a first part, we examine the value proposition of Net D clauses. Popular opinion is that these contracts, often used in retail supply chains, hurt suppliers. Using cash-flow NPV modelling we can get insight into why this would be, and when this popular opinion is incorrect. In the second part, we show how cash-flow NPV modelling helps to design so-called mixing contracts to establish perfect coordination in such supply chains. A mixing contract includes elements of discount, buyback, and revenue sharing contracts, and the increased flexibility means that the firms can also aim to achieve good performance on other criteria besides profit. In the case of information asymmetry, we show that with a dishonest firm, both parties can still benefit. In the final part, we develop an NPV model that captures non-stationary demand over a selling season and solve it with backward induction. Firms could benefit from this approach to help exploit their knowledge about product sales variations during the normal selling period, and/or about the conditions for selling during a discount/sales period.

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Symbols/Abbreviation

Chapter 1

SCM	**S**upply **C**hain **M**anagement	
NPV	**N**et **P**resent **V**alue	
SC	**S**upply **C**hain	
NP	**N**ewsvendor **P**roblem	
TP	**T**ransfer **P**ayment	
TVM	**T**ime **V**alue **M**oney	
EOQ	**E**conomic **O**rder **Q**uantity	
CR	Trade Credit	
AC	**A**verage **C**ost	
AS	**A**nnuity **S**tream	

Chapter 2

x	Quantity demand	unit
$f(x)$	The probability density funtion of x	
$F(x)$	The cumulative distribution function of x	
α_g	Shape of gamma distribution	
τ	scale of gamma distribution	
$f(x)$	The probability density funtion of x	
p	Selling price	£/item
w	Wholesale price	£/item
c	Production cost	£/item

v	Salvage value	£/item
Q_a	Newsvendor's order quantity	units
Q	NPV-I newsvendor's order quantity	units
Q_{II}	NPV-II newsvendor's order quantity	units
α_r	Continuous capital rate for retailer	
α_s	Continuous capital rate for wholesaler	
n	Number of selling period in an accounting year	
T	Selling period	time unit
lz	Time unit for the retailer to pay to the wholesaler	days
lc	Time unit for the wholesaler to start the investments	days
lv	Time unit for the occurrence of salvage value	days
L_z	Time unit for the retailer to pay to the wholesaler	year
L_c	Time unit for the wholesaler to start the investments	year
L_v	Time unit for the occurrence of salvage value	year
$P(Q_a, x)$	Retailer's expected sales revenue	
$E[\Pi]_r$	Retailer's expected profit for traditional model	£
$E[\Pi]_s$	Wholesaler's expected profit for traditional model	£
PG_r	Percentage gain for retailer	%
PG_s	Percentage gain for retailer	%
	NPV-I Model	
$E[R]_1$	Retailer's expected NPV of sales revenue	£
$E[AS_1]_r$	Retailer's expected annuity stream of sales revenue	£
$E[AS_2]_r$	Retailer's expected annuity stream of salvage	£
$E[TP]$	Expected transfer payment from retailer to wholesaler	£
$E[AS]_r$	Retailer's expected total annuity stream profit	£
$E[AS_1]_s$	Wholesaler's expected annuity stream of production	£
$E[AS_2]_s$	Wholesaler's expected annuity stream of revenue	£
$E[AS]_s$	Wholesaler's expected total annuity stream profit	£

	NPV-II Model	
$E[R_1]^b$	Retailer's expected NPV of sales revenue	£
$E[AS_1]_r^b$	Retailer's expected annuity stream of sales revenue	£
$E[AS]_r^b$	Retailer's expected total annuity stream profit	£
Chapter 3		
TNP	**T**raditional **N**ewsvendor **P**roblem	
$NPVNP$	**NPV N**ewsvendor **P**roblem	
RS	**R**evenue **S**haring	
v	Wholesaler's salvage value for unsold items	£/item
w	wholesale price under wholesale price contract	£/item
w_m	wholesale price under mixing contract	£/item
β	the portion of revenue sharing, $0 \leq \beta \leq 1$	
b	buyback price for returned items	£/item
Q	Retailer's order quantity in wholesale price only contract	units
Q_t	retailer's order quantity for traditional Newsvendor model.	units
Q_{sc}	Order quantity for integrated NPV model.	units
Q_m	retailer's order quantity under mixing contract.	units
lb	The buyback time from wholesaler to retailer	days
lr	The time for retailer to pay RS to wholesaler	days
L_b	The buyback time from wholesaler to retailer	year
L_r	The time for retailer to pay RS to wholesaler	year
$E[AS_d^r]$	Retailer's expected annuity stream for wholesale price only contract	£
$E[AS_d^s]$	Wholesaler's expected annuity stream for wholesale price only contract	£
$E[AS_{sc}]$	The expected annuity stream for integrated firm	£
$E[AS_m^r]$	The expected annuity stream for retailer in mixing contract	£

$E[AS^{s}_{m}]$	The expected annuity stream for wholesaler in mixing contract	£
$E[AS^{sc}_{m}]$	The expected total annuity stream in mixing contract	£

Chapter 4

T	Time horizon	time unit
N	Number of days within the selling season	days
w	Wholesale price.	£/item
h	per unit inventory cost, .	£/item/day
v	Salvage value.	£/item
K	Fixed cost.	£
Q	Order quantity.	units
D_t	possible maximum demand at time t	
lz	time units for the retailer to pay to the wholesaler.	days
lv	time units for the occurrence of salvage value.	days
L_z	time units for the retailer to pay to the wholesaler.	year
L_v	time units for the occurrence of salvage value.	year
$Peak$	The peak day in selling season.	
α_r	continuous capital rate for retailer(opportunity cost).	
$f_t(.)$	Probability density function.	
$F_t(.)$	Cumulative distribution function.	
CD	Cumulative maximum demand over T	
$E[AS_r]$	Expected annuity steam of profit	
$O_t(.)$	cost function at time t	
$H_t(.)$	holding cost function at time t	
$r_t(.)$	revenue function at time t	
	Case 1: Single pricing model	
p	Selling price per unit.	£
x_t	Random demand at time t	
	Case 2:Price adjustmet model	

p_0	Selling price	£/item
p_1	Selling price after price adjustment	£/item
θ	price discount percentage	
ϑ	price sensitive parameter of demand,	$\vartheta > 0$
β_p	price elasticity, $0 < \beta_p < 1$	
a_0	maximum possible demand at highest peak at first time interval $[0, t_1]$	units
a_1	maximum possible demand at highest peak at second time interval $[t_1 + 1, T]$	units
t_1	price adjustment point	unit time
ξ	random demand	
$y(p_i)$	deterministic decreasing function	
	Markov Decision Process	
t	Set of decision epochs	
S	State at each decision epoch	
$f_t(k\|S, a)$	Probability the system is in state $k \in S$ at $t + 1$ given action a in state S at time t.	
$\mathbb{E}[SSU_t(S, a)]$	expected rewards receives at time t given action a in state S	
$\mathbb{E}[R_t(S, a)]$	expected NPV of profit receives at time t given action a in state S	
$g(S)$	Salvage function	

Declaration of Authorship

I, Hafizah binti Zulkipli, declare that this thesis entitled *Analysing Newsvendor Problems: A Cash-flow Net Present Value Approach* and the work presented in it is my own and has been generated by me as the result of my own original research. I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. None of this work has been published before submission.

Signed:

Date:

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Chapter 1

Overview

A supply chain (SC) involves an entire network of firms that work either directly or indirectly in serving the same end customer. It consists of suppliers that supply raw material, manufacturers who produce finished products from the material, warehouses and distributions centres that keep products and deliver them to retailers, and retailers who deliver the products to the end-customers (Simchi-Levi et al. 2008).

Supply chain management (SCM) can be defined as the management of activities across the entire supply chain to maximise customer value and attain a continuous competitive advantage. SC firms strive to have the most effective and efficient ways to develop and run the SC. The concept of SCM is based on three core flows in SC; the physical flows, information flows, and financial flows from the upstream suppliers to the end-customers.

Physical flows generally move downstream (forward) from the supplier to the end-customers. However, the flow can also go backward (upstream), due to product returns. The physical flows involve goods and material that require a process of transformation such as from raw-material to finished products, the movement of goods from supplier to customers, storage, and returns or rejections of products. Information flows are the invisible piece of the supply chain. A variety of information involves in

SCM, namely, pricing, bills of materials, order information and financial information. Within the sharing of information, the firms in the SC can coordinate their current and future plans, and are able to execute the daily routine of physical flow throughout the supply chain.

Lastly, financial flows are related to the cost, investment, and flow of cash. Profitability in the SC is significantly related to the optimisation of the total supply chain cost. In addition, the optimisation of the return on the capital engaged in a firm corresponds to the optimisation of investment in the SC. Firms in a supply chain have both cash in-flow (receivables) from the end-customers going backward to the other supply chain member (retailers, manufacturers, and suppliers), and cash out-flow (payable). For a better understanding of financial flows in a supply chain, the cost of capital also gives clear insight into the flow's representation. By definition, the capital costs are the monetary expenses incurred by delays in time between cash in-flow and out-flow in the firms, inclusive of value-loss and interest charges (Grubbström & Thorstenson 1986). Thus, it is essential to manage these flows properly with minimal effort to gain an effective and efficient supply chain.

However, SCM also recognises that in most situations, various parts within a supply chain are being operated by independent firms. Each firm controls only a part of the whole supply chain and has its own specific objectives. The classic players include suppliers, manufacturers, retailers, and final customers, but one can also identify many third parties playing a role in supply chains, such as the transport companies that link the various parts of the chain.

In a world where firms within a supply chain are only concerned about the optimisation of their supply chain as a whole, and where all the information needed is shared with a central decision-maker having the same aim, the management of a supply chain can be controlled by this central decision-maker, who can delegate to each of the firms the actions they need to undertake to realise the best possible plan.

When independent firms do not make any effort to coordinate their actions, they make decisions that are typically only locally optimal, and they impose constraints onto other supply chain members. As a consequence, the channel profit (i.e. the total profit across their supply chain) is typically not as high as it could be under the centralised decision maker setting, where internal constraints are not imposed, and where the objective is to maximise the profits across the supply chain.

If a centralised decision-making process can be installed, then it is possible to investigate, based on cooperative game theory principles, how to distribute the gains of centralised optimisation among the participating firms such that they are each better off participating.

In most SCs, however, the installation of a central decision-making unit is difficult to implement, and the decision-making remains decentralised. As the different parties involved in a SC are part of different individual firms, they may each want to keep control over certain aspects relevant to their own operations within the SC, they may be unwilling to share sensitive information with other firms, and in aiming to maximise their own individual profits, they may account for firm-specific opportunity costs of both monetary and resource usages that would be difficult to account for in a centralised solution.

However, the centralised optimal solution may still be used as a benchmark against which any decentralised way of working can be compared. A decentralised SC is considered to be perfectly coordinated if the channel profits achieved are the same as in the centralised situation. Simple transaction-based decentralised operations in SCs cannot usually achieve perfect coordination, and hence a more sophisticated coordination mechanism is needed. A large part of the literature is therefore concerned with identifying which types of mechanisms can achieve perfect coordination under which assumptions.

In this dissertation, we examine newsvendor-type situations. Not only are such models a good characterisation of many SC settings, but they have also been used extensively in the study of supply chain coordination through various types of contracts.

Where this study deviates from the main literature on this topic is that the modelling methodology is based on cash-flow functions. In principle, this approach has several advantages:

- A firm's cash-flow function is dependent on both the financial arrangements adopted in any contract and the logistics decisions, the impact of the timing of both processes on the net present value of the firm is explicitly incorporated. This makes it a more accurate approach compared to models that do not explicitly account for the timing of events.
- The Laplace transform of the cash-flow function is the net present value of the modelled activity for the firm. This approach therefore automates the derivation of profit functions. It automatically accounts, for example, for any opportunity costs of inventories as a consequence of the firm's order pattern from suppliers, and it automatically includes the opportunity rewards from customers' order pattern. This makes it a more robust approach than models that postulate profit functions based on intuitive principles, such as average cost methods, which often do not account for opportunity rewards.
- The relevant terms in cash-flow functions of firms that exchange cash are, under mild assumptions, skew-symmetric. This greatly facilitates the study of various coordination mechanisms as well as the game theoretic study of coordination mechanisms. Skew-symmetry is often not present in classic models of firms in supply chains in which inventories are considered.
- This study's approach is expected to lead to refinements in the theory of supply chain coordination due to the explicit modelling of not only the timings of events, but also of the opportunity costs of independent firms.

1.1 Study Background

This section provides background information regarding this study. It begins by reviewing the definition and formulation of the traditional single-period newsvendor problem (NP). In particular, Section 1.1.1 to 1.1.3 gives a detailed description of the works that have extended the traditional NP. Section 1.1.4 provides an example of problem related to the asymmetry information. Section 1.1.5 briefly defines the price-dependent demand problem. Section 1.1.6 examines the existing work on dynamic demand NP, and in particular a model of the multi-period NP that has been proposed for non-stationary demand. Finally, section 1.1.7 briefly defines net present value.

1.1.1 The Single-Period NP

The NP is an important topic in the field of inventory management to determine the optimal order quantity involving stochastic demand over a short selling season (Nahmias & Olsen 2015). The present study deals mainly the uncertain demand of perishable, or seasonal products (Fisher et al. 1994).

Hill (2011) has proposed five situations that undergo the NP requiring a one-time business decision:

- Selling seasonal goods;-
- Determining safety stock levels;-
- Making a final production run;-
- Setting target inventory levels; and-
- Making capacity decisions.

The idea of the NP started as follows: a newsvendor (retailer) needs to decide how many newspapers to order, with unknown demand. If the newsvendor orders too many, at the end of the day he may have excess items, thus acquiring holding costs if he keeps the items or recoups salvage value for the excess inventory. If the newsvendor orders too few, on the other hand, he loses sales and profit, as he may not be able to meet all his customers' demand. Since the NP is the basis of our proposed model, a short introduction and general formulation of the model is described in Section 1.1.2 below.

1.1.2 Underlying Newsvendor Problem

Suppose that the retailer decides to order quantity Q_a at the beginning of the selling season and to pay at the wholesale price w per item ordered. If the realised demand in this period does not exceed the order quantity, then the retailer's revenue is px, and the unsold items in inventory are salvaged at $v(Q_a - x)$. If the realised demand is greater than Q_a, the retailer's revenue is pQ_a.

The retailer's revenue function is given as;

$$P(Q_a, x) = \begin{cases} px + v(Q_a - x), & \text{if } x \leq Q_a, \\ pQ_a, & \text{if } x \geq Q_a. \end{cases} \tag{1.1}$$

Let the subscripts r and s denote the retailer and wholesaler in the supply chain, respectively. The retailer's objective function is to maximise the expected total profit. The equation is:

$$\begin{aligned} \max_{Q_a} E[\Pi]_r &= p\Big(\int_0^{Q_a} xf(x)dx + Q_a \int_{Q_a}^{\infty} f(x)dx\Big) + v\int_0^{Q_a}(Q_a - x)f(x)dx \\ &- TP. \end{aligned} \tag{1.2}$$

The wholesaler's expected profit is given as:

$$[E[\Pi]_s = TP - Q_a, \tag{1.3}$$

where TP is the transfer payment made from the retailer to the wholesaler. Let consider a wholesale-price only contract. The wholesaler charges w per item to the retailer. Therefore, $TP = wQ_a$. By substituting TP into Eq.(1.2), we have:

$$\begin{aligned} \max_{Q_a} E[\Pi]_r &= p\Big(\int_0^{Q_a} xf(x)dx + Q_a \int_{Q_a}^{\infty} f(x)dx\Big) + v\int_0^{Q_a} (Q_a - x)f(x)dx \\ &- wQ_a. \end{aligned} \tag{1.4}$$

The first order condition of $E[\Pi]_r$ w.r.t Q_a for a given w is

$$dE[\Pi]_r/dQ_a = p\Big(\int_0^{\infty} f(x)dx - \int_O^{Q_a} f(x)dx\Big) + v\int_0^{Q_a} f(x)dx - w$$

.

Since $\int_0^{\infty} f(x)dx = 1$, and $\int_0^{Q_a} f(x)dx = F(Q_a)$, then the equation above can be simplified to;

$$dE[\Pi]_r/dQ_a = p - w + F(Q_a)(-p + v)$$

.

The second order condition of $E[\Pi]_r$ w.r.t Q_a is;

$$dE^2[\Pi]_r/dQ_a^2 = -(p - v)f(Q_a) < 0,$$

respectively. Thus, the optimal order quantity Q_a^* that maximises the objective function is given by;

$$Q_a^* = F^{-1}\Big(\frac{p-w}{p-v}\Big), \tag{1.5}$$

where F is the cdf of the demand distribution and F^{-1} is its inverse.

The wholesaler's expected profit function is given by;

$$E[\Pi]_s = (w-c)Q_a. \tag{1.6}$$

However, when decisions in a supply chain are made individually, the system optimal profits cannot be achieved due to the double marginalisation problem. To rectify this, various supply contracts are needed. Section 1.1.3 provides more details on the double marginalisation problem and contracts to coordinate the supply chain. All models analysed in this thesis follow a NP.

1.1.3 Coordination and Contract

Spengler (1950) was the first to introduce the concept of ***vertical integration*** and the term ***double marginalisation***. Double marginalisation occurs when at least two independent firms exist in a supply chain, instead of a single firm. Each firm applies its own markup in price, resulting in ordering too little, as compared with an integrated supply chain.

Perfect coordination happens when the channel profits in a decentralised supply chair are similar to those in the centralised situation. Thus, the optimal order quantity in an integrated supply chain is used as a benchmark to coordinate the supply chain. In addition, Cachon (2003) highlights two points that are needed to coordinate the supply chain:

- The set of supply chain optimal actions is a Nash equilibrium – that is, no firm has a profitable unilateral deviation from the set of supply chain optimal actions.
- The action to coordinate is the retailer's order quantity.

The model in Chapter 4 is developed based on coordination in SC.

Below, we introduce a simple example to introduce a key concept in the study of coordination. Specifically, this example illustrates the classic double marginalisation problem and why it is a common root cause of inefficiency in decentralised supply chains.

Example 1 Consider a supply chain with a wholesaler delivering goods to a retailer. The retailer's demand Q is price-sensitive according to the function $Q(p) = a - bp$, where p is the retailer price and a and b are positive constants ($0 < p < a/b$). The wholesaler's unit cost to produce a product is c and charges the retailer a transfer payment TP. The retailer does not incur any other costs. Both firms are monopolists in that they can set their sales prices. Since $c < p$ is a necessary condition to ensure profitability in the supply chain, if follows that $c < a/b$.

In this decentralised supply chain, the sequence of events is as follows. In step 1, the wholesaler sets TP. In step 2, the retailer observes T and sets p.

The profit function of retailer and wholesaler is, respectively, given by;

$$\pi_r = pQ(p) - TP \tag{1.7}$$

and

$$\pi_w = TP - cQ(p). \tag{1.8}$$

Vertical integration. In the vertically integrated supply chain, the power of decision making is given to an unbiased central decision maker, who has perfect information and aims to maximise the channel profit that is given as;

$$\pi_{SC} = \pi_r + \pi_w = (p - c)Q(p) = (p - c)(a - bp). \tag{1.9}$$

The first order condition gives the optimal price:

$$p^*_{SC} = \frac{a + bc}{2b} = \frac{a}{2b} + \frac{c}{2}. \tag{1.10}$$

The channel profits when setting the retail price at p^*_{SC} are therefore:

$$\pi_{SC}(p^*_{SC}) = (p^*_{SC} - c)Q(p^*_{SC})$$

$$= (p^*_{SC} - c)(a - bp^*_{SC}) = \frac{1}{4b}(a - bc)^2. \tag{1.11}$$

The wholesale-price contract. In this case, the wholesaler charges the retailer a unit price w, and thus $TP = TP_w = wQ(p)$. The profit function of retailer and wholesaler is, respectively, given by:

$$\pi_r = (p - w)Q(p) \tag{1.12}$$

and

$$\pi_w = (w - c)Q(p). \tag{1.13}$$

In order to determine the wholesaler's optimal decision in step 1, we work by backward induction. Given a wholesale price w, the retailer's decision in step 2 is to set a retail price that maximises π_r. From the first order condition, we derive this optimal price to be:

$$p(w) = \frac{a + bw}{2b}, \tag{1.14}$$

and therefore, this corresponds to a demand:

$$Q(w) = \frac{a}{2} - \frac{bw}{2}. \tag{1.15}$$

So, the wholesaler's profit function:

$$\pi_w = (w - c)Q(p(w)) = (w - c)(\frac{a}{2} - \frac{bw}{2}), \tag{1.16}$$

is optimised by taking:

$$w^* = \frac{a}{2b} + \frac{c}{2}. \tag{1.17}$$

Therefore, in step 2, the retailer will set the optimal retail price based on w^*, and finds:

$$p^* = \frac{3a}{4b} + \frac{c}{4}. \tag{1.18}$$

The channel profits in the decentralised scenario are:

$$\pi_{SC}(p^*) = (p^* - c)Q(p^*) = \frac{3}{16b}(a - bc)^2. \tag{1.19}$$

Comparison with the vertically integrated solution gives;

$$\frac{\pi_{SC}(p^*_{SC})}{\pi_{SC}(p^*)} = \frac{4}{3}, \tag{1.20}$$

or the benefit of the centralised decision making process is that it increases the channel profit with 33.3%. A contract based on a wholesale price cannot perfectly coordinate the supply chain.

Proof. It is easy to verify that $p^*_{SC} < p^*$ if and only if $c < a/b$; and then $Q(p^*_{SC}) > Q(p^*)$. The vertically integrated supply chain acting as a monopolist will set the retail price below the retail price set in the decentralised supply chain. As a result, more products will be sold to the market. ■

The example above provides insights into how to overcome the double marginalisation problem. With the 'right' contract, perfect coordination can be achieved. Cachon (2003) briefly reviews the contracts in the NP. For a better understanding of the wholesaler-retailer contract, a sequence of events relative to the time frame is presented in Figure 1.1.

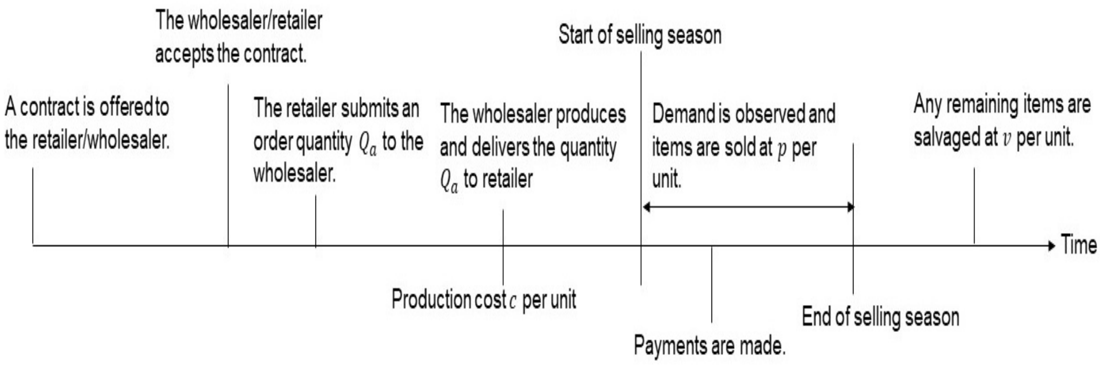


Figure 1.1: Sequence of events within wholesaler-retailer contract.

A number of contracts that coordinate the newsvendor setting have been studied in the literature. These contracts are characterised based on different parameters used to design the model. We refer interested readers to Khouja (1999), Petruzzi & Dada (1999), Lariviere & Porteus (1999), Cachon (2003), and Qin et al. (2011) for detailed reviews of the NP and contracts of coordination.

1.1.3.1 Wholesale-Price-Only Contract

The wholesale price contract cannot coordinate a supply chain (see Bernstein & Federgruen (2005); Cachon (2003);Lariviere & Porteus (2001)). However, due to its simplicity, this contract has received much attention and is set as a benchmark for the other contracts proposed in the literature. In this contract, the retailer is given the wholesale price, w per item from the wholesaler. The transfer payment TP from retailer to wholesaler is $TP = wQ_a$. Restating the Eqs. (1.4), (1.5), and (1.6), we have:

$$E[\Pi]_r = p\Big(\int_0^{Q_a} xf(x)dx + Q_a\int_{Q_a}^{\infty} f(x)dx\Big) + v\int_0^{Q_a}(Q_a - x)f(x)dx - wQ_a,$$

$$Q_a^* = F^{-1}\Big(\frac{p-w}{p-v}\Big),$$

and

$$E[\Pi]_s = (w-c)Q_a,$$

which represents retailer's expected profit, optimal order quantity, and wholesaler's expected profit, respectively.

1.1.3.2 Buyback Contract

Under a buyback contract, the wholesaler first sells a product to the retailer at a unit wholesale price w. w is assumed to be greater than the wholesaler's operation cost c. The retailer can then return any unsold items to the wholesaler at a unit buyback price b, which is $b > v$, and the wholesaler salvages the excess inventory at salvage value v per unit. The transfer payment in this setting is $TP = wQ_a - b(Q_a - x)^+$. By substituting TP into Eqs. (1.2) and (1.3), we have:

$$E[\Pi]_r = p\Big(\int_0^{Q_a} xf(x)dx + Q_a\int_{Q_a}^{\infty} f(x)dx\Big) + b\int_0^{Q_a}(Q_a-x)f(x)dx - wQ_a$$

and

$$E[\Pi]_s = (w-c)Q_a + (v-b)\int_0^{Q_a}(Q_a-x)f(x)dx,$$

which give;

$$Q_a^* = F^{-1}\Big(\frac{p-w}{p-b}\Big).$$

This type of contract has been implemented in industries such as publishing, apparel, and cosmetics (Kandel 1996, Emmons & Gilbert 1998).

1.1.3.3 Revenue-Sharing Contract

Under a revenue-sharing contract, the retailer shares a percentage of β of her revenue with the wholesaler. In return, the wholesaler lowers the price to w_m per unit purchased. The transfer payment is $TP = w_mQ_a + \beta p\Big(\int_0^{Q_a} xf(x)dx + Q_a\int_{Q_a}^{\infty} f(x)dx\Big)$. Thus, we have:

$$E[\Pi]_r = (1-\beta)p\Big(\int_0^{Q_a} xf(x)dx + Q_a\int_{Q_a}^{\infty} f(x)dx\Big) + v\int_0^{Q_a}(Q_a - x)f(x)dx - w_mQ_a$$

and

$$E[\Pi]_s = (w_m - c)Q_a + \beta p\Big(\int_0^{Q_a} xf(x)dx + Q_a\int_{Q_a}^{\infty} f(x)dx\Big),$$

which give;

$$Q_a^* = F^{-1}\Big(\frac{(1-\beta)p - w_m}{(1-\beta)p - v}\Big).$$

Giannoccaro & Pontrandolfo (2004), Cachon & Lariviere (2005), and Yao et al. (2008) are among of the authors that investigate this type of contract on various aspects such as inventory, risk adverse, and incomplete information.

1.1.4 Information Asymmetry

We first introduce a simple example of a profit-sharing contract to discuss the issue of dishonest firms. This topic has gained in importance in the recent literature, and it is related to the study of the value of information sharing in supply chains. A part of Chapter 3 assumes that there is information asymmetry in the wholesaler's production cost and opportunity cost.

Example 2 The following illustrates how the decentralised supply chain may arrive at the centrally optimal solution by a profit-sharing contract. The idea is to make the profit function of the decision maker in step 2 be an affine transformation of the supply chain profit function. Therefore, let:

$$\pi_w = \beta \pi_{SC}, \tag{1.21}$$

where β ($0 \leq \beta \leq 1$) represents the fraction of channel profits awarded to the wholesaler. Then $(1-\beta)$ is the fraction awarded to the retailer. Indeed, we can rewrite (1.21) to:

$$\Leftrightarrow \pi_w = \beta(\pi_w + \pi_r)$$

$$\Leftrightarrow \pi_r = (1-\beta)(\pi_w + \pi_r) = (1-\beta)\pi_{SC}.$$

Therefore, the retailer in step 2, in maximising her own profit function π_r, will arrive at the optimal p^*_{SC}.

