# FLEXIBLE AND SINGLE-IMPULSE TRANSFERS FOR ASTEROID RETRIEVAL USING THE INVARIANT MANIFOLDS OF THE CIRCULAR-RESTRICTED THREE-BODY PROBLEM

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Recent studies have used the invariant manifolds of the Circular-Restricted Three-Body Problem to retrieve Near-Earth Objects into Lagrange point orbits of the Sun-Earth system. Objects retrievable below a cost of 500m/s have been classified as Easily Retrievable Objects (EROs). In this work we significantly extend on the previous literature, both in the number of EROs and their optimal solutions: 44 EROs are presented, including four new EROs below 100m/s. We also study the Pareto fronts of the EROs for the first time, demonstrating that the retrieval cost is approximately constant for any transfer time, including single-impulse transfers onto a capture orbit for only marginally higher cost than two-impulse transfers for many EROs.

## **INTRODUCTION**

In recent years, Near-Earth Objects (NEOs) – asteroids and comets which pass within 1.3 AU of the Sun – have received considerable attention for scientific exploration and resource utilisation. This has lead to major space agencies either conducting or planning missions to investigate asteroids, including multiple missions to return samples of asteroids back to the Earth for further analysis. However, many scientific investigations require in-situ experiments, and the mass of samples that has been retrieved to date is small.<sup>1</sup>

To increase yield for commercial exploitation, attention has been given to retrieving entire asteroids, which aim to place a NEO in an orbit that keeps it closer to the Earth for access by subsequent missions.<sup>2</sup> Several methods of deflection have been proposed to alter the path of the NEO. For example, the gravity tractor uses the mutual attraction between a NEO and a nearby spacecraft to achieve a small deflection in the overall trajectory of the NEO, and the ion-beam shepherd<sup>3</sup> instead uses the momentum contained in a jet of plasma to alter the path of the NEO. These methods have also been investigated for planetary protection, where NEOs are deflected away from a collision course with the Earth.<sup>4–7</sup>

Previous research has studied using the invariant manifolds of the Circular-Restricted Three-body Problem (CR3BP) to facilitate asteroid retrieval, using the periodic orbits about the equilibria of the CR3BP as the destinations for the retrieved NEOs (Figure 1).<sup>8</sup> In particular, Reference 9 used the Sun-Earth CR3BP to identify a new class of 'Easily Retrievable Objects' (EROs), a subset of NEOs

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which may be retrieved into periodic orbits about the Sun-Earth  $L_1$  or  $L_2$  points using the invariant manifolds of the CR3BP for a total retrieval cost of less than 500 m/s. They find 17 EROs among the approximately 16000 NEOs known at the time.

We extend the previous literature in the field by first adjusting parameters used in the identification of the family of EROs, following analysis of the methods and results found in previous work. We go on to construct and study the Pareto fronts for these EROs for the first time through an extensive global optimisation procedure, trading off transfer time and transfer cost. This analysis shows that the transfer cost is approximately constant for all values of transfer time for the majority of EROs. This extends to zero-time transfers, where many EROs can be captured with a single impulsive manoeuvre at only marginally higher cost than the optimal two-impulse transfer. These discoveries greatly increase flexibility for mission designers. We also find 27 more EROs and significantly improve on previous solutions, including 17 new capture trajectories with a transfer  $\Delta v$  of less than 100 m/s.

This paper extends the previous literature in the field by performing a more extensive global optimisation procedure to construct transfer trajectories for these NEOs and to examine their Pareto fronts for the first time. We demonstrate that the transfer cost for the majority of EROs is approximately constant, and this extends to zero-time, single-impulse transfers, where many EROs can be captured with a single impulsive manoeuvre at only marginally higher cost than the optimal two-impulse transfer. This greatly increases flexibility for mission designers. In the process, 27 more EROs are found, and as a result of leveraging high-performance computing, 17 new capture trajectories with a retrieval cost of less than 100 m/s are identified.

# THE CIRCULAR-RESTRICTED THREE-BODY PROBLEM

The Circular-Restricted Three-Body Problem (CR3BP) is an autonomous dynamical system which studies the motion of an object of negligible mass under the influence of two far larger masses, known as primaries, which orbit each other in circles. The CR3BP is parameterised by  $\mu$ , the ratio of the masses of the two primaries. For this study, we use the Sun-Earth system, excluding the Moon, such that  $\mu = 3.0032080443 \times 10^{-6}$ .

