Education Transmission and Network Formation*

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Abstract

We propose a model of intergenerational transmission of education wherein children belong to either highly educated or low-educated families. Children choose the intensity of their social activities while parents decide how much educational effort to exert. Using data on adolescents in the United States, we structurally estimate this model and find that, on average, children's homophily acts as a complement to the educational effort of highly educated parents but as a substitute for the educational effort of low-educated parents. We also perform some counterfactual policy simulations. We find that policies that subsidize kids' socialization efforts can backfire for low-educated students because they tend to increase their interactions with other low-educated students (i.e., homophily), which reduces the education effort of their parents and, thus, their chance of becoming educated. On the contrary, policies that increase heterophily by favoring friendship links between kids from different education backgrounds can be effective in reducing the education gap between them.

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JEL classification: D85, I21, Z13.

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Replication codes are available here:
https://github.com/vincentboucherecn/CulturalTransmission

1 Introduction

School quality, family background and parents' investments, and peers have all been shown to have a significant and positive impact on children's educational attainment (see e.g., **?**, **?**, **?**, and **?**). However, while research has studied these components individually, it remains unclear how they interact. To the extent that parent's decisions are related to a child's peers (and vice versa), understanding this link is important for better implementing appropriate education policy. In this paper, we investigate this link by studying how the joint decisions of parent's investment in their children's education and children's choice of friends have an impact on the intergenerational transmission of education. We also evaluate different policies aimed at improving the education outcomes of more disadvantaged children.

We propose a simple model with two types of parents, high educated and low-educated, and, among each family type, we examine an education transmission mechanism wherein children decide how much effort to put into making friends. This determines the probability of making friends with other students and thus their degree of *homophily* with respect to their own family type. Parents choose how much effort to invest in their child's education by trading off the cost of such an effort against the benefit of having an educated child. What is key and new in this transmission process is that parents' investment behavior is a function of the (expected) degree of homophily of their children (i.e., the degree to which they become friends with children of their own types). We show that the education outcomes for both types of children depend on the education and investment of their parents, their peers' socialization efforts (i.e., how much effort their friends are making to make friends), as well as the interaction between parental investment and the quality of their peers.

We then structurally estimate this model using the AddHealth data, which provide information on the impact of the social environment (i.e., friends, family, neighborhood, and school) on adolescents' education outcomes in the United States. We find that both types of parents prefer their children to interact with other children from highly educated families. We also find that, on average, highly educated parents exhibit cultural complementarity (i.e., the more their children are making friends with children from highly-educated families (similar to their own), the more educational effort the parents invest in their child's education), whereas low-educated parents exhibit cultural substitutability (i.e., the more their children are making friends with children from highly-educated families (unlike their own), the more the parents invest in their child's education).

We then use this model to examine the likely effects of a variety of policies. First, we consider a policy that makes it easier for students to make friends, for example through subsidizing school clubs and sports' teams. This is a *non-targeted socialization policy.* We show that the implementation of such a policy reduces the educational attainment of children from low-educated families, while it increases the educational attainment of children from highly educated families. This is because the policy increases socialization efforts and homophily for both types of children. This leads educated parents to increase their education investments (cultural complementarity) but uneducated parents to decrease their education investments (cultural substitutability).

We then consider a *targeted socialization policy* that consists of subsidizing the private returns of socialization for low-educated children only; an example of this would be to subsidize extracurricular activities in which each student is involved. We show that this policy yields even worse education outcomes for children from low-educated families. They socialize even more with other low-educated kids and, therefore, their parents reduce their education effort.

Finally, we examine a policy aimed at directly increasing social interactions and heterophily between kids from different backgrounds. An example would be a tutoring program. This is a *targeted-bias reduction* policy that can be achieved by promoting targeted interactions. We show that this policy mostly favors children from low-educated families at the expense of children from high-educated families, reducing the education gap between them.

In summary, with our model, we show that policies that faciliate socialization can backfire for students from low-education families because they increase homophily and reduce the education investment of their parents, even if parents of both types value higher education over less education for their offspring. In contrast, policies that facilitate social interactions between kids from different backgrounds can be effective in reducing the education gap between children from low- and high-educated families.

1.1 Related literature

There is a significant body of theoretical and empirical literature on cultural transmission, initiated by the seminal papers of **??**. In this research stream, cultural transmission is conceptualized as the result of interactions between purposeful socialization decisions inside the family (*direct vertical socialization*) and other socialization processes, including social imitation and learning, which govern identity formation (*oblique and horizontal socialization*). These two types of socialization are cultural substitutes or complements if the level of parents' incentive to socialize their children depends negatively or positively on how widely dominant their values are in the population. Allowing for interesting socio-economic effects interacting with the socialization choices of parents, the basic cultural transmission model of Bisin and Verdier has been extended in different directions and been tested from different perspectives.[1]

In contributing to this literature, we endogenize the social network, which is formed by socialization efforts from children.[2] In other words, we consider a model in which children play an active role in the socialization process by choosing their socializing activities but the exact identity of a child's friends is not an object of choice.[3] In this respect, we provide one of the very few models that endogenizes oblique socialization using an explicit network-formation framework. We also establish a setting in which children are first and temporarily socialized in accordance with the parental trait (early socialization). Children choose effort and parents exert an educational effort that takes into account their offspring's choice. This allows us to have the parental socialization effort depend on the homophily choices of their children, which is crucial for understanding our policy experiments.

We also contribute to the education literature by examining the determinants and consequences of parental decisions on their offspring's outcomes using a model that includes the direct impact of children's social networks and homophily decisions on their parents' investments in education.[4] There is a recent body of literature, surveyed by **?** and **?**, that also models the interplay between parents' education effort and children's choices. This literature has focused on the various parenting styles and estimated different models of children's accumulation of cognitive and noncognitive skills in response to parental inputs. For example, **?** develop a model that allows for both altruism (parents care about their children's utility) and paternalism (parents care about their children's actions in ways that potentially conflict with the children's own preferences) and study the parent–child interactions by allowing for children taking actions on their own. This research has also modeled the parents' choice in terms of neighborhood, which influences with whom their children interact. In this literature, the closest paper to ours is that of **?**, who study the interaction between parents and children where children's skill accumulation depends on both parental inputs and peers, and parents can affect with whom children can interact. The model, empirical strategy, and focus are, however, very different from ours.

[1] For an overview, see **?**.

[2] For overviews on the social network literature, see **?**, **?**, **?**, **?**, and **?**.

[3] This approach, initiated by **?**, provides a simple way of solving the multiple equilibria issue that plagues network-formation models. See also **?** and **?**, who use a similar approach.

[4] **?** propose two channels of why parental education should be important for children's education: time allocation and higher productivity in child-enhancing activities. Here, the focus is different because parental education depends on the quality of children's peers and, thus, more educated parents may or may not have higher productivity in child-enhancing activities.

Our approach is different but complementary to the approach taken in this literature. We only have one parenting style (paternalism) and focus on the friendship (network) formation of children and how it influences the choice of education effort of their parents. We are aware that parents can partially anticipate children's choices by choosing the neighborhood where their children live or their school. While this is, of course, of extreme importance, in this paper, we take as given the neighborhood's structure and school choice and analyze how parents react to children's choices in terms of friendship formation.

Finally, our model and estimation are related to a broader literature estimating the effects of various inputs on the development of cognitive and non-cognitive skills. See, for example, **?**, **?**, **?**, **?**, **?**, and **?**. Our model and estimation also concern how these skills impact educational outcomes. Indeed, in our model, parents' investments are solely "educational" (or *cognitive*), whereas the child's investments are solely "social" (or *non-cognitive*); both cognitive and non-cognitive (or social) skills have an impact on the child's outcomes and can be complements or substitutes. This is in accordance with the findings of **?**, who document large spillover effects of a large-scale early childhood intervention from treated to untreated children who live near treated children. They show that the spillover effect on *non-cognitive* scores operates through the child's social network while parental investment in education is an important channel through which *cognitive* spillover effects operate. We complement this literature by providing a theoretical framework modeling these issues with an explicit network formation model and showing how different counterfactual policies may affect the cognitive and non-cognitive (social) skills of children.[5]

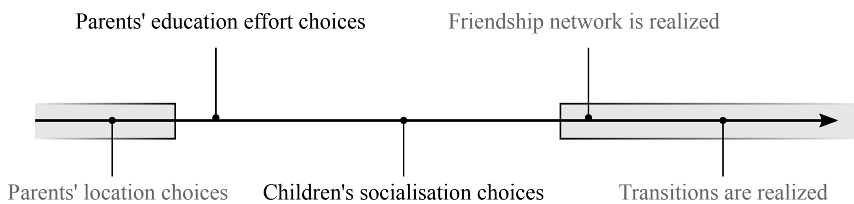
The rest of the paper unfolds as follows. In the next section, we develop and solve our theoretical model. In Section 3, we describe our dataset, explain our empirical strategy, and discuss our results. In Section 4, we implement different policies aiming at improving the education outcomes of children. Finally, Section 5 concludes this work.

2 The Model

Consider a cultural transmission model with a *two-cultural-trait population* of individuals. We build on the model of cultural transmission of **?**, in which vertical socialization inside a family interacts with horizontal socialization outside a family. However, contrary to **?**, we assume that children are active in the socialization process. Specifically, we assume that children choose

[5]See also **?** for a recent theoretical model of parent-child interactions with the formation of cognitive and non-cognitive skills; however, in his model, there is no network.

Figure 1: Timeline of the Model



how much social interaction they have with other children. Together with their preference biases toward other children, this socialization endogenously determines the role of horizontal socialization on cultural transmission.

To be more specific, define T as the set of possible types of traits in the population. Assume $T := \{H, L\}$, where H refers to *highly educated* (e.g., college degree) and L to *low educated* (e.g., less than a college degree). In our model (also in the data), children are still at school and have, therefore, not yet been educated. As a result, when we say that "a child has trait $t \in T$," it means that this child has a parent who is of type t. Families are composed of one parent and one child; hence, reproduction is asexual.

Socialization operates in two stages. First, children are temporarily socialized to the trait of their parent (*early socialization*) in the sense that they are exposed to the education level of their parents without acquiring their education level. Second, children choose their social interactions with other children of different types. Interaction choices are strategic and are a function of children's preference biases (e.g., homophily) and their socialization preferences (e.g., some children may like to interact more with other children, all else being equal).

Before the network is realized, parents anticipate their child's choices and choose the level of education effort to exert. Parents have explicit preferences regarding their child's friends. For example, parents may have lower (or higher) costs of exerting educational effort if their children make socialization choices that are in line with their type (i.e., homophily). In the end, the probability of a child becoming educated depends on the parent's education effort and the average type of the youngster's friends if the parent fails to transmit his or her trait.[6] Figure 1 summarizes the timeline.

The two choices of interest are the parents' education effort choices and the children's so-

[6] We assume that children are not farsighted. Indeed, children make socialization choices that have an impact on the probability of having some types of friends, but do not anticipate the impact on their parents' educational choices.

cialisation choices. The order of these two choices can be inverted (or be simultaneous) without any change to the analysis, as long as they both happen before the friendship network is realised and after the location choice of the parents.

Figure 1 explicitly shows that our analysis takes place *after* location decisions have been made (by the parents). Indeed, as we discuss in Section 3.1, our data set unfortunately does not allow us to study this important issue. Crucially, however, parents' location decisions are likely to be endogenous, and we discuss how we control for it in Section 3.4.2.

2.1 Children's socialization choices and network formation

We, first, describe the network-formation process, taking the children's socialization choices as given. Let $\mathbf{s} := (s_1, ..., s_n)$ be a profile of children's socialization efforts, where $s_i \geq 0$ for each child $i = 1, , ..., n$. The probability p_{ij} that a child i creates a link with a child $j \neq i$ is given by

$$p_{ij} = \frac{1}{c} d_{ij}(t_i, t_j) s_i s_j. \tag{1}$$

In (1), $d_{ij}(t_i, t_j) \in [0, 1]$ represents the preference bias between i and j, that is, how much i of type t_i likes or dislikes interacting with j of type t_j (**?**). In our empirical application, we allow $d_{ij}(t_i, t_j)$ to depend on the observable characteristics of i and j, such as their age, gender, race, and geographical proximity. It is natural to assume that $d_{ij}(L, L) > d_{ij}(L, H)$ and $d_{ij}(H, H) > d_{ij}(H, L)$, so that the model features *homophily* with respect to the children's types.[7] Thus, socialization decisions do not fully determine social connections. In this model, two socially active members with different characteristics will not necessarily link.

From (1), we can see that the greater both socialization efforts s_i and s_j are, the more likely a link will be formed. Note that the $c > 0$ is a normalization scalar that ensures that p_{ij} is always between 0 and 1.[8]

As in **?**, in (1), the exact identity of a child's friends is not an object of choice.[9] Rather, each child i chooses an aggregate level of socialization effort s_i. This total effort is then distributed

[7] *Homophily* is the tendency of agents to associate with other agents who have similar characteristics. It refers to a fairly pervasive observation in social networks. Having similar characteristics (age, race, religion, profession, education, etc.) is often a strong and significant predictor of two individuals being connected (**?**).