To find the payment structure T that can realise this, we work out condition (1.21):

$$\Leftrightarrow T - cQ(p) = \beta(p-c)Q(p)$$

$$\Leftrightarrow T = (\beta p + (1-\beta)c)Q(p).$$

Choosing $T = T_p = w_p Q(p)$, we get:

$$\Leftrightarrow w_p(p) = \beta p + (1-\beta)c = c + \beta(p-c). \tag{1.22}$$

The decentralised supply chain can thus arrive at the optimal integrated solution if a wholesale price w_p is set as the wholesaler's marginal cost plus a fraction of the supply chain marginal profit. Rearranging (1.22):

$$w_p(p) = \beta p + (1-\beta)c = \gamma + \delta p. \tag{1.23}$$

The wholesaler could thus *perfectly coordinate* the actions of the retailer by specifying a contract in which the wholesale price is given by (1.23). This contract could also distribute the gains of reaching the integrated channel profits in any acceptable manner by specification of $\delta(=\beta)$.

There are numerous ways in which the contract can be implemented. The wholesaler could charge w_p upon delivery of the batch. This corresponds to the classic wholesale-price contract arrangement. Another way is to let the wholesaler deliver all the goods for free initially, and receive payments at the level of w_p each time the retailer sells the products. Or, the wholesaler could charge initially c per product when delivering the batch, and receive a fraction of the profits $\gamma(p-c)$ upon sales. In practical terms these different forms of implementation will have different cash-flow consequences for the firms which are not explicitly incorporated in the model.

Information asymmetry. The cost structure of a firm is one of the most sensitive pieces of information that a firm is not readily willing to reveal. In this example, the wholesaler might for example not be willing to tell the retailer its real marginal cost c. However, assuming that the retailer knows the modeling logic and thus (1.23), the wholesaler, by specifying the contract for the *optimal* values of γ and δ, *reveals* his true marginal cost c to the retailer since this is specified from the system of equations:

$$\delta = \beta,$$

$$\gamma = (1-\beta)c.$$

For example, if the wholesaler sets the contract at $w_p(p) = 0.6p+4$, the retailer derives $\beta = 0.6$ and $c = 10$. If this contract is to perfectly coordinate the supply chain, then the true marginal cost of the wholesaler needs to be 10.

Dishonest firms. Information problems between the two firms leads to divergence from the integrated supply chain solution. Suppose, for example, that the wholesaler wishes to cheat by setting the contract at $w_p(p) = 0.6p + 5$, while claiming that this would perfectly coordinate their supply chain. Then the retailer would derive $c = 12.5$. If the wholesaler's true marginal cost is 10, however, then the contract would no longer perfectly coordinate the supply chain. However, the wholesaler might still wish to cheat, as illustrated in the numerical example below.

Numerical example. Let $a = 200$, $b = 5$, and $c = 10$.

In the vertically integrated solution under perfect information, channel profits are optimised for $p^* = 25$, giving $Q^* = 75$. The channel profit is $\pi^*_{SC} = 1125$.

With a wholesale-price contract, we find $w^* = 25$, $p^* = 32.5$, $Q^* = 37.5$. The profits of the firms are $\pi_r = 281.25$, $\pi_w = 562.5$, and the channel profit is thus $\pi_{SC} = 843.75$. As proven earlier, $\pi^*_{SC}/\pi_{SC} = 4/3$.

Under a profit-sharing arrangement with $\beta = 0.6$ and perfect information, the wholesale price is set in the contract to $w_p(p) = 0.6p + 4$. From maximising its own profit function, the retailer derives the optimal sales price $p^* = 25$, and thus $w^*_p = 19$ and $Q^* = 75$. The profits of the firms are $\pi_r = 450$, $\pi_w = 675$, and the channel profit equals the profit obtained in the vertically integrated solution, $\pi^*_{SC} = 1125$. The contract thus perfectly coordinates the supply chain. With $\beta = 0.6$, both firms are also better off than in the decentralised setting.

Now assume that the retailer does not know the wholesaler's marginal cost, and that the wholesaler proposes the contract $w_p(p) = 0.6p + 5$. From maximising its own profit function, the retailer then derives the optimal sales price $p^* = 26.25$, and thus $w^*_p = 20.75$ and $Q^* = 68.75$. The retailer's profit is then $\pi_r = 378.125$. The wholesaler, claiming this contract to be the one that perfectly coordinates the channel, signals to the retailer that his marginal cost is $12.5 = c + 2.5 = c'$. The retailer would then assume that the wholesaler makes a profit $\pi'_w = (w^*_p - c')Q^* = 567.1875$ and that the optimal channel profit is $\pi_r + \pi'_w = 945.3125$.

Also, according to this misinformation that c' is the wholesaler's marginal cost, the retailer could calculate that in the uncoordinated decentralised setting, one would find $w^* = 26.5$, $p^* = 33.125$, $Q^* = 34.375$, and that the profits of the firms would be $\pi_r = 236.3281$, $\pi_w = 558.5938$, and the channel profit $\pi_{SC} = 794.9219$. According to the retailer, it would hence make sense to adopt the wholesaler's contract $w_p(p) = 0.6p + 5$. In fact, the retailer would observe a much more generous increase in its profits that the increase in profits that the wholesaler would make.

In reality, however, the wholesaler's profit under the offered contract would be $\pi_w = (w_p^* - c)Q^* = 739.0625$. The channel profit under this contract thus amounts to $\pi_r + \pi_w = 1117.188$. The wholesaler would not mind deviating from the optimal integrated solution since his profits are significantly higher than those he makes in the profit-sharing contract under perfect information (739 $>>$ 675).

Also note that the optimal quantity in the dishonest scenario is lower than in the perfect information case. If the wholesaler would be a manufacturer producing at marginal cost c, then by being dishonest, he would not only make more profits, but also use up less of his production capacity, which reduces his opportunity cost of resource usage. Finally, the wholesaler by being dishonest has given the retailer misinformation about his true marginal cost. A naive retailer will assume this marginal cost to be 12.5 and thus higher than the true value, and a smarter retailer, suspecting the wholesaler might be dishonest, only knows that the true marginal cost is not higher than 12.5. Keeping his true marginal cost as private information might be valuable to the wholesaler in future negotiations. All in all, it seems that the wholesaler has very good reasons for being dishonest.

1.1.5 Price-dependent Demand

Many studies consider pricing and order quantity as decision variables (Petruzzi & Dada 1999). The randomness of price-dependent demand is modelled using either an additive case Mills (1959) or a multiplicative case (Karlin & Carr 1962). For an additive case, demand is defined as $x(p, \xi) = y(p) + \xi$, and for a multiplicative case, $x(p, \xi) = y(p)\xi$. $y(p)$ denotes a decreasing function that expresses the dependency between selling price and demand, and ξ represents a random variable defined in the range of $[A, B]$. $y(p)$ in additive case follows a linear form, $y(p) = a - bp$, and in a multiplicative case it follows an isoelastic form with constant elasticity, $y(p) = ap^{-b}$, where a, and b are given parameters.

This scope of modelling is presented in Chapter 5 considering a multiplicative case only, and assuming that the selling price is a known parameter.

1.1.6 Dynamic Programming

Dynamic programming (DP) is based on the recursive process that describes the relationship between the value of being in a state at one point in time and the value of being in the states that we will visit next, following our decision. The Markov decision process (MDP) method is used to solve this problem. The MDP consists of five elements: decision epoch, state, action, transition probabilities, and rewards. The optimal policy for MDP is one that provides the optimal solution to all sub-problems of the MDP (Bellman 1957).

Example 3 Consider a model of an inventory control system where a manager needs to decide whether or not to order additional stock from the supplier based on the current inventory at period of time t (month) (Puterman 2014). Let s_t denotes the inventory on hand and let D_t be the random demand at the beginning of month t. The demand is assumed to be independent and identically distributed (i.i.d) with a known probability distribution $p_j = P(D_t = j)$, $j = 0, 1, 2, ...$. At decision epoch $t + 1$, the inventory s_{t+1} is related to the inventory at decision epoch t, s_t, with the equation given below:

$$s_{t+1} = max\{s_t + a_t - D_t, 0\} = [s_t + a_t - D_t]^+, \tag{1.24}$$

where a_t denotes the action at state t, which represents the additional stock to order at time t from a certain set of A. The transition probability of moving from state s_t to state s_{t+1} is $p_t(s_{t+1}|s_t, a_t)$. When the reward depends on the state of the system at the next decision epoch, let $r_t(s_t, a_t, s_{t+1})$ denote the value at time t of the reward received at the decision epoch t. Thus, the total expected reward is given as:

$$r_t(s_t, a_t) = \max_{a_t \in A}\{r_t(s_t, a_t) + \mathbb{E}[\sum_{j \in S} r_t(s_t, a_t, j)p_t(j|s_t, a_t)], \} \tag{1.25}$$

which follows Bellman's equation. In Chapter 5, we consider a dynamic demand problem where the objective function is to find the optimal order quantity that maximises the expected annuity stream of profit. Later, we modify Bellman's equation to the case in which we only need one action at one decision epoch.

1.1.7 Net Present Value for the Inventory Model

The time value of money (TVM) is the concept that money at the present time is worth more than the same amount of money in the future. In other words, it refers to a present discounted value or NPV. Money has time value because of the unpredictable risk and uncertainty in the future. In addition, the TVM is related to the opportunity cost of not having the money earlier.

For a long time, the NPV principle has been studied in a mass of production-inventory problems, such as in (Trippi & Lewin 1974). For a comprehensive review related to NPV, see Beullens (2014). As stated in Van der Laan & Teunter (2002), there are three main arguments against the average cost function, which is mainly applied as an approximation to the NPV approach. First, the TVM is not explicitly taken into account, but only implicitly by including opportunity costs. Second, the traditional approach only considers the opportunity cost of holding inventory and disregards all other cash-flows (fixed costs, sales, etc.) that generate opportunity costs. Lastly, the conditions of; 1. fast moving items, 2. lower interest rates, and 3. the payment structure of customer does not depend on the inventory policy, are neglected. The authors conclude that when comparing the NPV approach with the average cost (AC) approach, the class for the transformation of model parameters either exists (both approaches are identical) or does not exist. An example of the latter class is a system with manufacturing, re-manufacturing, and disposal.

Grubbström (1980) concluded that the NPV framework should be considered rather than the traditional cost approach, since the economic consequences of production planning decisions need to be known; this is more realistic. Later, Gurnani (1983) presented the NPV analysis for various inventory systems assuming a given constant planning horizon. The results in Gurnanis' paper show that in the NPV model, the optimal order quantity and the length of the period constantly vary with the discount rates.

Grubbström (1980) states that the NPV of an activity for firm i follows the Laplace transform of a cash-flow function $a_i(t)$), where the Laplace frequency is the continuous capital rate α_i of the firm:

$$NPV_i = \int_0^\infty a_i(t)e^{-\alpha_i t}dt. \tag{1.26}$$

For a discussion of the Laplace transform, see, e.g., (Buck & Hill Jr 1971, Hill Jr & Buck 1974).

The annuity stream function AS_i, for an infinite horizon model is defined as $AS_i = \alpha_i NPV_i$. Thus;

$$AS_i = \alpha_i \int_0^{\infty} a_i(t) e^{-\alpha_i t} dt. \tag{1.27}$$

Then, the generalisation of the annuity streams (AS) function is based on the Maclaurin expansion of the exponential terms in the decision variable, which can later be directly compared to the classical inventory functions. The linearised NPV and AS functions have been adopted in many studies related to NPV, see (Kim et al. 1984, Teunter & Van der Laan 2002, Grubbström 2010, Beullens & Janssens 2011, 2014).

In brief, the main advantage of deriving profit functions from the NPV approach instead of the traditional inventory framework is that the appearance of cash-flows (inflow and out-flow) differs with the physical transactions that move products through the system. Although the NPV approach is often rather complicated, one can derive the NPV of the cash-flows by using AS functions, which are linear approximations in the discount factor. All models developed in this thesis adopt the NPV approach, which optimises a discounted cash-flow of future revenues and costs.

1.1.7.1 Payment Structure

At a particular moment in time, a cash-flow function may have a discrete payment or continuous payment over a period of time intervals. Beullens & Janssens (2014) provides the details of payment structures in the NPV approach. When there is more than one firm, the payment structures between these firms specify at what future point in time, relative to the event time t, which amount of the event payment w is paid out by the retailer; and for the wholesaler, at certain future times relative to t, which amount of w arrives at the wholesaler. Let the total amount paid by the retailer be w' at period t'. The payment is considered to be symmetric if the retailer pays an amount w' at time t', and the wholesaler receives an amount w' at time t'. The symmetric terms imply that the cash-flow of the payment for the retailer (negative) is the opposite of the wholesaler cash-flow (positive). Otherwise, the payment is asymmetric. The payment is called asymmetric if the wholesaler incurs a cost c' or there is a delay in payment at $t' + d'$ on receiving the amount w'.

The last stream of related research involves trade credit in production-inventory management. Trade credit is a short-term business loan given by a wholesaler to a retailer who purchases products, allowing the retailer to delay the payment. The net-term policy is a basic form of trade credit (e.g., Net 30 days). It means that the wholesaler allows the retailer to delay the payment until the net date. For example, under trade credit, the retailer pays a total amount of w' at time $t + L_z$,its NPV is $-w'e^{-\alpha_r(t+L_z)}$, and the wholesaler's NPV is $w'e^{-\alpha_r(t+L_z)}$ where L_z is the time for retailer to pay to the wholesaler in time unit year.

We consider a firm supplying a newsvendor-type retailer under a Net D clause, meaning that both parties agree that the payment will be made D days after the invoice (i.e. supply) date. The retailer orders a quantity that maximises his expected net profits over the selling season. We are interested in establishing the factors that affect whether the supplier would prefer a low or high value of D. How do the supplier's profits depend on the method the retailer uses to establish the order quantity? We reformulate the classic newsvendor supply chain in terms of cash-flow functions, and we use this to derive insights into the impact of Net D contract clauses on the NPV of profits of each of the firms.

In their contracts with their suppliers, large retailers commonly include a Net D clause, meaning that both parties agree that the payment will be made D days after the invoice date. The EU Late Payment Directive (2011/7/EU) states that the period for payment in a business-to-business contract should never exceed 60 days, unless expressly agreed by both parties. In the aftermath of the 2009 crisis, some large businesses in the UK shifted payment terms from 30 days to well over 100 days, in some cases. In 2012, for example, supermarket chain Morrisons extended its payment terms to Net 90 for most of its suppliers, and around the same time, Marks and Spencer extended their Net 60 to Net 75 for clothing and general merchandise suppliers (Steiner 2015). In March 2017, ASDA announced that they were extending their terms from Net 60 to Net 90 days for their clothing suppliers because of the lower value of the pound (Steiner 2017).

Goyal (1985) was the first to develop an economy order quantity, EOQ model with a constant demand rate and a permissible delay in payments. Ever since its introduction, many studies have used it, for example , in deteriorating items (Aggarwal & Jaggi 1995, Jamal et al. 1997, Jonas 2013, Chung et al. 2014). These models mainly involve two types of interest: interest earned I_e and interest charged, I_c. The retailer earns a return rate on investment I_e when the payment is made at the end of the permissible delay term. However, the wholesaler charges the retailer a rate of I_c if the payment is made after the agreed period. Gupta & Wang (2009) developed an inventory model under stochastic demand with the existence of trade credit, and derived the optimal policy. Lee & Rhee (2011) examined the trade credit from a wholesaler's perspective, but they presented trade credit as a tool for SC coordination. Thangam (2012) considered both upfront payment and a two-echelon trade credit term in a supply chain with perishable items. Seifert et al. (2013) provided a comprehensive review of the literature addressing trade credit. The authors posed a question about the important of accounting for cash-flow timing in the objective function. They highlighted that the comparison between traditional and NPV approaches is needed. In all models studied in this thesis, we applied this type of payment.

1.2 Research Aim and Objectives

The main aim of this thesis is to provide insight into the NPV framework in the NP. In line with this aim, this research is divided based on three objectives. Objective 1:

To examine the behaviour of supply chain parties in making decisions in the proposed NPV NP and the classical NP.

Research questions :

1. *Is there a difference in results between the proposed Newsvendor problem model and the classical model?*
2. *Is the wholesaler always disadvantaged when the Net-D contract is implemented?*
3. *Do parameters such as opportunity cost, Net-D terms, and variability of demand determine the parties's optimal decision?*

Objective 2: To examine the coordination and asymmetric information mechanism in the NP.

Research questions:

1. *Is there a difference in results between the NPV model and the classical model?*
2. *Does the proposed model achieve perfect coordination?*
3. *Under what circumstances does the wholesaler benefit from delayed payment?*
4. *How does dishonesty in information asymmetry affect the retailer's optimal order quantity and both SC members' expected annuity stream of profit?*

Objective 3: To examine the non-stationary demand in the NP in NPV analysis.

Research questions:

1. *Is there a difference in optimal results between the NPV model and the traditional model?*
2. *Is there a difference in optimal results when varying demand patterns?*

3. *Is the difference in optimal results between constant price and price-dependent demand significant?*

4. *Do changes in parameter values determine the firm's optimal decision?*

1.3 Thesis outline

The study is conducted based on the following outline, of which the first three chapters present the material to examine each of the three objectives stated in the previous section, respectively:

- *Chapter 2* first describes the problem and then develops the NPV model in the NP with a wholesale price only contract to analyse the effect of payment structures under Gamma-distributed demand. The model is based on individual policy. Numerical results and sensitivity analysis are provided to observe the performance behaviour of change in parameters.

- *Chapter 3* The concept of the NPV approach is introduced for the wholesaler-retailer coordination in the NP with the combination of wholesale price discount, buyback, and revenue contracts with symmetry and asymmetry of information. Then, numerical analysis is performed to compare the performance of the models with that of the classical framework.

- *Chapter 4* considers the single-echelon NP in a dynamic demand environment. The mathematical model is developed based on non-stationary demand in one selling season. Two cases are considered: a single pricing model and a price adjustment model. Then, the performance of these models is evaluated based on numerical results, and conclusions are drawn from the outcomes.

- *Chapter 5* concludes the study; it summarises the findings and proposes future research to extend the present work.

1.4 Expected Research Deliverable

Firstly, we propose a model of the NP from the NPV perspective with the addition of payment terms under Net D contract clauses to analyse and investigate the gap between the classical model and the proposed model. The outcome of this study can help us to better understand the advantages and disadvantages from both supply chain perspectives, as the impact of giving delayed payment has not been thoroughly explained in the literature. In particular, we expect that suppliers may in some circumstances benefit from being paid later if the additional order quantity from the retailer more than compensates for this.

Secondly, we extend the first model to one that can coordinate the supply chain. We introduce a mixing contract that includes the basic contract types: wholesale price discount, buyback, and revenue sharing. These contracts are expected to be able to perfectly coordinate the supply chain. Because of additional degrees of freedom they may also offer opportunities for tailoring contracts to specific needs. Then, we investigate the case in which the wholesaler has the option to deviate from having perfect coordination to gain benefit from giving delayed payment to the retailer. Extending this model, we investigate dishonest behaviour when there is asymmetry information. Can both parties still benefit from a mixing contract if one party is dishonest?

Lastly, the final outcome of the research lies in the third model of dynamic demand. We introduce two cases: time-dependent demand, and price- and time-dependent demand with price adjustment. Both cases assume a random non-stationary demand. The option of a second period (with e.g. 'discount' sales) is included. The outcome of this model can help to establish the value for the retailer of using knowledge about the anticipated evolution of the stochastic demand pattern over the selling season. We compare in particular the difference between a downward or upward demand pattern – and under which conditions the retailer should expect more profit. In addition, the model with price adjustment is believed to be superior to the price-dependent demand model without price adjustment. Sensitivity analysis is conducted for all of the developed models to address the way their behaviour depends on their parameters.

Chapter 2

Supplying to the Newsvendor under Net D Clauses - A Net Present Value Analysis

Abstract

This study considers a NP consisting of a retailer and a wholesaler and considering the timing of events and the opportunity costs of independent firms. The objective of this model is to determine the optimal order quantity, to maximise the expected total profit. The effects of variability of demand, opportunity costs, and delayed payment are discussed. Numerical analysis is used to illustrate the implementation of the solutions and the degree to which the parameter system affects decision -making and expected profit. The results show that the retailer is always better off using the NPV model instead of the traditional NP, and vice-versa for the wholesaler. We also conclude that with delayed payment, the benefit gain increases for both parties when the demand is highly variable, and the retailer's opportunity cost is greater than the wholesaler's.

2.1 Introduction

A Net D clause with a large D value equates to the buyer receiving an interest-free loan from the supplier. It is clear that delaying payments to suppliers may hurt them in their ability to pay their own bills and invest in new business developments; it may also force them to take on short-term loans and increase their risk of bankruptcy. In fact, some supermarkets offer suppliers access to credit from a bank instead of paying more quickly, but the bank will charge interest on the loan at, for example, 1.2 % per month (Steiner 2015). Note that this corresponds to an annual (compounded) interest rate of 15.4%.

A study by the Federation of Small Businesses (FSB, 2016) concludes that, in addition to demanding Net D agreements with large D values, many (large) firms settle bills on average well beyond these payment terms. Rebecca (2015) reported that in 2014, UK supermarkets paid on average 33 days beyond payment terms. The FSB study found that about 80 % of small businesses did not charge interest for these late payments, although according to the EU Directive, they would be entitled to do so at a rate at least 8% above the European Central Bank's base rate. The FSB report also presents findings from a European Commission impact assessment: 'Exercise of the rights conferred by the Directive is not widespread due to fear of damaging good business relationships. Rather than legislation, business culture, economic conditions, and power imbalances in the market are driving factors of payment behaviour.'

In this paper, we aim to better understand the impact of the above practices on the newsvendor problem. We provide an NP that considers elements such as delayed payment and the time value of money (TVM), which have not been discussed directly in NPs. Finally, we provide solutions that take into account these characteristics, and we analyse the implications of our assumptions.

2.2 Related literature

In the past decades, the issues related to supply chain management have been widely studied with the objective of minimising expected cost or maximising expected profit. However, most of these studies consider that the wholesaler receives payment from the retailer as soon as the items have arrived, which neglects the regular component of business, that is, delayed payment (trade credit). However, in reality, when delayed payment is offered, the wholesaler is thought to be disadvantaged or to not benefit from the scheme. Large retailers tend to pay the wholesaler later than the agreed due date to alleviate their own financial management. The aim of this paper is to analyse whether the wholesaler will always be in a difficult position when delayed payment is implemented, and, if not, in what setting he will benefit from the scheme. The newsvendor model is considered because of its simplicity in terms of stochastic demand.

Inventory models involving stochastic demand have received much attention in the inventory management literature. Reviews of the NP include Khouja (1999), Petruzzi & Dada (1999), and Qin et al. (2011). Lariviere & Porteus (2001) studied the price-only-contract in the NP as the Stackelberg game between the wholesaler and the retailer explicitly financial constraint. Cachon (2003) noted that a simple contract is always favourable for the contract maker. In practice, the wholesale-price-only contract is used in the supply chain due to its simplicity, where the wholesaler decides the wholesale price per unit of the products he sells. In addition, researchers always use the wholesale price contract as a benchmark for their proposed model. Some of the most interesting works on the NP are those of Wang & Webster (2009), Cachon & Lariviere (2005), and Xinsheng et al. (2015).

All the aforementioned studies omit the TVM, which refers to the interest one possibly gains when a payment is received today as opposed to some future time. There are very few articles that account for TVM in inventory and production problems. This is due to its complexity, which hinders research (Sun & Queyranne 2002). Seifert et al. (2013) hconducted a comprehensive review of the literature on trade credit, and highlighted the urgent need to compare the results of an opportunity cost approach and an NPV approach, which would provide a valuable base for further research.

The NPV may be adapted if the aftermath of economic in production planning decisions needs to be considered (Grubbström 1980). Grubbström (1967, 2007) provided a close-form solution of the discounted cash-flow analysis using Laplace Transform. Subsequent works on the NPV model include those of Kim et al. (1984), Beullens & Janssens (2011), Beullens & Janssens (2014), and Marchi et al. (2016).

Grubbström (1980) showed that the capital costs of inventory in the various stages of an SC can be restored from the linearised NPV or annuity stream of the cash-flow function. Later, Gurnani (1983) compared the NPV model with the classical inventory framework. The author concluded that there are inaccuracies in the classical model, since the holding cost is assumed to reflect only certain items, namely storage space and deterioration. Disney & Warburton (2012) show that by using the Laplace transform and the Lambert W function in two types of EOQ models, they can get the exact, explicit optimal results. Later, Disney et al. (2013) extended the EPQ model to incorporate the NPV in the model.

Beullens & Janssens (2014) developed the NPV equivalence analysis (NPVEA) under various payment structures – conventional payment, delayed payment, and advance payment – and applied it to a few classic inventory models. The results lead to different perceptions of, or alternatives to, the classical inventory model. While the NPV analysis in production-inventory systems has already been widely studied in deterministic demand, Grubbström (2010) is the only one to have considered the NPV approach in the NP. However, no attention has been paid to the impact of payment structures on the SC under TVM.

The second dimension that differentiates our work from the literature is how we confer delayed payment terms. Under a delayed payment or trade credit framework, the retailer is allowed to make payments to the wholesaler by a certain time at no additional charge. Goyal (1985) was among the first to propose an EOQ model with a permissible delay in payment. Since then, many articles have been published related to the delay in payments by the wholesaler under the lot sizing inventory problem, including Jamal et al. (1997), Lou & Wang (2013), and Teng et al. (2014). Recently, Chen & Teng (2015), and Wu et al. (2016) examined trade credits under deteriorating items considering discount cash-flow analysis on all relevant revenue and costs. Furthermore, a few papers have attempted to integrate the financial and production decisions. Kouvelis & Zhao (2011) and Kouvelis & Zhao (2012) considered the simple NP to incorporate the bankruptcy cost with the structure of optimal trade credit. However, these articles do not take into account the payment terms in every cash transaction.

In particular, the wholesaler benefits from offering delayed payment within a certain period by increasing sales and transferring inventory to the retailer. However, in reality, timely payment is a nightmare for small and medium-sized enterprises. These issue shows that wholesalers are in the worst position because of bad cash-flow. Thus, we seek to determine under which circumstances the wholesaler will benefit from giving/obtaining delayed payment. We find these interesting issues in the above article, where the result using the NPV objective function is observed from the retailer making a delayed payment to the wholesaler, which discounts these costs over the time periods. Regarding the aforementioned question, our analysis yields meticulous conclusions about the implications of using an NPV model, and it allows comparison to the traditional NP. In addition, it provides insights into how the wholesaler reacts when delayed payment is implemented. To make the model more relevant and applicable in practice, we amend the traditional NP by developing a generalised version using the NPV of the expected total profit as the objective function, and we compare the results with those obtained using the traditional newsvendor model.

The remainder of the chapter is organised as follows. Section 2.3 presents the benchmark model and the proposed model. Section 2.4 provides solution procedures to solve the problem. Section 2.5 demonstrates numerical examples to illustrate the model and generate managerial insights. Finally, Section 2.6 includes some concluding remarks and future research suggestions.

2.3 The Mathematical Model

We consider a two-echelon NP consisting of one wholesaler and one retailer. The retailer orders from the wholesaler to satisfy the uncertain demand x at time T. Without loss of generality, we assume that the wholesaler delivers the product to the retailer. The wholesaler only sells a single product to the retailer. The retailer needs to decide the optimal order quantity in order to maximise the expected profit function.

2.3.1 Assumptions

- The demand distribution is known.
- The selling price and the wholesale price are given (based on market price).
- No shortage is allowed.
- There are n sequential selling periods within an accounting year.
- The time unit is one accounting year.

To begin, we consider a traditional NP with a wholesale price-only-contract as a benchmark case of this model.