The system has a single constant of motion which can be considered analogous to energy, known as the Jacobi constant J.

There are five equilibria of the CR3BP, the well-known Lagrangian points. Numbered  $L_1$  through  $L_5$ , these points admit periodic orbits which may be classified into certain families (Figure 1).<sup>10</sup> These periodic orbits have several types of invariant manifolds associated with them; the stable hyperbolic invariant manifold is formed of the infinite set of trajectories which asymptotically approach the orbit in forward time, and will transport an object onto a periodic orbit using only the natural dynamics of the system.

# **RETRIEVING NEAR-EARTH OBJECTS**

The method used to find EROs is divided into a three-step 'pipeline' that proceeds largely as in Reference 9, but with some minor alterations. An overview of the procedure is given here and is discussed in more detail in the following subsections.

We start by computing stable hyperbolic invariant manifolds associated with periodic orbits about the Sun-Earth  $L_1$  and  $L_2$  points. Planar and vertical Lyapunov, and Northern and Southern 'Halo' orbits are generated. The trajectories that form the manifolds are integrated backwards to their



**Figure 1**: The orbit families studied in this paper: planar and vertical Lyapunov, and Northern and Southern 'Halo' orbits. Highlighted in red are portions of the retrieval trajectories for asteroids 2020 DW (planar Lyapunov), and 2006 RH120 (Northern Halo). Only orbits about  $L_2$  are shown to aid clarity.

intersection with a fixed section forming a  $\pm \pi/8$  angle with the +x-axis of the synodic frame of the CR3BP. Keplerian dynamics are assumed outside of the 'cone' formed by the two fixed sections (Figure 2). The NEO database is downloaded using the JPL HORIZONS system \*.

An approximation of the transfer cost is then used to pre-filter the NEO catalogue to obtain retrieval candidates. The approximation uses an analytical Hohmann transfer between osculating NEO orbits and intersection points of the manifold with the  $\pm \pi/8$  sections. Any transfer below a given pre-filter threshold qualifies a NEO as a retrieval candidate.

Finally, two-impulse Lambert transfers are constructed between the retrieval candidate's osculating orbit at a given departure epoch and a point on a target invariant manifold outside of the  $\pm \pi/8$ cone. The departure epoch, transfer time  $t_t$ , and the target manifold and target manifold point are all varied in an optimisation procedure to achieve the minimum capture  $\Delta v$ . Any retrieval candidate with a capture  $\Delta v$  of less than 500 m/s is classified as an Easily Retrievable Object.

## **Generating the Invariant Manifolds**

The periodic orbits are pre-computed within specific ranges of Jacobi constant, given in Table 1. These ranges are chosen to be consistent with previous literature to provide a like-for-like comparison.

The first orbit of every family was found by correcting an initial guess, generated using the method outlined in Reference 11, into the full CR3BP dynamics. Other orbits in each family were

<sup>\*</sup>horizons.jpl.nasa.gov

Orbit Family	Jacobi constant Ranges
$L_1$ Planar	$[3.0003, \ 3.00087]$
$L_2$ Planar	$[2.99985, \ 3.00087]$
$L_1$ Halo	$[3.00042, \ 3.00082]$
$L_2$ Halo	$[3.00025, \ 3.00082]$
$L_1$ Vertical Lyapunov	[3.0002, 3.00087]
$L_2$ Vertical Lyapunov	$[2.99935, \ 3.00087]$

**Table 1**: The ranges of Jacobi constant for which orbits were generated, inclusive. 1000 orbits were generated with equi-spaced x-coordinates between the extrema given above. The energy ranges for Halo orbits extend to both the Northern and Southern types.



**Figure 2**: The  $\pm \frac{\pi}{8}$  sections used as the reference fixed sections in this paper:  $+\frac{\pi}{8}$  is for manifolds belonging to orbits about  $L_2$  and  $-\frac{\pi}{8}$  is for orbits about  $L_1$ . Outside of the 'cone' formed by the two sections, the dynamics are assumed to be Keplerian about the Sun; inside of the cone, full CR3BP dynamics are used.

then found by incrementing the initial x-coordinate for each orbit, with the increment chosen to give 1000 orbits equispaced in x in the relevant energy ranges. Orbits are assigned a unique index F, such that  $1 \le F \le 8000$ .

