ARTICLE IN PRESS
JID: EOR [m5G;October 6, 2021;8:38]

European Journal of Operational Research xxx (xxxx) xxx

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing, Transportation and Logistics

Waste reduction in the supply chain of a deteriorating food item – Impact of supply structure on retailer performance

Patrick Beullens[a,b], Yousef Ghiami[c,*]

[a] Mathematical Sciences, University of Southampton, SO17 1BJ, United Kingdom
[b] Southampton Business School, University of Southampton, SO17 1BJ, United Kingdom
[c] Department of Operations Analytics, Vrije Universiteit Amsterdam, Amsterdam, the Netherlands

ARTICLE INFO

Article history:
Received 29 April 2020
Accepted 10 September 2021
Available online xxx

Keywords:
Inventory
Net present value
Deterioration
Waste
Collaboration

ABSTRACT

In this article, we examine how an Operational Research (OR) modeling approach can help in identifying how structural components in the supply process of a food product subject to a small probability of almost immediate failure affects the amount of waste arising at the retailer. This process can be viewed as the cumulative effect of various possible causes, including (apparent) product flaws and breakage. This category of waste, in contrast to products that are removed based on reaching their best before or use-by date, are also having little potential for redistribtion, and may thus be most targeted in future waste reduction legislative initiatives. We develop some relatively easy to calculate measures to help a retailer with identifying the financial implications of waste production in relation to some supply source characteristics, the financial motivation of its supplier to tackle item deterioration at the retailer level, and how this is affected by the level of logistics collaboration. We also discuss how the model can help in deriving the relative benefit of technological, logistical, supplier selection, and marketing strategies available to the retailer to meet future legislative waste reduction targets, and derive conclusions with respect to the design of legislative instruments.

1. Introduction

One particular area where product deterioration is highly relevant is food waste. The United Nations estimates that roughly 33% of food produced for human consumption is wasted or lost along the supply chain. The EU Commission estimates this figure to be 20% in the EU, with more than half of the 47 million tonnes of food waste being generated by households, increasing to 70% when including food service and retail. According to Lebersorger & Schneider (2014) and sources cited therein, the percentage of food waste produced by retail is estimated to be 5% in Germany and 7.6% in the UK. The costs associated with food losses are significant. Buzby & Hyman (2012) estimate the total value of food loss in 2008 at the retail and consumer level in the U.S.A. to have been $165.6 billion (at retail prices), with the following top three food groups: meat, poultry and fish (41%), vegetables (17%) and dairy products (14%)

Tackling food losses in supply chains is important for several reasons, including: to prevent food insecurity issues in the light of a growing world population; to reduce the direct costs of used resources; to avoid the opportunity cost of land, labor, energy, water, and other inputs which could be used in other valuable activities; and to reduce the impact of food waste disposal on human health and the environment (Lebersorger & Schneider, 2014).

The Revised EU Waste Legislation, adopted on 30 May 2018, calls EU countries to reduce the waste of food at each stage of the supply chain and monitor food waste levels. The EU's Sustainable Development Goal 12.3 in particular calls for *halving* per capita food waste at the retail and consumer level by 2030, and reduce food losses along the food supply chain. By the end of March 2019, the Commission seeks to adopt legislation of food waste measurement, and by the end of 2023, to have a proposal of how to set-up an EU-wide food waste reduction target to be met by 2030.

Given the above, the pressure on supply chains and in particular on retailers to reduce food waste can be expected to grow. While empirical research helps us to understand some of the causes of food waste and possible remedies, we argue (see Section 2) that an enriched understanding can be obtained by in addition also deriving insights from (simple) OR models. The purpose of these models is to increase understanding of the links between operational conditions, profit maximizing decisions, and waste production. While an extensive body of OR modelling re-

* Corresponding author.
E-mail address: Y.Ghiami@vu.nl (Y. Ghiami).

https://doi.org/10.1016/j.ejor.2021.09.015
0377-2217/

search has been conducted on item deterioration (see Section 2), arguably much of this research has either not focused on the impact of decisions on waste production itself, or has adopted complex supply chain settings or coordination mechanisms that may make the insights only applicable to very specific situations.

The aim of this article is to contribute to our understanding by addressing the question how future legislation on food waste reduction may impact the way retailers think about the organization of their supply. Rather than aiming to devise an optimal contract or coordination mechanism with a given supplier for example, we wish to identify the factors in the supply structure that make it more likely that a retailer can achieve waste reduction while remaining financially viable. We believe that there is scope for an analysis of a relatively simple supply model that can provide such fairly general insights with respect to some of the root causes of food waste arising at retailers, and how to address them. This may help decision makers (retailers, legislators) identify how best to make changes in this structure to help meeting the expected waste reduction targets, while preserving also the financial sustainability of this sector. It can also help to direct some of the research on OR models with item deterioration towards exploring alternative avenues with a focus on waste prevention and reduction strategies.

2. Related research and modelling assumptions

The purpose of this section is to present an overview of relevant research but also to justify the main assumptions that will be adopted in the main model (Section 3). Which modeling assumptions would be reasonable as a representation of significant food waste generated at retailers?

2.1. Root causes of food waste at retail level: Empirical findings

Empirical research to date indicates that the two main types of reasons for food waste arising at the retail level are product “deterioration” on the one hand, leading to products being rejected by consumers or store personnel, and “erratic” demand, leading to products not being sold in time relative to their best before/use-by/sell-by date.

Empirical studies on food waste at retailers have identified several sources. Buzby & Hyman (2012) report the following potential causes: damaged items from damaged packaging, inappropriate storage conditions or technical malfunctions, customer preferences for unblemished foods, and difficulty in predicting customer demand. Similarly, according to the study of Teller, Holweg, Reiner, & Kotzab (2018), the root causes of food waste (as mostly identified by store managers and further through simulation) are mostly related to undesirable customer behaviour, erratic demand, inefficient store operations and replenishment policies, and elevated product requirements of both retail organisations and customers.

Lebersorger & Schneider (2014) report, based on a study of over 600 retail outlets in Austria, that across all investigated assortment groups, 67% of the food mass removed had developed apparent flaws of the product, while 34% were considered too close to their best before/use-by/sell-by date, 5% had damaged packaging, as well as other less significant reasons (but multiple reasons were possible). They also find that the main root cause depends on the food category. For example, 75% of fruit & vegetables and 0% of bread & pastry and dairy had been removed from sales solely on the ground of apparent flaws, but 9% of fruit & vegetables, 78% of dairy, and 96% of bread & pastry is removed solely due to the best before/use-by/sell-by date.

Which of the food categories are the most important? Cicatiello, Franco, Pancino, Blasi, & Falasconi (2017), reporting on a study of an Italian store, identify that of the 70 tons of food waste produced in a year, the largest amounts arose from fruit & vegetables with 24 tons, bakery with 21 tons, and dairy with 6.6 tons of waste. Combined, this is close to 70% of total food waste. In financial terms, the losses over the three categories are, respectively, about 21%, 13% and 14%, or combined close to half of the total financial loss. The other half was roughly equally distributed over deli, fresh seafood, groceries and fresh meat. Lebersorger & Schneider (2014) report that fruit & vegetables account for close to half, and combined with bread & pastry and dairy to 81% of the financial loss from food waste. These three categories are thus likely significant food waste generators for retailers. The major root cause between these categories differs. For fruit & vegetables, this is mainly due to apparent flaws, while reaching a particular removal date is the main reason for the other two categories.

Besides the financial loss for the retailer, another important perspective is the social and environmental cost of the waste. Products removed due to best before or sell-by date are often still safe for human consumption, and thus have the potential for redistribution through e.g. donation to food banks or social services. From this perspective, it thus seem more important for any legislation (and for retailers in terms of compliance) to focus waste reduction efforts on those food categories for which the potential for redistribution is minimal.

Cicatiello et al. (2017) report that 99% of unsold bakery was redistributed, while almost none was possible for the dairy and fruit & vegetable categories. The dominance of the “apparent flaws” root cause for fruit & vegetables may indeed severely hamper redistribution. Lebersorger & Schneider (2014) find that 54% of removed product had changed colour, 52% had dents, 39% were overripe, 34% were mouldy, 29% were withered, and 18% moist (and multiple criteria possible). Mattsson, Williams, & Berghel (2018) did a study on three large retail stores in Sweden, focussing on characterising the impact of fresh fruit & vegetables waste. They identified so-called “hot spots”, that is, a collection of about 20 types of commonly found fruit and vegetables which produce roughly 80% of the impact on waste mass, economic cost and climate. The top 10 of items include: apple, banana, grape, lettuce, pear, sweet pepper, and tomato. There is no intuitive reason to assume that demand for hot-spot fruit and vegetables would be more difficult to predict than any other fruit or vegetable. The hot-spot study thus supports the findings from Lebersorger & Schneider (2014) in identifying that the major source of waste in fruit and veg are phenomena of deterioration.

The regression models in Lebersorger & Schneider (2014) show that about a third of food loss waste variation can be explained by the sales volume, and number of purchases, with a further weak impact from sales area. While these regression models are insightful, much of the variation remains unexplained. They thus conclude that food loss rates must be further influenced by other factors such as organizational aspects and individual behavioural aspects of the staff and situation specific aspects. Largely similar conclusions are made in Eriksson, Strid, & Hansson (2014), who also identify sales volume as having the most influence on food losses in meat and dairy, while differences in food loss rates between individual retail outlets are, based on interviews with industry, attributed to differences in work routines or demand planning and ordering, turnover, shelf-life, packaging size, customer preferences or accidental causes.

Notwithstanding the value of empirical studies, there is less insight so far developed from these studies in relation to how the organization of the supply process of retailers may influence waste production. As this process is typically organized in support of minimizing the costs to meet customer demand, operational research models may contribute in this area. While such models are necessarily a simplification of reality in many aspects, they may provide sufficient insight into how changes in operational conditions may affect profitability and thus decision making for the firms involved.

It seems fair to conclude that a significant fraction of food waste at retailers occurs due to products having or developing one or more of a list of possible defects, and that make consumers avoiding them, and store personnel to remove them from the aisles. Roughly estimating, this may well be close to 75% of all food waste arising, by taking the sum over apparent flaws, damaged packaging, and breakage over all categories (Table 6 in Lebersorger & Schneider (2014), last column). These three causes may well cover close to all of the waste from fruit & vegetables, 20% of dairy, and close to half of the 'other' product category. These products will not be sold and become lost. In order to satisfy a given consumer demand, and knowing there is this kind of loss, the retailer will thus have to order a higher amount from its supplier. This extra amount will become the waste production of the retailer, and it is this quantity that we want to study in this paper. The empirical studies thus suggest that models which assume food loss from deterioration rather than assuming a fixed shelf-life assumption, remain highly relevant to the study of food waste at retail level.

A second important root cause, and dominant for bread & pastry & dairy, relates to the expiration date. Because of the much larger redistribution potential, however, we will devote considerably less time in this paper on this cause (i.e. only in Section 8).

2.2. Deteriorating items inventory models

The modeling literature on deteriorating inventory research by now represents an extensive body of research that is still expanding at an increasing rate. Review articles published over the years include Nahmias (1982), Raafat (1991), Goyal & Giri (2001), Li, Lan, & Mawhinney (2010), Bakker, Riezebos, & Teunter (2012), and Janssen, Claus, & Sauer (2016).

Cohen (1977) is one of the earlier papers adopting this assumption, and considers the joint optimization of ordering decisions and pricing for a deteriorating item in the context of no shortages and backordering, respectively. In the context of food retail, however, the backordering assumption is arguably not that relevant. Rajan, Rakesh, & Steinberg (1992) examined a similar situation where prices could be dynamically altered with the age of items on the shelf. They showed that the problem can be decomposed such that an optimal pricing structure can be developed in a first stage, and optimal ordering decisions in a second stage. Following these early studies, a stream of research emerged focusing on the adoption of pricing strategies to tackle the food waste. In this stream, a group of studies considers the combination of fixed shelf-life and discounting, see, e.g., Berk, Gürler, & Yıldırım (2009) and Zhang, Wang, Lu, & Tang (2015b), and another group focuses on item deterioration along with price discounting, e.g., Qin, Wang, & Wei (2014) and Buisman, Haijema, & Bloemhof-Ruwaard (2019).

Although the strategy of reducing prices for items close to their use-by time is relevant to dairy products, it is much less applied for other food categories, see Lebersorger & Schneider (2014). Furthermore, most food categories in retail are governed by fierce market competition so that most retailers are largely price takers, not price setters. It also remains an open question to which degree customers actually (dis)like the idea of dynamically changing prices for food items. For these reasons, we will focus on a model where the consumer price is given, items do not have a sell-by date, deteriorated items are removed from the inventory rather than reduced in price, and backordering is not considered.

One particular reason for food waste mentioned in Buzby & Hyman (2012) is the difficulty of predicting demand. This would be a motivation to develop stochastic inventory models. It is also one of the recommendations made in the review article of Bakker et al. (2012). While it is undoubtedly true that increased demand variability will increase food waste, the question is to what degree it may affect optimal supply chain structure, in particular supplier order quantity and frequency. Some progress in this area is made in the research of Ghoreishi (2018) and Ghoreishi (2019). Using both stochastic simulation models and semi-Markov decision theory, she examines the robustness of deterministic inventory models under stochastic conditions of demand, supplier lead-time, and item deterioration. In this research, she finds that if the expected values of these three stochastic elements are constant, then policies derived from deterministic models with constant demand, a deterministic deterioration rate, and a constant lead-time are very accurate with respect to determining the optimal pricing and inventory lot-size decisions. These findings thus seem to underline, and contrary to expectations, the surprising relevance of (simpler) deterministic inventory models with item deterioration. We will thus also adopt deterministic assumptions, but recognize the value of further research into stochastic models for examining particular aspect of food waste management, such as in relation to examining suitable policies for reduced pricing and product substitution effects, see also Bakker et al. (2012).

The meaning of 'structure of supply' in this paper differs from the notion of *supply chain structure* in the literature. We focus in this paper on waste production at an individual retailer, and supply structure refers to a set of strategies by which the retailer can supply a deteriorating product to its consumers which helps to reduce waste production at the retailer. Supply chain structure typically refers to the number of firms involved in a supply chain and their business relationships, or to spatial characteristics of various supply chain functions. Supply chain structure will also impact waste production. Belavina (2021), for example, examines how store density makes an impact on food waste, and concludes that higher density of stores in a region would reduce food waste up to a threshold density, while exceeding the threshold would result in an increase in waste levels. Other research focuses on optimising the operations in relation to a secondary supply chain that aims at recycling the waste, as in Iqbal, Kang, & Jeon (2020) and Saxena, Sarkar, & Singh (2020). A model between supplier and retailer alone can, if desired, still assess the impacts of some supply chain structure components on the waste production at this retailer (see also Section 4).

