Multi-Pair Two-Way Massive MIMO DF Relaying Over Rician Fading Channels Under Imperfect CSI

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***Abstract*—We investigate a multi-pair two-way decode-and-forward relaying aided massive multiple-input multiple-output antenna system under Rician fading channels, in which multiple pairs of users exchange information through a relay station having multiple antennas. Imperfect channel state information is considered in the context of maximum-ratio processing. Closed-form expressions are derived for approximating the sum spectral efficiency (SE) of the system. Moreover, we obtain the power-scaling laws at the users and the relay station to satisfy a certain SE requirement in three typical scenarios. Finally, simulations validate the accuracy of the derived results.**

***Index Terms*—Massive MIMO, rician fading channels, decode-and-forward, two-way relaying, power-scaling law.**

I. Introduction

Driven by the dramatically increasing tele-traffic requirements, massive multiple-input multiple-output (MIMO) techniques have been extensively studied [1], where the users exchange their information via a base station (BS) equipped with hundreds of antennas. Compared with traditional systems, massive MIMO systems substantially increase the spectral efficiency (SE) and energy efficiency (EE) owing to their reduced transmit power [2]. Hence, massive MIMO techniques play an important role in the current/next-generation networks.

The integration of massive MIMO applications with relay protocols can increase the network capacity and extend the coverage [3]. The power scaling laws of a one-way (OW) relay network were studied [4]. However, OW relaying has the drawback of low SE, which may potentially be doubled by two-way (TW) relaying protocols. Explicitly, in TW relaying systems, multiple user pairs communicate with each other in a unique bidirectional channel [5]. Thus, mult-pair TW relaying systems have been developed to further improve the SE by adopting maximum-ratio (MR) processing at the relay.

This work was supported in part by the NSFC under Grant 61701307, and the open research fund of National Mobile Communications Research Laboratory, Southeast University under Grant 2018D14. The work of Lajos Hanzo was supported by the European Research Councils Advanced Fellow Grant QuantCom (Grant No. 789028). (*Corresponding author: Cunhua Pan.*)

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Generally, two main relaying protocols are widely used: amplify-and-forward (AF) and decode-and-forward (DF). DF relays decode the received signals before forwarding the re-encoded signals from a lower distance, which avoids interference and noise amplification [6]. Additionally, DF TW relaying is capable of performing independent precoding and power allocation in each communication direction [7]. In practice, massive MIMO systems generally operate in line-of-sight (LOS) propagation conditions [8], and Rician fading accurately models both LOS and diffuse scattered components [9]–[11]. Despite this, there are a paucity of analytical contributions under Rician fading channels for massive MIMO aided TW relaying system with imperfect CSI.

II. System Model

We first study a multi-pair TW massive MIMO half-duplex DF relaying system that has N pairs of users each employing a single antenna and an M-antenna relay (T_R) under imperfect CSI. The users at both ends are denoted by $U_{A,i}$ and $U_{B,i}$, for $i = 1, ..., N$. Additionally, none of the communicating users has direct LOS links and can only exchange information through the TW relay. The relay operates in time-division-duplex (TDD) mode. We assume the reciprocity of the channels, and denote the uplink (UL) and downlink (DL) channels between $U_{X,i}$ and T_R by $\mathbf{h}_{XR,i}$ and $\mathbf{h}_{XR,i}^T$, respectively, where $X = A, B$ and $i = 1, \ldots, N$. Additionally, the channel matrix is formed as $\mathbf{H}_{XR} = [\mathbf{h}_{XR,1}, \ ..., \ \mathbf{h}_{XR,N}]$, $X = A, B$. Then, the channel vector $\mathbf{h}_{XR,i}$ is expressed as $\mathbf{h}_{XR,i} = \mathbf{g}_{XR,i}\sqrt{\beta_{XR,i}}$, where $\mathbf{g}_{XR,i}$ represents the fast-fading element, while $\beta_{XR,i}$ is the path-loss coefficient. We assume that all the channels obey Rician distribution, and they are expressed as [9]

$$\mathbf{g}_{XR,i}=\sqrt{\frac{K_{XR,i}}{K_{XR,i}+1}}\bar{\mathbf{g}}_{XR,i}+\sqrt{\frac{1}{K_{XR,i}+1}}\tilde{\mathbf{g}}_{XR,i}, X\in\{A,B\}\,, \quad (1)$$