360 points per orbit are used to generate the structure of the manifold, and are located equi-spaced in time at regular intervals of  $\frac{\tau}{360}$  for an orbit with period  $\tau$ . These points are parameterised with an integer  $n_{\rm mnfd}$  for the  $F^{\rm th}$  orbit, such that  $1 \le n_{\rm mnfd} \le 360$ .

The manifolds are then integrated backwards to their respective fixed  $\frac{\pi}{8}$  section using a Dormand-Prince numerical integration scheme with relative and absolute tolerances of  $10^{-13}$ , and transformed from the synodic frame of the CR3BP into a Sun-centered inertial frame to simplify further analysis. Time in the inertial frame is measured in ephemeris seconds – seconds past an epoch, without leap seconds – such that its epoch is January 1<sup>st</sup> 2000, 12:00 (J2000), to simplify interactions with the SPICE library.

# **Downloading the NEO Database**

The Minor Bodies Database provided by the Jet Propulsion Laboratory was used as the reference database in this study. The ephemeris for each of the 22406 NEOs found in the Database as of the 24<sup>th</sup> Februrary 2020 was downloaded via the HORIZONS system. The ephemeris were downloaded between Jan 1, 2025, and Jan 1, 2100, with the default time-step. The coordinate system selected was the ecliptic J2000 frame, centred on the Sun. The downloads were performed following access limits for HORIZONS.

# **Removing unsuitable NEOs from the Database**

To reduce computational expense, we pre-filter the NEO database with a fast approximator to remove NEOs unlikely to become EROs. An idealised Hohmann transfer, between the NEO orbit and the equivalent Keplerian elements of the intersection of the manifolds with the relevant  $\pm \pi/8$  section, is used to approximate the optimal transfer. This greatly simplifies the orbital transfer by assuming that the aphelion and perihelion of the NEO and target manifold orbit lie on the line of nodes, and ignoring the relative orientation of the semi-major axes, but is sufficient for reducing a list of  $10^4$  NEOs to a list of  $10^2$  retrieval candidates.

The method for filtering unsuitable NEOs is given below, and largely follows Reference 9. However, we use a higher threshold of 3 km/s for classifying a given NEO as a retrieval candidate to prevent prematurely excluding EROs, and we use the ephemeris obtained earlier rather than pretabulated non-osculating orbital elements.

The cost of matching the aphelion and perihelion of the NEO's orbit and the equivalent orbit of the target manifold point is calculated first:

$$\Delta v_{a, 1} = \sqrt{\mu_s \left(\frac{2}{r} - \frac{1}{a_{\text{int}}}\right)} - \sqrt{\mu_s \left(\frac{2}{r} - \frac{1}{a_0}\right)}$$
$$\Delta v_{a, 2} = \sqrt{\mu_s \left(\frac{2}{r} - \frac{1}{a_f}\right)} - \sqrt{\mu_s \left(\frac{2}{r} - \frac{1}{a_{\text{int}}}\right)}$$
$$a_{\text{int}} = \frac{a_0 + a_f}{2}$$

where  $\mu_s$  is the Sun's gravitational parameter in km/s, and is r the radial distance to the Sun at which the transfer is made, in km. The variables  $a_0$ ,  $a_{int}$  and  $a_f$  are the initial, intermediate and final semi-major axes, respectively. Differences in inclination between the NEO orbit and target manifold point are considered by performing an inclination change manoeuvre  $\Delta v_i$  through an angle  $|i_f - i_0|$ :

$$\Delta v_i = 2\sqrt{\frac{\mu_s}{a}r^*}\sin\left(|i_f - i_0|/2\right)$$

where  $i_0$  and  $i_f$  are the initial and final orbit inclinations, respectively. The variable  $r^*$  is the ratio of aphelion distance to perihelion distance if the inclination change occurs at perihelion, or its inverse if the change occurs at aphelion. The variable *a* is the semi-major axis of the orbit whose inclination is being changed.

The total estimated capture cost is then computed using

$$\Delta v_t = \sqrt{\Delta v_{a,1}^2 + \Delta v_{i,1}^2} + \sqrt{\Delta v_{a,2}^2 + \Delta v_{i,2}^2}$$

where the inclination change is only performed once (i.e. one of  $\Delta v_{i, 1}$ ,  $\Delta v_{i, 2}$  is zero.)