2.3.2 Traditional Newsvendor Problem

The retailer decides order quantity Q_a at the beginning of the selling season and pays the wholesale price w per unit ordered. The unsold products at the end of the selling season are sold with a salvage value v. If the realised demand in this period does not exceed the order quantity, then the retailer's revenue is px, and the unsold items in inventory are salvaged at $v(Q_a - x)$. If the realised demand is greater than Q_a, the retailer's revenue is pQ_a.

The retailer's problem is given as;

$$P(Q_a, x) = \begin{cases} px + v(Q_a - x), & \text{if } x \leq Q_a, \\ pQ_a, & \text{if } x \geq Q_a. \end{cases} \tag{2.1}$$

The retailer's objective function is

$$\begin{aligned} \max_{Q_a} E[\Pi]_r &= p\Big(\int_0^{Q_a} xf(x)dx + Q_a \int_{Q_a}^{\infty} f(x)dx\Big) \\ &+ v\int_0^{Q_a} (Q_a - x)f(x)dx - wQ_a, \end{aligned} \tag{2.2}$$

where $E[\Pi]_r$ represents an expected profit function for the retailer. The first and second order conditions of $E[\Pi]_r$ w.r.t Q_a for a given w are;

$$dE[\Pi]_r/dQ_a = -(p - v)F(Q_a) + (p - w)$$

and

$$dE^2[\Pi]_r/dQ_a^2 = -(p - v)f(Q_a) < 0,$$

respectively. Thus, the optimal order quantity Q_a^* that maximises the objective function is given by;

$$Q_a^* = F^{-1}\Big(\frac{p - w}{p - v}\Big). \tag{2.3}$$

The wholesaler's expected profit function is given by;

$$E[\Pi]_s = (w - c)Q_a. \tag{2.4}$$

From Eq.(2.3), the wholesaler can predict the retailer's order quantity for any given wholesale price, w.

2.3.3 NPV-I Newsvendor Problem

We first derive a NP based on the NPV perspective. The sequence of events is as follows.

1. Before the beginning of the selling period, for instance at a period of lc days or $L_c = lc/365$ year, the retailer initiates an order from the wholesaler, given by Q_{I}. For every unit of product, the wholesaler incurs an operation cost, c.
2. When the orders arrive at the beginning of the selling period, the wholesaler charges the retailer w per unit of product. The retailer can pay w at period of lz days which equal to $L_z = lz/365$ year depends on what type of payment structures involved. The selling price of the product is p per unit.
3. At lv days or $L_v = lv/365$ year after the end of the selling season, the retailer earns a salvage value, v per unit unsold by selling these products to the end-customers.

We now examine our new divergence from the problem. We introduce the payment term to account for the time of event. In the real market, all of the cash-flow transactions between retailer and wholesaler are based on the timing of revenue and cost cash-flows. In the traditional model, the wholesaler invests in Q_{I} order quantity and the retailer pays the wholesaler at the start of the selling season. Any unsold inventory is assumed to be at the end of the selling period. This assumption is used in most of the literature. In reality, the retailer can pay at different moments in time, either before or after the selling period starts. The investments from the wholesaler will start before the beginning of the selling period, and the salvages occur after the end of this period.

We will assume without loss of generality that one accounting year is 365 days. Note that a Net-D contract with payment terms of e.g. $l_z = 60$ days then corresponds to taking a value for $L_z = 60/365$. A situation with two selling seasons (summer and winter, for example) will have $n = 2$ and $T = 365/2$ days, or $T = 1/2$ in accounting years; and in general the adopted assumptions imply $nT = 1$.

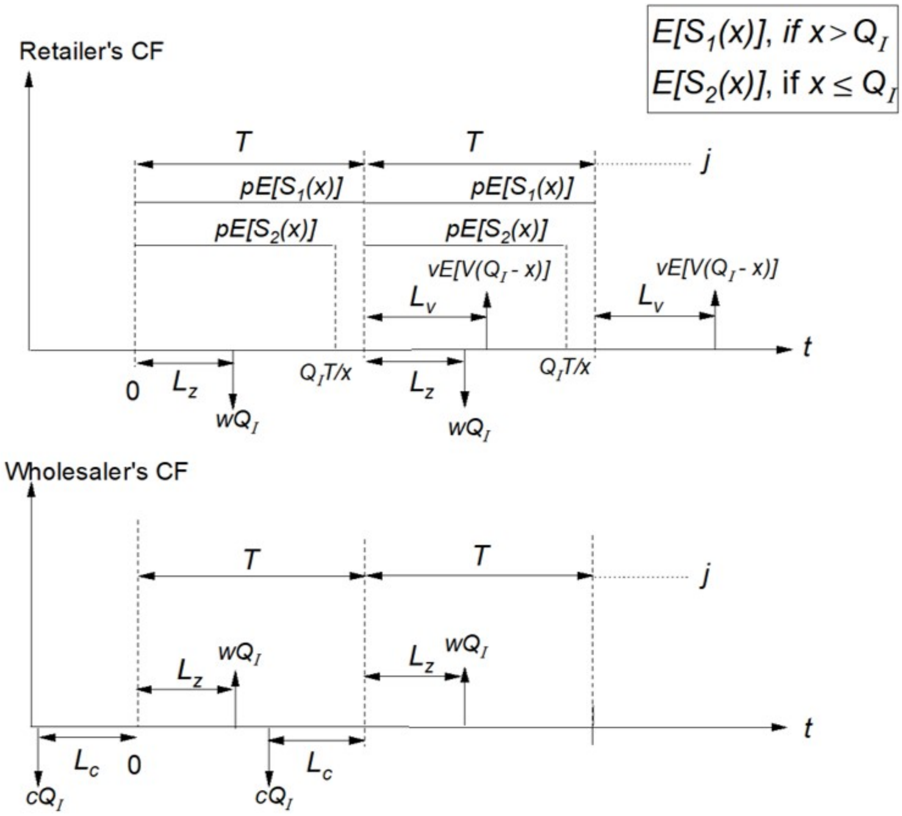


Figure 2.1: Cash-flows of retailer and wholesaler for NPV-I model

If $L_c, L_c, L_v = 0$, the investment occurs at the beginning of the selling period, and salvage revenue is achieved at the end of this period. It is more sensible to consider values of L_c, $L_v > 0$. In our case, we only consider $L_z > 0$, which happens when the retailer makes a delayed payment to the wholesaler.

2.3.3.1 Retailer's Profit Function

Figure 2.1 represents the cash-flow of the profit maximisation model for the wholesaler and retailer. From the figure, the expected present value of sales revenue at the rate of px/T throughout one selling period is given by;

$$E[R_1] \quad = \quad \frac{p}{\alpha_r T}\Big[\int_0^{Q_I}\Big(1-e^{-\alpha_r T}\Big)xf(x)dx + \int_{Q_{\rm I}}^{\infty}(1-e^{-\alpha_r Q_{\rm I}T/x})xf(x)dx\Big] \tag{2.5}$$

The two brackets in Eq. (2.5) indicate, first, that when $x \leq Q_{\rm I}$, the sales revenue is continuously earned from 0 until T ; and second, that when $x > Q_{\rm I}$, the sales revenue at the rate of px/T is continuously earned at the start of the selling period until $Q_{\rm I}T/x$.

The expected annuity stream of the sales revenue earned in that accounting year is subject to the sum of the time-adjusted expected value over n selling periods each of duration T over an infinite of the future accounting year. Then, the equation is;

$$\begin{aligned} E[AS_1]_r &= E[R_1]\Big(\sum_{i=0}^{n-1}\alpha_r e^{-i\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_r nT}\Big) \\ &= \frac{p}{T}\Big[\int_0^{Q_{\rm I}}\Big(1-e^{-\alpha_r T}\Big)xf(x)dx \\ &+ \int_{Q_{\rm I}}^{\infty}(1-e^{-\alpha_r Q_{\rm I}T/x})xf(x)dx\Big]\Big(\sum_{i=0}^{n-1}e^{-i\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_r nT}\Big). \end{aligned} \tag{2.6}$$

Since $nT = 1$, the equivalent annuity stream for Eq. (2.6) is;

$$E[AS_1]_r = \frac{p}{T}\Big[\int_0^{Q_{\rm I}}xf(x)dx + \frac{1}{(1-e^{-\alpha_r T})}\int_{Q_{\rm I}}^{\infty}(1-e^{-\alpha_r Q_{\rm I}T/x})xf(x)dx\Big]. \tag{2.7}$$

After the end of the selling season, the retailer will receive salvage revenue at L_v. The corresponding expected annuity stream of the salvage activities is;

$$E[AS_2]_r = v\int_0^{Q_{\rm I}}(Q_{\rm I}-x)f(x)dx(e^{-\alpha_r L_v})\Big(\sum_{i=0}^{n-1}\alpha_r e^{-(i+1)\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_r nT}\Big), \tag{2.8}$$

leads to:

$$E[AS_2]_r = v\gamma_v E[V(Q_\text{I})], \tag{2.9}$$

where $\gamma_v = \frac{\alpha_r e^{-\alpha_r(L_v+1/n)}}{1-e^{-\alpha_r/n}}$, $E[V(Q_\text{I})] = Q_\text{I} - E[X] + \int_{Q_\text{I}}^{\infty}(x - Q_\text{I})f(x)dx$, and $E[X]$ represents the expected value of X.

For a wholesale price contract, the transfer payment TP made by the retailer is the investment of the n period of orders Q_I which happens at L_z over an infinite accounting year. Thus, the expected transfer payment is given by;

$$E[TP] = -wQ_\text{I}e^{-\alpha_r L_z}\Big(\sum_{i=0}^{n-1}\alpha_r e^{-i\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_r nT}\Big), \tag{2.10}$$

or equivalently:

$$E[TP] = -wQ\gamma_{z_r}, \tag{2.11}$$

where $\gamma_{z_r} = \frac{\alpha_r e^{-\alpha_r L_z}}{1-e^{-\alpha_r/n}}$. Hence, by adding Eq. (2.7), Eq. (2.9), and Eq. (2.11) together, the expected total annuity stream for the retailer is given by;

$$\begin{aligned} E[AS]_r(Q_\text{I}) &= \frac{p}{T}\Big[\int_0^{Q_\text{I}} xf(x)dx + \frac{1}{(1-e^{-\alpha_r T})}\int_{Q_\text{I}}^{\infty}(1-e^{-\alpha_r QT/x})xf(x)dx\Big] \\ &+ v\gamma_v E[V(Q_\text{I})] - wQ_\text{I}\gamma_{z_r}. \end{aligned} \tag{2.12}$$

To find the optimal order quantity, Q_I^*, we differentiate Eq. (2.12) with respect to Q_I, and we have:

$$\frac{dE[AS]_r}{dQ_\text{I}} = v\gamma_v\int_0^{Q_\text{I}} f(x)dx + \frac{\alpha p}{1-e^{-\alpha T}}\int_{Q_\text{I}}^{\infty} e^{-\alpha Q_\text{I}T/x}f(x)dx - w\gamma_z^r. \tag{2.13}$$

The above equation cannot be solved analytically. Thus, the optimal order quantity is obtained by solving the following nonlinear program:

$$\begin{aligned} \underset{Q_{\mathrm{I}}}{\text{maximise}} \quad & E[AS]_r(Q_{\mathrm{I}}), \\ \text{subject to} \quad & Q_{\mathrm{I}} \geq 0. \end{aligned} \tag{2.14}$$

A corresponding algorithm is detailed in Section 2.4.1.

2.3.3.2 Wholesaler's Profit Function

Before the beginning of the selling period, the wholesaler produces each of the n orders Q_{I} over an infinite accounting year, incurs a production cost c for each unit of order at time L_c, and receives a payment of w for each unit of order at time L_z at/after the start of the selling season. The expected annuity stream is given by;

$$E[AS_1]_s = -cQe^{-\alpha_s(-L_c)}\left(\sum_{i=0}^{n-1}\alpha_s e^{-i\alpha_s T}\right)\left(\sum_{j=0}^{\infty}e^{-j\alpha_s nT}\right). \tag{2.15}$$

The negative sign of L_c denotes the time of payment occurs before the start of the selling season. Hence, Eq. (2.15) can be rewritten as;

$$E[AS_1]_s = -cQ\gamma_c, \tag{2.16}$$

where $\gamma_c = \frac{\alpha_s e^{\alpha_s L_c}}{1-e^{-\alpha_s/n}}$. The expected annuity stream of revenue for the wholesaler is the transfer payment made by the retailer. By replacing α_r with α_s from Eq. (2.10), we have:

$$E[AS_2]_s = wQ\gamma_{z_s}, \tag{2.17}$$

where $\gamma_{z_s} = \frac{\alpha_s e^{-\alpha_s L_z}}{1-e^{-\alpha_s/n}}$.

Adding Eq. (2.16), and Eq. (2.17) together, the expected profit function for the wholesaler is:

$$E[AS]_s(Q_{\rm I}) = (w\gamma_{z_s} - c\gamma_c)Q_{\rm I}. \tag{2.18}$$

2.3.4 Simplified NPV-II Model

The model developed in the previous section had the feature that sales occurred only as long as stocks last. This is realistic in most circumstances. In this section we develop a simple model of rationing. In this alternative model, illustrated in Figure 2.2, we assume that the retailer's revenue is always received at a continuous rate for the whole of the selling season. This would only be realistic in the case of stock-outs if the firm would ration the available stock each day of the selling season so that a fraction Q/X of daily demand would be sold only. The notation of $E[S_1(x)]$, and $E[S_2(x)]$ in Figure 2.2 represent retailer's revenue.

2.3.4.1 Retailer-II's Profit Function

The difference between NPV-I and NPV-II is in the expected NPV of sales revenue in an accounting year. Other formulations remain the same. In this model, we assume that the retailer's sales revenue is continuously received between 0 and T (see Figure 2.2). The expected NPV of the sales revenue in an accounting year is given by;

$$E[R_1]^b = \frac{P}{T}\Big[\int_0^{Q_{\rm II}} \Big(\frac{1-e^{-\alpha_r T}}{\alpha_r}\Big) x f(x)dx + \int_{Q_{\rm II}}^{\infty} \Big(\frac{1-e^{-\alpha_r T}}{\alpha_r}\Big) Q_{\rm II} f(x)dx\Big]. \tag{2.19}$$

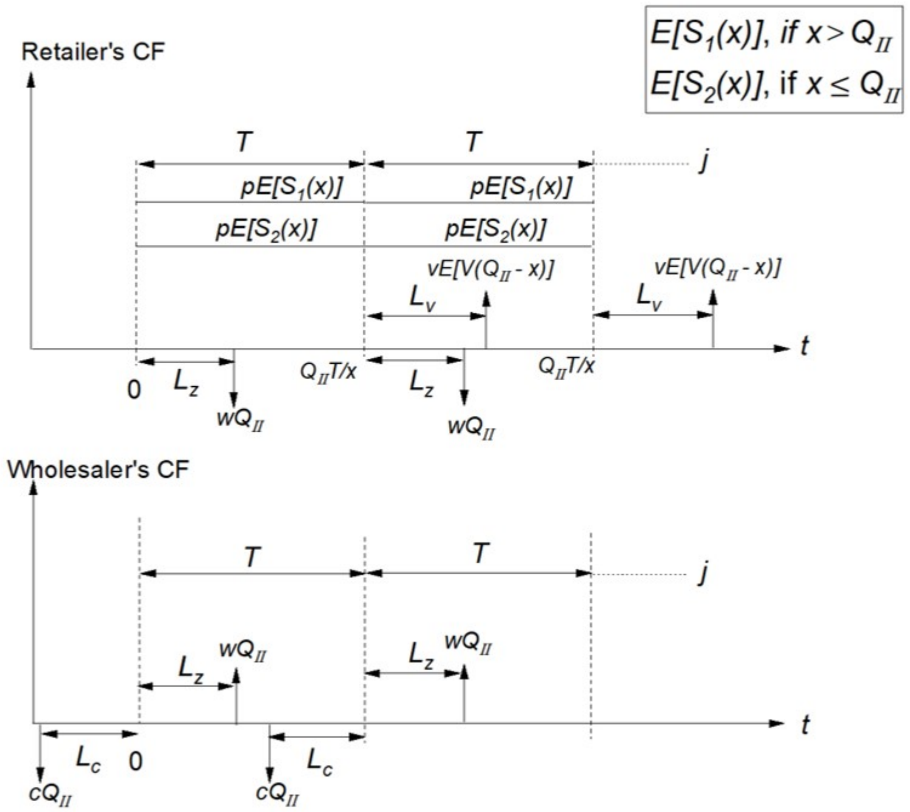


Figure 2.2: Cash-flows for the retailer and wholesaler over time, t

Then, the expected annuity stream of revenue for the retailer in n period of an accounting year over an infinite accounting year is;

$$\begin{aligned} E[AS_1]_r^b &= \frac{P}{T}\Big[\int_0^{Q_{\rm II}}\Big(\frac{1-e^{-\alpha_r T}}{\alpha_r}\Big)xf(x)dx \\ &+ \int_{Q_{\rm II}}^{\infty}\Big(\frac{1-e^{-\alpha_r T}}{\alpha_r}\Big)Q_{\rm II}f(x)dx\Big]\Big(\sum_{i=0}^{n-1}\alpha_r e^{-i\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_r nT}\Big) \end{aligned} \tag{2.20}$$

This leads to the following linearisation in α_r of the annuity stream:

$$E[AS_1]_r^b = \frac{P}{T}\Big(\int_0^{Q_{\rm II}} xf(x)dx + \int_{Q_{\rm II}}^{\infty} Q_{\rm II}f(x)dx\Big). \tag{2.21}$$

The expected total annuity stream for the retailer is the sum of (2.21), (2.9), and (2.11):

$$E[AS]_r^b(Q_{\rm II}) = \frac{P}{T}\Big(\int_0^{Q_{\rm II}} xf(x)dx + \int_{Q_{\rm II}}^{\infty} Q_{\rm II}f(x)dx\Big) + v\gamma_v E[V(Q_{\rm II})] - wQ_{\rm II}\gamma_{z_r} \tag{2.22}$$

By using the fundamental theorem of calculus, we first find Q_{II} that satisfies $dE[AS]_r^b/dQ_{\text{II}} = 0$.

$$\frac{dE[AS]_r^b}{dQ_{\text{II}}} = pn\int_{Q_{\text{II}}}^{\infty} f(x)dx + v\gamma_v\int_{Q_{\text{II}}}^{\infty} f(x)dx - w\gamma_{z_r}.$$

By setting $\frac{dE[AS]_r^b}{dQ_{\text{II}}} = 0$, the optimal Q_{II}^* satisfies

$$Q_{\text{II}}^* = F^{-1}\left(\frac{pn - w\gamma_{z_r}}{pn - v\gamma_v}\right). \tag{2.23}$$

Proof. To check whether $E[AS]_r^b$ has a unique maximum, we find the second derivative of $E[AS]_r^b$,

$$\frac{d^2E[AS]_r^b}{dQ_{\text{II}}^2} = (v\gamma_v - pn)f(Q_{\text{II}}).$$

Since $(v\gamma_v - pn) < 0$, $\frac{d^2E[AS]_r^b}{dQ_{\text{II}}^2} < 0$, therefore $E[AS]_r^b$ is a concave function on $[0, \infty)$. Thus, the optimal order quantity Q_{II}^* is given in (2.23). ■

2.3.4.2 Wholesaler-II's Profit Function

The expected annuity stream for the wholesaler is the same as in (2.18).

2.4 Model Comparison

In this section, we present numerical examples of the single-period newsvendor model. We assume that demand uncertainty, x is Gamma distributed. To illustrate the benefits of the NPV-I model, we compare the solution with the T-newsvendor model. Nine instances are used in this section, as shown in Table 2.1. The other parameter values are stated here: $p = 15, w = 10, c = 4, v = 2, n = 1$.

Table 2.1: Problem instance characteristics

Instance	shape, α_g	scale, τ	μ	σ	α_r	α_s
1	4	250	1000	500	0.2	0.2
2	4	250	1000	500	0.05	0.2
3	4	250	1000	500	0.2	0.05
4	3	333.33	999.99	577.34	0.2	0.2
5	3	333.33	999.99	577.34	0.05	0.2
6	3	333.33	999.99	577.34	0.2	0.05
7	2	500	1000	707.11	0.2	0.2
8	2	500	1000	707.11	0.05	0.2
9	2	500	1000	707.11	0.2	0.05

2.4.1 Algorithm

The NPV-I model is the (corrected) model based on the real-world problem. To see the differences in result between the T-NP and the NPV-I and NPV-II models, we derive the following algorithm and use the software Mathematica as a solution tool:

Algorithm 1

Input. List of parameters.
Output. A local maximum Q^*_{I}, and Q^*_{II} with $E[AS]_r(Q^*)$, $E[AS]_s(Q^*)$, and $E[AS]^b_r(Q_{\mathrm{II}})$.
Step 1. Compute Q^*_a from Eq. (2.3). Then, set $Q^* = Q^*_a$.
Step 2. Compute $E[AS]_r(Q^*)$, and $E[AS]_s(Q^*)$ in Eq. (2.12) and Eq. (2.18).
Step 3. (Local search.) Find local maximum of Q^* of function in Eq. (2.12) using *FindArgMax* function in software Mathematica.
Step 4. Go to *Step* 2.
Step 5. Compute Q^*_{II} from Eq. (2.23). Then, set $Q^* = Q^*_{\mathrm{II}}$.
Step 6. Compute $E[AS]^b_r(Q^*_{\mathrm{II}})$, and $E[AS]_s(Q^*)$ from Eq. (2.22) and Eq. (2.18).

2.4.2 Comparison of NPV-I and NPV-II

For this comparison, we set values of $lz = 0$, $lc = lv = 30$ days. PG_i denotes a percentage gain for $i = r$ and $i = s$ with:

$$PG_i = 100 \times (E[AS]_i(NPVI) - E[AS]_i(NPvII))/E[AS]_i(NPVII)$$

Table 2.2 presents the optimal solution of expected profit functions for NPV-I and NPV-II.

Table 2.2: Optimal solution expected profit functions for NPV-I and NPV-II

No.	Model	Q^*	Retailer	PG_r	Wholesaler	PG_s
1	NPV-I	668.31	2211.21	13.99	4375.31	-3.27
	NPV-II	690.92	1939.90		4523.31	
2	NPV-I	755.65	2581.57	2.47	4947.15	-0.85
	NPV-II	762.10	2519.29		4989.37	
3	NPV-I	668.31	2211.21	13.99	4099.65	-3.27
	NPV-II	690.92	1939.90		4238.32	
4	NPV-I	621.19	1965.78	16.46	4066.81	-2.62
	NPV-II	637.91	1687.94		4176.26	
5	NPV-I	710.96	2294.50	2.92	4654.57	-0.75
	NPV-II	716.37	2229.39		4689.93	
6	NPV-I	621.19	1965.78	16.46	3810.58	-2.62
	NPV-II	637.91	1687.94		3913.14	
7	NPV-I	543.36	1585.44	21.06	3557.30	-0.97
	NPV-II	548.66	1309.64		3591.99	
8	NPV-I	633.47	1852.36	3.75	4147.25	-0.48
	NPV-II	636.55	1785.44		4167.36	
9	NPV-I	543.36	1585.68	21.08	3333.18	-0.97
	NPV-II	548.36	1309.64		3365.67	

The table reveals that for any given instance, the difference of optimal order quantity between both models is not considered small. For example, in instance 1, the increment in order quantity when using NPV-I is 22.61(=690.92-668.31). The percentage increase is about 3.38%. It seems reasonable to expect that the profitability of ordering less than actual demand is now higher since revenues happen at the start of the period up to $Q/x < T$; thus, optimal Q^* in NPV-I is lower than in the NPV-II model of sales. It can also be deduced from the table that the optimal expected profit function for the wholesaler is sensitive to changes in optimal order quantity. Moreover, Table 2.2 indicates that the difference in PG for the retailer between both models is relatively high. Therefore, we choose NPV-I for the next examples.

2.4.3 Comparison of NPV-I and the Newsvendor Model

Now, we set the values of $L_z = L_c = L_v = 0$. PG_i denotes the percentage gain for $i = r$(retailer), and $i = s$(wholesaler) as:

$$PG_i = 100 \times (E[AS]_i(Q^*) - E[AS]_i(Q_a))/E[AS]_i(Q_a)$$

.

Table 2.3: Optimal solution expected profit functions in NPV model

No.	NPV-I	Q^*	$E[AS]_r$	PG_r	$E[AS]_s$	PG_s
1	Q_I	668.98	2212.80	3.90	4428.65	-14.84
	Q_a	785.58	2129.68		5200.53	
2	Q_I	755.9	2582.22	0.20	5004.03	-3.78
	Q_a	785.58	2577.04		5200.53	
3	Q_I	668.98	2212.80	3.90	4115.08	-14.84
	Q_a	785.58	2129.68		4832.3	
4	Q_I	621.93	1967.51	4.35	4117.14	-16.23
	Q_a	742.45	1885.42		4915.01	
5	Q_I	711.23	2295.20	0.23	4708.36	-4.20
	Q_a	742.45	2290.02		4915.01	
6	Q_I	621.93	1967.51	4.35	3825.62	-16.23
	Q_a	742.45	1885.42		4567	
7	Q_I	544.17	1587.57	5.09	3602.5	-18.31
	Q_a	666.20	1510.62		4410.24	
8	Q_I	633.78	1853.11	0.27	4195.61	-4.87
	Q_a	666.20	1848.16		4410.24	
9	Q_I	544.19	1587.57	5.09	3347.42	-18.31
	Q_a	666.20	1510.62		4097.96	

From Table 2.3, we see that Q_a^* is the same when α_g, and τ are the same regardless of any values of opportunity costs. The newsvendor model does not consider the opportunity cost, so this does not affect Q_a. In addition, the optimal Q_I differs for each instance except for the case in which α_g, τ, and α_r are fixed, and α_s change. Comparing the expected profit function, the NPV-I(Q_I) model generates a higher profit for the retailer compared to NPV-I(Q_a) for all instances. For example, from instance 9, when the NPV model is used instead of the traditional model, the retailer earns 5.094% more profit. However, the opposite is true for the wholesaler, with a profit loss of 750.54 (a decrease of 18.314%).

This comparison shows that when adopting the NPV model, the retailer benefits, but the wholesaler does not. The differences in PG for both parties are considered high. Thus, adopting the NPV model is a correct way to see how much the firms earn in every transaction.

2.5 Insight from NPV-I on Payment Terms

To study the effect of the expected annuity stream of profit for $E[AS]_r$, and $E[AS]_s$ of the NPV model, we perform a numerical analysis by varying value of lz, lc, α_g, τ, n, α_r, and α_s. To be more realistic, $lv = 30$. The other parameters remain the same as before.

2.5.1 Influence of Variability of Demand, Number of Period, and Delayed Payment

By setting $\alpha_r = \alpha_s = 0.2$, the optimal Q^* and PG for retailer and wholesaler are shown in Table 2.4. The PG_i is denoted as $PG_i = 100 \times (E[AS]_i((L_z \neq 0)) - E[AS]_i(lz = 0))/E[AS]_i(lz = 0)$.

As expected, the optimal order quantity is sensitive to the variability of demand. When L_z and n are fixed, the optimal order quantity decreases when the variability of demand increases. This indicates that the retailer will not take the risk of ordering more when there is high variability of demand.

However, when α_g, τ, and n are fixed, the optimal Q^* increases when L_z increases. This is because the retailer will order more as the delayed payments reduce the amount of capital invested in stock during the delayed payment time. In addition, when α_g, τ, and L_z are fixed, $E[AS]_r$ and $E[AS]_s$ increase with the increment of n. However, the increment of n will reduce the PG for retailer and wholesaler.

From the wholesaler's perspective, delayed payments are only beneficial when the variability of demand is high. Therefore, to benefit both retailer and wholesaler, we present the next examples by setting $\alpha_g = 2$, and $\tau = 500$.