where $\bar{\mathbf{g}}_{XR,i}$ denotes the LOS part representing the deterministic component, $\tilde{\mathbf{g}}_{XR,i}$ denotes the scattered part representing the random component, and $K_{XR,i}$ is the Rician K-factor. Perfect CSI is challenging to obtain for all the antennas. We use the MMSE estimator at T_R to estimate $\mathbf{H}_{AR}$ and $\mathbf{H}_{BR}$ [9], where we have $\mathbf{h}_{AR,i} = \hat{\mathbf{h}}_{AR,i} + \mathbf{e}_{AR,i}$ and $\mathbf{h}_{BR,i} = \hat{\mathbf{h}}_{BR,i} + \mathbf{e}_{BR,i}$; $\hat{\mathbf{h}}_{AR,i}$ and $\hat{\mathbf{h}}_{BR,i}$ are the ith columns of the estimated matrices $\hat{\mathbf{H}}_{AR}$ and $\hat{\mathbf{H}}_{BR}$; $\mathbf{e}_{AR,i}$ and $\mathbf{e}_{BR,i}$ are the ith columns of the estimation error matrices $\mathbf{E}_{AR}$ and $\mathbf{E}_{BR}$, respectively. $\hat{\mathbf{H}}_{XR}$ and $\mathbf{E}_{XR}$ ($X = A$ or B) are independent. Based on the assumption of the worst-case uncorrelated Gaussian noise, we can respectively obtain the variance of the estimation error vector elements $\mathbf{e}_{AR,i}$ and $\mathbf{e}_{BR,i}$ as $\sigma^2_{AR,i} = \frac{\beta_{AR,i}}{(1+\tau p_p \beta_{AR,i})(K_{AR,i}+1)}$ and $\sigma^2_{BR,i} =$

$\frac{\beta_{BR,i}}{(1+\tau p_p\beta_{BR,i})(K_{BR,i}+1)}$, where τ denotes the channel training interval and p_p is the transmit power of each pilot symbol.

The data transmission process is composed of two separate phases. First, all N user pairs $(U_{A,i}, U_{B,i})$ simultaneously transmit their signals, $\left(\sqrt{p_{A,i}}x_{A,i}, \sqrt{p_{B,i}}x_{B,i}\right)$, to T_R in the UL phase. Thus, the UL signal received at T_R is expressed as

$$\mathbf{y}_r = \sum_{i=1}^{N}\left(\sqrt{p_{A,i}}\mathbf{h}_{AR,i}x_{A,i} + \sqrt{p_{B,i}}\mathbf{h}_{BR,i}x_{B,i}\right) + \mathbf{n}_R, \quad (2)$$

where $p_{A,i}$ and $p_{B,i}$ are the UL transmit powers of $U_{A,i}$ and $U_{B,i}$, respectively. The variables $x_{A,i}$ and $x_{B,i}$ respectively denote the signals transmitted by $U_{A,i}$ and $U_{B,i}$ with $\mathbb{E}\{|x_{A,i}|^2\} = \mathbb{E}\{|x_{B,i}|^2\} = 1$, where $\mathbb{E}\{\cdot\}$ represents the expectation operator. The vector $\mathbf{n}_R \sim \mathcal{CN}\left(\mathbf{0}, \sigma_r^2\mathbf{I_N}\right)$ is the additive white Gaussian noise (AWGN) at T_R. The UL signal received at T_R is decoded by multiplying it with the linear processing matrix $\mathbf{F}_u$, yielding

$$\mathbf{Z}_r = \mathbf{F}_u\mathbf{y}_r, \quad (3)$$

where we have $\mathbf{F}_u = \left[\hat{\mathbf{H}}_{AR}, \hat{\mathbf{H}}_{BR}\right]^H$. From (2) and (3), we can derive the received signal of the ith pair of users after linear processing [7].

By contrast, in the DL phase, the signals received from all the users are decoded at the relay before transmission, while $\mathbf{F}_d$ is the linear precoding matrix which is applicable for the decoded signal $\mathbf{x}$. Therefore, the DL signal transmitted from the relay T_R is given by

$$\mathbf{y}_t = \rho\mathbf{F}_d\mathbf{x}, \quad (4)$$

where $\mathbf{x} = \left[\mathbf{x}_A^T, \mathbf{x}_B^T\right]^T$, $\mathbf{F}_d = \left[\hat{\mathbf{H}}_{BR}, \hat{\mathbf{H}}_{AR}\right]^*$, and ρ is adjusted for satisfying the transmit power constraint at the relay, i.e., $\mathbb{E}\left\{\|\mathbf{y}_t\|^2\right\} = p_r$ and $\rho = \sqrt{\frac{p_r}{\mathbb{E}\left\{\|\mathbf{F}_d\|^2\right\}}}$.

Finally, the signals are forwarded to their respective destinations by the relay and the DL signal received at $U_{X,i}$ ($X = A$ or B) is given by

$$z_{X,i} = \mathbf{h}_{XR,i}^T\mathbf{y}_t + \mathbf{n}_{X,i}, \quad (5)$$

where $\mathbf{n}_{X,i} \sim \mathcal{CN}\left(0, \sigma_{X,i}^2\right)$ is the AWGN at $U_{X,i}$.

III. Spectral Efficiency Analysis

In this section, we investigate the SE of the TW half-duplex DF relaying system when imperfect CSI is considered at T_R. The achievable sum SE of the system is defined as

$$R = \sum_{i=1}^{N} R_i \quad (6)$$

where R_i is the SE of the ith user pair, and it is defined as

$$R_i = \min\left(R_{1,i}, R_{2,i}\right) \quad (7)$$

In (7), $R_{1,i}$ is the SE of the ith user pair in the UL phase and $R_{2,i}$ is the SE of the ith user pair in the DL phase. Without loss of generality, we will derive its closed-form approximations for the ith user pair.