The routines for the fast approximator are implemented in Fortran and linked to the SPICE library<sup>\*</sup> for access to the high-precision ephemerides. The filtering is performed such that the orbital elements of the NEOs are as of January 1<sup>st</sup> 2025 00:00. Little difference was found when repeating the pre-filter at regular intervals compared to the increased computational expense.

### Capture trajectory optimisation

Two-impulse heliocentric Lambert arcs in the inertial frame are optimised between the NEO and a target point in a target manifold, to determine capture trajectories below the ERO threshold of 500 m/s. The epoch of the first transfer manoeuvre, the transfer time, the target manifold, and the target point in the manifold are varied to give insertion onto the manifold for the minimum  $\Delta v$ . Note that we assume the transfer complete after the second Lambert impulse, and do not examine the motion of the NEO onto the final periodic orbit. Up to and including 3 full Lambert arc revolutions are considered.

Five design variables define the optimisation problem:  $t_0, t_t, t_{end}, n_{mnfd}$ , and F.

- $t_0$  is the epoch of the first manoeuvre of the Lambert arc and determines the initial position of the candidate from the ephemeris.
- $t_t$  is the duration of the Lambert arc, measured in days.
- F identifies a periodic orbit to target, as introduced previously.
- $n_{mnfd}$  identifies a trajectory on the stable manifold of the *F*-th periodic orbit, as introduced previously.
- $t_{end}$  is the time in the non-dimensional units of the synodic frame to arrive at the  $\pm \pi/8$  section from the manifold insertion point.

Since the manifolds are computed at discrete grid points, for non-integer values of  $n_{mnfd}$  or F the manifolds are interpolated using a cubic B-spline between the adjacent grid points to determine the properties of the manifold at that point. Non-zero values of  $t_{end}$  are integrated backwards from the  $\pm \pi/8$  section as part of the cost function.

Two objectives are considered in the optimisation problem: the optimal capture  $\Delta v$ , and the transfer time  $t_t$ , measured between the first manoeuvre and the insertion into the target manifold. This allows the determination of the Pareto fronts in the transfer cost *versus* transfer time space later in the study.

Constraints are introduced to restrict the optimisation. The epoch of the transfer was constrained such that the transfer must begin between calendar dates Jan. 1, 2025, and Jan. 1, 2100. This is consistent with previous studies on this topic, although the lower bound has been increased from Jan. 1, 2020, for practical reasons. The transfer time was also constrained to have a maximum length approximately equal to that in the previous literature, at 1500 days.

The optimisation is performed using MIDACO, the Mixed-Integer Distributed Ant Colony Optimiser<sup>12</sup> on 2.0 GHz Intel Xeon E5-2670 processors. The optimisation proceeds in two discrete

<sup>\*</sup>https://naif.jpl.nasa.gov/naif/toolkit.html



**Figure 3**: Number of retrieval candidates and EROs for a given pre-filter threshold. The use of 700 m/s as the pre-filter threshold, as in the previous literature, limits the final number of EROs by approximately 30%.

steps: a short, initial global search occurs first, which is followed by a series of more rigorous optimisations to generate the Pareto fronts with the transfer time  $t_t$  set fixed for an integer number of days between 0 and 1500. The second stage of optimisation, to generate the Pareto fronts, proceeds only for the retrieval candidates where the initial global search yields a solution below the ERO threshold, to save computation time. Approximately 5 days of CPU time is required to perform the initial global search for all the retrieval candidates found, and 450 days of CPU time is required to generate the Pareto fronts for the 44 EROs. All of the computational work in this study is performed on the IRIDIS5 High-Performance Computing Cluster at the University of Southampton.

# EASILY RETRIEVABLE OBJECTS

Applying the pre-filter to the NEO database identifies 792 retrieval candidates. The use of 3 km/s as the pre-filter threshold was selected as no EROs were discovered for thresholds between 3 km/s and 5 km/s.

Of these 792 retrieval candidates, 44 classify as EROs with an optimal transfer cost of less than 500 m/s. This is an increase of 27 on the previous literature. Transfers are found with capture  $\Delta v$  between 13.67 m/s and 445.35 m/s. 17 of these transfers are below 100 m/s, which represents significant improvements on the previous best-known capture trajectories for these objects.

