**Limits for leak noise detection in gas pipes using cross correlation**

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# Abstract

The cross correlation method is widely used in leak detection and location for pipes. The model of the cross correlation function of the leak noise for water pipes has been established in an earlier study, while there is no such model for gas pipes. In this study, a model of the correlation function of the leak noise in gas pipes is developed. The characteristics of wave propagation in gas pipes, combined with the leak noise spectrum, are incorporated into the model to consider the effects of physical parameters, i.e., turbulence parameters, pipe parameters and flow parameters, on the cross correlation function. This model is capable of describing the main features of the correlation results in gas pipes. Moreover, the model gives an estimation of the detection limits of the leak noise in the absence of noise, which is crucial to the deployment of sensors in real gas pipelines. The findings of this study provide theoretical insight and experimental evidence for optimizing the cross correlation method when conducting leak detection and location in gas pipes.

**Keywords:** Leak detection; Cross correlation; Gas pipes; Acoustic methods; Detection limits.

# 1. Introduction

Gas pipelines are an essential part of modern infrastructure allowing the distribution of natural gas to homes, offices, and commercial buildings. However, due to factors such as corrosion, construction defects and mechanical or material failure, natural hazards, leakage sometimes occurs, leading to significant economic and energy losses. Therefore, the effective and accurate detection and location of gas leaks are of great importance [1–3].

Numerous methods have been developed for leak detection and location in gas pipes, including the acoustic method [4], real-time transient models [5], negative pressure wave method [6], and optical fiber method [7]. Among these, the acoustic method has been shown to be one of the most effective methods as a result of its high sensitivity, efficiency, accuracy and low false alarm rate [8,9].

The acoustic method focuses on the correlation-based method for leak location, which estimates the time difference between two signals either side of a suspected leak and the sound speed along the pipe. Gao et al [10] first proposed a model of the cross correlation function of the leak noise in water pipes, which was found to be mainly influenced by the decay rate of sound in the pipe due to losses at the pipe wall. This model was later extended to assess the effect on the correlation due to the introduction of a bandpass filter, the effects of wave reflections, the soil surrounding the pipe and resonances in the pipe [11–13]. Gao et al [14] also investigated the effectiveness of various time delay estimators in addition to the basic correlation method. It is now well established that the frequency bandwidth over which the signal is contained is crucial to location accuracy. Almeida et al [13] also proposed a scheme to select the lower and upper cut-off frequencies of a band-pass filter to optimize the current location method based on the coherence function between the two sensors**.** However, the above work is restricted to water pipes where the leak noise spectrum at the sensor is predominantly influenced by losses in the pipe wall.

It will be seen, in this paper, that the situation in gas pipes is fundamentally different due to much weaker coupling between the gas and the pipe wall. Existing work related to the leak location in gas pipes was carried out using the basic correlation or combining it with signal decomposition techniques. For instance, Wan et al. [15] used cross correlation analysis to estimate the time delay between two signals combined with a weighted average localization algorithm to determine the leak position. Similarly, Meng et al. [8] applied the basic correlation to estimate the time delay and considered the effect of the internal gas flow velocity to improve the location accuracy. However, these studies don’t take into account the effect of propagation characteristics of the leak noise. Li et al. [1] proposed a location scheme based on measurements of the cross-time–frequency spectrum, which is a kind of Cohen class distribution and can be used to extract the time delay and its corresponding spectral components simultaneously, to estimate the time delay where the spectrum reaches its maximum value. The wavespeed was chosen from the known dispersive curves, which helped to improve the location accuracy. Sun et al. [16] used ensemble local mean decomposition (ELMD) to derive multiple production function (PF) components. The selected characteristic PF components were then analyzed to extract the time delay. Liu et al. [17] performed wavelet transform (WT) to extract the target leak noise, which was then used to estimate the time delay between two sensors. The proposed method was validated by experimental results. Wang et al. [18] proposed a scheme to select the optimum mother wavelet, with which the acoustic signals were de-noised using WT. The time delay was then estimated from the appropriate WT coefficients.

None of the studies reviewed above involve a theoretical investigation into the cross correlation function of the leak noise in gas pipes. The predominantly fluid-borne wave responsible for the propagation of the leak noise in water pipes was taken as the basis for establishing theoretical expressions of the cross correlation function in water pipes [10,19]. However, such treatment might be problematic as a much weaker coupling between the gas and the pipe wall in gas pipes. Another critical difference between leaks generated in gas and water distribution pipes is that in gas pipes, it has been shown by Xiao et al [20] that the leak noise spectrum closely follows the Liepman spectrum for isotropic turbulence while in water pipes the leak noise spectrum was assumed to be ‘flat’ over the relatively small frequency range in which the leak signal is concentrated (typically less than 100Hz). By contrast, as shown in Section 4.2, the leak spectrum in gas pipes extends over a much wider frequency range.