1) As is in practical cases, the relay uses the estimated channel for signal detection. Then, for the imperfect CSI case, $R_{1,i}$ is obtained as

$$R_{1,i} = \lambda\mathbb{E}\left\{\log_2\left(1 + \frac{A_i + B_i}{C_i + D_i + E_i}\right)\right\}, \quad (8)$$

where we have $\lambda = \frac{T-\tau}{2T}$, while A_i and B_i represent the signals which $U_{A,i}$ and $U_{B,i}$ want to receive. Furthermore, C_i, D_i and E_i represent the estimation error, the inter-user interference and the compound noise, respectively. The expressions of these five terms are given by

$$A_i = p_{A,i}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\hat{\mathbf{h}}_{AR,i}\right|^2 + \left|\hat{\mathbf{h}}_{BR,i}^H\hat{\mathbf{h}}_{AR,i}\right|^2\right), \quad (9)$$

$$B_i = p_{B,i}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\hat{\mathbf{h}}_{BR,i}\right|^2 + \left|\hat{\mathbf{h}}_{BR,i}^H\hat{\mathbf{h}}_{BR,i}\right|^2\right), \quad (10)$$

$$C_i = p_{A,i}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{e}_{AR,i}\right|^2 + \left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{e}_{AR,i}\right|^2\right) + p_{B,i}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{e}_{BR,i}\right|^2 + \left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{e}_{BR,i}\right|^2\right), \quad (11)$$

$$D_i = \sum_{j\neq i} p_{A,j}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{AR,j}\right|^2 + \left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{h}_{AR,j}\right|^2\right) + \sum_{j\neq i} p_{B,j}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{BR,j}\right|^2 + \left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{h}_{BR,j}\right|^2\right), \quad (12)$$

$$E_i = \left\|\hat{\mathbf{h}}_{AR,i}\right\|^2 + \left\|\hat{\mathbf{h}}_{BR,i}\right\|^2. \quad (13)$$

Furthermore, the SE of the link $U_{X,i} \rightarrow T_R$ ($X = A$ or B) is given by

$$R_{XR,i} = \lambda\mathbb{E}\left\{\log_2\left(1 + \frac{X_i}{C_i + D_i + E_i}\right)\right\}. \quad (14)$$

In the DL phase, we can express the signals processed at $U_{X,i}$, ($X = A$ or B), after partial SIC according to (21) in [7].

The SE of the link $T_R \rightarrow U_{X,i}$ can be expressed as

$$R_{RX,i} = \lambda\log_2\left(1 + \mathrm{SINR_{RX,i}}\right), \quad (15)$$

where $X \in \{A, B\}$ and $\mathrm{SINR_{RX,i}}$ defined in (16) (at the bottom of the next page) is the corresponding SINR of $U_{X,i}$. In (16), $\{\bar{X}\} = \{A, B\} \setminus \{X\}$. Thus, $R_{2,i}$ is defined as

$$R_{2,i} = \min\left(R_{AR,i}, R_{RB,i}\right) + \min\left(R_{BR,i}, R_{RA,i}\right). \quad (17)$$

2) In the following theorem, the closed-form approximation of R_i under imperfect CSI is formulated.

Theorem 1: In the imperfect CSI scenario, when the numberof the relay antennas, M, tends to infinity, the SE of the ith user pair employing MRC receivers is approximated as

$$\tilde{R}_i = \min\left(\tilde{R}_{1,i}, \tilde{R}_{2,i}\right), \quad (18)$$

where

$$\tilde{R}_{1,i} = \lambda\log_2\left(1 + \frac{Mp_{A,i}\omega_{AR,i}^2 + Mp_{B,i}\omega_{BR,i}^2}{\left(\omega_{AR,i} + \omega_{BR,i}\right)q_i + Q_i}\right), \quad (19)$$

$$\tilde{R}_{2,i} = \min\left(\tilde{R}_{AR,i}, \tilde{R}_{RB,i}\right) + \min\left(\tilde{R}_{BR,i}, \tilde{R}_{RA,i}\right), \quad (20)$$

with

$$\tilde{R}_{XR,i} = \lambda\log_2\left(1 + \frac{Mp_{X,i}\omega_{XR,i}^2}{\left(\omega_{AR,i} + \omega_{BR,i}\right)q_i + Q_i}\right), \quad (21)$$

$$\tilde{R}_{RX,i} = \lambda\log_2\left(1 + \frac{Mp_r\omega_{XR,i}^2}{\sum_{j=1}^{N}\left(\omega_{AR,j} + \omega_{BR,j} + Z_{ij}\right)}\right), \quad (22)$$

$$Q_i = \sum_{j\neq i}^{N}\left(p_{A,j}(\xi_{AR,ij} + \xi_{BR,ij}) + p_{B,j}(\chi_{AR,ij} + \chi_{BR,ij})\right), \quad (23)$$

