UNIVERSITY OF SOUTHAMPTON
FACULTY OF MATHEMATICAL SCIENCES
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Net Present Value Models to help determine the Economic Operational Speed of a Chartered Ship

by

Fangsheng Ge

A thesis submitted for the degree of
Doctor of Philosophy in Mathematical Sciences

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Abstract

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Maritime shipping, while acting as the most important role in the world transportation system, witnesses the boom of Operational Research (OR) that has been applied to support the decision making towards not only higher profitability from the financial perspective but also better sustainability in the sense of green shipping, due to the sharply fluctuations of fuel price and newly approved emission measures. However, due to the complexity of operational scenarios, existing speed optimisation modelling frameworks are developed on a case by case basis, and the preference between them is yet well-explained in literature. This has now caught the attention to some researchers in line with the needs of evaluating other related problems, e.g., environmental measures, from the perspective of profit-seeking. This thesis, in particular, adopts the Net Present Value (NPV) cash-flow approach that explicitly accounts for each payment made for the decision maker and it can also be used to study the classic modelling frameworks via the technique of NPV Equivalent Analysis (NPVEA) developed in literature.

Using our NPV framework, this thesis established how to explicitly consider contracts details, e.g., for time charter contract and voyage-alike charter contract, into speed optimisation models, and how these affect optimal decisions and profitability for a ship operator (or the ship owner). The focus lies on decisions in the deterministic setting that maximises the decision maker's profitability. In particular, we look at the comparison between our proposed NPV modelling framework with classic speed optimisation models from both the mathematical analysis and the numerical experiments.

Factors other than contractual details are considered in this thesis as well. In particular, the decision maker's expectations about future is modelled as the Future Profit Potential (FPP), which is the NPV of profits within the relavent future. We thus aim to demonstrate how the setting of FPP will impact the optimisation, and clarify what is the corresponding underlying assumption.

Through this work, we also examine some of the selected topics that are marked in literature or yet studied in the context of maritime shipping to provide a modelling

framework as well as managerial insights that what is important for the decision maker to consider.

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List of Symbols

Chapter	3
H	initial approximate duration of charter party (in days)
H^*	agreed duration of charter party (in days)
t_{FS}	agreed start time of charter relative to signing contract (in days)
f^{TCH}	agreed daily charter rate (USD/day)
n	number of legs considered
i	$= 1, \ldots, n$, index of legs
v_i	average ship speed (in kn) on from port $i-1$ to i
V_i^-	$= \{v_1, \ldots, v_i\}$, the set of speed choices from leg 1 to i
$T_i^s(v_i)$	average sea voyage time (in days) of speed v_i to port i (decision variable)
T_i^l	loading time (in days) at port $i-1$
T_i^u	unloading time (in days) at port i
T_i^w	waiting time at (in days) port i
$T_i(v_i)$	$= T_i^l + T_i^s(v_i) + T_i^w + T_i^u$, leg time (in days) spent from port $i-1$ to i
$L_j(V_j^-)$	$= \sum_{i=1}^{j} T_i(v_i)$, the completion time (in days) of leg j from the start of first leg
S_i	distance (in nm) from port $i-1$ to i
R_i	revenue received at port i
C_i^u	unloading cost at port i including port related costs and unloading charges
C_i^h	the sum of port associated costs with the loading operations at port $i-1$
C_i^l	loading cost at port $i-1$ including C_i^h, and the fuel consumption costs
c_i^f	the fuel price at port $i-1$
w_i	the dwt carried on leg i including cargo, ballast water, supplies, and etc.
$F(v_i, w_i)$	the fuel consumption function (tonne/day)
d_i^c	the delay days of costs paid after leaving port $i-1$
d_i^r	the advance days of revenues received before arriving at port i
α	opportunity cost of capital (per day)
Chapter	4
m	number of legs considered
n	number of repetition of the m-leg journey
f^{TCH}	agreed daily charter rate (USD/day)
$\mathbb{P}$	$= \{P_0, \ldots, P_m\}$, a sequence of ports to visit

i	$= n, \ldots, 1$, index of journey repetition
j	$= 1, \ldots, m$, index of legs of the journey
v_j^i	average ship speed (in kn) on from port P_{j-1} to P_j on i-th repetition
$V^{i,j}$	$-\{v_1^i, \ldots, v_j^i\}$, the set of speed choices from leg 1 to j of i-th repetition
$T_j^{s,i}(v_j^i)$	average sea voyage time (in days) to port P_j on i-th repetition (decision variable)
T_j^l	loading time (in days) at port P_{j-1}
T_j^u	unloading time (in days) at port P_j
T_j^w	waiting time at (in days) port P_j
$T_j^i(v_j^i)$	$= T_j^l + T_j^{s,i}(v_j^i) + T_j^w + T_j^u$, leg time (in days) spent from port P_{j-1} to P_j on i-th repetition
$L_j^i(V^{i,j})$	$= \sum_{k=1}^{j} T_k^i(v_k^i)$, the total duration (in days) for completing the journey with index i
$\sum_i^n L_m^i$	the termination time (in days) when ship becomes available for future tasks
Γ_i	$:= \{T_1^i, \ldots, T_m^i\}$, the shorthand notation of the corresponding travel time decisions
S_j	distance (in nm) from port $j-1$ to j
R_j	revenue received at port P_j
C_j^u	unloading cost at port P_j including port related costs and unloading charges
C_j^h	the sum of port associated costs with the loading operations at port P_{j-1}
C_j^l	loading cost at port P_{j-1} including cargo loading costs, and the fuel consumption costs
c_j^f	the fuel price at port P_{j-1} at the time of bunkering
w_j	the dwt carried on leg j including cargo, ballast water, supplies, and etc.
$F(v_j^i, w_i)$	the fuel consumption function (tonne/day)
G_0	the Future Profit Potential (FPP)
α	opportunity cost of capital (per day)

Chapter	5
H	agreed duration of charter party (in days)
f^{TCH}	agreed daily charter rate (USD/day)
n	number of legs considered
$\mathbb{P}$	$= \{P_0, \ldots, P_n\}$, a sequence of ports to visit
S_i	distance (in nm) from port $i-1$ to i
v_i	average ship speed (in kn) on from port $i-1$ to i
V_i^-	$= \{v_1, \ldots, v_i\}$, the set of speed choices from leg 1 to i
$T_i^s(v_i)$	average sea voyage time (in days) of speed v_i to port i (decision variable)
T_i^l	loading time (in days) at port $i-1$
T_i^u	unloading time (in days) at port i
T_i^w	waiting time at (in days) port i
$T_i(v_i)$	$= T_i^l + T_i^s(v_i) + T_i^w + T_i^u$, leg time (in days) spent from port $i-1$ to i
$L_j(V_j^-)$	$= \sum_{i=1}^{j} T_i(v_i)$, the completion time (in days) of leg j from the start of first leg
$R_i(L_{i-1})$	revenue received at port i which is based on the freight rate agreed at time L_{i-1}
C_i^u	unloading cost at port i including port related costs and unloading charges
C_i^h	the sum of port associated costs with the loading operations at port $i-1$
C_i^l	loading cost at port $i-1$ including C_i^h, and the fuel consumption costs

$c_i^f(L_{i-1})$	the fuel price of i-th leg at time L_{i-1}
w_i	the dwt carried on leg i including cargo, ballast water, supplies, and etc.
$F(v_i, w_i)$	the fuel consumption function (tonne/day)
α	opportunity cost of capital (per day)
Chapter	6
Γ_i	$:= \{T_1^i, \ldots, T_m^i\}$, the shorthand notation of the corresponding travel time decisions
$\mathbb{P}$	$= \{P_0, \ldots, P_m\}$, a sequence of ports to visit

Declaration of Authorship

I, Fangsheng Ge, declare that this thesis titled, 'Net Present Value Models to help determine the Economic Operational Speed of a Chartered Ship' and the work presented in it are my own, and have been generated by me as the result of my own original research. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed : **Fangsheng Ge**

Date : **August 2020**

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To my beloved family, friends and colleagues

Chapter 1

Introduction

1.1 Research background

Maritime shipping, while acting as the most important role in the world transportation system, witnesses the boom of Operational Research (OR) that has been applied to support the decision making towards not only higher profitability from the financial perspective but also better sustainability in the sense of green shipping, due to the sharply fluctuations of fuel price and newly approved emission measures. As the ship operators wish to maximise the net revenues by making smart decisions on different planning levels (namely, strategic, tactical and operational planning), current studies focus on the logistics problems such as fleet selection, route selection and speed selection. However, the development of new technology and environmental policy now requires further understanding of the incentives underneath of maritime shipping which are based on the operational scenarios such that the ship operators could evaluate the possible impact of those changes also from the perspective of investment as well as the regulators. Therefore, this thesis investigates, within a cash-flow framework that incorporates the traditional logistics speed optimisation models with the consideration of payments (including both the timing and the amount), how does the optimal speed choice of the ship operators and therefore the profitability and emission interacts with the market conditions under specified charter contracts.

Due to the complexity of maritime shipping scenarios, current literature has been developed on a case by case basis. For example, there are liner shipping and tramp shipping where the ships act like buses and taxis respectively. When it comes to the charter contract, there are voyage contracts and time contracts, where the ship is specified either to be used on a given route or within a given period. Clearly, this makes the evaluation of any investment of new technology and application of new policy difficult as they have to choose the most suitable general model alone from all kinds of speed optimisation or related models available, while the boundary between them are misty (e.g., one could call a taxi and use it as well as a bus, but what model should be used then?). Roughly, these models can be divided into two criteria, namely *USD per trip* and *USD per unit time*. The discussion on which of them is superior has now caught the attention to some researchers (Psaraftis, 2017) in line with the requirement of evaluating environmental measures based on financial perspectives (Johnson & Styhre, 2015).

Despite the cash-flow approach has been widely used in financial investment and other OR aspect such as inventory theory (which will be further reviewed in Chapter 2), the OR study in the area of maritime shipping has paid scant attention to this method. Partly, the road transportation studies are often referred by researches in maritime logistics, and it is intuitive to conclude that the time impact of cash-flows by delaying or advancing a few hours is negligible, as the total transportation time of vehicles is often short. Indeed, by using the cash-flow approach, the objective function is no longer linear and thus brings in higher computational cost. But for intercontinental shipping, which is one of the most commonly discussed shipping scenarios, the travel time could be weeks or even months, and taking into account the payments in line with the logistics tasks could help the decision makers to understand what is going on based on the principle of finance as ship itself is actually an expensive investment (no matter either to buy it or to charter it).

Market conditions, e.g., the economic parameters such as freight rate, fuel price and charter hire as discussed in Psaraftis and Kontovas (2014), can now be analysed dynamically from time to time by evaluating all the cash-flows relevant. Efforts made towards the understanding of these parameters based on either deterministic or stochastic settings lack the bridge between them such that one can gain managerial insights such as how does the future fluctuation impact current optimal choice and why. As a consequence, building up the understanding towards the dynamic natural of the market is essential for both decision makers and researchers in the sense of further studies.

On the other hand, contractual clauses, which are specified in charter party contracts, have yet been taken into account into any OR models (Adland & Jia, 2018). Present models could fail in practice where the contractual clauses impose a stricter restriction, e.g., the Shell Time 4 redelivery clause will bring in a different time constraint, and thus the best decision has to be changed accordingly. A study on different charter contracts and clauses based on varied measures such as financial and environmental perspectives is therefore also timely.

The study of this thesis is also encouraged and sponsored by Shell, who is facing a variety of real life problems in maritime shipping as one of the oil and gas “supermajors”. Particularly, the need of fulfilling logistics demand of transporting crude oil worldwide and making decisions of investing ships (e.g., either to charter in & out existing ships or order new ships) requires more fundamental understanding of the shipping activities based on operational level. Driven by the inspiration to provide more managerial insights into maritime transportation, the simple but yet typical shipping scenarios such as laden-ballast are studied as they can be easily extended to any practical scenarios.

1.2 Research aims

The thesis contributes to the area of maritime speed optimisation, whereby a cash-flow framework based on Net Present Value (NPV) is developed to explicitly and dynamically account for both economic and contractual parameters that maximises the net profitability of decision maker, and to derive the insights into their theoretical and practical impacts under specified charter contracts.

The research aims of main chapters are given as follows.

Chapter 3 studies the problem in a time charter contract scenario, where a fixed time horizon is given. If the company is about to enter a time charter contract to charter a ship for a time period with particular logistics tasks, what is the best length of the contract and what are the best choices about the speed of the ship on each of the voyages? The research objectives are:

- to develop a particular type of speed model (P_M) for the problem, where we assume the ship will repeat the given logistics tasks a number of times (which can also be a decision) and determine the optimal ship speed on each leg;
- to consider two special cases P_1 and P_∞ where the journey is only executed once or repeated infinitely, respectively, and to compare them with conventional models in literature via NPVEA analysis;
- to provide further insights about maritime economic theory that (partly) explains the empirical studies found in literature.

Chapter 4, based on the profound findings of previous studies, extends the analysis to a voyage-alike contract scenario. In this scenario of shipping there is no time constraint, but a number of profitable opportunities may arise for a (chartered) ship. The research question is then what would be the best speed choices for the ship on each of the legs of a journey and how does the number of journey repetitions affect optimal speeds? The research objectives are:

- to presents a novel formulation of the ship speed optimisation problem in the particular case that the ship will execute a series of multiple-leg journey;
- to demonstrate a Chain Effect when executing a series of identical journeys;

- to illustrate that the optimal speeds are in general highly dependent on the decision maker's view about the ship's Future Profit Potential;
- to better understand the applicability of conventional models, e.g., Ronen (1982), analytically and numerically.

Chapter 5 specifically investigates the contractual details, namely the redelivery clause, at the end of contract by allowing the economic parameters freight rate and bunker price to be time-dependent. This is a special case where particular time constraints are imposed on general speed optimisation models. We aim to show how the firm can use the ship optimally towards the end of contract: is an additional laden leg still possible? The research question is to determine what is the preference of different types of redelivery clauses, e.g., Shell Time 4 versus ExxonMobil 2000, when selecting a charter contract. What would be the impact of market conditions in this decision? The research objectives are:

- to develop speed optimisation models with particular redelivery clause;
- to provide analytical derivation of time-varying economic parameters with managerial insights;
- to determine the preference of the redelivery clauses with numerical examples that based on different criteria.

1.3 Outline of the Thesis

Figure 1.1 outlines the main topics discussed in this thesis and the underlying connections are marked by the overlapping areas. In addition, the reminder of the thesis is then formulated as follows. Chapter 2 gives a review of the literature of classic maritime topics plus the essential techniques used in this thesis, namely NPV framework, Chain Effect and dynamic programming, as the precursor work needed to identify the gap on the study of maritime shipping logistics problems, and thus in particular the incentives of our research work.

In Chapter 3, we clarify the rationale of whether, and if so, the optimal speed decisions should vary for identical ships on identical routes in deterministic settings under a time charter contract (i.e., there is a time limit for the ship to be used). We argue that our proposed model provides higher profitability when compared to traditional models both theoretically (from the NPV equivalence analysis) and numerically. Furthermore, it reveals that the optimal speed decisions could vary for identical ships on identical route with deterministic settings, due to the impact of end of contract scenario and decision

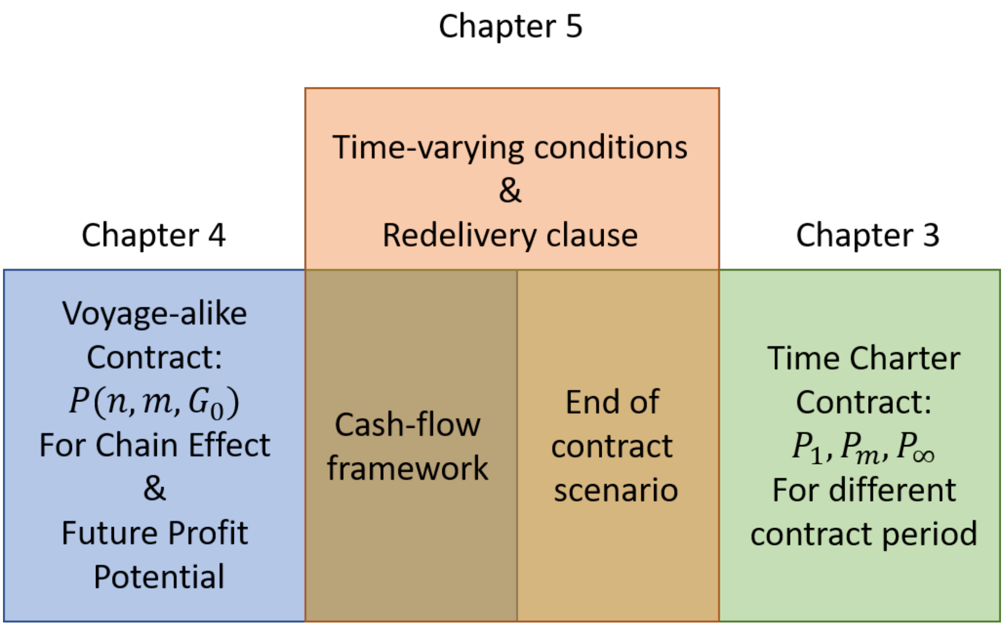


Figure 1.1: Main topics discussed in this thesis

maker's utility function. This chapter has been formulated as a paper submitted to peer review journals, see Beullens, Ge, and Hudson (2020).

Then, in Chapter 4, we address the Chain Effect under voyage-alike contracts (i.e., there is no time constraint) and contribute to state-of-art general speed optimisation model where a mild scenario of multiple voyages is formulated. A Chain Effect is introduced from the area of maintenance and financial investment to show the equivalence of classic criteria are two special cases in our proposed cash-flow framework. We then propose and study the Future Profit Potential (FPP), as it serves similar to the 'termination value' in finance that reflects the decision maker's expectations about future. Managerial insight derived by adopting numerical examples from literature suggests a huge impact of it. Again, this chapter has been formulated as a paper submitted to peer review journals and accepted, see Ge, Beullens, and Hudson (in press).

Next, we study the end of contract scenario as specified in redelivery clause with the consideration of dynamic market conditions in Chapter 5. The theoretical analysis of economic parameters is present to explain the rationale of their impact on optimal speeds. Then the comparison of different types of redelivery clause based on scenarios is given to illustrate when and how any particular clause is favored based on either financial or environmental criteria. Practical insights are discussed to provide the understanding of contractual details in OR literature.

Some selected topics in Chapter 6 are briefly discussed including the *harbour time*, the *Revenue Sharing contract* (RS), the *Contract of Affreighments* (COA), and the *Markov Decision Process* (MDP). These are the potential future works that have been (partly) studied during the research progress of the main chapters.

The harbour time, or the harbour efficiency, though serves as an important role in 'green shipping' and port operations, is claimed to be independent of optimal speeds in maritime speed optimisation literature. However, this leads to a pitfall that one can hardly explain why reducing harbour time is not yet implemented in practice if it is so cheap and effective. Thus we investigate its impact in the context of ship speed optimisation for profit-seeking decision makers in our NPV framework. For RS, it is well studied in the context of supply chain and inventory problems. In this thesis, we try to capture the applicable value of RS for both the ship owner and ship charterer within the charter party. The COA, which is commonly adopted in practice, creates a flexible scenario that can be either treated like liner shipping, tramp shipping or even as a 'mix' of them. We thus want to investigate the optimisation problems for the ship operator with a COA contract. As to MDP, it is widely applied in stochastic setting and already been studied in a few OR literature in shipping. It can generate the optimal policy based on the status whenever a decision needs to be reviewed in the cash-flow framework.

Lastly, conclusions, implications and possible future research directions are summarised in Chapter 7.

Chapter 2

The Precursor Work

2.1 Introduction

The study of maritime transportation has grown exponentially with the global supply chain since the sea or ocean going ship acts as its workhorse. This industry, which has a history of centuries, still faces miscellaneous challenges. To better understand the importance of this thesis and its contribution to not only the society but also the academia, a general but compact review of relevant topics in OR literature is needed here to illustrate the evolution of maritime speed optimisation. The introduction of techniques that have been adopted in this thesis is also provided, such that the inner logic of the incentives is presented explicitly.

We first review maritime speed optimisation literature to provide a comprehensive understanding of the academic contributions so far with respect to yet to be closed gaps or future challenges. Motivations of this thesis are subsequently presented, followed by a presentation of key ingredients of the methodology used almost throughout this thesis, namely the cash-flow-based Net Present Value approach as a means to construct objective functions, the importance of anticipating future scenarios and in particular the so-called Chain Effect, and dynamic programming as the general framework for optimisation.

2.2 Maritime OR literature: speed optimisation

This section presents a compact review of maritime literature with respect to ship speed optimisation. Other studies relevant to more specific topics will be further reviewed in the chapters devoted to our contributions (Chapters 3 to 6).

Seaborne transportation attracts many different types of research addressing various topics. The OR-related literature is primarily concerned with (computer-assisted) decision support and optimisation related to the efficient planning and execution of operational and logistics processes. Impactful reviews such as Christiansen, Fagerholt, Nygreen, and Ronen (2013); Christiansen, Fagerholt, and Ronen (2004); Ronen (1983, 1993) track important publications within this area decade by decade and classify the studies into different topics. These reviews make clear that Maritime OR is an active and growing research field.

Maritime speed optimisation, or the decision about how long a ship is to take on a leg between an origin and destination port, has always played a vital part in the industry according to Psaraftis and Kontovas (2013). However, in many Maritime OR transportation or vehicle routing models in the literature, ship speed is taken as a constant. They argue that assuming fixed and known speeds, instead of treating it as a decision variable, would remove the flexibility of the decision making and render the solutions sub-optimal in many practical applications.

One can consider speed as part of optimisation problems in both liner shipping and tramp shipping applications, see e.g. Ronen (1993). In the former, the shipping company has to design and maintain a service between a set of ports, much like a bus service offered to citizens. In tramp shipping, ships are used to execute particular tasks of limited duration, where the completion time of the current task may impact which future tasks it may perform.

Speed optimisation models can also be divided into general speed models and emission related models, see Psaraftis and Kontovas (2013). General speed models usually have a single objective related to maximising economic earnings or minimising costs to execute a given set of tasks, while emission related models may replace this with minimising environmental impact or at least consider pollution and other green aspects as an integral part of the model's objectives or constraints.

Christiansen, Fagerholt, Nygreen, and Ronen (2007) classify speed-related decision problems into strategic, tactical, and operational. Strategic to tactical problems may be for example about ship design, fleet composition and deployment, including problems that combine aspects of transportation and inventory management. Operational problems tend to focus more on (short-term) vehicle routing and planning of optimal port arrival and berthing times.

A famous and often referred to modelling approach for general ship speed optimisation is the work of Ronen (1982). Similar to the well-known Economic Order Quantity (EOQ) model of Harris (1913) in inventory management, Ronen's analytical framework captures the fundamental economic trade-off in maritime speed optimisation between traveling faster, and thus consuming more fuel, but then also reducing the revenue potential of the ship. Ronen presents three scenarios, each with a different model: the income generating leg, the (re)positioning leg, and the speed related income leg (or penalties). The income generating leg model in particular has been adopted in many studies for representing the basic scenario that a ship transports cargo for a reward. The (re)positioning leg scenario is developed for the case where the ship has to move without cargo on board to seek a next job, but has not yet been widely used, perhaps in part due to the fact that the term defined as *alternative daily value of the ship* may be hard to understand and thus to give a particular numerical value. The third scenario, the income related speed situation, would let the ship travel faster in order to enjoy an additional bonus if arriving earlier, for example, or a penalty if arriving later. This is, however, quite similar to the income generating leg scenario, and thus could be considered a special case of the first model.

It is now well-known that the price of bunker fuel cannot be considered as constant, but instead is affected by geopolitical events and global economic trends. Devanney (2010) studies the optimal speed from the angle of industry as influenced by both fuel price and freight rate dynamics for the case of Very Large Crude Carriers (VLCC). Because of the impact of fuel price on short-term mile-ton demand-and-supply curves, an increase in bunker price may sometimes be more than offset by an increase in freight rate, which may then result in ships in the market travelling at full speed. Devanney's model is based on Ronen and is used to demonstrate that the impact of fuel price on a ship's speed should not depend on whether it is operated in the spot market, or on term charter, or oil company owned.

The decision problems concerned with liner shipping are various. For general review articles with a focus on liner shipping, we refer to Meng, Wang, Andersson, and Thun (2014) and Christiansen, Hellsten, Pisinger, Sacramento, and Vilhelmsen (2019). The main stream in this area focuses on the network design and fleet deployment problems with solving techniques including but not limited to mathematical programming (e.g., integer programming and linear programming) and heuristics (e.g., tabu search), where the speed optimisation problem is not always explicitly included as a sub-problem.

Table 2.1: Speed optimisation related papers in liner shipping

Paper	Logistical context	Approach
Notteboomand and Vernimmen (2009)	liner shipping service design	Case study
Lang and Veenstra(2010)	Berth allocation	Simulation
Golias et al.(2010)	Berth allocation	Genetic algorithm
Ronen (2011)	liner shipping service design	Analytical
Meng and Wang(2011)	liner shipping service design	Branch and bound
Du et al.(2011)	Berth allocation	Second order cone programming
Cheaitou and Cariou(2012)	liner shipping service design	Analytical
Wang et al.(2013b)	Berth allocation	Quadratic outer approximation(B&B)
Wang(2016)	liner shipping service design	Analytical and Mixed-integer linear pogramming
Wang and Wang(2016)	liner shipping service design	Mathematical programming
Mulder and Dekker(2019)	liner shipping service design	Markov decision process

Table 2.1 lists speed optimisation related papers in liner shipping. The topics can be divided into berth allocation and liner shipping service design, where the former considers the problems as jointly shared between the ship and port, and the latter mainly focus on delivering an optimal liner shipping schedule. It is recognised that the problems studied in this area are very similar, such that the solution approach is the main difference.

Generally speaking, the berth allocation problem aims to minimise the total cost that includes both the fuel consumption cost paid by ship operator and operational cost charged to port operator. Though the harbour related costs are not impacted by speed choices, the fuel consumption cost is directly affected by the ship's speed choice, and thus the speed optimisation problem is solved as a sub-problem within this topic. In particular, the main difference is how to deal with the fuel consumption that is known to be a nonlinear function of speed. Lang and Veenstra (2010) assume the fuel consumption varies linearly with speed, while other studies adopting the power function between speed and fuel consumption, which is also known as "cubic law" in maritime engineering (Stopford, 2009), are trying to solve the nonlinear problem by various means, see Du, Chen, Quan, Long, and Fung (2011); Golias, Boile, Theofanis, and Efstathiou (2010); S. Wang, Meng, and Liu (2013b).

As to liner shipping service design, the aim is to determine the fixed service frequency to fulfil certain demand and service window (which could be deterministic or stochastic) while minimising the total operational cost, i.e., the fuel consumption cost. In this sense, the optimal speed is not directly impacted by the economic profits, but indirectly affected by the time window imposed. The fixed service window, somehow, limits the flexibility of speed choices, and thus liner companies are not encouraged to change the ship speed as much as in tramp shipping.

However, affected by the fuel price fluctuation over the last decade, an interesting investigation for liner shipping is to find out the impact of speed reduction, see Cheaitou and Cariou (2012); Meng and Wang (2011); Notteboom and Vernimmen (2009); Ronen (2011); S. Wang (2016); S. Wang and Wang (2016). The key point of these investigations lies between the trade-off with slow steaming and additional ships. Thus the companies are encouraged to bring in more ships into service in accordance with the speed reduction as it is shown to be economically beneficial. Another area of study is to design the service with the consideration of uncertainties, e.g., the ship may not be able to travel as scheduled and the service window is often violated in practice. For instance, Mulder and Dekker (2019) study the potential actions to manage such delays in the stochastic environment.

We now turn our attention to tramp shipping. The main focus here is to determine the optimal usage of the ship, including the selection of best available tasks, often including also deciding the optimal ship speed. The tramp shipping literature that considers ship speed optimisation can be further divided into categories as displayed in Table 2.2: (a) logistical context; (b) number of ships; (c) payload function included; (d) fuel price included; (e) freight rate included.

A decent proportion of tramp shipping OR literature consider pickup and delivery problems. These problems deal with situations in which certain amounts of cargo are waiting

Table 2.2: Speed optimisation related papers in tramp shipping

Paper	Logistical context	Number of ship	Payload function	Fuel price	Freight rate
Norstad et al.(2011)	Pickup and delivery	Many	No	Implicit	No
Hvattum et al.(2013)	Fixed route	One	No	Implicit	No
Fagerholt and Ronen(2013)	Pickup and delivery	Many	No	Implicit	Spot cargo
Psaraftis and Kontovas(2014)	General problem	One	Explicit	Explicit	Explicit
Magirou et al.(2015)	Route selection	One	No	Explicit	Explicit
Wen et al.(2016)	Pickup and delivery	Many	Implicit	Implicit	Implicit
Wen et al.(2017)	Pickup and delivery	Many	Explicit	Explicit	Explicit
He et al.(2017)	Fixed route	One	Implicit	Implicit	Implicit

to be transported, each having their individual origin and destination pairs, and available time windows for pickup and/or delivery. The decision maker aims to find out the optimal order of cargo and associated speed to maximise his profitability. As this problem is often faced by shipping companies with fleets, the studies relevant either assume homogeneous or heterogeneous ships available. The difference between them are based on which characteristics are included in the modelling framework, primarily related to: fuel price; freight rate; and the payload function. The latter function, not frequently encountered in the present literature, aims to more accurately model the relationship between dead weight carried, ship speed and fuel consumption (Psaraftis & Kontovas, 2014). The readers are referred to Fagerholt and Ronen (2013); Norstad, Fagerholt, and Laporte (2011); Wen, Pacino, Kontovas, and Psaraftis (2017); Wen, Ropke, Petersen, Larsen, and Madsen (2016).

In the fixed route problem, on the other hand, the ship has to visit a series of predetermined ports with (or without) time and other constraints. The aim is to find out the optimal speed, perhaps for different ships, on each leg that maximises the profitability or minimises the total cost, see He, Zhang, and Nip (2017); Hvattum, Norstad, Fagerholt, and Laporte (2013) for instance. It is recognised that Magirou, Psaraftis, and Bouritas (2015)'s route selection problem is a special case within this topic but in a dynamic setting. They maximise the net daily revenue by finding the equilibrium of traveling between ports on a possible route and thus by comparing this net daily revenue one can find the best route that contains the optimal visiting sequence of the ports.

Psaraftis and Kontovas (2014) present a general speed optimisation model that can be modified for any particular route. However, their main focus is to provide more fundamental insights into the modelling approach. Notably, they were perhaps first in the OR speed optimisation literature to present a calibration of the fuel consumption function as to include the ship's payload more accurately. Economic parameters such as fuel price, freight rate and charter hire are also included explicitly in their model. Furthermore, they propose a *Decomposition Principle* that each segment (trip between two consecutive ports) of a sequenced voyage can be solved individually and independently. This reduces the computational cost of the speed optimisation algorithm considerably.

Meng, Du, and Wang (2016) use the real log data obtained from container liner shipping company to examine the relationship between fuel consumption and speed, displacement, sea conditions and weather conditions. Here the term displacement is actually a measure to reflect the ship's deadweight carried. When assuming the lightweight of a ship is fixed in mild conditions, this term can be further simplified to payload. Their study thus provides empirical support for the calibration of the payload function in Psaraftis and Kontovas (2014).

In bunker management problems, speed decisions also play an important role as it directly affects fuel consumption, and indirectly the ship's net cargo carrying capacity (S. Wang, Meng, & Liu, 2013a). On the one hand, bunkering should be done as late as possible and only covering the next leg, so as to minimise inventory carrying costs and maximise a ship's fuel efficiency and carrying capacity. On the other hand, bunkering may also be performed earlier in the journey, at a port where the fuel is cheaper. These decisions will further be influenced by fuel price dynamics and uncertainties, as well as ultimately by the economics of freight rates, as shown in the basic models of Ronen (1982) and Devanney (2010), discussed earlier.

Table 2.3: Speed optimisation related papers in bunker management

Paper	Logistical context	Fuel price	Fuel consumption	Port time	Approach
Besbes and Savin(2009)	Liner and tramp	Stochastic	Deterministic	No	Heuristic
Oh and Karimi(2010)	Liner	Stochastic	Deterministic	No	Scenario based
Yao et al.(2012)	Liner	Determinisitc	Deterministic	Determinisitc	Mixed integer linear programming
Kim(2014)	Liner	Determinisitc	Determinisitc	Determinisitc	Lagrangian heuristic
Vilhelmsen et al.(2014)	Tramp	Determinisitc	Determinisitc	Determinisitc	Column generation
Meng et al.(2015)	Tramp	Determinisitc	Determinisitc	Determinisitc	Branch and price
Sheng et al.(2015)	Liner	Stochastic	Stochastic	Determinisitc	Progressive hedging algorithm and Rolling horizon solving approach
Aydin et al.(2017)	Liner	Stochastic	Stochastic	Stochastic	Deterministic approximation of dynamic programming
Wang et al.(2018)	Liner	Stochastic	Determinisitc	Determinisitc	Mixed integer second order cone programming

Table 2.3 summarises a selection of relevant papers that consider speed optimisation in the context of the bunker management. It is noted that only when given the scenario of tramp shipping, e.g., Besbes and Savin (2009); Meng, Wang, and Lee (2015); Vilhelmsen, Lusby, and Larsen (2014), the economics of freight rate is considered. For liner shipping bunker management, the objective is always to reduce the operational cost, i.e., the fuel consumption cost, while maintaining the fixed service frequency or meeting the pre-agreed service window. Factors such as the fuel price, fuel consumption on sea and harbour time, though known to fluctuate over time and specific to a port (or route), are often treated as deterministic parameters that are fixed and known in advance, as in e.g., H. Kim (2014); Yao, Ng, and Lee (2012). The stochastic nature of fuel price is examined by Oh and Karimi (2010); Y. Wang, Meng, and Kuang (2018), while Sheng, Chew, and Lee (2015) further allow the fuel consumption to be random because of sea or weather conditions: Due to the dynamic nature of bunker management, it is recognised that the optimal bunker policy should be an order-up-to or (s, S) policy, whereby the ship shall only bunker while the inventory is lower than a certain level s and bunker up to the level S. Aydin, Lee, and Mansouri (2017), in addition, relax the harbour time and service time window to be stochastic.

Maritime transportation studies in which emissions and the environmental impact are taking the forefront are becoming more numerous, and may also be known under the labels of “green shipping” or “sustainable shipping”. The latter is defined by the Brundtland commission in 1987, and is a holistic management concept that includes environment, economy and society. This aims to maintain changes in a homeostasis environment for future generations. Therefore the development of “sustainable shipping” relies on the improvements of “green shipping” but looks at a more general and wider scope of the problem. For comprehensive reviews of “green shipping” and “sustainable shipping”, the readers are referred to Psaraftis (2019b); Shi, Xiao, Chen, McLaughlin, and Li (2018).

Table 2.4: Emission related speed optimisation papers

Paper	Research theme	Particular concern
Corbett et al.(2009)	Green shipping practice	The relations between airborne emissions and speed and thus the impact of speed reduction
Kontovas and Psaraftis(2011)	Green shipping practice	The impact of speed reduction of container ships incorporated with reducing harbour time
Devanney(2011)	Green policy	The impact of EEDI on VLCC as well as carbon context fuel tax
Gkonis and Psaraftis(2012)	Relations between economic and emissions	The emissions led by optimal economic speed of tanker ships
Fagerholt and Psaraftis(2015)	Green policy	The impact of ECAs on optimal speeds
Fagerholt et al.(2015)	Green policy	Routing and speed optimisation with consideration of ECAs

Unlike the taxonomy of emission related speed optimisation models in Psaraftis and Kontovas (2013), Table 2.4 presents a selection of emission related speed optimisation models based on its research theme within the context of green shipping.

Green shipping practice is defined as the industrial measures with the aim to reduce waste and pollution for the protection of the environment (Lai, Lun, Wong, & Cheng, 2011). Speed reduction is particularly examined as the primary tool in green shipping by Corbett, Wang, and Winebrake (2009); Kontovas and Psaraftis (2011). In addition, Gkonis and Psaraftis (2012) claim that the high fuel price has already brought in the effect of speed reduction and thus the optimal economic speed can lead to certain emission reductions. Of course, fuel prices can also go down significantly, as in the period of 2020, and is not the only factor of influence, recall e.g. Devanney (2010).

Green policy, on the other hand, refers to the mandatory regulations with an emphasis on contributions towards environmental friendly industry and society. Devanney (2011), from the perspective of industry, examines the impact of Energy Efficiency Design Index (EEDI) on carbon emissions for VLCCs. EEDI is a technical measure for improving energy efficiency made mandatory for new ships with the adoption of amendments to MARPOL Anne VI in 2011. Devanney argues that EEDI will inadvertently induce more CO2 emissions and instead proposes that a carbon context bunker fuel tax is the more powerful measure. His main argument is that because the EEDI will only reduce the maximum power installed on a ship, while a well-designed VLCC only operates at full speed in a full-scale boom, which typically corresponds to less than 10% of the ship’s life. However, requiring ship owners to use less powerful ships, he argues, will bring about a larger fleet size. As a consequence, EEDI, as an indirect policy to limit carbon context emissions, may not work as hoped for. Carbon context fuel tax, on the other hand, is a more directly measure and thus should be favoured, according to Devanney.

MARPOL Annex VI, the international convention, adopted at International Maritime Organisation (IMO) also defines Emission Control Areas (ECAs) as a list of sea areas with a global limit of airborne emissions. It is recognised that ships need to use more expensive but environmental friendly bunker fuels within these areas, which can be interpreted as fuel tax to some degree. As a consequence, its impact on maritime shipping is studied by Fagerholt, Gausel, Rakke, and Psaraftis (2015); Fagerholt and Psaraftis (2015).

To conclude, this short review demonstrates that OR decision models and ship speed optimisation are gaining more recognition in both the academic OR literature and the maritime policy and legislation areas. Most shipping related problems, in liner and tramp shipping situations, can be formulated on the operational planning level as voyage- based scenarios in which the sequence of port visits is either pre-determined or part of the decisions. Additional constraints are added to represent the practicalities of a particular application context. An integral part of these models should be the accurate modelling of the fuel consumption versus the revenue potential trade-off, and the dependency of this trade-off on important parameter and variables such as ship payload and speed. Many of the current OR decision models, however, do not include this level of detail, and it may thus be questionable if the conclusions reached from these studies provide sufficiently accurate insight into the impact of decisions on the optimal economic (and environmental) usage of a ship.

2.3 Motivations

In this thesis, we do consider ship speed as in integral part of the decision problems considered. The faster a ship travels, the more fuel it will consume but also the more revenue generating work the ship can perform per unit of time. For instance, an oil ship travelling 1500 nm at 12 kn instead of 15 kn may save almost half of its fuel consumption per day (approximately from 25 ton per day to 13 tone per day on ballast leg) but will take roughly 25% more days (from 4 days to 5 days) to complete the leg.

Given the area of application of the sponsor organisation, we focus in this thesis on ships travelling under charter contracts. This includes both time charter contracts and voyage charters, as well as investigate situations that may also capture certain other contract types like COAs (Contract of Affeightment). These situations are neither liner nor tramp shipping situations, but exhibit some aspects of both.

We aim to develop general speed optimisation models derived from cash-flow functions. The economic objective is then to maximise the Net Present Value (NPV) of the ship's cash-flow function over a relevant time horizon. This approach to developing OR decision models appears to be novel at the time of starting this research (May 2016). It will in particular be suited to help determine optimal ship speeds, but other types of decisions can be incorporated into this framework as well.

The model developed will account more accurately for conditions that affect a ship's fuel consumption, such as accounting for its actual deadweight carried, as first advocated in Psaraftis and Kontovas (2014). We do not yet consider environmental constraints or objectives explicitly. However, the models can be used to minimise fuel consumed, which often directly relates to emissions saved. Furthermore, this methodology, as it aims to be able to more accurately predict the impact of operational usage on a ship's performance, should be highly suited to also consider a decision maker's environmental aims in future research.

The majority of models in this thesis will focus on determining the impact of economic and contractual parameters on the optimal travel times of a given ship between ports as part of any arbitrary but given (roundtrip) journey. The order in which to visit the ports, as well as the freight it will carry from one port to another, are all pre-determined. This research thus fits into the fixed-route category reviewed in Section 2.2. This framework can be extended to include also port sequencing decisions, or be part of pickup and delivery problems.

We focus in particular on how the execution time of one leg of the journey will affect the timing of other legs of the journey. The optimal execution of a series of repeated (identical) journeys has, to our knowledge, also not received explicit attention. The models as such investigated are aimed at giving a sound foundational insight and understanding into the ship speed optimisation problem, and should provide a sound foundation for further research into more tactical and strategic issues such as ship design and selection for optimal operational performance to execute an intended (set of) journeys.

Because of the importance of Ronen (1982) to the literature, our research will also devote considerable attention to comparing our models and findings to Ronen's modelling approach with the general aim to increase our understanding of this model's area of applicability. Indeed, the cash-flow NPV approach has been shown to have the potential to increase such understanding in the area of inventory management, as further explained in Section 2.4.2.

In ship speed optimisation models from the literature, the optimal ship speed is determined from a set of parameters about the ship, the cargo that needs to be transported, and the routes between the various ports the ship needs to visit. If the same set of data presents itself some time later, it is reasonable to assume to lead to the same solution. In reality, this may however not be the case, perhaps because the decision makers may consider the impact from factors not captured in the model, or perhaps because the decision makers may use other performance criteria.

The AIS data collected by Prakash, Smith, Rehmatulla, Mitchell, and Adland (2016) shows that the observed travel speeds of real ships vary from the minimum to the maximum within a ship's technically feasible range. These authors claim that the underlying reasons that determine the speed of a ship are hard to understand and translate into mathematical logic. To the best of the our knowledge, this gap has indeed not been closed as no theories or models in maritime shipping can explain or describe this observation. The main aim of this dissertation is to explore a few key features of reality that may have impact on ship speed decisions yet are not included in the models that are available in the literature.

By developing mathematical models based on the general framework of Net Present Value (NPV), this thesis aims to increase our understanding how a decision maker would have to decide on ship speed as to maximise the NPV of the ship's economic function. In particular, we wish to investigate to what degree the specifics of the (charter or voyage) contract influences this optimisation model. We also wish to pay attention to comparing this method with existing modeling techniques from the literature as to extract the added value of this approach.

For example, it seems within reason to assume that the time horizon in a time charter contract, as well as the details of the redelivery clause, will affect how the ship charterer will decide on the amount of work the ship can undertake during on this contract, which is affected by the ship speed as well as the time spend in harbours. There is to date not much research published on the impact of the contract on ship speed decisions. The NPV methodology adopted is particularly suited to study the interactions between logistics decisions and contracts, i.e. the payment structures that firms adopt to reward each other for rendered services.

An often encountered situation is that a ship may be used to execute a number of repetitions of a particular kind of roundtrip journey between a set of ports. This may be in the context of a time charter contract, but could also be encountered in other contract settings where the completion time of the final repetition can be freely chosen. In the latter situation, we are interested to study an important yet not well known phenomenon that we call the Chain Effect.

The Chain Effect can be tracked back to Preinreich (1940), who studied the single machine replacement problem. It is a phenomenon that is not well known, except perhaps in the context of investment appraisal, and it describes that the optimal economic life time of identical projects executed in series tends to increase for each successor project. The Chain Effect is a result of NPV modelling and it is able to explain that our assumptions of the relevant future of the ship must influence our present decisions about how to use the ship, including how fast is should travel. It is an aspect that is often not well formulated in current speed optimisation models from the literature. This is another motivation of why the want to use the NPV methodology: it helps to better understand potential shortcomings of existing modelling approaches from the literature, and how to make improvements.

Next to modelling the contract and the importance of how we account for how we intend to use the ship in the future, we also consider the importance of how key data, such as freight rates and fuel prices, may change over time. This may lead, for example, to the ship speeding up or slowing down over future repetitions of a roundtrip journey. (Or even to lay-up, when economic conditions worsen sufficiently.) To this end, we envisage that the technique of dynamic programming would be able to help obtaining optimal solutions or strategies. In particular, formulations of the problem where the ship is to execute a number of n future port to port legs, can be solved by dynamic programming in order to identify e.g. optimal ship speed policies. The technique can also handle non-linear objective functions, and towards future further research may prove fruitful to tackle stochastic conditions, or lead to reinforcement learning and other approaches.