This paper presents a model of the correlation function of the leak noise in gas pipes. The characteristics of wave propagation in gas pipes are first investigated. The wave attenuation in gas pipes is shown to follow a fundamentally different frequency dependence compared to that in water pipes. To consider the effect of turbulence parameters, the theoretical leak noise spectrum is invoked. The detection limits can be estimated and the main features of the cross correlation are well captured using the established model.

# 2. Leak location using the cross correlation method

Gas leaving the pipe under pressure generates noise at the leak orifice, which then propagates upstream and downstream of the leak. Two sensors located a known distance apart *d*, as shown in **Fig. 1**, can then be used to estimate the leak position from the peak in the correlation function between their measured signals.



**Fig. 1**. Schematic of leak location in gas pipes with a leak bracketed by two sensors.

Signals *x*1(*x*,*t*) and *x*2(*x*,*t*) measured at the two sensors comprise the sum of signals from the leak noise *s*1(*x*,*t*) and *s*2(*x*,*t*) and background noise *n*1(*x*,*t*) and *n*2(*x*,*t*) unrelated to the leak,

The leak position *d*1 relative to sensor 1 can be determined by,

where *c* is the wavespeed of the leak noise along the gas pipe and *T*0 is the time delay. **Eq. (3)** suggests that the time delay and wavespeed are the two of the main factors that affect the accuracy of the leak location estimate. The wavespeed can be estimated using theoretical expressions in terms of the pipe properties or directly obtained from in-situ measurements on the pipe [9,13]. The time delay *T*0 can be estimated from the peak in the cross correlation function *Rx*1*x*2(**) between the two signals *x*1(*x*1,*t*) and *x*2(*x*2,*t*) defined by,

where *E*[ ] is the expectation operator.

The cross correlation coefficient is therefore given by,

where *Rx*1*x*1(0) and *Rx*2*x*2(0) are the autocorrelation functions of the overall noise *x*1(*x*,*t*) and *x*2(*x*,*t*) at ** = 0, i.e., their variance **2 *s*1+**2 *n*1 and **2 *s*2+**2 *n*2, where **2 *s*1, **2 *s*2 **2 *n*1 and **2 *n*2 are the variances of the leak noise *s*1(*x*,*t*) and *s*2(*x*,*t*) and background noise *n*1(*x*,*t*) and *n*2(*x*,*t*) respectively.

In practice, measurements of the correlation coefficients are influenced by the presence of low frequency background noise that can propagate long distances along the pipe, as well as being pervasive along the pipe length. A high pass filter is therefore often applied to remove this contamination by low frequency noise for leak detection in water distribution pipes [9,13] but remains to be investigated for gas leaks. The effect of this filter on the correlation function is explored in Section 4.3.

# 3. Model of the correlation function

The pipe diameter 2*R* of most gas distribution pipes is limited to a few centimeters and therefore the frequency range, over which only plane waves are present (*k*0*R* < 1.84, where *k*0 = **/*cf*, *ω* and *cf* are angular frequency and speed of sound in the medium respectively), can therefore be up to several kHz. In this frequency range, therefore, the acoustic pressure at any frequency ** and location *x* from the leak location can be expressed in the form,

where *s*0(**) is the frequency spectrum of the leak noise at the leak position *x* = 0 and *k* = **/*cf* − i** is the complex acoustic wavenumber, whose real part is related to the propagation wave speedand the imaginary part represents the rate of decay **of the acoustic pressure due to losses in the pipe. In this paper, the appropriate Fourier spectrum and the Power Spectral Density (PSD) of the time-domain signal *x*(*t*) are represented by lower case *s*(**) and upper case *S*(**) respectively.

**3.1. Wave propagation characteristics in gas pipes**

The problem of wave propagation in gas pipes has been found to be quite different from that in water pipes, as the difference in impedance between the pipe wall and the gas is much higher. Therefore, the coupling between the gas and the pipe wall can be ignored and the pipe wall can be regarded as rigid [21,22]. The wave attenuation in the gas pipe is then determined from the attenuation of sound in rigid-wall pipes, i.e., intrinsic to the gas itself. A comprehensive review of the models for predicting wave attenuation in gas pipes was presented by Lahiri et al. [23]. Losses in the pipe ** were found to occur in the pipe wall*wall* and within the fluid *cl*. Losses at the pipe wall were shown to be due to the effects of viscosity and thermal conductivity effects in the fluid close to the pipe wall, whose rate of attenuation *wall* can be written in the form,

where ** is the kinematic viscosity, ** is the heat capacity ratio and ** is the thermal diffusivity.