One of the key issues addressed in this paper is the examination of how a retailer may benefit from lot-size coordination with its supplier. How does this impact the amount of waste produced by the retailer and the retailer's ability to meet future waste reduction targets? Is the supplier, next to lot-size coordination, also motivated to help address the item deterioration process at the retailer?

Relatively few papers in the area of item deterioration have addressed supply chain models. The review of Bakker et al. (2012) identified about 20 articles addressing the two-echelon inventory settings, and about 4 articles considering more echelons. The traditional multi-echelon setting, however, does not explicitly address the situation in which the echelons each represent individual firms, and the cost functions associated with each echelon do not map onto the actual profit functions of independent firms, as shown in Beullens (2014). Rather than multi-echelon inventory theory, the explicit modelling of interactions between supply chain parties, including how they rewards each other, is necessary to understand the true impact of logistics actions (and deterioration phenomena) on the profits of the firms involved.

Bai, Xu, Xu, & Wang (2016) identified 17 prior articles in the literature addressing the value of collaboration between independent firms in the supply chain of a deteriorating item under a variety of different situations, including with respect to demand, production

Table 1
Instance data experiments Section 3.3.

Instance	s_R	θ	f	d_R
μ_1	80	2	0.1	0.1
μ_2	40	2	0.1	0.1
μ_3	80	1	0.1	0.1
μ_4	40	1	0.1	0.1
μ_5	80	2	0.1	0.2
μ_6	40	1	0.2	0.1

rates, and supply strategies. Ghiami, Williams, & Wu (2013), Jonas (2019) (and four prior studies reported in Table 1 in that paper), and Ghiami & Beullens (2020) investigate the benefits from adopting the joint optimal inventory policy in a supplier-retailer setting in which the retailer can make use of two warehouses: one warehouse OW that is owned and in which the retailer can only store up to a limited amount, and a second rented warehouse RW for excess stock. Lin, Jia, Wu, & Yang (2019) argue that the deterioration directly makes an impact on the supplier's capacity utilization, hence should be accurately measured. They model an integrated two-echelon supply chain with trade credit and consider in-transit deterioration next to deterioration at the retail level.

In all but one of the above studies in the supplier-retailer setting with item deterioration, models have been developed based on the average cost approach. Only the recent study of Jonas (2019) has developed the model by taking the Net Present Value (NPV) of cash-flow functions. The review by Bakker et al. (2012) (Section 4.6) states that the motivation for adopting the NPV methodology is that inflation in certain countries can be significant, which warrants the development of models that consider the time value of money. In this paper, we also derive the profits functions of the firms from the NPV of cash-flow functions. One of the aims of this paper is to demonstrate its value not only because of the fact that inflation could be high. Rather, we wish to demonstrate its methodological value in the study of the (financial) motivations of independent firms in a supply chain of deteriorating items.

The only NPV study in this area, Jonas (2019), considers a lot-for-lot producer producing a deteriorating item at a finite production rate, delivering the products to a distributor in the OW/RW setting who aims to fulfill demand where shortages are fully backordered. The adoption of a lot-for-lot production assumption is quite reasonable. Indeed, if products are subject to deterioration, suppliers will want to produce towards meeting delivery dates and quantities so that the products are ready the moment the delivery is due, as this ensures that the products retain maximum freshness. While overproduction is a well-known issue for livestock farmers (Garrone, Melacini, & Perego, 2014), a lot-for-lot assumption is quite reasonable in the context of most greenhouse growers, vegetable producers, slaughter houses, and dairy product producers. The model in Jonas (2019) incorporates a trade credit policy, allowing the distributor to delay payment to the supplier, and considers that over time the cost of renting the RW decreases. While providing interesting results, the relative complexity of the model does limit the intuitive understanding of key features. In particular, by focusing on minimising costs in the supply chain, the model does not allow getting insight into the impact of item deterioration and collaboration on the profits of the two firms involved. Such an understanding is, in our view, vital when it comes to assessing the degree by which agreements or legislation can be successful.

The majority of the studies identified in Bai et al. (2016) and Jonas (2019) focus on deriving a joint supply chain cost or profit function, developing algorithms to derive optimal policies, and providing a sensitivity analysis. Some of the studies focus on developing a contract mechanism that can lead the parties to adopt the supply chain optimal solution, ranging from revenue sharing contracts to Stackelberg solutions, see also Bai et al. (2016). Most of these studies find that deterioration increases the cost in the supply chain, and affects the optimal lot-sizes transferred to the retailer.

Recent research is aiming to address the question how the deterioration rate of products can be optimally improved as to minimise costs or maximise profits. Blackburn & Scudder (2009) is one of the first studies examining supply chain strategies to address loss of quantity and quality of products. They develop a two stage approach based on the concept of a product's marginal value of time, while Cai, Chen, Xiao, & Xu (2010) address a similar problem in a newsvendor problem setting. The use of preservation technology is further investigated in a series of articles, including Hsu, Wee, & Teng (2010), Dye & Yang (2016), and Yang, Chi, Zhou, Fan, & Piramuthu (2020) and references therein, adopting a single-company focus. Zhang, Liu, Zhang, & Bai (2015a) consider the supply chain of a manufacturer and retailer with price-sensitive demand who may cooperatively invest in preservation technology to reduce deterioration. They design a revenue sharing contract to determine optimal inventory, preservation investment, and pricing decisions. They also identify that, in absence of coordination by revenue sharing, only the manufacturer (acting as a Stackelberg leader) would have a financial benefit from the cooperative investment in preservation technology. Mohammadi, Ghazanfari, Pishvaee, & Teimoury (2019) have similar aims and also adopt revenue and investment sharing as the strategy for profit maximisation and waste reduction.

With the exception of Zhang et al. (2015a), none of the above OR modelling studies are concerned about the amount of waste produced in the supply chain as a consequence of item deterioration in relation to possible future legislation. In this paper, we wish to investigate how the move from a single firm focus to a collaborative supply chain focus may impact the amount of waste produced in the supply chain, as well as how it may alter the supplier's motivation to address the item deterioration process at the retailer. We also aim to present and discuss a wider range of possible structural remedies and improvements that retailers may (independently) make than those studied in the OR literature so far.

We can summarise the topic of this paper as follows. We consider a retailer selling a deteriorating product to consumers and purchasing the product in lot-sizes from a supplier. The retailer wishes to obtain insight into the following main questions: (1) What is the financial incentive for the supplier to reduce the deterioration rate of the product? (2) How will the adoption of a collaborative focus impact the amount of waste arising at the retailer? (3) How will the introduction of waste reduction legislation impact the value of collaboration in the supply chain? (4) What kind of structural change adopted by the retailer with respect to its supply chain can help improve waste management?

3. A NPV supply chain model with deterioration

In this section, we develop a model of deterministic demand for a deteriorating item in the supply chain of a single retailer and its supplier.

3.1. Modelling assumptions and notation

In Section 2, we discussed the literature and also provided a rationale for the adoption of reasonable modelling assumptions, including for the adoption of deterministic models of item deterioration in a logistics and supply chain context, and for developing objective functions based on Net Present Value (NPV) principles. We can summarise the main assumptions as follows:

1. Consumer price is given and demand is known and constant;

2. The supplier knows the order delivery pattern, and produces the items in batches of the required size so that the items are ready by the delivery date (lot-for-lot, make-to-order);
3. The supplier is paid for each batch upon delivery to the retailer;[1]
4. Items start to deteriorate as soon as they arrive at the retail stage;
5. Deteriorated items are removed from inventory rather than reduced in price;
6. Items do not have a sell-by date, or are sold or removed prior to this date.

The model for the retailer is based on Cohen (1977) (Section 1) (extended to include out-of-pocket holding costs and disposal costs), while the supplier's model is based on Monahan (1984) (extended to the case of deterioration). Instead of using the average costs method as in those earlier works, we derive the profit functions of the firms as the NPV of their cash-flow functions. A list of the main notation is given below.

Parameters

x	Superscript to indicate the scenario, $x \in \{o, I\}$
p	Unit price for consumers, paid to retailer
p_R	Unit price for retailer, paid to supplier
p_W	Unit price for supplier, paid to external party ($p > p_R > p_W > 0$)
θ	Deterioration rate at retailer (quantity loss)
s_R	Setup cost per delivery to retailer, paid by retailer to external party
s_W	Setup cost per delivery to retailer, paid by supplier to external party
f	Out-of-pocket holding cost for retailer, paid to external party
d_R	Unit disposal cost for retailer, paid to external party
y	Consumer demand rate (annual rate)
$\hat{Y}$	Waste target (upper bound on the waste as a fraction of consumption sales)
$I_R(t)$	Inventory level at retailer at time t
ρ	Fraction of total supply chain gains from collaboration awarded to retailer
α	Opportunity cost of capital rate (per year)

Decision Variables

T_R	Inventory cycle time (time between two successive deliveries to retailer)
Q_R	Batch quantity per delivery to retailer

Performance measures

AS_R	Annuity Stream profit function of retailer (£ per year)
AS_W	Annuity Stream profit function of supplier (£ per year)
AS_{SC}	Annuity Stream profit function of supply chain (£ per year)
y_R	Waste production rate (annual rate)

3.2. NPV model

The retailer sells a product at price p to satisfy a demand rate y without shortages. The firm procures the item in batches of Q_R from a supplier at price p_R per product with cycle time T_R. The on hand inventory at the retailer $I_R(t)$ at time t ($0 \le t \le T_R$) satisfies:

$$\frac{dI_R(t)}{dt} = -y - \theta I_R(t), \tag{1}$$

where θ denotes the product's deterioration rate. We call a process *exponential decay* where deterioration occurs proportional to the on hand inventory. It is arguably the most widely considered deterioration process in the literature. The majority of studies, including the current paper, assume that such process starts as soon as the items arrive at the retail stage, see, e.g., Yang & Wee (2002), Skouri, Konstantaras, Papachristos, & Ganas (2009), Wu, Ouyang, Cárdenas-Barrón, & Goyal (2014), Hsieh & Dye (2017), and Saha, Chatterjee, & Sarkar (2021). This is an intuitive assumption for a lot of perishables that are highly sensitive to handling activities and keeping conditions, or where microbiological processes are triggered as soon as they are exposed to undesirable conditions.[2]

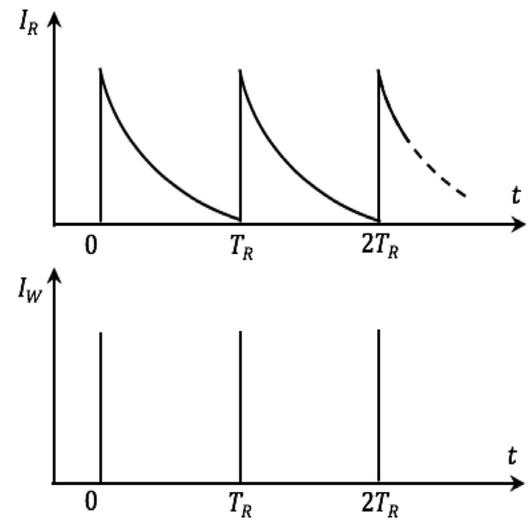

Fig. 1. Inventory level at the retailer I_R and supplier I_W.

Since $I_R(0) = Q_R$ (and $I_R(T_R) = 0$), this gives:

$$Q_R = \frac{y}{\theta}\left[e^{\theta T_R} - 1\right], \tag{2}$$

$$I_R(t) = \frac{y}{\theta}\left[e^{\theta (T_R - t)} - 1\right]. \tag{3}$$

For the supplier (wholesaler), we assume the case of lot-for-lot production at infinite rate. This model, without deterioration, can be traced back to Monahan (1984), and has been adopted in a large stream of inventory literature to get insight in particular phenomena, as in e.g. Jonas (2019). Note that the supplier incurs no holding costs nor deterioration as any batch produced is instantaneously moved to the retailer, see Fig. 1.

We proceed developing profit functions for both firms from NPV principles. Since the process is an infinite repetition of inventory cycles, we calculate the Annuity Stream functions of the relevant cash-flows for each party for an opportunity cost of capital rate α. For the retailer, we assume that set-up costs s_R and payments $p_R Q_R$ to the supplier are incurred whenever a batch arrives. In addition, out-of-pocket costs are incurred instantaneously based on the inventory level of products in the retailer's warehouse, while disposal costs are instantaneously incurred based on the amount of quantity lost due to deterioration, see Fig. 2. The Annuity Stream profit function for the retailer is then:

$$AS_R = py - \left[s_R + p_R Q_R - (f + d_R\theta)\int_0^{T_R} I_R(t)e^{-\alpha t}dt\right]\sum_{i=0}^{\infty}\alpha e^{-i\alpha T_R}. \tag{4}$$

According to the above made assumptions, the supplier receives the revenue $p_R Q_R$ from producing a batch at the same time as it incurs the set-up cost s_W. Assuming that the cost $p_W Q_R$ is also incurred at that time, the Annuity Stream profit function for the

[1] This follows the implicit assumption adopted in classic inventory models, see Beullens & Janssens (2014). They also show how to adapt the model to incorporate different payment structures. If, for example, the supplier wishes to be paid a deposit D per unit of product ordered, and the production lead-time is L, then the model in this paper is valid by replacing the unit price p_R by $(De^{\alpha L} + (1 - D))p_R$.

[2] Some authors adopt the assumption that the deterioration starts only after some time that the items have been on the shelves. Some of these studies provide insight into the benefits of using preservation technology to prolong the period with no deterioration, see, e.g., Dye (2013), Shah, Soni, & Patel (2013), and Mishra, Wu, Tsao, & Tseng (2020).