To enable this study, a few key techniques and methodologies will thus be relied upon. The purpose of the remainder of this chapter is to introduce these areas, and point out the role they will fulfill. In particular, we will first introduce cash-flow Net Present Value modelling as the main methodology to constructing speed optimisation models. Within this framework and important yet not well known phenomenon, which we refer to as the Chain Effect, naturally arises and will also be discussed. Finally, we present the dynamic programming technique as a means to solve the NPV ship speed optimisation models.

2.4 Cash-flow NPV modelling

From the perspective of finance, a ship executing a voyage can be viewed as an investment project, where a cost occurs at the start of the activity and the payoff is collected at the end of it. If we ignore the physical limitations of maritime shipping, then a shipping route selection problem, for example, can be viewed as an investment project scheduling problem. We could then rely on project scheduling literature, such as e.g. Creemers (2018); Leyman and Vanhoucke (2017).

To model ship speed decisions, however, we want to make sure to use an accurate representation of how the timing of cash-flows are affected by ship speed decisions. Instead of the traditional NPV approach based on discrete time periods, we will therefore use the continuous time cash-flow function modelling approach.

2.4.1 Origin and approach

The cash-flow framework has been widely used in finance. The opportunity cost of capital or interest rate r is used to calculate the Net Present Value of a series of payments which occur in a discrete time framework as described in Figure 2.1:

$$NPV(r) = -v_0 + \sum_{i=1,2,3,\ldots} \frac{v_i}{(1+r)^i}, \tag{2.1}$$

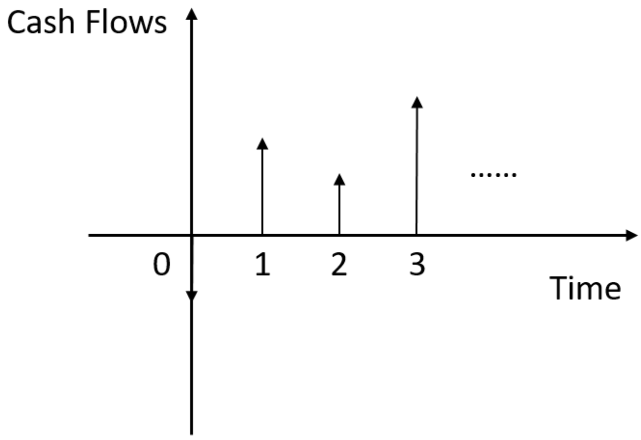


Figure 2.1: Cash-flows of an investment

where v_i is the cash amount received at the end of time period i.

When having the option to invest in one or more projects, calculating the NPV of the future money streams of each of these projects allows us to determine and compare their attractiveness. In absence of other strategic considerations, only those projects that have a non-negative NPV are worthwhile to consider, because otherwise the firm would be better to invest in the next best alternative that would produce a return as expressed through the value of the opportunity cost. Also, if only one project can be executed, the one having the highest non-negative NPV is preferred.

The NPV is one of the most widely applied approaches to appraising investment proposals. If can, however, also be applied to determine the best ways to execute business activities that generate future cash-flows. The best approach to the execution of an activity would then be that approach that maximises its NPV.

Many operational decisions affect the duration of processes in a way that is not well captured by a discrete or periodic approach. That is why the continuous-time formulation based on cash-flow functions, while not widely used in finance, is now becoming more readily used in the study of operational decisions to optimise the economic performance of e.g. production-inventory systems. To distinguish this case from the discrete case, we denote the continuous-time opportunity cost of capital rate by α.

An early general continutous-time formulation based on the cumulative cash-flow function can be traced back to at least Preinreich (1940). Let there be a cash-flow function $V : \mathbb{R} \to \mathbb{R}$, where the domain represents time t and $V(t)$ the cumulative amount of money received before or at time t. The Net Present Value of a cash-flow function $V(t)$ subject to a rate of interest $\alpha(t)$, $t \in \mathbb{R}$, is then given by the Stieltjes integral:

$$NPV(\alpha) = \int_0^\infty e^{-\int_0^t \alpha(\tau)d\tau} dV(t), \tag{2.2}$$

where the improper integral is interpreted as a limit of a definite integral over $[0, t]$ where $t \to \infty$.

As can be observed, the NPV does not depend on the level of $V(t)$, and is unaffected by the cash-flow accumulation prior to $t = 0$.

We can easily work with a set of separate functions for various revenue streams and various cost streams, since the NPV is a linear operator:

$$\int_0^\infty e^{-\int_0^t \alpha(\tau)d\tau} d(\sum_{i=1}^{n} c_i V_i(t))$$
$$= \sum_{i=1}^{n} c_i \int_0^\infty e^{-\int_0^t \alpha(\tau)d\tau} dV_i(t). \tag{2.3}$$

If $V(t)$ is differentiable over $[0, \infty)$, then let:

$$\frac{\partial V(t)}{\partial t} = v(t). \tag{2.4}$$

The NPV of a differentiable cash-flow function $V(t)$ can now be expressed as the Riemann integral:

$$NPV(\alpha) = \int_0^\infty v(t) e^{-\int_0^t \alpha(\tau)d\tau} dt. \tag{2.5}$$

In most applications it is often reasonable to assume a constant rate of interest, $\alpha(t) = \alpha > 0$. The NPV can then be viewed as the Laplace transform of the time function v, and properties of the Laplace transform can be exploited. This approach can be traced back to at least Grubbström (1967):

$$NPV(\alpha) = \int_0^\infty v(t) e^{-\alpha t} dt. \tag{2.6}$$

As explained in Grubbström (1980), Figure 2.1 is the case where the cash-flow function $v(t)$ contains multiple Dirac delta functions at each time point, denoting a series of finite payments. Calculating the NPV of discrete cash-flows then reduces to:

$$NPV(\alpha) = -v_0 + \sum_{i=1,2,3,\ldots} v_i e^{-\alpha t_i}, \tag{2.7}$$

where t_i measures the time at which the amount of cash v_i is exchanged, relative to decision time 0.

The Laplace transform also finds use in the study of economic decision problems with stochastic cash-flows, see Grubbström (2007).

2.4.2 NPV in operational decision models

Though NPV has been mostly developed as a tool in finance, it has by now also received a fair amount of attention applied in OR studies as well.

The benefits of NPV modelling for the study of production-inventory systems was perhaps first convincingly demonstrated in Grubbström (1980). In particular he showed how the method could lead to the accurate determination of the holding cost of keeping raw materials, work-in-process, and final products at the various stages in supply chains. It also leads to a method by which the applicability of classic inventory models can be better understood, since these models are often based on unit holding cost parameters, of which the meaning is often not well enough understood. This limits the practical applicability of these classic models since it is difficult to know how to choose appropriate numerical values for these parameters. NPV can thus help to increase the classic model's applicability to practical situations.

At other times, the meaning of the classic model's parameters remains ambiguous. In other words, the mapping of traditional parameters to the AS functions is not necessarily injective as shown by Çorbacıoğlu and van der Laan (2007); R. Teunter and Van der Laan (2002); R. H. Teunter, van der Laan, and Inderfurth (2000); Van der Laan and Teunter (2002). This research deals with remanufacturing processes in which final products are produced from either virgin materials or reused materials. The inventory costs of products and intermediate components in those situations cannot always be easily determined when working in the classic inventory modelling framework.

It was also noted in the literature that the interpretations of classic models through NPV modelling in the literature sometimes led to controversial interpretations. For example, Grubbström (1980) already found that in a simple production-inventory model where the production is re-initiated after a batch has been produced and then shipped, the inventory holding costs of the intermediate products, while the batch is being produced, is to be valued at sales price rather than at the investment cost. The latter, however, is the only interpretation found in the traditional treatment in textbook descriptions concerning the meaning of inventory holding costs.

Some progress in our understanding of why this may be was made in Beullens and Janssens (2011). They introduce the concept of the Anchor Point as to be able to model both push and pull conditions in NPV models. It is used in their demonstrations about how the assumption about how cash-flow functions change with operational decisions needs to be incorporated with some care so as to reflect reality most closely, and they used it to explain some of the differences that had been observed between the NPV modelling approach and the more traditional OR modelling techniques based on average cost functions. In particular, they found that classic modelling techniques based on average cost function often implicitly assume pull conditions, whereas most NPV models to date assume push conditions[1]. Applied to the batch production problem introduced in Grubbström (1980), they found that Grubbström's interpretation applies under push conditions, but the classic interpretation would be arrived at when adopting pull conditions.

The approach of using NPV modelling to investigate the applicability or adaptability of classic decision models to a specific cash-flow context has been formalised in Beullens and Janssens (2014) and is called NPV Equivalence Analysis (NPVEA). The approach may consist of taking the Maclaurin expansion of exponential terms in α so as to linearise the NPV function, since the classic models often are linear. However, the key to make NPVEA modelling work well, they argue, is to start from cash-flow functions to represent the financial implications of how an activity is executed. A benefit of the NPV approach is that it allows for a more seamless integration of both operational decisions and decisions about how firms arrange contracts between each other stipulating also when they will financially compensate each other for services rendered. These are referred to by these authors as payment structures. NPV modelling is in that sense much more powerful than the classic average cost models, since the timing of when firms exchange cash-flows may deviate significantly from the timing that important logistical events occur in the system. As the classic modelling techniques often assume implicitly that the payments occur with these events, they require more special attention to get these situations correctly modelled if the payment arrangement are very different. The NPV methodology, in contrast, makes little formal distinction between any of these different cases, and may thus be a more powerful approach to dealing with an array of different possible payment structures or contracts between the parties.

Cash-flow based NPV modelling and NPV Equivalence Analysis was used in Ghiami and Beullens (2016) to examine production-inventory systems where part of shortages may be lost and the other part results in backorders. They demonstrated how the approach can lead to insight into the value of planning for shortages and other strategic choices about the design of the system. In particular, they found that if customers would be financially compensated for incurring a backorder, then the classic unit backorder cost and unit lost sales cost parameters would have to depend on a set of non-financial systems design choices such as the production rate and the probability of whether a shortage would result in a backorder or in a lost sale. In other words, these models would not give a reliable insight from sensitivity analysis since these parameters are thought of as

[1]Under pull conditions, decisions need to be taken now about the operational use of system such that its generated output is guaranteed to start at some fixed (but arbitrary) time in the future and under push conditions, however, decisions are to be taken now about the operational use of a system when the first activity of the system needs to start at some fixed (but arbitrary) time in the future (Beullens & Janssens, 2011).

exogeneously determinable. The application of NPVEA has thus demonstrated that in the system under study the classic modelling framework has serious shortcomings.

The importance of payment structures, or properly accounting for the timing of when firms rewards each other relative to the time of execution of rendered services, also applies to the situation where these events occur stochastically, but much less research has been conducted in this area. Porteus (1985) examines such situations, and points out the importance of accounting the timing of income and outcome in the regenerative process with an example of Harris (1913)'s EOQ model.

The NPV methodology is slowly gaining momentum in the inventory modelling literature. Andriolo, Battini, Grubbström, Persona, and Sgarbossa (2014) is the first review paper providing also a comprehensive review on classic economic inventory models in which the cash-flow NPV approach is prominent. A complementary article is Beullens (2014), who reviews the lot sizing problems with further fundamental insights provided by adopting the NPV framework. In particular, he finds that the textbook approach to inventory management for firms forgets to include the "inventory rewards" for suppliers from the inventory kept by their buyers. This phenomenon was first proposed by Crowther (1964), without providing a mathematical proof. In the literature, it was thought to be only relevant in the context of quantity discounts, and then only viewed as an alternative modelling approach that did not attract much followers. Beullens' article does contain a proof that it is in general as relevant to inventory management theory as the EOQ model of Harris (1913).

There is thus a body of evidence in the literature that the cash-flow based NPV framework may be a more robust approach to investigating how system performance can be optimised in problems where how you execute the activity over time also affects the magnitude and timing of associated cash-flows.

The application of inventory management concepts and modelling approaches in the maritime shipping context has been limited. Psaraftis and Kontovas (2014) have defined a conceptual term in maritime shipping, namely the "in-transit costs", that incorporates the financial opportunity cost of both the cargo loaded and the cargo to be loaded in next ports. This concept, which is similar to the financial opportunity cost from investments in transitional inventory models that implicitly accounts for the time value of costs, is arguably true only if the model is designed from the perspective of both the ship operator and the shipper. Thus the model looks at no longer a simple transportation problem but the whole supply chain. In other words, back to the operational level, neither the shipper nor the ship operator would consider this cost unless the payment is occurred and interacts with other parties in line with the shipping activities.

Putting aside the argument, it is still not clear when and how this cost is paid. As a consequence, the *payments structure* of all the cash-flows, as introduced in Beullens and

Janssens (2014), may not necessarily be occurring at the same time as the logistics activities, and there may be other revenues and expenditures of importance that are triggered at various times and which may not be accounted for. For example, it is known that the revenue is normally but not necessarily collected from the shipper before unloading at the destination port. The question is not (only) to what degree a change in the payment structure would affect the financial performance for the shipping company, but also to what degree it may influence operational execution of the activity including the ship's optimal average speed (or travel time between the ports). There are also many other kinds of payment structures present in the maritime context. The harbour associated costs, for example, are often paid with a few days delay. Charter hire fees, likewise, may be paid in particular ways not accurately reflected by a daily expense. Therefore, it is safer to start the model development by including such payments structures explicitly in the corresponding cash-flows. This may lead to a better understanding of how to interpret the relative importance of the different financial contractual elements and the different logistical decisions to the overall net present value of the shipping activity. It also offers a formal mathematical procedure for capturing the financial consequences, from the viewpoint of the decision maker, of holding stocks at any point in the system.

Reduction of the model complexity can then be done with better assurance that only less significant aspects of the problem context are simplified. One way in which this can done is through linearisation, as shown in e.g. Grubbström (1980) and Beullens and Janssens (2014). This process can also be used, in the process of NPVEA, to increase our understanding of existing models in the literature that were not based on NPV principles.

2.4.3 Summary of findings

Although there are numbers of classic maritime speed optimisation models in the literature, they do not offer a method by which one can explicitly estimate the impact of model parameter choices. There is less assurance for such models on whether all the relevant factors have been included with respect to the impact of payment structures on optimal logistics decisions.

Cash-flow NPV modelling, in contrast, provides an easier way to evaluate shipping related problem by using cash-flow functions for model construction. It then provides the option to explicitly account for both the amount and timing of payments exchanged between relevant stakeholders within e.g. the charter party. It offers a way to investigate how the set-up of particular clauses in the contract may affect what would be the optimal operational execution of the activity by the ship.

The objective function of the ship operator is expressed as the NPV of all future cash-flows that are discounted to the decision making time point. By solving the model, the aim is to find out the best strategy that maximises this NPV value, which offers the chance for firm to evaluate logistics decisions as well as financial investments.

2.5 The Chain Effect

The Chain Effect, proposed by Preinreich (1940), is described in Figure 2.2. The "chain" represents the execution of an activity a number of times, where the decision task is to determine the optimal duration of each replication in order to maximise the Net Present Value of the whole chain. Later this concept has been widely applied in the area of equipment replacement (Bethuyne, 2002) and further introduced to financial investment by Götze, Northcott, and Schuster (2015, Chapter 5.3) for a project investment problem. We notice that authors in both streams conclude that the optimal life time of the activity should become longer and longer with every repetition. When the number of repetitions tends to infinity will the optimal life time be equal for each successive execution of the activity.

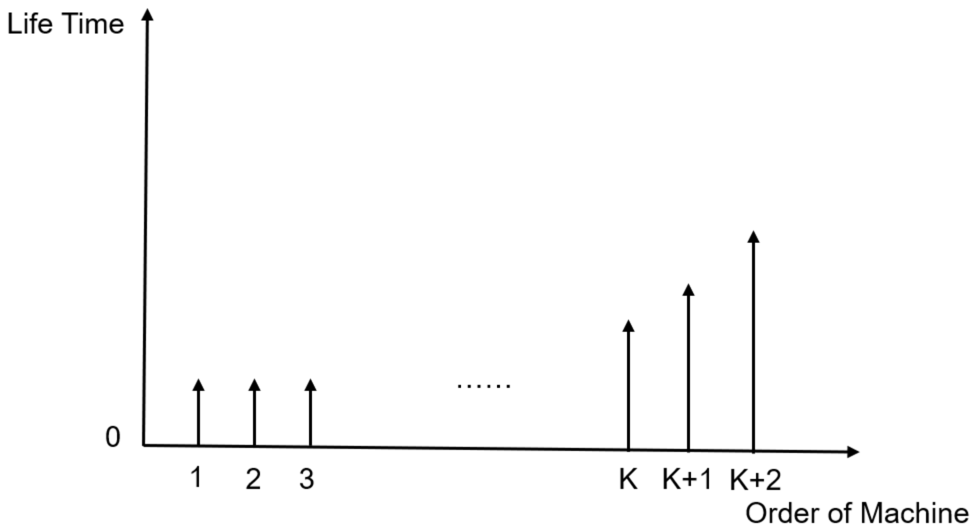


Figure 2.2: Chain Effect in single machine replacement

2.5.1 Equivalence of project investment and machine replacement

The original problem in Preinreich (1940) can be expressed as a problem in which a machine for production is to be replaced as follows:

$$V = \int_0^T (zQ(t) - E(t))e^{-\int_0^t i(r)dr}dt + Se^{-\int_0^t i(r)dr}. \tag{2.8}$$

V is the unknown capital value of a single machine, T is the unknown date at which the machine should be discarded, z is the known market price of the product, $Q(t)$ is the rate of production, $E(t)$ is the combined rate of all expenses, $i(t)$ is the rate of interest, and S is the scrap value of the machine when discarded.

Then he argues that, when only one machine can be kept in operation, (2.8) can be re-written as $V = B+G_j$, where B is the known cost of a single machine, and G_j is the *good will* of the j-th last machine, which consists the present value of aggregated capital values of all future replacements. For the simplicity of calculation, the "order" of a machine is counted backwards in time. The *good will* of machine j is the accumulated NPV from all machines $j, j-1, ..., 1$ that will be used in the relevant future, and discounted to the moment that machine j is introduced:

$$G_j = \int_0^{T_j} (zQ(t) - E(t))e^{-it}dt + (S + G_{j-1})e^{-iT_j} - B. \tag{2.9}$$

The time "0" in (2.9) also corresponds to the moment that machine $j+1$ was discarded, or the start time of the chain when j is the first.

For an investment project problem, we can formulate the problem in the same way:

$$G_j = \int_0^{T_j} R(t)e^{-it}dt + G_{j-1}e^{-iT_j} - C. \tag{2.10}$$

Here the $R(t)$ is the rate of rewards and C is the initial investment. Thus we can take (2.10) as a special case of (2.9), where the rate of expenses and discarded value $E(t) = S = 0$.

2.5.2 Derivation of the Chain Effect

In this section, we derive the Chain Effect from (2.9) to show its mathematical root and explain why it would be claimed to be exactly as Figure 2.2.

Preinreich claims that finding the maximum of G_j is equivalent to solving $\frac{\partial G_j}{\partial T_j} = 0$, viz.:

$$(zQ(T_j) - E(T_j))e^{-iT_j} - i(S + G_{j-1})e^{-iT_j} = 0, \tag{2.11}$$

By re-arranging the order of terms, we have:

$$G_{j-1} = \frac{zQ(T_j) - E(T_j)}{i} - S. \tag{2.12}$$

Now by assuming $G_0 = 0$, indicating that there is no future machine being replaced and thus no relevant cash-flows involved, one can solve the problem recursively with (2.9) and (2.12).

Actually, the term $zQ(T_j) - E(T_j)$ is the net profit of a single machine by selling its productions and deduct the relevant expenditures, which shall always be positive, and otherwise the machine should be replaced. Thus according to (2.9), the *good will* is a non-decreasing term. Because, in the context of machine replacement problem, the production rate $Q(t)$ is normally assumed to be decreasing with its life time, while the expense rate $E(t)$ should be increasing with t, thus by noticing (2.12), Preinreich observes the life time of successive machines tends to be longer, i.e., $T_j > T_{j-1}$.

When the chain, or the number of machine replaced, extends to infinity, Preinreich argues that a limit eventually emerges, such that $G_k = G_{k+1}$ and $T_k = T_{k+1}$. That is, we can omit all the subscripts and substitute (2.12) into (2.9), viz.:

$$z[Q(T) - Q(0) - \int_0^T Q'(t)e^{-it}dt] = E(T) - E(0) - \int_0^T E'(t)e^{-it}dt - i(B - S) \quad (2.13)$$

This implies, as shown in Figure 2.2, once the length of the chain exceeds K, where K is a large positive integer, the optimal economic life time of the first few machines should be equal. For the later machines, their economic life time will get longer and longer.

We will henceforth refer to this as the Chain Effect, however extend our definition to cases whenever this relationship of successive optimal life times is non-decreasing or non-increasing.

2.5.3 Summary of findings

Back to the discussion of the speed choice of ships: if each voyage is viewed as an identical machine or investment project, the same methodology could be applied to maritime shipping when we use the NPV framework. We could thus expect to observe the Chain Effect in the shipping context as well. Within the context of shipping, however, the scenarios are more complicated and we need to examine the Chain Effect carefully with specified conditions.

The Chain Effect clearly shows the impact of the relevant future on current optimal decisions. For example, the Chain Effect suggests that a ship's current optimal speed to travel between two ports is determined by the future journeys we consider for the ship. The concept of the Chain Effect applied to maritime shipping may lead to an increased understanding of whether the traditional models are accurate enough with respect to how they do or do not account for the relevant future usage scenarios of a ship.

2.6 Dynamic programming

Dynamic programming was developed into a formal methodology by Bellman (1957) for certain complex optimisation problems of an appropriate structure. It is a method that breaks down a complicated problem into a series of simpler sub-problems. Typically, it starts by solving a small trivial sub-problem, and then recursively considering larger and larger sub-problems in a recursive manner, in which the optimal structures already identified in earlier addressed sub-problems can be re-used. Once all the sub-problems are solved, we have the optimal solution to the original problem.

To be more specific, each of the sub-problem is defined with a stage of the dynamic programming procedure and policy decisions are needed for each stage. Within a stage, there could be several states that provide the information required for policy decisions to be made. Once a policy decision is decided, it transform the states at a previous stage into states at the successive stage. The recursion that relates the cost or profit at the current stage to the previous stage is also called the *bellman equation*. As a consequence, the technique is suitable for problems with multiple-stages in nature.

The *Principle of Optimality* is a fundamental property of dynamic programming as presented in Bellman (1954), and states that the optimal policy decisions at a current stage are independent of the decisions made at previous stages. This also limits the technique to problems of a corresponding appropriate structure.

Back to the definitions of dynamic programming in Bellman (1954), if we define the system of a deterministic process by $p = \{p_1, \dots, p_n\}$, representing an n stage problem. A policy consists of a selection of n transformations $T = \{T_1, T_2, \dots, T_n\}$, resulting in a successively states:

$$\begin{aligned} p_1 &= T_1(p),, \\ p_2 &= T_2(p_1), \\ &\dots \\ p_n &= T_n(p_{n-1}). \end{aligned} \tag{2.14}$$

If an optimal policy exists, such that the maximum n-stage return with optimal policy from initial state p is given as $f_n(p) = \max_p R(p_n)$, where $R(p)$ is a scalar function of the state p. We can express the problem equivalently as its bellman equation:

$$f_n(p) = \max_{p_1} f_{n-1}(T^*(p)), \tag{2.15}$$

where T^* is the optimal policy chosen at initial state p and result in a new state $T^*(p)$.

2.6.1 Dynamic programming in OR

The application of dynamic programming can be rooted back to the last century with focus on topics such as shortest path problems (Bellman, 1958; Dreyfus, 1969), equipment replacement (Cooley, Greenwood, & Yorukoglu, 1995), knapsack problems (Martello, 1990) and other problems including but not limited to scheduling (Held & Karp, 1965).

In particular, the shortest path problem (SPP) defined in graph theory refers to the problem of finding a proper subset of nodes such that the total cost (e.g., the path length between a start and finish node) is minimised. This problem can be viewed as an abstract representation of many transportation problems found in real applications, such as vehicle navigation. For a comprehensive review of the fundamental problem, readers are referred to textbooks, e.g., Bertsekas (1995) and surveys, e.g., Gallo and Pallottino (1988).

The equipment replacement problem, which is also studied in Preinreich (1940), deals with the economic decision of choosing the best time to replace an existing machine by a new one. In the reality of business, the production rate of a machine and the competitiveness of its production normally ease with time as new technology and other factors such as operating cost and maintaining cost will have their impact on it. Hartman and Tan (2014) provide a literature review of machine replacement problems, where dynamic programming serves as an important tool in related studies.

In the standard knapsack problem, a set of items, each with individual weight and value, need to be selected such that the total weight is limited by a given capacity, e.g., a knapsack, while maximising the value. A vast number of studies exist, see e.g., Andonov, Poirriez, and Rajopadhye (2000); Martello, Pisinger, and Toth (1999); Pisinger (1995). Its application, according to Bertsimas and Demir (2002), can also be extended to vehicle routing problem as in Laporte (1992), where the knapsack problem is used as a sub-problem during the solution. Updated research including the consideration of stochastic item size can be found in Blado and Toriello (2019).

In the maritime literature, the application of dynamic programming can be divided into deterministic models and stochastic models. In the deterministic setting, a maritime optimisation model proposed by Norstad et al. (2011) has applied it to solve a tramp ship routing and scheduling problem. The technique is used in particular to discretise the arrival time at each port and thus transform the maritime speed optimisation problem into a shortest path problem (SPP). Similarly, Lo and McCord (1995) discretise not only arrival time but also longitude and latitude to find the best shipping route that enjoys the benefit of ocean current. The calibration of Lo and McCord (1995) is discussed by S. Wang (2017) and he finds it lacking in accuracy and thus advocates for this dynamic programming approach to be further improved.

On the other hand, the stochastic nature of shipping is often discussed in the context of bunker management problems, e.g., Zhen, Wang, and Zhuge (2017). Fuel consumption and/or fuel price, for example, are often known to be uncertain in reality as discussed in Section 2.2. An analogous example can be found in S. Wang, Zhen, and Zhuge (2018), where the waste disposal problem for a cruise is investigated. It is assumed that the waste generated on board is random and thus similar to the case of fuel consumption during the journey. In the context of maritime speed optimisation, Magirou et al. (2015) use dynamic programming to deal with the stochastic nature of fuel price and solve the optimal speed for each origin and destination pair in an infinite time horizon where a equilibrium of the net profitability for a given route is reached.

Yet, to the best of the our knowledge, the application of dynamic programming is not found in any general speed optimisation models. The reason could be that traditional models often consist of only one stage. In Psaraftis and Kontovas (2014), they decompose a multiple stage problems into a series of single stage sub-problems and thus an algorithm for solving the single stage sub-problem is sufficient. However, in the NPV framework, it is questionable whether this "decomposition principle" still applies. Indeed, the Chain Effect discussed earlier seems to indicate that this is no longer valid in the exact sense, but at best perhaps still useful as a first approximation. As a consequence, it will be necessary to optimise over all future voyages simultaneously. The technique of dynamic programming seems appropriate to handle these situations.

2.6.2 Summary of findings

Due to its ability to solve complicated problems, dynamic programming is often used in cases where the principle of optimality applies. This in essence means that it can apply to problems that can be decomposed into stages.

In Preinreich's early study of equipment replacement problems and the related Chain Effect, he formulated the problem with the concept of *Good Will*. This property aggregates the impact of all future payments to the NPV at the start of the current stage. Although Preinreich did not solve the problem as such, we now can recognise that it fits the assumptions needed for it to be solved by dynamic programming.

Dynamic programming has been used in the literature about optimisation of routing and scheduling in shipping. It has not yet been examined from the NPV framework as developed by Preinreich. One of the tasks in this dissertation is to accurately reformulate Preinreich's formulation to the shipping context, and then solving it by dynamic programming.

Dynamic programming is also highly suited to tackle problems with time-varying parameters as well as stochastic problems. In shipping, both characteristics are present.

2.7 Conclusion

The literature review reveals that no study yet incorporates the cash-flows associated with the logistics activity. Doing so will offer the ability to explicitly capture the relationships between contracts, payment structures, and operational decisions such as speed choices. The objective of maximising the NPV of this activity is also a well accepted approach in the management of businesses.

In the basic probability theory, all assumptions introduce chances of error, especially if the assumption fails to hold accurately. This so-called *law of parsimony* suggests starting from the most basic scenario first, and only after sufficient understanding is reached, to expand the investigation to including more complex situations.

Therefore the NPV framework applied in this thesis is to construct the models of which the components and behaviors are compared with conventional maritime speed optimisation models based on different scenarios, and starting from simple scenarios first.

If incorporating a new angle, i.e. integrating cash-flows and logistics decisions, can bring new insights, either fundamental or practical, we are at a better position to contribute to the maritime shipping literature and practise.

Chapter 3

The Economic Ship Speed under Time Charter Contract - A Cash Flow Approach

Abstract

How to determine the optimal length of a charter contract is actually a practical problem faced by many shipping companies when about to enter a time charter contract to charter a ship for a time period with particular logistics tasks.

In this chapter, we thus hope to develop a particular type of speed model (P_M) for the problem, where we assume the ship will repeat the given logistics tasks a number of times (which can also be a decision to make) and determine the optimal ship speed on each leg.

It would then be natural to also consider two special cases P_1 and P_∞ where the journey is only executed once or repeated infinitely, respectively. From NPVEA analysis we found these to be equivalent to classic speed optimisation models found in literature i.e. Psaraftis and Kontovas (2014) and Ronen (1982) (Model I) , respectively.

Some empirical studies (e.g., Prakash et al. 2016) show that ship speeds are widely distributed in the available speed range, even for identical ships doing identical work under identical circumstances. This cannot be fully explained by conventional models and existing maritime economic theory. By including a new dimension, namely the time, we hope to better understand whether this could (in part) be due to decision making about optimal ship usage and thus contribute also to the maritime economic theory.

keywords: charter party contracts, bulk and tanker shipping, ship speed optimization.

3.1 Introduction

The optimisation of seaborne transportation has received much attention in the literature. We refer to the systematic reviews in Ronen (1983, 1993), Christiansen et al. (2013, 2004), S. Wang et al. (2013a) and Meng et al. (2014). Most of the studies focus on ship routing and scheduling, where ship speed is either a constant or determined approximately from a cost structure that is linear in ship speed. This assumption removes the flexibility of decision making deemed important in many practical applications, according to Psaraftis and Kontovas (2013, 2014), who also provide comprehensive reviews on speed optimisation models in maritime OR literature.

Close to 80 % of world trade is carried by ships. Bulk carriers and tankers make up 32% of the total merchant fleet of 53,000 vessels, and 70% of the global fleet capacity of 1190 million deadweight tonnage (maribus, 2010). Two main actors in this industry are ship owners (operators) and charterers. The charter contract specifies how the owner hires out the ship (and its crew) to the charterer. Several types of contract exist. In a time charter contract, the charterer rents the ship for a certain time and compensates the owner with a daily hire (in USD/day). The time charterer determines the vessel's route(s) but pays for certain operational costs, including fuel costs. In this chapter we focus on the speed optimisation problem for the ship charterer in this industry when the ship is subject to a time charter contract. Ships under these contracts are commonly used for repetitive tasks.

It is known that ship speed affects overall profitability. Lowering ship speed reduces daily fuel consumption, but less profit generating trips can be undertaken by the ship over time. If speed is a variable, it should thus be set in the context of this economic trade-off. Economic theory to date (Devanney, 2010; Psaraftis, 2017; Ronen, 1982; Stopford, 2009) finds that the optimal speed is largely determined from a non-linear function of the ratio between the freight rate and the fuel price (Adland & Jia, 2018; Psaraftis, 2019b). The aim of this chapter is to present a modelling approach and analyse its properties so as to investigate how this economic principle plays out for ships under time charter contracts.

To this end, we follow the OR approach and develop a framework of speed optimisation models P_M with the consideration of time charter contracts. To capture the impact of the relevant future, we view the determination of optimal economic speeds as a problem of how to maximise the Net Present Value (NPV) of this activity for the decision maker. We adopt the continuous time formulation based on cash-flow functions presented in Grubbström (1967). While the techniques of NPV evaluation and cash-flow accounting are not new to the economics of shipping, see e.g. Stopford (2009) (Chapter 6), the approach has, to our knowledge, not yet been applied to the shipping industry as a methodology to find optimal ship speed decisions. To the best of our knowledge, ship operators do currently not make explicit use of the NPV methodology for speed decisions. Our discussions with an international ship charterer, operating a large number of chartered vessels, give us a good level of confidence that the NPV models developed, and the insights derived from them, exhibit the characteristics and trade-offs experienced decisions makers in this industry take into account.

In this chapter we will show how two special cases of P_M, namely P_1 and P_∞, map under certain mild conditions onto the two main types of 'classic' (not based on NPV principles) ship speed optimisation models from the literature. The P_1 class shows equivalence to speed models like Psaraftis and Kontovas (2014), where the objective function is to maximise the *total* profit or to minimise the total cost per trip, per set of routes, or per nautical mile. We henceforth denote with 'Model PK' the type of models from the literature using this kind of objective function. We can recognise Model PK also in e.g. Corbett et al. (2009); Fagerholt et al. (2015); Norstad et al. (2011); Wen et al. (2017). The P_∞ class, on the other hand, shows equivalence to another class of speed models, where the objective function is to maximise the *daily* profit, or minimise the daily cost, as in Ronen (1982), and also Devanney (2010); C.-Y. Lee and Song (2017); Magirou et al. (2015); Ronen (2011). Models in this class divide the profit (or cost) earned on a route by total travel time of the route to arrive at their objective function. 'Model R' denotes models that follow Ronen's approach to constructing the objective function. Because we show the equivalence of Model PK to P_1 and of Model R to P_∞, we can from the study of the properties of the NPV models learn about the properties and underlying assumptions of the classic models[1].

The differences between Model PK and Model R have not been explicitly discussed until Psaraftis (2017). He shows that models that maximise profits per unit of time (Model R) can account for the additional revenue income from more trips if the ship speed increases, capturing one of the fundamental facets of shipping industry behaviour, namely that the state of the (spot) market, along with the price of fuel, are the two main determinants of ship speed. He shows that models with objective functions as in Model PK, cannot capture this behaviour accurately.

We can support these conclusions reached in Psaraftis (2017), but some further modifications developed in this chapter may be fruitful to further increase our appreciation of the limitations of both approaches. Both Psaraftis (2017) and Devanney (2010) show that charterer and ship owner have the same reaction to spot market signals. We demonstrate in this article that such a conclusion is reached because Model R (P_∞) is an approximation of the real world. By adopting the more general model P_M developed in our framework, the mathematical equivalence of the ship owner problem and ship charterer problem is challenged. This brings the contract type to our attention. We show that Model PK (P_1) is correct under certain contracts such as a short-term voyage contract. Model R (P_∞), however, is an approximation for other types of contract of

[1]The study of equivalence between models derived from classic principles and models based on NPV principles finds its roots in Grubbström (1980) (in the context of production-inventory systems), see also Beullens and Janssens (2014).

a long horizon and with repetitive logistics tasks, and may not always work well as a representation of reality because of the underlying assumptions.

The general model P_M proposed in this chapter captures a time charter scenario, where the charterer has to decide the optimal usage of the ship over the duration of the charter contract. Prior to signing the contract, deciding on the duration of the contract forms part of the decision whether to hire this ship and under these charter hire conditions. Once the contract is signed, the charterer typically has limited flexibility with respect to when to return the ship to the owner, as specified in the redelivery clause.

The main scientific contributions of this chapter can be summarised as follows. (1) A class of models based on *cash-flow functions*, novel to the shipping industry, is introduced that captures the impact of operating conditions on the Net Present Value (NPV) of the ship's relevant future, when subject to a generic time charter contract. The modeling of ship speed optimisation in the context of time charter contracts is novel. (2) The current literature on speed optimisation modelling can be roughly divided into two strands. Under mild assumptions, we prove that each strand maps onto one of two special cases of our model, respectively. This produces new insights about the applicability of the models from the literature in various charter contract situations, as well as insights into the conditions needed so that these classic models, while not derived themselves from NPV principles, will yet be (close to) maximising the NPV of the activity for the firm. (3) Analysis of the models with respect to contract type and various economic parameters is used to extract insights into their impact on optimal speed values. This leads us to conclude that the contract type has an important impact on optimal economic speed.

3.2 Background literature

Optimal ship speed in maritime economics and operations research literature, as well as in this chapter, is to be seen as *an average speed value over a particular leg*, and a proxy for the time that a ship should ideally take when traveling from one particular port to another particular port, so as to help maximise the ship's commercial success.

It is observed that ships tend to travel at a wide range of different speeds. Psaraftis (2019a), for example, reports average speed of 17.5 knots eastbound between Asia and South America, and 12.5 knots westbound on a particular transpacific service. These speeds are much lower than the speed for which these containerships were originally designed (typically up to 25 knots). This slow steaming is observed in all markets, including in the bulk and tanker industries, and typically in periods of depressed conditions (Psaraftis, 2019a). New engines and techniques (e.g. electronically controlled engine, slow steaming mods, waste heat recovery) or retrofit options (Easton, 2015) are helping to extend the feasible range in which ships can sail. This allows a Very Large Crude Carrier (VLCC) to extend its range to about 7 knots, from about 9.5 to 16.2

knots when loaded (Devanney, 2010). See also Psaraftis (2019b). Traditionally, this may have been only between 11 and 15 knots (Stopford, 2009). This section presents background literature concerning the usefulness of having ship speed flexibility.

3.2.1 Ship speed as an economic variable

Ship speed is deemed an important decision variable used by bulk ship and tanker operators worldwide. Slow swings in favourable versus unfavourable economic conditions, which Stopford (2009) refers to as the '7-years freight cycles', greatly impact the shipping industry structure, including the volume of ships being scrapped and new ships being built, which new designs will come out of shipyards, and at which ranges of speed these ships are designed to travel. At the strategic level, the demand for shipping changes rapidly but supply is slow as it may take two to three years for new ships to be delivered. It therefore takes some time for supply and demand to be adjusted, and the long-term evolution in freight rates signals how this process unfolds. In times when the supply of ships is tight, freight rates tend to be favourable on average, and ships will tend to travel at higher speeds on average, and vice versa.

Insight into how fast ships travel on a more day-to-day basis is arrived at from understanding short-term equilibria. This happens in time frames in which ships have the ability to readjust their position across the globe, their speed, and time in lay-up. The supply curve for a particular ship type (freight rate as a function of total supply capacity) has a characteristic J-shape, while short-term demand is largely inelastic (a vertical line). As explained in Stopford (2009), short-term freight cycle peaks and troughs superimposed on the larger freight cycles are caused by how demand travels slowly along the supply curve. If demand level is low, freight rates are low and inefficient ships are forced into lay-up, while more efficient ships will travel at lowest possible speeds. As demand increases, freight rates go above operating costs of most ships, and ship will come out of lay-up and also speed up. Beyond this point, the steep supply and demand curves may intersect at very different freight rate points, and actual rates are governed by contractual negotiations.

By far the largest part of operating costs is the cost of fuel. This is, next to the already discussed freight rate, generally thought to be the other main factor that affects speed decisions. Reducing a mid-sized oil tanker's speed down from its nominal speed by 20% may well reduce fuel consumption by 45%, saving 30 ton of fuel or (at 63 USD/barrel) $15,000$ USD per day at sea. The savings made may more than compensate for the running cost over the now increased journey time. Longer journey time, however, also implies that less revenue generating activities can be undertaken. The benefit of slowing down depends on whether the savings in costs compensate this revenue loss. Ronen (1982) presents a set of analytical models for understanding this trade-off and calculating optimal ship speeds at any fuel price point.[2] Ceteris paribus, higher bunker prices tend to slow down ships. However, Devanney (2010) argues that a raise in bunker price also tends to move the short-term supply J-curve upwards, in other words also increases the short-term freight rates. He uses this mechanism to explain the boom in 2007/2008 for the VLCC market, where higher bunker prices were more than offset by higher freight rates, giving ship operators the incentive to keep steaming at full speeds, and earning temporarily a tenfold increase in Term Charter Equivalent (TCE) earnings.

According to Psaraftis (2019a), ships sailing much slower than their design speed is typically observed in periods of depressed market conditions or high fuel prices, and this is reported in every market. Slow steaming not only saves fuel but also reduces emissions. In the context of ongoing climate change concerns, ship speed has thus become an important consideration as well (Psaraftis & Kontovas, 2014). Psaraftis (2019a) investigates whether reducing speed by imposing a limit is better than doing the same by imposing a bunker levy, and concludes that having ship speed flexibility remains important (see also Section 3.6.4). As a general conclusion, speed flexibility and speed optimisation may continue to serve an important role in maritime optimisation (Psaraftis, 2017).

3.2.2 Empirical evidence for speed variability

Prakash et al. (2016) have conducted an extensive study of the Panamax and Capesize dry bulk and Suezmax and VLCC tanker charter markets, using empirical data from 2012 to 2015. They observed that actual speeds on both ballast and laden legs vary widely, with interquartile ranges often above 2 kn, and ships found travelling at all available speeds within the vessel's range. They conclude that: 'Vessel speed remains a variable that is hard to fully explain or attribute, and significant variability exists within fleets of similar ship type, size, and technical specifications. Given speed's significance to operational energy efficiency, further work that examines the drivers of speed will be important for understanding the sector's GHG emissions.' Prakash et al. (2016) suggest, although do not further investigate, that contract type may be a determinant of operational performance and speeds. The example given is that differences in available speeds, imposed by ship owners in fuel clauses of charter party contracts, may explain differences in observed operational speeds.

Adland and Jia (2018) performed a linear regression analysis on empirical data of bulk carriers in the period 2011 to 2012. They found no significant evidence to support the argument that ship operators would adjust ship speeds under different market conditions. In fact, they found a result opposite to economic theory, namely that fuel price leads

[2] To model fuel consumption, he uses the well-known 'propeller' law. This can be easily replaced with a refined function of Psaraftis and Kontovas (2014) that also accounts for actual deadweight conditions (see also Section 3.3.3). This refinement would not alter Ronen's main argument.

to higher speeds, and freight rate to lower speeds. The model fit was not great, which makes these conclusions not well supported. They conclude that the ship speeds may largely be impacted by other factors not included in the analysis, such as the weather, charter clauses, and technical and organisational constraints.

It is important to note that the authors of above studies do not refute the main economic theory, but call for further refinements. In particular, they both suggest further research is needed into how charter contracts may affect speed. In this context, the basic question examined in this chapter can be stated as follows: If we assume identical conditions with respect to the ship's characteristics and how it can be used in operations, the journey travelled and ports visited, and the revenues and costs involved, can we find compelling economic reasons why such ships would travel at a wide range of speeds?

Many other explanations could potentially explain the observations in the empirical studies. For example, ships are often technically very different even when of the same type and age, and may have been travelling under different weather conditions, etc. However, if the mechanisms in the modeling framework presented show that optimal speeds depend on the time charter contract details and show very different sensitivity to exogeneous (market) signals, it would support the idea that the observed mixed signals in the empirical data could in part be explained by profit-maximising behaviour.

3.3 Problem description

This chapter considers the time charter agreement in the bulk cargo/tanker industry. Figure 3.1 illustrates the players and their interactions. The three major players are ship owners, ship charterers, and shippers. The charterers rent ships (including crew) from ship owners and receive revenues from shippers for the transport of cargo between ports.

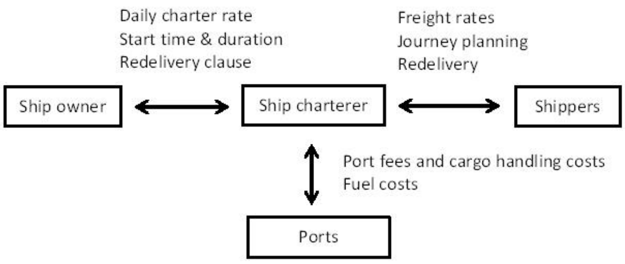


Figure 3.1: Time-charter players and interactions

We are concerned with determining the profitability of a particular time charter offer available to the ship charterer. The time charter party is the contract between ship owner and charterer, and typically allows charterers to decide on which journeys are undertaken, and at which speeds the ship will travel.

Our primary purpose is to develop an understanding of the degree by which the time charter contract is important in determining how fast a vessel should ideally travel on the different journeys. We assume that the ship is not limited in its choice of speeds by time windows imposed by external players (e.g. ports, shippers). At the moment of considering chartering a vessel in this industry, time windows are not an exogenous input. Only when (voyage) contracts with shippers are signed may these become (soft) constraints, see also Psaraftis (2017). At this point the ship is already planned to travel according to a chosen optimal speed range. This still allows for the charterer, if so desired, to make changes to the speeds during later voyages of the ship on the time charter contract[3].