The wave decay rate *cl* due to viscothermal absorption within the fluid is given by [23],

where ** is the dynamic viscosity, *f* is the density of fluid, Pr = *c*p** is the Prandtl number, ** is the thermal conductivity and *c*p is the specific heat capacity at constant pressure per unit of mass.

**Eq. (8)** indicates that attenuation within the fluid is proportional to **2, which is a significantly higher frequency dependence than the loss at the pipe wall in **Eq. (7)** which is proportional to .

The rates of decay versus frequency are compared in **Fig. 2** for losses at the pipe wall (**Eq. (7)**) and in the fluid (**Eq. (8)**) for the pipe properties listed in **Table 1**. Losses within the fluid are clearly seen to be much smaller compared to those at the pipe wall and can therefore be neglected. Based on the above results, the complex acoustic wavenumber of the leak noise in the pipe can be given by,



**Fig. 2**. Losses at the pipe wall and within fluid.

**Table 1**. Material properties used for calculation of losses.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *R*(m) | **  (kg/m3) | ** (m2/s) | ** | ** (m2/s) | ** (kg/(m·s)) | Pr | ** (W/(m·k)) | *c*p (J/(kg·K)) |
| 0.1 | 2.338 | × | 1.402 | × | × | 0.730 | 0.0255 | 1006 |

**3.2. Model of the correlation function**

Based on **Eqs. (6)** and **(9)**,theFrequency Response Function (FRF) *H*(**, *x*) between the leak noise at the leak position *x* = 0 and at the sensor *x* is given by,

where.

The attenuation of the amplitude of the leak noise in dB/m is given by,

The cross-spectral density (CSD) between signals *x*1(*x*1,*t*) and *x*2(*x*2,*t*) can be expressed in the form,

where is the PSD of the leak noise at the leak position *x*=0, and *T*0 = −(*d*2−*d*1)/*cf* is the time delay.

The phase spectrum *x*1*x*2(**) in the absence of reflections can be obtained from the phase of the CSD through **Eq. (12),** given by,

The time delay *T*0 can therefore also be calculated from the gradient of the phase spectrum with respect to the frequency. Previous work by Brennan et al. [24] has demonstrated that *T*0 obtained from the frequency domain estimate in **Eq. (13)** is entirely consistent with the time delay estimated from the time domain peak correlation.

In this paper, a more physical form for the leak noise spectrum *S*0(**) proposed by Xiao et al. [20] is assumed to predict the correlation function, which will be shown, has a strong influence on the form of the correlation function. Based on the measured spectrum of the leak noise in a gas pipe for different leak radii *a* and exit flow speed *U*, Xiao et al. [20] found that a good fit to the measured leak noise spectrum is the expression for isotropic turbulence of the form,

where is the normal component of the turbulence mean square velocity over the leak orifice area, which relates to the exit flow speed *U* by , where *I* is the turbulence intensity and *U* is a function of the pipe pressure, temperature and the properties of gas, and can be predicted using the gas release model [25,26], Λ is the turbulence integral length-scale at the leak orifice. In ref [20], and Λ were obtained from the values that gives best fit to the measured data.

Combining **Eqs.** **(12)** and **(14)**, the CSD between the two pressure measurements can be expressed as,

Multiplication in the frequency domain corresponds to the convolution in the time domain and hence the cross-correlation function *Rx*1*x*2(**) is given by,

where *F*−1{} denotes the Inverse Fourier Transform (IFT), and is the convolution operator.

The expression for the correlation function in a gas pipe of **Eq. (16)** is fundamentally different from that in a plastic water-filled pipe, which is controlled by ** (*d*1+*d*2), where ** is the decay rate [10] defined such the decay of the pressure/velocity over a distance *x* in the pipe as given by *e*−*x*. Note that ** is the coefficient of the attenuation linked to ** by , while in water pipes **=** . Performing the convolution and taking the IFT gives,

Unlike in the case of water, **Eq. (17)** for the cross correlation function in gas pipes has no analytic solution, and is therefore calculated numerically in this paper.