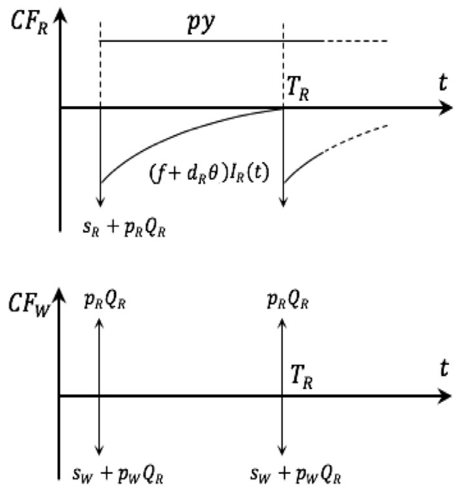

Fig. 2. Cash flows at the retailer and supplier.

supplier is given by:

$$AS_W = \Big[(p_R - p_W)Q_R - s_W\Big]\sum_{i=0}^{\infty}\alpha e^{-i\alpha T_R}. \quad (5)$$

The profit functions (4) and (5) can be further worked out, see Appendix A. The Annuity Stream profit function for the supply chain is the sum of (4) and (5):

$$AS_{SC} = py - \Big[s_R + s_W - p_W Q_R - (f + d_R\theta)\int_0^{T_R} I_R(t)e^{-\alpha t}dt\Big]\sum_{i=0}^{\infty}\alpha e^{-i\alpha T_R}, \quad (6)$$

and is independent of the cash-flow exchanged between the parties (and thus independent of the price p_R). We thus have:

$$AS_{SC}(.) = AS_R(p_R) + AS_W(p_R). \quad (7)$$

3.3. Scenarios: No collaboration vs collaboration

We develop two scenarios 'o' and 'I'. Let $x \in \{o, I\}$. In scenario $x = o$, there is no collaboration, and the retailer determines the order policy that maximises its profit function (4). In scenario $x = I$, retailer and supplier collaborate and establish the order policy that maximises the supply chain profit function (6). Given (2), let the optimal policy be Q_R^x and T_R^x, and corresponding profit functions be AS_R^x, AS_W^x, and AS_{SC}^x.

In order to establish the impact of collaboration on the profit functions of the two firms, we adopt the pragmatic approach that the firms agree to split the total gain achieved from collaboration ($AS_{SC}^I - AS_{SC}^o$) by agreeing on a new price p_R^I such that ($0 \le \rho \le 1$):

$$AS_R^I(p_R^I) = AS_R^o(p_R^o) + \rho(AS_{SC}^I - AS_{SC}^o), \quad (8)$$

$$AS_W^I(p_R^I) = AS_W^o(p_R^o) + (1-\rho)(AS_{SC}^I - AS_{SC}^o). \quad (9)$$

For any value $0 < \rho < 1$ that the parties can agree on, both firms will thus always be encouraged to adopt a collaborative focus whenever ($AS_{SC}^I - AS_{SC}^o$) > 0. (Note that ($AS_{SC}^I - AS_{SC}^o$) $\ge$ 0.)

The average rate of waste produced y_R to meet a consumer demand rate y is given by the excess amount of products per unit of time delivered to the retailer, or, using substitution with (2):

$$y_R = \frac{Q_R}{T_R} - y = y\Big[\frac{(e^{\theta T_R} - 1)}{\theta T_R} - 1\Big]. \quad (10)$$

Note that (10) is valid for any choice of T_R. Intuition says that y_R^x, the value of y_R when using T_R^x, will depend on the scenario. T_R^o, for example, will be a function of the price p_R^o, while T_R^I will be independent of the price p_R^I.

Table 2
Results for no collaboration scenario.

Instance	T_R^o (days)	Q_R^o	$AS_R^o(p_R^o)$	$AS_W^o(p_R^o)$	$AS_{SC}^o(.)$	y_R^o (%)
μ_1	26.65	786.2	5,817.5	7,014.7	12,832.2	7.67
μ_2	18.65	538.0	6,468.9	6,604.3	13,073.1	5.29
μ_3	36.65	1056.4	6,408.6	7,000.2	13,408.8	5.19
μ_4	25.65	728.1	6,881.3	6,706.2	13,587.5	3.60
μ_5	25.65	754.6	5,743.6	6,971.1	12,714.7	7.37
μ_6	24.65	698.7	6,845.5	6,672.0	13,517.6	3.45

The above approach can be viewed as an application of cooperative game theory in which the potential of collaboration between a set N of players is examined through the characteristic function $v(S), S \subseteq N$. In particular, $N = \{R, W\}$; $v(\{R\}) = AS_R^o$, $v(\{W\}) = AS_W^o$, and $v(\{R, W\}) = AS_{SC}^I$. The mechanism for splitting the rewards from collaboration presented in (8) and (9) extends the well-known Shapley Value method, which would set the value of ρ exactly to 0.5. The sharing of benefits through an adjustment of unit price seems a practical and simple approach. In reality the firms, even when collaborating, will not want to reveal all financial details.[3] Therefore, it is difficult in practise to establish a correct approach to sharing profits according to an exact ratio ρ by actually applying the mechanism as given by (8) and (9). However, agreeing on a (new) price p_R is part of the business processes of most supply chain members and it does not require the full sharing of sensitive data. A price adjustment also gives the benefit that the same invoicing method can be used.

Finally, we assume that in both scenarios, supplier and retailer sign a contract which specifies delivery pattern (order quantities Q_R^x, delivery dates) as well as the unit price p_R^x and the product quality.[4] The agreement about delivery pattern is an important feature as it allows the supplier to plan production and delivery, giving the best chance to offer the products in the best possible condition to the retailer and thus consumers. Because of this, there is little room for players to deviate from the agreed delivery strategy.

4. Supply structure

As stated in the introduction, part of the purpose of this article is to identify factors in the supply structure that make it more likely that a retailer can achieve waste reduction or prevention while remaining financially viable. To aid this analysis, the model presented in Section 3 has a limited number of parameters. For the retailer, the parameters or variables of influence on profits and waste production are basically choosing the scenario x, and then establishing values for θ^x, s_R^x, y^x, p^x, p_R^x, d_R^x, ρ, and the adopted Q_R^x and T_R^x. Despite the fact that the model is relatively simple and has few parameters, it still offers a way to assess the potential value of pathways towards improvements from modifying structural elements in the retailer's supply chain. We provide here examples of such strategies, some of which will be revisited in later sections (see also Table 12.)

Technological innovations. Both the retailer and the supplier may have some control over the deterioration rate θ. It may be possible to improve this rate by retailers through, for example, the method of displaying the products. While Aldi in the UK stores meat in cooled open shelves, Lidl in the UK and the Netherlands

[3] Notwithstanding the usefulness of principal-agent theory and its revelation principle, the study of this topic under assumption of asymmetric information is complex and to be left for further research.

[4] Product quality is not explicitly incorporated in our analysis, but consistency of quality is in general a requirement in the food sector, and making this assumption also helps justify why the model can work with a specific θ value. In Section 6, we investigate whether the supplier would have incentives to lower its quality (resulting in an increase of θ).

uses closed refrigerators, and Colruyt (Belgium) and Dirk (The Netherlands) place part of their fresh produce in a cooled walk-through area separated from the main store.

The supplier may affect the rate of deterioration at the retailer by e.g. the choice of packaging to help control oxygen levels, or minimise the chance of bruising or direct contact with customers. If the problem is too much variation in size or visual appeal[5], such that some products are not desirable, as may be case for certain vegetables, they may be packaged for being sold as a bundle. Suppliers may improve the situation at the retailer by changes in production method. For example, growing produce in greenhouses, with artificial light at night, climate control, and automated feeding circuits, allows for better control and reduction of production lead-times, and gives a more uniform quality; as a result the produce can be harvested just before delivery in optimal conditions and thus lower θ.

Any changes introduced to improve one parameter can, and often will, change other parameters too. Investment in technology that reduces the deterioration rate (θ) may increase prices (p_R and p) which may reduce sales (y), unless it is countered by an uptake in sales from an improved product quality. Changes in production approach can also affect y through changes in product size, taste, look, packaging, or nutritional value.

Supplier selection and logistics. Changing supplier can affect many parameters at once. Of prime importance is to ensure that standards of food quality and safety are followed. However, there can still be significant differences in the product due to differences in the production method, technology used, scale of operation, location (e.g. soil quality), packaging, etc. Besides the potential impact of this on θ and y, it may affect the efficiency of the logistics. If a supplier is located closer to the retailer, it may lower the delivery cost s_R. Note that s_R is the set-up cost of one individual type of item in our model. Therefore, if the retailer selects a supplier who can produce more types of product needed at the retailer and have these delivered in one trip, the delivery costs per product type s_R could be much lower.

Marketing and store concepts. Marketing efforts and store reorganisation can all affect a great number of parameters in the model. The image of the store (brand name, etc) determines how y is affected by product quality (and θ) and by pricing p. Demand y can be increased by advertising or price reductions.

As product range and store organisation form an integral part of a retailer's image and brand, care is needed before introducing great changes. Demand y in the model can be affected by how many different variations are offered of the same kind of product. A reduction in material handling by staff is possible by keeping the number of variations the same, but having the supplier deliver a pre-mix of these variations in a bigger carton or box, and placing this box on the shelf without separating the variations. This is a strategy used in stores like Aldi for e.g. displaying dairy products of different fat content (e.g. 0%, 10%, 20%) or flavours (e.g. natural, strawberries). In general it requires less shelf space and thus can be expected to lower f in the model. As the set-up cost s_R for the retailer is also in part covering the cost of the time that store keepers need to replenish the shelves, pre-mixed batches lower s_R. Such a strategy is best trialled first, as it is possible that the increased consumer handling of the product when selecting from the pre-mixed box could affect θ.

Limiting the number of product variations is an alternative strategy that will simplify the store organisation as well, but will primarily affect the demand y in the model. For example, if demand for yoghurt with 0%, 10% and 20% fat content is y_0, y_1 and y_2, respectively, then reducing the range to only the 10% product may change the demand y'_1 in the new model to, for example, $y'_1 = (0.8)y_0 + y_1 + (0.95)y_2$, if (most) of the customers are prepared to swap their choice.

Reducing the product range can also be done by limiting the number of different brands on offer. It may also reduce the number of suppliers, and thus impact logistics structure and costs. The retailer could decide to reduce the number of different brands competing with products simultaneously on the shelf, and review contracts more rigorously regularly so that brands still compete over time. This strategy will concentrate the demand and thus increase y in the model in a similar manner as illustrated above, and also reduce f. Brand switching is however not always easily adopted by consumers.

Advertising the shop's efforts to reduce waste, and the important contributions customers can make, may improve shopping behaviour affecting θ and attract demand for organically produced and responsibly sourced products. Introducing a separate range of 'funky' looking vegetables, branded with suitable name, may help sell what might otherwise be rejected. These odd shapes are more likely to occur for natural and organically produced products. Morrisons' (UK) Wonky range of fruits and vegetables (all about freshness and flavour) is a perfect example. Establishing links to organisations that can reuse products taken from the shelves which are still safe to consume (e.g. food banks), can affect the cost d_R, and reduces the fraction of the waste y_R that is to be accounted for in meeting any legislative waste target requirements (introduced in Section 7). This may also feed back positively into the advertisement effort in the shop.

Competitive landscape. The relative negotiation strength of the supplier will typically affect many of the parameters in the model. Does the supplier serve other retailers, and are you as retailer a big or a small client to your supplier? Do you as retailer see many potential alternative suppliers available and are you prepared to swap?

The number of competing retailers in the neighbourhood is an important consideration that may affect demand y and pricing p. Characteristics of the local population and the road infrastructure and local traffic regulations (e.g. in cities) are likewise affecting demand, product range, pricing decisions, and delivery costs.

Finally, the contract conditions adopted may have impact on profit and waste production. Does the contract, in addition to the delivery pattern and pricing, offer sufficient guarantees of product quality? Small retailers may not be in such a strong position as the big retailers. Suppliers delivering to multiple retailers may be tempted to stratify their produce according to what they see as most profitable, in effect leading to higher θ values for one retailer compared to the other.

In the case of products with high deterioration rates, it may make sense for large retailers to vertically integrate with the suppliers. In the USA market, for example, the large retailers operate their own milk processing plants. This not only removes the double marginalisation effect and thus can boost the profitability of the retailer, it also allows for full control of the production process, can avoid any potential opportunism (establishing full control over θ).

The above descriptions make us appreciate that there exist many influencing external factors. Trying to include these explicitly in the model is difficult because of the sheer number of possible functional forms to choose from - it may indeed very much depend on the particular retail store considered which aspects should be included, and how. Therefore, as also argued in Sections 1 and 2, it seems worthwhile instead to study the impact of the various parameters in the simpler model as presented here, and

[5] FAO (2011) report how unreasonable quality standards are a major cause of waste post harvest in medium- and high-income countries. The example given is that carrots that are not straight have to be diverted to livestock feed. Retailers who can let their customers embrace these instead will reduce a large source of waste arising in its supply chain.

Table 3
Results for collaboration scenario ($\rho = 0.5$).

Instance	T_R^I (days)	Q_R^I	$AS_R^I(p_R^I)$	$AS_W^I(p_R^I)$	p_R^I	$AS_{SC}^I(.)$	y_R^I (%)
μ_1	44.65	1,386.1	5,942.1	7,171.6	1.158	13,113.7	13.30
μ_2	36.7	1,112.1	6,660.2	6,812.3	1.148	13,472.5	10.75
μ_3	61.7	1,840.3	6,488.8	7,122.2	1.170	13,611.0	8.94
μ_4	50.7	1,488.7	7,012.4	6,860.5	1.162	13,872.9	7.27
μ_5	41.7	1,281.9	5,858.5	7,126.4	1.162	12,984.8	12.33
μ_6	47.7	1,394.7	6,971.1	6,831.0	1.164	13,802.1	6.82

from this, infer strategies of structural change for improved waste prevention.

5. Impact of supply structure - Numerical examples

To establish an initial insight into the impact of supply structure on firm profits and the amount of waste produced, we present results of the model developed in Section 3 applied to six well-chosen instances. We compare the optimal decisions for the retailer in the case of no collaboration and in the case of collaboration with its supplier.

Data about the instances μ_i $(i = 1, 2, ..., 6)$ is summarised in Table 1. In each case, we take $y = 10,000$ per year, $p = 2$, $p_R^o = 1.2$, $p_W = 0.5$, $s_W = 40$, and $\alpha = 0.1$ (per year). If we take μ_1 as the base case setting, then μ_i $(i = 2, 3, 4)$ differ in the values assigned to s_R and θ: μ_2 considers the situation where the retailer has found a way to half its set-up cost, while μ_3 is the case where the deterioration rate has been halved, and μ_4 represents the case when both improvements have occurred.[6] μ_5 is like instance μ_1 but where the disposal costs are doubled, while μ_6 is as μ_4 with doubled out-of-pocket holding costs.

Table 2 summarizes the results in the scenario of no collaboration. The optimal value for T_R is established via grid search as the value that maximizes (4), using the function given in Appendix A. The table reports the optimal cycle time in days (i.e. the value of T_R multiplied by 365), the optimal lot-size, and the AS profit functions for both firms (in monetary value per year). The final column lists the waste as a percentage of y (e.g. a value of 7.67% corresponds to 767 products that have to be discarded per year).