3.3.1 Charter party characteristics

When a ship charterer hires a ship from an owner through a typical time charter contract, the ship owner carries the costs of crew, repair, maintenance, lubricants, supplies, and capital costs. The charterer is concerned with revenues and costs associated with cargo handling, including costs of loading and unloading at ports, and main and auxiliary fuel costs. The agreement is made at decision time 0 for a certain duration during which the charterer pays a daily rate to the owner. The main variables involved are:

- H: initial approximate duration of charter party (in days);
- t_{FS}: agreed start time of charter relative to signing contract (in days);
- f^{TCH}: agreed daily charter rate (USD/day);
- H^*: agreed duration of charter party (in days).

The rate f^{TCH} is known as the Time Charter Hire (TCH), and t_{FS} as the Forward Start. TCH is a function of charter time duration and market conditions. In late November 2017, for example, TCH estimates for a Suezmax oil tanker published by the Hellenic Shipping News Worldwide were increasing with contract duration: $18,750$ (USD/day) for 1 year, $20,000$ (USD/day) for 2 years, and $23,500$ (USD/day) for 3 to 5 years, respectively. At other times, these estimates may be constant or decreasing. TCH can be a function of t_{FS}. Angolucci, Smith, and Rehmatulla (2014) found, for example, that

[3] In liner services, speed optimisation is also important, but additional constraints, see e.g. **?**, may make it difficult to implement optimal speeds identified in less constrained model as presented in this chapter.

rates in the Panamax market (for the period 2008-2012) reduced on average by 60 USD per extra day Forward Start. The agreed TCH will also depend on the ship's condition and negotiation.

The process of negotiating the time charter contract may follow the following logic. Considering the ship and contract characteristics, the type of logistics activities to be executed, and future spot market rate predictions, the ship charterer arrives at a first estimate H of a desirable total charter time duration. This decision is quite strategic. Using H, both parties arrive at an initial agreement on the charter hire $f^{TCH}(H)$. The ship charterer may use this value to come to a more specific duration H^* and start time t_{FS}. If the differences with the initial estimate is small, the contract can be signed for $f^{TCH}(H^*, t_{FS}) = f^{TCH}(H)$. A high-level description of the problem context is presented in Figure 3.2.

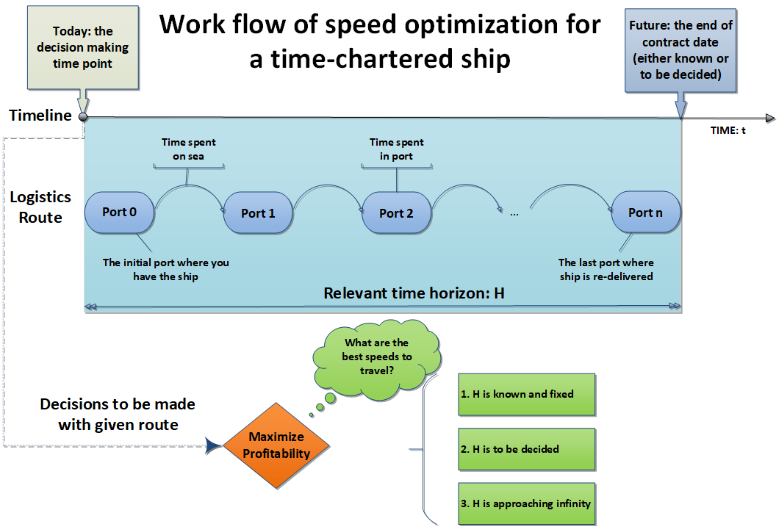


Figure 3.2: The timeline and workflow of the speed optimisation problem for a time-chartered ship

Example (Single iteration). The charterer considers hiring a particular ship for a set of journeys and estimates it may take $H = 550$ days (approximately 1.5 years), starting on 30 April. For this duration and start time, the charterer obtains an initial agreement with the owner of $f^{TCH}(550) =$ USD $19,500$ per day. As f^{TCH} affects the optimal ship speeds, the charterer may use this more accurate value to arrive at an optimised charter duration of, for example, $H^* = 500$ days, and a start time of 15 April. The owner accepts and the contract is signed for 500 days at USD $19,500$ per day.

It may be the case that they cannot yet agree and that a few iterations of this process may be needed. The ship charterer can work with a newly negotiated TCH based on readjusting to the newly proposed duration H, and re-iterate the process of finding the new best duration H^*. This process is expected to lead to better agreeable TCH values based on the actual number of days the contract will run, and may thus lead to agreement.

In this article, we assume that the above process does not require iterations. That is, the f^{TCH} is agreed based on initial values of H and t_{FS}, and will be accepted for a final value H^*. The model developed, however, can be used in an iterative fashion in case negotiations require this.

The contract will also include a redelivery clause, including details about the period around the agreed termination date in which the charterer can return the vessel. We will address this clause in Section 3.6.1.

3.3.2 High-level description of the optimisation problem

Without loss of generality, we assume the situation of market TCH values *increasing* with charter duration. The ship owner then typically gets agreement from the owner on signing a contract at $f^{TCH}(H)$ of actual duration H^* equal *or smaller* than H. In the ship charterer's problem, H acts as an upper bound for optimal values of H^*. In the case where TCH market values show a downward trend, H will act as a lower bound.

After the completion of the work at time H^*, the charterer will return the vessel to the owner and stops paying the time charter hire. We assume that other (future) projects for the charterer are independent and do not depend on the completion time of this project[4]. The criterion to charter this vessel or not must thus be based on considering the present values of all relevant revenue and cost cash-flows associated with this activity, where the times when these cash-flows arise are affected by decisions regarding e.g. the ship's speed. In general, the charterer's objective function is thus to maximise the Net Present Value (NPV) of the activity by taking a set of decisions $x \in X$. Let $\mathrm{CF}(t,x) = r(t,x) - c(t,x) - c_{TCH}(t,x)$ be the cash-flow function of the activity at time t ($t \geq 0$, in days), where the terms on the right-hand side represent the revenues, operational costs, and charter party costs, respectively. These terms will be further specified in Sections 3.3.3 and 3.3.4. Following Grubbström (1967), the optimisation problem is therefore[5]:

$$X^*(\alpha) = \arg\max_{x\in X, H^*(x)<=H} \int_0^{H^*(x)} \mathrm{CF}(t,x)e^{-\alpha t}dt, \tag{3.1}$$

[4] This is valid if the charterer has access to capital so it can hire yet other ships at any other time if other profitable business is on the horizon.

[5] Eq (3.1) formulates the charterer's optimization problem by applying the Net Present Value (NPV) definition given by Grubbström (1967): $\mathrm{NPV}(\alpha,x) = \int_0^\infty \mathrm{CF}(t,x)e^{-\alpha t}dt$, where $\mathrm{CF}(t,x)$ describes all relevant cash-flows associated with project execution, including also all costs and potential investments needed, and arising from decision time 0 and into the future. The criterion to accept this project is whether $\mathrm{NPV}(\alpha,x) \geq 0$.

where α is the (continuous) opportunity cost of capital against which the charterer wishes to evaluate this activity. Note that α is not an interest rate; rather it represents the opportunity cost rate corresponding with the next best alternative available to the decision maker (Grubbström, 1967), here the ship charterer. This value is typically larger than the interest rate on a risk-free investment. As the most appropriate time unit in this study is per day, we use a daily rate for the cost of capital $\alpha = 0.000219$ (per day), which is equivalent to a cost of capital at 8% (per annum), a value consistent with current marine industry values, see also Yagerman (2015). See also Section 3.3.6, where we examine the model's sensitivity to α.

A performance measure of value in the industry is known as Time Charter Equivalent (TCE) earnings. Like TCH, it is expressed in (USD/day). The TCE, according to the definition used in industry, is the difference between voyage revenues and voyage costs (excluding charter costs), divided by the total number of voyage days.

We will extend this definition to the NPV framework. The TCE earnings are the net profits earned as a cash-flow per day over the charter contract, and such as to produce an equivalent NPV as the real cash-flows of the project. The TCE earning are thus equal to the project's annuity stream value, with time and α expressed in days. For a project with Net Present Value NPV of duration L, its annuity stream value equals AS $= \alpha\text{NPV}/(1 - e^{\alpha L})$. The formula for arriving at the charter party TCE earnings f^{TCE} (USD/day) is therefore given by:

$$f^{TCE}(x, \alpha) = \frac{\alpha \int_0^{H^*(x)} (r(t,x) - c(t,x))e^{-\alpha t} dt}{1 - e^{-\alpha H^*(x)}}. \tag{3.2}$$

Maximising $f^{TCE}(x, \alpha)$ is not necessarily equivalent to solving problem (3.1), but will produce the same result if $H^* = H$ as in that case the total charter party costs are fixed.

Let $r^*(t)$ and $c^*(t)$ represent the revenue and cost cash-flows, and H^{**} the charter duration when adopting a solution $x^* \in X^*$, obtained from solving problem (3.1). The TCE earnings corresponding to this solution are then found by evaluating (3.2).

3.3.3 Journey characteristics

During the time of the charter, the charterer will use the ship to execute cargo transport between ports. The charterer can adjust the speed of the ship on its voyages within an allowed range of speeds. This freedom, as discussed in the introduction, is important in order to help maximise the charterer's profits.

To characterise the logistics tasks to be undertaken, we adopt a modelling logic based on Psaraftis and Kontovas (2014), but which extends this in various respects.

We define a *journey* of the ship as a scenario in which the ship visits a set of $n+1$ ports numbered 0, 1, 2, ... to n in this given order, see also Figure 3.3. Based on demand from an origin port i to a destination port j, the ship will, at each port i, unload cargo destined for this port and picked up from previous ports $j < i$ and pickup loads to be delivered to ports $j > i$. These amounts are such that at any point in the journey the cargo load remains feasible with respect to the ship's deadweight carrying capacity.

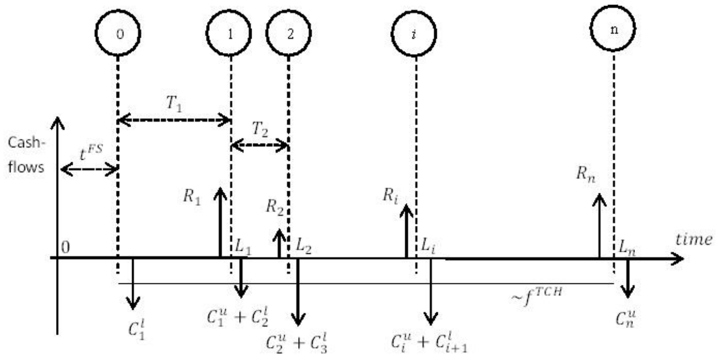


Figure 3.3: Cash-flows for a Journey of n-legs between $n+1$ ports

We define a *leg* i ($i = 1, ..., n$) as the process of the ship moving from port $i-1$ to port i. The time T_i (in years) corresponding to leg i consists of: loading time T_i^l at port $i-1$, sea voyage time to port i denoted by T_i^s, waiting time T_i^w at port i, and unloading time T_i^u at port i:

$$T_i(v_i) = T_i^l + T_i^s(v_i) + T_i^w + T_i^u, \tag{3.3}$$

where v_i the sailing speed (in kn) on leg i. The (un)loading time at a port is linearly dependent on the tonnage to be (un)loaded. For oil, for example, those times depend on the pump capacity of the harbour and the ship, respectively. During unloading and loading at port i, the ship will also replenish supplies and remove waste, take the main fuel and auxiliary fuel, and adjust the amount of ballast water needed to cover leg $i+1$. The travel time (in days) is given by:

$$T_i^s(v_i) = \frac{S_i}{(24)v_i}, \tag{3.4}$$

where S_i is the distance (in nm) from port $i-1$ to i.

Each visit to a port incurs costs. Without loss of generality, we will split the total costs incurred at a port i into two components. The first component C_i^u represents costs associated with entering the port for unloading, the second component C_{i+1}^l represents costs at port i associated with loading so that the ship is ready to undertake the sea voyage of leg $i+1$.

The process of how to allocate costs incurred at a port between these two components can be made arbitrarily as it will not affect the solutions to the (undecomposed) models presented. However, for the purpose of insight and comparison with (decomposition) models from the literature, it will be beneficial to make a division of costs that reasonably represents which costs are mostly associated with undertaking a leg. Therefore, we arbitrarily take C_i^u at port i to include the costs of waiting to enter port and dock at berth, fixed port charges, any mooring dues, and unloading charges. Cost C_{i+1}^l incurred at port i include cargo loading costs, cost of readjusting ballast, and the costs of main and auxiliary fuel intake.

The amount of main fuel to cover leg i depends on the sailing speed v_i, and the deadweight tonnage (dwt) w_i carried, which includes cargo, ballast, supplies, and main and auxiliary fuels carried. We adopt the fuel consumption function (tonne/day) proposed in Psaraftis and Kontovas (2014):

$$F(v_i, w_i) = k(p + {v_i}^g)(w_i + A)^h, \tag{3.5}$$

where w_i is the dwt carried on leg i, A the lightweight of the ship, and typically $g \geq 3$ and $h \approx 2/3$. We impose a stability constraint that dwt carried should be at least a given percentage of the design dwt of the vessel, see also David (2015). Any shortcoming arising from zero or low amounts of cargo carried is to be made up from ballast water. The loading cost can be stated as:

$$C_i^l(v_i) = C_i^h + c_i^f F(v_i, w_i)\frac{S_i}{24v_i}, \tag{3.6}$$

where c_i^f is the fuel cost (USD/tonne) at port $i-1$, and C_i^h represent the sum of other costs associated with the loading operations.

The sum of C_i^u and C_{i+1}^l is due within a number of d_i^c days after leaving the port i. The time lapse of this payment (in days), relative to the *completion time* of the leg i, is given by:

$$\epsilon_i = T_i^l + d_i^c. \tag{3.7}$$

Revenues are based on freight rates, i.e. (market) unit prices of the cargo type and the distance between origin and destination ports, and negotiations with shippers. For the journey described with known tonnages transported, we can thus calculate the total revenues R_i from unloading cargo at each port i. This value is not dependent on the actual sailing distances or leg speeds. Most ship charterers will demand payment a number of days d_i^r prior to the start of unloading the cargo at the destination port. Let us thus define the time of payment, relative to the completion time of leg i, as :

$$-\delta_i = -(T_i^u + d_i^r). \tag{3.8}$$

The negative signs indicate that payments are made earlier. The time charter hire payments can be modelled as an annuity stream at rate f_{TCH} over the period of the contract[6], see also Figure 3.3.

Note that where we differ from Psaraftis and Kontovas (2014) in the modelling of the logistics process is primarily in the following aspects: (a) we include the costs and times of harbour activities, (b) we include the revenues earned from cargo transport, (c) we specify the actual timing of when costs and revenues are occurring, (d) we do not explicitly incorporate the carrying charges of payload and goods yet to be picked up. We postpone a discussion on these differences to further sections.

The above *journey* model set-up can be used to model (as in Section 3.3.4) the logistics tasks to be covered by the ship over the whole envisaged horizon H of the charter party. However, H is in most time-charter parties a long period, during which the ship often will be used to execute a round-trip journey repeatedly.

3.3.4 Model formulations: $\mathbf{P}_M$, $\mathbf{P}_1$ and $\mathbf{P}_\infty$

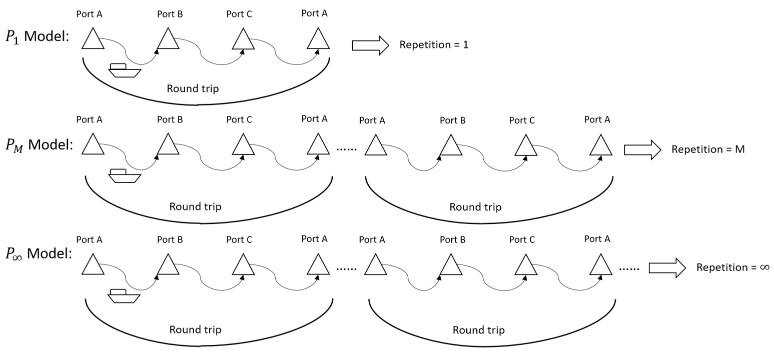


Figure 3.4: Example of the journey of Port A to B to C

We translate the general description (3.1) to the situation (as illustrated in Figure 3.4) in which the charterer desires to use the ship for the repeated execution of the journey between ports. As the journey represents a round-trip, port n equals port 0 and, without loss of generality, we can choose $R_0 = C_0^u = C_{n+1}^l = 0$.

We develop one general model P_M, and derive two special cases. Define $V_i^- = \{v_1, ..., v_i\}$ and $V_i^+ = \{v_i, ..., v_n\}$ $(i = 1, 2, ..., n)$. Let us define variable L_j $(j = 1, ..., n)$ as the time (in years) from the start of the charter party t_{FS} to the moment that leg j is completed:

$$L_j(V_j^-) = \sum_{i=1}^{j} T_i(v_i). \tag{3.9}$$

[6]The actual charter hire payment times could be e.g. twice every month on day 1 and 15. These details are not relevant since the payments scheme is not linked to any events in the model. Any actual payments plan can be converted to an equivalent annuity stream value in the model, similar to the approach used to arrive at TCE in (3.2).

We refer to these as *leg completion times*, and L_n represents the duration of the journey. The Net Present Value (NPV) of the first journey relative to decision time 0 is therefore:

$$\begin{aligned}\mathrm{NPV}_1(V_n^-,\alpha) = \Big[&\sum_{j=1}^{n}\Big(R_j e^{-\alpha(L_j-\delta_j)} - C_j^u e^{-\alpha(L_j+\epsilon_j)} - C_j^l e^{-\alpha(L_{j-1}+\epsilon_{j-1})}\Big) \\ &- \frac{f^{TCH}(1-e^{-\alpha L_n})}{\alpha}\Big]e^{-\alpha t_{FS}}. \qquad (3.10)\end{aligned}$$

Let integer M represent the number of times the ship will repeat this journey. The NPV of this is given by:

$$\mathrm{NPV}_M = \mathrm{NPV}_1\Big[\sum_{i=0}^{\infty} e^{-i\alpha L_n} - \sum_{i=0}^{\infty} e^{-(i+M)\alpha L_n}\Big] = \mathrm{NPV}_1\frac{1-e^{-\alpha M L_n}}{1-e^{-\alpha L_n}}. \qquad (3.11)$$

Note the dependencies of loading costs and completion times on speed variables, i.e. $C_i^l(v_i)$ and $L_i(V_i^-)$, whereas R_i and C_i^u are constants $(i = 1, 3, ..., n)$. Let v^- and v^+ indicate the minimum and maximum sailing speeds allowed for the ship.

We can now rewrite problem (3.1) as problem P_M:

$$\text{Max } \mathrm{NPV}_M(V_n^-, M), \qquad (3.12)$$

and subject to

$$\begin{aligned} v^- \le v_i \le v^+, \forall v_i \in V_n^-, \qquad (3.13)\\ ML_n(V_n^-) <= H, \qquad (3.14)\\ V_n^-, M >= 0. \qquad (3.15)\end{aligned}$$

Note that, with respect to (3.1), decisions $x \in X$ correspond to a choice of values for v_i and M where $ML_n(V_n^-) = H^*(x)$.

Taking $M = 1$ and dropping constraint (3.14) (as if taking $H \to \infty$), we get as a special case the decision problem P_1. This model optimises the NPV of a single journey. Decisions about ship speed will affect the duration L_n of the journey. If the ship charterer would like to undertake further journeys with this ship, it would have to account for this in the formulation. The fact that P_1 does not do so means that after completion of the journey, the ship is returned to the owner and that $L_n = H^*$, i.e. L_n is the duration of the charter contract. P_1 is thus a model for what is known as a *trip time charter*, which is a time charter contract of comparatively short duration agreed for a specified route, where route in this context refers to the n-leg journey.

With $H \to \infty$ in P_M, but M yet unspecified, we arrive at a second special case. For a charter contract to be considered, it must be that repeating the journey once must be profitable, i.e. $\mathrm{NPV}_1 \geq 0$, for at least some speed values. Then (3.11) shows it is optimal that the journey is infinitely repeated ($M \to \infty$, $H^* \to \infty$). We call this model P_∞. For NPV problems with infinite horizons, it is more convenient to maximise the Annuity Stream (AS) value, where AS $= \alpha$ NPV, representing the constant stream of cash (USD/day) with the same NPV value. This problem P_∞ has the objective function:

$$\mathrm{AS}(V_n^-) = \alpha \mathrm{NPV}_\infty = \mathrm{NPV}_1 \frac{\alpha}{1 - e^{-\alpha L_n}}. \tag{3.16}$$

P_∞ represents an idealised case of a time charter of very long duration.

Table 3.1: Types of Contracts for Models

Model	Decision Variables	Contract Horizon H^*	Possible Contract Types ([1])	Decision Maker
2*P_1	2*V_n^-	2*Not a decision variable but follows from solving P_1, since $H^* = L_n$, where L_n is a function of V_n^-	Trip Time Charter	Charterer
			Spot Market or various Voyage Charters (before retiring the ship) ([5])	Owner
4*P_M	4*V_n^-, M	4*Initial agreement H negotiated to $H^* = ML_n$, where M is solved from P_M	Time Charter	Charterer
			Consecutive Voyage Charter	Charterer
			Contract of Affreightment ([3])	Charterer
			Spot Market or various Voyage Charters (limited availability to H) ([4])	Owner
4*P_∞	4*V_n^-	4*Infinite H^*	Time Charter (long duration)	Charterer
			Consecutive Voyage Charter (long duration)	Charterer
			Bare Boat Charter	Charterer ([2])
			Spot Market (voyage charters)	Owner ([2])

([1]) For definitions, see Stopford 2009 p. 176; ([2]) The time charter hire f^{TCH} in the models is replaced by the operations costs, see Stopford, 2009, p. 182; ([3]) When using only a single ship. ([4]) H: e.g. the time at which the owner is required by law to lay ship up for major overhaul/maintenance operations, or the time at which the owner is bound to deliver the ship to a charterer under an agreed time charter contract. ([5]) The last leg includes the cost (or revenue) to the owner of dispositioning the ship.

Table 3.3.4 lists a number of key model characteristics. In P_M, the decision variables are the ship speeds on the legs of a journey V_n^-, and the number of times the journey is to be repeated M. The final contract horizon length then follows from: $H^* = M^* L_n(V_n^{*-})$. While we can expect that optimal ship speeds of different legs will differ, i.e. $v_j^* \neq v_i^*$, the model assumes that the chosen speed on any leg j is the same in every repetition of the journey. In the special cases P_1 and P_∞, only the leg speeds are the decision variables, and there is no constraint on the horizon.

The table also lists how the three models may be matched to other contract types found in Stopford (2009), and the corresponding decision maker. For example: Model P_∞ can also represent the situation in which the ship owner would not engage in a time charter party, but use the ship to attract business from shippers and plan its own (infinite series of) *voyage charters*. Then f^{TCH} represents the owner's daily costs of crew, lubricants, supplies, etc. If, however, the owner has signed a contract for a time charter to *start* at a time H into the future, then model P_M is a better representation of the owner's decision problem of how to use the ship up to that time H.

3.3.5 Algorithms

For solving P_1 or P_∞ models, use Algorithm 1 where $M = 1$ and $H \to \infty$, or $M \to \infty$ and $H \to \infty$, respectively, and where minimum and maximum speeds on each of the legs correspond to the minimum and maximum speeds allowed for the ship, v^- and v^+, respectively. For the model in Section 3.3.4, use Algorithm 1 where M is the required number of repetitions, and $H \to \infty$.

Algorithm 1: Grid search algorithm to solve P_1, P_M (with given M), or P_∞

Result: $\{v_1^*, \dots, v_n^*\}$, F^*, H^*
(F^* is the optimal NPV and H^* is the number of days used by the ship)
Input: $M, \quad H, \quad \{(v_i^{min}, v_i^{max}), i = 1, \dots, n\}$;
Step 1: For $i = 1, \dots, n$, set $\epsilon > 0$ and $\mathbb{V}_i =: \{v_i^{min}, v_i^{min} + \epsilon, \dots, v_i^{max}\}$;
Step 2: Let F be given by equation (3.11) (If $M \to \infty$, let F be given by equation (3.16)), and L_n is given by equation (3.9);
Step 3: $\{v_1^*, \dots, v_n^*\} =_{v_i \in \mathbb{V}_\beth, \forall i} F(v_1^*, \dots, v_n^*)$ s.t. $ML_n \leq H$;
$F^* = F(v_1^*, \dots, v_n^*)$;
$H^* = \lceil (ML_n(v_1^*, \dots, v_n^*)) \rceil$

Finding the maximum of $F(v_1^*, \dots, v_n^*)$, for small values of n, can be done by grid search. For a 7 knot range and a laden-ballast journey, as considered in this chapter, this requires $(7/\eta)^2$ function evaluations, where the factor η (knots) is speed accuracy required. Note that M and H values are given, and even large values for these will not complicate the algorithm. For cases where $H \to \infty$, the constraint does not need to be checked.

If the ship cannot run at certain intermediate values of speed, the algorithm can be adjusted in Step 1, by eliminating these invalid speed ranges in $\mathbb{V}_\beth$. Likewise, if the ship has different upper- and lowerbounds on speed for different legs of the journey, it is straightforward to make the necessary adjustments.

For large values of n, one may need to resort to other approaches, for example a dynamic programming formulation as in Ge et al. (in press).

For solving the general time charter contract problem, one can use Algorithm 2. It will generate speed menu profiles as in e.g. Figure 3.7. H^{max} is the latest possible redelivery date considered. The algorithm iteratively calls Algorithm 1, each time feeding it specific values for M, H, and minimum and maximum speeds on each of the legs.

Algorithm 2: Grid search algorithm to solve time charter P_M (with unknown M)

Result: $\{v_1^{'*}, \ldots, v_n^{'*}\}$, $M^{'*}$, $F^{'*}$, $H^{'*}$

Input: H^{max}

Initialisation: Set $\theta > 0$. Call Algorithm 1 $(1, H^{max}, \{(v_i^-, v_i^+), \forall i\})$ and record $H = H^*$, $M^{'*} = 1$ and $\{v_1^{'*}, \ldots, v_n^{'*}\} = \{v_1^*, \ldots, v_n^*\}$;

while $H \leq H^{max}$ **do**

  Call Algorithm 1 $(M^{'*}, H, \{(v_i^-, min(v_i^+, v_i^{'*} + \theta)), \forall i\})$;

  $F^{'*} = F^*$, $H^{'*} = H^*$, $\{v_1^{'*}, \ldots, v_n^{'*}\} = \{v_1^*, \ldots, v_n^*\}$;

  Call Algorithm 1 $(M^{'*} + 1, H, \{(max(v_i^-, v_i^{'*} - \theta), v_i^+), \forall i\})$;

  if $F^* > F^{'*}$ **then**

    $F^{'*} = F^*$, $H^{'*} = H^*$, $\{v_1^{'*}, \ldots, v_n^{'*}\} = \{v_1^*, \ldots, v_n^*\}$, $M^{'*} = M^{'*} + 1$;

  end

  $H = H + 1$;

end

In Algorithm 2, we are considering that the journey takes, at maximum sailing speeds, considerably more time than 1 day. Therefore, when extending the allowable horizon with 1 additional day in the next iteration of the while loop, the optimal number of journey repetitions either stays the same, or increases at most with one.

When extending the allowable horizon with one day, and if we keep the number of repetitions the same, then intuitively there is no point for the ship to speed up a lot. Likewise, when extending the allowable horizon with one day but the number of journey repetitions increases, then intuitively it does not make sense that the ship would slow down a lot. For this reason we can adjust the speed boundaries in the two calls to Algorithm 1 and using θ in the range of 1 knot or lower.

3.3.6 Sensitivity to the opportunity cost α

It is useful to give an indication of the importance of using appropriate estimates for the value of α. Sensitivity to other key parameters is presented in Sections 3.4.3 and **??**.

We consider the following example: a Suezmax ship of 157,880 dwt is to transport 1,200,000 barrels of light crude oil over a distance of 8,293 nm, and return to the first port in ballast. Minimum and maximum speeds are 10 and 17 kn. Further details are given in Appendix A. We will further refer to this as our *base case example.*

In Table 3.2 we report the optimal speeds in the base case example for various values of α ranging from 0 to 0.3 (per year). Next to P_1 and P_∞, we also report results for the model P_{10} $(H \to \infty)$ representing the situation where the journey is to be repeated exactly 10 times (and the contract length is then determined by when the last journey finishes). Note that the speed v_1 on the laden leg is lower than the speed on the ballast leg v_2.

Table 3.2: Sensitivity analysis of α for $M = 1$, $M = 10$ and $M \to \infty$

Variable / Model	P_1	P_{10} ($H \to \infty$)	P_∞ ($M \to \infty$)
$\alpha = 0$ (/year)			
v_1 (kn)	10.8	10.8	14.0
v_2 (kn)	12.5	12.5	16.3
NPV (USD)	1,630,374	16,303,740	10,084,697*
$\alpha = 0.05$ (/year)			
v_1 (kn)	10.8	11.0	14.0
v_2 (kn)	12.5	12.7	16.3
NPV (USD)	1,621,467	15,568,458	10,075,487*
$\alpha = 0.1$ (/year)			
v_1 (kn)	10.9	11.2	14.0
v_2 (kn)	12.5	12.9	16.3
NPV (USD)	1,612,675	14,885,105	10,066,268*
$\alpha = 0.2$ (/year)			
v_1 (kn)	11.0	11.5	14.0
v_2 (kn)	12.5	13.1	16.3
NPV (USD)	1,595,239	13,654,616	10,047,317*
$\alpha = 0.3$ (/year)			
v_1 (kn)	11.1	11.8	14.0
v_2 (kn)	12.4	13.4	16.2
NPV (USD)	1,578,092	12,577,983	10,027,717*

(*) AS (USD/year)

We observe that profitability (NPV or AS) is determined by the value of α. If the next best available alternative is more profitable, and thus α is higher, then the relative attractiveness of this given project reduces, and the NPV therefore reduces. This matters most in the case of P_{10}, and much less in the cases P_1 and P_∞. In P_1 the timeframe is relatively short (less than 70 days) and thus we can expect reduced sensitivity. In P_∞, there is very little change in the AS values, in part due to the optimal speeds in that model being quite insensitive to α.

Optimal speeds on both laden and ballast legs are typically increasing with α in P_1 and P_{10} models. Interestingly, the difference in optimal speeds of these models in comparison to P_∞ is the largest when α is the smallest, while the difference between P_1 and P_{10} is the largest for the largest values of α.

From the table, we can infer that for an estimate of α (per year) within an accuracy of ± 0.05 (or $\pm 5\%$), we find optimal speeds (in knots) within a $\pm 5\%$ accuracy. Given that optimal speeds thus remain fairly insensitive to the value of α, we conduct the remainder of this study using the marine industry average value of α at 8% (per year) reported in Yagerman (2015).

3.4 Special cases P_1 and P_∞

In our quest to identify if optimal speed decisions depend on the charter contract, we start with a thorough analysis of these two special cases. This shows that, and explains why, (1) speed decisions will vary largely between them, (2) they each represent the NPV equivalent of two distinct classes of models from the literature, Model PK and Model R (see also Section 3.1) and (3) key parameters play a very different role in each model.

By demonstrating this equivalence, we can develop a deeper understanding about the differences between Model PK and Model R by studying the differences between P_1 and P_∞. It may also lead to recommendations about how best to construct models following the approaches of Model PK or Model R as to bring these in close agreement with the principle of NPV maximisation.

3.4.1 Illustrative example

In Section 3.4.2 we will formally derive that P_1 and P_∞ are, under mild conditions, the NPV equivalent models of two distinct classes of speed optimisation models from the literature, called Model PK and Model R, respectively. In this section we introduce the main characteristics of these models. Using an example, their performance is numerically compared with P_1 and P_∞. Both Model PK and Model R address the problem of determining the optimal speed of each leg of an n-leg journey, and propose to solve it by decomposing it into n smaller problems.

Model PK refers to the class of models as presented by Psaraftis and Kontovas (2014). These authors define the principle of decomposition in this context. This principle states that the optimal sailing speed on a leg j can be found from the optimisation of a sub-problem of which the modelling properties would not depend on the sailing speeds adopted on the other $n-1$ legs of the journey. They show that this principle holds in their model. Using our notation, their formulation (after Eq.(1) on p. 59) can be stated as:

$$\text{Min } \frac{1}{v_j}\Big[c_j^f F(v_j, w_j) + f^{TCH} + \alpha_s u + \beta_s w\Big]. \tag{3.17}$$

In this model, u refers to volume of cargo yet to be picked up in ports $j+1$ to $n-1$ that would be waiting in these ports, and w to the volume of the cargo currently on the ship. The α_s and β_s represent the carrying charges for the shipper, and from the example on p.64 in that paper, we observe that e.g. $\beta_s = \alpha G$, where G is taken (in the examples on p. 64 of that paper) to be the CIF value of the cargo. The objective function (USD/nm) expresses costs per unit of distance travelled on the leg. Since distance is a constant, this is equivalent to minimising (USD/leg). The Model PK is adopted in e.g. Wen et al. (2017). When not considering holding costs ($\alpha_s = \beta_s = 0$) and replacing $F(V, W)$

by the simpler propeller law $F(V) \sim V^g$, the model is also representative of those used to optimise speeds in earlier literature, as in e.g. Corbett et al. (2009). More generally, Model PK represents the models from the literature that use the criterion (USD/nm), (USD/journey), or (USD/leg).

Model R refers to another class of models following the approach developed in the revenue generating leg model in Ronen (1982). When we replace the propeller law by (3.5), and use our notation, Ronen's problem statement is that optimal speed for leg j is found from:

$$\text{Max}\frac{1}{T_j(v_j)}\Big[(R - T_j(v_j)C - c_j^f F(v_j, w_j)T_j^s(v_j)\Big], \tag{3.18}$$

where R is the income from the leg, and C the (constant) daily cost of the vessel (which can be dropped from the objective function). This formulation thus seeks to maximise the profits per day on the leg (USD/day). Model R, using the propeller law, can be recognised in e.g. the models of Devanney (2010) and Fagerholt and Psaraftis (2015).

Table 3.3 compares four speed optimisation models when the journey starts with the laden leg. In Model PK, inventory costs are at CIF value of 63 USD/barrel and an opportunity cost of 0.08 per annum. In Model R, we use the revenue generating leg model for both laden and ballast legs. To determine journey times L_n, we evaluate the solution of Model PK and Model R into our journey model (which also accounts for harbour times). We evaluate the speed values obtained from Model PK and Model R using the NPV and AS functions of models P_1 and P_∞, respectively.

Observe that the models arrive at different speed recommendations. Both P_1 and P_∞ find ballast optimal speeds to be higher than laden leg speeds, corresponding to observed industry practice. P_∞ arrives at much higher speeds however, completing the journey (including port times) in about 13 days less, and also the gap between the optimal laden and ballast speed is widened.

Comparing journey times and speeds, we observe that Model PK is closer to P_1 and Model R is closer to P_∞. As P_1 maximises (USD/journey) while P_∞ maximises (USD/year), this is not entirely surprising. While Model PK arrives at a higher TCE than P_1, it is the result of shorter journey time, and its NPV value is lower. Model R arrives at a journey time close to that of P_∞, but the very different laden and ballast speeds will achieve one million USD less in AS value. Both Model R and Model PK find ballast speeds to be lower than laden speeds.

Table 3.4 shows results when the journey starts with the ballast leg. P_∞ and Model R give each the same speed results they obtained under the laden-ballast case, while P_1 slightly raises the speed on the ballast leg. The largest difference is observed in Model PK, where ballast speed shows a large increase and laden speed is slightly lowered. NPV and AS values are lower because costs arise earlier and revenues later compared to the

Table 3.3: Optimal speeds for a laden-ballast journey (base case example)

Variable / Model	P_1	P_∞	Model PK	Model R
v_1 (kn) (laden)	10.9	14.0	12.9	17.0
v_2 (kn) (ballast)	12.5	16.3	12.5	12.5
L_n (days)	66.9	53.4	62.0	55.5
NPV (USD)	$1,616,189$	$10,069,976$ (*)	$1,576,804$	$8,828,658$ (*)
f^{TCE} (USD/day)	$44,344$	$47,589$	$45,621$	$44,188$

(*) AS (USD/year)

laden-ballast situation. TCE values are therefore also lower, except in Model PK due to the much smaller journey time.

Table 3.4: Optimal speeds for a ballast-laden journey

Variable / Model	P_1	P_∞	Model PK	Model R
v_1 (kn) (ballast)	12.6	16.4	15.1	12.5
v_2 (kn) (laden)	10.9	14.1	12.5	17.0
L_n (days)	65.7	52.1	57.2	55.5
NPV (USD)	$1,608,351$	$10,246,502$ (*)	$1,517,854$	$8,674,053$ (*)
f^{TCE} (USD/day)	$44,672$	$48,073$	$46,701$	$43,765$

(*) AS (USD/year)

The example illustrates that optimal speeds depend on the model used. The differences between Model PK and Model R and our NPV models requires further investigation in following sections. With respect to the differences between P_1 and P_∞, we formulate the insight that charter contract type matters. Optimal speeds on a time charter contract of very long duration (P_∞) are significantly higher (in this example) than those on a trip time charter (P_1), and the gap between laden and ballast speed significantly larger. Whether or not the decision maker has an interest in what happens with the ship after completion of a journey, and thus the type of the charter contract, influences speed decisions.

3.4.2 Equivalence analysis

In this section we derive properties regarding the level of equivalence between Model PK and P_1, and between Model R and P_∞. We aim to provide rationales of how the two conventional models are mapped onto P_1 and P_∞ mathematically. This will lead to a better understanding of the conditions under which the classic modeling approaches represented by Models PK and R, while they were not derived from NPV principles, can still produce speeds that help to maximise the NPV of this activity for the ship charterer.

We adopt an abstract representation of journey models with cash-flow functions $a(t, x)$ at times t and (speed) variables $x \in X$ (as in Section 3.3.2). We introduce $\delta(x)$ to

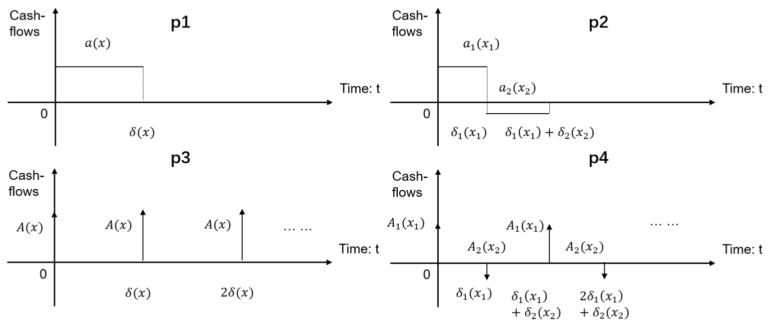


Figure 3.5: Cash-flows functions of the abstract representations of 4 different ship journeys $p1$, $p2$, $p3$ and $p4$

represent leg durations. We define four projects with the following cash-flow structures (see also Figure 3.5):

1. p1: $a(t, x) = a(x)$ for $t = [0, \delta(x)]$, and $a(t, x) = 0$ $(t \geq \delta(x))$;
2. p2: $a(t, x) = a_1(x_1)$ for $t = [0, \delta_1(x_1)]$, $a(t, x) = a_2(x_2)$ for $t = [\delta_1(x_1), \delta_1(x_1) + \delta_2(x_2)]$, and $a(t, x) = 0$ $(t \geq \delta_1(x_1) + \delta_2(x_2))$.
3. p3: an infinite series of discrete cash-flows arising at times 0, $\delta(x)$, $2\delta(x)$, ..., of magnitude $A(x)$, where $A(x)$ is the NPV of p1;
4. p4: an infinite series of discrete cash-flows $A_1(x_1)$ arising at times 0, $(\delta_1(x_1) + \delta_2(x_2))$, $2(\delta_1(x_1) + \delta_2(x_2))$, ... and $A_2(x_2)$ arising at $\delta_1(x_1)$, $\delta_1(x_1) + (\delta_1(x_1) + \delta_2(x_2))$, $\delta_1(x_1) + 2(\delta_1(x_1) + \delta_2(x_2))$, ..., where $A_1(x_1)$ is the NPV of $a_1(x_1)$ and $A_2(x_2)$ is the NPV of $a_2(x_2)$ of p2.

Projects p1 and p2 are simplified representation of a (one-leg) journey and two-leg journey model, respectively. Most models from the literature do not specify when revenues and costs are incurred. The more it is reasonable to assume that profits earned from a leg i arrive at a uniform rate a_i over the leg's duration, for which only its magnitude would depend on $x_i \in X$, the more p1 and p2 are fair characterisations of such models.

Note that Model PK, viewed as p1, maximises $a\delta$ (USD/leg), while when viewed as p2, maximises $a_1(x_1)\delta_1(x_1)$ (USD/leg) and $a_2(x_2)\delta_2(x_2)$ (USD/leg), by decomposition. Because of the separability of the formulation, this is equivalent to maximising $a_1(x_1)\delta_1(x_1) + a_2(x_2)\delta_2(x_2)$. Viewed as p1, Model R maximises the daily profit $a(x)$ (USD/day) over the finite journey time $\delta(x)$, and in p2, maximises $a_1(x_1)$ (USD/day) over the leg time $\delta(x_1)$, and $a_2(x_2)$ (USD/day) over the leg time $\delta(x_1)$, when using decomposition. Again, this is equivalent to maximising $a_1(x_1) + a_2(x_2)$.

Projects $p3$ and $p4$ represent infinite repetitions of the one-leg journey project $p1$ and the two-leg journey project $p2$, respectively.

Let $\overline{Y(\alpha)}$ represent the linear approximation of a function in α; $x \equiv y$ denote equivalence between statements x and y, and $x \not\equiv y$ otherwise; and 'Max x' shorthand for maximising a function x.

Theorem 3.1. *(proof in Appendix 8.1.3):*

- *(I) Max $a\delta$ of p1 $\equiv$ Max $\overline{NPV(\alpha)}$ of p1;*
- *(II) Max $a_1\delta_1 + a_2\delta_2$ of p2 $\equiv$ $lim_{\alpha\to 0}$ Max $\overline{NPV(\alpha)}$ of p2. ($\overline{NPV(\alpha)}$ contains an additional term in α and inseparable components of x_1 and x_2);*
- *(III) Max $a(x)$ of p1 $\equiv$ Max $AS(\alpha)$ of p3;*
- *(IV) Max $(a_1\delta_1 + a_2\delta_2)/(\delta_1 + \delta_2)$ of p2 $\equiv$ $lim_{\alpha\to 0}$ Max $\overline{AS(\alpha)}$ of p4 ($\overline{AS(\alpha)}$ contains additional terms in α, one of which inseparable);*
- *(V) (a) Max $a_1 + a_2$ of p2 $\not\equiv$ Max $\overline{AS(\alpha)}$ of p4, even if $\alpha = 0$, and (b) Max $a_1 + a_2$ will skew the optimisation by placing too much emphasis on the cash-flow a_i of leg i with the smallest value for $\delta_i/(\delta_i + \delta_j)$ $(i, j \in \{1, 2\})$.*

In the theorem, the left-hand sides of $\equiv$ or $\not\equiv$ represent (possible) methods we can apply when the objective function is set up following the classic models. The right-hand sides follow the NPV approach - but possibly applied to the linear approximation of the NPV function. Because this approximation is often of good accuracy, a result of equivalence ($\equiv$) implies a good correspondence between the classic method and the NPV approach. We can then also expect to be able to learn about the classic method through the study of the equivalent NPV model. See also Beullens and Janssens (2014).

In particular we derive the following insights. (I) and (II) indicate that an approach as in Model PK that maximises profits per nautical mile per leg (by decomposition), will come close to maximising the NPV for the ship charterer of this journey *only*. This implies that Model PK *implicitly assumes that the time of completing the journey does not affect future investment opportunities for the ship charterer* (as this is indeed what optimising the NPV of p1 or p2 establishes).

(III) to (V) show properties of models such as R that maximise daily profits over legs of a single finite journey. (III) indicates that R is equivalent to maximising the AS (USD/year) of a model *where this journey, that R considers, is in fact infinitely repeated.* It is thus important to realise that such models implicitly adopt this assumption. In contrast to PK models, the time of finishing the first journey now greatly affects speed choices, as these times affect the NPV value of the profits of future repetitions. Furthermore, if follows that R-like models can only accurately account for *round-trip* journeys. Consider a journey from port 0 to 1, 1 to 2, ..., and finishing at port n. In an R-like model, we aim to maximise the profits per unit of time. Figure 3.6 illustrates the equivalent model of maximising the NPV of an infinite repetition of this journey. It must thus follow that port 0 and port n are the same single port, since ships cannot move at zero cost and time from the final port in the journey back to the starting port, unless these ports are equal.