$$Z_{ij} = p_r(\zeta_{AR,ij} + \zeta_{BR,ij}), \quad (24)$$

and $X \in \{A, B\}$, $\omega_{XR,i} = \frac{\beta_{XR,i}}{K_{XR,i}+1}(K_{XR,i} + \eta_{XR,i})$, $\eta_{XR,i} = \frac{\tau_p p_p\beta_{XR,i}}{1+\tau_p p_p\beta_{XR,i}}$, $q_i = p_{A,i}\sigma_{AR,i}^2 + p_{B,i}\sigma_{BR,i}^2 + 1$. The terms $\xi_{AR,ij}$ and $\chi_{AR,ij}$ in (23) are respectively defined by (25) and (26) at the bottom of the next page, while $\zeta_{XR,ij}$ in (24) is defined by (27) at the bottom of the next page.

Proof: See Appendix A. ■

Theorem 1 presents the approximate expression of the SE for the ith user pair under imperfect CSI. When $\beta_{AR,i}$, $\beta_{BR,i}$, $K_{AR,i}$, $K_{BR,i}$, $\sigma_{A,i}$ and $\sigma_{B,i}$ are kept fixed, the SE is determined by the number of the user pairs N, the number of the relay antennas M and the transmit power $p_{A,i}$, $p_{B,i}$ and p_r. For fixed $p_{A,i}$, $p_{B,i}$, p_r and p_p, we can see that $\tilde{R}_{1,i}$ and $\tilde{R}_{2,i}$ increase unboundedly in both the UL and DL phases as the number of relay antennas increases. Furthermore, in following Section IV, we will use *Theorem 1* to investigate how the powers can be scaled down when the number of the relay antennas increases infinitely. It can be found that the sum SEs will converge to the upper limits for three typical cases, as $M \to \infty$. Additionally, the simulation in Section V verifies that the SE of the ith user pair increases with the Rician K-factor.

IV. Power-Scaling Laws

In this section, we quantify the power-scaling laws explicitly, we analyze how the powers can be scaled down upon increasing M, while maintaining a certain SE. Additionally, the transmit power of all users is set to be the same, i.e., $p_{X,i}$ = p_u, $X \in \{A, B\}$.

We have $p_u = \frac{E_u}{M^{\alpha}}$, $p_r = \frac{E_r}{M^{\varepsilon}}$, and $p_p = \frac{E_p}{M^{\gamma}}$, while E_u, E_r and E_p are all constants, $\alpha > 0$, $\varepsilon > 0$, and $\gamma > 0$. Then, it can be obtained from *Theorem 1* that as $M \to \infty$, we have $\eta_{XR,i} \to 0$, $Q_i \to 0$, $Z_{ij} \to 0$, $q_i \to 1$. Thus, as $M \to \infty$, $\tilde{R}_i$ defined by (18) in *Theorem 1* converges according to

$$\tilde{R}_i \xrightarrow{M\to\infty} \min\left(\bar{R}_{1,i}, \bar{R}_{2,i}\right) \triangleq \bar{R}_i, \tag{28}$$

where

$$\bar{R}_{1,i} = \lambda \log_2\left(1 + \frac{E_u}{M^{\alpha-1}} \frac{\psi_{AR,i}^2 + \psi_{BR,i}^2}{\psi_{AR,i} + \psi_{BR,i}}\right), \tag{29}$$

$$\bar{R}_{2,i} = \min\left(\bar{R}_{AR,i}, \bar{R}_{RB,i}\right) + \min\left(\bar{R}_{BR,i}, \bar{R}_{RA,i}\right), \tag{30}$$

with

$$\bar{R}_{XR,i} = \lambda \log_2\left(1 + \frac{E_u}{M^{\alpha-1}} \frac{\psi_{XR,i}^2}{\psi_{AR,i} + \psi_{BR,i}}\right), \tag{31}$$

$$\bar{R}_{RX,i} = \lambda \log_2\left(1 + \frac{E_r}{M^{\varepsilon-1}} \frac{\psi_{XR,i}^2}{\sum\limits_{j=1}^{N} \left(\psi_{AR,i} + \psi_{BR,i}\right)}\right), \tag{32}$$

$$\psi_{XR,i} = \frac{\beta_{XR,i} K_{XR,i}}{K_{XR,i} + 1}. \tag{33}$$

We observe that the asymptotic SEs of the ith user pair are closely related to the values of α and ε. Moreover, we find that R_i is independent of γ when M becomes large. Next, we analyze the effect of α and ε on the SE.

- When $\alpha > 1$ or $\varepsilon > 1$, R_i converge to zero. When p_u or p_r is reduced excessively, the asymptotic SEs will tend to zero.
- When $0 < \alpha < 1$ and $0 < \varepsilon < 1$, R_i grow without limit. This suggests that p_u and p_r can be reduced more drastically to obtain fixed SEs.
- When $\alpha = 1$, $0 < \varepsilon \leq 1$ or $\varepsilon = 1$, $0 < \alpha \leq 1$, R_i converge to a positive limit. We now study how much p_u or p_r or both can be scaled down, while maintaining a certain SE. We focus on three cases: 1) Case I: $\alpha = \varepsilon = 1$; 2) Case II: $\alpha = 1$, and $0 < \varepsilon < 1$; 3) Case III: $0 < \alpha < 1$ and $\varepsilon = 1$.