Comparing μ_i $(i = 2, 3, 4)$ with μ_1, it can be observed in Table 2 that the retailer increases profits and reduces waste considerably if it can, ceteris paribus, reduce set-up cost and/or deterioration rate. Note, however, that reducing the set-up cost alone (μ_2) results in *smaller cycle times and batch quantities*, while the reduction of deterioration rate alone (μ_3) *increases cycle times and batch quantities*. These instances further illustrate that the impact on supplier profits from increased efficiency at the retailer can be negative. In other words, the supplier seems to derive benefit from the waste that arises in the downstream supply chain, as well as from high set-up costs that drives retailers to order in larger quantities. We analyse these 'supplier's rewards' further in Section 6.

Comparing μ_5 with μ_1 shows that the increase of disposal costs, while reducing the retailer's profits, has a small impact on the optimal inventory policy and the amount of waste. Comparing μ_6 with μ_4 shows that the increase of the out-of-pocket holding costs also affects the optimal policy and the amount of waste produced, but to a much lesser degree than s_R and θ.

Table 3 lists the results in the scenario of collaboration, following the format of Table 2. The optimal value for T_R is established via grid search as the value that maximises (6), using the sum of functions for AS_R and AS_W given in Appendix A. Once the optimal cycle time and lot-size are determined, a second grid search on p_R^I then establishes its value as to satisfy (9) for $\rho = 0.5$. From this, the profits $AS_R^I(p_R^I)$ and $AS_W^I(p_R^I)$ are calculated.

Table 3 shows that conclusions reached for scenario $x = 0$ transfer to scenario $x = I$: the retailer benefits from reducing the set-up cost and/or deterioration, but the supplier's profits are negatively affected. The impact of increased disposal costs or increased out-of-pocket costs is more pronounced but remains small. Comparing Table 3 with Table 2 shows, not surprisingly, that both firms can never do worse by adopting a collaborative focus. The increase in total profits in the supply chain depends on the instance, but ranges between 1.4% to 3%. An equal split of supply chain gains can be achieved by p_R^I values that stay close to p_R^o.

The most significant differences between the two scenarios is the large increase in optimal cycle times, lot-sizes and waste produced when adopting a collaborative focus. The amount of warehouse space the retailer would need would thus also increase significantly. The potentially large practical implications in the warehouse and the increased environmental burden may perhaps not make it strategically worthwhile for the retailer to collaborate in order to gain the 1.8% increase in profits[7]?

From the perspective of the supplier, the highest profits are obtained in μ_1 in both scenarios. The supplier would in this case not be inclined to encourage the retailer to reduce set-up cost or deterioration rate.

6. Waste production and supplier's incentives: Analysis

The examples from Section 5 illustrate that the supply chain structure may make it more or less difficult for retailers to manage product deterioration. In particular, there are scenarios in which the supplier receives financial benefit from product deterioration in the downstream supply chain. Also, the amount of waste produced is drastically increased when adopting a collaborative focus.

The purpose of this section is to derive analytical results. These will also help to identify which supply chain structure would give a retailer the best possible vantage point to meeting legislative waste targets (addressed in Section 7) and which kinds of supply chain structural strategies may help with minimising waste production (further discussed in Section 9). In particular, Section 9.1 provides a further discussion of how some of these results can be used.

The easiest situation for a retailer to be in, when selling a deteriorating product, is to have a supplier who would not derive any benefit from the waste arising at the retailer. In this section we present some results to help assess this, as well as develop strategies to working towards establishing this situation.

Definition 1. The **supplier's sales reward SSR** is a measure of the additional amount sold of a particular (deteriorating) product from selling it in discrete amounts to the retailer as to meet a given final demand rate.

[6] Practical strategies to reduce set-up costs or deterioration rates are suggested in Section 4, and further discussed in Section 9.

[7] Note that profit increase as a percentage can be made arbitrarily large by considering other fixed costs which typically arise to manage a contract.

Table 4
SSR, SOR and SR values (unapproximated model).

Instance	SSR^o	SSR^I	SOR^o	SOR^I	SR^o	SR^I
μ_1	+767	+1329	+14.7	+591.6	+564.5	+920.6
μ_2	+529	+1075	-395.7	+332.3	+389.1	+732.7
μ_3	+519	+894	+0.2	+422.2	+400.6	+661.0
μ_4	+360	+727	-293.8	+240.5	+277.4	+530.8
μ_5	+737	+1233	-28.9	+506.4	+542.3	+858.9
μ_6	+345	+682	-328.0	+191.0	+266.3	+499.4

As Q_R is given by (2), the SSR is given by ($x \in \{o, I\}$):

$$SSR^x = \frac{Q_R^x}{T_R^x} - y. \tag{11}$$

Given (10), we see that $SSR^x = \overline{y_R^x}$. The supplier enjoys increased sales due to the product deterioration at the retailer. This is a consequence of the logistics process only: if the supplier would supply the retailer at a constant rate, the retailer selling each product immediately, then the quantity loss, waste and SSR are all equal to zero.

Definition 2. The **supplier's operational reward SOR** is a measure of the financial benefit in the annuity stream profit of the supplier from selling a particular (deteriorating) product in discrete amounts to the retailer as to meet a given final demand rate.

In this model[8], the SOR is given by ($x \in \{o, I\}$):

$$SOR^x = AS_W^x - (p_R^x - p_W)y. \tag{12}$$

If we instead subtract the marginal revenues plus the logistics costs of the supplier, we obtain:

$$SR^x = AS_W^x - \left[(p_R^x - p_W)y - s_W \sum_{i=0}^{\infty} \alpha e^{-i\alpha T_R^x}\right], \tag{13}$$

which we call the ***supplier's reward*** *in the case of deterioration with exponential decay*. SOR equals SR plus the supplier's logistics costs, and thus SOR ≤ SR.

Table 4 shows SSR, SOR and SR values for the instances in Table 1. In the analysis that follows in this section, it will be shown that SSR and SR values increase with deterioration rate; SR gives an upper bound on the financial value to the supplier as a consequence of the deterioration process at the retailer. Further, we will show that whether the supplier will profit from increased deterioration can be approximately deduced from the sign (and magnitude) of SOR values.

To derive analytical results, we approximate the functions (4), (5), (11)–(13). To simplify notation, we set $T_R \equiv T$, $Q_R \equiv Q$. We take the Maclaurin expansion[9] of exponential terms in α and θ, and retain only the low order terms. As the impact of both d_R and f was shown in Section 3.3 to remain modest, we set both values at zero. This produces, after some algebraic manipulation, the following functions:

$$\overline{SSR} = \overline{y_R} = \theta \frac{yT}{2}, \tag{14}$$

$$\tilde{\alpha} = \alpha(1 + \frac{\overline{SSR}}{y}). \tag{15}$$

$$\overline{SR} = (p_R - p_W)\left[\tilde{\alpha}\frac{yT}{2} + \overline{SSR}\right] \tag{16}$$

$$\overline{SOR} = \overline{SR} - \frac{s_W}{T} - \alpha\frac{s_W}{2}, \tag{17}$$

$$\overline{AS}_R(p_R, \theta) = (p - p_R)y - \frac{s_R}{T} - \alpha\frac{s_R}{2} - p_R\left[\tilde{\alpha}\frac{yT}{2} + \overline{SSR}\right], \tag{18}$$

$$\overline{AS}_W(p_R, \theta) = (p_R - p_W)y + \overline{SOR}, \tag{19}$$

We observe that the term in the retailer's function (18):

$$\left[\alpha p_R(1 + \frac{\overline{y_R}}{y}) + \theta p_R\right]\frac{yT}{2}, \tag{20}$$

generalises the unit holding cost established in Harris (1913) ($\theta = 0$) to the case of deterioration with exponential decay ($\theta > 0$). The first term in (20) shows that the capital cost of inventories, traditionally αp_R, increases by a factor dependent on the fraction of waste produced. The second term θp_R indicates the cost of having procured products at price p_R which are then to be discarded.

The role of SR (16) in (19) shows that the supplier enjoys financial benefits from selling to the retailer in batches. For given y and T, these financial benefits increase with the marginal revenue $(p_R - p_W)$, the opportunity cost of capital $\tilde{\alpha}$, and the deterioration rate θ. We observe that SR generalises the concept of the supplier's reward identified in Beullens & Janssens (2011) ($\theta = 0$) to the case of deterioration with exponential decay ($\theta > 0$). SR provides an upperbound on the financial value to the supplier of delivering (deteriorating) products in batches to the retailer (i.e. before considering the impact on the supplier's own logistics costs.)

The waste fraction $\overline{y_R}/y$ is a function of T and θ, see (14). In many practical situations, the value of $\theta T/2$ in optimal solutions remains small. Reasonable estimates for optimal cycle times can thus be established by taking $\tilde{\alpha}$ constant.[10] In the scenario of no collaboration, the optimal cycle time maximises (18):

$$T^o(p_R, \theta) = \sqrt{\frac{2s_R}{y(\theta + \tilde{\alpha})p_R}}, \tag{21}$$

and in the scenario of collaboration, the optimal cycle time maximises the sum of (18) and (19):

$$T^I(\theta) = \sqrt{\frac{2(s_R + s_W)}{y(\theta + \tilde{\alpha})p_W}}. \tag{22}$$

Let $Z^x(\delta_\theta\theta, \delta_s s_R)$ be the value of Z under scenario x with deterioration rate $\delta_\theta\theta$ and retailer's set-up cost $\delta_s s_R$. (The δ parameters represent positive deviations, increasing the deterioration rates.)

Lemma 1. *For $x \in \{o, I\}$, $\delta_\theta > 1$ and $\delta_s > 1$:*

$$\frac{\overline{SSR}^x(\delta_\theta\theta, \delta_s s_R)}{\overline{SSR}^x(\theta, s_R)} = \frac{\overline{y_R}^x(\delta_\theta\theta, \delta_s s_R)}{\overline{y_R}^x(\theta, s_R)} = \gamma_s^x \frac{\delta_\theta}{\gamma_\theta} > 1, \tag{23}$$

where

$$\gamma_\theta = \sqrt{\frac{\tilde{\alpha} + \delta_\theta\theta}{\tilde{\alpha} + \theta}} > 1;\ \frac{\delta_\theta}{\gamma_\theta} > 1;\ \gamma_s^I = \sqrt{\frac{s_W + \delta_s s_R}{s_W + s_R}} > 1;\ \gamma_s^o = \sqrt{\delta_s} > \gamma_s^I > 1.$$

In both scenarios, the supplier's sales reward and waste production rate increase with deterioration rate and retailer's set-up cost. For $\delta_\theta = 2$, $\delta_s = 1$ at $\tilde{\alpha} = 0.1$, the lemma predicts an increase by 45%. (In Section 3.3, the move from $\mu 3$ to $\mu 1$, for example, shows an actual increase of about 48% and 49% in the cases of no collaboration and collaboration, respectively.)

[8] In more general models with fixed costs at annuity rate FC, this would be $SOR^x = AS_W^x - (p_R^x - p_W)y - FC$

[9] Thus: $e^{-\alpha T} = 1 - \alpha T + \alpha^2 T^2/2! - \ldots$, $e^{\theta T} = 1 + \theta T + \theta^2 T^2/2! + \ldots$, and $\alpha/(1 - e^{-\alpha T}) = 1/T + \alpha/2 + \alpha^2 T/12 + \ldots$

[10] To test the validity of adopting a constant for $\tilde{\alpha}$, we compared using (21) and (22) with $\tilde{\alpha} = \alpha = 0.1$ to using a grid search optimisation on (18) and (19) on numerical examples for which the fraction of waste remains below 20% and optimal cycle times below 60 days. Differences between these methods in optimal cycle times remain typically well below 1%.

The next lemma shows how the amount of waste changes when the firms move from no collaboration to collaboration, while at the same time also making adjustments to relevant parameters. Let $\overline{y_R}^x(X)$ indicate the waste production in scenario x, where X denotes the set of relevant parameters.

Lemma 2. *Moving from scenario $x = o$ to $x = I$, the waste production rate changes by the factor:*

$$\frac{\overline{y_R}^I(\theta^I, s_R^I, s_W^I, p_W^I)}{\overline{y_R}^o(\theta^o, p_R^o, s_R^o)} = \left[\frac{\theta^I}{\theta^o}\sqrt{\frac{\tilde{\alpha}+\theta^o}{\tilde{\alpha}+\theta^I}}\right]\sqrt{\frac{s_R^I+s_W^I}{s_R^o}\frac{p_R^o}{p_W^I}}. \tag{24}$$

If the parameters in the rhs of (24) remain unaltered when moving from no collaboration to collaboration (thus $\theta^I = \theta^o$, ...), then the waste will always increase since $p_R^o > p_W$ and $(s_R + s_W) > s_W$. This waste increase can then be significant. For $\mu 3$ data in Table 1, for example, the proposition predicts an increase of 189%. (The actual increase, in the unapproximated model, is 172%.)

If $\theta^I < \theta^o$, then the first factor in the square brackets of (24) is strictly smaller than 1. The lemma thus shows that limiting this waste increase would require at least one of the following strategies: (a) $\theta^I < \theta^o$, (b) $s_R^I + s_W^I < s_R^o + s_W^o$, (c) $p_W^I > p_W^o$. Approach (c) would increase the production cost and thus from a financial point only makes sense if this would be more than compensated for from improvements in other areas, for example, if it would be in support of (a).

We now focus on determining whether the supplier might profit from the deterioration process at the retailer. Let $\text{SR}^x(p_R, T^x, \delta_\theta\theta)$ be the SR value as given by (16) when using the value $T^o(p_R, \delta_\theta\theta)$ given by (21) or $T^I(\delta_\theta\theta)$ given by (22) at price p_R and deterioration rate $\delta_\theta\theta$ ($\delta_\theta \geq 1$).

Lemma 3. *For $x \in \{o, I\}$, $\delta_\theta > 1$:*

$$\frac{\overline{\text{SR}}^x(p_R, T^x, \delta_\theta\theta)}{\overline{\text{SR}}^x(p_R, T^x, \theta)} = \gamma_\theta > 1, \tag{25}$$

where γ_θ is as in (23).

Lemma 1 shows that the supplier's sales increase for $\delta_\theta > 1$ with a factor $\delta_\theta/\gamma_\theta$. Lemma 3 shows that the supplier's reward also increases but with a smaller factor γ_θ (since $\delta_\theta > \gamma_\theta^2$).