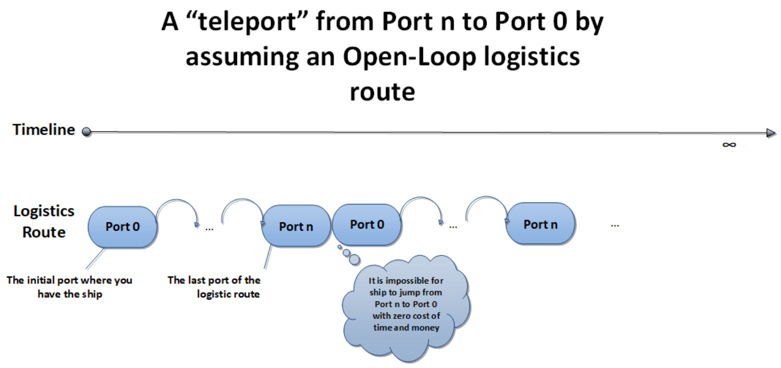


Figure 3.6: Time-flows of an open loop logistics route in infinite time horizon

(IV) and (V) show how to best construct R models when legs are not uniform in their profit characteristics and/or duration. (V) indicates that decomposition will not work well. For example, if leg 1 requires 1 month and leg 2 half of that, (III) indicates that the decomposed model will account for 12 repetitions of the first and 24 repetitions of the second leg in one year. This is wrong, since both legs can only be repeated an equal number of times (8 in a year). Using decomposition will thus skew the results by putting too much weight on the importance of leg 2. (IV) shows that an approach which maximises the weighted profits over the total journey duration will achieve much better results. For further discussions of the implications on the literature, see Section 3.4.4 and Section 3.5.3.

Table 3.5: Comparison between P_1, P_∞ and P_M models

Contract Length	*Objective*	*Model*
Short or Undefined (a few months, e.g. Spot Contract)	maximise the profitability over given leg(s) (USD/ Nautical Miles)	P_1
Very long (> 5 years, e.g. the decision maker owns the ship)	maximise the daily profitability over a round-trip (USD/ Day)	P_∞
In between, with speficied duration H (e.g. Time Charter)	maximise the profitability over given horizon H (USD/ Journey)	P_M

Table 3.5 summarises the main relationship between objective type and contract length. When optimising speeds over a journey choosing between the criterion (USD/nm) or (USD/day) is crucial as they imply adopting very different underlying assumptions about the future use of the vessel. In the former, the decision maker has no interest at all in its future use, while in the latter, it is assumed to be repeatedly used for an infinite repetition of identical journeys under identical circumstances. This difference in assumptions is only implicitly present in Model PK and Model R, but made explicit in P_1 and P_∞. In addition, the theorem has made clear that Model PK works well for any type of journey, and that optimising speeds on each leg separately through decomposition remains a fairly accurate approach in terms of maximising the NPV. In contrast, Model R only works on round-trip journeys, and decomposition will not be a good approach if the aim is to maximise the NPV.

3.4.3 Drivers of ship speed decisions

This section explains why speed decisions in trip charter contracts differ from those in time charter contracts of very long duration. We do this by, with further support from a sensitivity analysis, analytically deriving which journey and other model characteristics are most influential in driving speed decisions in models P_1 and P_∞, respectively.

3.4.3.1 Model $\mathbf{P}_1$

The role of speed variables in P_1, and which parameters mostly influence their optimal values, is not obvious from (3.10), as it is expressed through exponential terms in leg completion times.

We thus take the Maclaurin expansion of these exponential factors, and subsequently approximate the function by ignoring the second and higher order terms in α. Application of this process to (3.10) gives:

$$\overline{\mathrm{NPV}}_1 = \Big[\sum_{j=1}^{n}\Big(R_j(1+\alpha\delta_j-\alpha L_j) - C_j^u(1-\alpha\epsilon_j-\alpha L_j) - C_j^l(1-\alpha\epsilon_{j-1}-\alpha L_{j-1})\Big) - f^{TCH}L_n(1-\frac{\alpha L_n}{2})\Big]e^{-\alpha t_{FS}}. \tag{3.19}$$

Using (3.9), and rearranging terms, we can write $\overline{\mathrm{NPV}}_1 = \sum_{j=1}^{n}\overline{\mathrm{NPV}}_{1(j)}$, i.e. as a sum of terms with information that is mostly related to each of the legs. For leg j $(1 \le j \le n)$, and making explicit which components are functions of speeds, this gives:

$$\overline{\mathrm{NPV}}_{1(j)}(V_n^-) = \Big[R_j(1+\alpha\delta_j) - C_j^u(1-\alpha\epsilon_j) - C_j^l(v_j)(1-\alpha\epsilon_{j-1}) \tag{3.20}$$

$$- f^{TCH}T_j(v_j)(1-\alpha\frac{L_n(V_n^-)}{2}) \tag{3.21}$$

$$- \alpha T_j(v_j)\sum_{i=j}^{n}\Big(R_i - C_i^u - C_{i+1}^l(v_{i+1})\Big)\Big]e^{-\alpha t_{FS}}. \tag{3.22}$$

We can associate the following interpretation to this result. The approximate NPV contribution of leg j is determined by: (3.20) the revenues from completing the leg minus the costs for loading and unloading; (3.21) charter party costs which are proportional to the duration of the leg, but which also depend to some degree on the speed on all other legs through L_n; (3.22) the *opportunity costs* of deferred revenues earned from port visits yet to be completed, and *opportunity rewards* from delayed costs to execute the future legs on the journey, of which the loadings costs are functions of the speed on future legs $j+1$, $j+2$, ... to $n-1$.

So far, we have not removed any constant terms, and would thus still arrive at an accurate evaluation of $\overline{\text{NPV}}$ when using the above functions. For the sake of identifying the main drivers of speed decisions, we now proceed removing *constant* terms. Observing (3.20), we drop the constant terms in revenues and unloading costs. We substitute using (3.3), (3.4), (3.6), and can then remove the constant term C_j^h. Define for convenience $\overline{T}_j = T_j^l + T_j^w + T_j^u$ (a constant, measured in years). We separate the factor $S_j/24v_j$ from all remaining terms, of which for the optimisation we can drop the constant $S_j/24$, and find the following cost minimisation problem:

$$\text{Min } \frac{1}{v_j}\Big[c_j^f F(v_j, w_j)(1-\alpha\epsilon_j) + f^{TCH}\Big(1 + \frac{\overline{T}_j(24)v_j}{S_j}\Big)(1 - \alpha\frac{L_n(V_n^-)}{2}) + \alpha\Big(1 + \frac{\overline{T}_j(24)v_j}{S_j}\Big)\sum_{i=j}^{n}(R_i - C_i^u - C_{i+1}^l(v_{i+1}))\Big] \tag{3.23}$$

The expression in the large brackets evaluates to (USD/day), and solving the problem thus establishes how to minimise the cost per distance travelled (USD/nm). This function is dependent on v_j, on V_n^- through L_n, and on V_{j+1}^+ through the C_{i+1}^l terms.

In most realistic problem settings, certain terms in this equation will have a limited impact on the optimisation. If fuel costs are to be paid close to the time of leaving the port, the term $\alpha\epsilon_j$ is small relative to 1. The term $\overline{T}_k(24)v_k/S_k$ is in the range of 0.10 for distances in a range of 10,000 nm. In (3.23), the term $\alpha L_n/2$, for a total journey time of say 60 days, is in the order of 0.0066. Dropping these less influential terms from the function gives the following simplified optimisation problem:

$$\text{Min } \frac{1}{v_j}\Big[c_j^f F(v_j, w_j) + f^{TCH} + alpha\sum_{i=j}^{n}(R_i - C_i^u - C_{i+1}^l(v_{i+1}))\Big]. \tag{3.24}$$

This function is dependent on v_j, and only on V_{j+1}^+ through the C_{i+1}^l terms. The impact of the opportunity costs of future profits remains small when n is small and freight rates are not in a boom.

The function (3.24) shows that important drivers of speed decisions in P_1 are fuel price, deadweight carried, and time charter hire. Due to the opportunity cost term, we can also expect to see some influence from revenues earned on subsequent legs. From the opportunity rewards of future costs, we may see some dependency on the (un)loading costs. As harbour times and leg distances are not present, we expect not to see much influence from these factors.

To further qualify the direction and magnitude of the impact of input variables on optimal speeds and NPV, we have conducted a sensitivity analysis on two to four leg journey models of the Suezmax ship used in Section 3.4.1, under various plausible conditions. Note that to determine optimal speed decisions, we optimise the original unapproximated model P_1. The main drivers of optimal speed values and profitability for P_1 are reported in columns two and three of Table 3.6.

In addition, the sensitivity analysis adds the following insights. The effects of the opportunity costs and rewards related factors increase with the number of legs in the journey. If revenues are linearly increasing with leg distance, leg distance will have a small increase in optimal speeds through the opportunity costs of revenues increasing. There is no significant impact from changing the actual timing of payments of revenues and costs on speed decisions, and only a small impact on the NPV of the activity.

We also found that harbour times have no effect on speed decisions. Reducing harbour times as a means to stimulate slower steaming does therefore not seem to be an effective instrument for ships running under trip time charter contracts.

Table 3.6: Main drivers of ship speeds and profitability

Parameter / Model	P_1 speeds	P_1 NPV	P_∞ speeds	P_∞ AS
Fuel prices	$--$	$--$	$--$	$--$
Dwt carried	$--$	n/a	$--$	n/a
Freight rates	$+$	$++$	$++$	$++$
Time Charter Hire	$++$	$--$	0	$--$
Leg distance (with linearly increasing revenues)	$+\epsilon$	$+$	$+$	$+$
Leg distance (at constant revenues)	0	$--$	$--$	$--$
Fixed (un)loading costs	$-\epsilon$	$--$	$--$	$--$
Harbour times	0	$-$	$-$	$-$

How an increase in parameter value (e.g. Freight rates) affects model outcome (e.g. optimal speeds):
$--$, $-$ and $-\epsilon$ indicate, respectively, a large, small, or minimal reduction in outcome value;
$++$, $+$ and $+\epsilon$ indicate a large, small, and minimal increase in outcome value.
0 indicates no impact.

3.4.3.2 Model P_∞

Using (3.10), the profit function (3.16) of P_∞ can be re-written as:

$$\mathrm{AS} = \Big[\sum_{j=1}^{n}\Big(R_j e^{-\alpha(L_j-\delta_j)} - C_j^u e^{-\alpha(L_j+\epsilon_j)} - C_j^l e^{-\alpha(L_{j-1}+\epsilon_{j-1})}\Big)\Big]\frac{\alpha e^{-\alpha t_{FS}}}{1-e^{-\alpha L_n}} - f^{TCH} e^{-\alpha t_{FS}}, \tag{3.25}$$

showing, logically, that the sailing speeds of the ship do not affect the charter party costs. We can thus leave them out for the purpose of the optimisation.

The linear approximation of the Maclaurin expansion of (3.25), while substituting using (3.9), (3.3) and (3.6), and rearranging terms, leads to the approximation $\overline{\mathrm{AS}} = \sum_{j=1}^{n-1} \overline{\mathrm{AS}}_{(j)}$, where for each leg j $(1 \leq j \leq n)$:

$$\overline{\mathrm{AS}}_{(j)} = \frac{e^{-\alpha t_{FS}}}{\sum_{i=1}^{n} T_i(v_i)}$$

$$\Big[R_j(1+\alpha\delta_j) - C_j^u(1-\alpha\epsilon_j) - (C_j^h + c_j^f F(v_j, w_j)T_j^s(v_j))(1-\alpha\epsilon_{j-1}) \tag{3.26}$$

$$- \alpha T_j(v_j) \sum_{i=j}^{n} (R_i - C_i^u - C_{i+1}^l(v_{i+1}))\Big] \tag{3.27}$$

$$+ \frac{\alpha e^{-\alpha t_{FS}}}{2}(R_j - C_j^u - C_j^h - c_j^f F(v_j, w_j)T_j^s(v_j)). \tag{3.28}$$

This function tells us that the contribution to the $\overline{\mathrm{AS}}$ profits of leg j is the result of: (3.26), the profits made from executing the leg per unit of total journey time; (3.27), the opportunity costs from future leg profits per unit of total journey time; (3.28) the *charterer's rewards* earned from the profits on the leg. This term accounts for the fact that if the profitability from executing this leg would increase, the annual income for the firm will go up with more than what is given by (3.26) because of the leg being repeated.

In contrast to the case of model P_1, there is little that can be done to further simplify this result. Except for the time charter hire, we thus expect to see most factors playing their role in driving speed decisions. The results from a sensitivity analysis, using the unapproximated function P_∞, confirm this, and the impacts are reported in columns four and five in Table 3.6. Most factors influence speeds more significantly, in particular the impact from freight rates is much more pronounced.

Port events do effect speeds decisions, but not in the way as hypothesised in some of the literature (see Section 3.1). Indeed, these results indicate that slow steaming is encouraged by *increasing* port times, or *increasing* port costs, and be at the expense of reducing the profitability of the ship charterers.

3.4.4 PK and R revisited

The approximate optimisation function (3.24) established for P_1 and the objective function (3.17) of the PK model show great similarity. The interpretation we need to associate with the terms in α are, however, very different. In PK, these terms account for holding costs incurred by shippers. Although usually not charged to the charterer, they might be relevant to the charterer's speed decision, as argued in Psaraftis and Kontovas (2014), in a context where competition for demand makes transport time a competitive service component. In P_1, the opportunity cost rate is that of the charterer, and the opportunity costs are determined by the freight rate, rather than e.g. CIF value. In addition, the model also considers the opportunity rewards from deferred fixed (un)loading costs and fuel costs for future legs, a component that is missing from PK, but should always be relevant.

As discussed, the opportunity cost components of P_1 and the holding costs of PK model different aspects of a problem context. If service requires the consideration of holding costs for shippers, these should be added to the model *in addition to* the opportunity costs and reward terms identified in P_1. According to our understanding, the bulk cargo markets are not (yet) that sophisticated, and rather use the mechanism of changes to freight rates to implicitly reflect imbalances between ship availability and demand pressures. Model P_1 will then better reflect how speed decisions are made on trip charter journeys.

The optimisation function (3.26)-(3.28) for P_∞ differs in structure from the function (3.18) of the R model. From Theorem 1 (IV), it has already been established that contributions of individual legs should be valued per unit of time over the whole roundtrip journey, and such a modification can be easily implemented in the R model. In addition, (3.26) shows that it is best to take $R = R_k - C_k^u - C_k^h$ in the R model. When testing with these modifications only, we find that the so adapted R model will give speeds only marginally above those found from P_∞, with AS values that remain typically within 1%.

3.5 General model P_M

Speed decisions in general charter party contracts P_M differ from those in the two special cases P_1 and P_∞. We show that speeds decisions are strongly influenced by (1) horizon constraints of the contract, and (2) the charterer's attitude to the trade-off between profit maximisation and risk. While speed decisions are still influenced by all main drivers identified in Table 3.6, the influence is much less straightforward.

3.5.1 Illustrative example

To illustrate the impact of a finite time horizon on optimal speed choices, we reuse the base case example introduced in Section 3.4.1. We test for three different horizons: $H = 335$ days, or 5 times the journey duration obtained from model P_1; $H = 268$ days, or 5 times the journey duration obtained from P_∞; and finally, an arbitrary horizon set at $H = 365$ days.

In Table 3.7, we compare the performance of P_M with that obtained from using a feasible number of repetitions of optimal journey speeds obtained from P_1 and P_∞, respectively. For each of the three possible horizons, the table lists the optimal values of the agreed duration H^*, the corresponding number of feasible repetitions M^*, the leg speeds, and the NPV. For P_1 and P_∞, the optimal leg speeds are as reported in Table 3.3. In P_M, optimal leg speeds are determined simultaneously with the optimal values for M^* and H^*.

Table 3.7: Performance of repeated laden-ballast journeys

Variable / Model	P_1	P_∞	P_M
$H = 335$ (days)			
H^* (days)	334.4	320.5	334.8
M^* (-)	5	6	6
v_1 (kn)	10.9	14.0	13.4
v_2 (kn)	12.5	16.3	15.5
NPV (USD)	7,828,927	8,514,068	8,829,191
$H = 268$ (days)			
H^* (days)	267.5	267.1	268.0
M^* (-)	4	5	5
v_1 (kn)	10.9	14.0	13.9
v_2 (kn)	12.5	16.3	16.3
NPV (USD)	6,308,705	7,136,261	7,158,921
$H = 365$ (days)			
H^* (days)	334.4	320.5	364.6
M^* (-)	5	6	7
v_1 (kn)	10.9	14.0	14.4
v_2 (kn)	12.5	16.3	16.8
NPV (USD)	7,828,927	8,514,068	9,618,736

It is a striking observation how much better a solution from P_M is compared to those obtained from P_1 or P_∞.

Note that in each solution, the charterer only pays the charter rate f^{TCH} during $H^*(\leq H)$. In P_∞ for $H = 335$, for example, the charter rate only applies to 320.5 days. P_M completes an equal number of journeys at lower speeds such that $H^* \approx H$. For $H = 365$ days, however, optimal leg speeds increase relative to the P_∞ solution as to include one additional journey within the time horizon, increasing NPV profits by over 1 million USD, or an increase of 15%.

Depending on the horizon, P_1 leads to solutions which are between 0.8 and 1.8 million USD worse in NPV, or about 11% to 23% worse. The reason for this bad performance of P_1 can by now be understood since trip time charter contracts assume the ship is returned to the owner after a single journey is completed, and does not account for future repetitions. Model P_∞ is on average better equipped as it considers journey repetition, although depending on the horizon constraint, can still be quite inferior relative to P_M.

In brief, the example illustrates that the horizon of the charter contract is a factor of significant impact on speed decisions. The impact of the horizon is not explicitly considered in the special cases: in P_1 it is given as a result of speed choices on the journey legs, i.e. $H \equiv H^* = L_n^*$, and in P_∞ is it taken as $H \equiv H^* \to \infty$. Model P_M, by explicitly accounting for the impact of H to determine H^* in conjunction with optimal leg speeds, can significantly outperform the more naive approaches of multiplying the optimal journey speeds obtained from either of the special cases.

3.5.2 Drivers of ship speed decisions

We develop a deeper understanding of how the horizon H and the associated redelivery clause in the contract affect speed decisions in P_M.

The profit function for P_M is given by (3.11) in which NPV_1 is given by (3.10). Separating the term in TCH gives:

$$\begin{aligned}\mathrm{NPV}_M = &\Big[\sum_{j=1}^{n}\Big(R_j e^{-\alpha(L_j-\delta_j)} - C_j^u e^{-\alpha(L_j+\epsilon_j)} - C_j^l e^{-\alpha(L_{j-1}+\epsilon_{j-1})}\Big)\Big]\frac{(1-e^{-\alpha M L_n})e^{-\alpha t_{FS}}}{1-e^{-\alpha L_n}} \\ &- \frac{f^{TCH}}{\alpha}(1-e^{-\alpha M L_n})e^{-\alpha t_{FS}}, \qquad (3.29)\end{aligned}$$

and the linear approximation:

$$\overline{\mathrm{NPV}}_M = \sum_{j=1}^{n}\overline{\mathrm{NPV}}_j\Big[M + \alpha\frac{ML_n}{2}(M-1)\Big] - f^{TCH}ML_n(1-\alpha\frac{ML_n}{2}), \qquad (3.30)$$

where

$$\overline{\mathrm{NPV}}_j = \Big[R_j(1+\alpha\delta_j) - C_j^u(1-\alpha\epsilon_j) - C_j^l(1-\alpha\epsilon_{j-1}) - \alpha T_j\sum_{i=j}^{n}(R_i - C_i^u - C_{i+1}^l)\Big]. \qquad (3.31)$$

3.5.2.1 When subject to a given number of journey repetitions

In contracts of finite duration, the journey in model P_M can only be repeated a finite number of times. Let us first examine briefly how optimal speeds vary in P_M ($H \to \infty$), i.e. in absence of a constraint on H but in which we adopt a given number of journey repetitions M (i.e. assuming H is found as ML_n^*.)

The linear approximation (3.30) can be re-written in terms of its components for each leg as follows:

$$\begin{aligned}\overline{\mathrm{NPV}}_{M(j)} = &Me^{-\alpha t_{FS}}\Big[\overline{\mathrm{NPV}}_j - f^{TCH}T_j(1-\alpha\frac{ML_n}{2})\Big] \\ &- \alpha e^{-\alpha t_{FS}}\frac{ML_n}{2}(M-1)(R_j - C_j^u - C_j^l). \qquad (3.32)\end{aligned}$$

Comparing (3.32) for P_M to (3.20)-(3.22) for P_1 and (3.26)-(3.28) for P_∞ shows that, for a given constant M value, the objective function of P_M can be viewed as a weighted average of the component functions of P_1 (with weight M) and terms proportional to the charterer's rewards of P_∞ (with weight $-M(M-1)$). For constant low M values, we thus expect the model to behave similarly to P_1, while with increasing values of M, to take over some of the behaviour of model P_∞. Recalling the main drivers of P_1 and P_∞ in Table 3.6, we thus expect to see at low M values a large sensitivity to TCH and small sensitivity to freight rates, and as M increases, that the sensitivity to TCH reduces and to freight rates increases. Since P_M optimises (USD/journey) rather than (USD/day), we expect however that its solutions should remain closer to P_1 than P_∞.

Table 3.8: Performance of P_M for a given number of journey repetitions

Variable / M-values	$M=5$	$M=10$	$M=20$	$M=30$
Base case				
H^* (days)	331.85	656.59	1,289.79	1,996.83
v_1 (kn)	11.0	11.1	11.3	11.5
v_2 (kn)	12.6	12.8	13.1	13.3
NPV (USD)	7,829,768	15,113,504	28,219,741	37,580,022
Freight rates x 1.5				
H^* (days)	322.12	637.58	1,218.99	1,770.21
v_1 (kn)	11.2	11.5	12.1	12.5
v_2 (kn)	12.8	13.2	13.9	14.5
NPV (USD)	19,826,413	38,300,872	71,709,624	101,078,409
TCH x 1.5				
H^* (days)	299.80	597.90	1,187.91	1,770.21
v_1 (kn)	12.3	12.3	12.4	12.5
v_2 (kn)	14.2	14.3	14.4	14.60
NPV (USD)	4,789,051	9,273,999	17,413,688	24,569,661

The results in Table 3.8 show solutions for increasing values of M in the base case example. We observe that the economic principle that ships react to improved market conditions (increased freight rate) by speeding up is recognised in P_M $(H \to \infty)$. We can infer from the result that the more the future is important (higher value of M), the more prominent the decision maker's reaction to changes in freight market conditions. In other words, for low M values, the model behaves similarly to P_1, while with increasing value of M, it takes over some of the behaviour of P_∞.

We have seen in Section 3.5.1 that for finite horizons, solutions from P_M are in general quite superior to adopting solutions based on P_1 or P_∞. This also holds with respect the approach used here and based on fixing the number of repetitions a priori in P_M. For $H = 331$ days, for example, the model P_M arrives at speeds of 13.5 kn and 15.7 kn in order to complete $M^* = 6$ journeys. The NPV of this is 8,751,115 USD. For 656 days available, speeds are 13.7 kn and 15.8 kn, with $M^* = 12$. The NPV is 16,769,194 USD. Compare this to the solutions in Table 3.8 for the base case scenario at $M = 5$

and $M = 10$, respectively. It is clear that given a certain number of available days H, the charterer will greatly benefit from seeking an optimal value M^*.

3.5.2.2 When subject to a finite horizon

In a general charter contract, the optimisation problem P_M is subject to (3.14), i.e. $H^* = M^* L_n^* \leq H$. We will now show why the value of H has a significant impact on optimal speeds. We also introduce a novel representation of optimal solutions to model $P_M(H)$ as a function of H that we refer to as 'speed menu curves'.

For the base case example, Figure 3.7 displays the optimal laden and ballast speeds and corresponding optimal number of journey repetitions as functions of integer values for H, where $H \in [65, 665]$. Figure 3.8 plots the corresponding NPV profits; this is a non-decreasing function of H. Optimal speeds, broadly speaking, follow a saw-tooth pattern where each cycle corresponds to a constant optimal M^* value. It is clear why: when M^* remains constant (and $H^* \approx H$), an increase in available days H allows the ship to travel slower and slower, up to the point where an increase in M^* would achieve a higher NPV value rather than further reducing the ship speed. In that case, however, the ship will have to speed up to accommodate the extra journey.

We refer to the solution presented in Figure 3.7 as a 'speed-menu curve'. We provide two examples to help clarify how these results can be utilised.

Example 1. The charterer considers hiring a ship from an owner, who needs the ship back in 200 days. From Figure 3.7 and Figure 3.8, we can see that the highest NPV that could be achieved is 4 repetitions of the journey, at high speeds of 15.7 kn and 16.9 kn for all laden and all ballast legs, respectively. If adopted, the ship will only complete on day 200, and it would not have the ability to make up any time lost during the four journeys. The more sensible solution is for the ship charterer to aim for 3 repetitions at the lowest optimal speeds of 11.5 kn and 13.1 kn, needing 192 days.

Example 2. The charterer seeks a ship for work that requires 6 repetitions of the laden-ballast journey, and considers an offer from a ship owner. From Figure 3.7, we deduce that, with this particular ship, it would be optimally executed at 12.6 kn laden speed and 14.9 kn ballast speed, requiring 349 days.

It is clear from these examples that the higher ranges of speeds displayed in the saw-tooth patterns of the speed menu curves, occurring when it is profitable to insert another repetition of the journey, are solutions that will not be as preferred to solutions in the lower speed ranges of a saw-tooth, if there is some flexibility in the contract horizon H. We point out that the two examples consider an identical ship, the same daily hiring cost, the same journey structure, and under identical economic conditions. Yet, the range of optimal speeds for the ship depends on the time charter contract horizon.

The saw-tooth pattern is not perfect. It may be cut-off from above or from below due to the ship's speed range limitations of 10 and 17 kn. For low H values, the optimal speed pattern may be flat at other than the minimum and maximum speeds. This happens for the range $H \in [67, 96]$, where $H^* = 67$ days. The corresponding NPV remains constant. A similar case arises when $H \in [134, 144]$, for which $H^* = 134$. The model indicates that it is not possible or worthwhile in these circumstances to use the extra available days by including another journey, nor that it is optimal to reduce the speed as the savings made from slower steaming are not compensating the extra days that the charter rate TCH would be incurred. The solution optimal for $H \in [67, 96]$ corresponds, as expected, to that of P_1.

We also observe that the amplitude of the saw-tooth slowly reduces for larger M^* values, i.e. that the range of optimal speeds within a cycle reduces with H. For H values around 5 years, although not shown in the figure, optimal speeds differ no more than 0.3 kn around the optimal ballast and laden speed values reported in Table 3.3. The maximum and minimum speeds of each saw-tooth converge, as expected, to the optimal speeds obtained from P_∞, although fairly slowly.

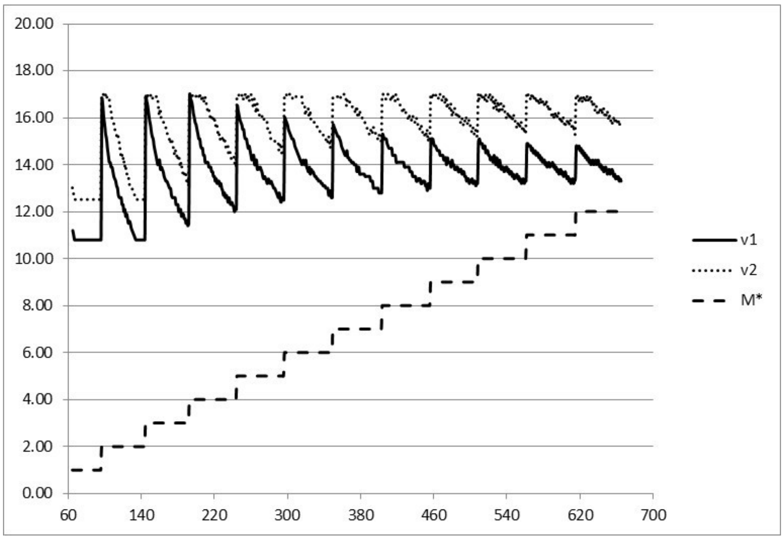


Figure 3.7: Speed-menu curve: Optimal leg speeds (kn) and repetitions as functions of H (days) in the base case example

In Section 3.4.3, we have identified the impact of drivers on speed decisions for models P_1 and P_∞. In the remainder of this section, we show that the constraint H in P_M greatly affects how changes to these parameters affect speeds.

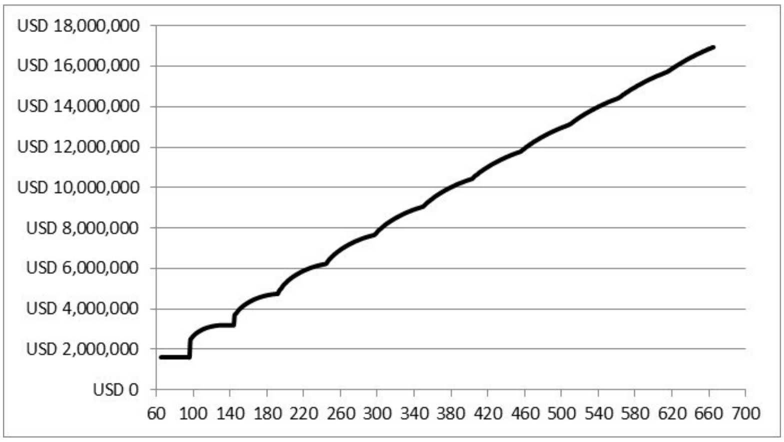


Figure 3.8: Optimal NPV profit (USD) as a function of H (days) in the base case example

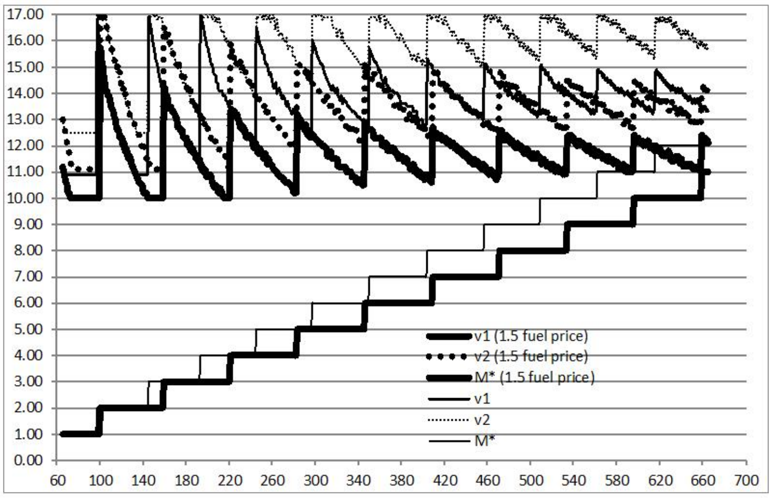


Figure 3.9: Optimal leg speeds (kn) and number of repetitions (fuel prices x 1.5)

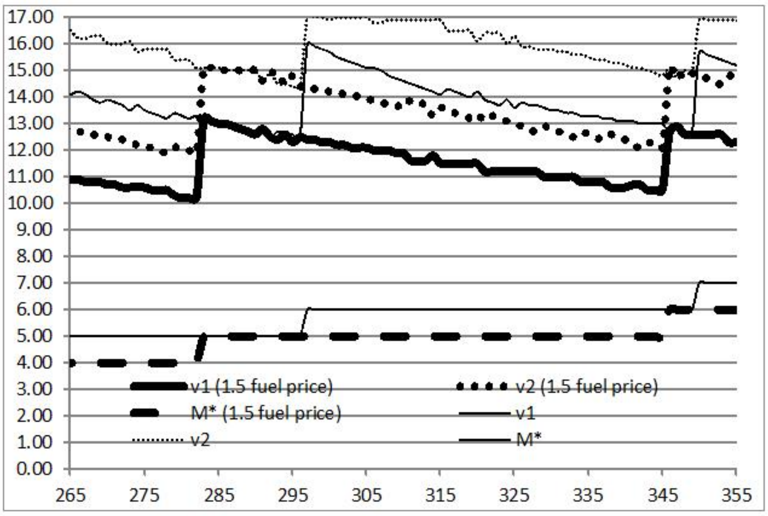


Figure 3.10: Close-up of Figure 3.9 for $H \in [265, 355]$.

Figure 3.9 illustrates the impact of a rise in fuel prices by a factor 1.5. Figure 3.10 is a close-up for $H \in [265, 355]$. For $H \in [297, 345]$, optimal speeds drop by 2 to 2.5 kn since M^* reduces with 1. For H around 460, the drop in optimal speeds is almost twice as large for the same fuel price increase, because M^* reduces by 2. In certain ranges, such as $H \in [283, 297]$ the optimal speeds are not significantly affected by the fuel price change. This is because M^* remains constant and also $M^*L_n^* = H^* \approx H$, so the journey time L_n^* is unaffected. The only degree of freedom available is thus to adjust the relative speeds on each of the legs in order to optimise the function $\sum_{j=1}^{n} \overline{\mathrm{NPV}}_j$. Such adjustments turn out to be quite insignificant. For fuel price rises much smaller than by a factor 1.5, the ranges of H for which we would see no significant impact from fuel price changes will become more prominent.

For some data inputs, $H^* << H$, i.e. the constraint is non-binding. Figure 3.11 illustrates in close-up a region of H where this is observed. In the base case scenario, such non-binding solutions arise for $H \in [134, 144]$. The increase in fuel prices moves the region of non-binding solutions to $H \in [145, 159]$. For $H \in [134, 144]$, the fuel price increase would thus result in the ship slowing down slightly and the solution moves from non-binding to binding, but for $H \in [145, 159]$, the ship's speed reduces drastically and the solution moves from binding to non-binding.

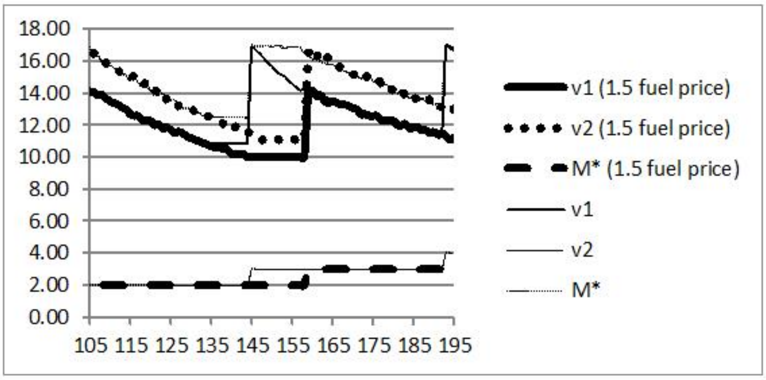


Figure 3.11: Close-up of Figure 3.9 for $H \in [105, 195]$

In both P_1 and P_∞, any rise in fuel prices always slows down the ship. In P_M, we have just seen that this is not always so, and that it depends, for given ship and journey characteristics, on the interaction between the magnitude of the fuel price increase and H.

Further sensitivity analysis indicates that we can draw similar conclusions with respect to the sensitivity to changes in other cost or revenue parameters. If as a consequence of the change in parameter value, the solution remains binding, and M^* keeps its original value, optimal speeds remain largely unaffected. If harbour *times* increase, however, then under these conditions the ship must speed up to realise the same total journey time. These insights are summarised in the second column of Table 3.9.

For some data inputs, a change in value of a parameter will produce a new optimal value for M^*. An increase in freight rates, ceteris paribus, will in that case include one or more extra journeys within H, with usually a significant increase in the speeds on both ballast and laden legs. Conversely, a decrease in M^*, with usually much lower speeds, will result from increases in fuel prices, TCH values, fixed (un)loading costs, or harbour times. This is summarised the last two columns in Table 3.9.

The above situations are most commonly arising. Less often occurring, but possible, are situations where an optimal non-binding solution remains non-binding at the same M^* value. Then, speeds tend to decrease with increases in fuel prices, and increase with increased TCH values. Our findings for this case are summarised in the third column of Table 3.9.

Table 3.9: Main drivers of ship speeds in model P_M

Parameter / Model	P_M speeds (*)	P_M speeds (**)	M^* (***)	P_M speeds (***)
Fuel prices	≈ 0	$-$	$-$	$--$
Freight rates	≈ 0	≈ 0	$+$	$++$
Time Charter Hire	≈ 0	$+$	$-$	$--$
Fixed (un)loading costs	≈ 0	≈ 0	$-$	$-$
Harbour times	$+$	≈ 0	$-$	$-$

(*) For constant M^* such that $H^* \approx H$;
(**) for constant M^* such that $H^* << H$;
(***) for changes leading to new optimal M^* value.

From the above analysis (and as summarised in the last row in Table 3.9) we also conclude that the impact of port times depends on the situation. If the constraint is binding, and the optimal M^* value remains unchanged by a decrease of port times, the extra time available would directly translate to increasing sailing times and speed reductions. If the constraint is non-binding and M^* remains constant, the impact of harbour times is negligible. If the optimal value M^* is impacted, however, then a decrease of the time in ports will in these cases usually result in much higher ship speeds as to allow the ship to make the extra journey(s).

3.5.3 Impact on related literature

We have spent considerable effort in analysing the differences between the decision context of P_1, P_∞, and P_M, rather than having concentrated all efforts on P_M only. We belief that this is important in order to help us better understand the results found in previous literature, and determine avenues for further research. We end this section giving three examples to illustrate this point.

Fagerholt and Psaraftis (2015) study speed choices for a ship using a model that maximises daily profit over the (variable) duration of a journey. The underlying assumptions are therefore that of a model like P_∞. They argue that the time in port can be left out as it is constant, and that their one-leg journey model can be adapted to account for roundtrips if the ship's payload on all legs would be the same. From Table 3.6, we conclude that harbour times would indeed have no effect in models akin P_1, but do affect speed choices in P_∞. Further, our discussions of Theorem 1 in Sections 3.4.2 and 3.4.4 show that erroneous speed decisions will be found in models that maximise daily profits of an individual leg.

As a second example, consider Corbett et al. (2009). The paper examines how speeds are to be found for a (variable) fleet of ships that must meet a certain annual demand rate for freight between origin and destination pairs. The model consists of modelling, for each ship, the objective function as maximising the profits per trip from a particular origin to a particular destination. The number of trips possible within a year is then determined after the optimisation of this function. The optimisation in essence thus considers a model such as P_1. We have seen in Section 3.4 the importance of accounting for the future use of a ship when it is to be used repeatedly, and thus a much better choice of model would be based on P_∞, maximising the annual profitability (or USD/day) from each ship instead.

As a final example, consider Devanney (2010), who claims that all ship operators (either being the owner or as a charterer) will basically react in the same manner to changes in freight rates, independent of the particular charter contract type and its duration. The proof consists of developing two objective functions, one from the viewpoint of a ship owner operating in the spot market, the second from the viewpoint of a charterer. However, both models are based on maximising (USD/day) of a roundtrip journey, which *implicitly* assumes an infinite repetition of this activity. As indicated in Section 3.3.4, model P_∞ can indeed also be interpreted as the situation of a ship owner, operating the ship on voyage charters in the spot market. However, our analysis of model P_M leads us to a very different conclusion when the time remaining on the charter contract would be finite, as then the charterer would face a different optimisation problem than that of the owner, with different optimal strategies to react to an increase in freight rates.

3.6 Speed adjustments under a time charter contract

The speed menu curves as in Figure 3.7 plot solutions to $P_M(H)$, i.e. the optimal speeds on each leg, and number of journey repetitions as a function of the available horizon H. As shown in Section 3.5.2.2, this can inform the decision maker about the profitability prior to accepting a time charter contract. In Sections 3.6.1 to 3.6.3 we discuss how model P_M and speed menu curves can find additional usage when the time charter contract is already in execution. In Section 3.6.4, we illustrate how it may help in the debate about finding solutions to reduce environmental impacts.

3.6.1 Redelivery clause

This section shows that the redelivery clause in a general time charter contract influences speed decisions.

As introduced in Section 3.3.1, the redelivery clause of a charter party stipulates when the charterer's right to use the vessel terminates. Redelivery clauses exist in many variations, but most include a certain degree of flexibility. We will consider here as an example the approach whereby the ship is to be returned up to 45 days earlier or later, at the charterer's discretion, relative to an agreed termination date.

To examine the impact of such a redelivery clause, we can re-use the results of Section 3.5.2.2. Figure 3.12, for example, which is a close-up of Figure 3.7, can be re-interpreted as displaying the range of optimal strategies available to the charterer when the agreed termination date is one year ahead (365 days), plus or minus 45 days. Depending on the actual completion date, possible speed choices for ballast legs range from 17 kn down to 15 kn, and laden leg speeds from 15.8 kn to 12.6 kn. Which speeds from these 'speed menus' are optimal?

Recall that the NPV within this range of available horizons is strictly increasing (see Figure 3.8). Does this mean that charterers, when having agreed to this contract, will select the solution as to arrive at day 410?

Perhaps. When adopting this solution, the ship would have to start travelling at 17 kn in ballast and 15 kn when laden. However, any delays means that that the ship would not be able to make 8 journeys within 410 days. Some charterers may be willing to take this risk, and the associated hassle and penalties associated with overrunning the contract duration. Others will not. If travelling at these high speeds, 7 journeys are completed within about 360 days (plus the delay). But such solutions are out-competed by targeting initially 7 journeys only, and travelling slower throughout the time of the contract.

When targeting 7 journeys, note that the solution of P_∞ ($v_1 = 14.0$ kn, $v_2 = 16.3$ kn) would lead to a completion by day 375. But this still gives 35 days of buffer relative to day 410. Some charterers may think this buffer can be smaller, since as the NPV curve indicates, travelling slower can increase profits. Therefore, a charterer may perhaps conclude that a solution ($v_1 = 13.0$ kn, $v_2 = 15.0$ kn), which will complete the 7 journeys by day 399, with a buffer of 11 days, would be a decent compromise between profit maximisation and risk.

Some charterers will conduct a more elaborate analysis to examine the impact of future uncertainties, and not only with respect to possible delays. There exist no commonly adopted approach to how to predict and account for future uncertainties. It is thus in the

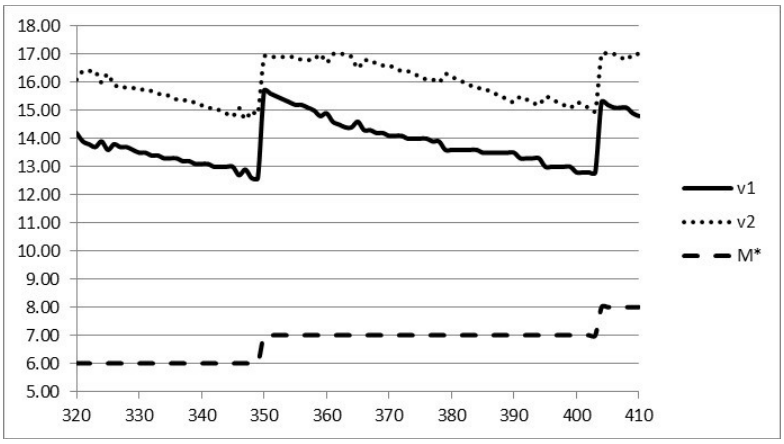


Figure 3.12: Close-up of Figure 3.7: $H = 365 \pm 45$ days

line of expectation that charterers may arrive at a different choice about the 'optimal' strategy to ship speed.

In conclusion, the redelivery clause of a charter contract gives rise to a range of available speeds ('speed menus'). Depending on the approach to risk taking, charterers may likely ending up making different choices about ship speeds for undertaking the (first few) journeys on the charter contract.

3.6.2 Risk attitude and utility theory

The previous section illustrates a ship charterer may reach different decisions when confronted with the same data, subject to their attitude to risk. In this section we further illustrate with a simple example that shows some of the underlying factors as well as linking risk attitude to utility theory.

It is in practice not always straightforward to maintain a given speed rather than to maintain a given engine power. The captain can only choose different ranges of engine power rather than selecting the exact travel speed. We can only predict ship speed with accuracy from engine power when in ideal or normal calm water conditions. If the ship maintains the engine power but it encounters bad weather, the ship will end up traveling slower. The captain may decide to turn up the engine so that the ship will make up some of the otherwise lost time, but this may not be the best strategy if the weather becomes too bad. The probability of encountering bad weather is also a function of the expected number of days the ship will (still) spend on sea, and thus of the initial speed choices and re-delivery strategy.