[8] We show in Appendix A that it is sufficient to impose that $c \geq \left(\bar{b} + \sqrt{\bar{b}^2 + 4\phi(n-1)}\right)^2 / 4$, where $\bar{b} = \max_i b_i$ and b_i are defined in (2) below.

[9] As an alternative, we could have considered a model of directed links g_{ij}, where each student i identifies a specific partner j to form a link g_{ij}. This model is intractable because the action space is very large with $2^{n(n-1)}$ potential directed links and, in the socialization effort decision, these links need to endogenously anticipate all pairs' strategic decisions over the entire network. Our framework instead allows for closed-form solutions and a simple realistic link formation process.

across every possible bilateral interaction, in proportion to the partner's socialization effort and the preference biases $d_{ij}(t_i, t_j)$. This interaction pattern arises naturally when meetings result from casual encounters rather than from an earmarked socialization process. In our context, children may participate in after-school activities (such as dance, music, honors club, foreign language clubs, etc.), and s_i may reflect the number of activities and how often they engage in these activities. Two children who spend a lot of time engaging in these after-school activities are then more likely to be friends than those who do not.[10] In this respect, equation (1) describes more a "meeting technology" that leads to an (expected) network formation through the game of socialization efforts rather than describing directly a "network formation process".

We consider the following linear quadratic specification of the expected utility of child i choosing socialization effort s_i. It is given by

$$\mathbb{E}_i[u_i] = b_i s_i + \phi \sum_{j \neq i} \mathbb{E}_i^{p_{ij}}[g_{ij}|s_i, s_j] - \frac{1}{2}s_i^2, \tag{2}$$

where the expectation is computed using distribution (1), $\phi \geq 0$, and $g_{ij} = 1$ denotes a link between i and j. Note that $g_{ij} = 1$ is the realization of the link ij, while p_{ij} is the probability of forming the link ij.

In (2), the (expected) utility of individual i who exerts a socialization effort s_i is the sum of a private component $(b_i s_i - \frac{1}{2}s_i^2)$ and a social component $(\sum\limits_{j \neq i} \mathbb{E}_i^{p_{ij}}[g_{ij}|s_i, s_j])$. The private benefit b_i of socialization may be a function of the ex-ante heterogeneity of child i (e.g., represented by the child's gender, race, etc.). Note that since the private cost is normalised to 1/2, the private benefit b_i is to be interpreted as a "net" private benefit. Note also that the quadratic specification implies linear best responses, as widely assumed in the literature on networks. See the overviews by **?** and **?**. Given $\phi \geq 0$, the benefit of socialization is due to the child i's expected number of friends $\sum\limits_{j \neq i} \mathbb{E}_i^{p_{ij}}[g_{ij}|s_i, s_j]$, which is a function not only of the child's own socialization effort but also of other children's socialization efforts. Using (1), we can rewrite (2) as:

$$\mathbb{E}_i[u_i] = b_i s_i + \frac{1}{c}\phi \sum_{j \neq i} d_{ij}(t_i, t_j) s_i s_j - \frac{1}{2}s_i^2. \tag{3}$$

Importantly, as discussed above, observe that each child i's socialization effort choice s_i is independent of the education effort of the parent. This assumption is made both for simplicity and credibility reasons. Note, however, that the parent's type *does* affect the child's payoff and,

[10] **?** provide a Lemma (see their Lemma 1), which shows how the functional form in (1) can be tied back to simple properties of the link intensity g_{ij}.

therefore, his or her choice, through its effect on b_i and $d_{ij}(t_i, t_j)$. By maximizing the expected utility (3) with respect to s_i, for each child i, we obtain[11]

$$s_i^* = \max\{b_i + \frac{1}{c}\phi \sum_{j \neq i} d_{ij}(t_i, t_j) s_j^*, 0\}. \tag{4}$$

If the solution is interior for all children, we can then write (4) in matrix form as follows:

$$\mathbf{s}^* = \mathbf{b} + \frac{\phi}{c}\mathbf{D}\,\mathbf{s}^*, \tag{5}$$

where $\mathbf{D}$ has zeros on the diagonal and $d_{ij}(t_i, t_j)$ off diagonal. By letting $\|\cdot\|$ be any sub-multiplicative matrix norm,[12] we obtain the following result:

Proposition 1. *If $\phi < c/\|\mathbf{D}\|$, then there exists a unique equilibrium of the children's socialization choices. If the equilibrium is interior, it is given by*

$$\mathbf{s}^* = \left(\mathbf{I} - \frac{\phi}{c}\mathbf{D}\right)^{-1}\mathbf{b}. \tag{6}$$

The proof relies on a standard contraction mapping argument and is therefore omitted (see, for example, **?**, for a proof). Note that Proposition 1 does not imply that the solution is necessarily interior, but merely states that *if* the unique solution is interior, the equilibrium socialization efforts are given by (6). A sufficient condition for interiority is $b_i \geq 0$ for all i. If the solution is interior, we can write the expected network structure in closed form as follows:

$$\mathbf{P}^* = \frac{1}{c}\mathbf{D} \circ \left[(\mathbf{I} - \frac{\phi}{c}\mathbf{D})^{-1}\mathbf{b}\mathbf{b}^T(\mathbf{I} - \frac{\phi}{c}\mathbf{D}^T)^{-1}\right], \tag{7}$$

where $\circ$ is the (Hadamard) element-wise product. If the solution is not interior, it can easily be computed iteratively by virtue of the contraction mapping theorem. We now turn to the parents' decision.

2.2 Parents' education effort

We assume that parents' incentives are partly driven by the expected education level of their child. Here, the effective education level of a child depends not only on the parents' education

[11]The solution may not be interior because b_i, which, in the data, captures the observable characteristics of individual i, may take negative values.

[12]That is, $\|\mathbf{AB}\| \leq \|\mathbf{A}\| \cdot \|\mathbf{B}\|$ for any two matrices $\mathbf{A}$ and $\mathbf{B}$.

level and effort (vertical socialization) but also on the education level of the parents of their child's friends (horizontal socialization). For each child i of type t, let

$$h_i^t = \frac{\sum_j g_{ij}.\mathbb{1}\{t_j = t\}}{\sum_j g_{ij}} \tag{8}$$

denote the fraction of i's friends who are of type $t = H, L$, where $g_{ij} = 1$ if i is friends with j, and $g_{ij} = 0$ otherwise. In (8), h_i^t captures i's *homophily* since it measures the fraction of same-type friends of individual i of type t. Note, however, that this notion of homophily is affected by the population's composition and should not be interpreted as a measure of bias. For example, if the population is highly educated then, mechanically, h_i^H will be large, while h_i^L will be low because the opportunity to meet children of low-educated parents is small.[13] In our context, it is the effective fraction of same-type friends that affects the transition probabilities, irrespective of how it is influenced by the population's composition.

Denote by $\pi_i^{tt'}$ the probability that a child from a parent of type t becomes of type t' when an adult. The education mechanism is characterized by the following transition probabilities:[14]

$$\begin{aligned}
\pi_i^{HH} &= \tau_i^H + (1-\tau_i^H)h_i^H, & (9)\\
\pi_i^{HL} &= (1-\tau_i^H)(1-h_i^H), & (10)\\
\pi_i^{LH} &= \tau_i^L + (1-\tau_i^L)(1-h_i^L), & (11)\\
\pi_i^{LL} &= (1-\tau_i^L)h_i^L, & (12)
\end{aligned}$$

where $0 \leq \tau_i^t \leq 1$ is the education effort of a type$-t$ parent who has a child i; τ_i^t is also the probability that *direct vertical transmission* to the parent's trait (t) will occur.

As an illustration, consider equation (9). Child i, whose parent is highly educated (type H), will be socialized to trait H if either the direct socialization from the child's parent H succeeds (which occurs with a probability of τ_i^H) or, if it does not succeed (which occurs with a probability of $1 - \tau_i^H$), the child i is subject to *horizontal socialization* captured by h_i^H, the fraction of the child's friends who are of type H. Here, the horizontal socialization is endogenous and determined by the network of social interactions described in the previous section. The probability that the horizontal socialization is successful is given by h_i^H, the fraction of friends of child i who have educated parents, and is defined by (8). The interpretation of (10) is similar.

[13] A notion of homophily that controls for composition is inbreeding homophily; see **?** for a discussion.

[14] As noted by an anonymous referee, these equations could be viewed as special cases of a linear probability model. For example, for equation (9), $\pi_i^{HH} = \beta_0 + \beta_1\tau_i^H + \beta_2 h_i^H + \beta_3\tau_i^H \times h_i^H$, where $\beta_0 = 0, \beta_1 = 1, \beta_2 = 1, \beta_3 = -1$.

Regarding the interpretation of equations (11) and (12), one needs to be careful because low-educated parents also want their children to be educated. Thus, τ_i^L, the low-educated parental effort, is the probability that a child from a low-educated family will become highly educated. Take, for example, π_i^{LL}. In this case, for a child from a low-educated family to stay low educated, both the vertical (parents) and horizontal (friends) socializations must fail, which is given by (12).

For children of educated parents, this means that *homophily* increases the probability that child i will become educated. The opposite is true for children of uneducated parents: homophily decreases the probability that their child will become educated. We will show that this fundamental difference has important consequences for the optimal choice of education effort for both types of parents.

Finally, instead of considering the average *population* with trait t as in **?**, we look at the average homophily among each child's *friends*. This has the striking implication of preventing us from formulating a unique equation that represents the entire set of agents of any given type. Accordingly, the transition probabilities are indexed by i since they depend on the social behavior of child i and not on the average population with trait t. In this respect, **?** can be seen as a *mean-field approximation* of this process, with the additional simplification of network exogeneity.

Observing that $h_i^H = 1 - h_i^L$, we can now define the expected utility of a parent of a child i of type $t = H, L$ as follows:

$$\mathbb{E}_i[U_i^t] = \mathbb{E}_i^{h_i^t} \left\{ \underbrace{\left(\pi_i^{tH} V_i^{tH} + \pi_i^{tL} V_i^{tL}\right) - \frac{1}{2}\left(\tau_i^t\right)^2}_{\text{Bisin-Verdier}} + \underbrace{\alpha^t \tau_i^t h_i^H}_{\text{Reciprocate Homophily}} \right\}, \tag{13}$$

where the expectation is taken with respect to the probabilities in (1) at equilibrium (see Proposition 1). The payoffs $V_i^{tH} > 0$ and $V_i^{tL} > 0$ denote the utility that a type$-t$ parent derives from having a child of type H and L, respectively, and $\frac{1}{2}\left(\tau_i^t\right)^2$ is a quadratic parental education cost function. For $t = H, L$, let us have the following notation: $\Delta V_i^t \equiv V_i^{tH} - V_i^{tL}$. Quite naturally, we assume that, for both $t = H$ and $t = L$, $V_i^{tH} > V_i^{tL}$, so that $\Delta V_i^t > 0$. That is, there is a positive utility associated with having a highly educated child for *both* types of parents.[15]

The (expected) utility (13) is composed of two parts: (i) the standard utility function used

[15]Note that as for the children's utility functions, the normalisation of the cost to 1/2 implies that $V_i^{tt'}$ is to be interpreted as a net benefit of having a child of type t' for a parent of type t. This allows low- and highly educated parents to have different costs of providing education effort.

in **?**, which depends on the benefits and costs of socialization, and (*ii*) a new part, which we refer to as *reciprocate homophily*. This is the utility that the parent derives from his or her child interacting with children of highly educated parents. If parents are homophilous, i.e., each type of parent wants their children to interact with those of the same type as their parents, we expect $\alpha^L < 0$ and $\alpha^H > 0$. However, it might also be possible, for example, that low-educated parents prefer their child to interact with children of high-educated parents, independently of the impact on the transition probability. This would imply that $\alpha^L > 0$. We do not make any assumption about the sign of α^H and α^L; these parameters will be estimated structurally.

Below, we show that the sign and magnitude of α^t have important consequences for the optimal choice of education effort each parent exerts. In particular, our utility function is more flexible than that of **?**, in which the high importance of the peer group (reciprocate homophily) necessarily translates into low effort by the parent.