Let L_W denote the lot-size dependent terms in $\overline{\text{AS}}_W(p_r, \theta)$:

$$\begin{aligned} L_W(p_R, T, \theta) &= -\frac{s_W}{T} + (p_R - p_W)(\tilde{\alpha}+\theta)\frac{yT}{2} \\ &= -\frac{s_W}{T} + \overline{\text{SR}}(p_R, T, \theta) = \overline{\text{SOR}}(p_R, T, \theta) + \alpha\frac{s_W}{2}. \end{aligned} \tag{26}$$

and $L_W^x(p_R, T^o, \theta)$ be the value of L_W in $\overline{\text{AS}}_W^x(p_R, \theta)$, when using the value $T^o(p_R, \theta)$ or $T^I(\theta)$.

Proposition 1. *For $x \in \{o, I\}$, $\delta_\theta > 1$:*

$$\begin{aligned} \overline{\text{SOR}}_W^x(p_R^o, \delta_\theta\theta) - \overline{\text{SOR}}_W^x(p_R^o, \theta) &= \overline{\text{AS}}_W^x(p_R^o, \delta_\theta\theta) - \overline{\text{AS}}_W^x(p_R^o, \theta) \\ &= (\gamma_\theta - 1)L_W^x(p_R^o, T^x, \theta), \end{aligned} \tag{27}$$

which will lead to increased annuity stream profits for the supplier in the scenario $x = o$ if and only if:

$$\frac{s_W}{s_R} < \frac{p_R^o - p_W}{p_R^o}, \tag{28}$$

and in the scenario $x = I$ when $p_R^I = p_R^o$ if and only if:

$$\frac{s_W}{s_R + s_W} < \frac{p_R^I - p_W}{p_W}. \tag{29}$$

While Lemma 3 shows that any $\delta_\theta > 1$ will increase the supplier's reward, due to $T^x(\delta_\theta\theta)/T^x(\theta) = 1/\gamma_\theta < 1$ (see proof of Lemma 1), the supplier's set-up costs per unit of time will increase. Whether or not the overall profits increase is determined by this trade-off. Conditions (28) and (29) show that the answer is not affected by θ and δ_θ, but by financial parameters (set-up costs and unit prices). Increased profitability from increased deterioration for the supplier is more likely in the case of a jointly optimised supply chain, as the right-hand side (rhs) in (29) is naturally larger (when $p_R^I = p_R^o$) and its left-hand side smaller when compared to (28).

The rhs in (27) is only dependent on δ_θ through γ_θ. Since $\gamma_\theta > 1$ it follows that if (28) is satisfied, then $L_W^o(p_R^o, T^o, \theta) > 0$ else $L_W^o(p_R^o, T^o, \theta) \leq 0$. The same dependencies hold between (27) and (29). Formulated differently, profits increase from increasing deterioration rate θ if and only if:

$$\overline{\text{SOR}}_W^x(p_R^o, \theta) > -\alpha\frac{s_W}{2}, \tag{30}$$

which follows from (17) and the definition of L_W.

Let us consider the situation that $\delta_\theta\theta = \theta + \delta$, i.e. we consider the case of an incremental increase of the deterioration rate by a constant (small) amount δ.

Corollary 1. *The difference in the supplier's annuity stream profits, for a given δ, reduces with an increase in θ. For $x \in \{o, I\}$ ($p_R^I = p_R^o$):*

$$\overline{\text{AS}}_W^x(p_R^o, \theta+\delta) - \overline{\text{AS}}_W^x(p_R^o, \theta) = C^x\sqrt{y}\left[\sqrt{\tilde{\alpha}+\theta+\delta} - \sqrt{\tilde{\alpha}+\theta}\right], \tag{31}$$

where C^x is a (positive or negative) factor dependent on set-up costs S_R and S_W and unit prices p_R and p_W.

$C^o > 0$ or $C^I > 0$ corresponds to (28) or (29) being satisfied, respectively. If $C^x > 0$, the supplier always benefits from an incremental increase δ, but the benefit will become smaller and smaller the higher the deterioration rate. If $C^x < 0$, the supplier always benefits from an incremental reduction in the deterioration rate, but the benefit is larger when the product has already got a lower deterioration rate.

The application of Proposition 1 to the instances in Table 1 (for $d_R = f = 0$) gives results listed in Table 5. The change in AS profit values as predicted by Proposition 1 are listed in columns 2 and 5. Columns 4 and 7 report, respectively, the 'actual' AS profit increase. i.e. when using the unapproximated model of Section 3.3. The proposition gives a fair indication of the direction and magnitude of the change in supplier profits when deterioration increases.

Proposition 1 gives us all the information needed in the case of no collaboration, but in the case of collaboration, it only gives a result for the case that the firms do not readjust the price p_R but decide to keep it at p_R^o. The next proposition will thus examine the impact on the supplier's profits when the firms agree to split the gains according to the mechanism given by (8) and (9).

Let p_R^I be the optimal value for p_R derived with this mechanism for deterioration rate θ as to split the gains according to a given ρ, and $p_R^{I\delta}$ be the optimal value for p_R when the deterioration rate would be $\theta + \delta$. Then (8) and (9) show that in general we should expect that $p_R^I \neq p_R^{I\delta}$, and that neither values are equal to the original price p_R^o adopted in the case of no collaboration. (This is also confirmed by the examples in Section 3.3, see Table 3.)

Let L_R denote the lot-size dependent terms in $\overline{\text{AS}}_R(p_R, \theta)$:

$$L_R(p_R, T, \theta) = -\frac{s_R}{T} - p_R(\tilde{\alpha}+\theta)\frac{yT}{2}, \tag{32}$$

and $L_R^x(p_R, T^x, \theta)$ denote the value of L_R in $\overline{\text{AS}}_R^x(p_R, \theta)$, when using the value $T^o(p_R, \theta)$ or $T^I(\theta)$.

Let L_{SC} denote the lot-size dependent terms in the supply chain profit function (the sum of (18) and (19)):

$$L_{SC}(., T, \theta) = -\frac{s_R + s_W}{T} - p_W(\tilde{\alpha}+\theta)\frac{yT}{2}, \tag{33}$$

Table 5
Application of Proposition 1.

Instance shift	(27)($x=o$)	(28)	$\triangle AS_W^o\,(x=o)$	(27)($x=I$)	(29)	$\triangle AS_W^I\,(x=I)$
μ_3 to μ_1	+18	$0.5<0.58$	+13	+234	$0.33<1$	+228
μ_4 to μ_2	-82	$1 \not< 0.58$	-44	+161	$0.5<1$	+151

Table 6
Application of Proposition 2 ($\rho = 0.5$).

Instance shift	rhs of (34)	rhs of (35)	Actual $\triangle AS_W^I$
μ_3 to μ_1	+70	1.25	+50
μ_4 to μ_2	-24	0.3	-22

and $L^x_{SC}(p_R, T^x, \theta)$ be the value of L_{SC} when using the value $T^o(p_R, \theta)$ or $T^I(\theta)$.

Proposition 2. *If adopting the scenario of collaboration leads to a strict increase in supply chain profits, then when the firms adopt the mechanism of splitting the gains as given by (8) and (9):*

$$\begin{aligned}&\overline{AS}^I_W(p_R^{I\delta}, \theta+\delta) - \overline{AS}^I_W(p_R^I, \theta)\\ &= (\gamma_\theta - 1)\left[\rho L^o_{SC}(p^o_R, T^o, \theta) + (1-\rho)L^I_{SC}(., T^I, \theta) - L^o_R(p^o_R, T^o, \theta)\right].\end{aligned} \tag{34}$$

which will lead to increased annuity stream profits for the supplier for any feasible choice of $0 \le \rho < 1$ if and only if:

$$\rho < \frac{L^o_R(p^o_R, T^o, \theta) - L^I_{SC}(., T^I, \theta)}{L^o_{SC}(p^o_R, T^o, \theta) - L^I_{SC}(., T^I, \theta)}. \tag{35}$$

The rhs of (34) shows that the increase (or decrease) in the supplier's profits from an increased deterioration rate under the profit splitting mechanism can be determined from information derived from the optimisation under both scenarios based on the original price p^o_R and deterioration rate θ, and without the need to establish the prices $p_R^{I\delta}$ and p_R^I. If the rhs of (35) is strictly negative, then the supplier will never benefit from increased deterioration rates; if it is equal to or larger than 1, then the supplier will always benefit no matter the choice of ρ; if it is at a value $0<a<1$, then benefit is extracted by the supplier if receiving at least the fraction $(1-a)$ of the total supply chain gains achieved from collaboration.

Table 6 reports the application of Proposition 2 in the setting as used to illustrate Proposition 1 in Table 5. It shows that, when starting from μ_3 data, an increase in deterioration rate would always improve the profits of the supplier for any $0 \le \rho < 1$. In particular, for $\delta = 1$ and $\rho = 0.5$, a profit increase of 70 (column 2) is predicted, which is close to the 'actual' value when using the unapproximated model (column 4). When starting from μ_4, improvements for the supplier would only arise when the retailer would be given less than $\rho = 0.3$ of the supply chain gains. At a value $\rho = 0.5$, the supplier would not benefit, as also indicated by the negative values in columns 2 and 4.

We end with a general interpretation of how these results can be used (see also the discussion in Section 9). Retailers selling a deteriorating product and concerned about tackling waste produced may want to find out how this is affected by the relationship with its supplier. When we ignore the impact of out-of-pocket holding costs and disposal cost (or assume they are very small), the above lemma's and propositions present easy to calculate tests that also show the main factors of influence. Since optimal cycle times will reduce when including out-of-pocket holding costs and disposal costs, these tests overestimate somewhat the financial value of deterioration for the supplier.[11] In other words, a retailer can use these tests to rule out if there would be a potential misalignment of incentives for the supplier to help tackle waste production. In particular: Lemma 3 gives an upper bound on the increase in SR; if conditions (28) or (29) of Proposition 1 are *not* satisfied, the retailer would know with a good level of confidence that the supplier would be motivated to help reduce the deterioration rate at the retailer as this would be in the supplier's own financial interest; and finally, if the retailer retains *more* of the profit gains that given by the rhs of (35), the supplier would be motivated to help tackle waste production at the retailer when adopting a collaborative focus.[12]

[11] This is easy to see as lower T values reduce SR and increase the supplier's own annual set-up costs. Also, with increasing θ, the decrease in T will be more pronounced when accounting for disposal costs.

Table 7
Examples of waste targets and solutions of Proposition 3.

$\hat{Y}$	$\widehat{\theta T_R}$	$\hat{Y}$	$\widehat{\theta T_R}$
0.01	0.0198	0.06	0.1154
0.02	0.0394	0.07	0.1338
0.03	0.0588	0.08	0.1518
0.04	0.0778	0.09	0.1698
0.05	0.0966	0.10	0.1876

7. Waste targets

In this section we consider the situation in which the retailer is required to meet a given *waste target*, expressed as a fraction of total consumption sales.[13] Such targets may arise, for example, through government or industry pressure. Recall that the waste per unit of time is given by (10), and thus the fraction of waste produced is given by:

$$\frac{y_R}{y} = \frac{e^{\theta T_R} - 1}{\theta T_R} - 1. \tag{36}$$

As (36) is monotone increasing in θT_R, the retailer must adopt a solution producing a value for θT_R that remains smaller than a maximum value $\widehat{\theta T_R}$ such that:

$$\frac{e^{\widehat{\theta T_R}} - 1}{\widehat{\theta T_R}} - 1 = \hat{Y}, \tag{37}$$

where $\hat{Y}$ is the imposed waste target.

Proposition 3.

$$\widehat{\theta T_R} = -\mathbf{W}_{-1}\left[-\frac{e^{-\frac{1}{\hat{Y}+1}}}{\hat{Y}+1}\right] - \frac{1}{\hat{Y}+1}, \tag{38}$$

where $\mathbf{W}_{-1}(.)$ is the solution on the secondary (or lower) branch of the Lambert W function.

While the Lambert W function offers an analytical solution, calculation of the function requires a computational device. Table 7 reports solutions for waste targets in the range of 1% to 10%. As observed from the table, as waste targets become more stringent, the

[12] This can also be seen in Tables 5 and 6, where the actual supplier benefits are lower than the predictions from the propositions.

[13] Note that waste targets could alternatively also be specified as a percentage of the total input, i.e. based on $y_R/(y+y_R)$; these numbers would 'look' better but are otherwise giving retailers an identical challenge.

Table 8
Waste targets: profit implications.

$\hat{Y}$	Instance	$\hat{T}_R$ (days)	$\widehat{AS}_{SC}(.)$	$\triangle AS^o_{SC}$ (%)	$\triangle AS^I_{SC}$ (%)
0.05	μ_1	17.52	12,159.1	-5.25	-7.28
0.05	μ_2	17.52	12,994.4	-0.60	-3.55
0.05	μ_3	35.04	13,371.6	-0.28	-1.76
0.05	μ_4	35.04	13,790.3	+1.49	-0.60
0.05	μ_5	17.52	12.109.5	-4.76	-6.74
0.05	μ_6	35.04	13,470.6	+1.65	-0.45
0.04	μ_1	13.87	11,572.8	-9,81	-11.75
0.04	μ_2	13.87	12,627.4	-3.41	-6.72
0.04	μ_3	28.11	13,138.9	-2.01	-3.47
0.04	μ_4	28.11	13,660.3	+0.54	-1.53
0.04	μ_5	13.87	11,533.9	-9.29	-11.17
0.04	μ_6	28.11	13,620.8	+0.76	-1.31

retailer may need to further reduce the product deterioration rate and the cycle time. Reducing the deterioration rate often requires *technological* innovation to the products itself or the facilities e.g. better temperature control in the warehouse. Reducing cycle times is a *logistical* approach and could be realized simply by restricting cycle times below what would be financially optimal in absence of waste targets. This may, of course, not be a financially viable approach. Thus, the retailer should aim to adjust its supply structure such that the economically optimal solution comes closer to, or falls under, the upper bound $\widehat{\theta T_R}$.

To see the impact of waste targets on profits when adopting logistical changes, and on the *potential of collaboration*, we examine the instances introduced in Section 3.3, and using the unapproximated model. Table 8 shows this impact for waste target values of 5% and 4%, respectively. Column 3 reports the maximum cycle time and column 4 the corresponding supply chain profits if this cycle time is used. The last two columns indicate the percentage difference of total supply chain profits when adopting this solution instead of the optimal solutions as reported in Tables 2 and 3.