Assume we have a ship under a one-year charter contract with plus or minus 45 days redelivery clause. We can now consider the following three scenarios:

1. the weather is all good, with probability p_1, and we achieve the calculated profit;

2. the weather is bad, with probability p_2, and we speed up as to meet the original delivery date with a profit loss of 5 %;

3. the weather is very bad, with probability p_3, and the ship will arrive 10 days later. Here we assume either a 10 % loss of total profit without detention penalty, or a 12 % loss of profit otherwise.

The individual probabilities p_1, p_2 and p_3 are actually functions themselves of the total travel time and thus speeds, as well as the actual start date, the actual route considered, etc. Here we only assume that with longer travel time, the chance of having bad weather would be greater and vice versa. Also with faster speeds, it is harder to further speed up, and hence we will have a larger p_3 along with a smaller p_2.

Now let us consider the following three options listed in Table 3.10. Remembering Figure 3.12, Option 1 is to choose a rather high speed in order to finish 8 journeys with a high probability of violating the redelivery clause, while Option 2 are adopting a relatively slow speed with a target of 7 journeys to finish. With the assigned probabilities for each situation, here we have calculated the expected return of each portfolio. Note that, for Option 1, there is a detention penalty included for being 10 days late.

Table 3.10: Strategy outcome portfolios

Original Plan		p_1	p_2	p_3	**Expected Value**	**Risk Averse**	**Risk Loving**
Option 1	405 days ($v1$=15.2 knot, $v2$=17 knot)	0.2	0.3	0.5	9.70	31.13	94.37
	profit (million dollars)	10.5	10.0	9.2			
Option 2	390 days ($v1$=13.5 knot, $v2$=15.3 knot)	0.3	0.4	0.3	9.70	31.14	94.24
	profit (million dollars)	10.2	9.7	9.2			

According to the *utility theory* developed in game theory, the risk neutral decision maker will value Option 1 indifferently to Option 2.

If the decision maker is risk-averse, however, we could use the utility function $U(x) = 10\sqrt{x}$, for example, to represent his Von Neumann-Morgenstern preference. This particular utility function captures that the decision maker will give higher priority to safe investments and thus has his utility function as a concave function. Then his expected utility, for example, of Option 2 is $10 * (\sqrt{10.2} * 0.3 + \sqrt{9.7} * 0.4 + \sqrt{9.2} * 0.3) \approx 31.14$. Note that the expected utility value represents the ordering of preference instead of the magnitude of favourite. As a consequence, Option 2 becomes the more attractive option, though intuitively a longer contract will bring larger profit (see Figure 3.8). Similarly, risk-loving decision makers may prefer Option 1 due to his own utility expectations, e.g., $U(x) = x^2$.

Because of differences about how to predict weather patterns and differences between how captains adjust to e.g. weather conditions, charterers will face different portfolios even if they were using identical ships on identical routes with identical tasks. The example illustrates that even if they would use the same data, utility theory can help explain that differences in risk attitude may make them prefer different portfolios, and thus adopt different speeds.

3.6.3 Decision dynamics

We have concluded that, under otherwise identical conditions, charterers may make different initial choices about vessel speeds. This difference in approach also sets them each on a different course towards addressing future dynamic changes.

Consider two charterers operating under identical conditions, aiming to complete 7 journeys in a one-year contract: charterer CI adopts ($v_1 = 14.0$ kn, $v_2 = 16.3$ kn, $L_n = 53.4$ days), but charterer CII adopts ($v_1 = 13.0$ kn, $v_2 = 15.0$ kn, $L_n = 57$ days).

Assume that, near the time of completing the first journey, fuel prices rise by, for ease of exposition, a factor 1.5 (so we can use Figure 3.10). CI then has a maximum of $410 - 53.4 = 356$ days left on the contract, and CII $410 - 57 = 353$ days. Having completed the first journey as planned, CI adjusts its estimate of needed buffer down to 25 days, while CII to 8 days. They thus target completion by days 332 and 345, respectively. Figure 3.10 shows that, as a consequence of the fuel price rise, they each can now only complete another 5 journeys, and CI selects ($v_1 = 11.0$ kn, $v_2 = 12.6$ kn, $L_n = 66.4$ days), while CII chooses ($v_1 = 10.5$ kn, $v_2 = 12.1$ kn, $L_n = 69.0$ days). The fuel price change has had a dramatic impact on the chosen speeds of both charterers.

Now assume another possible future pathway in which they each have been able to complete 4 journeys according to plan, without any changes to fuel price. They then have, respectively, 196 and 182 days left on the contract. While CI and CII have initially taken larger buffers to complete 7 journeys, it is reasonable to assume that with only 3 more journeys to go, all previous 4 journeys having been without disruptions, CI and CII may become embolden to reduce the buffer more drastically to, say, 6 days and 2 days, respectively. This means CI now targets completion by day $196 - 6 = 190$, and CII by day $182 - 2 = 180$. We see on Figure 3.11 that CI selects ($v_1 = 11.6$ kn, $v_2 = 13.3$ kn, $L_n = 63.3$ days), while CII chooses ($v_1 = 12.3$ kn, $v_2 = 14.2$ kn, $L_n = 60$ days). Note that CI now steers the ship at a lower speed than CII, and from Figure 3.11 it is also clear that a fuel price rise would now have no impact on their speed choices.

Charterers engage in charter party contracts at different times. Using the above example, it may e.g. be the case that at the time of the fuel price rise, CI would have completed only 1 journey, while CII would have already completed 4 journeys. The fuel price rise would then seem to have a large impact on CI's speed choices, while a much smaller impact on CII. As the example above shows, the latter conclusion would actually be invalid.

In reality, charterers differ in the frequency and the approach used towards revising their journey planning, and may use different methods than what the above example has considered. Also, if the first journeys do not go according to plan, charterers will revise and recalculate their options and speed choices using their own methods for accounting for the future. This can all lead to different speed menus and choices.

To summarise, if we assume identical ship, journey, and charter contract characteristics, we can expect different charterers to arrive at different strategies to adjust ship speeds when subject to a change in external data. The first important reason for this is that charterers may face a different relevant future for the chartered ship. As seen above, this may be due to differences in the time remaining on the contract arising from either initial choices about speeds or different contract starting times. A second important reason is that charterers differ in how they approach decision making when faced with the risks related to these (different) relevant futures for the chartered ship.

3.6.4 Environmental constraints: speed limits

The modeling approach developed in this study can help in the assessment of the potential of ongoing and future developments in emission related regulatory and technological initiatives, many of which will affect the conditions under which ship speed decisions will be made. Recall, for example, the discussion around harbour time impact on optimal speeds in Sections 3.4.3 and 3.5.2.2, from which we conclude that strategies to induce ships to travel slower voluntary by reducing the time of port visits would not be very effective in the time charter context (and may even have the opposite effect, all other factors staying the same).

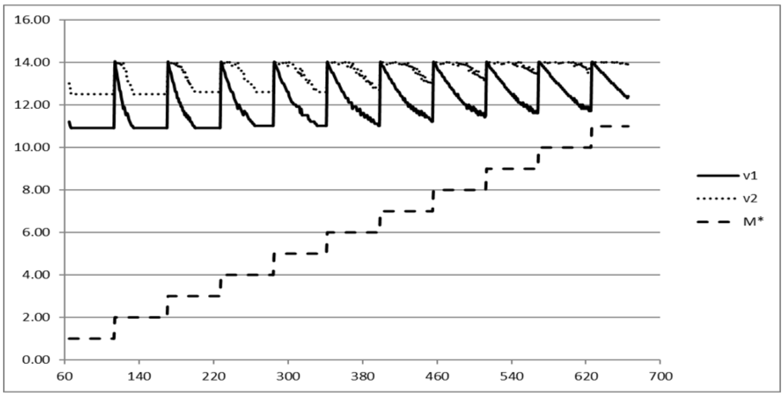


Figure 3.13: Optimal leg speeds (kn) and repetitions as functions of H (days) if maximum ship speed is limited to 14 knots

In another possible scenario, restrictions could be imposed on the maximum (average leg-) speed of a ship. Figure 3.13 plots the speed menu curve in the case that maximum speed v^+ is reduced from 17 kn to 14 kn. In comparison to Figure 3.7, the ship with this speed limitation can do, generally speaking, one less repetition when given the same number of days (H), or will need many more days for the same number of repetitions (M) as before. In any case, the charterer's Net Present Value (NPV) is much reduced (when rates remain the same), and the charter market would in total need more ships to perform the same amount of cargo movements per day.

Also notice in Figure 3.13 that on ballast legs in longer time charter contracts, as the ship already travels close to the maximum allowed speeds, that the ship may have greater difficulty making up for any lost time. When considering the potential profitability of a time charter contract under such speed limits, the ship charterer thus may feel it necessary to consider only solutions where v^+ is set even lower than the 14 kn. An alternative is for legislation to define the speed limit as an average over different legs, but this seems difficult to clearly define, check and enforce. For a thorough discussion of speed limits, see Psaraftis (2019b).

3.7 Conclusions

The bulk cargo and tanker shipping accounts roughly for 42% of the total merchant fleet, and 70% of the global fleet deadweight capacity. Ship speed is a variable used by ship operators as a means to maximise profitability. Lowering ship speeds reduces daily fuel consumption, but also daily revenue as less profit generating trips can be undertaken by the ship over time. Speed differences may arise from differences in freight rate, fuel prices, weather conditions, or the ship's actual operating performance. The importance of contracts in relation to ship speed decisions, while recognised in the literature as having a potential impact on speed choices, has not yet received explicit attention.

In this chapter we study the impact of the time charter contract. A methodology based on *cash-flow functions*, novel to the shipping industry, is introduced that captures the operating conditions when subject to a generic time charter contract. This leads to one generic time charter model P_M, and two special cases P_1 and P_∞.

These two special cases, P_1 and P_∞, arrive at two very different optimal speed profiles because they each adopt very different assumptions about the future use of the vessel. P_1 models a trip time charter, whereby the total time of the journey equals to duration of the contract. P_∞ assumes an infinite repetition of the considered roundtrip journey, and thus approximates conditions whereby the remaining time on the charter contract has to be of a long duration (typically 5+ years). By taking linear approximations of the objective function in our modelling framework, and deleting terms which are in many practical cases of lesser importance, we have derived that the influence of the main factors that drive speed decisions and profitability in both P_1 and P_∞ are very different.

The current literature on speed optimisation modelling has not yet paid explicit attention to contract type, and can be roughly divided into two strands PK and R: the main difference roughly being that PK maximises the criterion (USD/journey), while R uses the criterion (USD/day). Under mild assumptions, we prove that strand PK maps onto P_1, while R maps onto P_∞. We have illustrated how knowledge of this equivalence of PK and R models to P_1 and P_∞ may help in understanding the underlying implicit assumptions in some of the current models used in the literature. For example, it has been argued in the literature that R-like models have the benefit that they consider the impact of future revenue potential much better in comparison to PK-like models. Our analysis of P_1 and P_∞ confirms these conclusions from the literature, however because P_∞ assumes an infinite repetition of the same journey under the same economic conditions, literature that follow the R model, because it is equivalent to P_∞, may thus still not necessarily arrive at the best speed recommendations. Indeed, the ship may operate in still very different conditions in the relevant future. If the ship is to operate under a trip time charter, then P_1 or thus a PK model is arguably the better model, and the future revenue potential is only of minimal importance (see e.g. Table 3.6).

The decision environment changes drastically when considering the general model P_M, where the available time left on the charter contract is of a finite duration and within the range of several months up to a few years. We have demonstrated that it is in that case not recommended to use speeds derived from either P_1 or P_∞. The interplay between speed menu choice from the flexibility in redelivery conditions, the influence from the remaining time left on the shape of the saw-tooth functions, and the charterers' individual approaches to their strategy to risk, are shown to introduce a wide range of plausible speeds chosen within the overall context of maximising the NPV of the (future) profits earned from the chartered vessel. The impact of changes to fuel prices, charter party rates, freight rates, and harbour-related costs and residence times, becomes far less predictable than what one arrives at from models akin P_1 and P_∞.

In general, the adoption of the NPV framework helps to make explicit how the optimal current speed of a ship highly depends on what the decision maker considers to be the relevant future for the ship. We also demonstrate that decision makers, even when presented with the same set of data, may reach different conclusions about optimal speeds because they think differently about the associated risks.

Developments in (emission related) regulatory initiatives will affect the conditions under which ship speed decisions by operators will be made. While an analysis of environmental impact from speed optimisation, subject to various regulatory conditions, is not within the scope of this article, the models and insights developed may provide a foundation for further research in this area.

Chapter 4

Optimal Economic Ship Speeds, the Chain Effect, and Future Profit Potential

Abstract

In some scenarios of shipping commonly faced in practice the (chartered) ship does not travel on a time charter contract but a number of profitable opportunities may arise. This occurs in for example tramp shipping and consecutive voyage charters. It is then important to determine the optimal speed choices for the ship on each leg of the journey that would execute these opportunities. In the case where repetition occurs, this chapter also explores the impact of journey repetition on optimal speed decisions.

We study properties of optimal operational speeds obtained in the particular case that the ship will execute a series of (identical) multiple-leg journeys. The chapter demonstrates that the NPV framework presented exhibits two novel elements in comparison to existing speed optimisation modeling approaches in the literature: (a) When executing a series of identical journeys, optimal ship speeds from one execution of the journey to the next execution are shown to change. We refer to this as the *chain effect*. (b) The ship's optimal speed is in general highly dependent on the decision maker's view about the ship's *future profit potential*.

We present two efficient algorithms to solve the models. The methodology is applied to case studies based on the literature and the results are compared with several models from the classic modelling framework.

keywrods: maritime speed optimisation, Chain Effect, cash-flow framework, future profit potential.

4.1 Introduction

This chapter investigates the differences between two methodological approaches for the construction of ship speed optimisation models in maritime transportation: a novel approach based on the Net Present Value criterion versus (three flavours of) the 'conventional' approach that is not based on NPV principles. In the NPV approach, the objective function is the Laplace transform of the cash-flow function associated with the ship's future activities. This generic approach to the formulation of economic problems has been proposed in Grubbström (1967) and applied to the field of production and inventory system management in Grubbström (1980) to show how it can be used to gain insight into the applicability or limitations of classic model formulations not based on NPV principles. This kind of analysis is formalised in Beullens and Janssens (2014) and called NPV Equivalence Analysis (NPVEA). To our knowledge, this NPV approach has not yet been thoroughly applied, analysed, and compared with conventional model formulations of the ship speed optimisation problem[1].

In conventional approaches to ship speed optimisation modelling, profit or cost functions are typically expressed in one of two ways. A first 'conventional' speed optimisation method considers the criterion *dollars per journey* (or *per nautical mile*). This approach is found in e.g. Corbett et al. (2009); Fagerholt et al. (2015); Norstad et al. (2011); Psaraftis and Kontovas (2014); Wen et al. (2017). Importantly, Psaraftis and Kontovas (2014) formulate the *decomposition principle*: they prove that an m - leg journey in their model can be *decomposed* into m one-leg sub-problems, and each solved independently for the optimal ship speed on that leg[2].

A second popular 'conventional' speed optimisation method is based on the criterion *dollars per unit of time*, as perhaps first used in the *income-generating-leg model* in Ronen (1982), and adopted in many studies for representing the basic scenario that a (tramp) ship transports cargo for a reward, see e.g. Devanney (2010); Fagerholt and Psaraftis (2015); C.-Y. Lee and Song (2017); Magirou et al. (2015); Ronen (2011). It is a curious fact that the decomposition principle was in fact explicitly proposed in Ronen (1982) as being 'beneficial', although not formally proven[3]. Models following this method thus divide the profit earned on a leg by total travel time of the leg to arrive at the objective function.

[1] Further links to related literature on NPV, NPVEA, and other disciplines of research are discussed in Section 4.6. We also postpone the discussion of the limited literature of NPV in speed optimisation to Section 4.6.2, after we have been able to present our main results, as this helps us to better clarify our contribution.

[2] The optimality of decomposition holds in that conventional model. In a NPV model, this decomposition approach is no longer optimal, see Theorem 4.1, and Section 4.4.3.

[3] Ronen (1982): "Although it is common in the shipping industry to calculate profitability on a roundtrip basis, ships are often not operating in such a manner (...)... Moreover, the most profitable speed may be different for an empty and a full leg of the same roundtrip. Thus, it is beneficial to analyze the speed for each voyage leg separately and decouple the various legs of the voyage."

Ronen (1982) also presented a *positioning (empty) leg model*, minimising the total cost on the leg. We can consider this to be a third 'conventional' model. This model, however, has not received much actual application in the subsequent literature. The model crucially relies on a parameter C_a, which Ronen called the *Daily Alternative Value*, but for which he provided no further information about how to determine it. We speculate that this may have contributed to the model not being so popular.

It is a fact that these speed optimisation models produce different optimal speed values. This difference has not really been explained in the literature. Only Psaraftis (2017) points out the difference between the first two conventional approaches that we have discussed above. He argued in favour of Ronen's approach, as only here optimal speed is also a function of the revenue generated on the leg considered, which he believes captures a fundamental facet of shipping industry behaviour. Indeed, the importance of freight rates in guiding speed choice is also mentioned in e.g. Stopford (2009) and Devanney (2010).

In this chapter, we support the idea that if ships have speed flexibility, that optimal economic speeds should be based on the consideration of relevant future revenues and cost[4]. For instance, it seems natural for a ship to travel at higher average speed on a leg if the company knows this may gain it access to a future profitable job (e.g., due to the competition), while other companies without such information or opportunity may instead wish to safe fuel now by travelling slower. The class $\mathcal{P}(n, m, G_o)$ of NPV models presented incorporates these kinds of trade-offs between present and future. Models in this class are not solved on an individual leg-basis (i.e. by decomposition), but need to be solved by determining optimal speeds for all legs simultaneously across all journey(s) considered. Efficient algorithms are developed and run in (pseudo-) polynomial time.

The reason we introduced the repetition of a roundtrip journey is so that we can identify the properties of value to understand the importance of accounting for the future when planning the vessel's current activities. The repeated journey model, where then n grows from 1 to ∞, forms part of the mathematical machinery to help identify the chain effect. The chain effect show that the more repetitions considered, the faster the ship should travel on the first few trips. Secondly, the discovery of the chain effect has led us to introduce the FPP. This FPP in particular is of great significance in the context of the tramp shipping example. We explain this further in the next two paragraphs.

For tramp/bulk shipping, indeed the decision makers may make 'myopic' decisions due to the dynamic nature of market, as deciding the optimal speeds for a large number of repetitions would seem to be very hard. Actually, we believe this shows the practical value of our model, as the model parameter n and m can be freely adjusted. The decision makers can instead use a small journey model of one or a few legs only, say, and use

[4]As this accounts best for decision makers' objective to maximise shareholder wealth, one of the principles of corporate finance, see also Brealey and Myers (2003).

this to analyse different opportunities, but with the fair estimation of the FPP, they can be less myopic in the comparison. Such 'myopic' decisions can then be updated and adjusted frequently and dynamically. This type of decision making is important in practice, e.g., for voyage charter (HellenicShippingNews, 2015). It is noted that the models and algorithms developed in this chapter fully support this kind of decisions with rolling horizons. In particular, we present several examples in Section 4.5 to illustrate the value of considering the FPP in such 'myopic' (e.g. single leg, dual leg, etc) models. Examples 5.4 and 5.5 in particular illustrate the value of NPV modelling in a tramp shipping situation, and the value of a fair estimate of the FPP.

We provide here another context in which the modeling approach is applicable. In practice, the ship may sometimes only find the next job while executing the current job. But this is still captured in our modeling framework by say one ballast leg (to the next potential job), and then the laden leg (for bringing the cargo from pickup to destination port of that potential job.) What our model shows is that during the investigation of the profitability of this potential new job, one should also place a value on the future potential when this next job would be finished. This is captured by the FPP in our model. It is exactly because of dynamic conditions that it is impossible to consider many legs in advance explicitly and that a concept like the FPP is attractive in this environment. The FPP for one potential next job could be different from another potential next job simply because then the ship would be in another port, for example, or at another time in the future (think about e.g. seasonality of demand). Our analysis indicates clearly that not accounting for such differences, and only looking at the actual revenue of the potential job only, is too myopic. Furthermore, the NPV approach offers a robust comparison, while the concept of daily earnings, as in the TCE Earnings used in industry, could be dangerous as a measure to determine which job is most interesting. Indeed this is illustrated in Example 5.5.

The prediction of future is of course difficult due to lots of uncertainties. People in this industry, however, still calculate and use expected (or approximated) values, such as Time Charter Equivalence (TCE), that reflect the general opinions about future markets. We show in Section 4.5 how to approximate the G_0 value using such industry index, with the flexibility of decision makers' attitude towards future. What is thus important is to use the G_0 in a NPV model, rather than using the TCE values directly when comparing different possible future jobs. The NPV offers the most robust decision criterion for directing the ship to new jobs as well as for possibly adjusting speed. This view is consistent with what we have learned from our partner in the shipping industry.

If we do not restrict ourselves to spot market (tramp) shipping, but consider time charter contract conditions, or under COA, the ship may indeed be used in a repeated journey situation. Then the longer journey models that our framework allows for can also be considered and solved. If the (economic) conditions however also change in that context, then we would suggest that it may be better to use a shorter journey model with an appropriate FPP. Then the ship's travels may be readjusted in a rolling planning horizon approach. This is consistent with your comments, namely that in a dynamic environment, one must make myopic decision over only the next one or two legs. What we do add to the literature is that we propose it is maybe better to use a NPV model and use an FPP in the model. As we show in Section 4.4.4, for example, not appropriately accounting for the FPP can lead to very different NPV values and optimal speeds. For example, a job could offer good revenues but bring the ship in a bad location, so that would reduce the FPP of the current job considered. Of course, if one has more information of possible chains of multiple jobs, one could extend the NPV model to explicitly consider multiple legs, but still consider adding the FPP value each time in the last port. We present a class of models $\mathcal{P}(n, m, G_0)$ so each particular model should be adapted to the amount of information that the decision has available.

4.2 Assumptions, notation and problem description

In this research, we study at which speeds an oceangoing ship should *ideally* travel on each of a series of legs of a journey as to maximise the NPV of the ship. The decision maker is the ship operator (e.g. owner, time charterer). Our primary purpose is to develop greater understanding of the degree by which the future is important in determining the optimal current usage of the vessel, and to examine to what degree this accounted for in the conventional methods introduced in Section 4.1. For this reason, we assume that the ship is not constrained in its choice of speeds by time windows imposed by external influences (e.g. ports, shippers). In this way, we can study optimal speeds strategies in a context that sketches the ideal position of the operator.

This is less removed from practise then perhaps initially recognised, in particular in the bulk cargo and tanker industries. Here, time windows are not an exogenous input but rather agreed between shipper and operator before signing the contract, see also Psaraftis (2017). It should be of interest to the operator, prior to engaging in these negotiations, to know how speed choices affect the NPV of the ship's activities[5].

Because of the direct applicability to the sector, we adopted modeling assumptions in Sections 4.2.1 to 4.2.2, and in most numerical examples later, based on the operations of a Suezmax oil tanker. The theoretical results of Section 4.3 and 4.4, however, do not crucially rely on the particular details of the assumptions related to how the components

[5] In liner services, speed optimisation is also important, but additional constraints, see e.g. Christiansen et al. (2019), may make it difficult to implement the identified optimal speeds from an unconstrained model as presented in this chapter. Operators do implement the practice of slow steaming to save on fuel. Because the ships work in a group as to offer an agreed visit frequency, more ships may then be needed. For an insightful model explaining this trade-off, see Ronen (2011). We believe that the unconstrained model presented can still offer understanding of the fundamental economic forces when a containership would execute a series of (repeated) journeys, and may lead to refinements in the network optimisation of such services in future research.

of the revenue and cost cash-flows and their timing are exactly dependent on the journey data. The main results are of much more general applicability to any ship not subjected to time windows and that transports cargo for a revenue, where we only need mild conditions of a concave profit structure of a leg.

4.2.1 Model structure: example

In this chapter, we consider a single vessel, with a known sequence of port visits, and the decision maker is not restricted by e.g. time windows in ports or contractual limitations about the ship speed except for a known upper and lower bound on the ship's speed. This setting is similar to Psaraftis and Kontovas (2014).

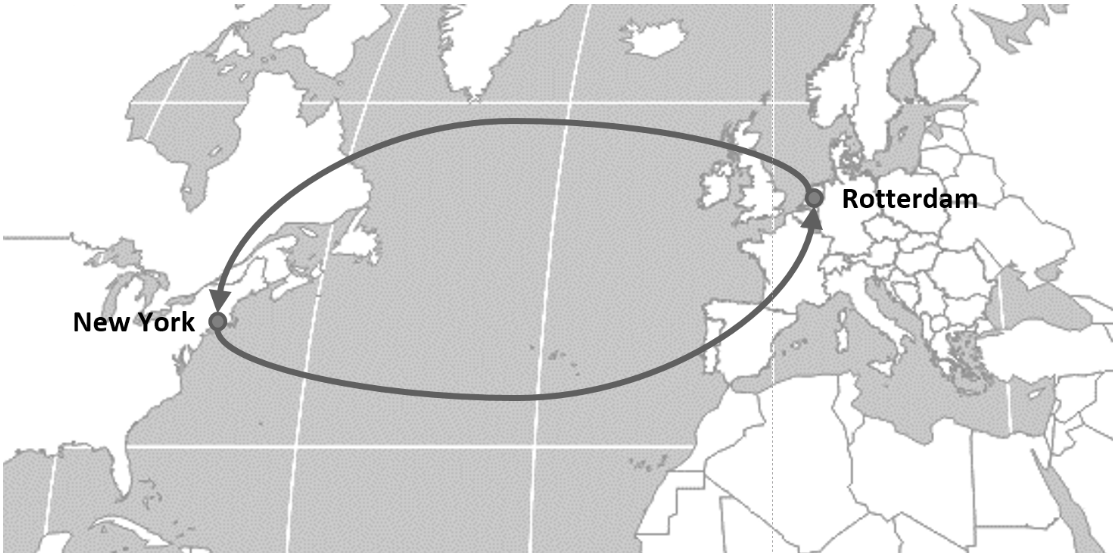


Figure 4.1: An example of roundtrip

The single ship will execute a *journey* n times, where n is allowed to take any value 1, 2, 3, ... We adopt the journey sequence $\{n, n-1, ..., 2, 1\}$, where n is the first and 1 is the last time that the journey is executed.

An often encountered situation is the *round-trip journey* between two ports. This is illustrated in Figure 4.1: the first *leg* of the journey goes from Rotterdam to New York, the second leg from New York back to Rotterdam. In general, a journey may consist of visits to a series of $m+1$ $(1 < m < \infty)$ ports in a certain order, covering m legs, where the last port not necessarily needs to be the point of origin. If the ship carries cargo on a leg, it is *laden*, otherwise it is a *repositioning* or *ballast* leg. Cases $n > 1$ cannot be realistically implemented unless the journey itself is a round-trip.

Journey repetitions of smaller roundtrips do occur as part of a time charter, a consecutive voyage charter, a contract of affreightment, and even liner shipping services, see also Stopford (2009). In the transport of oil or natural gas, the repeated ballast-laden journey is often encountered.

The organisation of repeated journeys may benefit in practical terms from adopting consistent speeds that do not change from one repetition to the next. However, how much would the company potentially gain from optimal speed choices, which may change over time? This is also of theoretical interest, since it can inform us about the accuracy of the decomposition approach used in Psaraftis and Kontovas (2014) and Ronen (1982). Allowing for journey repetitions in our models thus gives us the means to examine these questions.

It is clear that the model is very general by changing the values of n and m. In contrast to the example in Figure 4.1, we could also look at models where $n = 1$ and m is large. This is a situation where the journey is long, every leg may be unique, and there is no repetition, as in certain tramp shipping situations.

Returning to the example in Figure 4.1, let V^i_{AB} denote the average speed adopted by the ship on the leg from A (Rotterdam) to B (New York) on the i^{th} journey in the sequence $\{n, n-1, ..., i, ..., 2, 1\}$, and V^i_{BA} be the speed on the return leg. The general question addressed in this chapter on how to decide on the optimal ship speeds now becomes determining optimal values for $\{(V^i_{AB}, V^i_{BA})|i = 1, ..., n\}$ that maximises the NPV of all relevant cash-flows.

4.2.2 $\mathcal{P}(n, m, G_o)$: Problem description summary

We summarise the problem formulation as follows. A ship is to execute a journey n times. The journey consists of m legs. The problem is to find the optimal speed $v^i_j \in (v_{min}, v_{max})$ for each leg j of each repetition i that will maximise the Net Present Value (NPV) of all future operations of the ship. We call the class of problems considered henceforth $\mathcal{P}(n, m, G_o)$. The main assumptions are summarised below:

- For each day the ship is used during these journeys, including for the time spend in ports, we pay a fixed cost f^{TCH};
- At the start of a leg j, just prior to any loading activity at a port, the ship incurs a loading cost $C^l_j(v^i_j)$,
- The loading cost includes the cost of fuel intake to cover main and auxiliary fuel for the leg;
- The loading costs and time in the port increase with the amount of cargo to be loaded;
- After the execution of the sea voyage, the ship spends a given time prior to the start of unloading activities;
- At the end of a leg j, just after unloading is finished, the ship receives a revenue R_j and an unloading cost C^u_j;

- The unloading cost and time in the port increase with the amount of cargo to be unloaded;
- The ship must carry a minimum of 30% of its design DWT during a sea voyage, or make up the shortfall with ballast water;
- The fuel consumption per day at sea for the main bunker fuel is given by (**??**); w_j is the sum of cargo, ballast water, and main fuel at the port of loading.
- The parameter G_o is called the *Future Profit Potential*; its function is introduced in Section 4.3.2.

We will also look more specifically at some noteworthy special cases, as the model then reduces to (an NPV-formulation) of a specific type of speed optimisation model from the literature (see also Section 4.5). Examples of special sub-classes include $\mathcal{P}(1, 1, 0)$, $\mathcal{P}(1, m, 0)$, and $\mathcal{P}(\infty, m, -)$.

4.3 Model formulation and solution algorithms

4.3.1 $\mathcal{P}(n, m, 0)$: Dynamic Programming Formulation

We deploy dynamic programming to solve the model - this method can take into account dynamically evolving economic data (e.g. freight rates, fuel prices, ...). In this chapter, however, we examine the speed optimisation problem with port-specific yet constant freight rates and fuel prices, and study how the sequence of optimal speeds on each of the legs might evolve with growing values of journey repetitions n. This is also illustrated in Figure 4.2: it appears in this case that the ship will need to gradually slow down when values of i reach single digits and when the value of the FPP is zero. This does not need to be so, however, and it depends on a trade-off with the future profit potential (FPP).

The speed decision for leg j ($j = 1, 2, \ldots, m$) within the roundtrip journey i ($i = n, n-1, \ldots, 1$) will result in a *leg execution time* T_j^i, or *leg time* for short. Let $h_j(T_j^i)$ denote the profitability associated with this leg, discounted to the start time of execution of this leg:

$$h_j(T_j^i) = (R_j - C_j^u)e^{-\alpha T_j^i} - C_j^l - \int_0^{T_j^i} f^{TCH} e^{-\alpha t} dt. \tag{4.1}$$

The leg time T_j^i is a function of the chosen speed v_j^i as in (**??**), and as the ship's speed must remain between a given minimum and maximum value, say v_{min} and v_{max}, it is subject to the condition:

$$\frac{S_j}{365(24)v_{max}} \leq (T_j^i - T_j^l - T_j^w - T_j^u) \leq \frac{S_j}{365(24)v_{min}}. \tag{4.2}$$

Let $G_j^i(.)$ denote the accumulated profitability, associated with the sum of journeys $i-1, ..., 1$ up to and including of legs $m, m-1, ..., j$ of journey i, discounted to the start time of the leg j of journey i. This property of a NPV discounted to a future time is also defined as the *Goodwill* in Preinreich (1940). For the special case of $i = 1$ and $j = m$, i.e. $\mathcal{P}(1, m, 0)$, let:

$$G_m^1(T_m^1) = h_m(T_m^1), \tag{4.3}$$

and thus:

$$G_{m-1}^1(T_{m-1}^1, T_m^1) = h_{m-1}(T_{m-1}^1) + h_m(T_m^1)e^{-\alpha T_{m-1}^1}. \tag{4.4}$$

Let T_m^{1*} be the leg time value that maximises G_m^1, its maximum value denoted as $G_m^{1*}(T_m^{1*})$, then maximising (4.4) is equivalent to finding the value of T_{m-1}^1 that maximises (see also Section 4.4):

$$G_{m-1}^1(T_{m-1}^1) = h_{m-1}(T_{m-1}^1) + G_m^{1*}(T_m^{1*})e^{-\alpha T_{m-1}^1}. \tag{4.5}$$

Similarly, for all other (i, j) combinations the following recursion applies:

$$G_j^i(T_j^i) = h_j(T_j^i) + G_{j+1}^i(T_{j+1}^i)e^{-\alpha T_j^i}. \tag{4.6}$$

The value for the leg time T_j^i that maximises G_j^i, denoted as $G_j^{i*}(T_j^{i*})$, is not dependent on T_l^k $(k > i)$ nor on T_l^i $(l < j)$. Therefore, the *principle of optimality* holds, and we can find optimal speeds for each leg and journey with dynamic programming (Bellman, 1954) using Algorithm 3 (take $G_0 = 0$, see also Section 4.3.2). In the step of finding a new G value, we solve the following problem:

$$T_j^{i*} = arg \quad max \quad \left(h_j(T_j^i) + G_{j+1}^{i*}(T_{j+1}^{i*})e^{-\alpha T_j^i}\right). \tag{4.7}$$

Algorithm 3: Dynamic Programming to solve $\mathcal{P}(n, m, G_o)$.

Result: $\{T_1^{1*}, \ldots, T_m^{n*}\} = arg \quad max \quad G$

Initialisation: $i \leftarrow 1, \quad j \leftarrow m, \quad G = G_0$;

while $i \leq n$ **do**

$\quad G \leftarrow \max_{T_j^i}\left(h_j(T_j^i) + Ge^{-\alpha T_j^i}\right)$;

$\quad$ *Store optimal value* T_j^{i*};

$\quad$ **if** $j > 1$ **then**

$\quad\quad j \leftarrow j - 1$;

$\quad$ **else**

$\quad\quad j \leftarrow m$;

$\quad\quad i \leftarrow i + 1$;

$\quad$ **end**

end

This T_j^{i*} maximises G_j^i as given by (4.6). Note that G_{j+1}^{i*} at this time is already a known constant, and represents the optimal value of the accumulated profitability of all future journeys up to an including the leg following the currently considered leg. This optimisation problem may be implemented in linear time as a search over a finite number η of leg time values within the boundaries given by (4.2) to any desired degree of approximation[6]. Algorithm 3 then runs with a polynomial time complexity $O(\eta n m)$.

To aid the further analysis of the chain effect in Section 4.4, we consider also the following reformulation at the level of a journey. The profitability associated with the journey $h(T_1^i, \ldots, T_m^i)$, discounted to the start of execution of its first leg, is given by:

$$h(T_1^i, \ldots, T_m^i) = h_1(T_1^i) + h_2(T_2^i)e^{-\alpha T_1^i} + \cdots + h_m(T_m^i)e^{-\alpha(T_1^i + \cdots + T_{m-1}^i)}. \tag{4.8}$$

We will later use the shorthand notation $h(\Gamma_i)$ where $\Gamma_i := \{T_1^i, \ldots, T_m^i\}$.

Let $G_i(.)$ denote the accumulated profitability across journeys $i, i-1, \ldots, 1$ discounted to the start time of the journey with index i. The recursive formula can now be stated as:

$$G_i(\Gamma_i) = h(\Gamma_i) + G_{i-1}^*(T_1^{i-1*}, \ldots, T_m^{i-1*})e^{-\alpha L_m^i}, \tag{4.9}$$

where $G_{i-1}^*(T_1^{i-1*}, \ldots, T_m^{i-1*})$ indicates the optimal goodwill value obtained across journeys $i-1, i-2, \ldots, 1$.

4.3.2 Model Extension $\mathcal{P}(n, m, G_o)$: The Future Profit Potential G_0

In the last section, we have arrived at a quite generally applicable approach to represent the speed optimisation problem, with a straightforward algorithm to solve it, loosely speaking, to any desired level of accuracy. The NPV method applied, see (4.7), demonstrates that the optimal leg speed does not depend on the speeds and profitability achieved on legs that preceded this leg, but does depend on the goodwill to be realised on future legs and journeys. Recall that the goodwill G_{j+1}^{i*} in (4.7) is the NPV of all cash-flows associated with the ship's execution of future legs and journeys, discounted to the time that these activities start.

Consider the following extension of (4.3):

$$G_m^1(T_m^1) = h_m(T_m^1) + G_0 e^{-\alpha T_m^1}. \tag{4.10}$$

where G_o is defined as follows:

[6] The discretisation of T with any grid $\epsilon = 1/\eta$ would imply an upper bound ϵ on the error between the true optimum and the optimal solution calculated. For example, with $\eta = 330$, results are within 1 hour accuracy on arrival time, and correspond to specifying the ship's optimal speed with a range ± 0.02 knots. (On a journey of $8,000$ nm, $v^- = 10$ kn and $v^+ = 17$ kn).

Definition: Future Profit Potential (FPP).

The Future Profit Potential is the goodwill G_o of the ship at termination time. It is the NPV of all future cash-flows the ship is deemed to be able to generate, discounted to the moment that the ship completes its n journeys that are currently considered in the optimisation problem.

For practical purposes and in examples, we will often describe the FPP using the related αG_o or Annuity Stream (AS), the value of which can be expressed in USD/day.

Application of (6.17) will now lead to T_j^{i*} in (4.7) to to be influenced by G_o through G_{j+1}^{i*}. When $G_o = 0$, we revert back to the formulation developed in Section 4.3.1. This approach would only be accurate, therefore, if the future profit potential is considered to be zero. This may be the case when the decision maker has chartered the ship and will return it to the owner after completion of the n journeys. Solving this model with $G_o = 0$ implies that the charterer does not need to be concerned about what happens with the ship afterwards. There is no constraint or consideration placed on the finishing time of the last journey. Thus, the decision maker is not concerned about the exact time the ship is returned to the owner[7] other than that it will be such as to maximise the charterer's NPV across the n journeys considered.

The situation will in general be different if the decision maker is the ship owner operating the ship in the spot market. The optimal speeds for a series of planned legs currently considered will crucially depend on the profit potential that the owner considers the ship will have at the time and at the port of finishing this set of legs. This is exactly the function that G_o can fulfill. For any value $G_o \neq 0$, Algorithm 3 obviously still applies, but drastically different optimal speeds across the n journeys considered may be arrived at. We will further demonstrate in Section 4.4.4 the impact of G_o on optimal speeds, as well as discuss how the decision maker may arrive at estimates for G_o.

4.3.3 Specific case $\mathcal{P}(\infty, m, -)$: Value Iteration

We consider the problem class of a single journey $\mathcal{P}(1, m, G_o)$. We can then reformulate the journey representation (4.9) to consider the Future Profit Potential (FPP) in a similar fashion as in (6.17) to:

$$G_1(\Gamma_1) = h(\Gamma_1) + G_o e^{-\alpha L_m^1}. \tag{4.11}$$

[7] Obviously, the ship owner may think differently about this. In reality the owner may require an agreement about the return time, with perhaps penalties for late delivery etc. We consider here the situation of ship charterer having the power to first decide on what would be the best return time as to maximise its own profits, and use this result in the further negotations about redelivery time.

For any given feasible choice of the leg speeds Γ_1 producing the journey profitability $h(\Gamma_1)$, the corresponding AS value (in USD/day) earned over the journey up to the termination time is given by:

$$\mathrm{AS}(\Gamma_1) = \frac{\alpha h(\Gamma_1)}{1 - e^{-\alpha L^1_m}}. \tag{4.12}$$

Imagine the special case that the discounted FPP, i.e. αG_o, by either chance or design, is known to be equal to (4.12) for the leg speeds that would optimise (4.11). We thus consider the specific problem class $\mathcal{P}(1, m, \mathrm{AS}(\Gamma_1)/\alpha)$. For reasons that will become clear in Sections 4 and 5, we notate this special case as $\mathcal{P}(\infty, m, -)$.

Algorithm 4 describes how one can find the optimal leg speeds for this special case by repeatedly calling Algorithm 3 to solve a single journey problem, each time with an improved estimate for G_o. We found Algorithm 4 in our experiments to converge very fast, often requiring less than 4 iterations to achieve an accuracy of one USD/day. The proof of the optimality of the algorithm, and the various interpretations we could give this special case, will be given in Sections 4.4.2 and 4.5, respectively.

Algorithm 4: Value Iteration to solve $\mathcal{P}(\infty, m, -)$

Result: AS^*, Γ_1^*

Initialisation: $G_o \leftarrow 0$, $AS \leftarrow 0$, $Convergence \leftarrow FALSE$;

while $Convergence = FALSE$ **do**

  Call Algorithm 1 to solve: $G \leftarrow \max_{\Gamma_1}\left(h(\Gamma_1) + G_o e^{-\alpha L^1_m}\right)$;

  Store optimal values $h(\Gamma_1^*), {L^1_m}^*$;

  if $\left|\frac{\alpha h(\Gamma_1^*)}{1-e^{-\alpha {L^1_m}^*}} - \alpha G_o\right| \leq \epsilon$ **then**

    $AS^* \leftarrow \frac{\alpha h(\Gamma_1^*)}{1-e^{-\alpha {L^1_m}^*}}$;

    $Convergence \leftarrow TRUE$;

  else

    $G_o \leftarrow \frac{h(\Gamma_1^*)}{1-e^{-\alpha {L^1_m}^*}}$;

  end

end

4.4 Chain effect and Future Profit Potential in shipping: Analysis

In this section we study properties of $\mathcal{P}(n, m, G_o)$. We start in Section 4.4.1 with identifying the chain effect in the context of maritime speed optimisation by studying problems of the type $\mathcal{P}(n, m, 0)$. A possible solution is illustrated in Figure 4.2: in this case the ship is depicted to gradually slow down when values of i reach single digits.

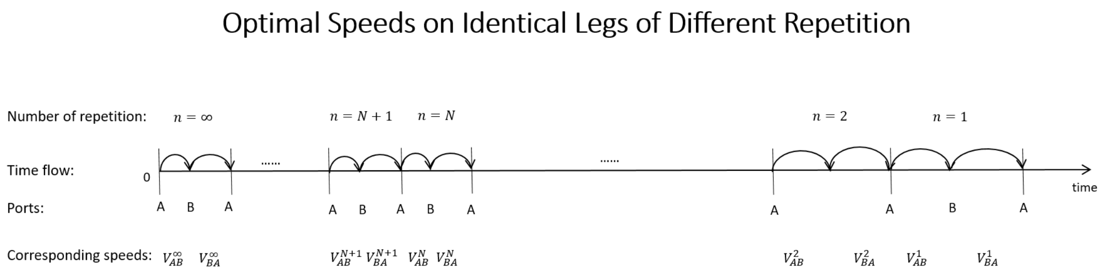


Figure 4.2: Optimal speeds on identical journeys

In Section 4.4.2 we consider taking the limit $n \to \infty$, which gives the infinite horizon case $\mathcal{P}(\infty, m, -)$ under steady state conditions.

In the final three sections, we provide more insight into the magnitude of the chain effect, the role of the FPP (G_o), and how it can be estimated.

4.4.1 Finite time horizon: $\mathcal{P}(n, m, 0)$ and $n < \infty$

In its original setting, the chain effect identifies a relationship between the optimal life-time of successive usages of a single machine (Preinreich, 1940). We study the relationship between the optimal lengths of successive repetitions of a journey by a ship, established through decisions on leg speeds. Where the single machine replacement problem has a fixed original cost (B) in the beginning of each replacement, we instead face a daily charter cost ($\int_0^{t_i} f^{TCH} e^{-\alpha t} dt$).

It can be shown that if daily fixed costs f^{TCH} or future revenues R_k ($k \geq j$) in a journey are not excessively large, then the profit structure of a single leg h_j given by (4.1) and the profit structure of a m-leg journey h given by (4.8) are concave, and otherwise the optimal solution is always travelling as fast as possible (see 8.2.3).

Before we present a general theorem, we provide an intuitive explanation why the decomposition principle does not apply in the NPV framework, and we instead observe a chain effect. We use for this Figure 4.3, which depicts the general case of a concave profit structure of a leg, and the situation that the optimal execution time T_j^{i*} of a leg (in the decomposition framework) is found within the ship's physical limits from setting the derivative $h_j' = \frac{\partial h_j(T_j^i)}{\partial T_j^i}$ to zero.