As noted above, the order in which the parents' education effort and the children's socialization effort are chosen does not matter for our result. This is because the children's socialization effort does not depend on parents' education effort *and* parents' education effort is independent of the realization of their children's network.[16] Thus, their expected utility and their education choices depend on the *expected* and not the *realized* homophily of their child, that is, $\bar{h}_i^t$, for $t = H, L$. The same is true for π_i^{tH} and π_i^{tL}. Indeed, using (13), we have:

$$\begin{aligned}
\mathbb{E}_i[U_i^t] &= \mathbb{E}_i^{h_i^t}\left\{\pi_i^{tH}V_i^{tH} + \pi_i^{tL}V_i^{tL} - \frac{1}{2}\left(\tau_i^t\right)^2 + \alpha^t\tau_i^t h_i^H\right\} \\
&= V_i^{tH}\mathbb{E}_i^{h_i^t}[\pi_i^{tH}] + V_i^{tL}\mathbb{E}_i^{h_i^t}[\pi_i^{tL}] - \frac{1}{2}\left(\tau_i^t\right)^2 + \alpha^t\tau_i^t\bar{h}_i^H.
\end{aligned}$$

For example, for a type $t = H$ parent, we have:

$$\begin{aligned}
\mathbb{E}_i[U_i^H] &= V_i^{HH}\mathbb{E}_i^{h_i^H}[\pi_i^{HH}] + V_i^{HL}\mathbb{E}_i^{h_i^H}[\pi_i^{HL}] - \frac{1}{2}\left(\tau_i^H\right)^2 + \alpha^H\tau_i^H\bar{h}_i^H \\
&= V_i^{HH}[\tau_i^H + (1-\tau_i^H)\bar{h}_i^H] + V_i^{HL}[(1-\tau_i^H)(1-\bar{h}_i^H)] - \frac{1}{2}\left(\tau_i^H\right)^2 + \alpha^H\tau_i^H\bar{h}_i^H.
\end{aligned}$$

One can see that this expected utility and the maximization of this utility to obtain the education effort of the parents is independent of the exact choice of their child's effort. It is only dependent on $\bar{h}_i^H$, which is taken as given by the parents when their decide upon their education effort.

[16]Importantly, parents observe $\mathbf{D}$, $\mathbf{b}$, ϕ, and c; thus, even if they do not observe $\mathbf{s}$ directly (e.g., if they play before their children), they can still compute the equilibrium value $\mathbf{s}^*$. In other words, parents know Proposition 1. This is important since it means that the expected homophily $\bar{h}_i^t$ of the children's network is a function of $\mathbf{s}^*$, the equilibrium socialisation level. For the ease of the exposition, we do not make this relation explicit, that is, we do not write $\bar{h}_i^t(\mathbf{s}^*)$. However, this is stated explicitly in our structural estimation; see Equation (21).

Thus, the key assumption that makes the timing inconsequential is the fact that the parent's effort choice is made before the realization of the network or, equivalently, the fact that the parents, when deciding their education effort, do not know exactly the network of their children but have some idea or some expectation of it, which is captured by $\bar{h}_i^t$. It is also due to the fact that the child's socialization effort decision is independent of the parent's education choice.

We have the following result:

Proposition 2. *Assume an interior solution, that is, $\tau_i^{t_i*} \in (0,1)$ for all i. Given the equilibrium distribution of the network-formation process (see Proposition 1), denote by $\bar{h}_i^t$ the expected value of h_i^t. Thus, we have the following:*

(*i*) *The optimal education effort of each type of parent is given by:*

$$\tau_i^{L*} = \Delta V_i^L \bar{h}_i^L + \alpha^L (1 - \bar{h}_i^L), \qquad \tau_i^{H*} = \Delta V_i^H (1 - \bar{h}_i^H) + \alpha^H \bar{h}_i^H. \tag{14}$$

(*ii*) *The behavior of type$-L$ parents exhibits cultural substitution (cultural complementarity) if and only if $\alpha^L > \Delta V_i^L$ ($\alpha^L < \Delta V_i^L$) since $\frac{\partial \tau_i^{L*}}{\partial \bar{h}_i^L} = \Delta V_i^L - \alpha^L$.*

(*iii*) *The behavior of type$-H$ parents exhibits cultural substitution (cultural complementarity) if and only if $\alpha^H < \Delta V_i^H$ ($\alpha^H > \Delta V_i^H$) since $\frac{\partial \tau_i^{H*}}{\partial \bar{h}_i^H} = \alpha^H - \Delta V_i^H$.*

Proposition 2 follows from the simple optimization of the parents' utility function and a simple comparative statics analysis. First, as shown in (14), parents of each type $t = H, L$ exert socialization efforts differently, which depends on their own α^t, that is, the extent to which they value the homophily or heterophily of their children's friendship network, the specific (expected) network their children belong to, and, thus, the homophily behavior of their children. Second, the parents' effort may exhibit either cultural complementarity (i.e., they exert *more* effort the more homophilous their children are) or substitutability (i.e., they exert *less* effort, the more homophilous their children are) depending on $\Delta V_i^t > 0$ (the benefits of having an educated child) and α^t (their preference regarding the fraction of high-type friends of their children).

Regarding low-educated parents, cultural substitution or complementarity depends on whether or not there is reciprocated homophily, that is, whether α^L is positive or negative. If $\alpha^L < 0$, which means that parents value homophily in their children's network (i.e., their children have a high fraction of low-educated friends), then there will always be cultural complementarity. If $\alpha^L > 0$, then, there will be a trade-off between the value of α^L and ΔV_i^L. The empirical estimation of our model will tell us the sign of α^L.

For highly educated parents, to exhibit cultural complementarity (substitutability), α^H must be large (small) enough and higher (lower) than ΔV_i^H. Indeed, if parents are very (not very) homophilous and/or the benefits of having an educated child is quite small (large), then they exert more (less) effort when their children increase h_i^H, their (expected) homophily network.

3 Structural estimation and empirical results

Let us now structurally estimate our model.

3.1 Data

We use a (relatively) well-known database on friendship networks from the National Longitudinal Survey of Adolescent Health (AddHealth). The AddHealth survey has been designed to study the impact of the social environment (i.e., friends, family, neighborhood, and school) on adolescents' behavior in the United States by collecting data on students in grades 7–12 from a nationally representative sample of roughly 130 private and public schools in years 1994–95. A subset of these adolescents, about 20,000 individuals, are also asked to complete a longer questionnaire containing sensitive individual and household information, which includes the geographical coordinates of their residential address.

From a network perspective, the most interesting aspect of the AddHealth data is the friendship information, which is based upon actual friends' nominations. For a subset of 16 schools,[17] all of the pupils were asked to identify their best friends from a school roster (up to five males and five females). From the data, one can reconstruct the entire friendship network. We follow **?** and **?** and restrict our analysis to friendship relations with students of the same school and the same grade level. We will refer to a student's school-grade level as their *group*. We retain groups of at least 10 students. The final sample comprises 3,471 students in 57 groups and 14 schools.[18]

In the context of these data, we say that a child is of type H if either parent (father or mother) is a college graduate. Otherwise, the child is of type L. We also use a series of

[17] These 16 schools are those from the saturated sample of Wave I, that is, the schools for which we have the whole network and each student in this sample completed both the in-school and in-home questionnaires. See also **?** who uses the same sample.

[18] The subset of 16 schools for which we observe the entire network includes 2 large schools and 14 small schools. We focus on the set of small schools. One should therefore be careful in applying our results to very large schools. Our focus on groups of at least 10 students makes more credible the modelling assumption that children socialization choices play an important role for friendship formation. Results are robust to alternative threshold choices.

students' individual characteristics, such as age, gender, racial group, as well as the (normalized) geographical location of their residence. The students' level of socialization (s_i for a student i in our model) is constructed from: (i) the number of extracurricular activities in which the child participates, (ii) the child's self-reported level of daily interactions with friends, and (iii) the child's self-reported level of interaction in their neighborhood. We then construct a composite index variable for each student i, which is equal to the sum of these three after-school activities of student i, and then we normalize this index s_i to be between 0 and 1.[19]

The parental education effort level τ_i is constructed from three types of questions: (i) parental control over the children's decisions, (ii) children's assessment of how much their parents care about them, and (iii) parents' involvement in the school-related activities of their child.[20] We take the average of the answers to these three questions and obtain a value of τ_i, which is between 0 and 1.

Summary statistics are presented in Table 1 for students from low-educated families and Table 2 for students from highly educated families for all our *individual* variables. Table 3 presents the summary statistics for our *pairwise* variables. Note that there are 2,360 low-educated students and 1,111 highly educated students; thus, 68% of students are low educated. The typical school grade will therefore have more low-educated students. The distribution of the share of highly educated students can be found in Figure C.1 of Appendix C.2.

Table 1: Summary statistics, individual variables – low-educated students

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
White	0.616	0.486	0	0	1	1
Black	0.135	0.342	0	0	0	1
Hisp.	0.267	0.443	0	0	1	1
Asian	0.112	0.315	0	0	0	1
Mother works	0.684	0.465	0	0	1	1
Female	0.494	0.500	0	0	1	1
Age	16.201	1.420	12	15.3	17.3	18
s_i	0.597	0.180	0.055	0.477	0.720	1.000
τ_i^t	0.449	0.176	0.000	0.325	0.587	0.952

Notes: Total number of low-educated students: 2,360 (out of 3,471). Only groups of size 10 or more have been kept. We also removed two small schools with only one grade level. Pctl(25) and Pctl(75) mean the 25th and 75th percentiles, respectively. Excluded racial groups are "Native American" and "Other."

First, we see that the percentage of Whites is higher among low-educated families than among the highly educated ones, while it is the opposite for the Black population. Hispanics

[19]See Appendix B.1 for additional details.
[20]See Appendix B.1 for additional details.

Table 2: Summary statistics, individual variables – highly educated students

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
White	0.500	0.500	0	0	1	1
Black	0.216	0.412	0	0	0	1
Hisp.	0.063	0.243	0	0	0	1
Asian	0.273	0.446	0	0	1	1
Mother works	0.823	0.382	0	1	1	1
Female	0.483	0.500	0	0	1	1
Age	15.938	1.599	12	15.1	17.2	18
s_i	0.621	0.184	0.055	0.497	0.754	1.000
τ_i^t	0.509	0.157	0.133	0.389	0.611	0.952

Notes: Total number of highly educated students: 1,111 (out of 3,471). Only groups of size 10 or more have been kept. We also removed two small schools with only one grade level. Pctl(25) and Pctl(75) mean the 25th and 75th percentiles, respectively. Excluded racial groups are "Native American" and "Other."

are much more likely to belong to low-educated than highly educated families (26.7% versus 6.3%). This is not surprising as the surveyed schools are spread around the United States, and many areas include poor White families. Even though there are (small) differences in the racial group compositions of low-educated and highly educated students, none of these differences is statistically significant. We observe that among the highly educated families, the mother is more likely to work.

Second, the average socialization effort s_i is 0.597 for low-educated students and 0.621 for highly educated students. This indicates that, on average, students from highly educated families socialize more than those from low-educated families, although there is substantial variation within each type.

Finally, the average value of τ^t is 0.449 for low-educated parents (type L) and 0.509 for highly educated parents (type H). This indicates that, on average, high-educated parents put more effort into education-related activities than low-educated parents do, although, here also, there is substantial variation within each type.

3.2 Empirical strategy: Children's decisions

Recall that the (conditional) network-formation process (1) is given by[21]

$$p_{ij,r} = d_{ij,r} s_{i,r} s_{j,r}, \tag{15}$$

[21] To facilitate the notations, we use: $d_{ij} := d_{ij}(t_i, t_j)$.

Table 3: Summary statistics, pairwise variables

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
g_{ij}	0.006	0.076	0	0	0	1
Both White	0.203	0.402	0	0	0	1
Both Black	0.053	0.223	0	0	0	1
Both Hisp.	0.127	0.333	0	0	0	1
Both Asian	0.093	0.291	0	0	0	1
Both mothers work	0.511	0.500	0	0	1	1
Same gender	0.500	0.500	0	0	1	1
Age difference	0.550	0.468	0.000	0.167	0.750	3.583
Geographical distance	0.031	0.046	0.000	0.012	0.034	1.000
$t = L$ with $t = H$	0.212	0.409	0	0	0	1
$s_i s_j$	0.348	0.152	0.003	0.236	0.445	1.000

Notes: Total number of individuals: 3,471; total number of pairs: 1,085,606. Only groups of size 10 or more have been kept. We also removed two small schools with only one grade level. Pctl(25) and Pctl(75) mean the 25th and 75th percentiles, respectively. Excluded racial groups are "Native American" and "Other."

where we normalize $c = 1$ and add the subscript r to denote the group (i.e., school-grade) $r = 1, ..., \bar{r}$.[22] We use the following parametrization: $d_{ij,r} = \Phi(\mathbf{z}_{ij,r}\boldsymbol{\gamma})$, where Φ is the standardized normal cumulative distribution, $\mathbf{z}_{ij}$ is a vector of pairwise characteristics of the directed pair ij in group r, and $\boldsymbol{\gamma}$ is a vector of parameters to estimate. Notably, $\mathbf{z}_{ij}$ includes t_i and t_j, but also other characteristics of i and j, such as their gender, age, geographical distance, or racial group, as well as a school fixed effect. Precise definitions for constructed variables are presented in Appendix B.1, and summary statistics are presented in Table 3.

Since we observe the network structure $\mathbf{G}$, if the socialization efforts $\mathbf{s}$ were exogenous, the parameters $\boldsymbol{\gamma}$ could simply be recovered using a simple maximum likelihood estimator (MLE) given by

$$\ln P(\mathbf{G}_r|\mathbf{s}_r, \mathbf{Z}_r; \boldsymbol{\gamma}) = \sum_{i \neq j} g_{ij,r} \ln[\Phi(\mathbf{z}_{ij,r}\boldsymbol{\gamma}) s_{i,r} s_{j,r}] + (1 - g_{ij,r}) \ln[1 - \Phi(\mathbf{z}_{ij,r}\boldsymbol{\gamma}) s_{i,r} s_{j,r}] \quad (16)$$

for any group r, which is a simple variation on a probit model. However, here, $s_{i,r}$ and $s_{j,r}$ are choice variables for any ordered pair (i, j). In particular, students choose their socialization efforts anticipating the network-formation process (15). As such, the equilibrium value of $\mathbf{s}_r$ is a function of $\boldsymbol{\gamma}$. Therefore, we need to estimate the network-formation model jointly with the model reflecting the optimal choice of $\mathbf{s}_r$.