For μ_i, $i = 1, 2, 3, 5$, Table 8 shows that the firms lose profits when compared to scenario-optimised logistics in absence of targets. The reason is that the waste target is set below the waste produced by the retailer in the scenario of no collaboration (and thus, given Lemma 2, also below the waste produced in the scenario of collaboration.) (Compare with results in Tables 2 and 3.) The reduction in the waste target from 5% to 4% would have great impact on the reduction of the firms' profits. Since the supply chain profit achievable under these waste targets is the same in both scenarios, the supply chain cannot leverage its profits from adopting a collaborative focus. The benefits of collaboration would thus have to be sought in structural changes rather than a supply-chain optimal lot-sizing process. (Recall the strategies discussed in relation to Lemma 2.)

For the case of no collaboration and μ_4 and μ_6, the total supply chain profit would increase when the retailer would adopt the cycle time $\hat{T}$. However, the retailer would not voluntary adopt this in scenario $x = o$ as it can make more profits at smaller cycle times, producing also less waste than the required target (see Table 2). As total supply chain profits in scenario $x = I$ under the waste targets are still higher than those achievable in scenario $x = 0$ in absence of the waste target, there would in these cases still be a financial benefit of adopting a collaborative focus in absence of further structural changes. To summarize, whether or not the retailer can benefit from logistics collaboration thus depends on the waste target value, and in particular whether the waste target is above or below the waste produced by the retailer in the scenario of no collaboration in the absence of targets.

Setting waste targets is one approach that governments or industries could adopt. An alternative approach would consist of not setting waste targets explicitly, but using a mechanism of *taxes on waste disposal*, in the hope that firms would then voluntarily reduce waste production as part of their quest to maximize profits. This corresponds in the unapproximated model to setting higher values for d_R. It is worthwhile to note that such taxes do not seem to work very well. For μ_1, for example, the retailer would voluntary decide to adopt a 5% waste fraction only for $d_R = 2.05$, while in the case of collaboration, this would require $d_R = 4.05$. Such high disposal cost values are unrealistic, and would also have a much larger impact on total profits in the supply chain, which would reduce by -12.72% and -22.22%, respectively.

Finally, we examine how the structure of the retailer's supply chain determines the ease by which waste targets can be met. Using the profit functions of Section 6, the retailer would not voluntarily meet a waste target $\hat{Y}$ in the case of no collaboration if (using (21)):

$$\theta \sqrt{\frac{2s_R}{y(\theta + \tilde{\alpha})p_R}} > \widehat{\theta T_R}, \tag{39}$$

which shows that, for any given price p_R, retailers which have a lower θ, lower s_R, or larger demand y will more easily, with less impact on their profits, be able to meet a given target $\hat{T}$.

Likewise, in the case of collaboration, the supplier and retailer working together will not voluntarily meet waste targets if (using (22)):

$$\theta \sqrt{\frac{2(s_R + s_W)}{y(\theta + \tilde{\alpha})p_W}} > \widehat{\theta T_R}. \tag{40}$$

It can thus be derived that supply chains for given price p_W that have lower θ, lower $s_R + s_W$, or larger demand y will more easily, with less impact on their profits, be able to meet a given target $\hat{T}$.

The above equations also shows that, ceteris paribus, (larger) *firms that sell more of the product will derive a competitive advantage through the introduction of waste targets*. For μ_1, for example, but with double the demand at $y = 20,000$, the profit losses of -5.25% and -7.28% reported in Table 8 would reduce to -0.4% and -2.0%, respectively. It also means that next to *technological* and *logistical* changes, a given firm may be able to meet waste targets better through *marketing* approaches which stimulate sales of deteriorating products, through e.g. reducing the number of alternative deteriorating products offered to meet a certain demand.

8. Products reaching an expiration date

We have so far concentrated on waste that arises from exponential decay. This corresponds to the assumption that each product placed in storage has at any time a small probability θ of almost instantaneous failure, which can be viewed as the cumulative effect of a myriad of probable causes, including breakage, wrong appearance, etc. We have argued in Section 2 for its importance both in terms of its financial and social/environmental impact due to the low redistribution potential.

Another source of waste is due to demand being "erratic" and thus difficult to predict, causing products to reach their expiration date. As seen in Section 2, this is a main cause for bread and pastry, also a financially important category for many retailers. While a detailed exposition cannot be included due to space limitations, Table 9 presents key results based on the well-known newsvendor model, while Table 10 reviews possible strategies to reduce this waste, and offers insights and directions for further research.

Tables 11 and 12 summarise similar results for the main model developed in previous sections, for ease of comparison. From these summaries we can draw the insight that waste is planned for at the retail level, whether it arises from deterioration or erratic demand. Both phenomena lead to oversupply in order to meet certain demand, and both also tend to lead to more waste in the integrated scenarios.

Table 9
Supplier to newsvendor model (Case: Fresh bakery).

	Main model components	Comments related to waste generated
	Unsold product at end of sales period is considered waste. Demand per period is a random variable $X \sim F(x) = Prob(X \leq x), x \geq 0$. Salvage value is $v = -d_R$ per unit of waste; $X_R(Q_R)$ is waste quantity when retailer orders Q_R product.	Fresh bread unsold by the end of the day is removed in Lebersorger & Schneider (2014). The loss of bread due to decay during the day is insignificant. High redistribution potential. For a profitable business $p > p_R$. In real situations $v < p_R$ or thus $-d_R < p_R$. We assume $F(0) = 0$ and $F(x) > 0, \forall x > 0$ and differentiable.
Scenario *o* (fresh bread delivered each morning)	Optimal Q_R^o is the solution to the classic newsvendor problem: $F(Q_R^o) \equiv Prob(x \leq Q_R^o) = \frac{p-p_R}{p+d_R}$ Expected amount of waste: $E(X_R(Q_R)) = \int_0^{Q_R}(Q_R - x)dF(x) \equiv E(Q_R - X)^+$	Since $0 < \frac{p-p_R}{p+d_R} < 1$, it follows $Prob(X \leq Q_R^0) > 0$, and also $E(Q_R^0 - X)^+ > 0$, or the optimal economic decision for the retailer always involves planning for waste. Since $F(x) > 0, \forall x > 0$ it holds that $E(Q_R - X)^+ > 0, \forall Q_R > 0$, thus any choice of $Q_R > 0$ will generate waste.
Scenario *I* (or bakery in-house)	Optimality criterion: $Prob(x \leq Q_R^I) = \frac{p-p_W}{p+d_R}$ Expected waste: $E(Q_R^I - X)^+$	As $p_W < p_R$, we get $Prob(X \leq Q_R^I) > Prob(X \leq Q_R^o) > 0$, and $E(Q_R^I - X)^+ > E(Q_R^o - X)^+ > 0$, or the integrated supply chain plans for more waste.
Scenario *o* with buy-back	Supplier refunds the retailer p_B per unit of unsold product, where $p_B \leq p_R$. The solution is as in Scenario *o* but where $d_R = -p_B$. Optimality criterion: $Prob(X \leq Q_B^o) = \frac{p-p_R}{p-p_B}$	For $0 < p_B < p_R$, $Prob(X \leq Q_B^o) > Prob(X \leq Q_R^o) > 0$, and $E(Q_B^o - X)^+ > E(Q_R^o - X)^+ > 0$, or more waste arises at the retailer under a buy-back scenario. When $p_B = p_R$, $Prob(X \leq Q_B^o) = 1$, or the retailer is incentivised to order up to largest possible demand, producing also the largest amount of waste. This corresponds to the case for unsold bread & pastry identified in Lebersorger & Schneider (2014) (p. 1917).

Table 10
Waste reduction strategies (Case: Fresh Bakery).

Strategies (for retailer)	Notes
Improve demand forecasts	May lead to smaller variability to be accounted for, and thus smaller waste fractions. May lead to finding better correlations to explanatory variables such as e.g. day of week, special (sport) events, etc. Leads to variable supply order sizes which requires flexible suppliers.
Redistribution	Donations to social services or food banks should not be penalised by legislation as severely as other waste, or not at all (this may depend on what happens with the food in those outlets).
Introduce additional orders during the sales period	Observed in e.g. the fresh bakery in LIDL stores in the UK and the Netherlands, where new batches are baked according to evolution of demand during the day, thereby improving profit and waste performance. May require in-house production.
Reduce the price of items close to their sell-by date	This is an alternative to donations, and may (not) generate additional profit. As a starting point, see Mitra (2018). However, in food we may have to account for the cannibalisation effect on the sales of regularly priced items as well.
Rely on the substitution effect	If one item sells out, some of the demand converts to buying a substitute product. Leads to more conservative order quantities and less waste, and higher profits. See Fang, Nguyen, & Currie (2021). The effectiveness of this complex strategy depends on identifying product baskets of substitute products and the correlation between their demand.

Table 11
Supplier to EOQ-type vendor (Case: Fresh fruit & vegetables).

	Main model components	Comments related to waste generated
	A product has a failure rate θ and is sold at a continuous demand rate y. Disposal of removed items costs d_R. The rate of waste generated from deterioration is $y_R(T_R)$ when T_R is the replenishment cycle time.	The majority of waste in this category in Lebersorger & Schneider (2014) arises due to flaws (change in colour, dents,...). Loss also occurs from handling by consumers and personnel. Few items have an explicit sell-by date. Low redistribution potential. For a profitable business $p > p_R$. In most situations $d_R > 0$.
Scenario *o*	The optimal supply cycle time: $T_R^o = \sqrt{\frac{2s_R}{y(\theta+\tilde{\alpha})p_R}}$ Waste rate for any cycle time T_R: $y_R = y(\frac{e^{\theta T_R}-1}{\theta T_R} - 1)$	Since: $y_R = y(\frac{e^{\theta T_R}-1}{\theta T_R} - 1) = y(\frac{1+\theta T_R+\frac{(\theta T_R)^2}{2!}+\ldots-1}{\theta T_R} - 1)$ $= y(\frac{\theta T_R}{2!} + \frac{(\theta T_R)^2}{3!} + \ldots) > 0$, any choice of $T_R > 0$ will generate waste, including choosing the optimal economic decision for the retailer, T_R^o.
Scenario *I*	Optimality criterion: $T_R^I = \sqrt{\frac{2(s_R+s_W)}{y(\theta+\tilde{\alpha})p_W}}$ Waste rate for any cycle time T_R: $y_R = y(\frac{e^{\theta T_R}-1}{\theta T_R} - 1)$	Since $p_w < p_R$ and $s_w > 0$ we get $T_R^I > T_R^o$, and thus $y_R(T_R^I) > y_R(T_R^o)$, or the integrated supply chain plans for more waste than in the no-collaboration scenario.

9. Summary and further discussion

The topic of supply chain collaboration or coordination for deteriorating items has received increased attention in the literature, but very few studies have adopted the NPV framework. In the NPV approach, one can find the profit functions of each firm in the supply chain, and the joint profit function from the sum of these functions. As shown in e.g. Beullens & Janssens (2014), NPV modelling can accurately integrate the payment structures used between firms into the logistics process. Sharing the benefits of collaboration in any desired split ρ can be accurately implemented by means of an adjusted price p_R^I. The price adjustment needed is typically small for ρ values close to 0.5. The practical value of this method is that in this way the firms can retain the invoicing processes they have used before the collaboration.

We now summarise and further discuss our findings in the context of the four questions at the end of Section 2.

Table 12
Waste reduction strategies (Case: Fresh fruit & vegetables).

Strategies (for retailer)	Notes (see also Section 4)
Reduce the deterioration or rejection rate	Technological innovations at store and product level, supplier selection, vertical integration, packaging innovation, training personnel for improved material handling, marketing for taste and health not product uniformity or shape, advertise appropriate self-selection methods to customers, ...
Lower cost of order delivery and material handling	Supplier selection and logistics, local sourcing, delivery consolidation, reduced material handling in store, adopt product-mix cartons, ...
Increase demand rate	Advertising, product and brand range rationalisation, price control, improve store image and (local) market share, ...
Develop strong negotiation position relative to suppliers	Become an important customer to your supplier(s), create supply competition through regular contract reviews, vertical integration for better product quality at price, avoid suppliers with potential financial misalignment, see Section 6.
Develop alternative usage of deteriorated or rejected food	Animal feedstock, compost & fertilisers, bio-fuel production, ...
Develop demand flexibility	Develop a (local) customer base signing up to weekly baskets (delivered at home, or via click and collect) of produce of maximal (seasonal) freshness, variable in type and quantities as chosen by store but within a customer-selected bandwith of preferences.

One particular strategy may affect multiple key parameters at the same time. Vertical integration, for example, may improve θ but the reduced unit production cost may otherwise stimulate the retailer to adopt economic optima that produce more waste, see also Section 4 and Lemma 2.

9.1. Supplier's financial motivation

With respect to question (1), we noted that very few studies so far have investigated how item deterioration affects the profits of firms upstream the supply chain. If more items go to waste for meeting a given demand, then upstream firms sell more. Could it thus be that suppliers may receive financial benefits from increased deterioration rates? The answer to this question may help retailers assess a supplier's motivation to help tackle the waste arising at the retailer. Zhang et al. (2015a) investigate the issue in a more complex single supplier single retailer model and find that the supplier will be motivated to help reduce deterioration.

Our analysis of the simple supply chain model with lot-for-lot production for the supplier adds to our understanding of the impact of deterioration on upstream suppliers. The model illustrates that suppliers sometimes do lack the financial motivation to invest in reducing deterioration rates. The NPV method allows us to define the concept of the supplier's reward SR (Beullens & Janssens, 2011), which extracts the financial benefit that is present in the annuity stream profit function of the supplier from selling (deteriorating) products in batches to retailers. Our analysis shows that item deterioration does play an important role in the magnitude of SR, and that increased deterioration rates at downstream firms always increases SR values (Lemma 3). The supplier's reward is a useful measure in the study of deterioration in supply chains as it gives an *upper bound* on the financial value for suppliers of deterioration arising at firms downstream its supply chain. If SR values are very small, then retailers may already deduce that suppliers will typically be motivated to tackle waste in the downstream supply chain. The examples examined, however, show that SR values may well contribute as much as 5% to 15% to the profits of a supplier. A further investigation is then needed to examine the impact of deterioration on the supplier's own cost structure.