**Eq. (17)** suggests that for a particular pipe flow speed and leak orifice, the correlation function is solely a function of *d*2−*d*1 (*T*0), **(*d*2+*d*1) and the turbulence length-scale  which determines the shape of the leak noise spectrum at the source. The exact dependence on these parameters can be made more explicit by expressing the various quantities in **Eq. (17)** in appropriate non-dimensional form, so that the cross correlation becomes,

where   ***U* is the non-dimensional frequency, **̃=*U*/ is the non-dimensional time lag and *T*̃0 =***U*/ is the non-dimensional time delay, and *d*̃*i* are the non-dimensional distances given by,

The auto-correlation functions of *x*1(*x*,*t*) and *x*2(*x*,*t*) can be obtained following a similar procedure and hence the cross-correlation coefficients of **Eq. (5)** becomes,

**Eq. (20)** demonstrates explicitly that the behavior of the cross correlation coefficients depends *only* on *d*̃*i*, thedistances of the sensors from leak normalized on a combination of the decay rate ** in the pipe, linked to the propagation, and the turbulence length-scale , which is only a function of the source spectrum. Clearly, therefore, this combination of ** and  determines the effective frequency bandwidth of the signal at the measurement locations.

At **̃ = −*T*̃0 the peak value of the cross-correlation coefficients is given by,

**Eq. (21)** makes explicit that the peak value of the correlation coefficients is only a function of *d*̃1+*d*̃2 and *d*̃1/*d*̃2 by virtue of the symmetry of **Eq. (21)** with respect to *d*̃1 and *d*̃2.

**3.3 Detection limits of leak noise**

**Fig. 3** shows the variation in the peak value *peak* of the correlation coefficients computed from **Eq. (21)** against *d*̃1+*d*̃2, and *d*̃2/*d*̃2.



**Fig. 3**. Peak value of the cross-correlation coefficients versus *d*̃1+*d*̃2 and *d*̃1/*d*̃2 (equals to *d*1/*d*2). The while dash line denotes the distance (*d*̃1+*d*̃2)**=0.5 under which the peak value is higher than 0.5, the red dot line is the asymptote of the curve when *d*̃1/*d*̃2→0 or ∞. Rig locations denote the values of *d*̃1+*d*̃2 and *d*̃1/*d*̃2 adopted in experimental measurements, which will be introduced in the experimental section.

This figure indicates that *peak* ~ 1 over the narrow range of sensor distances, 0.1 < *d*̃1/*d*̃2 < 10, where the leak signals at the two sensors are sufficiently similar that close to unity correlation is achieved at the appropriate time delay −*T*̃0. Clearly, therefore, the variation in the term in the numerator of **Eq. (21)** is negligible and therefore approximates close to 1.

For values of *d*̃1/*d*̃2 < 0.1 and *d*̃1/*d*̃2 > 10, the peak correlation coefficient *peak* steadily decreases with increasing values of *d*̃1+*d*̃2. A notable feature of **Fig. 3** is that the peak value of the correlation coefficients can be seen to be consistently high (> 0.9) and independent of *d*̃1/*d*̃2 for values of *d*̃1+*d*̃2 less than some threshold value referred to here as (*d*̃1+*d*̃2)*=thr*, where **thr is the specified threshold of the peak value of the cross correlation coefficients, indicated in **Fig. 3** as a horizontal red line at (*d*̃1+*d*̃2)*=*. Also shown in this figure, as the white dash line, is the contour corresponding to **thr = 0.5, which is chosen here arbitrarily, to be a minimum value that is sufficient for clearly locating the leak in the noise-free case. More generally, the contours of constant (*d*̃1+*d*̃2)*=thr* can be seen to be highly sensitive to the peak correlation **thr, i.e.,



**Fig. 4**. Variation of (*d*̃1+*d*̃2)*=thr* with the threshold of the peak value **thr.

A plot of (*d*̃1+*d*̃2)*=thr* against **thr between 0.01 and 0.99 calculated numerically from **Eq. (21)** is given in **Fig. 4**.

For **thr < 0.7, a useful approximation is that,

For **thr > 0.7, (*d*̃1+*d*̃2)*=thr* drops significantly below this approximate value. The derivation of **Eq. (23)** can be found in **Appendix A**. In the absence of noise, therefore, the detection limit, (*d*1+*d*2)****thr, in dimensional form, is given by,

**3.4. Effect of band-pass filtering**

The high-frequency components of acoustic signals due to leaks attenuate faster than low-frequency components, thus the lower frequency components can propagate over a long distance along the pipe and are most useful for leak detection and location [27,28]. A band-pass filter is usually introduced to remove components outside the frequency range of interest. Additionally, it also improves the sharpness of the correlation but lowers the signal-to-noise ratio (SNR). In this section, the effect of the band-pass filter on the form of the correlation coefficients in gas filled pipes is investigated.