In this section, we assume that the equilibrium socialization effort is interior. This is coherent

[22] The normalization of c follows from defining s_i as an index in $[0, 1]$.

with the data since all values of s_i are strictly above 0.[23] As such, the equilibrium socialization efforts are given by:

$$s_{i,r} = b_{i,r} + \phi \sum_j d_{ij,r} s_{j,r}, \tag{17}$$

for all i and r. We assume that $b_{i,r} = \mathbf{x}_{i,r}\boldsymbol{\beta} + \varepsilon_{i,r}$, where $\mathbf{x}_{i,r}$ is a vector of the characteristics of student i in group r (e.g., age, gender, racial group; see tables 1 and 2),[24] and where $\varepsilon_{ij} \sim N(0, \sigma^2)$. Then, following **?**, the likelihood of the students' socialization efforts, for any group r, is given by[25]

$$\begin{aligned} \ln P(\mathbf{s}_r|\mathbf{Z}_r, \mathbf{X}_r; \boldsymbol{\theta}) \quad = \quad & \frac{n_r}{2}\ln(\sigma^2) - \ln|\mathbf{M}_r(\boldsymbol{\theta})| - n_r \ln[\pi] \\ & -\frac{1}{2\sigma^2}\left[\mathbf{s}_r^T\mathbf{M}_r^T(\boldsymbol{\theta})\mathbf{M}_r(\boldsymbol{\theta})\mathbf{s}_r - 2\mathbf{s}_r^T\mathbf{M}_r^T(\boldsymbol{\theta})\mathbf{X}_r\boldsymbol{\beta} + \boldsymbol{\beta}^T\mathbf{X}_r^T\mathbf{X}_r\boldsymbol{\beta}\right], \end{aligned} \tag{18}$$

where $\mathbf{M}_r(\boldsymbol{\theta}) = \mathbf{I}_r - \phi\mathbf{D}_r(\boldsymbol{\gamma})$, and $\boldsymbol{\theta} = [\boldsymbol{\beta}, \boldsymbol{\gamma}, \phi, \sigma]$. The likelihood (18) is similar to the one in **?**, with the notable difference that the interaction matrices (here, $\mathbf{D}_r$, $r = 1, ..., \bar{r}$) are not row-normalized. While (18) can still be concentrated around ϕ, which facilitates the numerical optimization, we cannot adapt the within-group transformation used in **?**.[26]

Here, if $\mathbf{D}_r(\boldsymbol{\gamma})$ was known, then $\boldsymbol{\beta}$ and ϕ could be estimated by maximizing (18) under similar identification conditions as in **?** or **?**. However, since $\boldsymbol{\gamma}$, and therefore $\mathbf{D}_r(\boldsymbol{\gamma})$, are not known, the entire vector of unknown parameters $\boldsymbol{\theta} = [\boldsymbol{\beta}, \boldsymbol{\gamma}, \phi, \sigma]$ is likely *not* point identified using (18) alone.

Therefore, we propose to estimate $\boldsymbol{\theta}$ using the joint likelihood of the network and of the equilibrium socialization efforts, that is:

$$\ln P(\mathbf{G}_r, \mathbf{s}_r|\mathbf{Z}_r, \mathbf{X}_r; \boldsymbol{\theta}) = \ln P(\mathbf{G}_r|\mathbf{s}_r, \mathbf{Z}_r; \boldsymbol{\gamma}) + \ln P(\mathbf{s}_r|\mathbf{Z}_r, \mathbf{X}_r; \boldsymbol{\theta}), \tag{19}$$

for $r = 1, ..., \bar{r}$. Estimated coefficients are presented in Table 4. We present a discussion of the results in Section 3.4.

[23] The distribution of s is fairly continuous; there is no obvious mass point. See Figure C.2 in Appendix C.2.
[24] We also include a school fixed effect.
[25] Remember from the model that $\mathbf{D}$ has zeros on the diagonal and $d_{ij,r}$ off diagonal.
[26] Unfortunately, group sizes are too small to allow for a consistent estimation of group-level dummies. We therefore rely on school-level dummies.

Table 4: Estimation results: Joint likelihood of the network and the socialization efforts

Network formation			**Socialization effort**		
Variable	Estimate	S.E.	Variable	Estimate	S.E.
Both White	0.207***	(0.044)	White	0.005	(0.006)
Both Black	0.723***	(0.049)	Black	0.019	(0.012)
Both Hisp.	0.723***	(0.024)	Hisp.	-0.002	(0.012)
Both Asian	0.894***	(0.014)	Asian	-0.008	(0.016)
Both Mothers Work	0.081***	(0.011)	Mother works	0.005	(0.006)
Same Gender	0.281***	(0.014)	Female	-0.058***	(0.008)
Age Difference	-0.254***	(0.039)	Age	-0.023***	(0.003)
Geographic distance	-1.121	(0.477)	Type	0.020*	(0.011)
Type H with Type L	-0.032	(0.026)	ϕ	0.012**	(0.005)
Type L with Type H	-0.025	(0.025)	σ^2	0.029***	(0.001)
Type H with Type H	0.117**	(0.048)			

Notes: Estimation of (19). Both the network-formation specification (16) and the socialization effort specification (18) control for school fixed effects (one dummy for each school and no constant). Geographical distance is normalized (in the original data) between 0 and 1 for anonymity reasons. Standard errors are reported in parentheses and clustered at the group level. ***p<0.01, **p<0.05, *p<0.1.

3.3 Empirical strategy: Parents' decisions

Recall that, from Proposition 2, we have, for each parent i of type t and group (i.e., school-grade) r:[27]

$$\tau^t_{i,r} = \Delta V^t_{i,r}(1 - \bar{h}^H_{i,r}) + \alpha^t \bar{h}^H_{i,r}. \tag{20}$$

Here, we assume a simple linear specification for $\Delta V^t_{i,r} = \mathbf{w}_{i,r}\boldsymbol{\delta}_t + \eta^t_{i,r}$, where $\mathbf{w}_{i,r}$ is a vector of observable characteristics for parent i, $\boldsymbol{\delta}_t$ is a type-dependent vector of parameters to be estimated, and $\eta^t_{i,r}$ is unobserved error.

If $\bar{h}^H_{i,r}$, the expected fraction of friends of type-H, was observed, the model would be easily estimated by OLS. However, here $\bar{h}^H_{i,r}$ has to be constructed using the estimated parameters from $P(\mathbf{G}_r, \mathbf{s}_r|\mathbf{Z}_r, \mathbf{X}_r; \boldsymbol{\theta})$. Indeed, using the maximum likelihood estimator $\hat{\gamma}$, we compute the predicted probabilities

$$\hat{p}_{ij,r} = \Phi(\mathbf{z}_{ij,r}\hat{\gamma})s_{i,r}s_{j,r}, \tag{21}$$

which is a consistent estimate of the true probability $p_{ij,r}$.

Using these predicted probabilities, we can therefore simulate $\bar{h}^H_{i,r}$. Since the simulated value for $\bar{h}^H_{i,r}$ is consistent as the number of simulations increases, we simply replace $\bar{h}^H_{i,r}$ by its simulated value in (20). This can, in fact, be viewed as a two-step estimator, with the addition

[27] The interiority of the parents' education efforts is supported by the data; see Figure C.3 in Appendix C.2.

Table 5: Estimation results: Parents' education effort

Low-educated parents			**Highly educated parents**			**Diff.**
Variable	Estimate	S.E.	Variable	Estimate	S.E.	
White	-0.045*	(0.024)	White	-0.035	(0.043)	
Black	-0.097***	(0.034)	Black	-0.086*	(0.045)	
Hisp.	0.066***	(0.029)	Hisp.	0.036	(0.044)	
Asian	-0.013	(0.036)	Asian	0.039	(0.047)	
M. Works	0.096***	(0.016)	M. Works	0.038	(0.024)	*
Female	-0.020	(0.014)	Female	-0.007	(0.016)	
Age	-0.062***	(0.006)	Age	-0.043***	(0.008)	**
α^L	0.455***	(0.010)	α^H	0.524***	(0.014)	***
$P(\partial\hat{\tau}^L_{i,r}/\partial\bar{h}^L_i \geq 0 \vert \mathbf{W},\mathbf{X},\mathbf{Z})$	0.450		$P(\partial\hat{\tau}^H_{i,r}/\partial\bar{h}^H_i \geq 0 \vert \mathbf{W},\mathbf{X},\mathbf{Z})$	0.649		-

Note: Estimation of (20). Both specifications control for school fixed effects (one dummy for each school and no constant). The last row in the table shows the fraction of parents of each type for which education effort exhibits cultural complementarity. Standard errors are reported in parentheses, clustered at the group level, and bootstrapped using the procedure described in Appendix B.2. Column "Diff." tests whether the estimated parameters for low- and highly educated parents are statistically different. ***p<0.01, **p<0.05, *p<0.1.

of a simulation step. As such, we compute the standard errors using a bootstrap procedure, formally described in Appendix B.2. Since the first step relies on a computationally intensive numerical optimization (see Section 3.2), we use a conservative approach that relies on drawing parameters from the estimated asymptotic distribution.

The results are presented in Table 5. We discuss these results in the next section.[28]

3.4 Empirical results

3.4.1 Children's decisions and outcomes

We first discuss the children's decisions and outcomes. The results for the estimation of (19) are displayed in Table 4.

For the network-formation process (left panel of Table 4), the results show significant homophily behaviors for all observable characteristics. Ethnic bias appears to be more important for Asians, followed by Blacks and Hispanics. The labor market status of the mother appears to be of comparatively small importance; this is also true for the impact of the education level

[28]From the values of τ_i^t and the simulated values of $\bar{h}^H_{i,r}$, we can then calculate the *expected* transition probabilities π_i^{HH}, π_i^{HL}, π_i^{LH} and π_i^{LL} defined in equations (9)–(12). We could have used the *actual* college completion data available in the fourth wave of AddHealth to determine these transition probabilities. However, because of severe attrition bias, we prefer to use our estimated values, especially when we implement the different education policies in Section 4.

of the parents, *conditional* on the contribution of the other characteristics. We also see that there is strong homophily in terms of age, gender, and geographical location. In particular, we show that students living closer are more likely to form links. Other studies have also shown that social interactions decline with the geographic distance between locations (**??**).

Finally, students of educated parents are more likely to be friends. Note that the excluded category is $d(L, L)$, so students of low-educated parents do not exhibit homophilic preferences with respect to parents' types. This clearly shows the importance of distinguishing between homophilic preferences and the homophily level $h^t_{i,r}$. The homophily level is a function of homophilic preferences and of the group composition. Here, even if low-educated students have more homophilious networks (as measured by $h^t_{i,r}$), this is mostly due to the group composition. Low-educated students do not form more same-type friendships than they would if they chose their friends at random. However, highly educated students do form more same-type friendships than they would if they chose their friends at random. This is captured by a strictly positive value for $d(H, H)$.

If we now consider the right panel of Table 4, we see that the socialization effort is higher for Blacks than for any other ethnic groups (even if it is not significant), something that has been documented before (**?**). Girls and older students students exert less socialization effort. Children of highly educated parents (Type H) also socialise more on average. We also find that there is complementarity in socialization efforts since $\hat{\phi} > 0$ and is highly significant. Furthermore, we check whether $\phi < c/\|\mathbf{D}\|$, the equilibrium uniqueness condition in Proposition 1, is satisfied here. It is indeed satisfied since, with $c = 1$, the predicted upper bound based on the spectral norm of $\mathbf{D}$ is $\phi \leq 0.123$ and therefore relatively large compared to the estimated value of $\hat{\phi} = 0.012$. This means that preference biases (leading to $\mathbf{D}$) could, in principle, sustain much higher levels of complementarity between socialization efforts.

3.4.2 Parents' decisions and outcomes

We now discuss the parents' decisions and outcomes. Results for the estimation of (20) are displayed in Table 5. Low-educated parents (left panel) of White or Black students, as well as those of older students are less likely to exert an education effort, while those of Hispanic students are more likely to exert education effort. Highly educated parents (right panel) of Black or older students are also less likely to exert education effort, while those of Asian students are more likely to exert education effort (even if it is not significant).

Interestingly, we find that highly educated parents have *homophilous* preferences (i.e., $\alpha^H >$

0), while low-educated parents have *heterophilous* preferences (i.e., $\alpha^L > 0$). This means that even after controlling for the effects of their child's friends on their probability of becoming educated, both types of parents would prefer their child to spend time with children of highly educated parents. See (13).