Using the pre-filter threshold of 700 m/s found in the previous literature yields only 46 retrieval candidates, and reduces the number of EROs identified by approximately 30% (Figure 3). Thus, the authors believe the threshold used in previous literature was too low, and instead advocate for the higher threshold of 3 km/s to be used in future studies.



**Figure 4**: Pareto front for 2011 BL45 computed by MIDACO's in-built Pareto front generation and MIDACO's PlotTool. Only the points that define the front are included, and intermediate information on the dominating solutions is lost.

The optimal transfers found in this study are validated using a local optimisation to reproduce the transfer in the full dynamics of the CR3BP, including the arrival onto the final periodic orbit. There is little difference between the transfers in the Keplerian approximation used in the optimisation, and full CR3BP dynamics, validating the use of the Keplerian approximation outside of the  $\pi/8$  cone.

# PARETO FRONT GENERATION

In this section, we further examine the relationship between transfer time  $t_t$  and retrieval cost for the EROs found previously by generating the Pareto optimal solutions in the  $\Delta v$  versus  $t_t$  plane. To illustrate the difficulties encountered in this process, and the potential pitfalls, we present several approaches to the Pareto front generation before introducing our final method.

# Naïve MIDACO

MIDACO stores all the dominant solutions it finds during the optimisation process, and this can be extracted after the optimisation is complete. The number of points collected, and how points are collected, can be adjusted within the solver. MIDACO also provides PlotTool, a tool for plotting the Pareto fronts of the dominating solutions that MIDACO finds<sup>\*</sup>. However, the dominating solutions returned were clustered around the lower and upper end of the  $t_t$  range (Figure 4). While this is similar to the Pareto front for transfer times larger than about 200 days, as the points between the solutions in Figure 4 are dominated, the transfer cost for shorter transfers is vastly overestimated

<sup>\*</sup>http://www.midaco-solver.com/index.php/more/multi-objective



**Figure 5**: Preliminary transfer time  $t_t$  versus capture  $\Delta v$  space for 2011 BL45, found by continuing the solution at each value of  $t_t$  from the optimal solution at the previous  $t_t$ . This produces an approximately periodic cost across the range of  $t_t$ , with large differences between the lowest and highest capture  $\Delta v$ s.

in these results. The figure also provides no insight as to why the Pareto front has the resulting structure.

Therefore, we decided to generate denser Pareto fronts using a series of successive optimisations with the transfer time  $t_t$  fixed to an integer number of days between 0 and 1500, with all the other parameters introduced previously ( $t_0$ ,  $t_{end}$ ,  $n_{mnfd}$ , F) left free. Since all other parameters are left free, transfers for subsequent values of  $t_t$  may not be just continuations of each other, but instead spread widely around the parameter space. Careful implementation of the optimisation method is required to find the solutions across the full range of parameters.

Several techniques are used to alter the structure of the Pareto fronts and are discussed here with the example of 2011 BL45, which is not an ERO but exhibits the same behaviour. This is discussed in more detail later in this paper.

#### Naïve scanning

Our first attempt at Pareto front generation simply continued the search at each successive value of  $t_t$  from the optimal solution at the previous value of  $t_t$ . On the first iteration, the optimal solution found in the initial global search was used. The Pareto fronts are then 'grown' in two directions – increasing and decreasing transfer time – from this optimal solution. MIDACO was also configured such that its update rules are set to prioritise the currently best-known solution.

The results from this initial method are given in Figure 5, which displays a range of solutions between approximately 1000 m/s and 5000 m/s. The solution repeats almost periodically approx-



**Figure 6**: Preliminary structure of the  $\Delta v$  versus transfer time space for asteroid 2009 BD, when only the optimal solution at the previous transfer time is used as an initial guess for the parameters in the optimisation. The orbit of the NEO is in black, in red is the equivalent orbit of the manifold target point, and in blue is the transfer orbit between those points.

imately every 200 days, or half an orbital period of the NEO. However, we expected the transfer costs between different values of  $t_t$  to be similar, as the dense pre-computed grid of manifolds should contain a range of nearby points for which transfers exist with comparable  $\Delta v$ .

This is an issue with the use of a two-impulse Lambert arc in the optimiser cost function. As the transfer angle of the Lambert arc approaches  $\pm 180$  degrees, the Lambert arc approaches a singularity, where small changes in the start and endpoint of the arc yield very large changes in its inclination, potentially altering the cost of the transfer dramatically. This is shown graphically in Figure 6, which shows a preliminary Pareto front for asteroid 2009 BD with the transfers themselves shown in blue in insets.