For $n = 2$ we can write the aggregated G_2 function:

$$G_2 = h(\Gamma_2) + G_1^* e^{-\alpha L_m^2} = h(\Gamma_2) + h(\Gamma_1^*) e^{-\alpha L_m^2}. \tag{4.13}$$

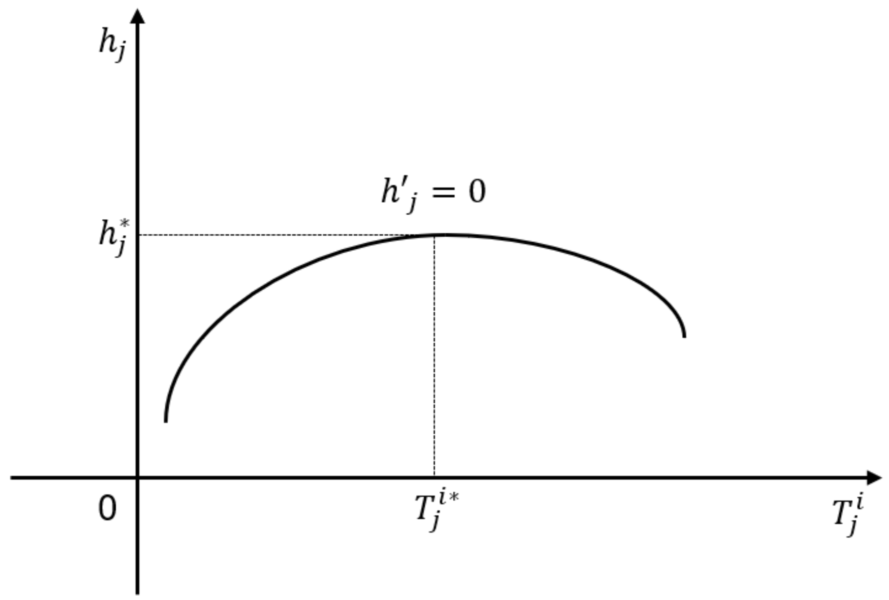


Figure 4.3: a general example of h_j

By taking the derivative with respect to leg $T_j^2 \in \Gamma_2$, we have:

$$\frac{\partial G_2}{\partial T_j^2} = \frac{\partial h(\Gamma_2)}{\partial T_j^2} - \alpha h(\Gamma_1^*) e^{-\alpha L_m^2}. \tag{4.14}$$

In the traditional models based on the *USD per journey* criterion, the decomposition principle implies that identical speeds are found for identical legs on identical roundtrips being repeated (see Section **??**). This would imply here that $\Gamma_1^* = \Gamma_2^*$ and $\frac{\partial h(\Gamma_2)}{\partial T_j^2} = \frac{\partial h(\Gamma_1)}{\partial T_j^1} = 0$. This is in contradiction to (4.14), unless the opportunity cost of capital α is zero. With $\alpha > 0$, $h'(\Gamma_2)$ is pushed away from zero to either positive or negative values.

Instead of the decomposition principle, we observe a chain effect. This is intuitively understood as follows. Consider the case $m = 1$ for a journey as in Figure 4.1, and assume $A - B$ and $B - A$ are identical laden legs. This can then be repeated for $n > 1$. We deduce the following phenomenon: (1) if $h(.) > 0$, then $h'(\Gamma_2) > h'(\Gamma_1) = 0$ (see (4.14)); and thus the optimal solutions for earlier legs would be to travel faster (see Figure 4.3); (2) if $h(.) < 0$, then $h'(\Gamma_2) < h'(\Gamma_1) = 0$, and thus the ship would travel slower on earlier legs. The case $h > 0$ typically corresponds to a laden leg or profitable journey, while the case $h < 0$ may represent a ballast leg or repositioning journey.

In general for $\mathcal{P}(n, m, 0)$, we formulate the following theorem (proof in 8.2.4).

Theorem 4.1. *(The Chain Effect) For a finite number of identical m-leg journeys indexed* $(n, n-1, ..., 2, 1)$*, when the opportunity cost of capital* $\alpha > 0$*, the following is an optimal strategy:*

- *If the journey is profitable ($h(.) > 0$), the ship travels faster on earlier journeys:* $T_j^{n*} < T_j^{n-1*} < T_j^{n-2*} \cdots < T_j^{1*}$, $j = 1, ..., m$;
- *If the journey is unprofitable ($h(.) < 0$), the ship travels slower on earlier journeys:* $T_j^{n*} > T_j^{n-1*} > T_j^{n-2*} \cdots > T_j^{1*}$, $j = 1, ..., m$.

Recall that we consider here the situation that optimal speeds can be found within the physical boundaries (v_{min}, v_{max}). In practice, it is of course possible to hit these boundaries. For example, if profits are very high, the optimal speed may reach v_{max} for certain j and repetitions higher than some number k: $T_j^{n*} = \ldots T_j^{k+1*} = T_j^{k*} < T_j^{k-1*} \cdots < T_j^{1*}$.

The theorem implies that we cannot let the ship travel at speeds that are optimal for the current journey (a myopic viewpoint), but have to consider the trade-off with postponing future profits. On profitable journeys, the ship travels fastest on the first journeys, and then slowly reduce speed somewhat. Repetitions of an unprofitable journey ($h < 0$) may not happen in practice unless the company would be forced to do so by e.g. contractual obligations. But if it would happen, then it is optimal to start at slow speed, and gradually increase speed. It also implies the practical consequence that one must always consider revenues, even if they are fixed, in deciding on optimal ship speeds. In traditional methods, the objective is often to minimise costs, and revenue structure is ignored, see e.g. S. Wang and Meng (2012).

Within each journey repetition, in when $m > 1$, legs speeds are also influenced by the profitability of later legs in the journey. For $m = 2$, for example, the optimal speed decisions for $n = 1$ are determined by:

$$\frac{\partial h(\Gamma_i)}{\partial T_2^i} = h_2'(T_2^i)e^{-\alpha T_1^i}, \tag{4.15}$$

$$\frac{\partial h(\Gamma_i)}{\partial T_1^i} = h_1'(T_1^i) - \alpha h_2(T_2^{i*})e^{-\alpha T_1^i}. \tag{4.16}$$

Optimal speeds are also pushed away from their myopic optimum by the profitability of later legs. We can thus also expect a difference in optimal speeds between the situation of a laden leg $h_1 > 0$ before a ballast leg $h_2 < 0$ and the situation where the ballast leg is first, but the difference in practice may be small (see also Example 5.10 in Section 4.5).

A numerical example illustrating the chain effect is given in Section 4.4.3.

4.4.2 Infinite time horizon: $\mathcal{P}(\infty, m, -)$

We consider here the case $\mathcal{P}(n, m, 0)$ for $n \to \infty$. We are thus asking what happens with the leg time sequences of Theorem 1. For example, when the journey is profitable ($h > 0$), is it possible that the leg times for higher n values reach a limit even if they are not constrained by physical boundaries (here v_{max})? This then implies that a stationary state is achieved, i.e. where the optimal speed of leg j is independent of its the repetition number. To our knowledge, no previous study has proven how this stationary (or steady) state is achieved or guaranteed.

Theorem 4.2. *(Convergence) If h is bounded in $\mathbb{R}$, i.e., there exists a positive number $M \in \mathbb{R}$, such that $|h| \leq M$, then there exists Γ^*, and $\Gamma^* := arg \quad max(G^* = G_n = h(\Gamma_n) + G_{n-1}e^{-\alpha L_m^n})$ with $n \to \infty$.*

Theorem 4.2 (proof in 8.2.5) shows the existence of a stationary set of optimal speed decisions and a maximum *Goodwill* in the infinite time horizon. The optimal speeds obtained in finite time horizon will converge to the stationary speed decisions with the increase of journeys.

To see the link with the traditional models based on the *USD per unit of time* criterion, we extend (4.13) to G_{k+1}:

$$G_{k+1} = h(\Gamma_{k+1}) + G_k^* e^{-\alpha L_m^{k+1}}. \tag{4.17}$$

Consider a large enough number N such that for $k > N$, the set of optimal speed decisions Γ_k have converged to Γ^*, then substitute this and $G_{k+1}^* = G^* = G_k^*$ into (4.17) to get:

$$\alpha G^* = \alpha \frac{h(\Gamma^*)}{1 - e^{-\alpha L_m^*}}, \tag{4.18}$$

which proves the correctness of Algorithm 4 (Section 4.3.3) for solving the $n \to \infty$ case. Using the linear approximation of the Maclaurin expansion: $e^{-\alpha L_m^*} \approx 1 - \alpha L_m^*$, we further obtain:

$$\alpha G^* = \alpha \frac{h(\Gamma^*)}{1 - e^{-\alpha L_m^*}} \approx \frac{h(\Gamma^*)}{L_m^*}, \tag{4.19}$$

which means that a good approximation for the optimal annuity stream value αG^* (or *daily profits*) is obtained by finding the speed decisions Γ that maximise the journey profits divided by the journey time. This is an approach similar to the *income-generating leg model* of Ronen (1982)[8].

[8] The differences with the NPV approach is that Ronen's model considers a single leg, not a roundtrip, and does not discount the costs and revenues arising during the journey based on the their relative timing of occurrence. The latter difference will typically produce only minor differences in speed. See also Section 4.5.1.

By adopting the concept of roundtrip, we know from the proof that G_i is monotone, since $h(\Gamma_i)$ is positive definite for a profitable journey and negative definite for a non-profitable journey. This property is not guaranteed if we consider the problem leg by leg, because negative T_j^i are not possible. If we consider a laden-ballast journey ($m = 2$), we get:

$$\begin{aligned} G_1^{k+1} &= h_1(T_1^{k+1}) + G_2^{k+1} e^{-\alpha T_1^{k+1}} \\ &= h_1(T_1^{k+1}) + h_2(T_2^{k+1}) e^{-\alpha T_1^{k+1}} + G_1^k e^{-\alpha(T_1^{k+1}+T_2^{k+1})}, \end{aligned} \tag{4.20}$$

where we are sure that for a sufficiently large N such that $k > N$, $G_1^{k+1*} = G_1^{k*} = G^*$ (but it is not guaranteed that $G_1^{k+1*} = G_2^{k+1*}$), giving as before:

$$\alpha G^* = \frac{h_1(T_1^*) + h_2(T_2^*) e^{-\alpha T_1^*}}{T_1^* + T_2^*} \neq \frac{h_1(T_1^*)}{T_1^*} + \frac{h_2(T_2^*)}{T_2^*}. \tag{4.21}$$

This indicates that an approach on a total journey basis is guaranteed to produce fair results, but that this is not so when optimising speed on an individual leg by leg basis, as proposed in Ronen (1982). We present a further analysis in Section 4.5.1.

4.4.3 Example of the Chain Effect in $\mathcal{P}(n, 4, 0)$

The following numerical example illustrates the chain effect in maritime shipping. The roundtrip journey of the ship consist of the sequence of port visits $A - B - C - D - A$, laden from A to B, ballast to C, laden to D, and laden to A. Further data are given in 8.2.1.

Table 4.1: Chain effect of $\mathcal{P}(n, 4, 0)$ example; (*) kUSD/year

n	G^*(kUSD)	V^n{AB}	V^n{BC}	V^n{CD}	V^n{DA}	V^2{AB}	V^2{BC}	V^2{CD}	V^2{DA}	V^1{AB}	V^1{BC}	V^1{CD}	V^1{DA}
1	2,378									10.9	12.6	11.9	11.1
2	4,689					11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
3	6,936	11.1	12.8	12.1	11.2	11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
4	9,121	11.2	12.8	12.1	11.3	11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
10	21,038	11.5	13.3	12.6	11.7	11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
20	37,084	12.0	13.9	13.1	12.2	11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
30	49,443	12.3	14.3	13.5	12.6	11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
40	59,022	12.6	14.6	13.8	12.8	11.0	12.7	12.0	11.2	10.9	12.6	11.9	11.1
∞	7,495*	13.4	15.6	14.7	13.7								

Table 4.1 reports the optimal speeds on each of the legs for various repetitions ($n = 1, 2, 3, ...$), obtained using Algorithm 3. As the result for $n = 1$ indicates that the journey is profitable at optimal speeds ($h > 0$), then in accordance to Theorem 4.1, we should observe that the ship travels faster on earlier repetitions, which is confirmed by these results. Optimal speeds obtained from Psaraftis and Kontovas (2014) correspond very well to the values for $n = 1$, and also provide a good lower bound on the speeds and profitability obtainable for $n < 10$ values.

When n values increase to large numbers, optimal speeds converge (Theorem 4.2). The results in Table 4.1 confirm this, and also show that in the limit for $n \to \infty$ (solved with

Algorithm 4), these speeds are about 2.5 knots above the optimal speeds when $n = 1$, yet the convergence is slow as can be observed from comparison with the results for $n = 10, 20, 30$ and 40. The *USD per unit time* approach as in Ronen (1982) produces a result very close to the case $n \to \infty$. For intermediate n values in the range $10, ..., 100$, neither of the traditional methods will produce good approximations of the NPV results in this example. This is a consequence of the chain effect.

4.4.4 Example of the impact of G_o in $\mathcal{P}(1, 4, G_o)$

In the problem class $\mathcal{P}(1, m, G_o)$, the profit function (4.11):

$$G_1(\Gamma_1) = h(\Gamma_1) + G_o e^{-\alpha L_m^1}, \tag{4.22}$$

requires an estimate for G_o.

If we solve this problem with Algorithm 4, however, we don't need to provide a value for G_o as it will be the value that maximises the expression (4.12). This, as seen in Section 4.4.2, corresponds to the solution in the steady state, having the G_o value, see (4.19):

$$\hat{G}^* = \frac{h(\Gamma_1^*)}{1 - e^{-\alpha L_m^1{}^*}}. \tag{4.23}$$

In the steady state, the daily profits that can be made in the future equal those made today, at optimal speeds to maximise the NPV of all future profits.

Notice that G_o in (4.11) fulfills a similar role as $h(\Gamma_1^*)$ in (4.13); positive G_o values will thus according to the same mechanism that led to Theorem 4.1 push speeds on the journey to higher values, while negative values will reduce these speeds. We can analyse the effects of different estimates by taking:

$$G_o = \beta \hat{G}^*, \tag{4.24}$$

and varying the value of β.

Table 4.2 summarises the optimal speed decisions and the corresponding journey time with different G_o values. We use the same settings as for the example presented in Section 4.4.3. For $\beta = 1$, we find the steady state solution, which corresponds to the result for the $n = \infty$ entry in Table 4.1. At $\beta = 0$, the solution is the same as for the $n = 1$ case in Table 4.1. These two solutions correspond to the traditional models based on the criteria *USD per unit time* models and *USD per trip*, respectively. At all different values for β, the ship's optimal speeds will be different.

Figure 4.4 illustrates the trade-off that is considered in the optimisation of the NPV criterion between the FPP and the current journey's profits. When the FPP is zero

Table 4.2: The impact of $G_o = \beta\hat{G}^*$ on optimal speeds for $\mathcal{P}(1, 4, G_o)$

β	**Duration (Days)**	V^1**{AB}**	V^1**{BC}**	V^1**{CD}**	V^1**{DA}**
-1	147.9	10.0	10.0	10.0	10.0
-0.5	147.2	10.0	10.2	10.0	10.0
$0^{(1)}$	129.7	10.9	12.6	11.9	11.1
0.5	116.6	12.3	14.2	13.4	12.5
$1^{(2)}$	108.0	13.4	15.6	14.7	13.7
1.5	101.7	14.3	16.7	15.7	14.7

$^{(1)}$ Equivalent to criterion *USD per trip.*

$^{(2)}$ Appromixating the criterion *USD per unit of time.*

($\beta = 0$), the decision maker does not consider the ship to have any value after the completion of the current journey. This corresponds to choosing ship speeds that will maximise the current journey's NPV. With positive or negative FPP values, the ship speeds on the current journeys adjust accordingly, which will reduce the NPV of the current journey. In particular for positive FPP values, we observe a dramatic decrease in NPV. In steady state ($\beta = 1$), for example, the current journey's profits are reduced by nearly 8%. Profit-seeking decision makers are thus willing to sacrifice current profits for longer-term benefit. At negative FPP values, the decrease is less dramatic, since the ship will soon hit the v_{min} boundary (see also Table 4.2), at which point the NPV will remain constant.

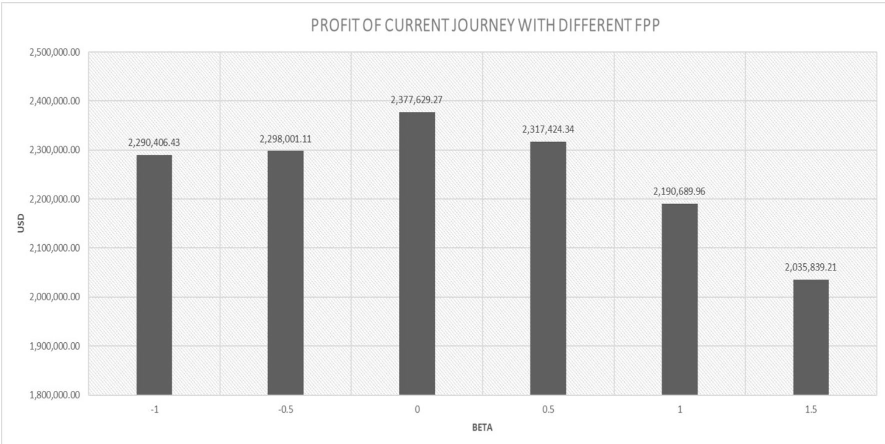


Figure 4.4: Profitability of the selected journey (NPV value, in USD) for different $G_o = \beta\hat{G}^*$ levels

4.4.5 Estimating G_o

Industry reports for various ship types, routes, and contract types estimates of current profitability. These are based on industry averages. For example, the *Time Charter Equivalent* (TCE) numbers published by various organisations represent the operational profits that can be realised before fixed costs, but accounting for the average freight rates achieved and operational costs (mainly port and fuel costs) incurred.

A simple approach consists of using this value in (4.23):

$$G_o = \beta \hat{G}^* = \frac{\beta}{\alpha}\big(\mathrm{TCE} - f_F^{TCH}\big), \tag{4.25}$$

where f_F^{TCH} is the future value of daily fixed costs, a value which is typically more private information to ship owners. The α value corresponds to the opportunity cost of capital of the decision maker, while β is as presented in Section 4.4.4, and represents the decision maker's level of optimism or pessimism with respect to the future evolution of this market.

Shipping companies would use these TCE values cautiously since it can depends highly on underlying assumptions such as the journey structure considered. See also e.g. Example 5.1.

4.5 Comparison with models from the literature

In this section we show how the models and algorithms presented in Sections 2 and 3 can help in assessing the applicability of a few important ship speed optimisation models presented in the literature. To this end, we use the technique of NPV Equivalence Analysis (NPVEA), see also Section 4.6, as to identify under which assumptions the classic method can produce results that are also (close to) optimal from the NPV perspective.

4.5.1 Model I of Ronen (1982): Income generating leg

Model I in Ronen (1982) is an analytical model to help finding the speed of the ship when executing a leg which will maximise the net profit associated with this leg. NPVEA's main question is the following: When Ronen's income generating leg model is used to solve a ship speed decision problem, what kind of assumptions are needed if this approach is to produce results in line with the optimisation of the NPV of the ship's activities?

The profits of one arbitrary leg j in $\mathcal{P}(n, m, G_o)$ are given by (4.1). This expresses the NPV of net earnings discounted to the start of loading operations for this leg. We assumed that unloading operations and revenues were received at the time of completion of this leg. We can consider arbitrary modifications to the timing of payments, for example as follows:

$$h(T_j, \Delta^u, \Delta^l) = (R_j - C_j^u)e^{-\alpha(T_j+\Delta^u)} - C_j^l e^{-\alpha\Delta^l} - \int_0^{T_j} f^{TCH} e^{-\alpha T} dt, \tag{4.26}$$

where values $\Delta^u > 0$ and $\Delta^l > 0$ would indicate further delays in time of cash-flows associated with the unloading and loading operations, respectively.

We now consider an infinite repetition of this profit generating leg as a problem in $\mathcal{P}(\infty, 1, -)$. An infinite repetition produces an infinite chain. We have seen in Section 4.2 that in $\mathcal{P}(\infty, m, -)$ the chain effect is not observable; optimal leg durations of each leg repetition are equal. This therefore certainly applies in $\mathcal{P}(\infty, 1, -)$. The corresponding Annuity Stream (AS) value achieved (dropping the index j) is then given by the equation:

$$\mathrm{AS} = h(T, \Delta^u, \Delta^l) \sum_{i=0}^{\infty} \alpha e^{-i\alpha T}. \tag{4.27}$$

The leg duration T that maximises this expression is optimal from the NPV perspective. Ronen's Model I, instead, considers as objective function the profits of the single leg divided by its duration.

Working out (4.27), substituting (4.26):

$$\mathrm{AS} = \Big[(R - C^u)e^{-\alpha(T+\Delta^u)} - C^l e^{-\alpha\Delta^l} - \frac{f^{TCH}(1 - e^{-\alpha T})}{\alpha}\Big]\frac{\alpha}{1 - e^{-\alpha T}}. \tag{4.28}$$

Maclaurin expansion of the exponential factors, and retaining only constant and linear order terms in α gives:

$$\mathrm{AS} \approx (R - C^u)\Big(\frac{1}{T} - \frac{\alpha}{2} - \frac{\alpha\Delta^u}{T}\Big) - C^l\Big(\frac{1}{T} + \frac{\alpha}{2} - \frac{\alpha\Delta^l}{T}\Big) - f^{TCH},$$

which reduces to:

$$= \frac{1}{T}\Big[R - C^u - C^l - f^{TCH}T\Big], \tag{4.29}$$

when $\Delta^u = -T/2$ and $\Delta^l = +T/2$. Recalling (??) and (??), it is easily verified that (4.29) is equivalent to the objective function of Ronen's Model I (equation (6) in that paper.) We thus get:

Lemma 4.3. *(Equivalence of Model I) (a) The objective function of Model I is a linear approximation of (4.27) when $\Delta^u = -T/2$ and $\Delta^l = +T/2$. (b) Solving Model I is equivalent (in approximation) to solving the infinite repetition on the leg in $\mathcal{P}(\infty, 1, -)$, i.e. maximising the AS of an infinite repetition of this leg under identical economic conditions; and such that all cash-flows associated with a leg are exchanged when the ship is halfway in leg duration.*

Because of the infinite repetition of the leg in this interpretation of Model I, it is difficult to assume other conditions but that the leg, somehow, would be a roundtrip journey all by itself. The single leg assumption explicitly made in Ronen is not compatible with this result. The result is, however, in alignment with Ronen's own comment that 'it is common in the shipping industry to calculate profitability on a roundtrip basis'. To make Ronen's model more practical can thus be done by replacing the single leg by a roundtrip as considered in this chapter. We can rewrite (4.27) using the journey profit expression (4.8):

$$h(T_1^i, \ldots, T_m^i) \sum_{i=0}^{\infty} \alpha e^{-\alpha i L_m}, \tag{4.30}$$

with the understanding that we now need to seek m optimal leg durations (or leg speeds), which are in general all different. The implications of this are profound, as illustrated by the next example.

Example 5.1: *Model I: Single leg vs Roundtrip.*

We use the example of a Suezmax with a speed range of 10-17 kn (see Appendix 1 for full data). The scenarios #1 and #2 examined in this example are listed in Table 4.3. Scenario #1 considers a single profitable leg, as Ronen intended. In #2 the ship must return in ballast in order to make it a roundtrip; an assumption in line with Lemma 3 (b). To calculate the optimal speeds in both scenarios, we use the NPV method and Algorithm 4 (infinite repetition of the journey), since the equivalence results indicate this to be a fair representation of the solutions obtained with Ronen's Model I.

The optimal solution in #1 is to travel at full speed of 17 knots. In #2, the ship travels at 13.61 knots on the laden leg, and at 15.91 knots on the ballast leg. In #1 the ship makes much larger profits per day, because no days have to be accounted for that the ship travels in ballast. The question is not in which scenario the profits per day are the highest, of course, but how plausible each scenario is. In scenario #1, the ship would have to travel back to the origin port at infinite speed and zero costs in order to start the next repetition. In #2, we do not face such difficulties. □

Table 4.3: Evaluating Ronen (1982) (I^1 and II^1: modified to multiple legs; (*) i.e. $C_a = 42,968$, see (4.37))

#	Model	Journey	αG_o (USD/day)	**Journey (USD/day)**	**Days at sea**	**Speed (knots)**
1	I	Laden	77,340	77,340	20.32	17.0
2	I^1	Laden; Ballast	12,968	12,968	25.40; 21.72	13.61; 15.91
3	II	Ballast	12,968 (*)	-65,022	21.72	15.91
4	II	Ballast	2,000	-58,976	23.81	14.52
5	II^1	Laden, Ballast	2,000	12,529	27.71; 23.81	12.47; 14.52
6	II^1	Laden, Ballast	20,000	12,826	24.26; 20.71	14.25; 16.68
7	II	Ballast	20,000	-68,787	20.71	16.68

Ships do not necessarily make roundtrips, however. This indeed was part of Ronen's motivation to contruct separate single leg models. He states: 'ships do (often) not return to the same origin to begin the next trip'. Is there an alternative interpretation of Model I that does not need to rely on the infinite repetition of a roundtrip journey? Let us thus return to the single leg interpretation, and rewrite (4.27) as follows:

$$\text{AS} = \alpha h(T, \Delta^u, \Delta^l) + h(T, \Delta^u, \Delta^l) \sum_{i=1}^{\infty} \alpha e^{-\alpha i T}$$

$$\text{AS} = \alpha h(T, \Delta^u, \Delta^l) + h(T, \Delta^u, \Delta^l) e^{-\alpha T} \sum_{i=0}^{\infty} \alpha e^{-\alpha i T}$$

$$\text{AS} = \alpha h(T, \Delta^u, \Delta^l) + \frac{\alpha h(T, \Delta^u, \Delta^l)}{1 - e^{-\alpha T}} e^{-\alpha T} \tag{4.31}$$

$$\text{NPV} = \frac{\text{AS}}{\alpha} = h(T, \Delta^u, \Delta^l) + G_o e^{-\alpha T} \tag{4.32}$$

Maximising (4.32) will maximise (4.27), and thus the equivalence with Model I, as given in Lemma 3 (a), still holds. Equation (4.32) can be recognised as the dynamic programming approach developed in Section 2, where G_o represents the future profit potential. Because in this case it must be, for the equivalence to hold, equal to the specific value as seen in (4.31), we can use Algorithm 4 to solve situations as considered in Ronen's Model I. (Alternatively, we can intuitively deduce this from Ronen's (implicitly adopted) steady state assumption and results of Section 4.4.2.)

More importantly in this context of how to interpret Model I, we see from the form of (4.31) that G_o can be interpreted as the NPV of an Annuity Stream (AS) value *in size* equal to the optimal value of the AS function of the single leg under consideration. We are thus solving a problem in class $\mathcal{P}(1, 1, G'_o)$, where G'_o refers to being a very specific value. Solving Ronen's Model I can thus be alternatively achieved by maximising (4.32) if we can correctly verify that the value of the future profit potential would be:

$$G'_o = G_o(T^*) = \frac{h(T^*, \Delta^u, \Delta^l)}{1 - e^{-\alpha T^*}}. \tag{4.33}$$

This means that Ronen's Model I would also be applicable when the future profit potential (expressed in USD per day) is in size equal to the profits per day of the current single leg, *while these future profits however may be reached undertaking other types of (non-)roundtrip journeys and under different economic circumstances*:

Lemma 4.4. *(Equivalence of Model I) (c) The objective function of Model I is a linear approximation of (4.32) when* $G_o = \frac{h(T, \Delta^u, \Delta^l)}{1 - e^{-\alpha T}}$*, which can be solved by Algorithm 4; (d) Solving Model I is equivalent (in approximation) to solving a problem in* $\mathcal{P}(1, 1, G_o(T^*))$*, i.e. maximising the Goodwill of the current leg when the future profit potential equals the specific value as given in (4.33).*

This would thus seem to allow for Ronen's argument in support of the single leg model for situations when the ship executes e.g. tramp operations. The situation, however, is very unlikely corresponding to reality. Indeed, adopting the assumption of a future producing *exactly the same* daily value as in the optimal solution of the current single leg means that the future would have to consist of profit generating legs that are *more* profitable per day than the current leg, given that it is not unlikely that the ship may have to incorporate ballast or repositioning legs, which are not profitable by themselves.

In conclusion, through NPVEA we have offered two possible interpretations of Ronen's Model I. As explained in the above paragraph, it seems fair to conclude that the first interpretation is much more plausible and acceptable. This means that we should consider replace the "leg" with a roundtrip journey of two or more legs, and assume this roundtrip is infinitely repeated under the same economic and operational conditions.

4.5.2 Model II of Ronen (1982): Positioning (empty) leg

We check under which conditions there is equivalence of Ronen's Model II with our approach. This also provides an interpretation for a parameter in Ronen's model termed the *Alternative Daily Value* of the ship, named C_a in that paper.

Model II considers a single leg that it used to reposition the ship. In the NPV framework, we consider a problem in $\mathcal{P}(1,1,G_o)$ with profits of the leg in general given by (4.26). Since the ship carries no cargo, we set the revenues R and cargo unloading costs C^u to zero. In addition, after the execution of this journey, the future profit potential can be realised. This gives the objective function:

$$G = -C^l_j e^{-\alpha\Delta^l} - \frac{f^{TCH}}{\alpha}(1-e^{-\alpha T}) + G_o e^{-\alpha T}, \tag{4.34}$$

of which the linear approximation is equal to:

$$G \approx -C^l(1-\alpha\Delta^l) - (\alpha G_o + f^{TCH})T + G_o. \tag{4.35}$$

Constant terms can be dropped for the purpose of determining optimal ship speed v. Only T (through T^s, the time at sea) and C^l (through the bunker fuel consumption function) are functions of the ship speed. Using (**??**) and (**??**), maximising (4.35) thus equals minimising:

$$c^f F(v,w)T^s(1-\alpha\Delta^l) + (\alpha G_o + f^{TCH})T^s. \tag{4.36}$$

It can be verified that (4.36) is equivalent to the objective function of Ronen's Model II (equation (8) in that paper) for $\Delta^l = 0$ and when Ronen's *Daily Alternative Value* is set to:

$$C_a = \alpha G_o + f^{TCH}. \tag{4.37}$$

Ronen's C_a parameter equals the annuity stream value of the goodwill of *future* profits after finishing this leg, plus the *current* daily charter hire value (or the daily fixed costs) that is due for every day the ship executes this leg.

Lemma 4.5. *(Equivalence of Model II) (a) The objective function of Model II is a linear approximation of the speed-dependent terms in (4.34) when $\Delta^l = 0$ and Ronen's Alternative Daily Value of the ship is set as in (4.37). (b) Solving Model II is equivalent (in approximation) to solving a problem in $\mathcal{P}(1, 1, G_o)$, i.e. maximising Goodwill of the current leg using (4.34) when $G_o = (C_a - f^{TCH})/\alpha$.*

Example 5.2: *Model II and C_a.*

In this example we focus on scenarios #3 and #4 of Table 4.3. In #3 we consider a single ballast leg for repositioning the ship and assume a value $C_a = 42,968$ USD/day. With a current hire cost $f^{TCH} = 30,000$ USD/day, this corresponds to a FPP of $\alpha G_o = 12,968$ USD/day, see (4.37). The optimal repositioning speed in #3 is 15.91 knots. This is the same as the optimal ballast speed in #2 because the ship faces at the start of either the ballast leg in Model I, or the repositioning leg in Model II, the same future profit potential. If the future looks less promising, as in #4, then optimal repositioning speed is also lower at 14.52 knots and the ship spends about 2 extra days at sea, saving on the other hand also about 6,000 USD/day during the repositioning journey. □

In the development of Model II, Ronen did not include the daily costs into the objective function, unlike what he did for Model I. This may seem to find support from intuition, since Model II investigates the marginal cost of an extra day sailing on a repositioning (non-earning) leg. In other words, it examines the trade-off about how slower steaming saves fuel on the one hand, but also means loosing out on realising the alternative value on the other hand, and during this time the ship's daily costs are obviously the same. At first sight, it may thus seem counterintuitive to have to add the daily fixed cost f^{TCH} of the current leg into the alternative daily value C_a, as indicated by (4.37), in order for this method to be leading to optimal decision making in the NPV framework. It is nevertheless correct, given the above mathematical logic leading up to (4.37). The reason is that the ship's daily costs are not necessarily remaining the same in the above trade-off as we need to compare *current* with *future* operations. A further clarification may help, and goes as follows. Goodwill G_o is based on net future profits, and thus the daily future profits are based on:

$$\alpha G_o = \textit{Future Revenues - Future Operational Costs - Future Fixed Cost}$$

The operational costs refer to fuel and port costs mainly, while the fixed costs represent either the time charter hire or the costs to cover crew, maintenance, lubricants, insurance, etc. Equation (4.37) can then be written as:

C_a = *Future Revenues - Future Operational Costs - (Future Fixed Cost - Current Fixed Cost).*

Let $\triangle$ = *Future Fixed Costs* $-$ *Current Fixed Costs*. If the future and current fixed costs (per day) are equal, which may be the case if the ship owner operates in the spot market, C_a would be a measure for the operational profits that can be realised (before fixed costs). In that case, C_a matches what is known in the industry as the *Time Charter Equivalent* (TCE), of which values are reported in industry. If future and current fixed costs per day are *not equal*, however, one should not forget to correct for this, i.e. $C_a = \text{TCE} - \triangle$. This situation may arise, for example, if the current (charter) contract ends after completing this leg, and the daily hire in the future changes.

In conclusion, Ronen's Model II did not include the daily hire cost; and we have given also an intuitive explanation why this may seem sufficient. However, the NPVEA method has given us a proof that this may be incorrect in the case of future hire cost being different, unless if we correctly account for this difference. The practical formula (4.37) derived from NPVEA can ensure that Ronen's Model II will be applicable in all cases, however, as it captures both eventualities automatically.

4.5.3 Ronen's switching strategy between Models I and II

Ronen (1982) recommends to switch between Model I and II to make the most profitable decision. In particular, if the maximum daily profit achieved on the profit generating leg using Model I is less than the daily alternative value C_a used in Model II, then it is advised to switch to Model II. We interpret this that in that case, the ship's next leg will not be a profit generating leg, but becomes a repositioning leg instead. Let Z^* indicate the optimal solution from Model I, then Ronen's advice is thus to use Model I when $Z^* > C_a$, and switch to Model II otherwise.

Given Lemma's 3 and 5, and (4.37), however, we must make the following adaptation to this rule if we want to make the NPV-optimal decision: Use Model I when:

$$Z^* \geq C_a - f^{TCH}, \tag{4.38}$$

where f^{TCH} reflects the *current* daily costs on the current leg, and check with Model II otherwise, *and take the scenario with best overall NPV*. Example 5.3 illustrates.

Example 5.3 *Switching Model I vs Model II: updated rule.*

Consider that the ship currently executes operations at profits of $12,968$ USD/day as in scenario #2 (Table 4.3), but it could potentially make the switch to #3. Current daily hire is $30,000$ USD/day, and the future daily hire in #3 will be $35,000$ USD/day, but with a Time Charter Equivalent value of $47,968$ USD/day. We then have: $C_a = 47,968 - 35,000 + 30,000 = 42,968$ USD/day. Note that $Z^* << C_a$, so according to Ronen's rule we should switch to undertake the repositioning leg instead, i.e. adopt #3. Equation (4.38), however, gives an equality; while the future in #3 promises equal daily value as in #2 at $12,968$ USD/day, the fact that we have a repositioning journey to make first makes its NPV lower (at only $12,580$ USD/day). We should thus stay in scenario #2. □

Ronen's models do not capture all possible 'switching' situations. In fact, the following example is not uncommon: It may be that in order to reach the area of the world in which the ship could realise C_a, the ship could travel along a profit generating route instead of a revenue-less repositioning leg. We would thus need to extend Model II (see also the example below) to a situation in which as part of the repositioning strategy, the ship may first make a laden leg, and then a ballast leg as to arrive at the area in which C_a can be realised.

Note that using Model I for this (once used) laden leg journey, in combination with the switching rule, as Ronen suggested, would not lead us to the correct decisions. Indeed, Lemma 3 and 4 show that Model I would work under the assumption that the economic conditions experienced during this laden, ballast journey would also be there in the future. Not only would then optimal speeds obtained from Model I be wrong, one would not be able to estimate the actual future profits (or losses) the ship could make. The following example illustrates.

Example 5.4 *Extending Model II with profit generating legs.*

Consider scenarios #4 and #5 in Table 4.3, and assume the ship is currently idle. In #4 the ship will face a loss of $-58,976$ USD/day for almost 24 days during a single repositioning leg, before arriving at a situation where it can make $2,000$ USD/day. In #5, the ship first executes a laden leg and then travels in ballast to the area of future profits. It takes the ship almost 28 days longer to arrive there, but it will make a profit of $12,529$ USD/day on the way. Here, #5 is clearly superior to #4, because the current daily earnings are higher than in the future. Note that the speeds obtained in #5 are optimal as to maximise the NPV of future profits of the ship. If we would follow Ronen and have used #1 for deciding on the laden leg as in Model I, it would travel too fast (as it implicitly assumes a much to optimistic future in this case). □

It is clear from these examples and elsewhere in this chapter that the NPV model we developed offers increased flexibility for considering multiple possible scenarios for the ship to balance current operations with future profit potential. When using Ronen's models, a decision maker needs to think about which model to use, and we have also shown that the switching rule of Ronen cannot guarantee arriving at optimal decisions. When adopting our modeling approach instead, all scenarios deemed possible can be modelled similarly and then compared based on which scenario has the highest NPV. The next example illustrates the power of the NPV criterion.

Example 5.5 *NPV as the ultimate arbiter.*

Consider scenarios #6 and #7 in Table 4.3. These are equal to #5 and #4, respectively, save for the fact that the future profit potential is $20,000$ USD/day. It is now not immediately clear from the numbers reported in the table which of these scenarios is the best: in #6 we make profits per day during the laden, ballast repositioning journey, but it takes us about 25 days longer to get to a future where we could earn even much more per day. In #7, we make a loss during the repositioning leg but we'll be 25 days earlier to the promising future of high earnings. As our method is based on maximising the NPV (or AS) of the ship's future activities, it can be easily decided which scenario is best. The AS of #6 is $7,270,608$ USD/year and of #7 is $7,146,125$ USD/year, and thus the laden journey is worth doing. (With altered data, #7 could have been better.) □

We hope to have convinced the reader of the following conclusions. First, we demonstrated that the application of Ronen's models may lead to erroneous decisions if the underlying assumptions are not well understood. With NPVEA we have been able to shed light on these assumptions, as well as provided a correct formula to help calculate Ronen's Daily Alternative Value (C_a) parameter. In particular, we derived that it will have to account for any differences in daily hire values between the current repositioning journey and the future. We have also demonstrated with some simple examples that in order to make better decisions, the shipping company may actually wish to extend the models of Ronen to include multiple legs, which goes against Ronen's own arguments. Finally, because of the equivalence results we have obtained with our modelling framework, Ronen's Models I and II can both be solved with our model (Algorithms 2 and 1, respectively), guaranteeing the optimality according to the NPV criterion, while at the same time it allows easily for extending these models to multi-leg journeys and journey repetition. Finally, our modelling approach seems also superior in that it can deal with a wide range of future profit potential values, which allows it to consider many more realistic situations. Not accounting for the future profit potential accurately leads to erroneous speed decisions and profit values that will not optimise the ship's net present value.

Table 4.4: Instance characteristics: Cases 2 to 10 are alternative plausible scenarios in the setting considered in Fagerholt & Psaraftis (2015)

#	Repetitions (n)	Journey Structure	FPP (USD/day)
1	∞	Laden	-
2	∞	Laden-Ballast	-
3	1	Laden	0
4	1	Laden-Ballast	0
5	1	Laden	-10,000
6	1	Laden-Ballast	-10,000
7	1	Laden	10,000
8	1	Laden-Ballast	10,000
9	∞	Ballast-Laden	-
10	1	Ballast-Laden	0

4.5.4 Fagerholt and Psaraftis (2015): optimal speeds in Emission Control Areas

In this last section, we consider the problem presented in Fagerholt and Psaraftis (2015). The study concerns the situation in which a ship has to travel through an Emission Control Area (ECA), introduced by the International Convention for the Prevention of Pollution from Ships (MARPOL). As a consequence, vessels have to burn cleaner but more expensive marine gas oil (MGO) when travelling within an ECA, while they can use heavy fuel oil (HFO) outside of such areas. Fagerholt and Psaraftis (2015) have built their model based on Ronen (1982), Model I, assuming a ship travelling a single journey starting outside and crossing into an ECA zone to end in the destination port. We illustrate how one could arrive at very different conclusions about optimal speeds, and thus also environmental impact, depending on the choice of model.

Within our modelling framework of Section 4.3, this type of problem can be viewed as a journey between three ports A, B and C, where the leg $A - B$ is outside of the ECA, and the leg $B - C$ is inside of the ECA, and B is a virtual port with zero residence time and costs, but where the fuel is switched. The impact of ECAs can be denoted by assuming a different fuel price for each leg, while the former is normally much cheaper than the latter.

In the following examples, we follow a ship with a speed range of 15-21 knots. Due to confidentiality, the full data used in the original study could not be obtained. We thus have used our own data for undisclosed parameters, see 8.2.2. Table 4.4 provides key data of ten different journey situations. In the first eight cases, the ship travels from A outside an ECA zone into an ECA zone laden to deliver in port C. In odd case numbers 1, 3, 5, and 7 there is no subsequent route out of C explicitly modelled. This corresponds in our model to a journey A-B-C ($m = 2$). In the even cases 2, 4, 6 and 8, a ballast return journey is added so as to get back out of the zone. This is modelled as the journey A-B-C-B-A ($m = 4$).

Example 5.6 *Using Ronen's Model I with two different journey structures.*

We compare the first two cases of Table 4.4. Case 1 replicates the model used in Fagerholt and Psaraftis (2015): a single income generating model without the consideration of a return journey. In Case 2, a ballast return journey is added. Both cases, as in all examples in this chapter, are modelled using our framework. In the spirit of Model I, they are solved using Algorithm 4 (see Lemma 4).

Figure 4.5 illustrates how optimal leg speeds inside and outside the ECA vary with the fuel price ratio MGO/HFO, where we keep the HFO price fixed. An increase in this ratio may, for example, be induced through taxation. In Case 1 of a single laden journey, the ship's speed inside the ECA is reduced from the maximum 21 knots down to 18.4 knots with increasing MGO/HFO, but outside the ECA the ship maintains maximum speed. At MGO/HFO = 2, the ship would make 44,319 USD/day, and it would be worthwhile for the ship to not slow down at all.

Case 2, however, shows much greater sensitivity to the (taxation) ratio. Most importantly, the speed inside the ECA on the laden is drastically reduced: at a ratio of 2.5 the speed is about 4.5 knots below the optimal speed in Case 1. At ratios above 3, also the laden leg speed outside the ECA would start to decrease. At MGO/HFO = 2, the ship makes only 8,278 USD/day, much lower than the profits in Case 1. The reason is that Case 2 also considers the ballast leg from C back to A, which reduces the average daily profits, and also greatly tempers optimal ship speeds.

The question here is not in which case the ship is most profitable but, as in Example 5.1, how plausible each scenario is. In both cases, since we use Ronen Model I's criterion of maximising USD/day, we must implicitly assume that the ship continues these activities with the same daily profits (see Lemma 3 and 4). In Case 1, this is difficult to imagine: even if the ship would have also a laden leg from C back to A, the fact the ship first needs to go through the ECA zone makes this not identical to the A-B-C laden leg, and thus the laden leg C-B-A would need to have a very specific revenue value as to counteract this (see also Lemma 4). In Case 2, the journey is a roundtrip so we don't need to ensure specific relationships between revenue and fuel cost parameters. □

Example 5.6 also illustrates (see Figure 4.5) how difficult it may become to determine quantitatively how taxation on bunker fuel prices may affect emissions, since optimal speeds both inside and outside the ECA highly depend on the case considered. We think that, at best, only a qualitative effect may be predictable. This point will be further strengthened by considering the next cases.