Note that the results from the left and right panels in Table 5 come from two separate regressions, each including school-level fixed effects. We are therefore controlling for any school (or neighborhood) characteristic that may explain (low- or highly educated) parents' location choices. In other words, school-level fixed effects control for any unobserved factor, common among parents of a given education level, which may explain both school choice (and therefore homophily) and their educational effort. Results in Table 5 are, hence, interpreted as being conditional on the parents' location choices.[29]

Moreover, in order to control for within school-type unobserved heterogeneity, we present a robustness analysis in Appendix C.1 in which we estimate a finite mixture model for the parents' decisions (instead of the OLS regressions). This allows us to additionally control for the omission of a binary variable (potentially correlated with location decisions), affecting the contribution of parents' observable characteristics (including expected homophily) on effort. The results displayed in tables C.1 and C.2 are in line with our analysis.

Using the estimated coefficients from (20), we are now able to compute the predicted cultural substitutability or complementarity for both types of parents (see Proposition 2, parts (*ii*) and (*iii*)), that is,

$$\frac{\partial \hat{\tau}_{i,r}^L}{\partial \bar{h}_{i,r}^L} = \mathbf{w}_{i,r}\hat{\boldsymbol{\delta}}^L - \hat{\alpha}^L,$$

and

$$\frac{\partial \hat{\tau}_{i,r}^H}{\partial \bar{h}_{i,r}^H} = \hat{\alpha}^H - \mathbf{w}_{i,r}\hat{\boldsymbol{\delta}}^H.$$

The results are displayed in the last row of Table 5 and Figure 2. On average, the socialization effort of highly educated parents is more than 15% more likely to exhibit cultural complementarity than that of low-educated parents. In particular, the fraction of highly educated parents for which there is cultural complementarity is greater than 64.9%, while, for low-educated parents, this figure is 45%, which means that they are more likely to exhibit cultural substitutability (last row of Table 5). This implies that, for both types of parents, the more their children interact with youngsters from high-educated families, the more likely they are to exert greater

[29]This is due to the lack of data required to fully describe the location choices. We are therefore likely to underestimate the overall impact of the parents' choices (including both location and education choices) on their children's transition probabilities.

educational effort. This result is also confirmed by Figure 2, where the values to the left of the 0–axis exhibit cultural substitutability, while those to the right of the 0–axis exhibit cultural complementarity. There is, however, substantial heterogeneity. In particular, note that the interquartile range is $[-0.087, 0.053]$ for low-educated parents and $[-0.021, 0.085]$ for highly educated parents. This means that most parents barely react to their children's (expected) network.[30]

In Figure 3, we plot the simulated distribution of the main outcome variables. In panel (a), we see that the students from highly educated families are more likely to exert higher socialization effort than those from low-educated families. In tables 1 and 2, we saw that this was true, on average, in the data, while, in table 4, this was also true, on average, in the simulations. Here, we plot the whole (simulated) distribution. In panel (b), we perform the same exercise for homophily. We see that low-educated students are more homophilous (i.e., have more same-type friends) than highly educated students. This is because among the low-educated children, there are many more Hispanics (26.7% versus 6.3%; see tables 1 and 2), and they tend to form links with each other (table 4). Among the highly educated children, there are more Asians (27.3% versus 11.2%; see tables 1 and 2), who also tend to form links with each other (table 4), but the difference is less important. Panel (c) confirms what we knew: highly educated parents are more likely to exert education effort than low-educated parents even if there is some variation.[31] Finally, in panel (d), we provide the (simulated) probability distribution of becoming educated. Even though children from highly educated parents are more likely to become educated, there is a wide dispersion.

The above discussion and, in particular, Figure 3 summarizes the predictions of the model. In the next section, we compare the predictions of the model with the data.

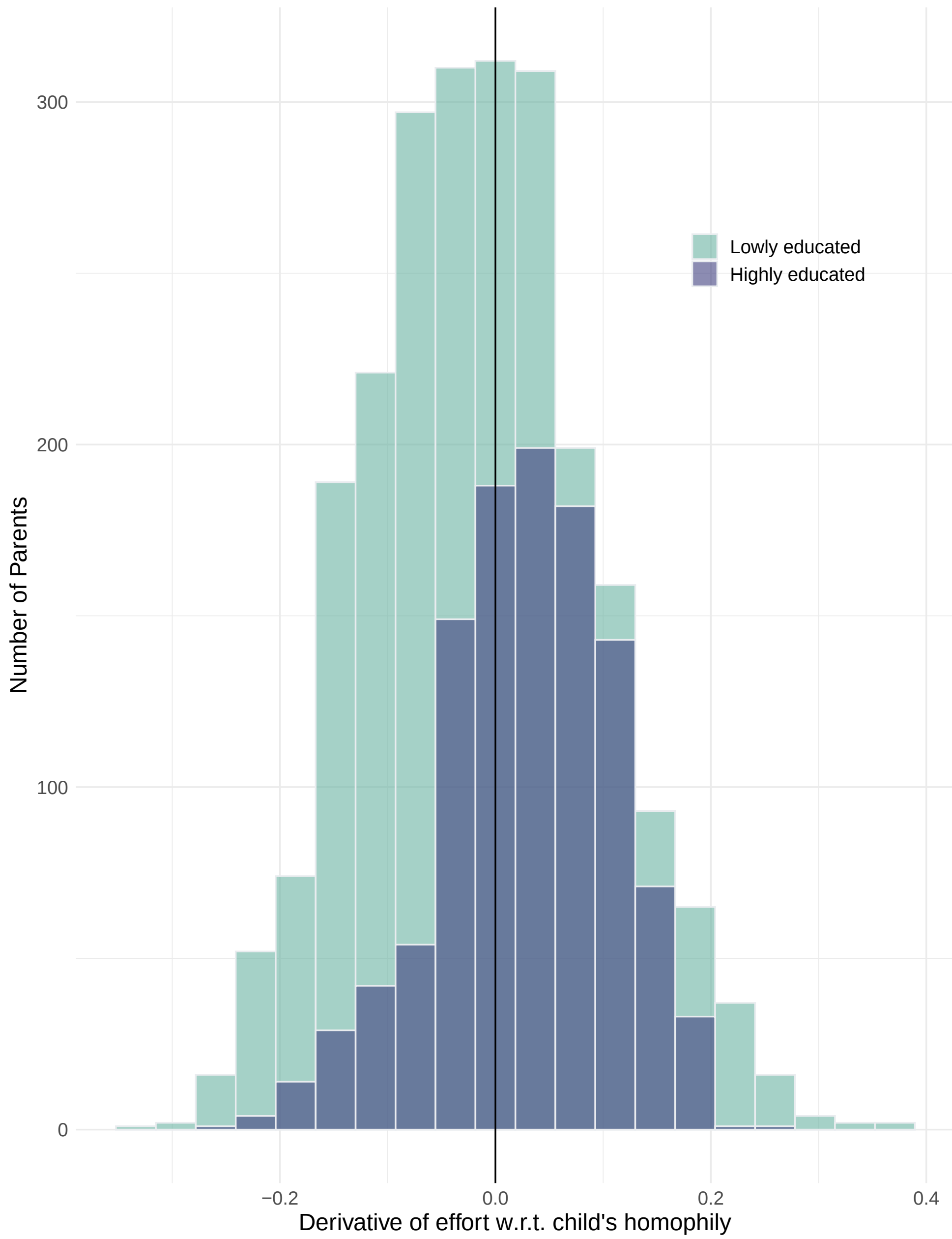
3.4.3 Model fit

We conclude this section with a discussion predicting the performance of the model. We focus on four main outcomes: Socialization effort (child), education effort (parents), the fraction of same-type links (homophily) and the probability that the child becomes educated. Figure 4 summarizes the results. Exact values can be found in Table C.3 of the Appendix. Overall, the model is able to replicate the key features of the data. In particular, socialization effort

[30] Observe that we have assumed that children are not farsighted, i.e., they don't take into account the impact of their socialization choices on their parents' educational effort, and they don't care about their future educational attainment. If we introduce this aspect in the model (this will clearly complicate the analysis), it is easily verified that it will reinforce even more our results in terms of education gap between low- and highly educated kids.

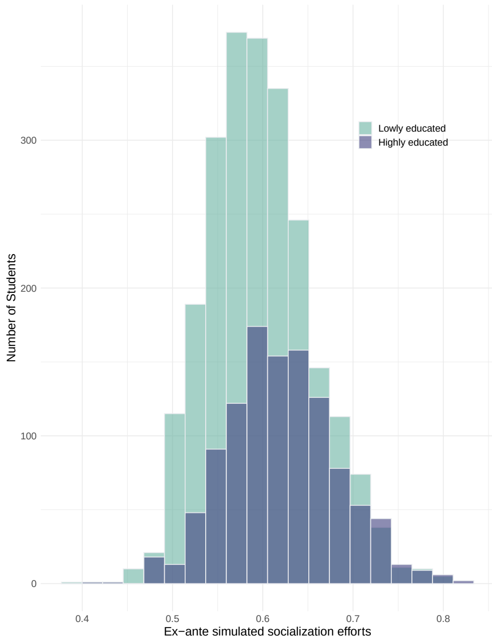
[31] For all simulations, we assume that η_i is normally distributed and homoscedastic.

Figure 2: Cultural substitution and cultural complementarity

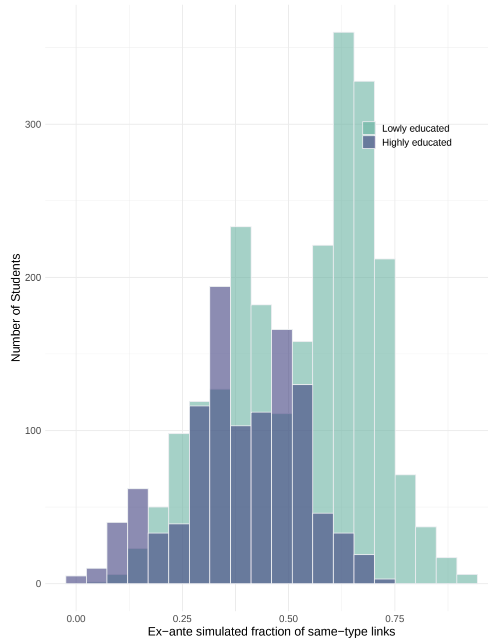


Note: Histogram for $\partial\hat{\tau}_i^t/\partial h_i^t$, for $t = L, H$. Parents to the left (right) of the 0.0−vertical line are predicted to exhibit cultural substitution (complementarity).

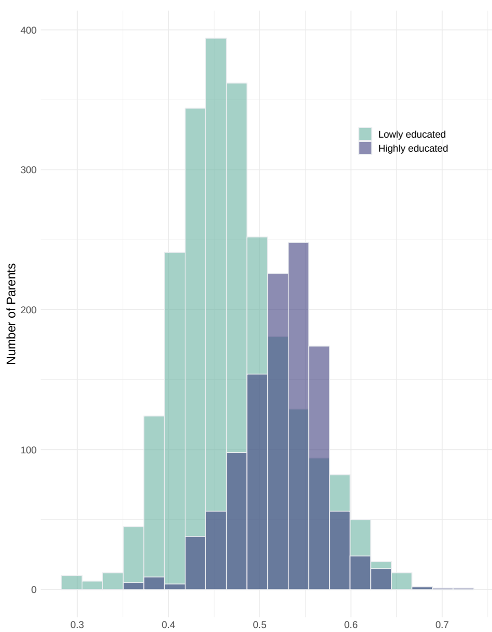
Figure 3: Ex ante simulated distribution for the main outcomes



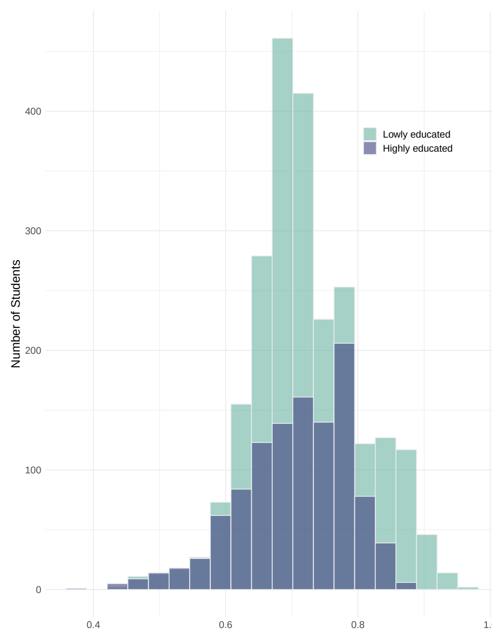
(a) Socialization effort (s_i)



(b) Fraction of same-type friends (h_i^t)



(c) Education effort (τ_i^t)



(d) Proba. of becoming educated (π_i^{tH})

Notes: Simulated values for the main outcomes of the model.

and education effort both increase with education. There is, however, substantial heterogeneity. This heterogeneity also explains why the model predicts, as in the data, that on average, the probability of becoming educated is slightly lower for children of educated parents. In the context of our model, this is rationalized as follows. Parents' education effort is larger for educated parents, but, and as we discuss further in the next section, this effort is not sufficient to compensate for the children's friendship choices.