A. **Case I:** $\alpha = \varepsilon = 1$.

For fixed E_u, E_r and E_p, by substituting $\alpha = 1$ and $\varepsilon = 1$ into (29) - (32), as $M \to \infty$, we can simplify the SE in (29) - (32) as

$$\bar{R}_{1,i} = \lambda \log_2\left(1 + \frac{E_u\left(\psi_{AR,i}^2 + \psi_{BR,i}^2\right)}{\psi_{AR,i} + \psi_{BR,i}}\right), \tag{34}$$

$$\bar{R}_{2,i} = \min\left(\bar{R}_{AR,i}, \bar{R}_{RB,i}\right) + \min\left(\bar{R}_{BR,i}, \bar{R}_{RA,i}\right), \tag{35}$$

with

$$\bar{R}_{XR,i} = \lambda \log_2\left(1 + \frac{E_u \psi_{XR,i}^2}{\psi_{AR,i} + \psi_{BR,i}}\right), \tag{36}$$

$$\bar{R}_{RX,i} = \lambda \log_2\left(1 + \frac{E_r \psi_{XR,i}^2}{\sum\limits_{j=1}^{N} \left(\psi_{AR,j} + \psi_{BR,j}\right)}\right). \tag{37}$$

Based on (34) - (37), the limit of R_i also increases with E_u and E_r, and decreases with N.

B. **Case II:** $\alpha = 1$, *and* $0 < \varepsilon < 1$.

For fixed E_u, E_r and E_p, substituting $\alpha = 1$ and $0 < \varepsilon < 1$ into (28) - (32), we can obtain that $\tilde{R}_i$ converges to $\bar{R}_{1,i}$ given by (34), i.e., $\tilde{R}_i \to \bar{R}_{1,i}$, as $M \to \infty$.

We find that the asymptotic SE of R_i is decided by the UL phase, which increases with E_u, while E_r has no effect on the asymptotic SE. Furthermore, as M increases, the SE of each user will become lower in the UL than that of the DL phase.

C. **Case III:** $0 < \alpha < 1$ *and* $\varepsilon = 1$.

For fixed E_u, E_r and E_p, by substituting $0 < \alpha < 1$ and $\varepsilon = 1$ into (28) - (32), $\tilde{R}_i$ converges to $\bar{R}_{2,i}$ given by (38) (at the bottom of the next page), i.e., $\tilde{R}_i \to \bar{R}_{2,i}$, when $M \to \infty$.

The asymptotic SE of R_i increases with E_r, while E_u has no effect on the asymptotic SE. Furthermore, as M tends to infinity, the SE of each user in the DL phase will be lower than that of the UL phase.

$$\mathrm{SINR_{RX,i}} = \frac{\left|\mathbb{E}\left\{\mathbf{h}_{XR,i}^T \hat{\mathbf{h}}_{XR,i}^*\right\}\right|^2}{Var\left\{\mathbf{h}_{XR,i}^T \hat{\mathbf{h}}_{XR,i}^*\right\} + Var\left\{\mathbf{h}_{XR,i}^T \hat{\mathbf{h}}_{\bar{X}R,i}^*\right\} + \sum\limits_{j\neq i}\left(\mathbb{E}\left\{\left|\mathbf{h}_{XR,i}^T \hat{\mathbf{h}}_{BR,j}^*\right|^2\right\} + \mathbb{E}\left\{\left|\mathbf{h}_{XR,i}^T \hat{\mathbf{h}}_{AR,j}^*\right|^2\right\}\right) + \frac{1}{\rho^2}} \tag{16}$$

$$\xi_{XR,ij} = \frac{\beta_{XR,i}\beta_{AR,j}}{\left(K_{XR,i}+1\right)\left(K_{AR,j}+1\right)} \times \left(\frac{K_{XR,i} + \eta_{XR,i}}{1 + \tau_p p_p \beta_{AR,j}} + K_{AR,j}\eta_{XR,i} + K_{XR,i}\eta_{AR,j} + \eta_{XR,i}\eta_{AR,j}\right) \tag{25}$$

$$\chi_{XR,ij} = \frac{\beta_{XR,i}\beta_{BR,j}}{\left(K_{XR,i}+1\right)\left(K_{BR,j}+1\right)} \times \left(\frac{K_{XR,i} + \eta_{XR,i}}{1 + \tau_p p_p \beta_{BR,j}} + K_{BR,j}\eta_{XR,i} + K_{XR,i}\eta_{BR,j} + \eta_{XR,i}\eta_{BR,j}\right) \tag{26}$$

$$\zeta_{XR,ij} = \frac{\beta_{AR,i}\beta_{XR,j}}{\left(K_{XR,j}+1\right)\left(K_{AR,i}+1\right)} \left(\frac{K_{XR,j} + \eta_{XR,j}}{1 + \tau_p p_p \beta_{AR,i}} + K_{AR,i}\eta_{XR,j} + K_{XR,j}\eta_{AR,i} + \eta_{XR,j}\eta_{AR,i}\right) \tag{27}$$