Table 2.4: Summary of experiments

No.	α_g	τ	lz	n	Q^*	$E[AS]_r$	$E[AS]_s$	PG_r	PG_s
	4	250	0	1	668.31	2211.21	4375.31	-	-
	4	250	30	1	682.95	2332.40	4348.32	5.481	-0.617
1	4	250	60	1	697.35	2454.87	4316.61	11.019	-1.342
	4	250	90	1	711.53	2577.49	4280.51	16.565	-2.167
	4	250	120	1	725.50	2700.52	4240.32	22.129	-3.085
	3	333.33	0	1	621.19	1965.78	4066.81	-	-
	3	333.33	30	1	636.92	2078.94	4055.23	5.756	-0.285
2	3	333.33	60	1	652.45	2193.01	4038.64	11.559	-0.693
	3	333.33	90	1	667.79	2307.92	4017.35	17.405	-1.216
	3	333.33	120	1	682.95	2423.56	3991.63	23.287	-1.849
	2	500	0	1	543.36	1585.68	3557.30	-	-
	2	500	30	1	560.39	1684.95	3568.01	6.260	0.301
3	2	500	60	1	577.31	1785.61	3573.53	12.608	0.456
	2	500	90	1	594.11	1887.56	3574.12	19.038	0.473
	2	500	120	1	610.81	1990.72	3570.00	25.544	0.357
	2	500	0	2	601.55	3510.70	7501.77	-	-
	2	500	30	2	619.04	3719.82	7507.70	5.957	0.079
4	2	500	60	2	636.40	3931.40	7503.73	11.983	0.026
	2	500	90	2	653.64	4145.26	7490.32	18.075	-0.153
	2	500	120	2	670.78	4361.25	7467.93	24.227	-0.451
	2	500	0	3	622.09	5453.32	11447.10	-	-
	2	500	30	3	639.71	5772.30	11447.90	5.849	0.007
5	2	500	60	3	657.20	6094.81	11434.00	11.763	-0.114
	2	500	90	3	674.60	6420.59	11406.30	17.737	-0.356
	2	500	120	3	691.85	6749.40	11365.40	23.767	-0.714

2.5.2 Influence of α_r, α_s, L_z, and L_c

This example serves to evaluate the relative performances with different α_r, α_s, L_z, and L_c. We set $\alpha_g = 2$, and $\tau = 500$, and the other parameters are the same as in the previous example. The optimal results of Q^* and percentage gain for both parties when varying these parameters are reported in Table 2.5. Table 2.5 shows that when L_z, L_c, α_r are fixed and α_s increases, the optimal Q^*, and PG_r remain the same. In addition, the PG_s decreases with the increases in α_s. Moreover, the wholesaler only benefits from delayed payment when α_s is low. . In conclusion, it can be determined from this data that the wholesaler can expect to benefit from the delayed payment when his opportunity cost is small, while the retailer always benefits from this type of payment.

Table 2.5: Summary of experiments

lz	lc	α_r	α_s	Q^*	$E[AS]_r$	$E[AS]_s$	PG_r	PG_s
0	30	0.05	0.05	633.47	1852.36	3885.96	-	-
30	30	0.05	0.05	637.92	1879.09	3886.39	1.443	0.011
60	30	0.05	0.05	642.35	1905.89	3886.51	2.890	0.014
90	30	0.05	0.05	646.78	1932.77	3886.32	4.341	0.009
120	30	0.05	0.05	651.20	1959.72	3885.82	5.796	-0.004
0	30	0.05	0.2	633.47	1852.36	4147.25	-	-
30	30	0.05	0.2	637.92	1879.09	4061.58	1.443	-2.066
60	30	0.05	0.2	642.35	1905.89	3976.15	2.890	-4.126
90	30	0.05	0.2	646.78	1932.77	3890.97	4.341	-6.180
120	30	0.05	0.2	651.20	1959.72	3806.04	5.796	-8.227
0	30	0.05	0.3	633.47	1852.36	4326.22	-	-
30	30	0.05	0.3	637.92	1879.09	4176.72	1.443	-3.456
60	30	0.05	0.3	642.35	1905.89	4029.07	2.890	-6.869
90	30	0.05	0.3	646.78	1932.77	3883.27	4.341	-10.239
120	30	0.05	0.3	651.20	1959.72	3739.30	5.796	-13.567
0	30	0.2	0.05	543.36	1585.68	3333.18	-	-
30	30	0.2	0.05	560.39	1684.95	3414.10	6.260	2.428
60	30	0.2	0.05	577.31	1785.61	3492.97	12.608	4.794
90	30	0.2	0.05	594.11	1887.56	3569.85	19.038	7.100
120	30	0.2	0.05	610.81	1990.72	3644.83	25.544	9.350
0	30	0.3	0.05	490.07	1439.73	3006.24	-	-
30	30	0.3	0.05	514.74	1581.36	3135.94	9.837	4.314
60	30	0.3	0.05	539.18	1726.29	3262.26	19.904	8.516
90	30	0.3	0.05	563.41	1874.22	3385.38	30.179	12.612
120	30	0.3	0.5	587.46	2024.87	3505.49	40.642	16.607
0	0	0.3	0.05	490.07	1439.73	3014.51	-	-
30	0	0.3	0.05	514.74	1581.36	3144.63	9.837	4.316
60	0	0.3	0.05	539.18	1726.29	3271.36	19.904	8.520
90	0	0.3	0.05	563.41	1874.22	3394.90	30.179	12.619
120	0	0.3	0.05	587.46	2024.87	3515.41	40.642	16.616
0	60	0.3	0.05	490.07	1439.73	2997.93	-	-
30	60	0.3	0.05	514.74	1581.36	3127.21	9.837	4.312
60	60	0.3	0.05	539.18	1726.29	3253.11	19.904	8.512
90	60	0.3	0.05	563.41	1874.22	3375.83	30.179	12.605
120	60	0.3	0.05	587.46	2024.87	3495.53	40.642	16.598

When lz, lc, and α_s are fixed, the retailer orders less if her opportunity cost increases. In addition, the highest benefit gain from the delayed payment happens when the difference between both opportunity costs is large: namely, the retailer has a high opportunity cost and the wholesaler has a low one. For example, when $lz = 120$, $lc = 30$, and $\alpha_r = 0.3$, the result shows an increment in expected annuity stream for the retailer by 40.642% compared to conventional payment, which boosts the order quantity from 490.07 to 587.46. Conversely, for the wholesaler, the expected annuity stream increases by 16.607% compared to conventional payment, so a large order quantity increases the benefit for the wholesaler in the NPV, despite the disadvantage of being paid late.

Furthermore, when lc changes and other parameters are fixed, the optimal order quantity remains the same. The impact of lc only relates to $E[AS]_s$. Increasing or decreasing lc does not affect the retailer's optimal order quantity or expected profit.

2.6 Conclusions

This study has focused on the cash-flow-based NPV for the NP in which the timing of events and the opportunity costs of two independent firms are considered. The objective was to examine how the retailers choice of optimal order quantity is affected by different Net D clause conditions, and how this affects profit for retailer and wholesaler. Numerical examples and analysis were used to illustrate the solution procedure and to show the difference between using the NPV model and using a traditional one, as well as the effect of parameters' value on decision-making and expected profit.

The findings of this paper show a difference between the results when using the NPV model and the traditional one. However, implementing the NPV model only benefits the retailer. This indicates that if the wholesaler has the power to make the decision, he will not consider using the NPV model.

We also conducted an experiment using a second NPV model for which an analytical solution can be readily obtained. The aim was to see whether the results would show a small difference compared to the first NPV model. Unfortunately, the results were not what we expected and the use of the second simpler NPV model will typically not be as accurate.

Regarding the effect of delayed payment transaction, the analysis of the NPV model showed that the benefit of higher D values increases for both parties when the demand is highly variable, and when the retailer's opportunity cost is greater than the wholesaler's. The optimal Net D clause is thus subject to the circumstances. These findings are important in order to provide a warning sign for both parties when including a payment term contract.

The cash-flow-based NPV model used in this paper, though a simplification of reality, can provide insight to both retailers and wholesalers based on the optimal solutions proposed in this paper, which show that the right value of opportunity cost and high variability of demand could result in significant profit enlargement. In fact, the NPV approach is the appropriate model to capture the time value of events, which appears significant to the real-life model.

There are several opportunities for further research. First, this paper considered an independent/sequential supply chain; a future study could consider a coordinated supply chain instead. Second, further research could examine the impact of lead time on production cost based on the NPV model.

Chapter 3

A Mixing Contract for Coordinating a Supply Chain with Stochastic Demand

Abstract

This study investigates a coordinated two-echelon supply chain model for the newsvendor problem in the presence of cash-flow. We introduce a mixing contract that combines the buyback, revenue-sharing, and wholesale price discount contract, which achieves perfect coordination. The impact of trade credit on supply chain coordination is investigated and a few adjustments to the current parameters are made, such as non-integrated optimisation. We then extend the model to the information asymmetry setting in which the wholesaler's operation cost and opportunity cost are unknown to the retailer. From numerical examples, we show that, in each setting, the retailer can reveal private information. The results still indicate a win-win situation for both retailer and wholesaler, and the wholesaler can gain more profit from cheating.

3.1 Introduction

With the increment of market competition among individual enterprises, it is essential for firms to cooperate in supply chain (SC) coordination to maximise their expected profits. In a traditional business setting, each SC member commonly acts independently and tries to maximise his own profit. This kind of setting is called decentralised SC. It leads to the concept of *double marginalisation* (Spengler 1950), which occurs if both firms charge a markup which will result in a higher selling price and a lower demand in comparison with vertically integrated SC. In contrast, in centralised SC, a decision-maker is responsible for making decisions to maximise profits from the whole SC's point of view.

Over the last decades, there has been extensive study of SC contracts, since they can improve the profit of the whole SC. *Double marginalisation* ccan be eliminated by providing an incentive mechanism among the members of the SC. According to Cachon (2003), three main objectives can be achieved through an SC contract: (i) the total SC profit is increased so as to make it closer to the profit resulting from a centralised SC (ii), the risks are shared among the SC members (iii), each SC member obtains an expected profit higher than he/she would do without the contract. The SC contract becomes more complex when the retailer does not know the exact cost components of the wholesaler, such as raw material costs, labour costs, and opportunity cost. In this case, the wholesaler's production cost or opportunity cost is privately known.

In addition, in most business activities, the wholesaler/retailer normally offers/requests a delayed payment period to/from the retailer/wholesaler. In this case, modelling a SC in the NPV approach is a 'correct' way to see what is happening to the cash-flow of both the wholesaler and the retailer. With delayed payment, the retailer will gain more profit as it frees up working capital for her. From the wholesaler's perspective, holding an unpaid debt for long terms can have a destructive effect on his cash-flow. Our proposed model includes the delayed payment terms to analyse the effect of making such a payment to the wholesaler.

Our first goal is to examine whether there is a difference in optimal solution when implementing the NPV of the newsvendor problem (NPVNP) instead of the traditional newsvendor problem (NP). Our second goal is to develop a mixing contract that will perfectly coordinate the SC. Our third goal is to analyse how the wholesaler should adjust the model to benefit from the delayed payment term. Our last goal is to analyse how dishonesty (i.e. false information) affects the retailer's optimal order quantity and impacts both SC members' expected annuity stream of profit. The benchmark of the problem is the case in which the wholesaler is honest by giving true information. Thus, our aim here is not to design the principal-agent framework to elicit truthful information, but to yield insights on optimal results when the retailer faces a dishonest wholesaler.

To reach the aforementioned goals, we study a two-echelon SC with one wholesaler and one retailer. The wholesaler manufactures a single item and sells it to the retailer. The retailer faces a random demand that follows a known distribution function. The wholesaler proposes a mixing contract, which is a combination of buyback and price discount, to the retailer to entice the latter to order more. However, the retailer negotiates and provides another contract: revenue sharing if the wholesaler offers delayed payment terms to the retailer. This model is developed considering two cases: complete information and asymmetric information.

Our work contributes to the literature in three ways. First, we derive the mixing contract in the NP under a few payment structures with time value of money (TVM). Under TVM, the results show an increment in the expected annuity stream of SC profit and a decrease in optimal order quantity compared to the traditional NP. Second, the proposed mixing contract perfectly coordinates the SC. Regarding the effect of giving delayed payment to the retailer, the wholesaler has an option to deviate from having perfectly coordinated results to non-integrated optimisation to benefit from the delayed payment. Third, we show that under asymmetric information, given a set of mixing contracts, the retailer can reveal private information. In addition, the retailer and wholesaler's expected annuity stream of profit is still higher than with a wholesale price contract when the wholesaler is dishonest.

The remainder of the paper is organised as follows. Section 3.2 reviews the relevant literature. Sections 3.3 to 3.6 provide a mathematical formulation to develop the proposed model. Numerical analysis is given in Section 3.7. Finally, conclusions are presented in Section 3.8.

3.2 Literature Review

This study is directly related to three streams of the production and inventory management literature: SC contracts, NPV, and information asymmetry.

There is a vast body of literature on SC contracts that coordinate the SC. The revenue-sharing contract is one of the most popular incentive mechanisms and has received much attention in the SC literature (Cachon & Lariviere 2005, Xu et al. 2014, Arani et al. 2016). Chen (2011) proposed a mixing contract based on two sub-contracts – wholesale price discount and return contracts – that can achieve perfect coordination. Chintapalli et al. (2017) propose a contract that involves both advance-order discount and a minimum order quantity requirement, which can coordinate the supply chain.

Recently, Tang & Kouvelis (2014) presented a contract with random yield based on a combination of buyback and revenue sharing that can coordinate the SC. In contrast to Chen (2011) and Tang & Kouvelis (2014), we propose a contract with a combination of buyback, revenue sharing, and price discount in the presence of different payment structure and NPV. Most buyback contract and revenue-sharing contract studies neglect TVM. If there is a payment structure, such as delayed payment terms, it is important to consider the TVM in the SC environment. In the past few decades, numerous studies have examined production-inventory systems incorporating the cash-flow-based NPV approach.

One of the earliest studies in this field was that of Goyal (1985), who developed an EOQ formula for the case in which the buyer may delay the payment by a specified number of days. Since then, many researchers have extended the model into different aspects. Seifert et al. (2013) conducted a comprehensive review of the literature addressing trade credit. The authors highlighted the importance of accounting for cash-flow timing in the objective function. Recently, Chen & Teng (2015) developed an EOQ model for deteriorating items with upstream and downstream trade credit financing considering the discounted cash-flow analysis. However, the aforementioned papers mainly use trade credit in deterministic demand. There are a few recent studies combining the financing and inventory environment considering trade credit under stochastic demand (Kouvelis & Zhao 2011, 2012). The authors developed an SC model considering that both wholesaler and retailer have limited working capital and assumed that the retailer can either choose to finance with the wholesaler by trade credit or borrow from the bank. In the present study, we omit the financing part and only consider a basic trade credit (delayed payment).

Surveying the literature on delayed payment reveals that no studies have examined the impact of payment structures on the SC under TVM. Most papers on newsvendor models have failed to take into account the TC and TVM. Furthermore, most papers on TC and TVM have not considered stochastic demand (newsvendor model). A model including stochastic demand and incorporating TC and TVM does not exist(see more details on the past literature in Section 2.2). Therefore, such a model should be developed.

The third aspect that differentiates our work from the literature is how we evaluate the asymmetric information in our model. The wholesaler's cost and the retailer's demand forecast are the common parameters that are set as a private information in the study of SC coordination with asymmetric information. Among others, Corbett & Tang (1999), Corbett & De Groote (2000), Ha (2001) considered a take-it-or-leave-it contract using the principal-agent framework and mechanism design principle to ensure information credibility. However, such a contract involves complexity and, in practice, the agreement of a contract is normally based on negotiations among the firms. Our model is developed based on the idea in Heese & Kemahlıoğlu-Ziya (2016). Instead of designing a contract principle to reveal true information, these authors investigated the revenue-sharing contract with dishonest information from the retailer to coordinate the SC. They showed that the wholesaler might benefit from the retailer's dishonesty when the retailer can exert the sales effort. In contrast, we are interested in the retailer's perspective: whether he would be better off with inaccurate information regarding the wholesaler's operation and opportunity costs. To our knowledge, no study has set opportunity cost as private information.

3.3 Mathematical Model

In this study, a two-echelon SC model is developed under the NP, which comprises a downstream retailer R and an upstream wholesaler W. The retailer faces a random demand x which is a non-negative random variable with a probability distribution $f(x)$ and corresponding cumulative distribution function $F(x)$.

The following assumptions are used in this model:

Assumptions

- The wholesale price is decided by the wholesaler.
- The demand follows a Gamma distribution.

- The selling price is exogenous determined and known.
- The selling season is in an accounting year.
- The retailer's sales revenue continuously received between 0 and T.
- $F(x)$ is a differentiable and strictly increasing function on its support.

The superscript * denotes the optimal solutions, and the use of subscripts s and r serves to distinguish between wholesaler and retailer, respectively.

The sequence of event is as follow. Before the start of the selling season, W propose a contract to R, and then R, negotiates with the wholesaler by offering another contract to increase R's expected profit. When both parties agree with the contract, the retailer decides how much quantity to order. The aim of proposing the contract is to coordinated the SC and maximise the entire SC's expected profit. In the next section, we first develop a general formulation of the expected annuity stream for both wholesaler and retailer.

3.3.1 Retailer's Expected Profit Function

Considering n selling periods of an accounting year over an infinite accounting year, the expected annuity stream of revenue received by the retailer at a continuous time from 0 to T is;

$$\begin{aligned} E[AS_1]_r &= \frac{P}{T}\Big[\int_0^Q \Big(\frac{1-e^{-\alpha_r T}}{\alpha_r}\Big) x f(x) dx \\ &+ \int_Q^\infty \Big(\frac{1-e^{-\alpha_r T}}{\alpha_r}\Big) Q f(x) dx\Big]\Big(\sum_{i=0}^{n-1} \alpha_r e^{-i\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty} e^{-j\alpha_r nT}\Big). \end{aligned} \tag{3.1}$$

Leading to the following linearisation in α_r of the expected annuity stream:

$$E[AS_1]_r = \frac{P}{T}\Big(\int_0^Q x f(x) dx + \int_Q^\infty Q f(x) dx\Big). \tag{3.2}$$

For simplicity, let $E[S(Q)] = \int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx$. Thus, we have;

$$E[AS_1]_r = \frac{P}{T}E[S(Q)]. \tag{3.3}$$

At $L_z = lz/365$ times after the end of the selling season T, the retailer receives the corresponding expected annuity stream from the salvage activities;

$$E[AS_2]_r = v\int_0^Q (Q-x)f(x)dx(e^{-\alpha_r L_v})\Big(\sum_{i=0}^{n-1}\alpha_r e^{-(i+1)\alpha_r T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_r nT}\Big), \tag{3.4}$$

leads to;

$$E[AS_2]_r = v\gamma_v\Big(Q - \int_0^\infty xf(x)dx + \int_Q^\infty (x-Q)f(x)dx\Big), \tag{3.5}$$

where $\gamma_v = \frac{\alpha_r e^{-\alpha_r(L_v+1/n)}}{1-e^{-\alpha_r/n}}$. For simplicity, let $E[V(Q)] = Q - \int_0^\infty xf(x)dx + \int_Q^\infty (x - Q)f(x)dx$. Thus, we have:

$$E[AS_2]_r = v\gamma_v E[V(Q)]. \tag{3.6}$$

The total expected annuity stream of profit for the retailer is the summation of Eqs. (3.3) and (3.6) minus the transfer payment (TP) from retailer to wholesaler. Thus, we have;

$$E[AS^r] = \frac{P}{T}E[S(Q)] + v\gamma_v E[V(Q)] - TP. \tag{3.7}$$

3.3.2 Wholesaler's Expected Annuity Stream

The wholesaler produces n orders Q over an infinite accounting year before the start of the selling period, with a production cost c per order at time $L_c = lc/365$. The expected annuity stream of total profit for the wholesaler is the TP made by the retailer minus the expected annuity stream of total cost, which is given by;

$$E[AS^s] = TP - cQe^{-\alpha_s(-L_c)}\Big(\sum_{i=0}^{n-1}\alpha_s e^{-i\alpha_s T}\Big)\Big(\sum_{j=0}^{\infty}e^{-j\alpha_s nT}\Big). \tag{3.8}$$

By taking an algebraic manipulation, we have:

$$E[AS^s] = TP - cQ\gamma_c, \tag{3.9}$$

where $\gamma_c = \frac{\alpha_s e^{\alpha_s L_c}}{1-e^{-\alpha_s/n}}$.

3.3.3 The Integrated Channel

In this section, we derive two integrated models: the traditional and the NPV NPs. Then, we compare the results of these two models using numerical experiments.

3.3.3.1 Traditional NP

From traditional NP, the expected total profit function for the integrated model is given below;

$$E[\Pi_{sc}] = p\Big(\int_0^{Q_t} xf(x)dx + Q_t\int_{Q_t}^{\infty} f(x)dx\Big) + v\int_0^{Q_t}(Q_t - x)f(x)dx - cQ_t. \tag{3.10}$$

It is relatively easy to prove that $E[\Pi_{sc}]$ is concave in Q_t (Silver et al. 1998). Thus, the first order conditions (FOCs) are necessary to determine the optimal Q_t. By taking the first derivative of Eq. (3.10) with respect to Q_t and setting it to zero, the optimal order quantity is given by;

$$Q_t^* = F^{-1}\Big(\frac{p-c}{p-v}\Big). \tag{3.11}$$

3.3.3.2 NPV NP

The integrated firm's expected annuity stream of total profit is the summation of Eqs. (3.7) and (3.9);

$$E[AS_{sc}] = \frac{P}{T}E[S(Q_{sc})] + v\gamma_v E[V(Q_{sc})] - cQ_{sc}\gamma_c. \tag{3.12}$$

By taking the first derivative of Eq. (3.12) with respect to Q_{sc} and setting it to zero, the optimal order quantity is given by;

$$Q_{sc}^* = F^{-1}\left(\frac{p - c\bar{\gamma}_c}{p - v\bar{\gamma}_v^r}\right), \tag{3.13}$$

where $\bar{\gamma}_c = \frac{\alpha_r e^{\alpha_r L_c}}{n(1-e^{-\alpha_r/n})}$, and $\bar{\gamma}_v^r = \frac{\alpha_r e^{-\alpha_r(L_v+1/n)}}{n(1-e^{-\alpha_r/n})}$. From Eq. (3.13), the existence of $\bar{\gamma}_c$ and $\bar{\gamma}_v^r$ lower the value of critical ratio $\frac{p-c\bar{\gamma}_c}{p-v\bar{\gamma}_v^r}$. Therefore, $Q_{sc}^* < Q_t^*$.

3.4 The Mixing Contract

The wholesaler first proposes a mixing contract, denoted by $C_s(b, w_m)$, combining two popular contracts: the buyback policy and wholesale price discount policy. Then, the retailer negotiates by adding another contract $C_r(\beta, L_z)$: a revenue-sharing contract if the wholesaler agrees with a delay in payment. Assuming both firms agree with these contract, the final contract follows $C(b, w_m, \beta, L_z)$. The graphical representation of this system is shown in Figure 3.1. We now describe in detail the contracts considered in this paper.

Before the start of the selling season, W makes two decisions: the unit wholesale price w_m, and the buyback price b per unsold item returned by R based on the given β and L_z. The retailer then decides the optimal order quantity Q_m based on w_m, and b from

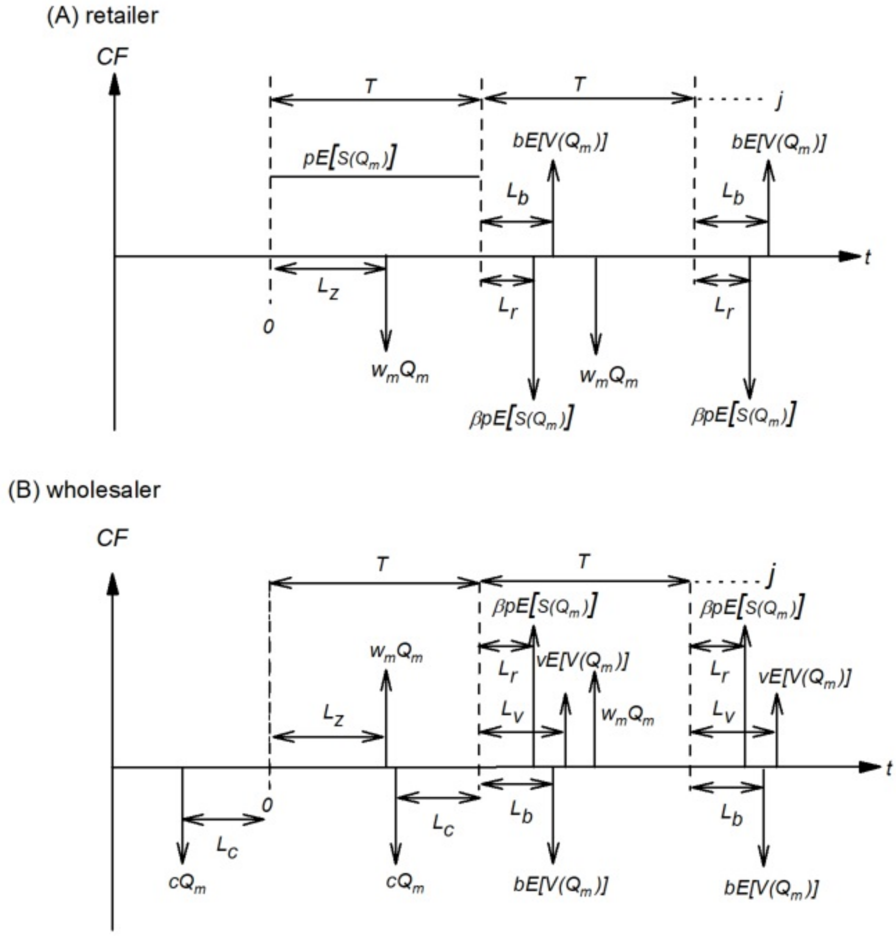


Figure 3.1: Cash-flows representation of the mixing contract

the wholesaler. At the start of the selling season, the sales revenue is continuously received until T. The first TP happens at $t = L_z$, where R pays $w_m Q_m$ to W. At the end of the selling period, the TP happens twice. First, at $t = L_r$, the retailer shares her revenue at $\beta pE[S(Q_m)]$ with the wholesaler. At $t = L_b$, W pays R for any unsold items at $bE[V(Q_m)]$ and sells the salvage, $vE[V(Q_m)]$ at $t = L_v$. The cash-flow is repeated until j times in the future accounting year.

Thus, the expected transfer payment for this model is developed as follow;

$$
\begin{aligned}
E[TP] &= \Big(\sum_{j=0}^{\infty} e^{-j\alpha_k nT}\Big)\Big[w_m Q_m e^{-\alpha_k L_z}\Big(\sum_{i=0}^{n-1}\alpha_k e^{-i\alpha_k T}\Big) \\
&- \frac{\beta p}{T}\Big(\int_0^{Q_m} x f(x)dx + \int_{Q_m}^{\infty} Q_m f(x)dx\Big)e^{-\alpha_k(L_r+T)}\Big(\sum_{i=0}^{n-1}\alpha_k e^{-(i+1)\alpha_k T}\Big) \\
&- b\int_0^{Q_m}(Q_m - x)f(x)dx(e^{-\alpha_k(L_b+T)})\Big(\sum_{i=0}^{n-1}\alpha_k e^{-(i+1)\alpha_k T}\Big)\Big], \qquad (3.14)
\end{aligned}
$$

where α_k denotes the opportunity cost for the retailer($k = r$) and wholesaler ($k = s$). Since we assume that the sales period occurs at an accounting year, $nT = 1$. Then, by using linear approximation, Eq. (3.14) can be written as;

$$
E[TP] = \beta pnE[S(Q_m)]\gamma_p^k + w_m Q_m \gamma_z^k - b\gamma_b^k E[V(Q_m)], \qquad (3.15)
$$

where,

$$
E[S(Q_m)] = \int_0^{Q_m} x f(x)dx + \int_{Q_m}^{\infty} Q_m f(x)dx
$$

$$
E[V(Q_m)] = \int_0^{Q_m}(Q_m - x)f(x)dx
$$

$$
\gamma_p^k = \frac{\alpha_k e^{-\alpha_k(L_r+1/n)}}{(1-e^{-\alpha_k/n})}, \gamma_z^k = \frac{\alpha_k e^{-\alpha_k L_z}}{(1-e^{-\alpha/n})}, and \gamma_b^k = \frac{\alpha_k e^{-\alpha_k(L_b+1/n)}}{(1-e^{-\alpha_k/n})}.
$$

Therefore the retailer's and wholesaler's expected annuity stream of profit is given by:

$$
E[AS_m^r] = (1-\beta\gamma_p^r)pnE[S(Q_m)] - w_m\gamma_z^r Q_m + bE[V(Q_m)]\gamma_b^r, \qquad (3.16)
$$

and

$$
E[AS_m^s] = \beta pnE[S(Q_m)]\gamma_p^s + w_m\gamma_z^s Q_m - c\gamma_c Q_m + (v\gamma_v^s - b\gamma_b^s)E[V(Q_m)], \qquad (3.17)
$$

respectively.