For 2011 BL45, this instability causes the optimiser to discover a new family of Lambert arcs at approximately  $t_t = 100$  days, which are themselves relatively unstable. These solutions are then continued down to a transfer time of zero days. This instability can be avoided by seeding the optimiser with multiple initial guesses to examine regions away from the instability, and potentially through the use of a three-impulse transfer.

#### **Global scanning**

To prevent premature convergence to local minima, and to avoid instabilities caused by the Lambert transfer, we include multiple initial guesses for each  $t_t$  to promote exploration of the parameter



Figure 7: Pareto front for 2011 BL45 when generated using multiple initial guesses for each value of  $t_t$ . There is a pronounced 'floor' in the optimisation but transfers with similar  $t_t$  can have significantly different  $\Delta v$ .

space by the optimiser. The guesses and subsequent sets of design variables are evaluated in parallel, such that the total number of guesses and design variable sets is equal to the total number of cores.

One of these guesses is set to the previous optimal solution at the previous time-step (or the optimal solution from the initial global search if this is unavailable). The remaining guesses are initialised randomly throughout the parameter space. Subsequent sets of design variables are also evaluated simultaneously. The parameters for MIDACO are adjusted to force the solver to be less 'greedy', and place less emphasis on the currently best-known solution.

Figure 7 shows the resulting Pareto front, again for asteroid 2011 BL45, using this modified strategy of including multiple initial guesses. Solutions are included with transfer costs between 625 m/s and 2650 m/s. With this strategy, there is a pronounced 'floor' in the Pareto front around the optimal capture  $\Delta v$  of approximately 625 m/s. However, there is also significant variance in the solutions identified, and solutions close to each other in terms of transfer time are often very different in both transfer cost and the values of the design variables.

This is a result of the optimiser no longer converging at every time step due to the larger search space. In this figure, each value of  $t_t$  is generated using 60 seconds of wall-time on 48 cores, or approximately  $70 \times 10^3$  function evaluations. Further analysis shows that approximately  $500 \times 10^3$  function evaluations gives convergence for the majority of candidates. The strategy was modified to run for 10 minutes per value of  $t_t$ . While MIDACO does provide the ability to terminate based upon an estimation of convergence, it was found to be simpler to interface the codes with the IRIDIS HPC scheduling system when fixed times were used.



**Figure 8**: Pareto front for NEO 2011 BL45. While not an ERO, with an optimal capture  $\Delta v$  of approximately 625 m/s, it still displays the same behaviour. Non-EROs may, therefore, offer similar flexibility to the mission designer as EROs.

## Fully converged global scan

The final Pareto fronts for asteroids 2011 BL45 and 2012 TF79 are given in Figures 8 and 9, respectively. 2012 TF79 admits an optimal capture  $\Delta v$  of 90.82 m/s, but is actually retrievable for any transfer time between 0 and 1500 days, including a zero-time, single-impulse transfer. In fact, 34 of the 44 EROs identified remain EROs for a zero-time, single impulse transfer, and remain generally low-cost solutions. The  $\Delta v$ -optimal solutions, and the ratio between the single-impulse  $\Delta v$  and the optimal two-impulse  $\Delta v$ , is given in Table 2 for all the EROs identified.

One object, SPK ID 3648046, has a single-impulse transfer far higher than the optimal  $\Delta v$ . This was found to be due to an unstable Lambert arc – as discussed previously – affecting convergence at the previous value of  $t_t = 1$  day. Given that there are only relatively few points in the manifold suitable for capture with a single impulse, the optimiser is dependent on continuing the solution from a transfer time of 1 day to 0 days. The optimiser was then unable to converge for the zero-impulse transfer. We believe this solution could be improved significantly by manual optimisation and thus represents an outlier.