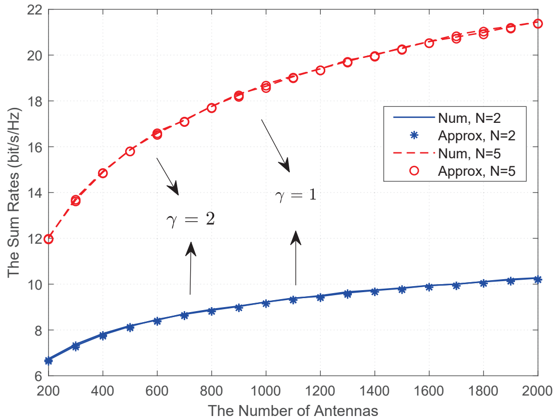

Fig. 1: Sum SEs versus M for $p_u = E_u$, $p_r = E_r$, and $p_p = \frac{E_p}{M^\gamma}$.

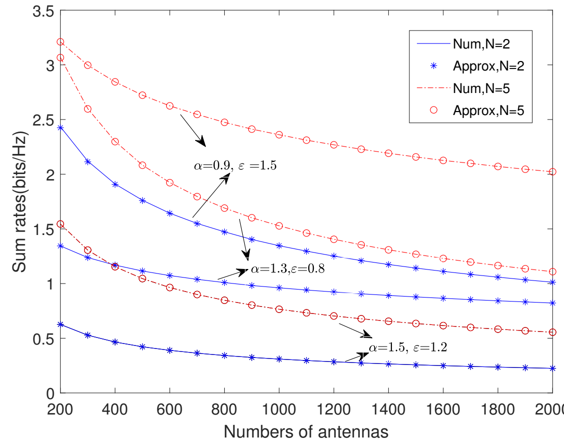

Fig. 3: Sum SEs versus M for $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\varepsilon}$, $p_p = \frac{E_p}{M^\gamma}$, $\alpha > 1$, or $\varepsilon > 1$.

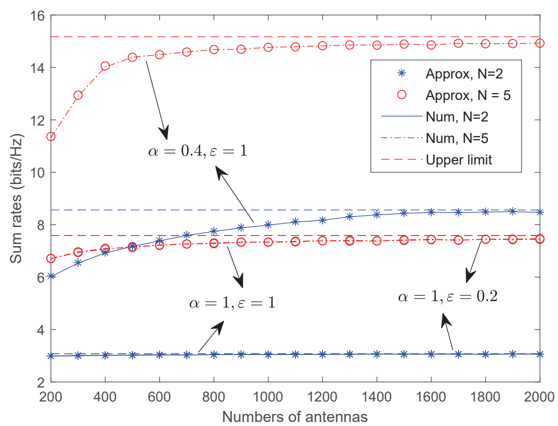

Fig. 2: Sum SEs versus M for $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\varepsilon}$, $p_p = \frac{E_p}{M^\gamma}$, $\alpha \leq 1$, and $\varepsilon \leq 1$.

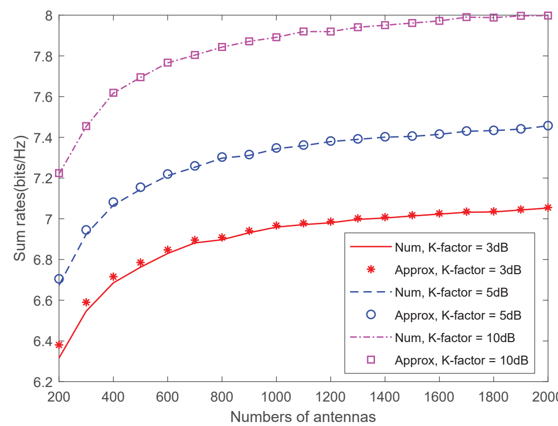

Fig. 4: Sum SEs versus M with $N = 5$ users and $p_u = \frac{E_u}{M}$, $p_r = \frac{E_r}{M}$, and $p_p = \frac{E_p}{M}$.

V. Numerical Results

In this section, we verify the main results of this letter by numerical results. The length of the coherence time is set to T = 196 symbols. For simplicity, we assume that $E_u = E_p = 10$ dB, $E_r = 20$ dB, $\beta_{AR,i} = \beta_{BR,i} = 1$. Furthermore, each Rician K-factor is set to the same value.