4 Policy experiments

We now study the impact of some policy interventions on the main outcomes of the model. Given these distributions displayed in 3, let us, now, implement two different types of policies. First, we consider policies that promote social interactions between students (Sections 4.1). Second, we consider a policy that promotes heterophily (Section 4.2).

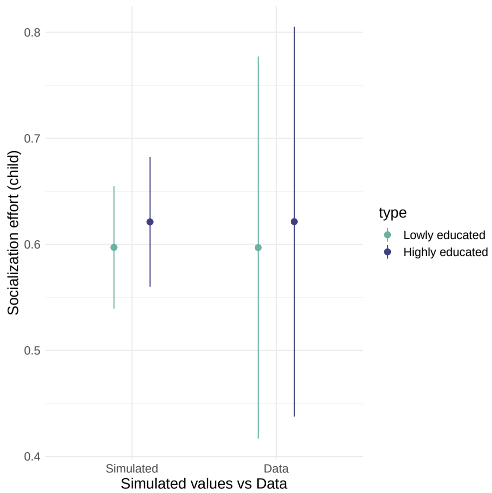
4.1 Policies that subsidize the private returns of socialization

Let us first consider policies that subsidize the private returns of socialization. For example, the government could subsidize the cost of extracurricular activities, such as sports' teams, chess clubs, etc. School administrators and teachers could also promote students' interactions by facilitating project-based learning or other approaches favoring teamwork.

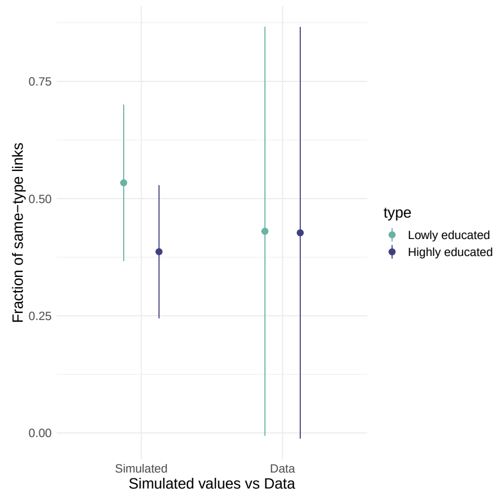
To study the impact of such policies, we exogenously increase b_i in our model, that is, the students' private benefit of socialization effort. We consider two policies, one uniform for all students (Section 4.1.1) and one targeting low-educated students (Section 4.1.2). In each of these policies, we increase b_i by the fraction of the standard deviation of s_i in the data, from 0 (no intervention) to 1 standard deviation. This means that, in the absence of network effects, $\phi = 0$, the maximal policy would increase the socialization effort by one standard deviation.

Unfortunately, b_i is in utility units and a function of observable characteristics over which the policy maker has little influence (e.g., gender, race, age...). However, recall that socialization is proxied using, among other variables, the number of extracurricular activities in which children are involved (see Appendix B.1). The median number of activities in which a child is involved is 2. Using only extracurricular activities, an increase in one standard deviation of s_i roughly corresponds to an increase in four extracurricular activities. Using this back-of-the-envelope computation, it means that a policy increasing b_i by one standard deviation of s_i is roughly equivalent to a policy increasing the number of extracurricular activities in which a child is

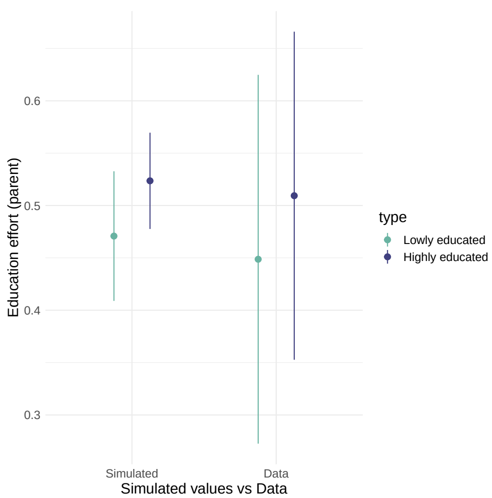
Figure 4: Contrasting simulated versus observed outcomes



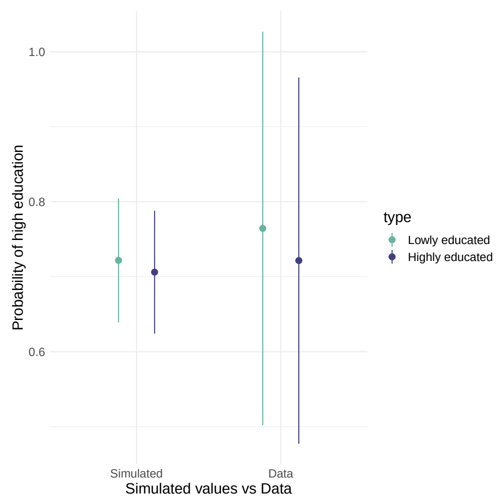
(a) Socialization effort (s_i)



(b) Fraction of same-type friends (h_i^t)



(c) Education effort (τ_i^t)



(d) Proba. of becoming educated (π_i^{tH})

Notes: Simulated values for the main outcomes of the model versus values in the data. Precise number are available in Table C.3 of the Appendix.

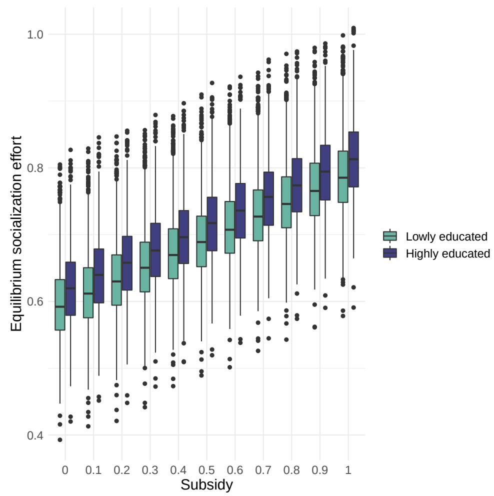
involved by four.

4.1.1 Increasing the private returns of socialization for all students

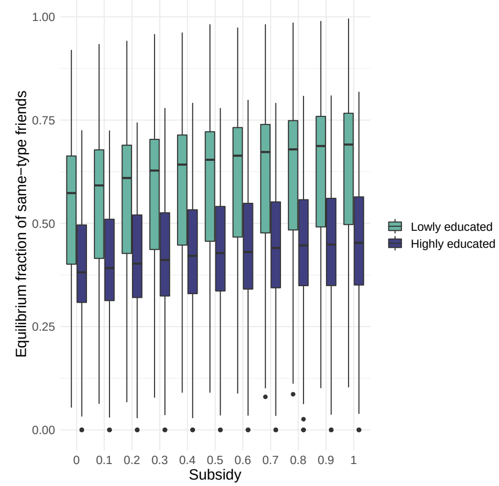
The first policy consists of increasing b_i by one standard deviation of s_i (that is, increase by four the number of extracurricular activities for each student) for *all* students. The results of this *uniform* policy are presented in Figure 5.

As expected, in panel (a), we see that s_i, the socialization effort of each student i, increases for both low- and highly educated families. The overall effect is slightly magnified by the effect of the complementarity of investments (i.e., positive ϕ), with a multiplier of roughly 1.05. This increase in socialization effort translates into an increase in the fraction of same-type links and, therefore, more homophily. See panel (b). Indeed, an increase of b_i from 0 (no intervention) to 1 of the fraction of the standard deviation of s_i, increases (the median of) the fraction of same-type friends from 58% to 71% for low-educated students and from 34% to 41% for highly educated students. In panel (c), we see that this increase in the expected fraction of same-type links barely affects the education effort of the parents. Finally, in panel (d), we study the impact of this policy on the (expected) probability for a child i of type t of becoming educated (π_i^{tH}). We see that this policy *reduces* the probability of becoming educated for children from low-educated families (from 71% to 66%), while it *increases* the probability of becoming educated for children from highly educated families (from 71% to 73%). This is because the policy increases socialization efforts and homophily for both types of children. Thus, for children from low-educated parents, there is a decrease in the average friend's "quality" because of more homophily. This leads to a decrease in the (expected) probability of becoming educated.

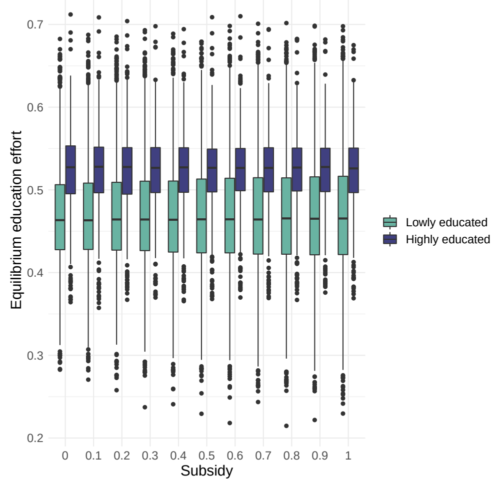
Figure 5: Increasing social interactions of all students



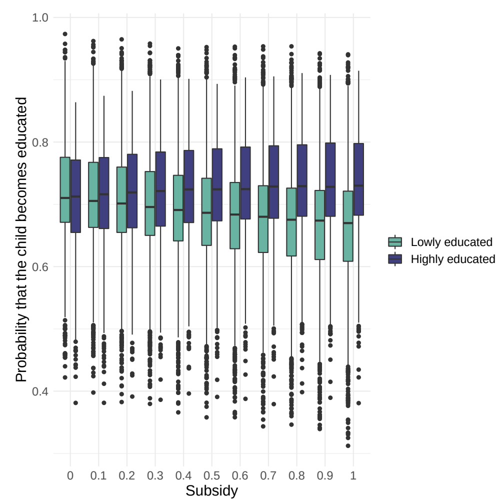
(a) Socialization effort (s_i)



(b) Fraction of same-type friends (h_i^t)



(c) Education effort (τ_i^t)



(d) Probability that the child becomes educated (π_i^{tH})

Notes: Counterfactual policy simulations for the main outcomes of the model. The policy corresponds to a uniform shift in b_i for all i: $b_i \rightarrow b_i + policy$, where *policy* ranges from zero to one standard deviation of s_i in the data ($\sqrt{Var(s_i)}$) by increments of $0.1\sqrt{Var(s_i)}$. Box plots show the 25th, 50th, and 75th percentiles.

For children from highly educated families, the effects are exactly the opposite: because of their increased homophily, the average "quality" of their friends increases. As a result, their probability of becoming educated increases.

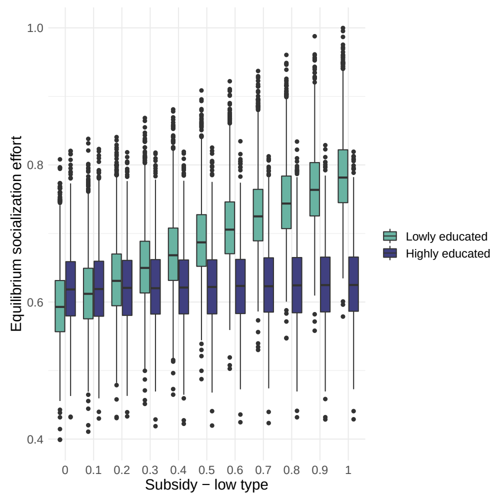
4.1.2 Increasing the private returns of socialization for students from low-educated families

Next, we consider a second policy that consists of subsidizing only children from low-educated families. The results of this *targeted* policy are presented in Figure 6.

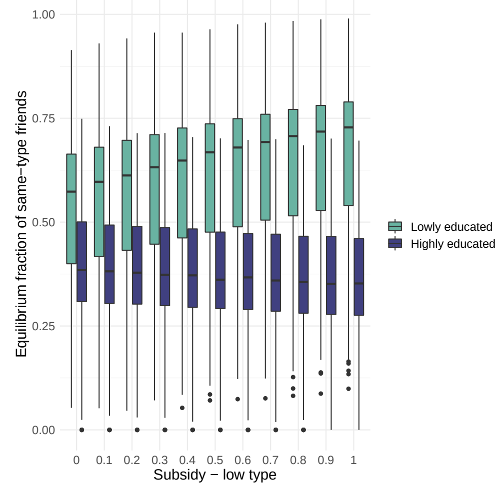
In panel (a), we see that the effect on socialization effort s_i is very significant for low-educated children. While their socialization effort is lower than that of highly educated children when there is no policy, under the targeted policy, they strongly increase their (median level of) effort from 0.59 to 0.78, while highly educated kids stay around 0.62. This leads to a large increase in homophily for low-educated kids and a slight decrease for highly educated kids. See panel (b). For low-educated students, homophily strongly increases because all low-educated students increase their socialization effort much more than do highly educated students and, thus, are more likely to form friendship links with other low-educated students. See (1) or (15). Here again, parents barely change their education effort. See panel (c). As a result, the (expected) probability of becoming educated for low-educated kids (π_i^{LH}) decreases from 71% to 65% because of the increased homophily. See panel (d). For highly educated children, this policy has a small negative effect on π_i^{HH} (from 70% to 69%), which is due to the slight decrease in homophily levels.