In general, the optimal capture  $\Delta v$  for a given ERO is not unique and is approximately constant through the  $t_t$  versus  $\Delta v$  space. This offers significant flexibility to the mission designer. These NEOs may also be more promising as retrieval targets since they are more resilient to changes or to disruptions affecting mission timelines. These objects may also allow for material samples to be returned over a longer time without the need for full capture, a strategy which would see samples returned at regular intervals via the invariant manifolds of the Sun-Earth system, and return to a Lagrange point of the system using only natural dynamics. The single-impulse transfers also often

SPK ID	$\Delta v[m/s]$	Transfer epoch	Transfer end	Arrival at $\pm \frac{\pi}{8}$ plane	Orbit	$\frac{\text{Single-impulse }\Delta v}{\text{Optimal }\Delta v}$	Single- impulse orbit
3005816	162.60	2084/12/21	2085/07/06	2086/02/01	L1V	3.86	L1P
3054374	27.39	2029/02/04	2030/10/05	2032/08/04	L1N	2.27	L1P
3076774	41.60	2091/09/26	2093/06/06	2095/07/07	L1V	15.48	L2N
3315004	27.55	2074/09/30	2075/03/22	2078/05/17	L1S	3.18	L1S
3390109	166.09	2090/01/13	2091/02/27	2092/02/19	L2V	2.48	L2P
3403148	63.83	2025/03/19	2027/03/11	2027/03/13	L2N	1.83	L2N
3405189	50.94	2087/10/22	2091/06/26	2093/01/24	L1N	4.48	L1P
3405338	13.97	2083/08/31	2086/11/12	2089/06/27	L1S	25.00	L2V
3435539	42.23	2029/06/16	2030/07/20	2030/08/04	L1V	8.76	L2P
3444297	136.52	2074/07/15	2076/10/29	2080/04/03	L1N	2.98	L2P
3519518	140.95	2057/09/13	2060/12/01	2061/09/02	L1P	2.36	L1P
3549639	146.66	2092/11/17	2095/12/09	2098/08/30	L2V	2.42	L1P
3550232	195.40	2059/10/03	2062/03/25	2065/01/24	L1N	1.71	L1N
3551168	180.83	2035/02/16	2036/08/25	2037/05/30	L2V	1.30	L2V
3556127	169.01	2073/01/15	2073/10/30	2073/11/24	L2V	1.54	L2P
3568303	426.35	2063/03/19	2065/12/07	2067/12/16	L2V	1.23	L2V
3582145	111.98	2039/11/11	2040/07/18	2040/07/18	L1V	1.74	L1V
3610175	90.82	2097/06/19	2100/12/02	2103/11/20	L2V	1.27	L2V
3618493	140.38	2085/10/31	2089/08/16	2090/11/08	L1S	2.17	L1P
3625129	203.01	2090/12/27	2092/02/26	2092/05/11	L1S	1.60	L1N
3648046	355.29	2041/11/26	2045/09/23	2049/04/05	L2V	2032	L2V
3696401	69.84	2073/11/14	2076/03/14	2079/12/15	L1N	1.74	L1P
3697803	115.13	2038/07/11	2040/03/31	2041/03/09	L1V	1.35	L1V
3698849	23.89	2091/05/27	2095/07/05	2096/02/06	L1S	18.27	L2P
3702319	56.93	2080/12/13	2084/01/05	2085/03/28	L1V	3.24	L1N
3719859	445.35	2036/03/14	2039/03/02	2040/11/08	L2N	1.35	L2N
3726012	108.25	2047/04/27	2051/01/14	2053/06/13	L1N	3.95	L2P
3733264	23.68	2037/10/07	2040/08/23	2044/02/08	L1V	2.40	L1V
3735181	123.71	2064/12/07	2068/09/14	2071/12/06	L1P	2.03	L1P
3745994	301.71	2030/01/29	2033/10/17	2035/07/21	L2S	1.76	L1P
3748467	332.02	2045/07/11	2047/07/20	2049/07/17	L1P	2.02	L1P
3759358	89.76	2048/06/09	2051/07/02	2052/04/18	L2N	3.82	L2V
3781977	305.79	2072/12/16	2073/07/23	2075/01/11	L2V	1.09	L2V
3781983	245.83	2099/06/01	2102/07/12	2105/07/20	L1P	1.65	L1P
3825158	170.93	2095/05/25	2099/04/20	2102/08/26	L1P	3.20	L1P
3826850	254.37	2069/08/12	2071/04/19	2074/05/24	L2V	2.20	L1P
3836857	51.52	2049/01/24	2051/10/17	2052/11/10	L2V	1.66	L2V
3840784	81.52	2083/02/05	2086/04/24	2089/10/10	L1V	7.80	L1P
3843512	33.06	2087/07/16	2090/04/14	2092/06/05	L1V	3.13	L1V
3843865	15.25	2093/06/22	2096/06/08	2096/08/01	L2S	30.62	L2S
3879383	128.16	2051/07/27	2053/12/07	2057/07/19	LIN	2.02	LIS
3893945	94.96	2099/10/21	2103/02/09	2106/12/15	LIN	3.64	LIP
3989308	116.72	2082/08/03	2086/07/09	2088/11/26	L2V	2.54	
3991650	112.27	2026/05/08	2028/03/07	2031/12/31	L2V	1.81	L2S