Numerical results are provided for the sum SE with $K_{XR,i} = 5$ dB for all i and $N = 2$ or 5 in Fig. 1 - Fig. 3. In Fig. 1, the exact expressions and the approximations are compared. It is observed that the pairs of curves match well for $N = 2$ and 5. The sum SE increases with M, as expected. Furthermore, when $N = 5$, the sum SE is almost twice as high as that for $N = 2$. This indicates that the sum SE increases with N. Additionally, when p_u and p_r are unchanged, the transmit power p_p of the pilot symbol is cut down, we find that the sum SE is independent of the choice of γ, when M becomes large.

In Fig. 2 and Fig. 3, the corresponding sum SEs and upper limits are presented when p_u and p_r are scaled down. Explicitly, Fig. 2 investigates three cases using different settings of α and ε. In line with Case I-III of Section IV, the sum SEs saturate as M tends to infinity in all three circumstances. We observe that $\alpha = 1$, $\varepsilon = 1$ and $\alpha = 1$, $\varepsilon = 0.2$ achieve the same sum SE, because it is decided by the UL phase. Fig. 2 also illustrates the corresponding upper limits $\bar{R}_i$ defined in (28) for the sum SEs. It can be observed that the sum SEs converge to the corresponding upper limits with the increasing M for these three different cases. Fig. 3 studies three different scenarios, i.e., 1) $\alpha > 1$, and $\varepsilon > 0$, 2) $\alpha > 0$, and $\varepsilon > 1$, 3) $\alpha > 1$, and $\varepsilon > 1$. As expected, the sum SEs converge to zero as M grows. The reduction of the sum SEs is faster for larger scaling parameters for different N.

Fig. 4 depicts the sum SE versus the Rician K-factor. Herein, we set $N = 5$, $p_u = \frac{E_u}{M}$, $p_r = \frac{E_r}{M}$, and $p_p = \frac{E_p}{M}$. We compare the sum SEs when $K_{XR,i} = 3, 5, 10$ dB. It is clear that the approximations match well with the exact expressions in all scenarios. The sum SE increases with M, as expected. As the Rician K-factor grows, the sum SE increases.

VI. Conclusions

We studied a multi-pair TW DF relay system using a massive MIMO scheme at the relay, upon adopting a MR receiver. Furthermore we derived the exact expressions and approximations of the SE over Rician fading channels for an imperfect CSI scenario. Finally, we quantified the trade-off between the SE, p_u and p_r. Additionally, the sum SE of the imperfect CSI scenario increases, as the Rician K-factor grows.

$$\tilde{R}_i \rightarrow \bar{R}_{2,i} = \lambda \log_2\left(1 + E_r \psi_{AR,i}^2 \Big/ \sum_{j=1}^{N} (\psi_{AR,j} + \psi_{BR,j})\right) + \lambda \log_2\left(1 + E_r \psi_{BR,i}^2 \Big/ \sum_{j=1}^{N} (\psi_{AR,j} + \psi_{BR,j})\right) \tag{38}$$

Appendix A
Proof of Theorem 1

From (8), by using Lemma 1 of [9], $R_{1,i}$ in (8) can be approximated as

$$R_{1,i}\approx \lambda\log_2\left(1+\frac{\mathbb{E}\left\{A_i\right\}+\mathbb{E}\left\{B_i\right\}}{\mathbb{E}\left\{C_i\right\}+\mathbb{E}\left\{D_i\right\}+\mathbb{E}\left\{E_i\right\}}\right)\triangleq\tilde{R}_{1,i}. \quad (39)$$

Then, we will calculate the terms $\mathbb{E}\left\{A_i\right\}$, $\mathbb{E}\left\{B_i\right\}$, $\mathbb{E}\left\{C_i\right\}$, $\mathbb{E}\left\{D_i\right\}$ and $\mathbb{E}\left\{E_i\right\}$.

By using Lemma 5 of [9] and retaining the dominant components, we can approximate $\mathbb{E}\left\{A_i\right\}$, $\mathbb{E}\left\{B_i\right\}$ and $\mathbb{E}\left\{C_i\right\}$ respectively as

$$\mathbb{E}\left\{A_i\right\}\approx M^2\omega_{AR,i}^2,\quad \mathbb{E}\left\{B_i\right\}\approx M^2\omega_{BR,i}^2. \quad (40)$$

$$\mathbb{E}\left\{C_i\right\}=M\left(\omega_{AR,i}+\omega_{BR,i}\right)\left(p_{A,i}\sigma_{AR,i}^2+p_{B,i}\sigma_{BR,i}^2\right). \quad (41)$$

From (12), $\mathbb{E}\left\{D_i\right\}$ can be written as

$$\begin{aligned}\mathbb{E}\left\{D_i\right\}=&\sum_{j\neq i}p_{A,j}\left(\mathbb{E}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{AR,j}\right|^2\right)+\mathbb{E}\left(\left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{h}_{AR,j}\right|^2\right)\right)\\&+\sum_{j\neq i}p_{B,j}\left(\mathbb{E}\left(\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{BR,j}\right|^2\right)+\mathbb{E}\left(\left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{h}_{BR,j}\right|^2\right)\right). \quad (42)\end{aligned}$$