As a result, subsidizing social interactions of children from low-educated families backfires because it decreases rather than increases their probability of becoming educated. This is because such a policy increases homophily among low-educated students, which means that they interact more with students of the same type. The small change in the parents' education effort does not compensate for the change in homophily. Thus, increasing socialization reduces their chances of becoming educated.

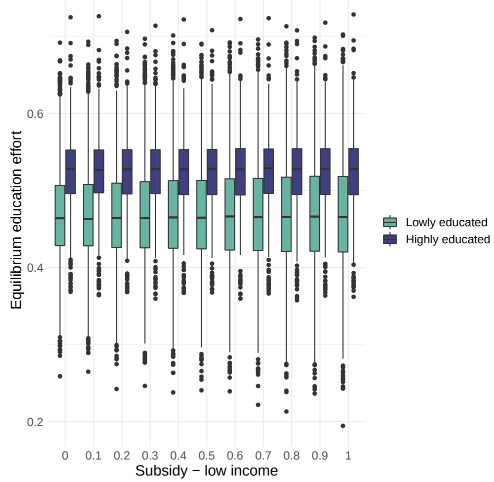
Figure 6: Increasing social interactions of students from low-educated families



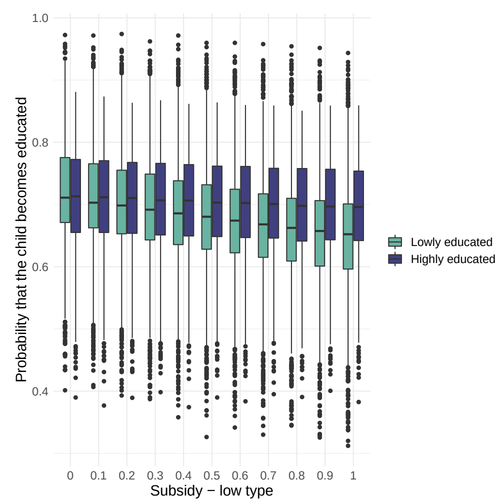
(a) Socialization effort (s_i)



(b) Fraction of same-type friends (h_i^t)



(c) Education effort (τ_i^t)



(d) Probability that the child becomes educated (π_i^{tH})

Notes: Counterfactual policy simulations for the main outcomes of the model. The policy corresponds to a uniform shift in b_i for all i such that the variable *Type* is equal to 0: $b_i \rightarrow b_i + policy$ for all $i : Type_i = 0$, where *policy* ranges from zero to one standard deviation of s_i in the data ($\sqrt{Var(s_i)}$), by increments of $0.1\sqrt{Var(s_i)}$. Box plots show the 25th, 50th, and 75th percentiles.

In summary, subsidizing social interactions (by targeting certain types of students or not) is detrimental to the possibility of education for children coming from low-educated families. This is because these children tend to react to this policy by increasing their social interactions mostly with children of the same type, which, in turn, causes their parents to reduce their education effort. Both effects have a negative impact on the probability of becoming educated because of negative vertical transmission (parents) and negative horizontal transmission (peer effects). Moreover, even in cases where the parents increase their education effort in response to their child's network, our simulations indicate that the magnitude of the parents' response is not enough to compensate for the child's network choices.[32]

4.2 Increasing heterophily

Let us now consider a "targeted policy" that would intentionally promote *heterophily*, for example, a tutoring program. Indeed, one could imagine another type of "targeted policy" where adolescent children from low-educated families are incentivized to (voluntarily) socialize more with children from highly educated families and vice versa. For example, the National Citizen Service (NCS) program in the United Kingdom is a large-scale, real-world policy, which purposely brings together groups of teenagers from different socio-economic backgrounds during the holidays and has them do various types of physical and team-building activities, social projects, etc.[33] Another policy that indirectly affects the social interactions between kids from different backgrounds is the Moving to Opportunity (MTO) program (**???**), which provides housing assistance (i.e., vouchers and certificates) to low-income families when they relocate to better and richer neighborhoods.

To simulate such a policy in our framework, we gradually increase $d(H, L)$ and $d(L, H)$, starting from their estimated values in Table 4, that is, $d(H, L) = -0.032$ and $d(L, H) = -0.025$, until they reach the estimated value of $d(H, H)$, that is, $d(H, L) = d(L, H) = d(H, H) = 0.117$. The results are displayed in Figure 7. This policy has a small positive impact on students' socialization choices (Figure 7a), since it increases the expected number of friendship relations. See equation (4). As expected, this policy decreases the fraction of same-type friends for both types of students and thus increases heterophily (Figure 7b). Because parents barely react to changes in the network (Figure 7c), the overall impact on the probability of education is

[32] In our simulations, the parents' education effort seems to play a minor role compared to the peer social-network effect.

[33] We thank a referee for making us aware of the NCS program. For a description of the NCS program and its impact on social integration, see **?**.

positive for low-educated students, but negative for highly educated students (Figure 7d). This is a standard concern that has been raised for this type of policies (such as, for example, the MTO program): it helps the low-educated kids at the expense of the highly educated ones. However, compared to the two previous policies, it reduces the education gap between kids from low- and highly education parents.

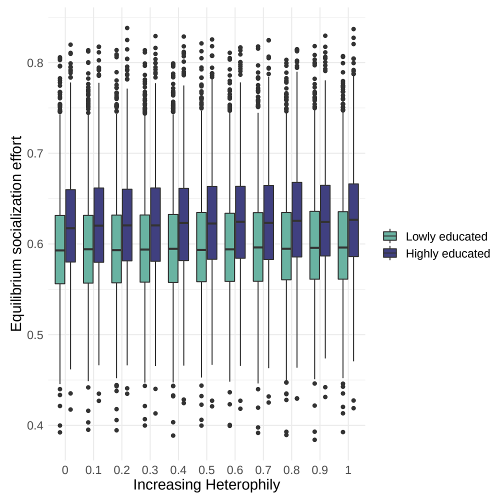
5 Concluding remarks

In this paper, we develop a model of intergenerational transmission of preference for high education, in the presence of an endogenously determined social context. In particular, we study the formation of a network of students as an equilibrium outcome of socializing activities between students. Parents observe the (expected) homophily of their children (i.e., the share of own-type friends) and decide accordingly how much educational effort to exert. We structurally estimated all parameters of the model using adolescent friendship networks in the United States. We find that, on average, children's homophily acts as a complement to the educational effort of highly educated parents but as a substitute for the educational effort of low-educated parents.

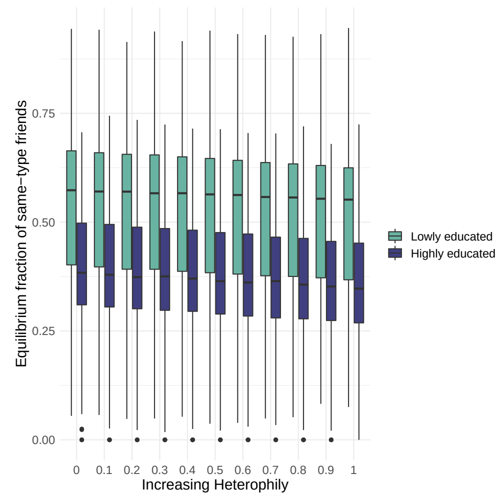
With the goal of increasing the probability of becoming educated for all students, we use the estimated parameters to run some policy experiments. We find that increasing socialization among students has a negative effect on the educational outcomes of low-educated students, while not necessarily improving those of highly educated students. This is due to the fact that, by subsidizing socialization, low-educated students become more "social" and, because of complementarity in socialization efforts, tend to interact more with other students of the same type. This is also true for highly educated students when subsidies are not targeted. However, the key difference is that there is *cultural complementarity* for highly educated parents, which means that more homophily from their children leads to greater parental education effort, while there is *cultural substitutability* for low-educated parents, which implies that more homophily from their children leads to a reduction in the education effort of these parents.[34] Thus, our results suggest that *socialization policies*, such as, for example, those that promote students' interactions in school by facilitating project-based learning or other approaches favoring teamwork, may not be as successful as expected in terms of educational outcomes.

[34]Interestingly, in a very different model, **?** also find that policies (such as busing) may have limited effects because of parents' negative reactions against the peer group in the new neighborhood.

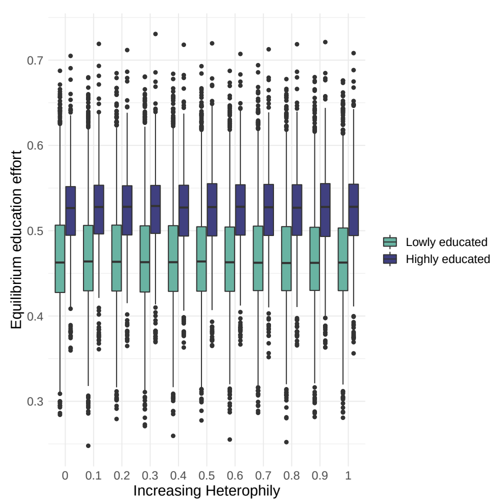
Figure 7: Increasing heterophily with respect to students' types.



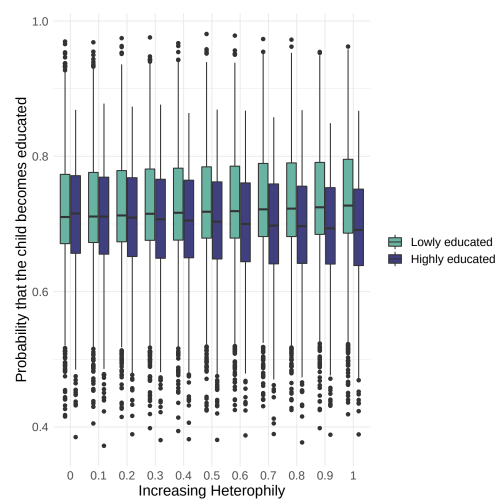
(a) Socialization effort (s_i)



(b) Fraction of same-type friends (h_i^t)



(c) Education effort (τ_i^t)



(d) Probability that the child becomes educated (π_i^{tH})

Notes: Counterfactual policy simulations for the main outcomes of the model. The policy corresponds to an increase in $d(L, H)$ and $d(H, L)$ from their estimated values in the data, to the estimated value of $d(H, H)$. Box plots show the 25th, 50th, and 75th percentiles.

We then consider other policies aiming at directly affecting social-mixing and, in particular, increasing social interactions and heterophily between kids from different backgrounds. We show that this policy mostly favors low-educated kids and reduces the education gap between the two types of children. An example of such policies is the Moving to Opportunity (MTO) program (**???**), which incentivizes low-income families to relocate to richer neighborhoods.[35] This policy tends to favor interactions between children of different backgrounds. However, it has been shown that there is a significant and positive long-term effect of neighborhoods on education (college attendance) and earnings *only* for children who move when they are younger than 13 years old (**?**). The main explanation of this result is that when the MTO program moves people (mostly low-educated families) from poor areas to richer areas, they do not interact much with their "new neighbors" but instead with their "old neighbors," who are their real peers. For example, **?** document the fact that many Black families who moved to richer areas thanks to the MTO program did not interact with their new neighbors because they felt rejected. In particular, on Sundays, they were still going to the church in their previous neighborhood, even though it was located very far away from their current residence. However, when young children (under 13) move to a new area, they have time to build a new network of friends; therefore, their new neighbors can become their peers and have a positive impact on their education outcomes.

The National Citizen Service (NCS) program in the United Kingdom is another example of a policy that increases social interactions and thus heterophily between kids from different backgrounds by bringing together groups of teenagers during the holidays. It is, however, quite different (and substantially cheaper) from the long-term MTO experiment. Furthermore, it has only been implemented for older adolescent children and not (yet) for younger children.

The question of which policy is best for improving the educational outcomes of children is very difficult. In this paper, we have highlighted one dimension related to the role of children's socialization and homophily behavior and of parents' effort in education outcomes. We believe that a successful education policy should therefore take into account its impact on children's social networks and on parents' education transmission.

[35] Observe that, in our model, location is fixed and our "heterophily" policy takes place in school. However, the MTO programs, by moving poor families to richer neighborhoods, affect the social interactions of kids from different backgrounds in school.

APPENDIX

A Theory

We look for an upper-bound $\bar{s}$ such that

$$\mathbf{b} - \bar{s}\mathbf{1} + \bar{s}\frac{\phi}{c}\mathbf{D}\mathbf{1} < 0$$

That is, the first derivative of the utility function is negative for all children. Let $c = \bar{s}^2 \geq d_{ij}s_is_j$. It is then sufficient to look for $\bar{s}$ such that:

$$\bar{s}\bar{b} - \bar{s}^2 + \phi(n-1) < 0,$$

where $\bar{b} = \max_i b_i$. It is therefore sufficient to have:

$$\bar{s} \geq \left(\bar{b} + \sqrt{\bar{b}^2 + 4\phi(n-1)}\right)/2,$$

Thus, a sufficient condition is $c \geq \left(\bar{b} + \sqrt{\bar{b}^2 + 4\phi(n-1)}\right)^2/4$.