**Table 2**: Table of the  $\Delta v$ -optimal solutions for the EROs found, and the ratio of single-impulse  $\Delta v$  to the optimal  $\Delta v$ ; many single-impulse transfers exist at similar or less cost than two-impulse transfers. 24 EROs are retrieved into different periodic orbits for the single-impulse transfer. Dates are given in the form YYYY/MM/DD.



**Figure 9**: Pareto front for 2012 TF79, an ERO with all transfers below the ERO threshold, including a zero-time, single impulse transfer. The Pareto front is shown as a black line.

have different transfer properties, including the final periodic orbit, than the optimal two-impulse transfer.

This behaviour is to be expected from the use of the invariant manifolds to facilitate NEO capture. In backwards time, the structure of the stable manifold 'stretches' and spans large regions of space in the Sun-Earth system. Thus, for any possible target point, there exist other points in the set of pre-computed manifolds that are retrievable for similar  $\Delta v$ . A dense underlying grid of manifolds and target points increases the likelihood of discovering low-cost transfers for all possible transfer times, provided the parameter space is not too large for the optimiser.

The near-constant transfer cost also extends to objects that do not meet the traditional definition of an ERO. 2011 BL45, the NEO introduced earlier to discuss the preliminary tuning of optimiser behaviour is not an ERO but exhibits the same behaviour as the EROs identified. The zero-time transfer and optimal capture  $\Delta v$  for this object differ by only 2.5%.

We remark that the orbit family used for capturing the NEO for minimum cost and the singleimpulse transfer differ for 24 of the 44 EROs found in Table 2. While this could be enforced in the optimisation procedure if a specific final periodic orbit is desired, we left the optimiser free to find the minimum capture  $\Delta v$ . The transfer epoch and insertion point onto the manifold can also be significantly different from the  $\Delta v$ -optimal solutions.

Note also the 'flat' Pareto front as a result of the optimisation strategy and optimiser parameter tuning, when contrasted to the first approach to the Pareto front generation in Figure 5. Based on our work, we believe that with further computation time, the lowest transfer  $\Delta v$  for some NEOs could be improved even further, which would also help 'flatten' the structure of their Pareto fronts. This includes the case of the single-impulse transfer for 3648046, introduced previously.

#### CONCLUSION

We have presented the Pareto fronts for 'Easily Retrievable Objects', NEOs which can be retrieved using the invariant manifolds of the Circular-Restricted Three-body Problem for a total capture cost of less than 500 m/s. We find that the optimal transfer cost for these EROs, and indeed more generic NEOs, is not unique. The existence of single-impulse transfers for a similar cost to two-impulse transfers could allow for significant flexibility in the design of space missions. In studying these Pareto fronts, we have identified an additional 27 EROs among the currently-known NEO population and improved on the majority of their best-known solutions.

While we have improved on the majority of the best-known solution for EROs, the transfers we find often have different properties, such as transfer time and epoch, than previously found; this may be explained by revisions to the states of the NEOs in the database as well as our more exhaustive global optimization.

The authors are also aware that the assumption of an impulsive transfer between the NEO and the target manifold point is a limitation in this study, as the mass of many EROs makes these impulsive transfers impossible. However, given that the length of many of the transfers is on the order of hundreds of days, these transfers could be recreated in a constant low-thrust transfer instead of using two impulsive approximations, as was done in Reference 9.

The authors also believe that the number of EROs among the NEO population is higher than is reported here and that more intensive computation would yield even more EROs from the pool of retrieval candidates.

To facilitate future analysis, the codes and data used to perform this study are openly available from the University of Southampton repository at https://doi.org/10.5258/SOTON/D1631. As further investigations proceed, the codebase will likely be updated; a more up-to-date version is available at http://doi.org/10.5281/zenodo.4317077.

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