Proposition 1. *$E[AS_m^r]$ is concave in Q_m.*

Proof. The first derivative of $E[AS^r_m]$ with respect to Q_m is;

$$Q_m = F^{-1}\Big(\frac{(1-\beta\gamma^r_p)p - w_m\bar{\gamma}^r_z}{(1-\beta\gamma^r_p)p - b\bar{\gamma}^r_b}\Big), \tag{3.18}$$

where $\bar{\gamma}^r_z = \frac{\alpha_r e^{-\alpha_r L_z}}{n(1-e^{-\alpha_r/n})}$, and $\bar{\gamma}^r_b = \frac{\alpha_r e^{-\alpha_r(L_b+1/n)}}{n(1-e^{-\alpha/n})}$. The second derivative of the retailer's expected annuity stream with respect to Q_m can be calculated as;

$$\frac{d^2E[AS^r_m]}{dQ^2_m} = (b\bar{\gamma}^r_b - (1-\beta\gamma^r_p)p)f(Q_m). \tag{3.19}$$

Note that the expected annuity stream for the retailer, $E[AS^m_r(Q_m)]$ is concave if $(1-\beta\gamma_r)p > b\bar{\gamma}_b$. To ensure a profitable return in business, we assume that $b\bar{\gamma}^r_b < w_m\bar{\gamma}^r_z < (1-\beta\gamma_r)p$. Thus, $E[AS^r_m]$ is concave in Q_m, and maximises the given annuity stream function. ■

The mixing contract is perfectly coordinated if $Q^*_m = Q^*_{sc}$. Thus, we have;

$$\frac{(1-\beta\gamma^r_p)p - w_m\bar{\gamma}^r_z}{(1-\beta\gamma^r_p)p - b\bar{\gamma}^r_b} = \frac{p - c\bar{\gamma}_c}{p - v\bar{\gamma}^r_v}. \tag{3.20}$$

Given w_m, β, and L_z, and rearranging Eq. (3.20) the optimal value of b^* is;

$$b^* = \frac{p(1-\beta\gamma^r_p)\lambda + w_m y\bar{\gamma}^r_z}{\varphi\bar{\gamma}^r_b}, \tag{3.21}$$

where $y = p - v\bar{\gamma}^r_v$, $\varphi = p - c\bar{\gamma}_c$, and $\lambda = v\bar{\gamma}^r_v - c\bar{\gamma}_c$.

3.5 Pareto Improvement

In this section, we provide an analytical condition for setting the sales target that can achieve Pareto improvement under the mixing contract. In a decentralised situation, both wholesaler and retailer make their own decisions to maximise the expected annuity stream of total profit. In particular, the wholesale price contract can usually be used as a benchmark to compare the channel performance. The wholesaler and retailer, each can provide an appealing sales target to induce both parties to accept the mixing contract, and thus, achieve Pareto improvement.

Under the wholesale price contract, the retailer's and the wholesaler's expected annuity streams of total profit are given as;

$$E[AS_d^r(Q)] = pnE[S(Q)] - w\gamma_z^r Q + vE[V(Q)]\gamma_v^r, \tag{3.22}$$

and

$$E[AS_d^s(Q)] = w\gamma_z^r Q - c\gamma_c Q, \tag{3.23}$$

respectively.

The optimal $E[AS_d^r]$ is concave in Q, thus, the optimal Q that maximises (3.22) is given by;

$$Q^* = F^{-1}\Big[\frac{p - w\bar{\gamma}_z^r}{p - v\bar{\gamma}_v^r}\Big], \tag{3.24}$$

where $\bar{\gamma}_z^r = \frac{\alpha_r e^{-\alpha_r L_z}}{n(1-e^{-\alpha_r/n})}$, and $\bar{\gamma}_v^r = \frac{\alpha_r e^{-\alpha_r (L_v+1/n)}}{n(1-e^{-\alpha/n})}$.

The condition for the retailer and wholesaler to accept the mixing contract is when the expected annuity stream of the mixing contract is higher than that of the wholesale price contract:

i. $[AS_m^r(b^*, Q_m^*)] \geq E[AS_d^r(Q)]$.

ii. $E[AS_w^m(Q_m, b^*)] > E[AS_w^d(Q)]$.

By substituting Eqs. (3.21), (3.16), and (3.22) into (i), we have;

$$w_m \leq \frac{pn(1-\beta\gamma_p^r)\Big(E[S(Q_m)]\varphi + E[V(Q_m)]\lambda\Big) - \varphi E[AS_d^r]}{\gamma_z^r(\varphi Q_m - yE[V(Q_m)])} = \overline{w}. \tag{3.25}$$

By substituting Eqs. (3.21), (3.17), and (3.23) into (ii), we have;

$$\begin{aligned} w_m &\geq \frac{\Big(E[S(Q_m)]pn\beta\gamma_p^s + E[V(Q_m)]v\gamma_v^s - cQ_m\gamma_c - E[AS_w^d(Q)\Big)\varphi}{\gamma_z^r\Big(yE[V(Q_m)] - Q_m\varphi\Big)} \\ &- \frac{E[V(Q_m)]np(1-\beta\gamma_p^s)\lambda}{\gamma_z^r\Big(yE[V(Q_m)] - Q_m\varphi\Big)} = \underline{w}. \end{aligned} \tag{3.26}$$

To avoid a trivial case, we assume that $c < b^* < w_m < w_d$. The equation above is solved numerically. Then $w_m \in [\underline{w}, \overline{w}]$. Thus, if the wholesaler choose $w_m \in [\underline{w}, \overline{w}]$, and b^* after the retailer decides β and L_z, the mixing contract can achieve perfect coordination and yield a higher expected annuity stream of profit than the wholesale price contract.

3.6 Asymmetric Information

In this section, we examine two cases of asymmetric information: first, when the wholesaler's operation cost c is unknown; and second, when the wholesaler's opportunity cost α_s is unknown.

Unknown operation Cost, c. From the mixing contract, we assume that the value of β is axiomatically shared information, and that the retailer knows the modelling structure of the problem. Therefore, specifying the contract of (β, w_m, b^*) reveals the wholesaler's true marginal cost, c to the retailer, since this is given from Eq. (3.21). Rearranging Eq. (3.21) to find c, thus, the value of c is given by;

$$c = \frac{p(b\bar{\gamma}_b^r - \bar{\gamma}_v^r v(1-\beta\gamma_p^r)) - \bar{\gamma}_z^r w_m(p - v\bar{\gamma}_v^r)}{\bar{\gamma}_c(b\bar{\gamma}_b^r - p(1-\beta\gamma_p^r))}. \tag{3.27}$$

Unknown opportunity cost of the wholesaler, α_s. The wholesaler's opportunity cost α_s is unknown. The value of β is necessarily shared information for revenue sharing to occur, and we assume that the retailer knows the modelling structure of the problem. Hence, by specifying the optimal value of w_m, b^* and claimingFalse Signal of c that the contract will achieve perfect coordination, the wholesaler reveals his true opportunity cost α_s to the retailer. The retailer can reveal α_s by rearranging Eq. (3.21) to find α_s. The equation is given below;

$$\bar{\gamma}_c = \frac{p(b\bar{\gamma}_b^r - \bar{\gamma}_v v(1 - \beta\gamma_p^r)) - \bar{\gamma}_z^r w_m(p - v\bar{\gamma}_v^r)}{c(b\bar{\gamma}_b^r - p(1 - \beta\gamma_p^r))}. \quad (3.28)$$

Since the RHS of the equation is known, the retailer can use optimisation to find α_s in $\bar{\gamma}_c$.

3.6.1 Dishonest Firm

In this section, we assume that the retailer has no prejudice regarding the wholesaler's behaviour, which means that she trusts the wholesaler. From the wholesaler's perspective, if he prefers to cheat, he needs to ensure that he meets all the previously stated conditions for the retailer to accept the contract. We investigate whether the retailer still benefits from the wholesaler's behaviour in this case.

3.6.1.1

Assume that the wholesaler's unit cost c is unknown and that the wholesaler wishes to cheat. Let c be the true marginal cost, while $c'' = c + \Delta c$ denotes the wholesaler's signal of his marginal cost to the retailer. Let b'' be the buyback price when cheating and let w_m be in the range of (w_l, w_u) from true c. Thus, b'' is given by;

$$b'' = \frac{p(1 - \beta\gamma_p^r)\lambda'' + w_m y\bar{\gamma}_z^r}{\varphi''\bar{\gamma}_b^r}, \quad (3.29)$$

where $\varphi'' = p - (c + \Delta c)\bar{\gamma}_c$, and $\lambda'' = v\bar{\gamma}_v^r - (c + \Delta c)\bar{\gamma}_c$.

The retailer's optimal quantity is Q''. Therefore, the expected annuity streams for the retailer and wholesaler are;

$$E[AS_m^r(Q'')] = (1-\beta\gamma_p^r)pnE[S(Q'')] - w_m\gamma_z^r Q' + b''E[V(Q'')]\gamma_b^r, \tag{3.30}$$

and

$$E[AS_m^s(Q'')] = \beta pnE[S(Q'')]\gamma_p^s + w_m\gamma_z^r Q'' - c\gamma_c Q'' + (v\gamma_v^s - b''\gamma_b^s)E[V(Q'')], \tag{3.31}$$

respectively.

Proposition 2. *Cheating decreases the buyback value for the wholesaler.*

Proof. Assume that b'' from Eq. (3.29) is less than b^* in Eq. (3.21). Thus we have:

$$\frac{p(1-\beta\gamma_p^r)(v\bar{\gamma}_v^r - (c+\Delta c)\bar{\gamma}_c) + w_m y\bar{\gamma}_z^r}{(p-(c+\Delta c)\bar{\gamma}_c)\bar{\gamma}_b^r} < \frac{p(1-\beta\gamma_p^r))(v\bar{\gamma}_v^r - c\bar{\gamma}_c) + w_m y\bar{\gamma}_z^r}{(p-c\bar{\gamma}_c)\bar{\gamma}_b^r}.$$

By solving this equation, we have $w\bar{\gamma}_z^r < p(1-\beta\gamma_p^r)$. This equation holds because if the marginal cost is greater than the marginal revenue, the retailer will not gain profit. ■

Proposition 3. *Cheating decreases the optimal order quantity of the retailer if and only if Proposition 2 holds.*

Proof. The optimal $Q_m(b'')$, and $Q_m^r(b^*)$ are:

$$Q_m(b'') = F^{-1}\Big[\frac{(1-\beta\gamma_r)p - w\bar{\gamma}_z}{(1-\beta\gamma_r)p - b''\bar{\gamma}_b}\Big], Q_m(b^*) = F^{-1}\Big[\frac{(1-\beta\gamma_r)p - w\bar{\gamma}_z}{(1-\beta\gamma_r)p - b\bar{\gamma}_b}\Big].$$

From the above Eqs., it can easily prove that $Q_m(b'') < Q_m(b^*)$ when $b'' < b^*$. ■

3.6.1.2 False Signal of the Wholesaler's Opportunity Cost

This scenario is only valid when $\alpha_s \leq \alpha_r$. For the SC to achieve perfect coordination, $\alpha_s = \alpha_r$. Therefore, let the true $\alpha_s < \alpha_r$. Let $\alpha^{s'}$ be the true opportunity cost, and let $\alpha^{s'} = \alpha_s + \Delta\alpha_s$ be the wholesaler's signal of his opportunity cost to the retailer. Thus we have $\bar{\gamma}_c' = \frac{\alpha^{s'} e^{\alpha^{s'} L_c}}{n(1-e^{-\alpha^{s'}/n})}$.Then, we set b from the mixing contract as;

$$b' = \frac{p(1-\beta\gamma_p^r\lambda' + w_m y\bar{\gamma}_z^r}{\varphi'\bar{\gamma}_b^r}, \tag{3.32}$$

where $\varphi' = p - c\bar{\gamma}_c'$, and $\lambda' = v\bar{\gamma}_v^r - c\bar{\gamma}_c'$.

The retailer now arrives at quantity Q' that optimises;

$$E[AS_m^r(Q')] = (1-\beta\gamma_p^r)pnE[S(Q')] - w_m\gamma_z^r Q' + b'E[V(Q')]\gamma_b^r. \tag{3.33}$$

The wholesaler's true expected annuity stream is hence given by;

$$E[AS_m^s(Q')] = \beta pnE[S(Q')]\gamma_p^s + w_m\gamma_z^r Q' - c\gamma_c Q' + (v\gamma_v^s - b'\gamma_b^s)E[V(Q')]. \tag{3.34}$$

Proposition 4. *Cheating decreases the wholesaler's buyback value.*

Proof. Assuming that b' from Eq. (3.32) is less than b^* in Eq. (3.21), we have:

$$\frac{p(1-\beta\gamma_p^r)(v\bar{\gamma}_v^r - c\bar{\gamma}_c') + w_m y\bar{\gamma}_z^r}{(p-c\bar{\gamma}_c')\bar{\gamma}_b^r} < \frac{p(1-\beta\gamma_p^r))(v\bar{\gamma}_v^r - c\bar{\gamma}_c) + w_m y\bar{\gamma}_z^r}{(p-c\bar{\gamma}_c)\bar{\gamma}_b^r}.$$

By solving this equation, we have $v\bar{\gamma}_v^r < p$. Therefore, $b' < b^*$. ■

Proposition 5. *Cheating decreases the retailer's optimal order quantity if and only if Proposition 4 holds.*

Proof. The optimal order quantity for $Q^{'}(b')$, and $Q_m^r(b^*)$ are

$$Q'(b') = F^{-1}\left[\frac{(1-\beta\gamma_r)p - w\bar{\gamma}_z}{(1-\beta\gamma_r)p - b'\bar{\gamma}_b}\right], Q_m^r(b*) = F^{-1}\left[\frac{(1-\beta\gamma_r)p - w\bar{\gamma}_z}{(1-\beta\gamma_r)p - b\bar{\gamma}_b}\right].$$

Since $b' < b$, then $Q'(b') < Q'(b*)$. ■

3.7 Numerical Examples

In this section, we present a numerical example of the single-period newsvendor model. We assume that demand uncertainty, x is Gamma distributed. . Five instances are used in this section, as in Table 3.1. The other standard parameter values are the following:

Table 3.1: Problem instance characteristics

Instance	shape, α_g	scale, τ	μ	σ
1	10	100	1000	316.23
2	8	125	1000	353.55
3	6	166.6667	1000	408.25
4	4	250	1000	500.00
5	2	500	1000	707.11

$$p = 15, w = 10, c = 4, v = 2, n = 1, \alpha_s = 0.2, \alpha_r = 0.2$$

3.7.1 Comparison between Traditional and NPV Model

To see the differences in results between the traditional NP and the NPV models, we derive the following algorithm and use the Mathematica software as a solution tool: We assume that there are no payment terms involved. Thus,

$$lc = lz = lv = 0$$

Algorithm 2

Input. List of parameters.
Output. A local maximum Q_t^*, and Q_{sc}^* with $E[AS_{sc}(Q_t^*)]$, and $E[AS_{sc}(Q_{sc}^*)]$.
Step 1. Compute Q_t^* from Eq. (3.11). Then, set $Q^* = Q_t^*$.
Step 2.Compute $E[AS_{sc}(Q^*)]$,using Eq. (3.12).
Step 3. Repeat Steps above by finding Q_{sc}^* from Eq. (3.13).

Table 3.2 compares the optimal order quantity and expected total profit of the traditional newsvendor and the NPV models. To represent the gains from using the NPV model versus the traditional model, we define $PG^{sc} = \frac{100(E[AS_{sc}(Q_{sc}^*)]-E[AS_{sc}(Q_t^*)])}{E[AS_{sc}(Q_t^*)]}$.

Table 3.2: The Expected Annuity Stream for Integrated SC (NPV framework)

No.	Q_{sc}^*	Q_t^*	$E[AS_{sc}(Q_{sc}^*)]$	$E[AS_{sc}(Q_t^*)]$	PG^{sc}
1	1255.08	1318.65	9337.67	9325.43	0.131
2	1282.70	1354.97	9180.63	9159.13	0.235
3	1321.95	1407.36	8947.87	8922.41	0.285
4	1384.18	1492.65	8551.56	8519.19	0.380
5	1505.29	1669.79	7641.26	7592.02	0.649

Firstly, it is clear that for each instance, the NPV model generates more profit than the traditional model. The highest PG^{sc} is at 0.649% when the variability of demand is the highest and the lowest PG^{sc} is at 0.131% when the variability of demand is the lowest. Overall, when moving down the rows, the expected annuity stream of SC for both cases decreases with the increase in variability of demand. This indicates that when the variability of demand is high, the risk of the item not being sold is high as well, thus lowering the expected profit. In contrast, the value of Q_{sc}^*, and Q_t^* show an opposite pattern when moving down the row. For example, at instance 1, $Q_{sc}^* = 1255.08$, and $Q_t^* = 1318.65$ are lower than $Q_{sc}^* = 1505.29$, and $Q_t^* = 1669.79$ at instance 5. This result shows that the retailer will order more when the variability of demand is high.

While $E[AS_{sc}(Q_{sc}^*)]$ is greater than $E[AS_{sc}(Q_t^*)]$, in fact, the optimal order quantity is smaller in the NPV model than in the traditional newsvendor model for each instance. For example, at instance 1, $Q_{sc}^* = 1255.08 < 1318.65(Q_t^*)$. This happens because the existence of the opportunity cost in Q_{sc}^* reduces the value (Proposition **??**) . This data shows that it is generally correct to consider the NPV framework, as it reflects the 'real world' situation that accounts for TVM.

3.7.2 The Mixing Contract.

To show the effectiveness of the proposed policies, we adopt the same instances as in Table 2.1. For ease of reference, other parameter values are restated as follows: $p = 15, w = 10, c = 4, v = 2, n = 1$. In this example, we consider $\beta = 0.4$, $lb = lr = lc = 10$, and $lz = 0$. We derive the following algorithm:

Algorithm 3

Input. List of parameters.
Output. b^*, a local maximum Q^*_m, and Q^*_{sc} with $E[AS^s_m(Q^*_m)]$ and $E[AS^s_m(Q_{sc})]$.
Step 1. (Local search). Find the value for $\underline{w}$,and $\bar{w}$ using *FindRoot* function in Software Mathematica.
Step 2.Choose w_m between the limit $\underline{w}$ and $\overline{w}$.
Step 3.Compute b^*, Q^*_m $E[AS^s_m(.)]$ and $E[AS^s_m(.)]$.

Assume that the wholesaler chooses $w_m = 5$, as this value lies within the boundaries of w_m for all instances. For example, for instance 1, $w_m \in [4.67, 5.07]$. Thus, the optimal $b^* = 4.9665$. Table 3.3 gives the optimal solutions for (a) the mixing contract, and (b) the wholesale-price only contract. The parenthesis represents the optimal value of Q^*, the expected annuity stream for the wholesaler, $E[AS^w_d]$ and the expected annuity stream for the retailer, $E[AS^r_d]$ in the wholesale-price-only contract. To represent the gain from using a proposed model versus a wholesale-price-only contract, we define the percentage gain as $PG^{r/s} = \frac{100(E[AS^{r/s}_m] - E[AS^{r/s}_d])}{E[AS^{r/s}_d]}$ where subscript (r/s) represents either the retailer or the wholesaler. Overall, the results show that both parties gain profit when using a mixing contract compared to a wholesale-price-only contract.

For example, in instance 1, the retailer gains about 37.70% and the wholesaler 5.95% more profit in the mixing contract compared to the wholesale price-only-contract. This is due to the increment of the order quantity, which is about twice the amount in the wholesale-price-only contract. Moving down the row in Table 3.3, as the variability

Table 3.3: The Expected Annuity Stream

No.	Q_m^*	$E[AS_m^r]$	$E[AS_m^s]$	PG^r	PG^s	$E[AS_m^{sc}] = E[AS^{sc}]$
1	1245.5 (813.28)	3596.21 (2611.70)	5641.3 (5324.44)	37.696	5.951	9237.51
2	1271.84 (789.01)	3533.93 (2467.41)	5543.59 (5165.53)	43.224	7.319	9077.52
3	1309.15 (752.85)	3441.67 (2262.99)	5398.86 (4928.76)	52.085	9.538	8840.53
4	1368 (690.92)	3284.76 (1939.9)	5152.72 (4523.31)	69.326	13.915	8437.48
5	1481.01 (548.66)	2925.17 (1309.64)	4588.64 (3591.98)	123.357	27.747	7513.81

of demand increases, the PG for both parties is highest in instance 5. Normally, when there is high variability of demand, the retailer will not take much risk and will thus order less to avoid unexpected loss. However, as we can see, the difference between w_m and b^* is only 0.03. Thus, the retailer confidently orders more, as she knows that she will definitely earn a high profit from unsold items. This is a win-win situation, as the wholesaler can still profit from the revenue sharing while giving a wholesale price discount and buyback to the retailer.

We also observe that the total expected profit from the mixing contract is the same as in the integrated SC for every instance. The results can be seen in the last column in the table above. This proves that the proposed contract achieves perfect coordination.

3.7.3 The impact of Delayed Payment

3.7.3.1 Perfect Coordination

Another objective of this study is to observe the impact of time of payment from the retailer to the wholesaler. For this example, we use $\alpha_g = 2$, and $\tau = 500$, and other parameter values are the same as the above example.

Table 3.4: The optimal results under the mixing contract

w_m	b^*	lz	Q_m^*	$E[AS_m^r]$	$E[AS_m^s]$
5.0	4.97	0	1245.50	3596.21	5641.30
			(1481.01)	(2925.17)	(4588.64)
		30	1269.48	3710.24	5524.39
			(1542.06)	(3063.05)	(4443.42)
		60	1294.12	3823.63	5402.31
			(1605.65)	(3202.25)	(4281.77)
		90	1320.30	3937.39	5273.41
			(1674.15)	(3344.94)	(4099.45)
4.8	4.66	0	1245.50	3790.10	5447.41
			(1481.01)	(3082.87)	(4430.94)
		30	1266.66	3898.55	5336.71
			(1534.84)	(3213.05)	(4295.03)
		60	1288.82	4007.06	5221.21
			(1591.91)	(3345.81)	(4144.28)
		90	1312.17	4115.71	5100.41
			(1652.78)	(3481.32)	(3977.07)
4.6	4.35	0	1245.50	3983.98	5253.53
			(1481.01)	(3240.58)	(4273.23)
		30	1264.74	4087.83	5147.81
			(1529.93)	(3365.13)	(4143.93)
		60	1284.77	4191.59	5038.31
			(1581.42)	(3491.74)	(4002.53)
		90	1305.72	4295.29	4924.67
			(1635.88)	(3620.52)	(3847.91)
4.4	4.04	0	1245.53	4177.92	5059.59
			(1481.01)	(3398.29)	(4115.52)
		30	1263.04	4277.19	4958.77
			(1706.32)	(3748.64)	(3672.02)
		60	1281.17	4376.23	4854.98
			(1770.34)	(3883.95)	(3480.44)
		90	1300.01	4475.07	4747.98
			(1839.64)	(4022.15)	(3268.83)

Table 3.4 gives the optimal values of buyback, order quantity, and expected annuity stream for the retailer and wholesaler under the proposed contract. Since $w_m \in [4, 5.06]$, we vary w_m accordingly, as in Table 3.4. The parenthesis represents the optimal solutions for instance 5. From the table, we can see that the value of b^* is proportional to the value of w_m. This is because b^* must always be smaller than w_m. As expected, $E[AS_m^r]$ increases when the delayed payment time lz increases, regardless of any circumstance – namely, low or high variability of demand, and high or low w_m. The reason for this is that the amount of capital invested in stock for the retailer during lz times is reduced. Thus, she will order more from the wholesaler and increase her

expected profit. In contrast, $E[AS_m^r]$ decreases with the increases in lz under any of the circumstances mentioned before. This is because the chosen w_m is too small to compete with the increment of lz. Thus, we can conclude that through perfect coordination, the wholesaler will not benefit from giving delayed payment to the retailer.

However, in reality, it always comes to the negotiation process between the parties: the retailer asks for a low wholesale price. For example, suppose that the variability of demand is high (instance 5). In the initial contract, both parties agree with $w_m = 5$, $b^* = 4.97$ $\beta = 0.4$, and the retailer agrees to pay $lz = 90$ days prior to the start of the selling season. The retailer and wholesaler will obtain $E[AS_m^r] = 3344.94$ and $E[AS_m^r] = 4099.45$, respectively. After a few negotiations, the wholesaler agrees to reduce the wholesale price from 5 to 4.6, with $b^* = 4.35$, but the retailer needs to pay at $lz = 30$ days after the start of the selling season. This new* contract benefits both parties, as it increases the expected profit for the retailer and wholesaler to $E[AS_m^r] = 3365.13$ and $E[AS_m^s] = 4143.93$, respectively. Thus, the wholesaler can increase his expected annuity stream. Note that, the results show no difference when the variability of demand is low. For example, consider the same situation as above, but using instance 1 instead of instance 5. From Table 3.4, the new* contract benefits the retailer, with $E[AS_m^r] = 4087.83 > 3937.39$. The wholesaler's new expected annuity stream is $E[AS_m^s] = 5147.81 < 5273.41$.

Sensitivity Analysis

To study the effect of b^*, Q_m^*, $E[AS_m^r]$,and $E[AS_m^s]$, we analyse these optimal values and objective functions by varying some parameter values. We perform a numerical sensitivity analysis by varying the value of Lc, and n. The following standard parameter values are used:

$$\alpha_g = 2, \tau = 500, lz = lb = lv = 30, \beta = 0.2, w_m = 6.28$$

The other parameter values remain the same as before. In this example, we vary the value of lc and lr from 0 to 60 to see the changes in the optimal values and objective functions. The results are illustrated in Figure 3.2.