The FRF of an ideal band-pass filter with lower and upper frequency limits of *ω*0 and *ω*1 is,

Substituting **Eq. (25)** into **Eq. (20)**, and neglecting the attenuation term, the cross correlation coefficients become,

where *i*  *i**U* are upper and lower frequency limits in non-dimensional form.

**Fig. 5** **(a)** and **(b)** are contour plots of the cross correlation coefficients as a function of the lower and upper frequency limits 0 and 1, for values of **̃ between −10 and 10. When the lower frequency limit 0 is fixed, there is nearly no change in the correlation with an increase of the upper frequency limit 1 as shown in **Fig. 5** **(a)**. In contrast, a significant difference can be observed in **Fig. 5** **(b)** as the lower frequency limit 0 increases. These results indicate the cross-correlation coefficients are mainly dominated by the lower frequency limit 0, which is consistent with the results in [10]. Thus **Eq. (26)** can be further simplified as,

In practice, the background noise contained in the very low frequencies may affect the time delayestimate and therefore the lower cut-off frequency must be chosen carefully.



|  |  |
| --- | --- |
| (a) 0 = 0.5 | (b) 1 = 10 |

**Fig. 5**. Effects of the frequency limits on the cross correlation coefficients.

* 1. **Width of the cross correlation function**

In the ideal noise-free case, the ability to detect and locate the position of the leak will depend on the width of the correlation peak Δ**̃0.5, and the width Δ**̃main between the main peak and the second peak of the correlation function. These characteristic time-scales are illustrated in **Fig. 6** for the correlation coefficients described by **Eq. (27).**



**Fig. 6**. Illustration of the three cross-correlation widths (*T*0 = 0).

Numerical investigations into the behaviors of Δ**̃0.5, Δ**̃main and the peak of the cross correlation function *Rx*1*x*2(−*T*̃0) obtained from **Eq. (18)**, indicate that they are inversely proportional to the lower frequency limit 0, i.e.,

where *C* is the coefficient tabulated in **Table 2**. **Eq. (28)** makes explicit that the lower cutoff frequency limit 0 should be kept as small as possible to maximize the separation of the main peak and side peaks in the cross correlation coefficients, which is beneficial for the determination of the time delay. In practice, of course, 0 will be chosen as a compromise between minimizing background noise whilst allowing the correlation peak to be as small as possible. The ability to locate the leak is related to the ‘energy’ (area under the curve) in the correlation function which can be approximated by the product of the peak of the cross correlation function *Rx*1*x*2(−*T*̃0) and the width, as given by,

**Table 2**. Coefficients of the quantities in **Eq. (28)**.

|  |  |
| --- | --- |
| Quantity | *C* |
| Δ**̃0.5 | 0.725 |
| Δ**̃main | 4.893 |

****

**Fig. 7**. Variation of Δ**̃main and Δ**̃0.5 with the lower frequency limit 

# 4. Comparison between the model and experimental results

This section deals with measurements of the cross correlation coefficients of a gas leak on a purpose-built pipe rig, starting with a description of the experimental pipe rig.

## 4.1 Description of the experimental setup

The test rig used in this study was adapted from an existing pipeline, which was also connected to the pipeline network. The schematic and photographs of the test rig can be seen in **Figs. 8** and **9** respectively. The test pipeline was composed of steel pipe with an external diameter of 150 mm and a thickness of 4.5 mm. The detailed distances between the valves, sensors and pressure gauges are indicated in **Fig. 8**. The pipe network was supplied by a mobile air compressor (Atlas Copco model of XRVS 1050) with an outlet pressure of 0.8MPa, as shown in the photograph of **Fig. 9 (b)**. A gas regulator station was used to adjust the operating pressure with an inlet pressure of 0.1 MPa and 0.3 MPa. A large reservoir was placed in the network to store gas and maintain a steady pressure. Artificial leaks were simulated by drilling a round hole with a radius of 1.5 mm in the pipe wall directly. The acoustic pressure was measured by two sensors (PCB with a sensitivity of 72.5 mV/kPa and a frequency range of 0.5 Hz ~ 40 kHz) labeled 1 and 2 in **Fig. 8**. The pressure sensors were inserted into the pipe using a mounting adaptor, positioned in the pipeline at distances *d*1, and *d*2 from the leak equal to 0.7 m, and 70.15m respectively. Three valves were used to change the flow rate and operating pressure in the test pipeline and two pressure gauges were placed upstream of the pipe section and downstream of Sensor 2 to monitor the operating pressure. **Table 3** summarizes the pipe system characteristics. Note that the non-dimensional distance *d*̃1, and *d*̃2 in this rig are generally very small but their ratios are of order 100. The sum and ratio of non-dimensional distances, *d*̃1+*d*̃2 and *d*̃1/*d*̃2 are indicated by the circles in **Fig. 3** where near unity correlation coefficients are predicted. However, the measured signals also included the additional pressure perturbations caused by the turbulence flow generated at the pipe bends, the branch and valves. Moreover, additional background noise could be generated by the nearby factory. Consequently, the background noise in the pipe rig is therefore relatively high and significantly affects the peak value of the correlation, which will be further evaluated in Section 4.3. In these measurements, therefore, the leak detection limits were affected by the background noise.