Proposition 1 considers this trade-off between supplier's rewards and the supplier's own cost structure. It shows that adopting the collaborative optimal lot-size is more likely to lead to a supplier becoming less willing to tackle waste production at the retailer. In both scenarios, retailers may expect suppliers to be more likely motivated to help address deterioration at the retailer if the supplier's own marginal profit $p_R - p_W$ is relatively small and/or if $s_R << s_W$. This implies that suppliers delivering to large retailers, who typically provide indeed only small profit margins to the suppliers, will be stimulated to help prevent waste at the retailer. Smaller retailers delivered by large suppliers, however, may not be so lucky, as they usually pay a higher profit margin to the supplier. Interpreting s_R as the cost of transport from supplier to retailer, willingness of a supplier to help tackle waste production at the retailer should thus be likely higher if being located closer to the retailer. If the supplier benefits from reducing the deterioration rate, it will be more motivated to help do so the less the product deteriorates, while if it benefits from increasing deteriorate rates, it will be more inclined to not help improve the deterioration rate the smaller it is (Corollary 1).

In the case of collaboration, however, the above conclusions from Proposition 1 only hold when the firms do not adjust the wholesale price. Proposition 2 refines this analysis for the case of collaboration where the firms use the proposed sharing mechanism p_R^I. It finds that suppliers tend to benefit more from increased deterioration rates if the fraction $(1-\rho)$ they receive is above a treshold (rhs of (35)). This thus leads to the insight that retailers can expect more willingness from suppliers in a collaborative scenario to help reducing deterioration at the retailer when the retailer is the dominant firm in the supply chain.[14]

9.2. Waste production at the economic optimum

We now turn to question (2) of Section 2 regarding the amount of waste at the retailer in the economic solution, and the impact of adopting a collaborative focus. Many studies in the literature have focused on the financial benefits of collaboration in the supply chain. We examined the impact of collaboration on the amount of waste produced in the supply chain.

The key result is most succinctly presented in Proposition 3. It says that in order to limit the waste production of a product subject to exponential decay to, say, less than 5%, the retailer should aim to keep θT_R less than 0.0966 (see Table 7). This result is independent of whether the retailer acts independently or collaborates with the supplier.

We hope that is clear from the paper that this does not have to simply mean forcing the supplier to give better products (lower θ) and deliver more just-in-time in smaller quantities (lower T_R), and thereby increasing the amount of transportation (and related emissions). The process of analysis this paper suggests is rather as follows. In the case of the independent scenario, the retailer can look at (39) and focus on these products where the 5% target is not achieved. The task would then be to make the waste target become economically viable. In simple terms, one thus aims to devise strategies that affect the values of θ, s_R, y or p_R, and such that the left-hand-side of (39) meets or falls under the target value of

[14] This may be counterintuitive at first sight, but is in line with the rhs of (29), since a higher ρ value implies a lower p_R^I.

0.0966. As parameter values are always difficult to determine accurately, this should still be seen as a qualitative exercise, i.e. in order to classify a strategy as warranting further investigation, it seems sufficient to be able to answer the question: Is the left-hand-side of (39) when implementing the strategy expected to go down when deploying a particular structural change in the supply of the product? See also Section 9.4.

In the collaborative scenario, the waste in the economic optimum, ceteris paribus, is higher. Lemma 2 shows that adopting the jointly optimized logistics process will often increase the waste by significant amounts. An intuitive explanation is that because of avoiding the double marginalisation effect in determining the supply chain optimal strategy, keeping stock at the retailer becomes much cheaper, and thus more waste is part of the economically optimal approach.

The analysis in Section 7 shows that the introduction of waste targets affects the value proposition of logistics collaboration in the supply chain. It demonstrates that retailers will no longer benefit from lot-size coordination with suppliers unless the waste targets would be above the waste fraction produced by the retailer in the case of no collaboration and in the absence of waste targets.

In a similar fashion as above discussed for the independent retailer, firms that do wish to collaborate and limit waste production at the retailer can use (40) to help assess how further structural changes can help address the waste production.

9.3. Legislative instruments

The third question (3) examined was related to the potential impact of waste reduction legislation. Insofar the model is a fair representation of wholesaler-retailer supply chains, the following general insights can be formulated with respect to how governments or industry-led initiatives can help tackling waste.

As the impact of (reasonable) disposal cost values remains small on changes in waste production, governments should not expect great improvements from imposing (reasonable levels) of waste disposal taxes. While the money raised from such taxes may benefit research into improved preservation technology for example, or subsidize retailers who invest in such technology, a potential drawback of such an approach is that it may distort market innovation by unintentionally promoting not the best technologies which would otherwise naturally arise.

Given the high sensitivity of waste production to s_R, great benefit may be found from stimulating retailers to find economic ways to increase stock turnover, reduce stock levels, or encourage making just-in-time deliveries more efficient. In order not to increase the environmental impact from transportation, this is best realised by also monitoring these emissions, and encouraging retailers to seek for creative solutions. In fact, by joining horizontal collaboration arrangements in the distribution chain (e.g. collaborative city logistics), or to seek more provisions of deteriorating products from fewer suppliers, or from more local suppliers, total transportation distance may actually reduce. Recall that there are also other means to reduce s_R, by e.g. reducing material handling in the store through e.g. pre-mixed batches. There are again potential difficulties with promoting any particular approach, as it may be that some other solutions may work better in some circumstances.

We have analysed the impact of setting waste targets as a percentage of retail sales. This seems to us a reasonable and pragmatic approach, since retailers can calculate actual values from accounting records. It also seems to be the approach currently envisaged to be adopted in the EU waste reduction framework.

As the discussion in Section 7 has highlighted, firms that have lower set-up costs or deterioration rates will naturally find it easier to meet these targets while remaining competitive. The benefit of setting such waste targets is that it would keep open the methods by which individual firms may seek to improve their performance. Each firm can thus decide how best to use the mix of *technological*, *logistical*, or *marketing* strategies. Waste targets can also be gradually tightened over time as actual waste figures in the industry improve.

As the ability to meet these targets depends on the deterioration rate (see Proposition 3), and this deterioration rate may to a large extend be dependent on the inherent characteristics of the product, different targets may be needed for different product categories.

We have also shown that meeting such waste targets is affected by sales volume (Section 7). Large retailers would find it, ceteris paribus, easier to meet these targets. If authorities wish to protect small businesses, they may consider introducing differentiated waste targets according to product category and firm type.[15]

9.4. Exploiting the supply structure

The final question (4) from Section 2 was asking what kind of structural change can bring the retailer in a better position such that, by pursuing its economic interests, the waste production can be managed?

There are many possible strategies retailers can investigate, as discussed in Section 4, and of which several have been suggested as potential solutions in the further analysis in later sections. Which strategy has potential will depend on the situation. We can imagine that large retailers may be much further in the path towards consolidated transport and thus might find more potential from boosting y for their most deteriorating products through e.g. marketing and product range strategies, while smaller retailers may still find scope in addressing s_R by e.g. developing ties with local suppliers, or adopting pre-mix batches, etc.

The benefit of simple models is that we do not have to know in which situation the retailer finds itself, but may still be able to derive general useful guidelines from the model. As discussed in Section 9.1, one can use the formula for SR, and Propositions 1 and 2, for example, to assess the supplier's financial preparedness to help reducing the waste. If this preparedness can be assumed, the retailer may more easily be able to convince the supplier to participate and invest. If this preparedness is not there, then perhaps adopting a different supply structure may help the retailer in getting more participation from the supplier. As discussed in Section 9.2, Proposition 3, Table 7, and Eqs. (39) and (40) can be used to help assess the likely benefit of different supply structure altering strategies. As long as the impact on the key parameters in these equations can be estimated, even only roughly and in direction, the results in this paper could be used as a guideline in evaluating the relative potential of different changes to the supply structure.

That said, there are also clear potential pitfalls from the model that are worth to point out. Although not incorporated into our model, suppliers may have other good reasons to reduce θ even when they would have a direct financial benefit from the deterioration process. Indeed, efforts to reduce item deterioration for retailers may be one of the many dimensions in which suppliers wish to compete in the market. The study of firm interactions in oligopoly situations could be one fruitful area for further research. Refinements to the model in the context of particular foods or food groups also offers scope for further research. Further research should also go into waste arising in models with uncertainty in various parts of the supply process as well as demand.

[15] Such a differentiation is already in place in the UK for the taxation system, where smaller firms can settle corporate tax and value-added tax in ways that help them improve their cash-flow management.

The examination of the environmental impact of various supply chain strategies is an important area for research. This is difficult because in many cases trade-offs and opportunity costs need to be considered between various stages in the supply chain. If increased transport prevents waste at the stage of retail or consumers, then less energy and emissions are needed in the farming and production stages, and more land may be usable for other productive means. A simple model as presented in this paper will surely fail to make reasonable suggestions.

Finally, the study of food waste prevention may be affected by new trends. The increased usage of online ordering and home delivery may help reduce the deterioration of the products at the retailers. Among consumers, there is also an increased awareness of the impact of food on human health, and the impact of food production on the environment. This is leading to local initiatives in various areas of the USA and Europe, including fresh produce boxes from organic fruit and vegetable farmers, and meat and dairy product packages from grass-fed livestock farmers (Harvey, 2016). Consumers with patience and a spirit for adventure (what will be in the package?) may thus not only improve their own health, but also help to eliminate much of the waste that would otherwise arise from products on shelves in stores being tried by many hands.

Acknowledgment

The authors thank the anonymous referees for their comments which have helped to improve this paper, in particular with respect to enhancing the content regarding supply structure improvements.

Appendix A. AS profit functions (4) and (5)

Using (2) and (3), profit functions (4) and (5) can be further worked out to, respectively:

$$AS_R = py - \frac{\alpha s_r}{1-e^{-\alpha T_R}} - p_r \frac{\alpha y (e^{\theta_r T_R}-1)}{\theta_r(1-e^{-\alpha T_R})}$$

$$-\Big[(\frac{f}{\theta_r}+d_r)y\Big]\Big[\frac{\alpha(e^{\theta_r T_R}-e^{-\alpha T_R})}{(\alpha+\theta_r)(1-e^{-\alpha T_R})}-1\Big]. \quad (41)$$

$$AS_W = \Big[(p_r-p_W)\frac{y(e^{\theta_r T_R}-1)}{\theta_r} - s_W\Big]\frac{\alpha}{1-e^{-\alpha T_R}} \quad (42)$$

Appendix B. Proof of Lemma 1.

As the cycle time when $x=o$ is determined from (21), while if $x=I$ it is determined by (22), we calculate the ratio ($x \in \{o, I\}$):

$$\frac{T^x(\delta_\theta\theta, \delta_s s_R)}{T^x(\theta, s_R)} = \frac{\gamma_s^x}{\gamma_\theta},$$

where γ_s^x and γ_θ are as defined in Lemma 1, and if $\delta_\theta > 1$ and $\delta_s > 1$, then $\gamma_s^x > 1$ and $\gamma_\theta > 1$. Note that the cycle time thus increases if and only if $\gamma_s^x > \gamma_\theta$. In the special case of $\delta_s = 1$ (no change in set-up cost) and $\delta_\theta > 1$, the cycle time strictly decreases with factor $1/\gamma_\theta$.

Given (14), we substitute the above and find:

$$\frac{\overline{\text{SSR}}^x(\delta_\theta\theta, \delta_s s_R)}{\overline{\text{SSR}}^x(\theta, s_R)} = \frac{\overline{y_R}^x(\delta_\theta\theta, \delta_s s_R)}{\overline{y_R}^x(\theta, s_R)} = \gamma_s^x \frac{\delta_\theta}{\gamma_\theta},$$

which is (23). To show that its right-hand-side is larger than one, we are left to prove that $\delta_\theta/\gamma_\theta > 1$. This is indeed always the case since:

$$\frac{\tilde{\alpha}+\delta_\theta\theta}{\tilde{\alpha}+\theta} < \delta_\theta.$$

Appendix C. Proof of Lemma 2.

This follows easily from (14), and using (21) for $x=o$ and (22) for $x=I$, while adding superscripts 'o' and 'I' to all parameters but $\tilde{\alpha}$ and y.

Appendix D. Proof of Lemma 3.

The proof of Lemma 1 shows that $T^x(\delta_\theta\theta)/T^x(\theta) = 1/\gamma_\theta$ for both scenarios $x \in \{o, I\}$. With SR calculated from (16), we thus get:

$$\frac{\overline{\text{SR}}^x(p_R, T^x, \delta_\theta\theta)}{\overline{\text{SR}}^x(p_R, T^x, \theta)} = \frac{(\tilde{\alpha}+\delta_\theta\theta)}{(\tilde{\alpha}+\theta)\gamma_\theta} = \sqrt{\frac{\tilde{\alpha}+\delta_\theta\theta}{\tilde{\alpha}+\delta}} \equiv \gamma_\theta > 1.$$

Appendix E. Proof of Proposition 1.

Let SC^x, $x \in \{o, I\}$, represent the annual set-up cost s_W/T^x Then according to the proof of Lemma 1:

$$\frac{\text{SC}^x(\delta_\theta\theta)}{\text{SC}^x(\theta)} = \frac{T^x(\theta)}{T^x(\delta_\theta\theta)} = \gamma_\theta > 1.$$

Therefore, given Lemma 3, the introduction of $\delta_\theta > 0$ increases both SR and SC by the same factor γ_θ, irrespective of whether T is determined from (21) or (22). This thus, since $L_W = -SC + SR$ and given (18) and (19), produces the result (27).

As SR is a revenue and SC a cost, the annuity stream profit function of the supplier will increase if and only if $L_W > 0$ or thus SR > SC in the first place. This condition SR > SC can be made explicit:

$$(p_R - p_W)(\tilde{\alpha}+\theta)\frac{yT}{2} > \frac{s_W}{T}$$

If T is determined from (21), substitution and algebraic simplification shows this condition equivalent to:

$$\frac{s_W}{s_R} < \frac{p_R^o - p_W}{p_W},$$

while if T is determined from (22), equivalent to:

$$\frac{s_W}{s_R+s_W} < \frac{p_r^I - p_W}{p_W},$$

when $p_R^I = p_R^o$.

Appendix F. Proof of Corollary 1.