The term $\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{AR,j}\right|^2\right\}$ can be expanded as

$$\begin{aligned}\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{AR,j}\right|^2\right\}=&\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\hat{\mathbf{h}}_{AR,j}\right|^2\right\}+\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{e}_{AR,j}\right|^2\right\}\\&+\mathbb{E}\left\{\hat{\mathbf{h}}_{AR,i}\hat{\mathbf{h}}_{AR,j}^H\hat{\mathbf{h}}_{AR,i}^H\mathbf{e}_{AR,j}\right\}+\mathbb{E}\left\{\hat{\mathbf{h}}_{AR,i}^H\hat{\mathbf{h}}_{AR,j}\hat{\mathbf{h}}_{AR,i}\mathbf{e}_{AR,j}^H\right\}. \quad (43)\end{aligned}$$

According to Lemma 5 of [9], an approximation of $\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{AR,j}\right|^2\right\}$ can be obtained as

$$\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{AR,j}\right|^2\right\}\approx M\xi_{AR,ij}. \quad (44)$$

Similarly, we can calculate the approximate expressions of the remaining three terms $\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{h}_{AR,j}\right|^2\right\}$, $\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{AR,i}^H\mathbf{h}_{BR,j}\right|^2\right\}$ and $\mathbb{E}\left\{\left|\hat{\mathbf{h}}_{BR,i}^H\mathbf{h}_{BR,j}\right|^2\right\}$. Then, the approximate expression of $\mathbb{E}\left\{D_i\right\}$ can be obtained.

From (13), by using Lemma 5 of [9], we can calculate $\mathbb{E}\left\{E_i\right\}$ as

$$\begin{aligned}\mathbb{E}\left\{E_i\right\}&=\mathbb{E}\left\{\left\|\hat{\mathbf{h}}_{AR,i}\right\|^2\right\}+\mathbb{E}\left\{\left\|\hat{\mathbf{h}}_{BR,i}\right\|^2\right\}\\&=M(\omega_{AR,i}+\omega_{BR,i}). \quad (45)\end{aligned}$$

By substituting the above results into (8) and (14), we can respectively approximate $R_{1,i}$ and $R_{AR,i}$ as $\tilde{R}_{1,i}$ in (19) and $\tilde{R}_{AR,i}$ in (21) with $X=A$. Then, we use a similar method to obtain the approximation of $R_{BR,i}$ as $\tilde{R}_{BR,i}$ in (21) with $X=B$. Thus, $R_{XR,i}$ in (14) can be approximated as $\tilde{R}_{XR,i}$ in (21).

Moreover, to calculate $R_{RX,i}$ in (15), we will first caculate $R_{RA,i}$. The term $\mathbb{E}\left\{\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{AR,i}^*\right\}$ is given by

$$\mathbb{E}\left\{\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{AR,i}^*\right\}=M\omega_{AR,i}. \quad (46)$$

Then, we derive the term $Var\left\{\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{AR,i}^*\right\}$ as

$$\begin{aligned}&Var\left\{\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{AR,i}^*\right\}\\&=\frac{M\beta_{AR,i}^2\left[2K_{AR,i}\eta_{AR,i}+\eta_{AR,i}^2+\frac{K_{AR,i}+\eta_{AR,i}}{1+\tau_p p_p\beta_{AR,i}}\right]}{\left(K_{AR,i}+1\right)^2}. \quad (47)\end{aligned}$$

Similar to (47), the term $Var\left\{\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{RB,i}^*\right\}$ can be expressed as

$$Var\left\{\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{BR,i}^*\right\}\approx M\xi_{BR,ii}. \quad (48)$$

Then, we derive the term $\sum\limits_{j\neq i}\left(\mathbb{E}\left\{\left|\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{BR,j}^*\right|^2\right\}+\mathbb{E}\left\{\left|\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{AR,j}^*\right|^2\right\}\right)$ similarly. For $j\neq i$, we can obtain

$$\mathbb{E}\left\{\left|\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{BR,j}^*\right|^2\right\}\approx M\xi_{BR,ij}, \quad (49)$$

$$\mathbb{E}\left\{\left|\mathbf{h}_{AR,i}^T\hat{\mathbf{h}}_{AR,j}^*\right|^2\right\}\approx M\xi_{AR,ij}. \quad (50)$$

Finally, the term ρ can be expressed as

$$\rho=\sqrt{p_r/\left(M\sum_{j=1}^N(\omega_{AR,j}+\omega_{BR,j})\right)}. \quad (51)$$

Substituting (46)-(51) into (15) and (16), we can obtain $\tilde{R}_{RA,i}$ as (22) with $X=A$. Then, $R_{RB,i}$ can be approximated by (22) with $X=B$ by using a similar method. Thus, $R_{RX,i}$ in (15) can be approximated as $\tilde{R}_{RX,i}$ in (22). Then, by substituting (21) and (22) into (20), we can obtain the expression of $\tilde{R}_{2,i}$.

Given the expressions of $\tilde{R}_{1,i}$ and $\tilde{R}_{2,i}$, we complete the proof.

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