**Fig. 8**. Schematic diagram of the experimental pipeline system.

|  |  |  |
| --- | --- | --- |
|  |  | |
| (a) | (b) |

**Fig. 9**. (a) Experimental setup, and (b) the mobile air compressor.

**Table 3**. Pipe rig characteristics.

|  |  |  |
| --- | --- | --- |
| Pipe system | Value range | Unit |
| *d*1 (*d*̃1) | 0.70 (9.199×10−4) | m (-) |
| *d*2 (*d*̃2) | 70.15 (9.220×10−2) | m (-) |
| External diameter | 150 | mm |
| Wall thickness | 4.5 | mm |
| Pipe pressure | 0.1, 0.3 | MPa |

A commercial data acquisition system (NI cDAQ 9178 compatible with the module NI 9234) was used to record and digitize the acoustic signals. The sampling frequency was set as 4 kHz and the time duration was 2 mins for all tests. A Hanning window with 50% overlap and power spectrum averaging were adopted in the spectral analysis.

## 4.2 Data analysis

**Fig. 10** shows Power Spectral Densities (PSD’s) of the acoustic signals *x*1(*t*) and *x*2(*t*). This figure indicates that the leak noise is primarily concentrated in the low frequency range below a few hundred Hertz. The measured leak noise spectral shape can be seen to be well captured by the isotropic spectrum model of **Eq. (14)**, with the turbulence parameters chosen to provide the best fit to the measured spectrum. The ratio Λ/*U* was found to be close to 0.02, which when substituted into **Eq. (24)** predicts a total separation distance of (*d*1+*d*2)** = 4.1km at which the correlation coefficient drops to 0.5 in the noise-free case. Clearly, this estimate is highly encouraging for practical applications.



|  |  |
| --- | --- |
| (a) *x*1(*t*) | (b) *x*2(*t*) |

**Fig. 10**. PSD’s of acoustic signals *x*1(*t*) and *x*2(*t*).

**Fig. 11** shows the coherence function between the two signals *x*1(*t*) and *x*2(*t*), which can be observed to be high (** > 0.9) in the frequency range between 5 to 100 Hz, where there is also a good linear relationship between the unwrapped phase with frequency shown in **Fig. 12**. The red line in **Fig. 12** represents the estimated phase for an infinite pipe in the absence of reflections in the pipe. The time delay obtained from the gradient of the phase spectrum is −0.19786s ± 0.00003s deduced over the frequency band, 10~120Hz, where the error is estimated from the least square fit corresponding to a confidence level of 95%. It has been shown in Ref [24] that the variance of the time delay estimate obtained from a least square fit to the phase spectrum is given by,

where *i* is the *i*th frequency point and **2 *i*denotes its corresponding coherence value, *r* is the number of window segments over which the PSD is taken and *N* is the frequency data points in the measurement window.

The confidence interval of the time delay can be estimated from [29],

where *t*( ) is the Student's *t*-distribution,1−** is the intended confidence interval, usually chosen to be 95%, with *m*−1 degrees of freedom. Note that the variance is now a function of the frequency bandwidth. Thus when applying different band-pass filters, the error will differ, as shown in Section 4.3.

Based on an assumed wave speed of 346.13m/s [30], the corresponding distance (*d*2) calculated according to **Eq. (3)** is 69.67m, which is less than 1% of the actual distance. A better estimate for the wavespeed could be obtained by direct measurement of the time delay between a source and receiver of known separation distance.



**Fig. 11**. Coherence function between signals *x*1(*t*) and *x*2(*t*).****

**Fig. 12**. Phase spectrum between signals *x*1(*t*) and *x*2(*t*).

In order to predict the correlation function from **Eq. (20)**, the rate of attenuation ** in the pipe is now estimated from estimates of the transfer function in the pipe. Combining **Eqs. (6)** and **(10)** gives the wave attenuation as,

The attenuation in dB/m is then calculated from 8.686**.