In Case 1 and 2, the ship is assumed to continue doing activities in the future producing the same USD/day value (or thus as if the journey is infinitely repeated in the same economic conditions.) An alternative situation arises if we imagine the journey A-B-C

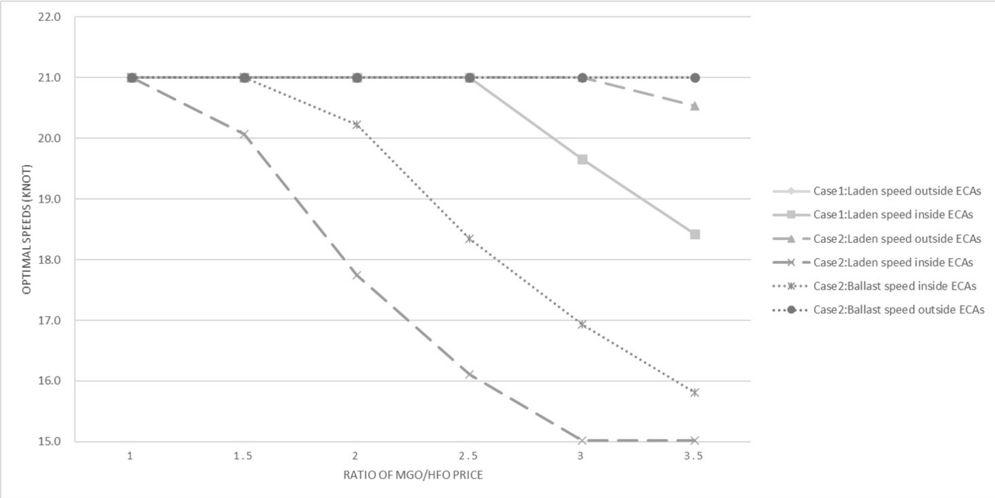


Figure 4.5: Optimal speeds within and outside the ECA for Cases 1 and 2

considered to be part of a tramp shipping scenario, where the ship aims to arrive in port C to realise a future profit potential. Cases 3 to 8 consider several scenarios in which different FPP values apply.

Example 5.7 *Using a model similar to Psaraftis and Kontovas (2014).*

Cases 3 and 4 from Table 4.4 describe the situation of a single laden journey A-B-C, and a two-leg laden ballast roundtrip A-B-C-B-A, respectively. We solve both cases with Algorithm 3. In approach, these NPV models with $n = 1$ and FPP $= 0$ are comparable to the classic multi-leg journey model described in Psaraftis and Kontovas (2014) where the criterion is to maximises total profits in USD per journey. Optimal speeds inside the ECA are depicted in Figure 4.6; the optimal speed outside the ECA is 21 knots at any ratio MGO/HFO.

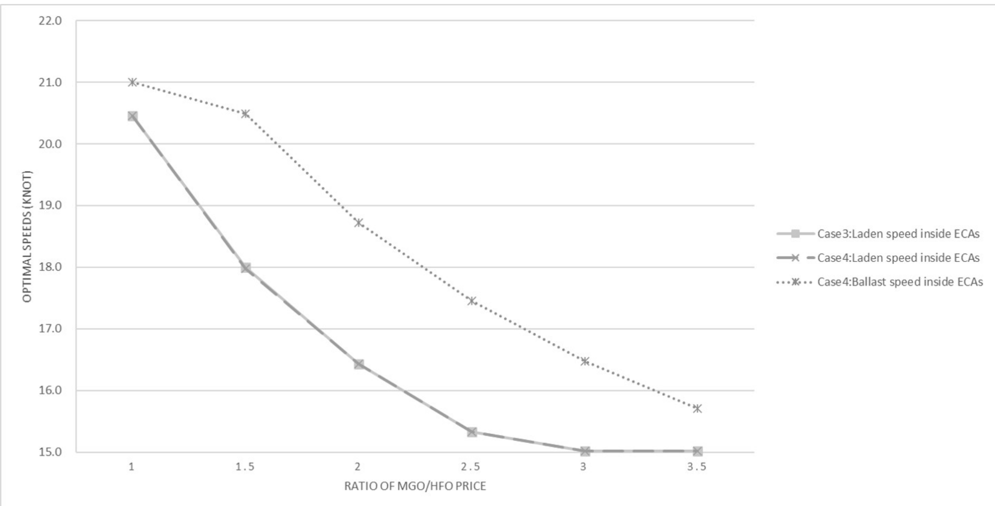


Figure 4.6: Optimal speeds within the ECA for Cases 3 and 4

Though the speeds outside of ECAs are affected by the fuel price inside ECAs for large n values (see e.g. Case 2), this is not observable here for $n = 1$: by changing the travel speeds, one cannot postpone or advance the future payments much. As a consequence, adding fuel tax will not change the optimal speeds outside of ECAs, and thus the ship always travels at maximum speed on $A - B$. Moreover, including a ballast leg will not urge the ship to travel slower on the laden leg, unlike its dramatic effect it had in Case 2. Because the FPP is not as attractive as in Case 1 and 2 (see Example 5.6), the ship travels slower on laden legs in Case 3 and 4, while however still showing a similar sensitivity to MGO/HFO, see Figure 4.6.

We can imagine a scenario in which the roundtrip journey of Case 4 would be repeated $n > 1$ times. The chain effect analysis has shown us that the future now becomes more and more attractive. This will have the effect of gradually moving the optimal speed curves observed as in Case 4 given in Figure 4.6 to the corresponding curves of Case 2 given in Figure 4.5. We would again have to conclude that the impact of taxation on optimal speeds inside and outside an ECA depend considerably on the scenario considered. □

In Case 3 and 4 we have considered FPP $= 0$. In the following example we compare this with the situation in which the decision maker has a more pessimistic expectation about the future. This could correspond to, for example, expecting a major global economic downturn (from e.g. the outbreak of a pandemic, such as was COVID-19 in 2020), or from the ship upon completion of the journey having to undergo a major overhaul which is expected to ground the ship for several months.

Example 5.8 *Facing a negative FPP: future losses ahead.*

Cases 5 and 6 describe the situation when the ship, upon completion of the laden leg to port C, or the laden-ballast journey back into port A, respectively, will run into the FPP of $-10,000$ USD/day. As examined in Section 4.4.4, we should expect the ship to travel much slower now in order to save fuel and to postpone the onset of the negative FPP. The optimal speeds shown in Figure 4.7 indeed confirm this (to ease the comparison, the Case 3 and 4 results are reproduced.) As the ship is already travelling at low speeds, increased taxation has only a small impact on additional speed reductions. □

Example 5.9 *Speeding towards a positive FPP.*

When the decision maker has a positive FPP, it is more likely that he will decide to let the ship travel faster in order to receive the future revenues earlier. Adding fuel tax, however, can obviously urge it to slow down.

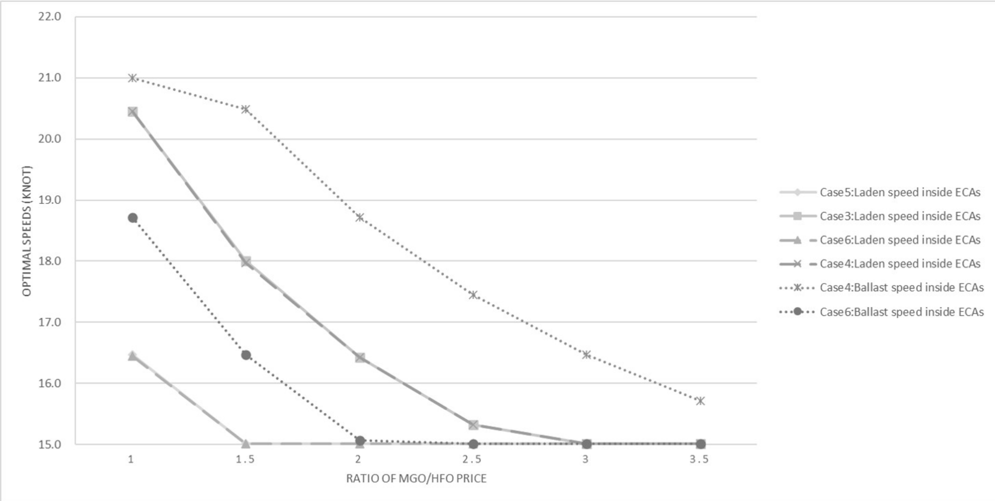


Figure 4.7: Optimal speeds within the ECA for Cases 5 and 6

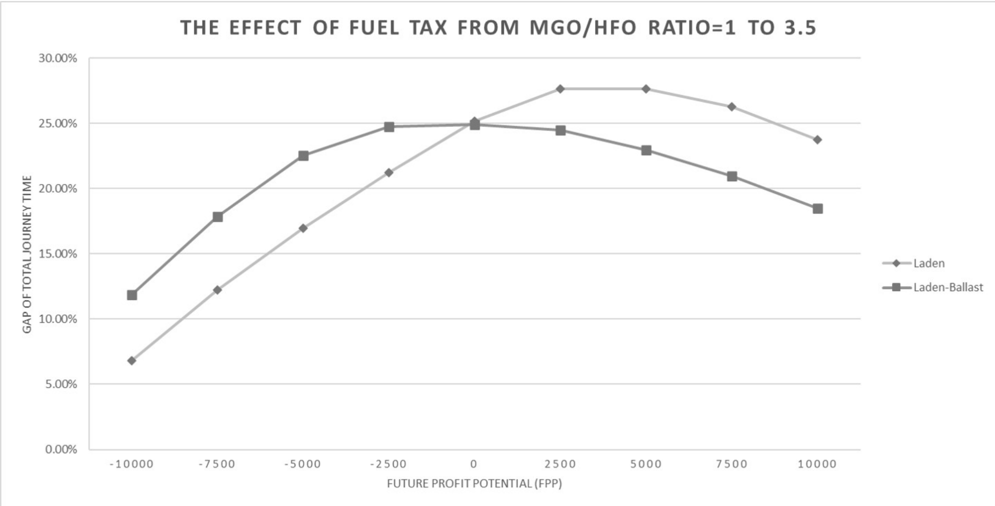


Figure 4.8: The impact of FPP on the power of fuel tax

In this last example about FPP, we summarise the impact of a taxation incentive in Figure 4.8, using the scenarios as in Cases 3 (laden) and 4 (laden-ballast), but for a range of different FPP values. The figure plots the gap when the MGO/HFO ratio is changed from 1 to 3.5. The percentage gap (y-axis) here is defined based on the difference of total journey time at a ratio of 3.5 and 1, divided by the total time at ratio 1.

It may seem intuitive that when the market is in prosperity, that adding a fuel tax will enlarge the gap of total time and thus contribute more to the emission reduction, as confirmed by initial increase observed. However, this is not true when the expectation of FPP is too high. With a promising economic future, the shipper expects to earn so much that he is willing to bear the cost of fuel taxes at high speed. Consequently, the power of fuel tax increases on emission reduction is then reduced. □

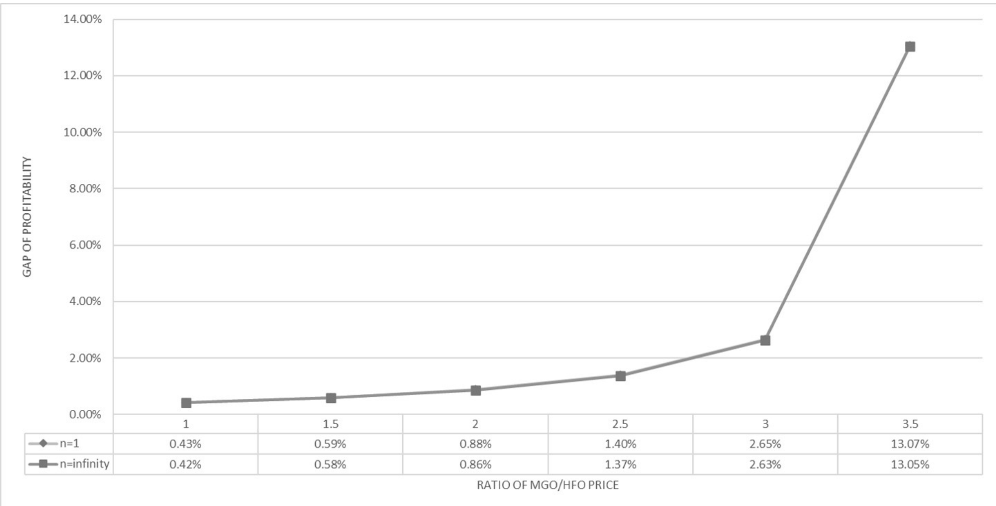
Example 5.10 *How ECA may affect competitiveness*

In all the above cases, the ship executes the profitable leg first, starting in port A. However, there could be another identical ship on the same route, but starting from the port C within the ECA. This alternative ship will do the journey C-B-A-B-C; the ballast leg first, then the laden leg. The latter situation is described by the Cases 9 and 10; the tasks in this set-up is to compare Case 9 to 2, and 10 to 4, respectively.

For Cases 9 and 10, the decision maker incurs the costs of the ballast leg first and receives the revenues of the laden leg later, and profitability is lower compared to Cases 2 and 4.

The ship also travels faster on the ballast leg to secure the job, and then slows down on the laden leg. Consequently, if the whole journey is still profitable, the ship is likely to spend less total travel time when doing the ballast leg first, and thus create more emissions. However, this time (and speed) difference is typically rather small, especially when the distance is short or the logistics task is not that attractive.

The difference in profitability is given in Figure 4.9. The percentage gap is based on the difference of net profits between Case 9 (or 10) and Case 2 (or 4), and divided by net profits of Case 2 (or 4). The gap in profitability in this example grows rapidly with the ratio MGO/HFO. It should be noted, however, that with growing ratio, the whole journey also reduces in profits for both ships, which tends to boost the gap measure. ☐



	1	1.5	2	2.5	3	3.5
n=1	0.43%	0.59%	0.88%	1.40%	2.65%	13.07%
n=infinity	0.42%	0.58%	0.86%	1.37%	2.63%	13.05%

Figure 4.9: The gap of profitability between Laden-Ballast and Ballast-Laden

This actually leads to an interesting problem for competitors using identical ships on the same route with different directions. Consider the example from the introduction given in Figure 4.1, and assume that the ECA is around Rotterdam. Whoever has a ship in New York can not only serve the laden journey much sooner, but will also be more profitable compared to those who have a ship in Rotterdam. Figure 4.9 suggests that the fuel tax will enlarge this profitability gap even further.

The examination of the Cases 1 to 10 illustrates how differently the same fuel taxation may impact a ship's execution of a journey into (and out from) an ECA. While all odd and even cases may seem very similar at least in the logistics of the journey and associated costs and revenues considered, the optimal speeds are yet very different, and thus the environmental impact as well. To a large degree the difference in these cases results from how the NPV model, through the value of n and the FPP value, is able to capture how the decision maker views this journey as being in a trade-off with the kind of activities the ship will be able to do upon its completion: the more optimistic/pessimistic about the future relative to the current economic conditions, the more willing one may forego making myopic optimal decisions about the current journeys. The ability to capture this effect in a model as we have developed, and show its importance by conducting comparisons with classic speed optimisation models has, to our knowledge, not previously been effectively demonstrated.

4.6 Further comments

4.6.1 Chain Effect

The NPV framework has been widely used in the study of replacement (Bethuyne, 2002). Preinreich (1940) is one of the earliest studies of the single machine replacement problem from the NPV perspective. Amongst his interests was the relationship between the optimal life-time of successive usages of a single machine. He derived the insight that "the latter machine would have a longer (re-)cycle life", and referred to it as a "chain rule". The chain effect was also identified in the area of financial investment in problems concerning optimal durations of a sequence of identical projects, see Götze et al. (2015, Chapter 5.3).

We study the relationship between the optimal lengths of successive repetitions of a journey by a ship, established through decisions on leg speeds. Where the single machine replacement problem has a fixed original cost (B) in the beginning of each replacement, we instead face a daily charter cost ($\int_0^{t_i} f^{TCH} e^{-\alpha t} dt$). The chain effect has, to our knowledge, not been reported as to also apply in ship speed optimisation problems, even though ships are often used for repeated logistics services in practice. We have thus investigated the chain effect in this chapter to determine its potential importance. We would conclude, see in particular Sections 4.4.1 and 4.4.2, that it has great value to help understand the transition from the finite journeys situation to the infinite horizon case.

The material presented Section 4.4.3 also helps us to understand that the two conventional modelling approaches first discussed in Section 4.1 each present solutions only valid in an extreme situation. One modeling approach considers the finite single journey situation. By using the optimisation criterion of USD over the journey, it implicitly ignores all of the potential relevant future. The other approach uses the optimisation criterion of USD per time, thereby implicitly assuming the infinite repetition of the journey over an infinite horizon, with constant economic conditions over time. The example in Table 4.1 illustrates that there are many plausible situations that do not fit either of these extremes.

4.6.2 Future Profit Potential

As a ship is an expensive investment, Net Present Value methods and discounting cash-flow projections over the life-time of the vessel form part of common business investment analysis, see e.g. Alizadeh and Nomikos (2007). It is in this application also part of the theory about maritime economics, see Stopford (2009). Yet, it has rarely been actually adopted at the operational level of ship journey and speed optimisation.

A notable exception is Magirou et al. (2015), who present a discounted profit model as an extension to their average profit model in the infinite horizon situation. They use the model to identify the optimal traversal cycle between ports that maximises the net profitability, although state that "the equations [*of the discounted model*] are more complicated and the optimal speed expressions are not as easy to interpret." They further identify a 'paradox' in which solutions violate "the principle that profitable voyages should be traversed at high speeds."

We are first to present the finite horizon case ($n < \infty$) and analyse in this context the chain effect and the impact of the future profit potential (FPP). This offers also a robust understanding about the paradox identified in the infinite horizon model of Magirou et al. (2015) by viewing the infinite horizon model as the limit of the n journey situation. It is by now hopefully clear to the reader that optimal speed on a current journey or leg is not determined by its profitability, but by the trade-off with future profit potential. We would thus argue that there is no paradox, but rather that the stated principle is simply not valid. It is worthwhile to point out that both the chain effect and the FPP concept are no longer observable in the infinite horizon case.

A pre-cursor idea of the FPP in the maritime shipping literature can be found in Brown, Graves, and Ronen (1987), who address a scheduling problem of crude oil transportation in shipping. They made the interesting suggestion at the end of their paper that, because of uncertainty in scheduling events near the end of the planning horizon, it may be better to discount the costs of future events to a present value to ease the comparison of alternative shipping plans. We have not actually seen the development of this idea in further literature on ship scheduling.

In corporate finance theory, a similar concept exists called *horizon value* and is defined as "the forecasted value of the business at the valuation horizon, also discounted back to present value" (Brealey & Myers, 2003) (p. 77). The horizon value, also known as *terminal value*, forms an important component in the method to value a business by the discounted cash-flow method, see e.g. Chapters 4 and 19 in Brealey and Myers (2003).

The *Future Profit Potential* (FPP) in this chapter can be viewed as a translation of this concept from corporate finance to the ship speed optimisation problem. The FPP then receives a very natural interpretation as the forecasted value of the ship at the time and location where the ship completes the journey. (But unlike the horizon value, it is not discounted back to the present, but to the completion time of the journeys.) We find in our numerical experiments (see Section 4.5) that the ship speeds on the legs of a currently planned set of journeys can be quite sensitive to the FPP.

4.6.3 Net Present Value Equivalence Analysis

In Section 4.5 we make use of the technique of *NPV Equivalence Analysis* (Beullens & Janssens, 2014). This approach has been pioneered in the field of production and inventory system management by Grubbström (1980). The NPV objective function in this theory is the Laplace transform of a cash-flow function of an activity. NPVEA is used to compare the outcomes from this NPV model to a "classic" optimisation model (not based on the Laplace transform of a cash-flow function) about the same activity.

Assuming that the cash-flow structure in the NPV model is a fair representation of reality, the NPVEA method asks the question whether the classic model can find its own optimal solutions to be also (close to) optimal from the NPV perspective. This equivalence may be subject to making specific assumptions or interpretations about the classic model's "structure" (e.g. its parameters), which helps us to better understand the classic model's applicability.

One of the techniques that can be used is based on Maclaurin expansion of exponential terms in α of the NPV objective function, and then linearisation of the resulting expression. This is often a fairly accurate approximation. More importantly, this linearised objective function is often close to the objective function developed in classic modelling approaches (not based on the NPV technique). This allows us to state with more confidence *under which conditions* there is "equivalence". Grubbström (1980), for example, was able to show that in some specific production systems, equivalence only exists if the holding cost of products in stock is based on the sales price rather than the cost price. In this chapter, for example, we identified that equivalence of Ronen's Model II with the NPV framework only exists when Ronen's parameter C_a receives a specific interpretation as given by (4.38).

Equivalence results not only tell us more about the applicability of classic models, but also allows us to use either the classic or the NPV model interchangeably. We make extensive use in Section 4.5 of equivalence results. For example, as we have established equivalence between our NPV model and Model I of Ronen (1982), we can obtain solutions of Ronen's model by solving our NPV model instead.

4.7 Conclusions

Generally stated, a first aim of this chapter is to present and apply a method by which one can identify the underlying (implicit) assumptions within which these conventional approaches will produce solutions that are also (close to) optimal when these solutions are evaluated in the NPV modelling framework. This helps us to better understand the applicability of these classic speed optimisation models. The complementary second aim is to highlight the situations in which the NPV modeling approach as presented in this chapter should be preferred.

we identify and study the phenomenon that the corresponding ship speeds should differ with each repetition, even in the static case of time-invariant economic conditions. This generalises the *Chain Effect* first observed in Preinreich (1940) in a study about the optimal renewal period of industrial equipment.

This chapter seems also first in the literature to formally define and recognise the importance of the concept of a *Future Profit Potential* (FPP) in ship speed optimisation models, and show how it is key in driving optimal decisions. The FPP is the present value, at the time and location of completion of the n journeys, of all future cash-flows associated with the ship. The use of a FPP we introduce here shows similarities with approaches used in Corporate Finance, in particular with the concept of the *horizon value*, or *terminal value*, which forms an important component in the method to value a business by the discounted cash-flow method[9].

It is shown in this chapter that in special cases of journey conditions a classic optimisation criterion is guaranteed to produce a result that is also close to being optimal from the NPV perspective. We demonstrate that the NPV-based approach shows greater flexibility in addressing various journey situations, and in particular can more rigorously account for the trade-off between maximising profitability of current activities versus maximising the ship's future profit potential.

Restricting ourselves to static conditions during the journey makes it much easier to observe the chain effect, as otherwise changes in speed might be attributed to the dynamic data. This setting also makes it much more clear how the FPP affects the chain effect as well as the optimal ship speeds on each of the legs. This restriction also leads us to present two easy to apply algorithms that can solve instances for any set of (n, m) values in polynomial time. It also allows us to make direct analytical and numerical comparisons with models from the literature based on the classic approaches, which also

[9] See e.g. Chapters 4 and 19 in Brealey and Myers (2003).

assume port-specific constant freight rates and fuel prices. We leave the extension of the models to dynamic (stochastic) conditions for further research, but point out that both the chain effect as well as the impact of the FPP will still be present in these models, albeit possibly in a more disguised manner.

We also note that while we use *round-trip* journeys in the study of the chain effect, the model and algorithms work equally well without imposing this requirement. In Section 4.5.2, for example, we examine single journeys of m legs with an arbitrary destination port.

Finally, we argue in favour of the NPV approach as developed in this chapter. We can summarise the reasons as follows. First, we demonstrate in this chapter that due to the chain effect, the *decomposition principle* no longer holds in the NPV framework. We would in general benefit from optimising the ship speeds jointly across all the (nm) legs of all journey repetitions. Second, the algorithm to solve for optimal ship speeds on (nm) legs simultaneously in the NPV framework runs in (pseudo-)polynomial time to any degree of accuracy. The reason that decomposition helps to reduce the computational burden is no longer a practical limitation with the availability of computing power. Third, we conduct an NPV Equivalence Analysis (NPVEA, see also Section 4.6) which leads us to conclude that we can replace the three conventional methods introduced above by the proposed unifying NPV modelling approach. Fourth, and the most important, we recognise that the NPV approach allows for much greater flexibility: being able to consider journeys of arbitrary composition and cash-flow structure, and evaluate their value relative to the decision maker's expectations about the future. We demonstrate that optimal speeds derived from the NPV perspective, when accounting for the effect of the FPP, may take on very different values as those derived from the two most popular conventional approaches[10]. This model, however, has not received much further actual application in the subsequent literature. We provide a practical interpretation of the somewhat mysterious *Daily Alternative Value* C_a parameter in that model, and demonstrate in which circumstances it may work well, and how it can be expanded.

[10]We show 4.5.2 that only Ronen's 'forgotten' *positioning (empty) leg model* would be capable to account for the FPP by setting its C_a parameter in a specific manner.

Chapter 5

Charter Contract under Time-Varying Economic Conditions: The Selection of Redelivery Clause

Abstract

This chapter studies a practical problem faced in charter contract negotiation that the shipping company needs to evaluate the redelivery clauses based on either financial or environmental criteria. This scenario can be viewed as a type of maritime speed optimisation problem with particular kinds of time constraints that depends on the contractual details.

In particular, two commonly adopted redelivery clauses, namely the Shell Time 4 and ExxonMobil 2000, are discussed here. The evaluation will focus on the end of contract scenario in which the ship could perhaps perform one or two additional laden trips before re-delivery according to the conditions set out in the contract.

The main decision is related to whether to execute these additional journeys, and at which speeds. When evaluating the redelivery clauses, the preference may also depend on the decision maker's view of market trend. We therefore further allow the economic parameters, e.g., freight and fuel price to be time dependent. That is, we would like to further investigate if the preference of redelivery clause may depend on market parameters.

The results of this chapter reveal that the selection of redelivery clause for journeys with moderate length needs to incorporate the consideration of market dynamics, while

for other cases. e.g., when the journey is rather short or long, the decision is easier to make. Numerical examples show that ExxonMobil 2000 clause often brings in financial benefit at the cost of emissions, while Shell Time 4 clause may be more environmentally friendly.

key words: dynamic market conditions, time charter contract, redelivery clause, maritime speed optimisation.

5.1 Introduction

Based on the view of contracts, maritime shipping is often known to be either time chartered or voyage chartered. In particular, the ship charterer, which is the person or company that operates the ship, needs to negotiate with the ship owner to reach an agreement for the time charter contract.

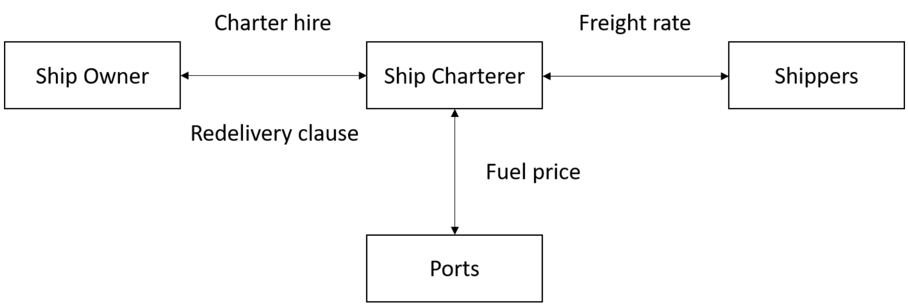


Figure 5.1: Factors within the charter party and ports

In a more general situation, as depicted in Figure 5.1, when the ship owner decides to charter the ship out with a predetermined charter hire to ship charterer, they will negotiate when and where the ship has to be returned once the contract is finished and specify it in the redelivery clause. Then the ship charterer will operate the ship to undertake some logistics tasks for the shippers with a freight rate specified per ship type, cargo and route. When the ship has berthed to a port, excluding the fixed dues, the charterer needs to purchase the bunker fuel there. The fuel price is thus normally specified per port and changes over time. The optimal operational strategy of the ship, e.g., the speed choices, is therefore constrained by the factors discussed above.

5.1.1 Example

A yet rarely discussed problem in literature that has been concerned in practice is how to evaluate the charter contract, especially the contractual clauses, e.g., redelivery clause. For example, if a shipping company operating mainly in Europe is required to return a chartered ship to somewhere in America, it is very likely for the company to reject such charter contract and look for a "better" one, Now, if there are two available contracts that allows the ship to be returned to the same port in Europe, but one is of the format of Shell Time 4 and the other one is using ExxonMobil 2000, which one is "better"?

Such question is hard to answer without knowing further information about the potential usage of the ship and the market conditions, but may impact the charterer's preference before entering the charter contract. For simplicity, we can assume the usage of the ship is identical for each contract before redelivery (as they have no restrictions on it). Thus, to distinguish the values of each clause, we can focus on the end of contract horizon, i.e., when the ship is exactly before redelivery.

For a time charter contract, if the contract duration is set long, e.g., 5 years, evaluating the end of contract scenario would naturally incorporate with uncertainties. The randomness includes the future evolution of market conditions, and also the timing of when the ship will arrive at the last port before redelivery.

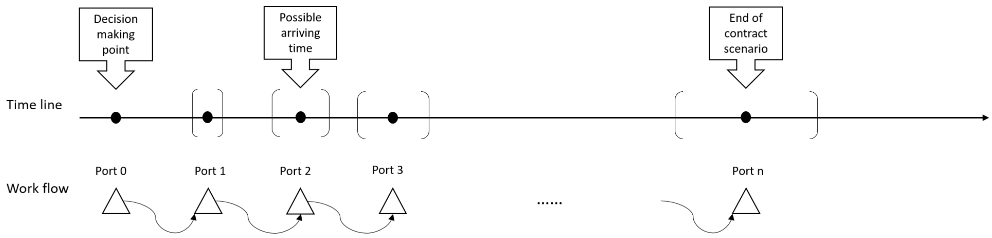


Figure 5.2: The uncertainty of assessing redelivery clauses

Figure 5.2 gives a simple illustration that the longer future we are looking at, the wider arriving time window we are facing. Without loss of generality, here we assume the chance for arriving Port n before redelivery is uniformly distributed. As a consequence, the problem is transformed into maximising the expected profitability of the ship in different end of contract scenarios, and one can thus compare their performances under certain market conditions and/or adopt different criteria, e.g., greenhouse emissions.

5.1.2 Related literature

The economic conditions, e.g., fuel price and freight rate, are often analysed in traditional maritime OR/MS literature. Ronen (1982) gives a cubic function of optimal speed affected by the freight rate and fuel price for an income-generating leg and implicitly assume the leg to be repeated infinitely, which is agreed by Magirou et al. (2015). However, later works in Psaraftis and Kontovas (2014) claim that the optimal speed is independent of income in a single voyage scenario. To compensate the incentive of freight rate for such scenario, Psaraftis and Kontovas further claim that there is a non-trivial component, namely the in-transit inventory cost, as it would force the ship to travel faster when transporting more expensive cargo. The observations in practice do agree with the consequence of fast steaming. However, the underneath reason might not be the in-transit inventory cost as claimed. In real world business of tramp shipping, we notice that the freight rate is not only route specified but also cargo type specified. So, for more valuable cargo (e.g., the demand or price is high), the shipper is willing to pay more (e.g., higher freight rate) as has been shown in Hummels, Lugovskyy, and Skiba (2009) and the so-called in-transit inventory cost is incorporated into that additional freight rate already.

These market parameters, though known to be fluctuating over time for each port, are normally treated as known constant in the model since the state of market is too complicated (Psaraftis & Kontovas, 2014). Few work known so far would allow the state of market to be dependent on time. Besbes and Savin (2009) have assumed the fuel price to be a Markovian process with stationary error term over time in a bunker management problem. Yao et al. (2012) have tried to include 5 simple fuel price evolution scenarios for a given route in their liner shipping bunker management model. Magirou et al. (2015) examine the impact of freight rate and assume it to be stochastic and either independent or dependent on its past value for a sequence of voyages. Their results show that the uncertainty of economic conditions would impact the speed decisions.

Unlike the studies of static market condition, e.g., facts such as high fuel price would advocate slow steaming can be easily observed and explained by models (see Ronen (1982)), these stochastic models fail to provide analytical insights of how future evolution of economic conditions will impact current speed decisions. As a consequence, the analysis of time-varying market conditions is in a timely need to fill the gap of literature.

Adland and Jia (2018), in addition, point out that there are contractual factors to be considered as well, which is a pity in maritime optimisation that rare studies have incorporated. When talking to the industry, we realise that whenever a company wants to charter a ship, the differences of clauses, especially redelivery clause, would actually lead to a preference. This is to say, the contractual details are affecting the profitability of the company as well (or at least the utility function of the company), otherwise company should be neutral to it.

Two commonly used redelivery clauses in industry, namely the Shell Time 4 contract and ExonMobil 2000 contract, are examined in this chapter. In particular, the Shell Time 4 contract does not specify a restrict return time window. Instead, it says that, if the ship is on a laden leg (e.g., she is undertaking some tasks with cargo loaded) on the end of contract date, then the charterer can keep the ship to finish that task and return it once the cargo are unloaded. As to the ExonMobil 2000 contract, there is a given time window, normally plus or minus 45 days at the end of contract. In other words, the main difference between them is the flexibility, or more specifically, the exact available time horizon in the end of contract scenario.

5.1.3 Summary of contributions

In the present chapter, we formulate the end of contract scenario with a general assumption that the ship will be used for profitable round trips. In particular, each round trip contains a *laden leg* followed by a *ballast leg*, indicating the ship will travel with cargo first and re-position back without cargo. The model is constructed based on the cash-flow Net Present Value (NPV) framework with redelivery constraints. The aim is to examine the impact of time-varying economic conditions and redelivery clauses on optimal speed decisions, while maximising the profitability of the ship.

The main contribution is then to answer the following questions: (1) what is today's optimal speed decisions, when given certain information (or expectations) of the future market conditions; (2) how to evaluate the redelivery clause based on either financial or environmental criterion.

5.2 Time-varying market conditions

In traditional models, the analysis of economic parameters only concerns their impact on optimal speeds in a static condition. For example, Ronen (1982) and Psaraftis and Kontovas (2014) claim the optimal speed decision of each leg is independent of other legs, and thus current speed choice is decided by current market state. Another type of analysis, e.g., in Magirou et al. (2015), considers a sequence of legs in the infinite time horizon. They believe the optimal speed decision is impacted by the net revenue of the whole journey rather than the profitability of each leg.

The former statement clearly violates the practice in shipping industry as company would always change their strategy according to future expectations. For example, when companies realise there is a competitor chasing for the same job, e.g., the freight rate would decrease over time (if the competitor arrives first, he will pick the better job), the corresponding choice is to speed up. Similarly when there is an expectation of fuel price drop, they are willing to slow down rather than waiting in harbour to enjoy that price.

As to the latter, their model suggests the impact of economic conditions on current speed choice is uniformly distributed from each leg within the journey. Implicitly it is to say that the current speed decision is also impacted by the profitability of earlier legs as they assume the journey is repeated in the infinite time horizon (without the consideration of NPV). Now back to the case of a time charter contract, we only consider the usage of the ship in a finite horizon H. In a general sense of speaking, if companies decide to re-optimise the speeds after executing a few legs, will they still consider the performance of earlier legs? The answer is probably not in practice. The impact of earlier legs is arguably imposed only by changing the starting time of later legs when encountering uncertainties, especially in the scenario of which the arrival time is predetermined.

In this section, we will derive the analytical results of the economic conditions in a timely manner under a time charter contract. For ease of notation, we adopt model (4.6) given in Chapter 4 with n legs in total, but ignore the index of repetition i as the market condition for each leg may vary. It therefore contributes to the understanding of speed optimisation models in a time-varying setting. The aim is to better capture the rationale of practical phenomenons mentioned above that are rarely discussed or explained in literature. In particular, fuel price and freight rate are examined in the following section.

5.2.1 Fuel consumption cost

Fuel consumption cost is the majority of the ship's total operational cost. Thus a lot of efforts have been made towards minimising the fuel consumption cost, especially when the fuel price is high. The main streams of the related literature encourage the ship to slow down in order to save fuel consumption. In particular, emission related maritime literature has paid a huge attention to the potential means for slow steaming. However, as pointed out by Johnson and Styhre (2015), such investigation should be considered with a profit-seeking behaviour. In order to provide a better understanding for these related researches, we present how fuel price would motivate the ship to either travel fast or slow in a time-varying setting.

The analysis of fuel consumption cost, or the fuel price if given the fuel consumption to be fixed for each leg, can be separated into two parts: (1), what if we know the fuel price of current leg; (2), what if we know (or expect) the fuel price will change for later legs?

5.2.1.1 The impact of current fuel cost

Recalling (4.6), fuel price only impact the c^l term where the bunker cost is included. Now if we only consider G_1. By taking the derivative of c^l with respect to T, we have:

$$\begin{aligned}\frac{\partial C_n^l(T_n, L_{n-1})}{\partial T_n} &= c_n^f(L_{n-1})[F_n'(T_n, w_n)T_n + F_n(T_n, w_n)] \\ &= c_n^f(L_{n-1})k(W+A)^h[p + (1-g)(\frac{S_n}{24})^g T_n^{-g}]. \end{aligned} \tag{5.1}$$

The above mathematics are based on the assumption that the current bunker price $c_n^f(L_{n-1})$ is independent of the current speed choice T_n.

By substituting the constant coefficients from Appendix, e.g. 8.1.1, into (5.1), it is a negative definite function unless c^f term, which is the fuel price, could be negative. This implies that, slow steaming is always encouraged to save fuel consumption cost.

With the increase of T_n from the minimum to maximum within physical limits, $\frac{\partial C_n^l(T_n, L_{n-1})}{\partial T_n}$ shall decrease exponentially to the power of g if fuel price c^f is fixed. In the sense of practice, the exponential change of fuel price is not easily observed. Thus it is unlikely that the ship would speed up just for the benefit of a better fuel price with small discount.

For example, without the consideration of other parameters, if the ship has a chance to speed up from 10 kn to 12 kn for a lower bunker price (from 700 USD/tonne to 560 USD/tonne), the marginal fuel consumption cost at given speed choice, i.e., (5.1) would actually decrease (approximately from $-21,658$ USD per day to $-32,707$ USD per day, when the ship is fully loaded). This is to say, even though the fuel price is cheaper when speed up, one could now save $32,707$ USD by slowing down and spend one more day on sea, which is a stronger signal for slow steaming compared to travelling at 10 kn with higher fuel price. Thus the impact of fuel consumption cost (as the product of fuel price and total fuel consumption) would suggest the ship to travel maybe even slower than 10 kn due to the saving of fuel consumption.

5.2.1.2 The impact of future fuel cost

Now if we extend the analysis to G_2, by taking its derivative with respect to T_{n-1}, we shall get a similar structure as $\frac{\partial C_{n-1}^l(T_{n-1}, L_{n-2})}{\partial T_{n-1}}$, plus the aggregated term of future $\frac{\partial C_n^l(T_n, L_{n-1}) e^{-\alpha T_{n-1}}}{\partial T_{n-1}}$:

$$\frac{\partial C_n^l(T_n, L_{n-1}) e^{-\alpha T_{n-1}}}{\partial T_{n-1}} = e^{-\alpha T_{n-1}} [F_n(T_n, w_n) T_n (c_n'^f(L_{n-1}) - \alpha c_n^f(L_{n-1})) - \alpha C_n^h]. \quad (5.2)$$

If $c^f(L_{n-1})$ is a differential-able function of T_{n-1}, we have $c_n'^f(L_{n-1}) = \frac{\partial c_n^f(L_{n-1})}{\partial T_{n-1}}$. This is the marginal fuel price at given time point L_{n-1}. The term $F_n(T_n, w_n)T_n$ is the total fuel consumption for the last journey and the term αC_n^h is a discounted harbour fixed costs (with fixed amount of cargo to be loaded & unloaded).

On the other hand, if the fuel price is independent of time, e.g., we assume the fuel price to be a fixed constant, the first part of the equation shall be ignored. The only part left is the contribution of aggregated fuel consumption cost from future journey and it is negative definite.

Now we simplify the notations of (5.2) by getting rid of the C_n^h term (since we only focus on the fuel consumption cost now). It is negative definite, if and only if:

$$c_n'^f(L_{n-1}) - \alpha c_n^f(L_{n-1}) < 0. \tag{5.3}$$

This is because the term F_n is the daily fuel consumption, which is positive. And the term T_n is the travel time for n-th leg, which is also positive (otherwise we could ignore the n-th leg as it is not performed).

Here (5.3) suggests that if the marginal price (or the increase rate) of future bunker fuel $c_n'^f(L_{n-1})$ (for the n-th leg) at time L_{n-1} is outperformed by a discounted fuel price $\alpha c_n^f(L_{n-1})$, the aggregated impact of future fuel consumption cost will enlarge the incentive of slow steaming. In other words, unless we believe the fuel price will rise dramatically for the next leg, the best strategy is to travel slower in the sense of fuel cost saving.

Similarly, if we now move from G_2 to G_3, we can get the aggregated term as follows:

$$\frac{\partial[C_{n-1}^l(T_{n-1}, L_{n-2}) + C_n^l(T_n, L_{n-1})e^{-\alpha T_{n-1}}]e^{-\alpha T_{n-2}}}{\partial T_{n-2}} \tag{5.4}$$

$$= e^{-\alpha T_{n-2}}[c_{n-1}'^f(L_{n-2})F(T_{n-1}, w_{n-1})T_{n-1} \tag{5.5}$$

$$- \alpha c_{n-1}^f(L_{n-2})F(T_{n-1}, w_{n-1})T_{n-1} \tag{5.6}$$

$$- \alpha c_n^f(L_{n-1})F(T_n, w_n)T_n e^{-\alpha T_{n-1}}]. \tag{5.7}$$

Recalling (5.7) is the discounted future fuel consumption paid for the n-th leg, which is a large constant at this stage. Now we can show that (5.4) is negative definite as long as:

$$c_{n-1}'^f(L_{n-2}) - \alpha c_{n-2}^f(L_{n-2}) - M < 0. \tag{5.8}$$

Here $M = \frac{\alpha c_n^f(L_{n-1})F(T_n,w_n)T_n e^{-\alpha T_{n-1}}}{F(T_{n-1},w_{n-1})T_{n-1}}$ is a positive number, which can be understood as a ratio of the discounted future fuel consumption divided by the fuel consumption of current leg. Since M is positive for each leg, c'^f has to be larger in (5.8) than in (5.3). As a consequence, it is even harder to observe the ship to speed up in the earlier legs for the benefit of fuel price.

When assuming fixed parameters, the M term is expected to only increase, if we move from G_1 to G_n, as more and more fuel consumption costs have been accumulated. Thus the more legs repeated, the larger signal of slow steaming for the earlier legs are observed from the perspective of fuel cost.

For example, if we assume fixed fuel price 600 USD/tonne, then $c'^f = 0$ and $c'^f - \alpha c^f = 0 - \frac{0.08}{365} * 600 \approx -0.13$. This shows that the aggregated future impact now agrees with the marginal effect of fuel consumption costs paid at current stage. As a result, we get a stronger signal of slow steaming for earlier legs, as M term is larger there. The reasoning could be, if we travel faster to enjoy a cheap fuel price, at the same time, we also pay the future fuel consumption cost earlier. If the latter term is large, for sure everyone wants to postpone these payments as late as possible, without the consideration of other parameters.

If we extend the accumulated marginal effect iteratively until G_n, we shall expect to get the same structure $c'^f - \alpha c^f - M$ for each decision making stage and the above analysis will still hold.

In addition, the analysis could be extended to infinite time horizon following Chapter 4. When assuming there exists a boundary for the evolving fuel price, the impact of future fuel cost can be taken as fixed in the stationary state, e.g., M term has reached its maximum as a large constant. As a consequence, the optimal speed decisions for each repetition are not likely to be changed by the fluctuations. This would be interesting in practice, especially for liner shipping and related bunker management problems.

To summarise it, when the fuel price is fixed over time, the total impact of fuel consumption cost would urge the ship to always travel slower in the earlier stages. However, if the fuel price is changing over time, and for cases that we find the fuel price is going to rise dramatically (for the following journey), it now makes sense to travel faster. In practice, the exact impact would depends on the evolution of fuel price, e.g., assumed to be fixed or stochastic, and how much we know about the future.

5.2.2 Revenue

The revenue is often treated as constant in traditional models, especially for liner shipping. However, the freight rate is actually negotiable and agreed per cargo type, ship and even route. This is equivalent to say, the available jobs in a port can change over time, and thus the freight rate is a function of time. In liner shipping, this could be understood as the case where the demand and/or the unit price is no longer fixed. Similarly the investigation are presented in two parts.