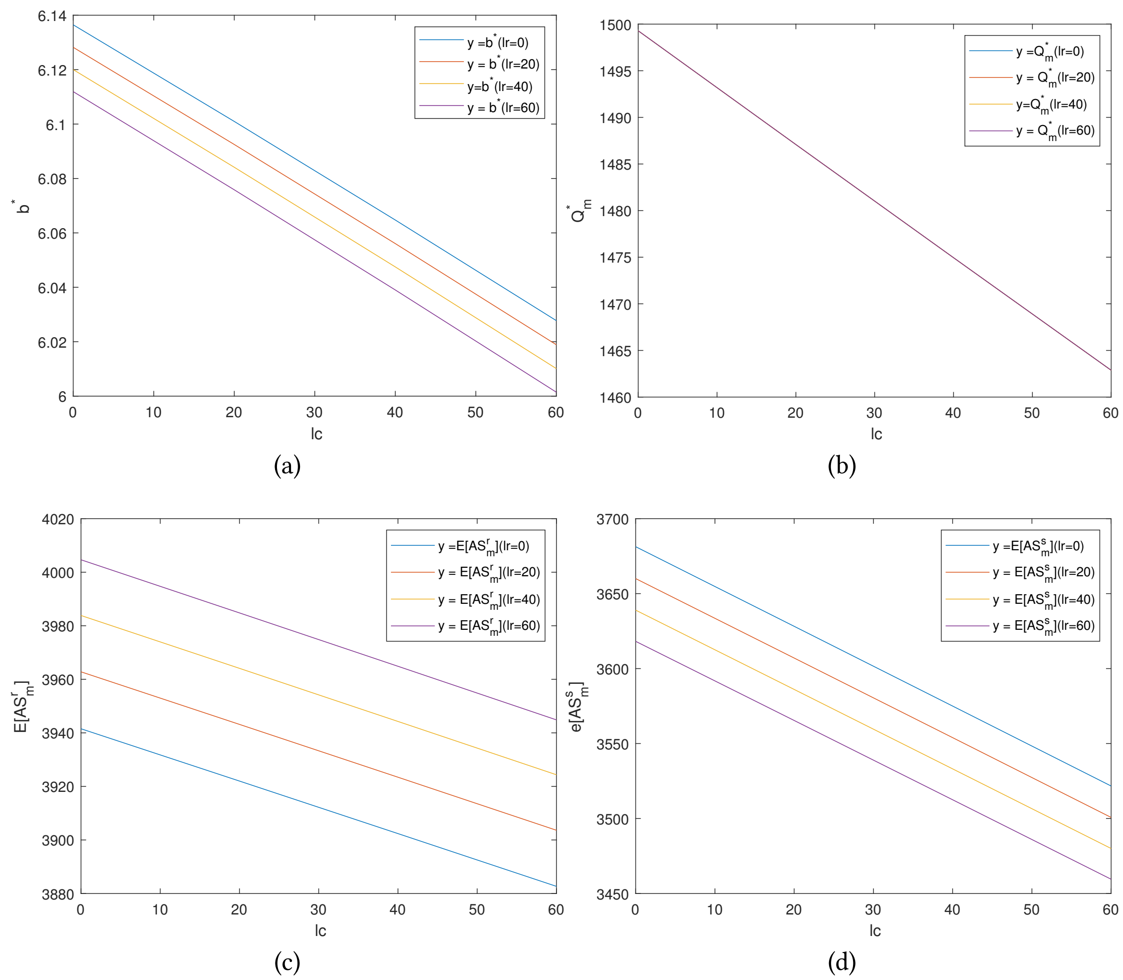


Figure 3.2: Optimal results of (a)b^*, (b) Q^*_m, , (c)$E[AS^r_m]$, (d) $E[AS^s_m]$ with different lb and n .

The blue, red, yellow, and purple lines represent the optimal values when $lr = 0, 20, 40,$ and 60, respectively. First, let lr be constant at value 20. Overall, the values for b^*, Q^*_m, $E[AS^r_m]$,and $E[AS^s_m]$ decrease over lc. As this contract is for perfect coordination, the value of L_c affects the value of b since the optimal order quantity under global optimisation depends on lc. Increasing lc will reduce the optimal b, thus lowering the value of optimal order quantity Q^*_m. When Q^*_m decreases, the expected annuity stream for both SC members will decrease too.

Now consider $lc = 20$. From Figure 3.2, when lr increases, b^* and $E[AS_m^s]$ gradually decrease, $E[AS_m^r]$ noticeably increases , and Q_m^* remains constant. The value of Q_m^* remains constant since $Q_m^* = Q_{sc}^*$, and Q_{sc}^* does not depend on lr. The drop in b^* is expected, as in Eq. (3.21), the optimal b^* decreases with the increasing lr. The values of $E[AS_m^s]$ fall for the wholesaler when lr increases because the wholesaler's amount of capital of investment is high as he needs to wait longer than that of small lr to obtain his payment. For example, $E[AS_m^r]$ stands at about 3922 at $lr = 0$ but rises to 3984.80 at $lr = 60$. In sharp contrast to this, the increment in $E[AS_m^r]$ is due to the reduced amount of capital invested in stock for the retailer, as she holds the revenue-sharing payment to the wholesaler for much longer that of small lr.

Suppose now that $\beta = 0.1$, $lc = 10$, $lr = 10$, and other parameter values remain constant. We vary the value of $n = 1$ up to 4, and $lb = 0$ up to 60. The results are shown in Figure 3.3.

The four coloured lines in Figure 3.3 represent the optimal values when varying lb. To begin, from Figure 3.3(a), for each constant lb ($lb = 0, 20, 40, 60$), the graph shows a similar pattern which first, the optimal b^* decreases from $n = 1$ to $n = 2$, and then gradually increases up to $n = 4$. The optimal b^* is the highest at $lb = 60$ than that of $lb = 0, 20, 40$ over n. For example, when $n = 4$, the optimal b^* when $lb = 60$ stands at value 6, greater than 5.9025 when $lb = 0$.

The optimal values of Q_m^*, $E[AS_m^r]$, and $E[AS_m^s]$ in Figure 3.3(b), (c), and (d), respectively remain the same when varying lb over n. This indicates that a change in lb does not affect the optimal order quantity and the retailer's and wholesaler's expected annuity streams. In addition, when lb remains constant, Q_m^* increases dramatically from $n = 1$ to $n = 2$, and continues to climb gradually until $n = 4$.

Similarly, $E[AS_m^s]$ sharply increases from 2676.20 when $n = 1$ to 24117.57 when $n = 4$. This result is predicted as the revenue-sharing is higher when n is larger. Thereafter, $E[AS_m^r]$ increases rapidly from 4910.33 to 9608.05 during the given n, except for a

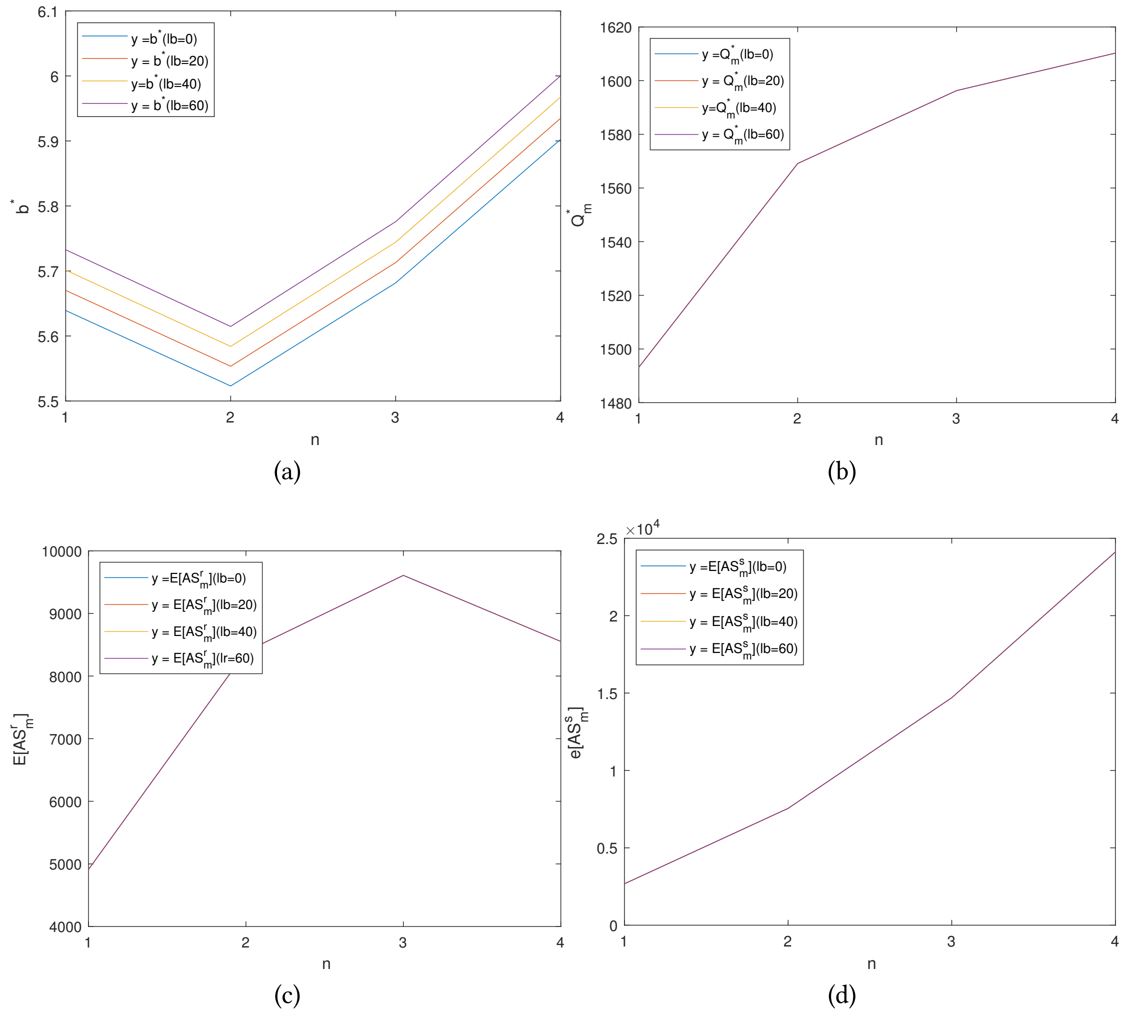


Figure 3.3: Optimal results of (a)b^*, (b) Q_m^*, , (c)$E[AS_m^r]$, (d) $E[AS_m^s]$ with different lc and lr .

slight drop to 8552.73 when $n = 4$. The result implies that when n is relatively large, the retailer needs to share more of her profit with the wholesaler, thus reducing her expected profit.

3.7.3.2 Non-integrated Optimisation

One way to increase w_m is by decreasing the value of β. We now assume that $\beta = 0.2$ and other parameter values are the same as in the previous example. Table 3.5 shows the optimal results of the mixing contract for both parties with (a) a perfect coordination policy (rows 1-2), and (b) a non-integrated optimisation policy (rows 3-4). The parenthesis denotes the optimal value of expected profits for both SC members in the wholesale-price-only contract.

Table 3.5: The optimal results of the mixing contract with delayed payment

w_m	b^*	lz	$Q^*_m[.]$	$EAS^r_m[.]$	$EAS^s_m[.]$	$EAS^{sc}_m[.]$
6.28	6.17055	0	1481.01	3842.05	3671.76	7513.81
		30	1537.63	4012.51	3494.96	7507.48
		60	1597.90	4186.69	3300.83	7487.52
		90	1662.50	4364.84	3087.36	7452.20
6.38	6.32501	0	1481.01	3763.19	3750.62	7513.81
		30	1539.82	3936.50	3570.49	7506.98
		60	1602.63	4113.83	3371.58	7485.41
		90	1670.21	4295.51	3151.57	7447.08
8	3.07	0	637.55	1309.74	4135.85	5445.59
		30	658.81	1403.02	4161.53	5564.55
		60	679.93	1497.77	4180.82	5678.59
		90	700.94	1593.92	4194.03	5787.95
8	5.07	0	759.93	1520.18	4553.35	6073.54
		30	788.13	1631.57	4567.64	6199.21
		60	816.45	1745.13	4573.38	6318.52
		90	844.94	1860.81	4570.86	6431.67
				(1309.64)	(3591.98)	

From the first two rows, we observe that the results show the same pattern as in Table 3.4. This indicates that even though the value of β decreases and w_m increases, the wholesaler still does not benefit from giving delayed payment to the retailer. However, let us assume that the wholesaler has the power to decide any value of w_m as long as both parties can still gain advantages from the contract compared to the wholesale-price-only contract. The value of w_m must be high to compete with the increment of lz. The underline value in Table 3.5 represents the chosen $w_m < w_d$ and the optimal b^*. Notice that, $E[AS^r_m] = 1309.74$ for the retailer and $E[AS^s_m] = 4135.85$ for the wholesaler. This means that both parties still accept this contract because it results in more profit than the wholesale-price-only contract. Suppose that the retailer asks to pay $lz = 90$ days later; the new expected annuity streams for the retailer and wholesaler are then $E[AS^r_m] = 1593.92 > 1309.64$, and $E[AS^s_m] = 4194.03 > 4135.85$, respectively. In addition, when b increases to 5.07 while w_m is constant, the $E[AS^r_m]$,

$E[AS^s_m]$, and $E[AS^{sc}_m]$ proportionally increase. This yields the insight that the wholesaler can increase the value of b to offer a more attractive contract to the retailer.

Overall, the wholesaler is happy to choose the non-integrated contract but not achieve perfect coordination, as he obtains more profit than in the case of perfect coordination. Furthermore, the wholesaler benefits from offering delayed payment. From the retailer's perspective, if she has the bargaining power, she will choose the case of perfect coordination, as it generates more profit for her compared to the non-integrated optimisation. Nevertheless, both cases benefit both SC members.

Sensitivity Analysis

A sensitivity analysis is performed to examine how the optimal results respond to some parameter values. We restate the standard parameter values as follow:

$$\beta = 0.2, w_m = 8, b = 5.07, \alpha_r = 0.2, \alpha_s = 0.2$$

First, we vary the value of α_s and lz while other parameter values remain the same. The line graphs in Figure 3.4 depict the optimal results of $Q^*_,$, $E[AS^s_m]$, and $E[AS^r_m]$ over different values of α_s and lz. From Figure 3.4(a), Q^*_m remains constant as α_s increases. This pattern is similar for any value of lz. However, when α_s is constant, Q^*_m increases as lz does. For example, when $\alpha_s = 0.3$, $Q^=_m 759.93$, and 844.94 when $lz = 0$ and 60, respectively.

As can be seen in Figure 3.4(b), when α_s is $\leq \alpha_r$, $E[AS^s_m]$ is the highest when $lz = 60$ and the lowest when $lz = 0$. In contrast, when $\alpha_s > \alpha_r$, the opposite result is observed. At $\alpha_s = 0.4$, $E[AS^s_m]$ is 4768.78 and 4412.46 when lz is 0 and 60, respectively. Based on the four lines in Figure 3.4(c), there appears to be a clear upward pattern for any given lz over α_s. According to the graph, $E[AS^r_m]$ is highest (2056.72) when $lz = 60$ at $\alpha_s = 0.4$, and the lowest(1428.23) when $lz = 0$ at $\alpha_s = 0.1$. This indicates that the

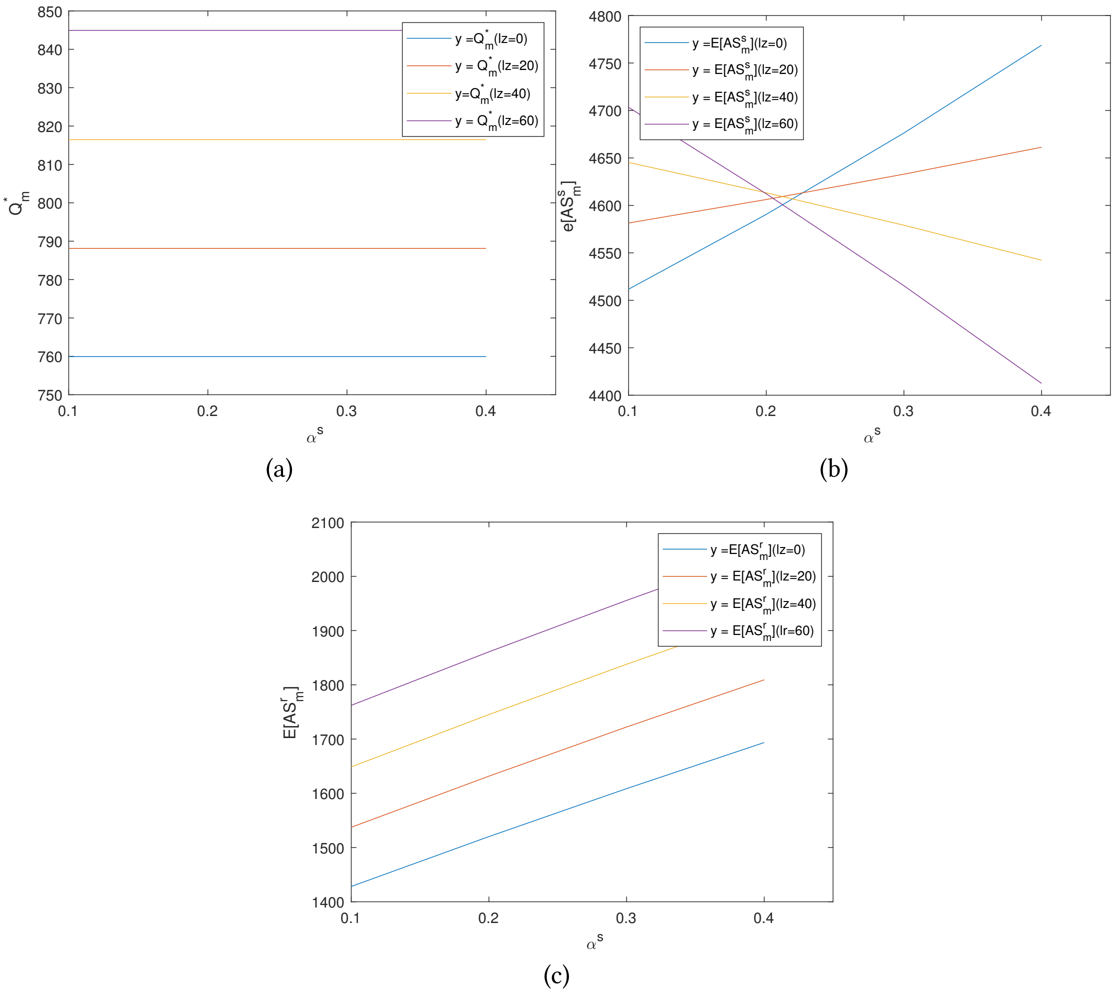
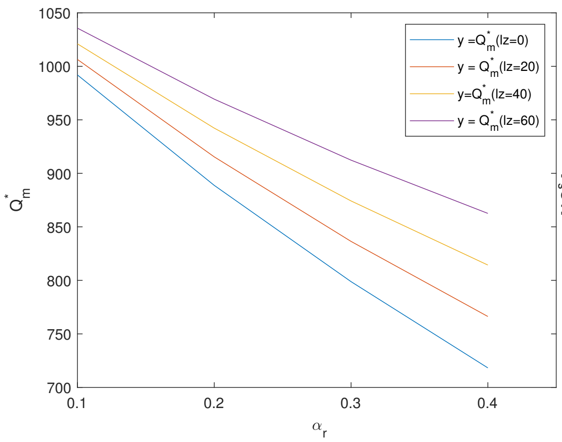


Figure 3.4: Optimal results of (a)Q^*_m, (b)$E[AS^s_m]$, (c)$E[AS^r_m]$ with different α_s, and lz.

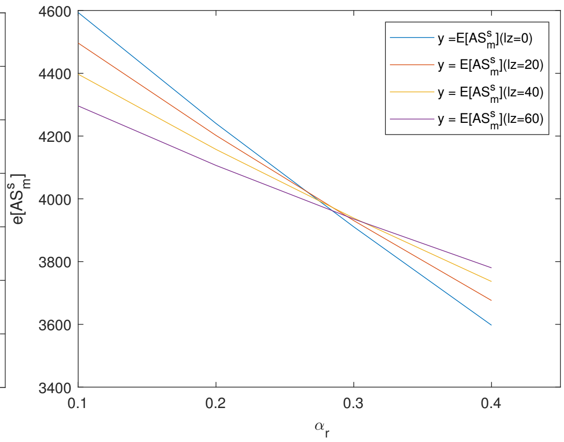
retailer will obtain the most profit gain when the value of α_s is high, with the longest delay payment time,is high with the longest delayed payment time, lz.

Next, we vary α_r from 0.1 to 0.4 and lz from 0 to 60. All the other standard parameter values remain the same. The corresponding results are displayed in Figure 3.5. Firstly, the results for Q^*_m in Figure 3.5(a) show a downward trend for any given lz over α_r. The highest value of Q^*_m(1035.67) is when $lz = 60$ at $\alpha_r = 0.1$ and the lowest Q^*_m(718.24) is when $lz = 0$ at $\alpha_r = 0.4$. This shows that the retailer will order more (less) when her opportunity cost is low (high) and there is a longer (shorter) delayed payment time.

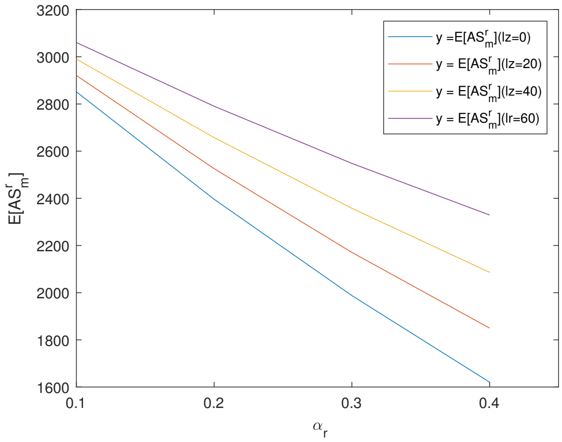
Figure 3.5(b) indicates that a decrease in $E[AS^s_m]$ happens over α_r for any constant value of lz. According to the figure, when $\alpha_r \leq \alpha_s$, $E[AS^s_m]$ is the highest when $lz = 0$



(a)



(b)



(c)

Figure 3.5: Optimal results of (a)Q^*_m, (b)$E[AS^s_m]$, (c)$E[AS^r_m]$ with different α_r, and lz.

and the lowest when $lz = 60$. For example, when $\alpha_r = 0.1$, $E[AS^s_m] = 4593.53$ and 4296.03 at $lz = 0$ and 60, respectively. Meanwhile, when $\alpha_r > \alpha_s$, the results show the opposite pattern, where $E[AS^s_m]$ is the smallest when $lz = 0$ and the highest when $lz = 60$. In sum, the wholesaler will benefit from delayed payment time when α_r is greater than α_s. Meanwhile, $E[AS^r_m]$ in Figure 3.5(c) shows the same decrease over α_r, reaching its highest value (3060.45) when $lz = 60$ at $\alpha_r = 0.1$ and the lowest (1620.47) when $lz = 0$ at $\alpha_r = 0.4$.

3.7.4 NPV of the Dishonest Firm

This section provides some numerical examples to evaluate the retailer and wholesaler's expected annuity stream under information asymmetry. The parameter values used are:

$$\beta = 0.4, lc = lb = lr = 10, lv = 30, lz = 0, c = 5$$

and other parameter values remain the same.

3.7.4.1 Unknown c

Suppose that c is unknown to the retailer. To see the effect of being dishonest on the retailer and wholesaler, we consider two scenarios:

1. The wholesaler is honest with the retailer;
2. The wholesaler is dishonest with the retailer.

Table 3.6 shows the parameter values and results for two instances when the value of c is true. The retailer can easily reveal the value of c.

Table 3.6: The optimal results with the true value of c

$Instance.$	$[w_l, w_u]$	w_m	b^*	lz	Q^*_m	$E[AS^r_m]$	$E[AS^s_m]$
1	[4.66,5.01]	5	4.99	0	1250.31	3603.21	5695.32
				30	1274.05	3716.72	5579.02
				60	1299.15	3830.55	5456.47
				90	1325.88	3944.76	5326.90
5	[4.66,5.01]	5	4.99	0	1493.18	2939.81	4646.72
				30	1553.81	3076.80	4502.60
				60	1618.75	3217.10	4339.69
				90	1688.87	3360.99	4155.51

Instance 5 represents a higher variability of demand compared to instance 1 (see Table 3.1). It is evident that whereas the optimal order quantity, Q^*_m increases over the instance, the others show a corresponding decline. Meanwhile, Q^*_m and $E[AS^r_m]$ rises when lz does but the opposite is true for $E[AS^s_m]$ for any given instance. The data from Table 3.6 is used later for comparison with the second scenario.

Suppose now that the wholesaler wishes to cheat. Note that $w_m \in [5, 5.9]$, and by setting the contract ($w_m = 5$, $b^* = 4.32$) with the agreed $\beta = 0.4$ from the retailer, the retailer reveals a value of $c' = 5$. However, the wholesaler's true marginal cost is $c = 4$. Table 3.7 represents the parameter values and optimal results for scenario 2. The parenthesis value denotes the optimal results under the wholesale-price-only contract. This result acts as a benchmark to accept the mixing contract. PG^s represents the

Table 3.7: The optimal results when the wholesaler is being dishonest

$No.$	w_m	b''	lz	Q^*_m	$E[AS^r_m]$	$E[AS^s_m]$	$E[AS^s_m(\alpha_s)]$	PG^s
1	5	4.32	0	1153.78	3449.62	4517.53	5797.53	1.79
			30	1170.21	3554.12	4411.30	5709.53	2.34
			60	1187.00	3658.40	4301.81	5618.67	2.97
			90	1204.21	3762.45	4188.92	5524.87	3.72
	5.8	5.69	0	1152.61	2704.09	5263.04	6541.74	14.86
			30	1177.02	2825.61	5138.12	6443.89	15.50
			60	1202.49	2947.70	5004.72	6338.75	16.17
			90	1229.24	3070.44	4862.09	6225.80	16.87
				(813.28)	(2611.70)	(4461.96)	(5364.18)	
5	5	4.32	0	1255.38	2627.56	3440.98	4833.69	4.02
			30	1294.84	2742.23	3322.18	4758.66	5.69
			60	1335.61	2858.58	3193.15	4674.87	7.72
			90	1377.85	2976.65	3053.40	4581.98	10.26
	5.8	5.69	0	1252.57	2057.76	4010.76	5400.36	16.22
			30	1311.31	2191.47	3868.82	5323.58	18.23
			60	1373.59	2329.21	3703.41	5227.26	20.45
			90	1440.07	2471.19	3511.97	5109.58	22.96
				(548.66)	(1309.64)	(3010.13)	(3618.81)	

percentage gain for the wholesaler when he cheats. We define the percentage gain as $PG^s = \frac{100(E[AS^s_m(c,b'',lc)]-E[AS^s_m(c,b^*,lc)])}{E[AS^s_m(c,b^*,lc)]}$. The retailer will accept the offer because she will earn more profit than with a traditional wholesale-price contract. For example, in instance 1, when $lz = 0$, the retailer obtains the expected profit ($3894.11 >> 2611.70$). In addition, the retailer will be happy to accept the contract with the delayed payment. In contrast, the wholesaler's real expected profit ($c = 4$) is in column 9 of Table 3.7. The wholesaler's expected profit is much higher than in scenario 1 (see Table 3.6). For example, in instance 1, the PG^s is highest at 3.72% when $lz = 90$.

According to the table, the values of w_l and w_u are higher than when the wholesaler is honest. This indicates that in the case of dishonesty, the wholesaler has more options in choosing the value of w_m to maximise his expected profit. As we can see, when $w_m = 5.8$, the wholesaler gains much profit. For example, in instance 2, when $lz = 90$, the $PG^s = 22.06\%$ higher than when the wholesaler is honest with the retailer. As stated in Proposition 1(a), $b'' < b^*$, and from Proposition 1(b) $Q_m(b'') < Q_m(b^*)$.

Unknown α_s

Let $c = 4$, while other parameter values are kept constant and are the same as in the above example. To study the effect of being dishonest, we consider the same two scenarios as before:

1. The wholesaler being honest with the retailer;
2. The wholesaler being dishonest with the retailer.

Table 3.8 provides the parameter values and results for scenario 1. From the chosen b^*, the retailer can reveal $\alpha_s = 0.1.$. In the table, we can see a similar pattern as in Table 3.6 regarding the optimal values of Q^*_m, $E[AS^r_m]$, and $E[AS^s_m]$ over the instances, except for the values of w_l, w_u, and the chosen w_m. The $E[AS^s_m]$ in this table will be compared to scenario 2 later.