**Fig. 13** provides a comparison between the wave attenuation obtained experimentally from **Eq. (32)** and the theoretically predicted attenuation from **Eq. (9)**. The theoretical prediction can be seen to provide a good match to the general behavior of the experimental estimate, which can be observed to fluctuate around its mean behavior caused by acoustic resonances in the pipe system. Note that the measured attenuation appears to drop to 0dB at frequencies above300Hz, due to poor coherence in this high frequency range, as can be seen in **Fig. 11**. In general, however, accurate measurements of the rate of attenuation are difficult in this relatively short pipe rig since losses in the pipe rig are small over these small distances.



**Fig. 13**. Wave attenuation in dB/m.

## 4.3 Comparison of the model with experimental results

As discussed above the lower frequency limit **0 should be set as low as possible to ensure that the main peak of the cross correlation coefficients is as large as possible. However, this will be achieved at the expense of increasing its bandwidth Δ**. The effect of the bandpass filter on the cross correlation coefficients is now investigated experimentally.

The total separation distance *d*1+*d*2=70.85m between sensors adopted in this experiment is substantially smaller than the detection limit, (*d*1+*d*2)** for the rig of 4.1km discussed in Section 3.3, suggesting that the peak of the correlation coefficients should be close to unity *s*1*s*2(−*T*0) ~ 1 in the absence of background noise. However, in practice, the correlation coefficients will be less than 1 due to the presence of background noise as indicated in **Eq. (5)**. The background noise is now included in the prediction of the correlation coefficients to allow direct comparison with the measured correlation.

The variances of the background noise **2 *n* at the two sensors were measured in the absence of the leak. The variances of the leak noise are then obtained by subtracting the variances of background noise **2 *n* from the autocorrelation functions *Rxx*() =**2 *n+*2 *s***. Table 4** lists the estimates of the variances of the leak noise and background noise **2 *s*1, **2 *s*2, **2 *n*1 and **2 *n*2 at the two sensors and the corresponding peak value *x*1*x*2(−*T*0) over eight different frequency bandwidths. The variances of background noise are between 20% and 30% of the acoustic signals, which predicts that the peak correction *x*1*x*2(−*T*0) should be about 70% to 80% of the noise-free estimates shown in **Fig. 3**. Additionally, the peak value *x*1*x*2(−*T*0) in the presence of noise can be determined from **Eq. (5)** as , where *s*1*s*2(−*T*0) ~ 1 and the values of **2 *n*1/**2 *s*1 and **2 *n*2/**2 *s*2 can be obtained as shown in **Table 4**. The following theoretical cross-correlation coefficients were thus adjusted accordingly to include background noise, whose values are listed in **Table 4**.

**Table 4**. Estimates of **2 *s*1, **2 *s*2, **2 *n*1 and **2 *n*2, and the crosspoding peak value *x*1*x*2(−*T*0) of the cross correlation coefficients.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **0 (Hz) | **1 (Hz) | **2 *s*1 | **2 *n*1 | **2 *n*1/**2 *s*1 | **2 *s*2 | **2 *n*2 | **2 *n*2/**2 *s*2 | *x*1*x*2(−*T*0) |
| 10 | 30 | 0.271 | 0.071 | 0.262 | 0.235 | 0.075 | 0.319 | 0.775 |
| 50 | 0.337 | 0.079 | 0.234 | 0.328 | 0.084 | 0.256 | 0.803 |
| 100 | 0.414 | 0.103 | 0.249 | 0.407 | 0.110 | 0.270 | 0.794 |
| 200 | 0.429 | 0.135 | 0.315 | 0.422 | 0.142 | 0.337 | 0.754 |
| 20 | 100Hz | 0.274 | 0.059 | 0.215 | 0.273 | 0.062 | 0.227 | 0.819 |
| 30 | 0.199 | 0.042 | 0.211 | 0.195 | 0.045 | 0.231 | 0.819 |
| 40 | 0.137 | 0.031 | 0.226 | 0.117 | 0.034 | 0.291 | 0.795 |
| 50 | 0.099 | 0.024 | 0.242 | 0.082 | 0.027 | 0.329 | 0.778 |