B Empirical Application

B.1 Constructed variables

To measure the *socialization effort* s_i of each child i, we take the average of three types of interactions of student i and then normalize them between 0 and 1 (log scale). These three types of interactions are as follows:

1. *The number of extracurricular (or after-school) activities* in which the student participates (normalized between 1 and 2, after censoring outliers). These activities are: dance, music, any kind of sports, writing or editing the school newspaper, honors club, foreign language clubs, participating in the school council, and other clubs.

2. *Involvement in daily activities*: The average value of the answer to the question: “During the past week, how many times did you hang out with friends?”. Range between 0 and 3.

3. *Average neighborhood participation*: The average of the two following binary variables:

"You know most of the people in your neighborhood and if, during the month before the interview, they stopped on the street to talk to someone they knew " (answer 0 or 1) and "Do you use a physical fitness or recreation center in your neighborhood?" (answer 0 or 1).

To measure the *education effort* τ_i^t of a parent i of type $t = H, L$, we average all the answers to questions asked to students regarding their relationship with their parents and to parents regarding their relation with their kids concerning school activities. These three types of questions are as follows:

1. *Decision variables*: The average of the answers from the child to the following questions: "Do your parents let you make your own decisions about..." (for each question, the answer is either 0 or 1). A value of 1 signals lower education effort.

2. *Caring*: The average of the answer from the child to the following question: "How much do you think she/he cares about you?" (range between 0 and 5).

3. *Activities related to school*: We take the average of three questions that were asked to the parents about the following topics: (i) whether they talked to the child about his or her grades, (ii) whether they helped the child with a school project, (iii) whether they talked to the child about other things he or she did at school.

B.2 Bootstrap procedure for parental effort models

Before we present our bootstrap procedure, we would like to have a conceptual discussion. Consider a general two step estimator:

1. $\hat{\theta}_1 = \arg\max_{\theta_1} Q_{1,n}(\theta_1)$

2. $\hat{\theta}_2 = \arg\max_{\theta_2} Q_{2,n}(\hat{\theta}_1, \theta_2)$

A standard bootstrap procedure for computing the standard errors would go as follows: (1) Resample the data; (2) estimate θ_1^*; (3) estimate θ_2^*, conditional on θ_1^*, and; (4) repeat (1)–(3).

In our case, this is too computationally demanding since step (2), the estimation of the children's model, takes a significant amount of time. Instead, we rely on the following strategy. Since we have (a consistent estimate of) the asymptotic distribution of $\hat{\theta}_1$, we proceed as follows: (1) Draw θ_1^* from the asymptotic distribution of $\hat{\theta}_1$; (2) Resample the data; (3) estimate θ_2^*, conditional on θ_1^*, and; (4) repeat (1)–(3).

Our approach is conservative with respect to the standard one (meaning that the standard errors are larger). This is because our approach neglects the fact that $\hat{\theta}_1$ and $\hat{\theta}_2$ are obtained from the same sample of the population. Indeed, in the standard approach, both θ_1^* and θ_2^* are estimated using the same (bootstrap) sample. In our approach, they are not.

With that in mind, we now present formally how we compute the standard errors:

1. A parameter $\tilde{\boldsymbol{\theta}}$ is drawn from the multivariate normal distribution $N(\hat{\boldsymbol{\theta}}, \hat{\mathbf{V}}_{\boldsymbol{\theta}})$, where $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{V}}_{\boldsymbol{\theta}}$ are the estimator and estimated variance-covariance matrix for the joint likelihood (19).

2. Given $\tilde{\boldsymbol{\theta}}$, we compute the predicted probabilities from (21). We then simulate $\bar{h}_{i,r}^H$ using 500 network draws.

3. We implement the following bootstrap procedure:

   (a) We first draw, with replacement, a sample of groups.

   (b) Within these sampled groups, we draw, with replacement, a sample of parents.

4. We estimate (20) for $t = L, H$.

5. We repeat 1–4 499 times.

C Additional Results

C.1 Heterogenous within-type reciprocate homophily

As described in Section 3.4, we find that $\alpha^L, \alpha^H > 0$, which implies that both types of parents would prefer their child to interact with children of educated parents. However, a concern is that there might be within-type heterogeneity so that, for example, $\alpha^L < 0$ for some parents, while $\alpha^L > 0$ for other parents.

To study this issue, we estimate a latent class model (**?**) with two components, for both types of parents. For each type of parents, the model sorts parents into two groups (or components) and estimate values of $\boldsymbol{\delta}$ and α for each component. These models famously suffer from a lack of identification with respect to the labels of the component (we can swap labels and obtain the same likelihood). In order to solve for this identification issue among bootstrap replications, we assume that the first component is the one with the lowest value of α, for both types of parents. (**?**)

Table C.1 and Table C.2 present the results. Our main result is confirmed: both types of parents would prefer their child to be friend with children of educated parents. We also measure substantial heterogeneity with low-educated parents in the second component having higher preferences for reciprocal homophily than high-educated parents in the first component ($\alpha_2^L = 0.507$ vs $\alpha_1^H = 0.420$).

Table C.1: Parents' education effort: First component (lowest α)

Low-educated parents			**Highly educated parents**		
Variable	Estimate	S.E.	Variable	Estimate	S.E.
White	0.045	(0.162)	White	-0.033	(0.131)
Black	0.039	(0.181)	Black	-0.100	(0.130)
Hisp.	-0.040	(0.129)	Hisp.	0.045	(0.150)
Asian	-0.022	(0.178)	Asian	0.025	(0.135)
Mother Works	0.312	(0.097)	Mother Works	0.032	(0.062)
Female	-0.088	(0.052)	Female	0.020	(0.061)
Age	-0.151	(0.033)	Age	-0.048	(0.020)
α_1^L	0.274	(0.063)	α_1^H	0.420	(0.071)
σ_1^L	0.109	(0.026)	σ_1^L	0.119	(0.032)

Note: Estimation of (20) using a latent class model with two components. Both specifications control for school fixed effects. Errors are assumed to be normally distributed and homoscedastic. Standard errors are reported in parentheses and bootstrapped using the procedure described in Appendix B.2.

Table C.2: Parents' education effort: Second component (highest α)

Low-educated parents			**Highly educated parents**		
Variable	Estimate	S.E.	Variable	Estimate	S.E.
White	-0.062	(0.110)	White	0.004	(0.196)
Black	-0.125	(0.109)	Black	-0.095	(0.187)
Hisp.	0.087	(0.076)	Hisp.	-0.062	(0.231)
Asian	0.026	(0.145)	Asian	0.031	(0.189)
Mother Works	0.031	(0.072)	Mother Works	0.008	(0.110)
Female	0.011	(0.033)	Female	-0.054	(0.081)
Age	-0.031	(0.015)	Age	-0.024	(0.025)
α_2^L	0.507	(0.030)	α_2^H	0.665	(0.065)
σ_2^L	0.151	(0.014)	σ_2^H	0.099	(0.038)

Note: Estimation of (20) using a latent class model with two components. Both specifications control for school fixed effects. Errors are assumed to be normally distributed and homoscedastic. Standard errors are reported in parentheses and bootstrapped using the procedure described in Appendix B.2.

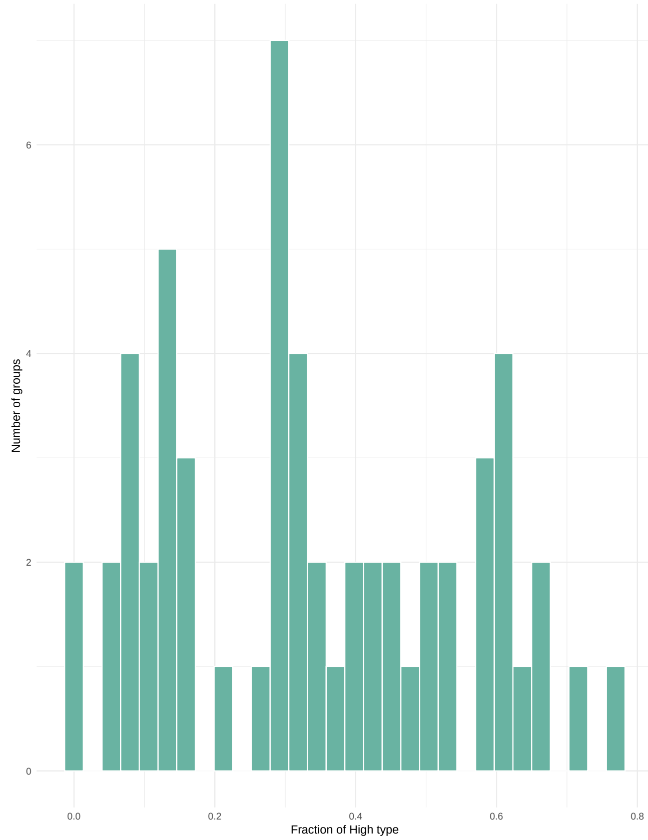


Figure C.1: Distribution of the share of high-type children among groups

C.2 Additional Figures and Tables

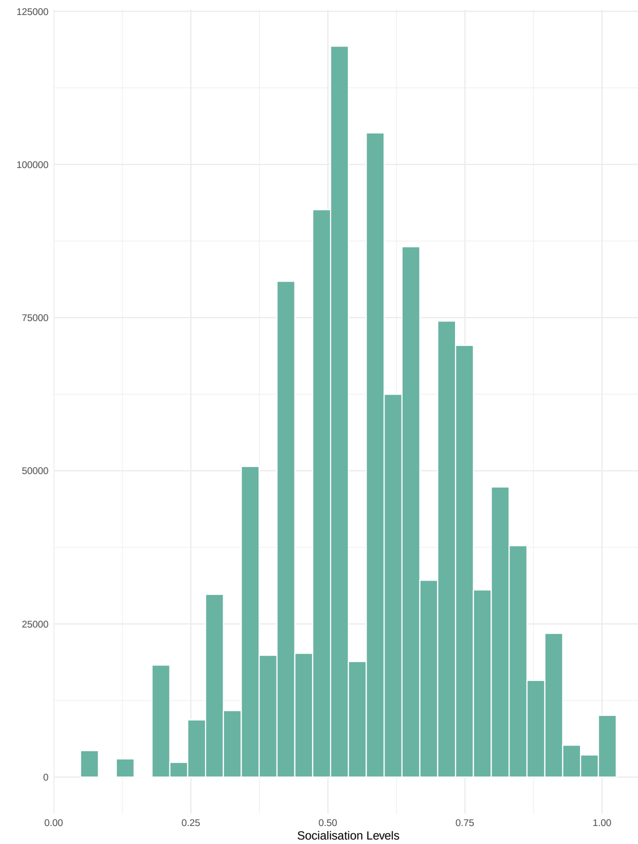


Figure C.2: Distribution of observed socialisation levels

	Simulated Values				**Data**			
	Low Education		High Education		Low Education		High Education.	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
Socialization effort	0.597	0.058	0.621	0.061	0.597	0.180	0.621	0.184
Education effort	0.471	0.062	0.524	0.046	0.449	0.176	0.509	0.157
Homophily	0.534	0.167	0.387	0.142	0.430	0.436	0.427	0.439
Probability of becoming educated	0.722	0.083	0.706	0.082	0.764	0.262	0.722	0.244

Table C.3: Model fit: Comparison between outcomes simulated using the model and the ones in the data.

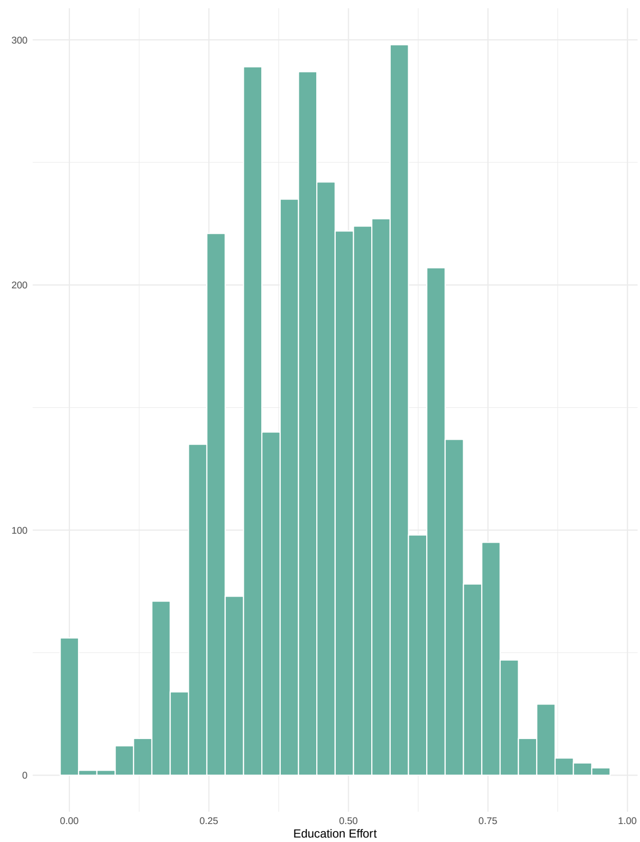


Figure C.3: Distribution of observed education effort