Given Proposition 1 and $\delta_\theta\theta = \theta + \delta$, the first-order difference in the annuity stream profits of the supplier is given by $(\gamma_\theta - 1)L_W^x(\theta)$, where $x \in \{o, I\}$ and:

$$\gamma_\theta = \sqrt{\frac{\tilde{\alpha}+\theta+\delta}{\tilde{\alpha}+\delta}}.$$

In the case of no collaboration, using (21), $(\gamma_\theta - 1)L_W^x(\theta)$ is equivalent to:

$$\Big(\sqrt{\frac{\tilde{\alpha}+\theta+\delta}{\tilde{\alpha}+\delta}}-1\Big)\Big[-\frac{s_W}{\sqrt{2s_R}}\sqrt{y(\tilde{\alpha}+\theta)p_R} + (p_R-p_W)(\tilde{\alpha}+\theta)\frac{y}{2}\sqrt{\frac{2s_R}{y(\tilde{\alpha}+\theta)p_R}}\Big],$$

from which (31) easily follows for:

$$C^o = \Big[-\frac{s_W}{\sqrt{2s_R}}\sqrt{p_R} + \frac{(p_R-p_W)}{2}\sqrt{\frac{2s_R}{p_R}}\Big],$$

which is strictly positive only when (28) is satisfied. A similar derivation applies for the case of collaboration using (22), resulting in (31) with $C^I > 0$ only when (29) is satisfied.

Let $g(\theta) = \Big[\sqrt{\tilde{\alpha}+\theta+\delta} - \sqrt{\tilde{\alpha}+\theta}\Big]$, then (31) shows that the first-order difference in profits only depend on θ through $g(\theta)$, and becomes smaller with increasing values of θ.

Appendix G. Proof of Proposition 2.

We first prove the following result: If the move from the scenario of no collaboration to the scenario of collaboration strictly improves the supply chain total profits, then:

$$L^o_{SC}(p^o_R, T^o, \theta) - L^I_{SC}(., T^I, \theta)$$

$$= (L^o_R(p^o_R, T^o, \theta) + L^o_W(p^o_R, T^I, \theta)) - (L^I_R(p^o_R, T^I, \theta) + L^I_W(p^o_R, T^I, \theta)) < 0. \quad (43)$$

This follows from substitution using (26), (32) and (33) in the total supply chain profit function, which is given by the sum of (18) and (19). The proof then easily follows, given that $L^o_{SC} < 0$ and $L^I_{SC} < 0$.

We now proceed with the main part of the proof. Recall (9):

$$\mathrm{AS}^I_W(p^I_R) = \mathrm{AS}^o_W(p^o_R) + (1-\rho)(\mathrm{AS}^I_{SC} - \mathrm{AS}^o_{SC}),$$

and, as in (7), $\mathrm{AS}^x_{SC}(p_R)$ $(x \in \{o, I\})$ takes the same value for any price p_R. Thus:

$$\mathrm{AS}^I_W(p^{I\delta}_R, \theta+\delta) - \mathrm{AS}^I_W(p^I_R, \theta) = \mathrm{AS}^o_W(p^o_R, \theta+\delta) - \mathrm{AS}^o_W(p^o_R, \delta)$$
$$+ (1-\rho)\Big(\mathrm{AS}^I_{SC}(\theta+\delta) - \mathrm{AS}^I_{SC}(\theta) - (\mathrm{AS}^o_{SC}(\theta+\delta) - \mathrm{AS}^o_{SC}(\theta))\Big). \quad (44)$$

From (26), (32) and (33), it follows that $(z \in \{W, R, \mathrm{SC}\})$:

$$\mathrm{AS}^o_z(p^o_R, \theta+\delta) - \mathrm{AS}^o_z(p^o_R, \delta) = L^o_z(p^o_R, \theta+\delta) - L^o_z(p^o_R, \theta)$$

$$= (\gamma_\theta - 1)L^o_z(p^o_R, \theta),$$

where the last step follows from the application of a similar process as used in the proofs of Lemma 3 and Proposition 1. Substitution into (44):

$$\mathrm{AS}^I_W(p^{I\delta}_R, \theta+\delta) - \mathrm{AS}^I_W(p^I_R, \theta)$$

$$= (\gamma_\theta - 1)\Big[(1-\rho)\Big(L^I_{SC}(p^o_R, \theta) - L^o_{SC}(p^o_R, \theta)\Big) + L^o_W(p^o_R, \theta)\Big]$$

$$= (\gamma_\theta - 1)\Big[\rho L^o_{SC}(p^o_R, \theta) + (1-\rho)L^I_{SC}(p^o_R, \theta) - L^o_R(p^o_R)\Big],$$

which gives (34). Since $\gamma_\theta - 1 > 0$, this difference is positive if and only if:

$$\Big[\rho L^o_{SC}(p^o_R, \theta) + (1-\rho)L^I_{SC}(p^o_R, \theta) - L^o_R(p^o_R)\Big] > 0,$$

which, given (43), produces the condition (35).

Appendix H. Proof of Proposition 3.

The Lambert W function is the solution to $xe^x = y$, given by $x = \mathbf{W}(y)$, see also Disney & Warburton (2012). Consider the problem:

$$\frac{e^x - 1}{x} - 1 = y,$$

then substitute $-t = x + \frac{1}{y+1}$:

$$e^{-t}e^{-\frac{1}{y+1}} = (y+1)(-t),$$

$$te^t = -\frac{e^{-\frac{1}{y+1}}}{(y+1)},$$

and thus:

$$t = \mathbf{W}\Big[-\frac{e^{-\frac{1}{y+1}}}{y+1}\Big],$$

The above problem corresponds to (37) for $x = \widehat{\theta T_R}$ and $y = \hat{Y}$. Substitution thus produces:

$$\widehat{\theta T_R} = -\mathbf{W}_{-1}\Big[-\frac{e^{-\frac{1}{\hat{Y}+1}}}{\hat{Y}+1}\Big] - \frac{1}{\hat{Y}+1}, \quad (45)$$

where the solution, given that the argument $\frac{-1}{e} < \frac{-e^{-\frac{1}{\hat{Y}+1}}}{\hat{Y}+1} < 0$, is to be located on the lower branch.

References

Bai, Q., Xu, X., Xu, J., & Wang, D. (2016). Coordinating a supply chain for deteriorating items with multi-factor-dependent demand over a finite planning horizon. *Applied Mathematical Modelling, 40*(21), 9342–9361.
Bakker, M., Riezebos, J., & Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research, 221*, 275–284.
Belavina, E. (2021). Grocery store density and food waste. *Manufacturing & Service Operations Management, 23*(1), 1–18.
Berk, E., Gürler, U., & Yıldırım, G. (2009). On pricing of perishable assets with menu costs. *International Journal of Production Economics, 121*(2), 678–699.
Beullens, P. (2014). Revisiting foundations in lot sizing - Connections between Harris, Crowther, Monahan, and Clark. *International Journal of Production Economics, 155*, 68–81.
Beullens, P., & Janssens, G. K. (2011). Holding costs under push or pull conditions - The impact of the anchor point. *European Journal of Operational Research, 215*, 115–125.
Beullens, P., & Janssens, G. K. (2014). Adapting inventory models for handling various payment structures using net present value equivalence analysis. *International Journal of Production Economics, 157*, 190–200.
Blackburn, J., & Scudder, G. (2009). Supply chain strategies for perishable products: the case of fresh produce. *Production and Operations Management, 18*(2), 129–137.
Buisman, M. E., Haijema, R., & Bloemhof-Ruwaard, J. M. (2019). Discounting and dynamic shelf life to reduce fresh food waste at retailers. *International Journal of Production Economics, 209*, 274–284.
Buzby, J. C., & Hyman, J. (2012). Total and per capita value of food loss in the United States. *Food Policy, 37*(5), 561–570.
Cai, X., Chen, J., Xiao, Y., & Xu, X. (2010). Optimization and coordination of fresh product supply chains with freshness-keeping effort. *Production and Operations Management, 19*(3), 261–278.
Cicatiello, C., Franco, S., Pancino, B., Blasi, E., & Falasconi, L. (2017). The dark side of retail food waste: Evidences from in-store data. *Resources, Conservation and Recycling, 125*, 273–281.
Cohen, M. A. (1977). Joint pricing and ordering policy for exponentially decaying inventory with known demand. *Naval Research Logistics Quarterly, 24*(2), 257–268.
Disney, S. M., & Warburton, R. D. H. (2012). On the lambert w function: Economic order quantity applications and pedagogical considerations. *International Journal of Production Economics, 140*(2), 756–764.
Dye, C.-Y. (2013). The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega, 41*(5), 872–880.
Dye, C.-Y., & Yang, C.-T. (2016). Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. *Omega, 62*, 52–67.
Eriksson, M., Strid, I., & Hansson, P. A. (2014). Waste of organic and conventional meat and dairy products-a case study from swedish retail. *Resources, Conservation and Recycling, 83*, 44–52.
Fang, F., Nguyen, T.-D., & Currie, C. S. M. (2021). Joint pricing and inventory decisions for substitutable and perishable products under demand uncertainty. *European Journal of Operational Research, 293*(2), 594–602.
FAO (2011). Global food losses and food waste – Extent, causes and prevention. http://www.fao.org/3/i2697e/i2697e.pdf.
Garrone, P., Melacini, M., & Perego, A. (2014). Opening the black box of food waste reduction. *Food Policy, 46*, 129–139.
Ghiami, Y., & Beullens, P. (2020). The continuous resupply policy for deteriorating items with stock-dependent observable demand in a two-warehouse and two-echelon supply chain. *Applied Mathematical Modelling, 82*, 271–292.
Ghiami, Y., Williams, T., & Wu, Y. (2013). A two-echelon inventory model for a deteriorating item with stock-dependent demand, partial backlogging and capacity constraints. *European Journal of Operational Research, 231*, 587–597.
Ghoreishi, M. (2018). *Joint optimization of pricing and operational decisions in generic inventory systems under stochastic conditions (PhD Dissertation)*. Denmark: Aarhus BSS, Aarhus University, Department of Economics and Business Economics.
Ghoreishi, M. (2019). Testing the robustness of deterministic models of optimal dynamic pricing and lot-sizing for deteriorating items under stochastic conditions. *Central European Journal of Operations Research, 27*(4), 1131–1152.
Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research, 134*, 1–16.
Harris, F. W. (1913). How many parts to make at once. *Factory, The Magazine of Management, 10*(2), 135–136,152.
Harvey, G. (2016). *Grass-fed nation: Getting back the food we deserve*. Icon Books.

Hsieh, T.-P., & Dye, C.-Y. (2017). Optimal dynamic pricing for deteriorating items with reference price effects when inventories stimulate demand. *European Journal of Operational Research, 262*(1), 136–150.
Hsu, P. H., Wee, H. M., & Teng, H. M. (2010). Preservation technology investment for deteriorating inventory. *International Journal of Production Economics, 124*(2), 388–394.
Iqbal, M. W., Kang, Y., & Jeon, H. W. (2020). Zero waste strategy for green supply chain management with minimization of energy consumption. *Journal of Cleaner Production, 245*, 118827.
Janssen, L., Claus, T., & Sauer, J. (2016). Literature review of deteriorating inventory models by key topics from 2012 to 2015. *International Journal of Production Economics, 182*, 86–112.
Jonas, C. P. (2019). Optimizing a two-warehouse system under shortage backordering, trade credit, and decreasing rental conditions. *International Journal of Production Economics, 209*, 147–155.
Lebersorger, S., & Schneider, F. (2014). Food loss rates at the food retail, influencing factors and reasons as a basis for waste prevention measures. *Waste Management, 34*(11), 1911–1919.
Li, R., Lan, H., & Mawhinney, J. R. (2010). A review on deteriorating inventory study. *Service Science and Management, 3*, 117–129.
Lin, F., Jia, T., Wu, F., & Yang, Z. (2019). Impacts of two-stage deterioration on an integrated inventory model under trade credit and variable capacity utilization. *European Journal of Operational Research, 272*(1), 219–234.
Mattsson, L., Williams, H., & Berghel, J. (2018). Waste of fresh fruit and vegetables at retailers in sweden–measuring and calculation of mass, economic cost and climate impact. *Resources, Conservation and Recycling, 130*, 118–126.
Mishra, U., Wu, J. Z., Tsao, Y. C., & Tseng, M. L. (2020). Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates. *Journal of Cleaner Production, 244*, 118807.
Mitra, S. (2018). Newsvendor problem with clearance pricing. *European Journal of Operational Research, 268*(1), 193–202.
Mohammadi, H., Ghazanfari, M., Pishvaee, M. S., & Teimoury, E. (2019). Fresh-product supply chain coordination and waste reduction using a revenue-and-preservation-technology-investment-sharing contract: A real-life case study. *Journal of Cleaner Production, 213*, 262–282.
Monahan, J. P. (1984). A quantity discount pricing model to increase vendor profits. *Management Science, 30*(6), 720–726.
Nahmias, S. (1982). Perishable inventory theory: A review. *Operations Research, 30*(4), 680–708.
Qin, Y., Wang, J., & Wei, C. (2014). Joint pricing and inventory control for fresh produce and foods with quality and physical quantity deteriorating simultaneously. *International Journal of Production Economics, 152*, 42–48.
Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. *Journal of the Operational Research Society, 42*(1), 27–37.
Rajan, A., Rakesh, & Steinberg, R. (1992). Dynamic pricing and ordering decisions by a monopolist. *Management Science, 38*(2), 240–262.
Saha, S., Chatterjee, D., & Sarkar, B. (2021). The ramification of dynamic investment on the promotion and preservation technology for inventory management through a modified flower pollination algorithm. *Journal of Retailing and Consumer Services, 58*, 102326.
Saxena, N., Sarkar, B., & Singh, S. R. (2020). Selection of remanufacturing/production cycles with an alternative market: A perspective on waste management. *Journal of Cleaner Production, 245*, 118935.
Shah, N. H., Soni, H. N., & Patel, K. A. (2013). Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. *Omega, 41*(2), 421–430.
Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2009). Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. *European Journal of Operational Research, 192*, 79–92.
Teller, C., Holweg, C., Reiner, G., & Kotzab, H. (2018). Retail store operations and food waste. *Journal of Cleaner Production, 185*, 981–997.
Wu, J., Ouyang, L.-Y., Cárdenas-Barrón, L. E., & Goyal, S. K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. *European Journal of Operational Research, 237*(3), 898–908.
Yang, P. C., & Wee, H. M. (2002). A single-vendor and multiple-buyers production-inventory policy for a deteriorating item. *European Journal of Operational Research, 143*, 570–581.
Yang, Y., Chi, H., Zhou, W., Fan, T., & Piramuthu, S. (2020). Deterioration control decision support for perishable inventory management. *Decision Support Systems*, 113308.
Zhang, J., Liu, G., Zhang, Q., & Bai, Z. (2015a). Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. *Omega, 56*, 37–49.
Zhang, J., Wang, Y., Lu, L., & Tang, W. (2015b). Optimal dynamic pricing and replenishment cycle for non-instantaneous deterioration items with inventory-level-dependent demand. *International Journal of Production Economics, 170*, 136–145.