**Fig. 14** shows a comparison of the predicted and measured cross correlation coefficients for different upper frequency limits of *f*1 = 30, 50, 100 and 200Hz (1 = 1.2, 2, 4 and 8) when the lower frequency limit *f*0 is set at 10Hz ( = 0.4). As the upper frequency limit is increased, a negligible difference in the cross correlation coefficients is observed, indicating that most of the leak ‘energy’ at the two sensors is concentrated at lower frequencies, and the main shape of the cross correlation coefficients is controlled by the lower frequency limit. The time delay given by the peak of the cross-correlation coefficients of the measurements in **Fig. 14** are −0.19975s ± 0.00019s, −0.19950s ± 0.00010s, −0.19825s ± 0.00004s, and −0.19825s ± 0.00002s respectively, for the four frequency bandwidths listed in **Table 4**, which can be seen to be within 99.2% of each other. Note that the presence of reflections in the cross correlation coefficients can be clearly seen at about *T*0 = 0.20075s ± 0.00004s, originating from reflections at the valve downstream of Sensor 2, which belonged to the original network and was not shown in **Fig 8**.



|  |  |
| --- | --- |
| (a) | (b) |

****

|  |  |
| --- | --- |
| (c) | (d) |

**Fig. 14**. Cross-correlation coefficients, the lower frequency limit *f*0 is set at 10 Hz, and the upper frequency limits *f*1 are: (a) 30 Hz, (b) 50 Hz, (c) 100 Hz, and (d) 200 Hz.

**Fig. 15** shows a comparison of the cross correlation coefficients for varying lower frequency limits *f*0 = 20, 30, 40 and 50 Hz (0 = 0.8, 1.2, 1.6 and 2) when the upper frequency limit *f*1 is set at 100Hz (1 = 4), where a clear main peak can be obtained for all frequency bandwidths under consideration. However, the correlation coefficients become more oscillatory and the width between the main peak and side peaks reduces as the lower frequency limit is increased, as can be seen from **Fig. 15 (b), (c)** and **(d)**. This behavior is accurately predicted by **Eq. (28)**. The differences between the experimental results and the model are mainly due to the effects of the background noise and pipe reflections. The small frequency bandwidth serves to increase the peaks in the cross correlation coefficients due to reflected waves in the pipe, thus the bandpass filter needs to be chosen with care. The time delay calculated from the cross correlation coefficients is to be −0.19825s ± 0.00003s, −0.19825s ± 0.00003s, −0.19775s ± 0.00003s and −0.19775s ± 0.00003s respectively.



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**Fig. 15**. Cross-correlation coefficients, the upper frequency limit *f* is set at 100 Hz, and the lower frequency limits *f*0 are: (a) 20 Hz, (b) 30 Hz, (c) 40 Hz, and (d) 50 Hz.

# 5. Conclusion

A model of the cross correlation function of the leak noise for leak detection and location in gas pipes has been presented which combines a theoretical model for the leak noise spectrum and a propagation for the acoustic wave attenuation. The model has been used to determine the detection limits of the leak noise in the absence of noise, with a useful approximation being derived. Besides, high correlation is predicted for the range of sensor distances, 0.1<*d*1/*d*2<10. The widths of the cross correlation coefficients are shown to be inversely proportional to the lower frequency limit. The main features of the cross correlation coefficients are shown to be controlled by the lower frequency limit. Good agreement is obtained between measurements and predictions with the main features being accurately predicted. Correlation measurements made in the pipe rig described in this paper show that the peak correlation is mainly determined by the presence of background noise in the pipe. Nevertheless, this study has shown the acoustic method is a promising tool for leak location in gas pipelines, with predicted detection limits up to several kilometers. Future work will be needed to examine the detection limits along with the practical issue of sensor deployment in a real gas pipe network.

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**Appendix A. Evaluating the coefficient in Eq. (23)**

The coefficient in **Eq. (23)** is obtained as follows,

The ratio *d*̃1/ *d*̃2 in **Eq. (21)** is set to be 0(or ∞), thus, *d*̃1+*d*̃2≈*d*̃2, then **Eq. (21)** becomes,

Letting , and writing the derivative with respect to *d*̃2 we have,

Similarly, we have,

Considering the first integral in **Eq. (A.3)**, and letting, we have,

The second integral in **Eq. (A.3)** is *I*(*d*̃2), substituting **Eq. (A.4)** into **Eq. (A.3)** gives,

When *d*̃2 is relatively large, the fourth derivative in **Eq. (A.5)** is quite smaller than*I*(*d*̃2), thus,

Similarly, we have,

Substituting **Eqs. (A.7)** and **(A.6)** into **Eq. (A.1)** gives,

which is the result given in **Eq. (23)**.

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