Table 3.8: The optimal results with asymmetry information

Instance	$[w_l, w_u]$	w_m	b^*	lz	Q^*_m	$E[AS^r_m]$	$E[AS^s_m]$
1	[4,4.78]	4.7	4.66	0	1273.22	3642.85	5583.75
				30	1295.74	3750.25	5523.29
				60	1319.51	3857.85	5457.87
				90	1344.78	3965.72	5386.92
5	[4,4.78]	4.7	4.66	0	1551.67	2986.71	4473.71
				30	1609.86	3117.25	4387.75
				60	1672.05	3250.70	4287.97
				90	1739.03	3387.28	4172.49

Now consider Scenario 2, where the wholesaler wishes to cheat. Note that $w_m \in [4, 5.01]$, and by setting the same value of $w_m = 4.7$, $b^* = 4.53$. The wholesaler's opportunity cost is unknown to the retailer. The retailer reveals $\alpha_s = 0.2$ by solving Eq. (3.28). However, the wholesaler's true opportunity cost is 0.1. Table 3.9 summarises the results obtained when there is dishonesty. The value in the parenthesis represents the wholesale-price-only contract. Again, the retailer will accept the offer regardless of any instance and lz, since she will obtain more profit than with the wholesale-price-only contract. From the wholesaler's perspective, he gains more profit than he would if he was honest with the retailer.

In addition, the effect of being dishonest increase the value of w_l and w_u while decreasing the value of b'' and Q^*_m. Again, the wholesaler will consider higher w_m to increase his profit.

Table 3.9: The optimal results under dishonest firm

No.	w_m	b''	lz	Q^*_m	$E[AS^r_m]$	$E[AS^s_m]$	$E[AS^s_m(\alpha_s)]$	PG^s
1	4.7	4.53	0	1250.00	3894.11	5404.42	5625.35	0.75
			30	1270.53	4000.66	5295.85	5571.59	0.87
			60	1292.00	4107.21	5182.88	5513.79	1.02
			90	1314.59	4213.82	5065.06	5451.57	1.20
	4.9	4.84	0	1250.43	3700.52	5598.01	5801.22	3.89
			30	1273.06	3811.72	5484.24	5744.12	4.00
			60	1296.90	3923.13	5364.90	5681.94	4.11
			90	1322.17	4034.81	5239.34	5614.15	4.22
				(813.28)	(2611.70)	(5364.21)	(5118.38)	
5	4.7	4.53	0	1492.39	3176.52	4410.00	4563.22	2.00
			30	1544.75	3304.88	4276.46	4494.67	2.44
			60	1600.16	3435.63	4129.14	4415.02	2.96
			90	1659.12	3568.93	3966.59	4323.08	3.61
	4.9	4.84	0	1493.48	3019.42	4567.10	4695.01	4.95
			30	1551.26	3153.58	4426.40	4620.81	5.31
			60	1612.88	3290.72	4268.72	4532.80	5.71
			90	1679.08	3431.07	4092.00	4429.20	6.15
				(548.66)	(1309.64)	(3618.81)	(3452.97)	

Thus, in both cases of information asymmetry, the wholesaler will not mind cheating. In addition, the optimal order quantity and buyback value in both cases is lower than in the perfect information case. By being dishonest, the wholesaler can use less of his production capacity and pay for fewer of the unsold items, which reduces his opportunity cost of resource usage and cash out-flow.

3.8 Conclusions

This chapter focused on the NP under the NPV framework. The optimal order quantities from the traditional problem were compared with those of the model derived from the NPV framework for a single buyer and single supplier under stochastic demand. By introducing the NPV framework to derive newsvendor models, the consideration of the timing of cash-flows – namely, delayed payment –influences the relative attractiveness of the different coordination contracts.

We considered a mixing contract that improved on the traditional contracts. We found that given equal opportunity costs of the wholesaler and retailer, a returns policy and revenue-sharing contract with a reduction of the wholesale price can coordinate the SC. When delayed payment is implemented in the system, the wholesaler will not benefit in a perfectly coordinated SC. Furthermore, we found that when the wholesaler's objective is to increase his expected annuity stream without perfectly coordinating the SC – that is, by setting a large w_m and a set value of $b < w_m$ with the given β, both firms still benefit from this kind of contract. Moreover, the wholesaler gains more than he would from perfect coordination. Finally, through analytical and numerical studies, we further confirmed that the retailer can reveal private information, and that under incomplete information, the wholesaler can gain advantages from cheating.

Chapter 4

Non-Stationary Demand in Newsvendor Problem under an NPV Approach

Abstract

The traditional newsvendor problem (NP) assumes that the demand for every period is independent and identically distributed, i.i.d. In this study, we relax these assumptions and consider that the demand in each period is independent but non-identically distributed. Based on the literature on the NP, we consider two types of multi-period NP: non-stationary demand and dynamic pricing. In the first case, we assume that the selling price is constant over a selling period. The second case extends the first model by assuming that the demand depends on the selling period, and the retailer makes a price adjustment at a certain point in time before the end of the selling season. In addition, we use a net present value (NPV) approach in developing the model. We solve the problem using the backward induction method. Then, we perform a series of sensitivity analyses to examine the impact of different values of parameters on the

optimal expected NPV and order quantity. The study concludes with a summary of the findings.

4.1 Introduction

This study considers a multi-period newsvendor problem (NP) with non-stationary demand using the NPV approach. The problem is deciding the optimal order quantity to meet the required demand for every period so as to maximise the expected NPV of total profit for the retailer. The practical relevance of the problem is conspicuous, since in today's market the life cycle of products is becoming shorter due to intense global competition. This phenomenon is happening not only with fashion apparel, but also with many high-tech electronic devices (Neale & Willems 2009).

As an example, we consider a UK retailer selling swimming attire. The sales of the item are strongly seasonal – they take place during the summer term only, from May to September. The retailer purchases the order before the start of the selling season, and reorders are impossible during the selling season, because the lead time is longer than this season. This scenario is in contrast to the traditional multi-period demand NP that considers a multiple lot sizing model. In addition, instead of having independent and identical demand distributions for every period, we assume that the demand for every period t is independent but not identically distributed. The importance of considering non-stationary demand in a stochastic demand environment has received much attention from researchers and practitioners (see, Morton & Pentico (1995) and Chen & Song (2001)). However, the difference in demand patterns may yield different optimal solutions. It is worth examining a new dimension of this problem: namely, considering time value of money since demand is non-identical across periods.

Our work is motivated by the fact that in a business industry, the retailer faces a certain type of demand pattern – an upward or downward demand pattern. For example, suppose that a new product is launched in the market. Since the end-customers are initially unfamiliar with the product, the demand at the beginning of the selling period is considered low. Over time, however, demand increases due to certain factors, such as an effective marketing strategy. In contrast, the downward demand pattern shows a very high demand at the beginning of the selling season and then decreases over time. This kind of situation happens often in today's business. Our aim in this study is to develop a model that can capture these two types of demand patterns and compare them to see under which conditions the results vary.

In addition, we consider another case in which demand depends on selling price: demand increases when the selling price decreases and vice-versa. The intention here is not to find the optimal pricing policy as provided in the literature; instead, we consider this assumption as an added parameter which seems realistic in real business, where the selling price is based on the competitive price. A well-known multiplicative case is included in our model to capture the price-demand relationship. In the traditional NP, the selling price is constant over the selling period. However, a retailer who is dealing with an overstocking situation before the end of the selling period can provide a price adjustment to clear the remaining stock. Hence, this model extends the traditional NP by assuming that the retailer can make a price adjustment before the end of the selling season.

This study is organised as follows. After the literature review in Section 4.2, Section 4.3 provides a problem definition related to two cases, a single pricing model (4.3.1) and dual pricing model (4.3.2). In Section 4.4, we introduce the general formulation of this model. In Section 4.5, we then present a numerical study to investigate the effectiveness of the model, based on assumed parameters and with the specific assumption that demands are uniformly distributed. A summary of findings, implications, and suggested directions for future research are described in Section 4.6.

4.2 Literature Review

In the classical NP, the retailer determines the optimal order quantity at the beginning of the selling season when the demand is stochastic and follows a known distribution. As the simplest and most fundamental problem in stochastic inventory control, this problem has been studied by many researchers in the past decades. Comprehensive reviews on this topic can be found in Khouja (1999), Petruzzi & Dada (1999), and Qin et al. (2011). Our works compress three stream: non-stationary demand, price-dependent demand, and the NPV approach. Thus, we only focus on the literature in these three areas.

Prior work on the multi-period NP has usually considered that the demand is i.i.d. However, today it is quite common to have a stochastic and non-stationary demand due to seasonality, short product life cycles, and customer buying patterns, among others (Neale & Willems 2009). Wagner & Whitin (1958) developed a dynamic programming model to deal with non-stationary demand.

Sox (1997) extended Wagner & Whitin (1958) with some additional feasibility constraints. Recently, Kim et al. (2015) developed a multi-stage stochastic use of the progressive hedging method to solve the lot sizing problem. Furthermore, Chen & Song (2001) proposed a periodic review model of non-stationary demand. The objective was to find the optimal total order quantity commitment that would minimise the expected total cost for the retailer. The authors assumed a smaller T, and the demand pattern increased and decreased with the same variance. Alwan et al. (2016) considered the dynamic demand problem of non-stationary demand, and assumed the demand over the time period to be autocorrelated.

Whitin (1955) presented the first mathematical formula for price-dependent demand, whereby the retailer has to decide both the price and the order quantity optimally. Karlin & Carr (1962) developed a price-dependent demand model in the form of multiplicative demand. In this model, the expected demand is given as the multiplication of the decreasing function and the stochastic factor. The authors proved that under uncertainty, the optimal selling price is strictly higher than when there is no uncertainty. For a review of price-dependent demand, the reader is referred to Petruzzi & Dada (1999). Subsequent works on multiplicative models include (Yao et al. (2006), Xu et al. (2011)). The studies reviewed above have assumed a single pricing policy. However, in reality, throughout one selling season, the retailer can provide a sales discount at a certain period of time to boost sales or clear stock. Bitran & Caldentey (2003) reviewed the literature on dynamic pricing policies and their relationship with revenue management. A few authors have considered markdown pricing. You (2005) proposed an inventory model under the condition that demand is time and price dependent. The authors assumed that a decision-maker has the opportunity to adjust prices during n period of time before the end of the sales season. They showed that the dynamic pricing strategy outperforms the static pricing strategy. However, they assumed the demand to be deterministic. Forghani et al. (2013) proposed a price-dependent demand model of the NP. They assumed demand to be dependent on price and to follow three types of function: linear, two-segment, and exponential. The objective was to find the optimal adjustment price, either a markup or markdown from the original price, that captured the unwanted costs due to the deviation between the actual demand and the given inventory at the beginning of the selling period. The authors restricted their study by assuming a known inventory at the beginning of the selling season, and assuming the demand to be continuous over the selling period. Recently, Ullah et al. (2019) proposed a free distribution NP considering price adjustment at the end of the selling period. The authors showed that the proposed discount policy increased the retailer's sales and expected profit.

A common characteristic of the above works is that the models consider optimal pricing with stocking policies. Typically, the lead time to replenish the stock is longer than the selling season. Therefore, reorders are not possible. Likewise, the selling price is typically a known parameter based on competitors' price. In our model, we assume that the price and discount rate are given, and we only consider a single-ordering policy. Furthermore, the aforementioned studies do not consider the TVM. Yet, time value of money is an indispensable element in business transactions, because a pound in hand today is more valuable than a pound received in the future: the pound in hand today can be used to invest and earn interest or capital gains. Several authors have emphasised the importance of the TVM in lot sizing decisions; see Section 2.2 for further details on the past literature.

This study contributes to the existing literature in the following ways. First, it analyses the behaviour of non-stationary stochastic demand which follows a demand pattern – a downward or upward pattern. Second, it considers the TVM with payment structures in the optimal solutions in the NP, which has so far been ignored in this problem. Lastly, the model is extended to the case in which the retailer makes an adjustment in price and the demand is assumed to be dependent on selling price.

4.3 Problem Definition

We consider the multi-period NP for a single product based on NPV analysis. The retailer faces random demand x_t and decides to order quantity Q at a price of w per unit before the start of the selling season. The ordered lots arrive at the beginning of the period to fulfil the random demand x_t of each period $t = 1, 2, ..., T$. The demand is realised when the selling season starts. We assume that x_t follows uniform distribution $U[0, Dt]$ and comprises independent but not identically distributed random variables, with $f_t(x_t)$ and $F_t(x_t)$ being their probability density and cumulative distribution function, respectively. Salvaging occurs in period $T + 1$. In this period, we assume that there is no demand or holding cost.

4.3.1 Case 1: Constant Price

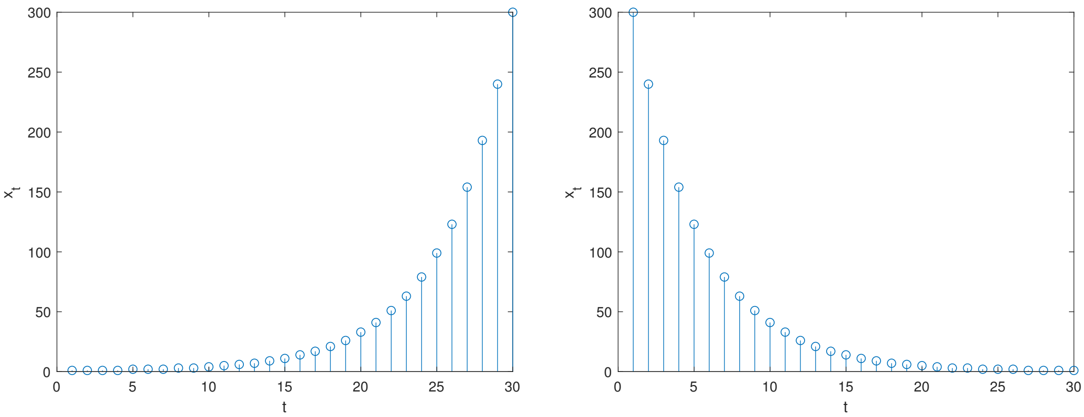
We first consider the case in which the selling price is constant, as modelled in the traditional NP. The stochastic demand x_t is dependent on time, where $x_t \in [0, Dt]$, and follows the generalised increasing failure rate (GIFR). This means that x_t can be any positive integer such that $x_t = j$, where $j = 0, 1, ..., D_t$. Then, the probability of demand scenario j at period t is denoted by $f_{t,j} = Pr[x_t = j]$. The retailer faces one type of demand profile, which is either an upward trend ($D1$) or a downward trend ($D2$). The formula to generate the demand pattern of $D1$ and $D2$ is given below:

For $t = 1, 2, ..., N$,

$$D_t = \begin{cases} \lceil a(1-\beta)^{(N-t)} \rceil, & \text{for } D1, \\ \lceil a(1-\beta)^{(t-1)} \rceil, & \text{for } D2. \end{cases} \tag{4.1}$$

where a, and β are known and constant, and $a > 0$, $0 < \beta < 1$ is the geometrical approach of demand a. This type of demand pattern is common in the literature (Panda 2013). The above formula is necessary to make both demand profiles comparable with each other. The demand functions must be symmetrical to be fair when making comparisons. The cumulative maximum demand over T is $CD = \sum_{t=1}^{N} D_t$.

Example 1. Suppose that $N = 30$, $a = 300$, and $\beta = 0.2$. By using Eq. (4.1) to generate D_t, we obtain the possible demand pattern for a 30-day selling period, as in Figure 4.1. The $CD = 1512$ for both demand profiles. As we can see from the figure, D1 and D2 are symmetrical in terms of D_t. Later, we provide insights regarding having different demand profiles.



(a) $D1$ (b) $D2$

Figure 4.1: x_t over t with varying demand profiles

4.3.2 Case 2: Price Dependent Demand with Price Adjustment

End of season sales are now a trend among retailers selling seasonal items. These retailers normally offer a discount at a certain time before the start of the next selling season. Typically, the selling season in the UK is classified as Spring or Summer, Autumn or Winter, or Halloween or Valentine's Day, among others. In contrast to the first case, we only assume that the demand pattern is going downward, (D2).

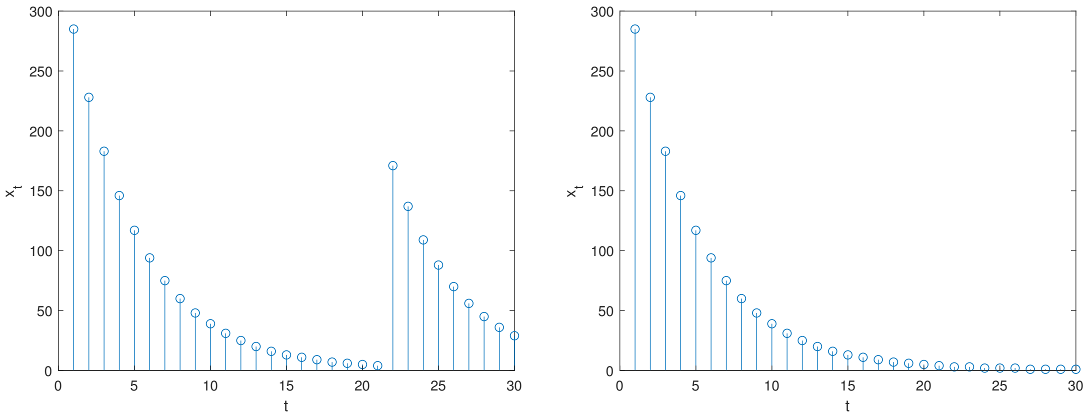
We consider two time intervals. First, during time interval $[1, t_1]$, items are sold at a competitive unit selling price $p_0 > 0$. Second, at time interval $[t_1 + 1, T]$, the retailer sells the remaining items at $p_1 = (1 - \theta)p_0$, where θ represents the price discount percentage. t_1 and θ are parameters. After the end of the selling period, any excess items are salvaged at value v per unit. We assume that $v < p_1 < p_0$.

Considering a multiplicative case (Karlin & Carr 1962), the demand is assumed to be dependent on times and selling price, which the demand is $x_t(p_i, \xi_t) = y(p_i)\xi_t$, where $y(p_i)$ is the deterministic part of x_t that decreases in selling price, and ξ_t captures the random factor of the demand model where $\xi_t \in [0, D_t]$. D_t denotes the possible maximum demand that depends on time t. Let $f_t(\xi_t)$ be the density function and $F_t(\xi_t)$ the cumulative distribution function. For simplicity, assume that $F_t(\xi_t)$ is continuous, differentiable, and strictly increasing. We let $y(p_i) = \vartheta p_i^{-\beta_p}$ where $\vartheta > 0$ is a price-sensitive parameter of demand, and $0 < \beta_p < 1$ is the elasticity of the price.

Eq. (4.2) provides the formula to generate D_t;

$$D_t = \begin{cases} \lceil a_0(1-\beta)^{t-1} \rceil, & \text{for } 1 \le t \le t_1, \\ \lceil a_1(1-\beta)^{t-t1-1} \rceil, & \text{for } t_1+1 \le t \le N. \end{cases} \tag{4.2}$$

where a_0 and a_1 represent parameters of the maximum possible demand (highest peak) at the first time interval and second time interval, respectively with $0 < a_1 < a_0$. $0 < \beta < 1$ is the elasticity of demand rate over time. If $\theta = 0$, $D_t = \lceil a_0(1-\beta)^{t-1} \rceil$ for $t = 1, 2, ..., N$.



(a) $\theta = 0.3$ (b) $\theta = 0$

Figure 4.2: x_t over t with varying β

Example 2. Suppose that $N = 30$, $t_1 = 21$, $p_0 = 8$, $\theta = 0.3$, $a_0 = 300$, $a_1 = 150$, $\vartheta = 2$, and $\beta = 0.2$. Thus we have $p_1 = 5.6$ and $t_1 = 21$ with the cumulative demand $CD(\theta = 0.3) = 2163$ and $CD(\theta = 0) = 1438$. Figure 4.2 shows two figures of x_t over selling period t corresponding to the given parameter values.

4.4 Model Development

The cost function of ordering Q units before the start of the selling season is $O_1(Q)$. It consists of a fixed cost $K > 0$ and a variable cost $w(Q)$. We assume that the retailer pays the ordering cost at l_z times after the selling period start. Let $L_z = l_z/365$. Therefore, $O_t(Q)$ is calculated using Eq. (4.3) below;

$$O_t(Q) = \begin{cases} (K + w(Q))e^{-\alpha_r L_z}, & \text{if } Q > 0 \ \& \ t = 1, \\ 0, & \text{if } Q = 0 \text{ or } t \neq 1. \end{cases} \tag{4.3}$$

For every period t, the present value of the inventory cost of Q unit inventory is represented by the non-decreasing function $H(Q)$, where $H(Q) = h(Q)e^{-\alpha_r t}$. Consider that the demand is j units. If the inventory is sufficient to meet demand j, the retailer receives revenue with present value $r_t(j)$ at time t. We assume that $r_t(0) = 0$.

The expected NPV of the sales profit received at time t is denoted as $E[R_t(Q)]$. If the demand j does not exceed the inventory Q ($0 \leq j < Q - 1$), the expected NPV of the sales revenue is $r_t(j) = pje^{-\alpha_r t}$ with probability $f_{t,j}$, otherwise, it is $r_t(Q) = pQe^{-\alpha_r t}$ with a probability of $q_{t,Q} = \sum_{j=Q}^{\infty} f_{t,j}$. Therefore, the total expected NPV of the profit received at time t when the inventory at t is Q is given by;

$$\mathbb{E}[R_t(Q)] = \begin{cases} \sum_{j=0}^{Q-1} r_t(j)f_{t,j} + r(Q)q_{t,Q} - H(Q) - O_t(Q), & \text{if } Q \leq D_t, \\ E[R_t(Q-1)], & \text{if } Q > D_t. \end{cases} \tag{4.4}$$

At $L_v = lv/365$ times after the end of the selling period $t = T+1$, any excess inventory is salvaged at a unit price v. Thus, the expected NPV of the salvaged revenue is given as;

$$g(Q) = vQe^{-\alpha_r (T+1+Lv)}. \tag{4.5}$$

Now, let S_t denotes the inventory in period t. The inventory for each period is reduced by satisfying a stochastic demand x_t. We illustrate the situation in Figure 4.3.

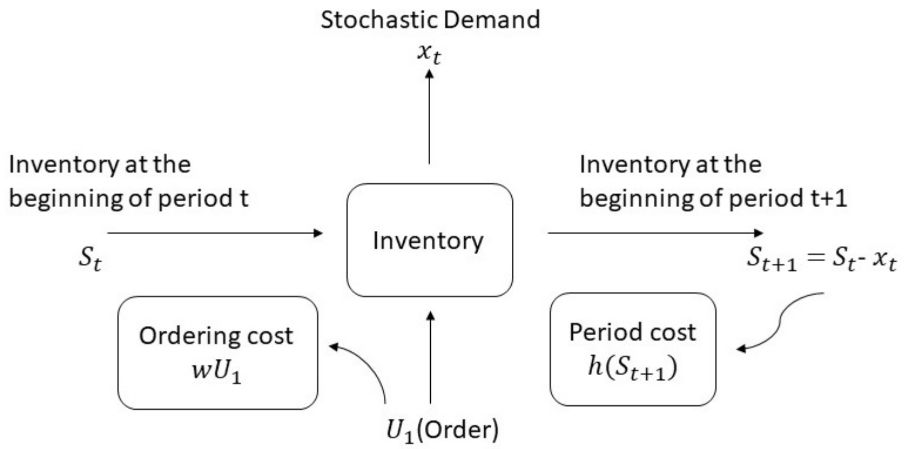


Figure 4.3: Inventory Management

From Figure 4.3, the inventory level in the first selling period (day 1) is given by $s_1 = Q$. Since x_t units are demanded during period t, the inventory state dynamic equation at the beginning of period $t + 1$ is;

$$S_{t+1} = S_t - x_t, \tag{4.6}$$

where $0 \leq S_{t+1} \leq CD$.

The formulation of our problem follows a finite horizon MDP. The components of an MDP consist of decision epochs, states, actions, transition probabilities and a rewards function (Puterman 2014). We describe these in detail below.

Decision epoch: Decisions are made at a point in time. Let t denote the set of decision epochs. The set of decision epochs is;

$$t = 1, 2, ..., N. \tag{4.7}$$

States: In a system, there are states in each decision epoch. In our model, the states represent the inventory level at the start of period t;

$$S = \{0, 1, ..., CD\}. \tag{4.8}$$

Actions set: By definition, for every states, the retailer may choose an action a from the set of allowable actions in state S, such as A_s (Puterman 2014). The action in our model represents the number of items to order before the start of the selling season. As mentioned before, the inventory level (state) when $t = 1$, S_1, is equal to the amount to order. Since we assume a single-ordering model, there is only one action which is the optimal state to maximise the overall system at $t = 1$. Following the definition, we assume $a \in A_s = 0$.

Transition probabilities: The system state in the next decision epoch $t + 1$ is determined by the probability distribution $f_t(.|s, a)$. As proposed in Puterman (2014), transition probability is given by;

$$f_t(k|S, a) = \begin{cases} 0, & \text{if } k > S, \\ f_{t,S-k}, & \text{if } k \leq S, \\ q_{t,S}, & \text{if } k = 0. \end{cases} \tag{4.9}$$

The explanation for the above transition probabilities is as follows. An inventory level of $k > 0$ at the start of period $t + 1$ requires a demand of $(S - k) \in [0, D_t]$ in period t with probability $f_{t,S-k}$. If the demand in period t exceeds S units, then the inventory at the start of period $t + 1$ is 0 units. This occurs with probability $q_{t,s}$. The probability is equal to 0 if the inventory level k exceeds S, since $S - k$ cannot be negative.

Expected rewards: The retailer receives a reward in every period t, and this reward depends on the state of the system at the next decision epoch. Thus, we have;

$$\mathbb{E}\, SSU_t(S,a) = \mathbb{E}[R_t(S,a)] + \sum_{k\in S} f_t(k|S,a) SSU_{t+1}(k,a)]. \tag{4.10}$$

Eq. (4.10) follows the Bellman equation without maximisation. We show the optimality of this equation later. The value of the terminal inventory (salvage) is given as;

$$\mathbb{E}\, SSU_{N+1}(S) = g(S),\ t = N+1. \tag{4.11}$$

Let us denote Eq. (4.12) as the expected annuity stream of profit over an infinite accounting year.

$$\mathbb{E}\, AS_r(Q) = SSU_1(S) \sum_{i=0}^{\infty} \alpha_r e^{-i\alpha_r(1)} \tag{4.12}$$

By using the Maclaurin series to solve the above equation, we obtain:

$$\mathbb{E}\, AS_r(Q) = SSU_1(S) \frac{\alpha_r}{1 - e^{-\alpha_r}} \tag{4.13}$$

The objective is to find the optimal order quantity Q, $Q \in (0, S]$ that maximises the expected annuity stream of profit, $\mathbb{E}\, AS_r(Q)$.

4.4.1 Optimality Solution

In this section, we introduce the optimality equation. Eq. (4.10) is restated below:

$$\mathbb{E}[SSU_t(S,a)] = \mathbb{E}[R_t(S,a)] + \sum_{k\in S} f_t(k|S,a) SSU_{t+1}(k)]$$

Theorem 1. $\forall S$, the function of $SSU_t(S,a)$ is decreasing in t.

Proof. We prove this by using induction (follow optimality solution in Bellman (1957)).

i. Base case: For completeness, we define $SSU_{T+1}(S) = g(S)$. We assume that $\mathbb{E}[R_T(S,a)] > \sum_{k \in S} f_t(k|S,a)g(k)$. By definition;

$SSU_{T+1}(S) = g(S) \leq \sum_{k \in S} f_t(k|S,a)g(S) + \mathbb{E}[R_t(S,a)] = \mathbb{E}[SSU_T(S,a)]$.

ii. Induction Hypothesis: $\mathbb{E}[SSU_{t+1}(S)] \leq \mathbb{E}[SSU_{t+2}(S)]$ for all S.

iii. Induction Step:

Proof.

$$\begin{aligned} \mathbb{E}[SSU_t(S)] &= \mathbb{E}[R_t(S) + f_t(k|S,a)SSU_{t+1}(k)] \\ &= \mathbb{E}[R_t(S) + f_t(k|S,a)[R_{t+1}(S) + f_{t+1}(k|S,a)SSU_{t+2}(k)] \\ &\geq \mathbb{E}[R_{t+1}(S) + f_{t+1}(k|S,a)SSU_{t+2}(k)] = \mathbb{E}[SSU_{t+1}(S)] \end{aligned} \tag{4.14}$$

■

Note that, in every period t, $\mathbb{E}[SSU_t(S,a)]$ follows a single-period NP with an optimal value of S^* that maximises the NPV of $\mathbb{E}[SSU_t(S^*,a)]$.

Theorem 2 $\forall t$, $SSU_t(S)$ is maximises with optimal value S^*.

Proof. Let consider one selling period with $t = 1$ The equation is given by:

$$\begin{aligned} \mathbb{E}[SSU_{t=1}(S)] &= p\sum_{j=0}^{S-1} jf_{1,j}e^{-\alpha_r t} + p\sum_{j=S}^{\infty} Sf_{1,j}e^{-\alpha_r t} - wSe^{-\alpha_r L_z} \\ &- hSe^{-\alpha_r t} + v\sum_{j=0}^{S-1}(S-j)f_{1,j}e^{-\alpha_r t+1+L_v} \end{aligned} \tag{4.15}$$

We know from economics that the optimal expected NPV of profit will be where the expected NPV of profit for ordering S units is approximately the same as for S + 1 units

(Hill 2011). Setting $\mathbb{E}[SSU_{t=1}(S)] = \mathbb{E}[SSU_{t=1}(S+1)]$ and applying Eq. (4.15) yields:

$$\Big[p\sum_{j=0}^{S-1} jf_{1,j} + p\sum_{j=S}^{\infty} Sf_{1,j} - hS\Big]e^{-\alpha_r t} - wSe^{-\alpha_r L_z} + v\sum_{j=0}^{S-1}(S-j)f_{1,j}e^{-\alpha_r t+1+L_v}$$
$$= \Big[p\sum_{j=0}^{S} jf_{1,j} + p\sum_{j=S+1}^{\infty}(S+1)f_{1,j} - h(S+1)\Big]e^{-\alpha_r t} - w(S+1)e^{-\alpha_r L_z}$$
$$+ v\sum_{j=0}^{S}(S+1-j)f_{1,j}e^{-\alpha_r t+1+L_v} \tag{4.16}$$

Rewriting the above equation, we have:

$$\Big[p\sum_{j=0}^{S} jf_{1,j} - pSf_{1,S} + p\sum_{j=S+1}^{\infty} Sf_{1,j} + pSf_{1,S} - hS\Big]e^{-\alpha_r t} - wSe^{-\alpha_r L_z}$$
$$+ v\sum_{j=0}^{S}(S-j)f_{1,j}e^{-\alpha_r t+1+L_v}$$
$$= \Big[p\sum_{j=0}^{S} jf_{1,j} + p\sum_{j=S+1}^{\infty}(S+1)f_{1,j} - h(S+1)\Big]e^{-\alpha_r t} - w(S+1)e^{-\alpha_r L_z}$$
$$+ v\sum_{j=0}^{S}(S+1-j)f_{1,j}e^{-\alpha_r t+1+L_v} \tag{4.17}$$

Combining the terms in Eq. (4.17) and defining the CDF as $F(S) = \sum_{j=0}^{S} f_{1,j}$ yields:

$$F(S) = \frac{(p-h)\gamma_t - w\gamma_z}{p\gamma_t - v\gamma_v} \tag{4.18}$$

where $\gamma_t = e^{-\alpha_r t}$, $\gamma_z = e^{-\alpha_r L_z}$, and $\gamma_v = e^{-\alpha_r t+1+L_v}$. Thus, for each period t, there exists an optimal S^* for discrete demand of NP by finding the smallest S such that $F(S) = \sum_{j=0}^{S} f_{1,j} = \frac{(p-h)\gamma_t - w\gamma_z}{p\gamma_t - v\gamma_v}$. Since we only consider single-ordering, at period $t = 2, 3, ..., N$, we can simply put $w = 0$ and no action taken at these periods. ■

4.4.2 Solution Algorithm

This section presents the backward induction algorithm to find the optimal solution. This method is well-known for solving the finite horizon discrete time of MDPs. The algorithm is shown in Algorithm 4.

Algorithm 4

Input. List of parameters.
Output. A local maximum A_1^*, and $SSU_1(A_1)^*$.

1: Set $t = N + 1$;

$$SSU_{N+1}(S_{N+1}) = g(S_{N=1}) \forall S_{N+1} \in S$$

2: $t - 1$ is substituted for t and computes $SSU_t(S_t)\ \forall\ S_t \in S$ by;

$$\mathbb{E}\, SSU_t(S_t) = E[R_t(S_t)] + \sum_{k \in S} f_t(k|S_t) SSU_{t+1}(k) - H_t(S_t)$$

3: If $t = 1$ stop; Otherwise return to step 2;
4: Compute $\mathbb{E}[AS_r(Q^*)]\ \forall\ Q \in S_1$ by;

$$\mathbb{E}[AS_r(Q^*)] = \max_{Q \in S_1} \Big\{ \mathbb{E}[SSU_1(Q)] \Big\}$$

Set;

$$Q^* = \underset{Q \in S_1}{\text{argmax}} \Big\{ \mathbb{E}[SSU_1^*(Q^*)] \Big\}$$

4.5 Numerical Experiments

The developed model is tested with a numerical experiment and sensitivity analysis of the input parameters. We provide two sections of numerical examples to cover Case 1 and Case 2. The parameter values used are as follows.

$$p_0 = p = 21, w = 15, v = 0.5, h = 0.01, \alpha_r = 0.1, l_z = 0, l_v = 0, N = 150,$$

$$a_0 = a = 300, K = 7, \beta = 0.2$$

4.5.1 Case 1:- Constant Price

It is worth noting that $Peak = 1$ represents the downward trend and $Peak = N$ the upward trend of the demand profile. Table 4.1 gives the optimal solutions of both types in the NPV model (NPVNP) and the traditional NP(TNP) including Δ_1 and Δ_2 which capture the difference between these two-demand patterns. When $\alpha_r = 0.00001$, the optimal Q is based on TNP. Then, the Q is substituted into the NPVNP model to find the optimal $\mathbb{E}[AS_r]$ of TNP.

$$\Delta_1 = 100 \times \frac{\mathbb{E}[AS_r^{NPVNP}] - \mathbb{E}[AS_r^{TNP}]}{\mathbb{E}[AS_r^{TNP}]}$$

$$\Delta_2 = 100 \times \frac{\mathbb{E}[AS_r^{Peak=1}] - \mathbb{E}[AS_r^{Peak=N}]}{\mathbb{E}[AS_r^{Peak=N}]}$$

As shown in Table 4.1, when p increases, Δ_1 decreases for both $Peak$. This indicates that when the retailer increases the price, the presence of opportunity cost in the critical ratio in Eq. (4.18) is diminished, which decreases the difference in the optimal Q between the traditional and NPV models. The results show similar pattern in Δ_2. When $p = 18$, Δ_2= 220.51% which is four times higher than Δ_2 when $p = 21$. This demonstrates that when the retailer faces a random demand that follows a type of demand pattern, the optimal results vary accordingly. Thus, it is crucial to examine this kind of problem, as it provides guidance to the retailer regarding how much she could earn if she knows that the random demand follows a certain type of pattern. In addition, it is necessary to apply the NPV approach since there exists a value in Δ_1 which represents the difference in expected annuity stream of NPV and traditional NP, and, as we will consider the payment structure, namely, delayed payment – the approach will capture the TVM.

Table 4.1: Optimal solution of different p, $Peak$ and α_r

p	$Peak$	α_r	Q	$\mathbb{E}[AS_r]$	Δ_1	Δ_2
18	1	0.1	658	1816.13	0.006	220.51
		0.00001	661	1816.02		
	150	0.1	575	566.64	2.147	
		0.00001	615	554.73		
21	1	0.1	719	3941.94	0.002	58.74
		0.00001	721	3941.84		
	150	0.1	683	2483.29	0.217	
		0.00001	700	2477.91		

4.5.1.1 Sensitivity Analysis

To see how the parameters' values affect the optimal solution, we run some additional tests. The base values of the parameters are the same as in the previous example. Now, Δ_1 represents the percentage change in expected profit between the base and the current parameters. Δ_2 represents the percentage change in expected profit between $Peak = 1, N$. The computational results are shown in Tables 4.2 to 4.4. The sensitivity analysis results from Table 4.2 reveal the following insights regarding the model parameters:

- • For both Peaks, the results of changing a are the opposite of changing β. For example, when Peak = 1, decreasing a by 50% reduces the expected profit by 45.13%, while decreasing β by 50% increases the expected profit by 104.32%. Decreasing β increases the maximum possible demand in each selling period and the optimal order quantity, thus leading to a greater expected profit. As a is the maximum possible demand, reducing it will reduce the maximum possible demand for every period, thus decreasing Q.

- When comparing the expected annuity stream between $Peak = 1$ and $Peak = N$, Δ_2 shows an increasing pattern when parameters a and β increases. However, when β increases by $+50\%$, Δ_2 is slightly lower than when the increment is $+25\%$. The difference in Q between $Peak$ becomes smaller as β increases, therefore giving the results.

Thus, when the retailer faces low random demand, he hopes for a smaller β to increase his expected profit. Regardless of any change in parameters, the result when the demand pattern is decreasing is always higher than when the pattern is increasing.

Table 4.2: Sensitivity analysis for demand parameter

Parameter	Peak	Percentage Change in Value	Q	Δ_1	Δ_2
a	1	-50	392	-45.13	52.64
		-25	556	-22.47	56.55
		25	884	22.66	60.12
		50	1048	45.33	61.07
	150	-50	376	-42.93	-
		-25	530	-21.39	-
		25	838	21.60	-
		50	993	43.23	-
β	1	-50	1424	104.32	54.73
		-25	954	34.83	57.66
		25	580	-20.57	58.95
		50	486	-34.39	58.72
	150	-50	1377	109.62	-
		-25	914	35.75	-
		25	547	-20.68	-
		50	456	-34.38	-

Table 4.3 provides a sensitivity analysis of the operational parameters. The following insights are obtained from the results:

- The effect of changing h to Δ_1 is greater when $Peak = 150$ compared to when $Peak = 1$. Since the demand pattern is upward, at the beginning of the selling period, the holding cost is higher as the range of demand is lower.
- The most effective parameter is w: decreasing w by 15% increases Δ_1 above 40% for both $Peaks$.
- Δ_2 shows a significant change at the lowest p with a 256.73% increment. This is because the small margin of revenue cannot cover the high holding cost in $Peak = 150$, thus drastically reducing the expected profit. This insight holds when the h increases to $+50\%$, where $\Delta_2 = 94.45\%$.

- When changing parameter v, there is only a small change in Q, Δ_1, and Δ_2. Increasing or decreasing v hence does not have much impact on the optimal result.

Thus, for the retailer facing any type of demand pattern with higher holding costs and a higher wholesale price w, it is better to increase p to cover the high holding and ordering costs. If the retailer is facing an upward demand pattern, it is worth identifying anything that can reduce the holding cost per day, as this will yield a significant change in result.

From Table 4.4, the results show that:

- When T decreases, Δ_1 for both types of demand pattern increases. However, when $Peak = 1$, there is a negative sign of Δ_1 which means a percentage loss from the original parameter. In contrast, the results show the opposite when $Peak = T$. Δ_1 is the highest when $T = 30$. Q does not change much when T increases. Δ_2 is the highest when $T = 120$, with an increment of 42.42%.

- When lz increases, the optimal Q rises slightly, Δ_1 increases, and Δ_2 decreases. The impact of lz on Δ_1 is higher when $Peak = T$. This provides the insight that when lz is longer, the retailer can buffer her capital pressure up to lz times.

- A change in lv does not affect Q, Δ_1, and Δ_2.

- When α_r increases, Δ_1 for $Peak = 1$ and $Peal = T$ increases and decreases, respectively. The results show the opposite when α_r decreases.

These results indicate that when the selling season is shorter, it is beneficial for the retailer who faces an upward demand pattern. At the beginning of the selling period, the retailer has higher working capital due to low sales, but he also has high liability, namely, ordering costs and holding costs. With a shorter selling season, the retailer accumulates sales faster to overcome the negative cash-flow. In contrast, when the selling season is longer, this is beneficial for the retailer who faces a downward demand pattern.

Table 4.3: Sensitivity analysis for operational parameters

Parameter	**Peak**	**Percentage Change in Value**	Q	Δ_1	Δ_2
h	1	-50	725	1.07	33.89
		-25	722	0.53	45.28
		+25	716	-0.52	74.85
		+50	713	-1.02	94.45
	150	-50	703	19.83	-
		-25	693	9.84	-
		+25	673	-9.68	-
		+50	662	-19.20	-
p	1	-15	654	-56.51	256.73
		-10	681	-38.17	119.55
		+10	747	39.52	39.65
		+15	758	59.64	34.31
	150	-15	563	-80.65	-
		-10	623	-55.29	-
		+10	721	58.59	-
		+15	735	88.68	-
w	1	-15	763	44.46	36.69
		-10	749	29.35	41.99
		+10	686	-28.10	96.56
		+15	667	-41.63	141.39
	150	-15	738	67.76	-
		-10	721	44.61	-
		+10	635	-41.94	-
		+15	603	-61.62	-
v	1	-50	718	-0.14	58.74
		-25	718	-0.07	58.74
		25	720	0.07	58.74
		50	720	0.14	58.73
	150	-50	682	-0.15	-
		-25	683	-0.07	-
		25	684	0.07	-
		50	685	0.15	-

Table 4.4: Sensitivity Analysis for time parameters

Parameter	**Peak**	**Change in Value**	Q	Δ_1	Δ_2
T	1	30	670	-7.93	6.63
		60	683	-5.86	16.91
		90	695	-3.85	28.75
		120	707	-1.89	42.52
	30	30	665	37.06	-
	60	60	670	27.81	-
	90	90	675	18.55	-
	120	120	680	9.27	-
lz	1	30	722	2.36	56.90
		60	724	4.70	55.18
		90	727	7.04	53.57
		120	729	9.36	52.06
	150	30	687	3.56	-
		60	690	7.11	-
		90	693	10.64	-
		120	696	14.16	-
lv	1	5	719	0.00	58.74
		10	719	0.00	58.74
		15	719	0.00	58.74
		20	719	0.00	58.74
	150	5	683	0.00	-
		10	683	0.00	-
		15	683	0.00	-
		20	683	0.00	-
α_r	1	0.05	720	-2.10	44.08
		0.075	720	-1.06	51.10
		0.125	719	1.06	67.08
		0.15	718	2.13	76.22
	150	0.05	692	7.86	-
		0.075	688	3.95	-
		0.125	679	-3.98	-
		0.15	674	-8.00	-

4.5.2 Case 2:-Price Dependent Demand with Price Adjustment

The additional data for the given examples is provided below.

$$a_1 = 150, r = 0.7, \theta = 0.1, \beta_p = 0.4$$

With these assumed parameter values, we evaluate the retailer's optimal decisions, subject to the expected NPV of profit maximisation in all combination scenarios. By putting the value of $\theta = a_1 = \beta_p = 0$, and $\vartheta = 1$, we obtain numerical results similar to Case 1. Table 4.5 summarises the optimal results for Cases 1 and 2. For Case 2, we examine the results for single pricing and dual pricing. From these results, it is clear that the expected NPV of profit for Case 1 is higher than for Case 2. This is because the demand is independent of the price. Comparing Case 2 when ($\theta = 0$) and ($\theta = 0.1$), the dual pricing ($\theta = 0.1$) assumption lead to ordering more stock and results in a higher expected profit compared to the single pricing ($\theta = 0$) assumption. This is because the price adjustment can stimulate the demand and reduce the excess inventory at the end of the selling season.

Table 4.5: Optimal results of Case 1 and Case 2

Optimal Solution	**Case 1**	**Case 2** ($\theta = 0$)	**Case 2** ($\theta = 0.1$)
Q	719	257	341
$E[AS_r]$	3934.58	1416.82	1729.50

4.5.2.1 Sensitivity Analysis

The sensitivity analysis is performed and the results are compiled in Tables 4.6 to 4.9 including Δ_1, and Δ_2 which capture the difference in $E[AS_r]$ between the changing parameters and base parameters, and between the two models, respectively. Note that the underlined values in all tables represent the optimal solution for the dual pricing model.

$$\Delta_1 = 100 \times \frac{\mathbb{E}[AS_r(change)] - \mathbb{E}[AS_r(base)]}{\mathbb{E}[AS_r(base)]} \tag{4.19}$$

$$\Delta_2 = 100 \times \frac{\mathbb{E}[AS_r(\theta = 0.1)] - \mathbb{E}[AS_r(\theta = 0)]}{\mathbb{E}[AS_r(\theta = 0)]} \tag{4.20}$$

Table 4.6: Sensitivity analysis for demand setting parameters

Parameter	**Change in value**	Q	$E[AS_r]$	Δ_1	Δ_2
ϑ	2	451	2479.30	74.99	28.74
		625	3191.75	84.55	-
	3	645	3532.56	149.33	31.56
		907	**4647.32**	168.71	-
β_p	0.3	327	1802.59	27.23	25.01
		442	2253.41	30.29	-
	0.5	206	1132.81	-20.05	18.72
		267	1344.89	-22.24	-
a_1	100	257	1416.82	0.00	15.82
		311	1641.03	-5.12	-
	200	257	1416.82	0.00	27.63
		374	1808.34	4.56	-

In Table 4.6, the results reveal the following insight regarding the parameters:

- When varying ϑ, by increasing ϑ, Q, $\mathbb{E}[AS_r]$, Δ_1, and Δ_2 increase accordingly. For example, when $\vartheta = 3$, under dual pricing, $\mathbb{E}[AS_r] = 4647.32 > 3934.58$ which is greater than the result in Case 1. This shows that increasing ϑ has a high impact on $E[AS_r]$.
- The impact of β_p on Δ_1 is almost symmetrical regarding both increasing and decreasing β_p. However, when β_p increases, Δ_2 is lower than when β_p decreases.
- Changing a_1 only affects dual pricing. Increasing a_1 will increase Δ_1 and Δ_2. This is expected, as a_1 represents the second peak of demand.

All in all, this insight indicates that if the retailer faces higher demand with lower elasticity in price, she will maximise her expected annuity stream of profit. It is always beneficial to consider dual pricing over single pricing since it leads to more profit.

Table 4.7: Sensitivity analysis for price setting parameters

Parameter	**Change in value**	Q	$E[AS_r]$	Δ_1	Δ_2
p	19	254	931.70	-34.24	17.98
		329	1099.23	-36.44	-
	23	258	1896.31	33.84	24.17
		345	2354.61	36.14	-
θ	0.05	348	1801.52	27.15	-
	0.15	333	1661.23	-3.95	-

Table 4.7 provides a sensitivity analysis of the price setting parameters. The following insights are obtained from the results:

- For Δ_1, the results are nearly symmetric when p increases and decreases. Increasing p leads to a greater Q, $\mathbb{E}[AS_r]$, and Δ_2. This is expected, as it produces a higher Q (based on Eq. (4.18)), thus increasing the $\mathbb{E}[AS_r]$.
- When θ increases, Q and $\mathbb{E}[AS_r]$ decrease (negative value of Δ_1). Giving a greater price discount reduces the $E[AS_r]$ of the retailer.

Table 4.8: Sensitivity analysis for operational parameters

Parameter	**Change in value**	Q	$E[AS_r]$	Δ_1	Δ_2
w	13	269	1970.35	39.07	25.33
		362	2469.40	42.78	-
	17	243	891.14	-37.10	16.85
		311	1041.30	-39.79	-
h	0.075	258	1428.21	0.80	23.32
		345	1761.29	1.84	-
	0.15	254	1394.85	-1.55	19.65
		334	1668.99	-3.50	-

A sensitivity analysis of operational parameters is given in Table 4.8. The results show that

- when w increases, Q, $\mathbb{E}[AS_r]$, Δ_1 and Δ_2 decrease. This is expected, since increasing ordering cost will decrease the optimal order quantity, thus decreasing the expected profit.
- The same happens when h increases. However, the impact of h on the optimal solutions is not as great as that of w.

The sensitivity analysis results in Table 4.9 reveal the following insights regarding the models' parameters:

- T is proportional to Q and $\mathbb{E}[AS_r]$. A decrease in T leads to a lower Q and $\mathbb{E}[AS_r]$. A longer selling season means higher sales. From Δ_2, the difference is larger when T is smaller. A shorter selling period means lower sales revenues. This shows that when T becomes shorter, price adjustment happens more quickly than when T is longer, and it boosts the demand faster. Thus, the difference in $E[AS_r]$ between dual pricing and single pricing is higher.
- lz influences Q and $\mathbb{E}[AS_r]$. An increase in lz leads to a higher $E[AS_r]$ and slightly higher$Q$. Therefore, with a longer delay in payment, the retailer can accumulate more profit without ordering much. The results do not vary much between Δ_1 and Δ_2 which means that lz has a similar impact on both the single pricing and dual pricing models.
- When α_r increases, Q decreases but $\mathbb{E}[AS_r]$ increases. This indicates that the retailer will order less when α_r is high to reduce her risk of investing in Q.
- When r changes, the impact on Q and $E[AS_r]$ under dual pricing is small. Changing r will not affect the single pricing.

In sum, this yields the insight that when the retailer faces a longer selling period with a lower opportunity cost, she can adjust the selling price earlier. Delaying the payment to the wholesaler is a bonus to increase the retailer's expected profit. Thus, negotiating with the wholesaler is necessary to allow a delay in payment.

Table 4.9: Sensitivity analysis for time parameters

Parameter	**Change in value**	Q	$E[AS_r]$	Δ_1	Δ_2
T	60	217	1181.10	-16.64	35.72
		308	1603.03	-7.31	-
	90	231	1263.89	-10.79	30.58
		319	1650.43	-4.57	-
	120	244	1342.64	-5.24	26.06
		330	1692.54	-2.14	-
lz	30	258	1450.11	2.35	22.32
		343	1773.72	2.56	-
	60	259	1483.22	4.69	22.55
		344	1817.74	5.10	-
	90	259	1516.16	7.01	22.78
		346	1861.58	7.64	-
α_r	0.08	257	1406.88	-0.70	22.61
		343	1724.95	-0.26	-
	0.12	257	1426.79	0.70	21.54
		340	1734.09	0.27	-
	0.14	256	1436.79	1.41	21.01
		339	1738.72	0.53	-
r	0.6	257	1416.82	0.00	22.12
		343	1730.18	0.04	-
	0.65	257	1416.82	0.00	22.08
		342	1729.68	0.01	-
	0.75	257	1416.82	0.00	22.08
		341	1729.61	0.01	-

4.6 Conclusions

We have presented and analysed the NP with non-stationary demand and two types of pricing schemes: constant price and price-dependent demand with price adjustment. The model was derived based on an NPV approach to capture the TVM. The numerical results proved that there is a difference in the optimal solution between the traditional NP and the NPV NP. Therefore, it is necessary to conduct the model based on NPV analysis. This model is divided into two large cases: 1) constant price, and 2) price-dependent demand with price adjustment. We showed that the optimality of the model can be proven analytically. For case 1, we analysed the model with two different demand patterns: upward demand and downward demand. The numerical results showed that the expected annuity stream of profit with a downward demand pattern is always superior to the case with an upward pattern, regardless of any changes in parameters.

In addition, we showed that with the right setting of parameter values, Case 2 can yield better results than Case 1. The sensitivity analysis conducted for this model confirmed the impact of TVM on optimal solutions, which has not been discussed in the literature. Besides the impact of TVM, the analysis can provide a signal to the retailer if she faces some kind of demand pattern under certain circumstances: a longer selling season, lower opportunity cost, and higher ordering cost. The retailer can then make any adjustment, for example by increasing or decreasing her selling price, considering dual pricing, or decreasing holding costs in order to maximise her expected annuity stream of profit.

Chapter 5

Conclusion and Recommendations

5.1 Introduction

The conclusions of this study are related to the research questions in Chapter 1. The chapter is organised as follows: the key findings and contributions are discussed in Section 5.2, and suggestions for future research are provided in Section 5.3.

5.2 Contributions of the Study

This study contributes to the existing literature in various ways. Its main contribution is the introduction of the new policies in the NP based on the NPV framework in SC management. We proved that the proposed models give more accurate results than the classical framework. The specific contributions of the chapters are listed below.

In chapter 3, we developed a general model of the NP with the simplest contract – the wholesale price contract. We showed that there was a difference in results between the proposed model and the classical model. The difference indicated that the NPV framework should be considered, as it provides more accurate results that capture

TVM. The first question was thus answered. Then, to address the second question, we found that the wholesaler might sometimes be better off if a Net D contract is considered in NPV thinking, but not in classical newsvendor thinking. This depends on variables such as demand variability, the opportunity costs, and the selling period, which all impact the wholesaler's optimal results. Thus, the third question was also answered. In summary, these results can help the retailer to decide which optimiser she should choose (as in this case, the NPV framework is the best) to give her the highest profit. Then, the wholesaler has two possible outcomes under the Net D contract, depending on the aforementioned variables.

Different models of the Net D contract derived from the NPV framework were compared with the simplified model of the NPV framework. It can be concluded that small changes in formulation can lead to a significant difference in results. It is necessary to formulate a model that gives more accurate results than the traditional model, though it cannot be simplified analytically. However, for ease of understanding, the simplified NPV model was used in the next chapter to examine how the contract works from the NPV perspective.

In Chapter 4, the previous NPV-II model was extended to include a combination of contracts. The first finding is that the contract achieves perfect coordination. In addition, when the delayed payment terms are included, only the retailer benefits from this option since the wholesale price is too low to cover the loss incurred by the wholesaler through delayed payment. Then, we showed that when perfect coordination is not necessary for both SC members to accept the contract, the wholesaler can benefit from offering delayed payment because he can increase the wholesale price. We also addressed the problem of the wholesaler being dishonest towards the retailer when there is asymmetric information: the analysis showed that both SC member still obtain more profit than with the wholesale-price-only contract. In addition, the wholesaler earns more than he would by being honest with the retailer.

In Chapter 3 and 4, NP models were developed based on continuous demand. However, in real situations, it is common to have non-stationary demand due to seasonality and product life cycles, among others. Thus, the NP with non-stationary demand was adopted in Chapter 5. The contribution of this model is its inclusion of different demand patterns. The assumption made is based on single-ordering policy, which is rarely found in the literature on non-stationary demand. In addition, the model also considers the case of price-dependent demand and price adjustment. The best solution of the expected annuity stream to the case of different demand patterns is when the demand pattern is going downward. In contrast, delayed payment is more beneficial to the retailer if she faces an upward demand pattern. When considering a price-dependent demand pattern, the expected annuity stream of profit with price adjustment can reach the highest value when the deterministic demand function is high. Assuming a longer selling period with a low opportunity cost, we suggest that the retailer adjust the selling price earlier to maximise the expected annuity stream of profit. These findings are a few of the solutions from the model that the retailer can consider when she knows some of the parameters.

5.3 Further Work

There are a few opportunities for further research to use the proposed NPV model in other cases to test its wider applicability. We recommend that further development be undertaken in the following area:

- The distribution used in chapter 3 and 4 is limited to a Gamma distribution. It would be interesting to study the model with different distributions, such as a Log-Normal distribution.

- In chapter 3, we mainly focus on deriving simple contract to solve the asymmetry information problem. For future research, it would be possible to derive a more complex contract such as mechanism design to find the optimal solution.

- The input data in chapter 4 is based on simulation. It would be more realistic to consider demand correlations such as ARIMA type demand model for further research.

- In reality, the SC system has a more complex and larger network than considered in this study. Therefore, it is necessary to extend the model to make it more realistic, for instance by adding multiple wholesalers, multiple retailers, or multiple products.

- Finally, it would be useful to study the inventory model related to transportation and outsources, as the NPV approach has the potential to consider more extended SC settings in which the roles of third parties (e.g. transport companies) and other external stakeholders (e.g. tax authorities) are explicitly incorporated. This could lead to the study of more realistic SC situations than those models studied in the mainstream literature.

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