5.2.2.1 The impact of current revenue

Now, we can take out the revenue part which is the R term from G_1. By taking its derivative with respect to T_n, we have:

$$\frac{\partial R(L_{n-1})e^{-\alpha T_n}}{\partial T_n} = -\alpha R(L_{n-1})e^{-\alpha T_n}. \tag{5.9}$$

If the freight rate available $R(L_{n-1})$ is independent of current speed choice T_n, (5.9) is negative definite unless the freight rate and the revenue $R(L_{n-1})$ can be zero or negative (freight rate and revenue are equivalent once the amount of cargo is fixed). This actually suggests that travel at a slower speed would decrease of time value of revenue received, and hence encourage the ship to travel faster. The discount term $e^{-\alpha T_n}$ then suggests that the longer travel time T_n we currently have, the less incentive we have to fasten the speed. For example, shortening a journey from 350 days to 340 days will have less improvements of the profitability than having the equivalent profitable journey being shortened from 50 days to 40 days.

5.2.2.2 The impact of future revenue

Assume the freight rate is a differential-able function of travel time T. If we look at the G_2, the aggregated marginal impact of future revenue is shown as follows:

$$\frac{\partial R(L_{n-1})e^{-\alpha(T_n+T_{n-1})}}{\partial T_{n-1}} = (R'(L_{n-1}) - \alpha R(L_{n-1}))e^{-\alpha(T_n+T_{n-1})}, \tag{5.10}$$

where $R'(L_{n-1}) = \frac{\partial R(L_{n-1})}{\partial T_{n-1}}$ is the marginal freight rate at time point L_{n-1} (and L_{n-1} is affected by T_{n-1}).

It is clear that (5.10) is negative definite if the freight rate is fixed (thus $R' = 0$). Then the ship is encouraged to travel even faster in the earlier legs to receive the aggregated future revenue earlier. With the larger discounted future rewards ($\alpha R(L_{n-1})$), we will have the stronger signal for fast steaming.

If we find the marginal price (the change rate) of freight rate $R'(L_{n-1})$ for the next leg outperforms the discounted freight rate $\alpha R(L_{n-1})$), then it would actually push the ship to travel slower on current leg in order to get better aggregated revenues. This is the case where we think the time of market is going to be prosperous and a better freight rate can be reached by waiting. Thus it makes the ship less willing to collect the revenue of current leg earlier.

However, according to the discount term $e^{-\alpha(T_n+T_{n-1})}$ above, if the future travel time T_n is too long, the accumulated revenue would have almost no marginal effect on current speed choices. For instance, assuming $T_n = 100$, then by shortening T_{n-1} from 50 to 1 will make the discount coefficient $e^{-\alpha(T_n+T_{n-1})}$ increase only from approximately 0.97 to 0.98. As a consequence, the change of T_{n-1} is not that important. Numerically, the discount coefficient equals to 0 when $T_n+T_{n-1} \to \infty$ and it equals 1 when $T_n+T_{n-1} = 0$. Since the future travel time, e.g., T_n here, is already decided, the longer it is, the less impact we shall expect from it on current speed decision.

By extending (5.10) to G_3, we will have a similar structure as follows:

$$\frac{\partial R(L_{n-1})e^{-\alpha(T_n+T_{n-1}+T_{n-2})} + R(L_{n-2})e^{-\alpha(T_{n-1}+T_{n-2})}}{\partial T_{n-2}}$$
$$= e^{-\alpha(T_{n-1}+T_{n-2})}[R'(L_{n-2}) - \alpha R(L_{n-2}) - \alpha R(L_{n-1})e^{-\alpha(T_n)}]. \tag{5.11}$$

Here term $\alpha R(L_{n-1})e^{-\alpha T_n}$ is a positive number, which is the discounted future revenues. By extending (5.10) and (5.11) iteratively to G_n, we shall get the similar structure $R' - \alpha R - FR$ recursively, if we use FR to denote the terms of discounted future revenues.

If the identical journey is assumed to be repeated, FR is guaranteed to be semi-positive and non-decrease (on ballast leg the revenue is 0 and on laden leg it shall be positive). This is to say, the more future laden leg that we consider, the larger incentive for ship to travel faster in the beginning stage (as the marginal effect of aggregated future revenues will tend to be negative more easily with a larger FR).

The analysis can be as well extended to the infinite time horizon following the work of Chapter 4. For example, if the liner shipping company already estimates or obtains the maximum FR in the stationary state, the impact of future revenue is then fixed for each leg. In such case, using same speeds on identical journey would be a good approximation, despite the market fluctuation.

To conclude it, the incentive from revenue will always push the ship to travel faster in the beginning, if the freight rate is assumed to be fixed. But if we expect a prosperity of market (e.g., a higher freight rate) in the near future, it now makes sense for ship to slow down and wait for that price. In practice, we notice that those impacts from fuel price and freight rate are often opposite and thus the mixed incentive is only known when we get the information of all the economic market conditions. With the knowledge about future, the optimal speeds would vary accordingly as present in this section.

5.3 Redelivery clause

The end of contract scenario, although seems to be far away from the negotiation of charter contract, it is non-trivial as normally one would re-optimise the operational plan once per period, and also there is a trend of short contracts in practice. When we are at the late stage of the contract to evaluate the redelivery clause, everything is more certain and we can calculate the performance as in a deterministic sense.

In this section, we will first introduce two typical types of redelivery clauses, namely ExxonMobil 2000 and Shell Time 4, and then use the numerical example to show the preference of them based on both economic and environmental criterion. Algorithm 2 given in Chapter 3 can be modified (with each iteration solved instead in a manner like Algorithm 3) and therefore used here to solve the optimisation problem with time constraints. The maximum of number of leg is limited to 4 in this chapter for the ease of computation. When the market parameters are time-dependent and known as a priori, the algorithm can solve the problem within a few minutes.

5.3.1 ExxonMobil 2000 clause

One of the most commonly used redelivery clause is the ExxonMobil 2000 clause, where an additional duration is specified for the charterer to return the ship. Normally, the duration could range from plus 45 days to minus 45 days.

Without any other requirements, it can be seen as an extension of the original contract duration, because the charterer could still use the ship at their own preference within the redelivery time horizon.

However, since the time horizon is clearly stated, it is not allowed to violate the final return date. Otherwise a huge penalty shall be paid to the ship owner. Though, arguably there is a chance that the charterer would like to violate the redelivery clause if he can gain profit against such penalty, in a deterministic setting, it is foreseeable and avoidable for such violation. For simplicity, in this chapter, the ship has to be returned in time and there is no allowance for late return.

The mathematical expression of ExxonMobil 2000 clause can be written as follows:

$$T^* := \{T_n^*, T_{n-1}^*, \dots, T_1^*\} = arg \ \ maxG := \{G_1, G_2 \dots, G_n\},$$

$$subject \ \ to : \frac{S_i}{365(24)v_{max}} \leq T_i^* \leq \frac{S_i}{365(24)v_{min}}, \tag{5.12}$$

$$L_n^* \leq H + H_{redelivery}, \tag{5.13}$$

where $H_{redelivery}$ is the additional redelivery duration. Constraint (5.12) is the physical limits of the ship speed. Then the total completion time of all the tasks is guaranteed to be within the total duration set in contract by constraint (5.13).

5.3.2 Shell time 4 clause

Shell Time 4 clause, on the contrary, would not specify a clear duration for the redelivery. It says that the charterer could still use the ship and return immediately if the ship is carrying cargo on the date when contract is expired. Otherwise, the ship needs to be returned on that date if it is on ballast leg.

This is to say, a profit seeking decision maker would split the whole problem into two sub-problems, where the first one is to use up the original time contract as much as possible with the last leg to be a ballast one. Then the second step is to optimise the single laden and ballast round trip scenario with infinite time horizon.

However, the tricky thing is, when to start the last laden leg. Only if the start time of last laden leg is decided, the final duration for the repeated journey problem is known. In this chapter, due to the flexibility of dynamic model, the sub-problems will be solved together. The whole problem can formulated as follows:

$$T^* := \{T_n^*, T_{n-1}^*, \ldots, T_1^*\} = arg \;\; max \;\; G := \{G_1, G_2 \ldots, G_n\},$$

$$subject \;\; to : \frac{S_i}{365(24)v_{max}} \leq T_i \leq \frac{S_i}{365(24)v_{min}}, \tag{5.14}$$

$$T_i^*(laden) = \begin{cases} T_i^*(laden) & if \;\; L_{i-1}^* < H; \\ 0 & \text{otherwise,} \end{cases} \tag{5.15}$$

$$T_i^*(ballast) = \begin{cases} T_i^*(ballast) & if \;\; T_{i-1}^*(laden) > 0; \\ 0 & \text{otherwise.} \end{cases} \tag{5.16}$$

Constraint (5.14) satisfies the physical limits of the ship speed. Here we introduce constraints (5.15) and (5.16) to guarantee the redelivery clause to be met. Constraint (5.15) would omit any laden leg that start no earlier than H, and constraint (5.16 indicates that the ship has to re-position herself every time after a laden leg.

5.3.3 Clause analysis

From the mathematical expressions above, the redelivery clause will not change the structure of the original problem without time horizon constraint. However, such constraint, when implemented strictly, will have an impact on the available region of the total completion time L. Thus, under each contract scenario, there is a chance that we are unable to reach the optimum, and hence make a difference. In another word, the difference is between (5.13) and (5.15)-(5.16).

For ExxonMobil 2000 clause, (5.13) has no specified restriction on laden or ballast leg. The constraint itself will either omit the later round trip due to insufficient time available or push the ship to travel faster if the time is sufficient but tight.

As to Shell Time 4 clause, (5.15) and (5.16) give no strict time window and allow the ship charterer to use it freely after H. In other words, we can consider this part as the same to the original problem without time constraints.

However, there is a condition to enjoy such benefit. The sub-problem before H can be considered exactly as the ExxonMobil 2000 clause but with a shorter time horizon. Hence we would expect the ship to travel faster on the earlier legs due to tight time window, and then slow down to fully exploit the benefit of the last journey after H.

The discussion above implicitly assumes that the ship could at least undertake one round trip before (for Shell Time 4 clause) or around (for ExxonMobil 2000 clause) H, and then perform the last round trip after H. It is equivalent to say, if the journey length of one round trip is approximately $H_{redelivery} = 45$ days, such that the ship can roughly operate one more round trip under ExxonMobil 2000 clause (compared to the original problem without redelivery), the comparison between the two types of redelivery clauses will depend on further information, e.g., logistics task, market condition, etc.

In addition, there are also two other scenarios to be clarified: (1) If the journey is short enough, according to Shell Time 4 contract, no more than one journey could be performed after H, while ExxonMobil 2000 contract gives such freedom. So if the ship can execute more than one round trip between the time $[H, H + H_{redelivery}]$, ExxonMobil 2000 clause is favored without questions; (2) If the journey is too long, such that the ship could only perform one round trip within the whole time window $[0, H + H_{redelivery}]$, the Shell Time 4 clause problem will now be transferred to the original problem without time constraint, while the ExxonMobil 2000 clause problem remains the same. Thus the optimum is easier to reach under Shell Time 4 clause rather than under ExxonMobil 2000 clause.

5.4 Scenario based performance

In this section, we would like to further validate the analysis discussed earlier for redelivery clauses based on operational scenarios. Without losing generality, assume the same ship is chartered ($f^{TCH} = 22,000$ USD/day) to transfer same amount of cargo ($d = 150,000$ tonnage) in 4 different scenarios that the ship is going to repeat the logistics tasks several times on a laden-ballast basis (e.g., $A - B - A$ scenario). The end of contract duration (i.e., the available time left in original contract H for us to consider the redelivery problem) is tested from 0 days to 45 days.

In addition to the financial objective function, which reflects the decision maker's profit-seeking behaviour, we also evaluate the redelivery clauses based on environmental criterion, which is often concerned in recent transportation literature, e.g., Bektaş and Laporte (2011); Corbett et al. (2009); Erdoğan and Miller-Hooks (2012). Since the emission of a ship is directly correlated to the fuel consumption, in this chapter, we adopt the *unit fuel consumption per tonnage cargo transported* as the standard for environmental performance. Here the higher ratio means worse energy efficiency as it would pollute more emissions when fulfilling fixed logistics demand.

5.4.1 Short journey

We start our investigation from the easy case first. Suppose the ship is traveling between Marseilles and Amsterdam with the distance being $S = 2079$ nautical miles. This is a representative of short-sea shipping that commonly seen in shipping practice. For simplicity, let the freight rate ($r = 12$ USD/tonne) and bunker price ($c^f = 600$ USD/tonne) to be fixed over time. This is the case when we lack the exact information about future market revolution but one can still estimate the future based on his own expectations (e.g., using the expected mean for economic parameters).

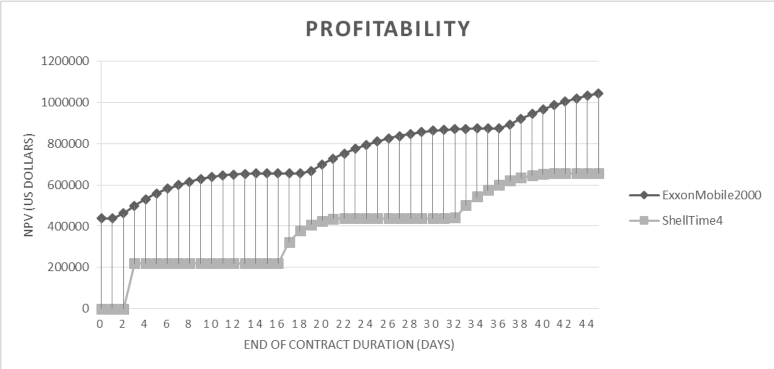


Figure 5.3: NPVs of two types of redelivery clauses between Marseilles and Amsterdam

In this example, the approximate travel time of one round trip is very small such that the ship can execute more than one round trip within the additional 45 days under ExxonMobil 2000 contract. As a consequence, Figure 5.3 suggests that Shell Time 4 contract is completely unfavorable.

Note that, for Shell Time 4 clause, when the end of contract duration is less than 2 days, the ship cannot perform any task simply because it takes approximately 3 days to load all the cargo. The actual loading and unloading time could be very different in reality, but here in this chapter, it will not change the results of comparison as these extreme cases are omitted later.

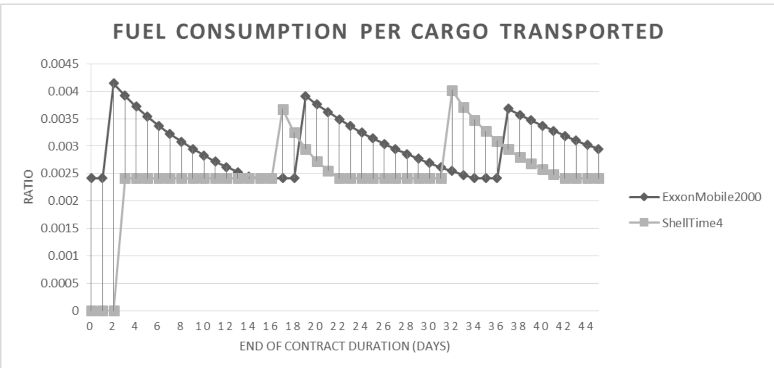


Figure 5.4: NPVs of two types of redelivery clauses between Marseilles and Amsterdam

Table 5.1: Expected performance for both contract between Marseilles and Amsterdam

	Short Journey	
Clause type	**Shell Time 4**	**ExxonMobil 2000**
E(NPV)	416,585.2	776,664.1
E(fuel)/cargo	0.0026	0.0030

According to the environmental performance, Figure 5.4 gives an suggestion that the economic advantage of ExxonMobil 2000 clause is based on the sacrifice of environmental efficiency. In addition, the saw tooth patterns indicates that, for several cases, ExxonMobil 2000 contract is more emission friendly (when a new round trip needs to be squeezed under Shell Time 4 contract), and thus it is hard to draw a general conclusion.

For the sake of fairness, we omit the cases that ship cannot execute any task under Shell Time 4 contract. Since the chance of arriving at each day is uniformly distributed, i.e., the chance of having different end of contract duration is equal, we can calculate the expected performance and summarise it in Table 5.1. It shows that choosing ExxonMobil 2000 contract will provide 86.4% more net revenues at the cost of 14.1% higher fuel consumption rate on average.

5.4.2 Long journey

Similarly, assume the ship is used to transport cargo between Shanghai and Santos ($S = 13043$ nautical miles), which is the case of deep sea shipping. Now the approximate travel time for a single leg is close to 45 days. In order to make the journey profitable, we increase the freight rate to $r = 25$ USD/tonne.

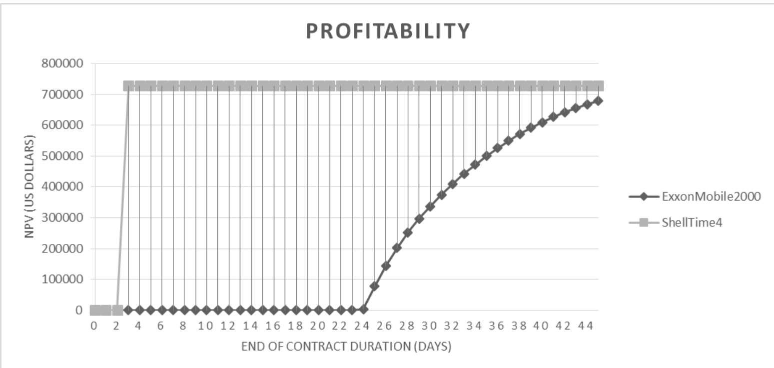


Figure 5.5: NPVs of two types of redelivery clauses between Shanghai and Santos

As expected, Figure 5.5 suggests that it is economically benefit to use Shell Time 4 contract as the operator could use up the flexibility as long as the ship can get loaded before contract expire date. On the contrary, ExxonMobil 2000 clause hardly provides the same profitability as the total duration available is not sufficient. Eventually they should reach the same profitability if more time is provided.

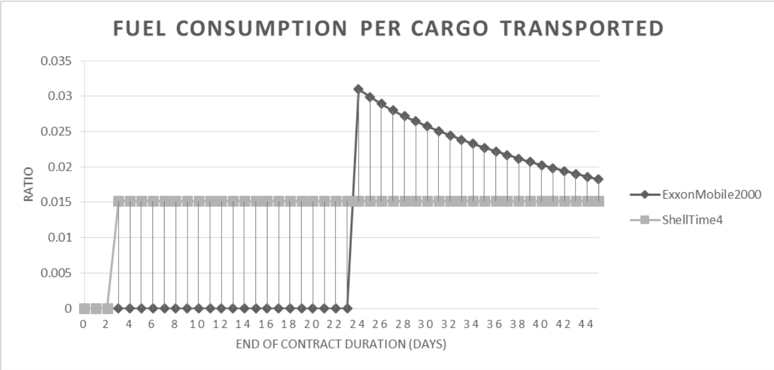


Figure 5.6: Fuel consumption per tonnage cargo transported between Shanghai and Santos

Furthermore, Figure 5.6 now agrees with Figure 5.5 that Shell Time 4 clause is also environmentally better than ExxonMobil 2000 clause, as the ship has to speed up in order to finish the round trip and gradually slow down now.

Table 5.2: Expected performance for both contract between Shanghai and Santos

	Long Journey	
Clause type	**Shell Time 4**	**ExxonMobil 2000**
E(NPV)	727,705.9	437,739.2
E(fuel)/cargo	0.0152	0.0235

If we again omit the extreme cases of which ship cannot perform any logistics task, Table 5.2 summarises the expected performance based on both criterion to give a general preference of Shell Time 4 clause as it provides better performance both in the sense of economic and environment.

5.4.3 Moderate journey

Figure 5.7: NPVs of two types of redelivery clauses between Basra and Hong Kong

Now move on to the case where the journey time is moderate. In this case, we use a route from Basra to Hong Kong (S = 6223 nautical miles), which is an important example of oil transportation. To make sure the whole round trip is profitable, we assume the freight rate to be $r = 22$ USD/tonne. This is the case of which approximately two round trips can be executed by the ship under both redelivery clause.

The economic performance of ExxonMobil 2000 clause, according to the optimal strategy, is always no worse than Shell Time 4 clause, as depicted in Figure 5.7. Due to the attractiveness of freight rate, the ship is encouraged to squeeze in the second round trip at a high speed, and thus we see the rise of profitability as a curve. However, when subjected to Shell Time 4 clause, there is less motivation to chase such financial benefit due to its tight time window for the first round trip.

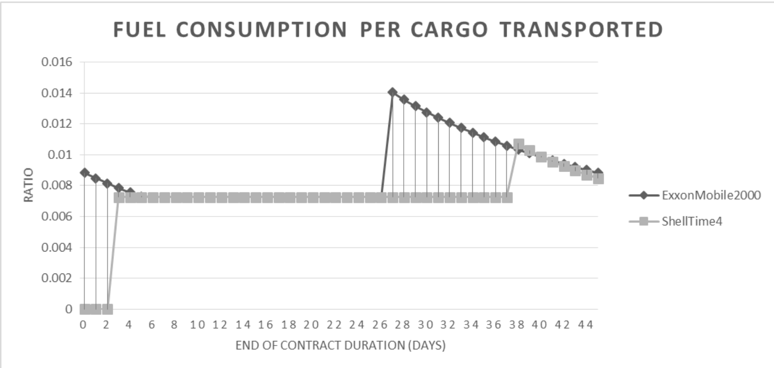


Figure 5.8: Fuel consumption per tonnage cargo transported between Basra and Hong Kong

On the other hand, based on the emissions, Figure 5.8 suggests that Shell Time 4 clause contributes more to the environment. However, with the extension of the duration, both of them would perform similarly.

Table 5.3: Expected performance for both contract between Basra and Hong Kong

	Moderate Journey	
Clause type	**Shell Time 4**	**ExxonMobil 2000**
E(NPV)	691,752.0	763,727.0
E(fuel)/cargo	0.0077	0.0090

Similarly we calculate the expected profitability and environmental performance and summarise in Table 5.3. The expected economic performance of ExxonMobil 2000 clause now brings in 10.4% more profits at the cost of 16.8% more fuel consumption per cargo transported, which is no longer as attractive as in the scenario of short journey.

As a matter of fact, the previous examples reveal that the main advantage of ExxonMobil 2000 clause is to better enjoy the economic benefit, when the market performance is good, as implicitly assumed in this chapter. Intuitively, one can then expect to see a larger difference when the market is in prosperity, e.g., freight rate is high or the fuel price is cheap, and a smaller gap if on the contrary. Thus the comparison of this scenario needs to also take into account the market conditions as it may change the preference.

5.4.4 Time varying market condition

To give an example of how time varying market conditions will change the preference of delivery clause, we further allow the fuel price to evolve over time based on the moderate journey case as discussed above.

For simplicity, assume the fuel price is fixed to be 600 USD/tonne for the first logistics leg, and then rises 5 USD/tonne per day over time. This is indeed a very simple linear assumption of the fuel price, as one can further extend it to a stochastic process or to follow any other distribution accordingly.

We argue here, for a given finite time horizon, the bunker price is less likely to "jump" between "bad" and "good" states frequently within a short period, see for instance SHIP & BUNKER (2020). Thus it is reasonable to assume certain information about market evolution is available, e.g., the expected mean value. Then it should give us a good approximation, even if the actual fuel price comes with a random but small error term, as examined in Magirou et al. (2015). An extension to allow freight rate changing over time can be easily implemented in the same manner.

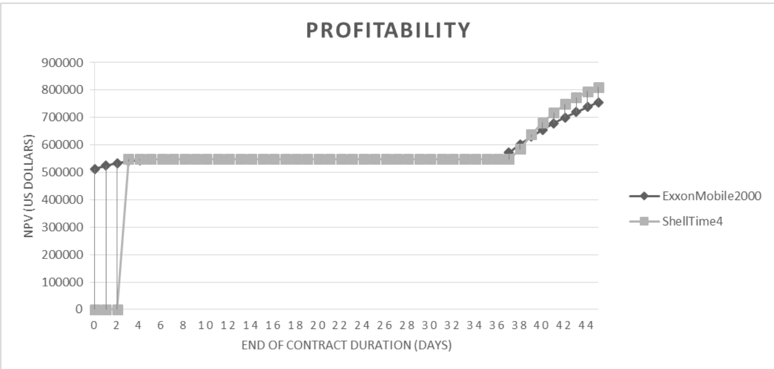


Figure 5.9: NPVs of two types of redelivery clauses between Basra and Hong Kong with varying fuel price

When other parameters remain fixed, a higher fuel price will surely push the ship to travel slower for the last round trip. In another word, it is no longer profitable to squeeze it in the limited time window at high speed, as confirmed by Figure 5.9. In particular, under Shell Time 4 clause, the ship is less interested in fast steaming even with fixed fuel price. Thus its optimal strategy of operation will not be hugely impacted compared to when subjected to ExxonMobil 2000 clause.

As to the earlier round trip, the high fuel cost will naturally encourage the ship to slow down. On the other hand, knowing we have to pay more fuel cost in the future brings in an opposite incentive that urges the ship to travel actually faster. As a consequence, the optimal speed decisions will highly depend on the ship's characteristics and other parameters such as profitability.

Actually, given such a scenario indicating bad economic conditions, the differences between the two clauses are reduced. The ship under ExxonMobil 2000 clause now decides to execute the second round trip only if the time is more sufficient, as the whole journey is now less profitable. As a consequence, the two clauses give almost identical performances based on either economic or environmental standard.

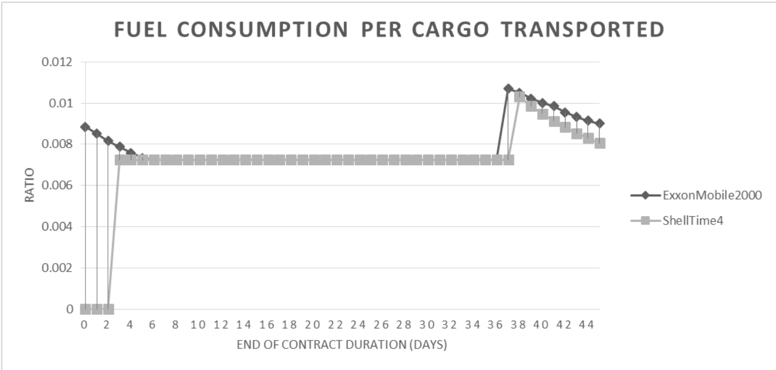


Figure 5.10: Fuel consumption per tonnage cargo transported between Basra and Hong Kong with varying fuel price

Table 5.4: Expected performance for both contract between Basra and Hong Kong with varying fuel price

	Varying Fuel Price	
Clause type	**Shell Time 4**	**ExxonMobil 2000**
E(NPV)	579,764.7	573,803.5
E(fuel)/cargo	0.0076	0.0078

By comparing Table 5.3 and Table 5.4, we notice that, as long as the ship is not tempted to speed up and undertake additional logistics tasks, e.g., when economic condition is bad, Shell Time 4 contract has a small advantage according to both criterion. However, in other situations, it is clear that the ExxonMobil 2000 clauses provides better flexibility such that the ship could react faster to enjoy the potential benefit of market prosperity.

In this scenario, 33.1% more profit can be earned by using ExxonMobil 2000 clause at the cost of 14.6% more fuel consumption per cargo transported, if the actual fuel price is not rising as expected and fluctuating around 600 USD/tonne. The corresponding figure under Shell Time 4 contract is 19.3% more profit with only 1.0% more fuel consumption per cargo transported.

To conclude it, for short and long journeys, the preference of each clause is more easy to decide, while it requires the decision maker to think more carefully about the scenario of moderate journey. We show that including the dynamic of future market conditions can change the performances of the redelivery clause. In particular, The market conditions and potentially the relevant policies such as carbon context fuel tax may thus enlarge or reduce the difference between them. In a general sense of speaking, the economic advantage of ExxonMobil 2000 clause often comes with the sacrifice of energy efficiency, while Shell Time 4 contract, purely based on environmental criterion, contributes more towards green shipping. Hence the exact selection of redelivery clause and thus charter contract would be considered only with specific conditions.

5.5 Conclusions

At the time of writing this chapter, the world is experiencing a global challenge of COVID-19. The crude oil trade price has first ever been negative. As a consequence, the bunker fuel price drops a lot worldwide. An earlier discussion in Devanney (2010) reveals that such change of fuel price will eventually impact the spot market, e.g., the freight rate.

For instance, the worldwide average price of MGO drops from 472.5 USD/tonne on April 6th to 399 USD/tonne on May 5th (SHIP & BUNKER, 2020), while the tanker spot rates of VLCC in May (Hellenic and International Shipping, May 05, 2020) are less than half of their past values in April (Hellenic and International Shipping, Apr 05, 2020). However, for smaller sizes of ship, e.g., Panamax, the freight rate actually rises. Naturally, shipping company that considers to charter different types of ship for various logistics tasks needs to evaluate his own redelivery problem under particular market dynamics.

To solve such maritime speed optimisation problem combined with the redelivery option, this chapter has provided a mathematical model based on cash-flow Net Present Value (NPV) approach.

In particular, the analytical results of economic parameters, e.g., freight rate and fuel price, are present for the better understanding of how and why the information of future market condition can impact today's optimal decision. The managerial insights can help, e.g., the company, to understand how and why should the ship react to certain market evolution, and thus improve the interpretation, particularly, in the stochastic setting.

The comparison between ExxonMobil 2000 clause and Shell Time 4 clause can be separated into three general scenarios: (1) For deep sea shipping, Shell Time 4 clause is preferred financially and environmentally; (2) For short sea shipping, ExxonMobil 2000 clause allows the ship to fully enjoy the economic benefit at the cost of polluting more emissions on sea; (3) For other type of shipping in a moderate journey duration, the selection of redelivery clause will highly depends on the market conditions.

Back to the example given above, a decision maker that considers to charter a Panamax may have a completely different preference than someone who needs to charter a VLCC, even if we assume the other settings to be identical. Similarly, people having different expectations about future evolution of the market may end up with different charter contracts as well.

We believe that the results provided have contributed to the understanding of market dynamics and charter contract agreements, e.g., redelivery clause, in the context of maritime speed optimisation. The further research direction includes other type of contractual analysis. Building up understanding towards stochastic environments could also lead to more accurate results as well.

Chapter 6

Selected topics

6.1 Introduction

In this chapter, we are going to investigate several topics across the maritime speed optimisations that are not yet fully explored, even in this dissertation, and thus leave room for further research. These topics may help to deepen the understanding of how best formulate decision models to help address both classic (e.g. route selection) and more recent optimisation problems (e.g. ship speed, contract choice). We hope to demonstrate that the consideration of the Net Present Value criterion, and the more explicit consideration of contract type, may lead to further refinements and improvements to the current models from the maritime optimisation literature.

First, we will investigate the role and importancec of the harbour time, or the time that the ship spends in a port in between ocean going journeys. On one hand, the speed optimisation models often treat it as a constant and thus claim it is independent on the optimal solutions. On the other hand, emission related studies, as classified in Psaraftis and Kontovas (2013), have investigated the potential environmental contribution by assuming a reduced harbour time. Also, there is a considerable number of OR literature in port operations. As a consequence, it is worthwhile to investigate how harbour time serves in the maritime speed optimisation, especially within our NPV framework. A numerical example adopted in literature (Corbett et al., 2009) is also present to further validate our findings.

An extension to the discussion earlier in Chapter 5 is to evaluate the impact of an alternative contract between ship owner and charterer, also known as the *revenue sharing contract*. We compare this type of contract with the classic fixed daily hire contract, from both the viewpoint of the charterer as well as the ship owner. The analytical result of how charter hire cost will impact speed decisions is also developed. We present numerical examples in simple yet representative scenarios with time-varying charter hire index.

Another topic is to evaluate the *Contract of Affreightment* (COA), which provides certain flexibility for the ship operator to transport the shipments. This problem is closely related to the route selection problem in which the ship operator may decide to pick up 'optional' cargoes, similar to tramp shipping, at some points in the execution of a COA. However, with the COA contract, the scenario is sort of a 'mix' of liner shipping and tramp shipping, in that a dedicated model for such situation is needed. In particular, we demonstrate the important of the explicitly recognising of the COA contract and the impact of the FPP.

In the final part of this chapter, we the extend the NPV/contract modelling framework developed in previous chapters to the case of stochastic data. By introducing the *Markov decision process* modelling technique, the probabilities of state transitions can be assigned to reflect the expectations of future events by decision maker, and thus the aim is to find an optimal policy. As an application, we discuss problems related to finding the optimal lay-up strategy. e.g., when and why it is better for ships to lay-up.

6.2 Harbour Time

Harbour time, or often known as port operation efficiency, itself attracts a great number of attentions. There is a vast number of literature on the port operation problems, including but not limited to berth allocations (e.g., Imai, Nishimura, and Papadimitriou (2001, 2003); Imai, Sun, Nishimura, and Papadimitriou (2005); K. H. Kim and Moon (2003)) and crane operations (e.g., Goodchild and Daganzo (2007); Zhang, Wan, Liu, and Linn (2002)), aiming to minimise the harbour time, whereby Du et al. (2011) further incorporate the airborne emissions to the optimisation. As a consequence, reducing harbour time is recognised to be the potential means within the shipping industry to make a contribution towards sustainability, e.g., Psaraftis and Kontovas (2015).

In particular, with the developing of global warming and associated emission problems, we notice the bloom of research with respect to improving environmental performance, especially by reducing harbour time, in maritime shipping (see Devanney (2011); Psaraftis and Kontovas (2013)). However, the lack of understanding towards the relationship between harbour time and optimal speed decisions leads to a pitfall in literature that one can hardly explain why this "cheap and efficient" means has not yet been adopted in practice.

Perhaps, by adding a new dimension in our cash-flow framework, namely the *time dependency* as discussed above, we are now at a better position to close this gap in literature, with the ability to present the throughout analysis of harbour time in the generalised shipping scenarios. This can potentially contributes to, including but not limited to, the area of green shipping by providing a numerical model that can reflect impacts of harbour time associated factors, and also the integral consideration of both the ship and the port to capture the value of harbour time saved, e.g., by updating technique, equipment or policy.

Adding the harbour time T^p, intuitively it seems to be equivalent to add a constant directly to the optimal travel time T^i_j, and thus has no impact on the speed decisions (because T^p is irrelevant to T^i_j). This is actually claimed by many studies in maritime literature, e.g., Fagerholt and Psaraftis (2015) and Magirou et al. (2015). However, it is not true from the perspective of cash flows.

Consider a general scenario of a round trip consisting m legs (from P_1 to P_m and back to P_1) being repeated n times with the revenue structure of each leg being $h_j(t_j)$ (t_j is the travel time of the j-th leg within the round trip) and the total revenue structure being $h(\Gamma)$. Note that actually the fuel consumption is irrelevant to T^p, thus one cannot directly add the T^p to the corresponding T^i_j. In addition, the harbour time T^p also port specific as each port would have different traffic volume and operation efficiency.

Without loss of generality, assume the harbour time T^p is added at a random port P_s. This could also be interpreted as the ship needs to be maintained there during each repetition of the round trip. The impacted revenue structure of the round trip h^s could be expressed as:

$$h^s(\Gamma'_i, T^p) = h_1(T^{i\prime}_1) + \cdots + h_{i-1}({T^i_{s-1}}')e^{-\alpha(T^{i\prime}_1 + \cdots + {T^i_{i-2}}')} \tag{6.1}$$

$$+ [(h_i(T^{i\prime}_s) + \cdots + h_m({T^i_m}')e^{-\alpha(T^{i\prime}_s + \cdots + {T^i_{m-1}}')})e^{-\alpha T^p} \tag{6.2}$$

$$- \int_0^{T^p} f^{TCH} e^{-\alpha t} dt]e^{-\alpha(T^{i\prime}_1 + \cdots + {T^i_{s-1}}')}. \tag{6.3}$$

For simplicity, denote (6.1) by Q_1 and (6.2) by $Q_2 e^{-\alpha T^p}$ Then (6.3) can be rewritten as:

$$h^s = Q_1 + (Q_2 e^{-\alpha T^p} - \frac{f^{TCH}(1 - e^{-\alpha T^p})}{\alpha})e^{-\alpha {L^i_{s-1}}'} \tag{6.4}$$

The corresponding aggregated net revenue can be written as:

$$G^s_i = h^s(\Gamma'_i, T^p) + G^s_{i-1} e^{-\alpha({L^i_m}' + T^p)}. \tag{6.5}$$

We now present the harbour time effect by giving Theorem 6.1 here:

Theorem 6.1. *(harbour time effect) For any fixed harbour time $T^p > 0$, the following statements hold in line with the sense of practice*[1]*:*

[1] (1) The decision maker would not repeat any journey that generates no profit (e.g., the net revenue of each trip is negative or zero); (2) The harbour time and travel time spent should be within a reasonable region (e.g., cannot be years long).

- *If* $Q_2 + f^{TCH} > 0$ *(i.e., the logistics tasks between Ps to* P_m *and back to* P_1 *generate profits or only small loss), reducing harbour time would improve the total net revenues, e.g.,* $G_i^s(\Gamma_i', T^p) < G_i(\Gamma_i)$*;*

- *Without repetition, i.e.,* $i = 1$*, the optimal speeds of the legs from* P_s *to* P_m *and back to* P_1 *are not affected by adding harbour time* T^p*, i.e.,* $T_j^{i'} = T_j^i$*;*

- *Without repetition, i.e.,* $i = 1$*, the optimal speeds of the rest legs from* P_1 *to* P_s *are impacted by harbour time* T^p *that:*

  i. If $Q_2 + f^{TCH} \geq 0$*, adding harbour time will urge the ship to travel slower;*

  ii. If $Q_2 + f^{TCH} < 0$*, the effect discussed above will be reduced with larger harbour time* T^p*;*

- *With repetitions, i.e.,* $i > 1$*, if* $Q_2 + f^{TCH} > 0$*, the ship will travel slower for each leg, when harbour time is added, i.e.,* $T_j^{i'} > T_j^i$*.*

The first statement of Theorem 6.1 (proof in 8.3.1) explicitly shows the trade-off between aggregated future net revenues and additional charter hire. By adding harbour time, part of the future profits are delayed (i.e., Q_2) and additional charter hire costs are added (i.e., $\frac{f^{TCH}(1-e^{-\alpha T^p})}{\alpha}$).

As a consequence, its impact will base on scenarios given. For example, if the term Q_2 here is a sum of loss (e.g., a series of re-positioning legs), it is desirable to postpone the due date to pay that future bill at the cost of extending charter contract. Thus reducing harbour time is not always optimal from the perspective of profit maximising.

Without the consideration of future repetitions, i.e., $n = 1$, as the additional charter cost is already paid at port P_s and the rest part of payments Q_2 remains the same, the original speed decisions after port P_s still work. This is exactly the case of *what's done is done*, and thus harbour time will not impact on the corresponding speed choices.

However, for the voyages before port P_s, the decision maker knows that the future is different. Additional charter cost $\frac{f^{TCH}(1-e^{-\alpha T^p})}{\alpha}$ and the delayed future payments Q_2 together change the total profitability, and thus the speed decisions have to be re-optimised accordingly.

In the case that the total profitability is reduced by adding harbour time (i.e., $Q_2 + f^{TCH} > 0$), such incentive of advancing the future rewards at the cost of bearing a worse profit of current voyage is reduced as well. For example, if the decision maker realises that only $500,000$ USD, instead of $1,000,000$ USD, are waiting to be collect at the destination port, it makes no sense for him to travel as fast as before on current leg, especially when fast steaming is expensive.

As a final point, to evaluate the impact of harbour time, it is necessary to clarify the impacted profit structure first and then examine the effect accordingly with given conditions. In this section, some mild constraints are introduced to better match the interest of practical issues. Within this context, Theorem 6.1 reveals that, for most profitable scenarios, reducing harbour time would lead to higher economic benefit, which explains explicitly why should we improve port efficiency and port-ship coordination as suggested in literature, but for different reasons. Only if the profitability of known future is reduced by adding harbour time, the ship is willing to travel slower, and thus consume less bunker fuel and contributes more to the environment. This, to a degree, explains why there exists 'unknown' barrier, arisen by Johnson and Styhre (2015), that stops the adoption of slow steaming by reducing harbour time. An example obtained from Corbett et al. (2009) will be discussed further in Section 6.2.1 to validate the key points given here.

6.2.1 The impact of harbour time

An example given here is Corbett et al. (2009), where they have examined the speed reduction given a Contracts of Affreightment (COA) scenario with a profit-seeking operator to study its effectiveness and cost for better understanding the emissions from ocean-going shipping. According to the COA, the fleet has to satisfy a total amount of cargo to be transported annually, so as a consequence of speed reduction, the operator might use more ships to fulfill the contract requirement. They assume pretty much everything to be known and fixed over time and present their numerical example with real data collected. In the formulae, the optimal speed and hence the travel time is obtained for a single journey and then the number of trips per year made by this ship is accounted to calculate the maximised profit per year. Thus the underlying assumption is a single journey scenario even though the criterion seems to be revenue per time unit. We notice that harbour time is not accounted in their paper while it is shown to be non-negligible for *revenue per unit time* criterion in the section above.

In this section, we simplify their problem as a single $A-B$ route representing the origin and destination pair ports. There are two options given to examine the effective and cost of speed reduction. The first option assumes less transport frequent, e.g., keep the same number of trips with speed reduction and the second option is to add more ships in accordance with speed reduction. With arbitrary numbers, we now show the impact of harbour time if the duration of COA is assumed to be infinity or sufficiently long.

Assume the ship described in Section 4.5.4 has been chartered to undertake the same logistics task with identical charter contract. The distance between port A and B is $S_{AB} = 3000$ nautical miles. The market is static with the freight rate equal to $r = 75$ USD/tonne and the baseline fuel price equal to 300 USD/tonne. The whole scenario is thus profitable in this example even with fuel tax. In addition, we assume the harbour time to be $T^p = 0.5$ days.

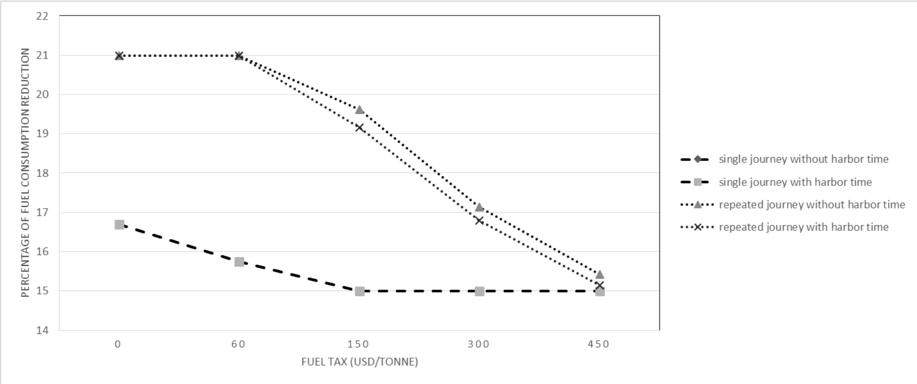


Figure 6.1: Optimal speeds with and without harbour time under different assumptions

By solving the problem under either single journey assumption (i.e., $i = 1$) and repeated journey assumption (i.e., $i \to \infty$), Figure 6.1 shows the optimal speeds correspondingly with and without the consideration of harbour time. The repeated scenario can be understood as a laden-laden round trip with identical settings, e.g., $A - B - A$. As expected, with a single journey assumption, the harbour time has no impact on the optimal speeds. However, if the trip will be repeated, adding harbour time would lower the profitability and thus the optimal speeds. One shall notice that such impact is not negligible.

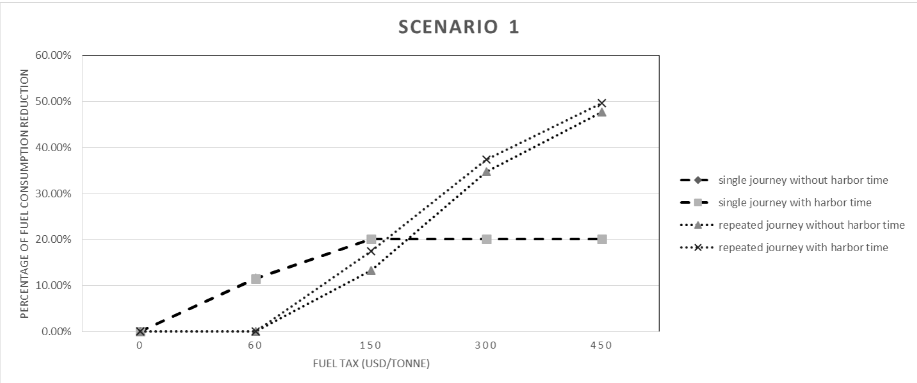


Figure 6.2: Fuel consumption reduction with and without harbour time for option 1

Since the emission is directly correlated to the fuel consumption, for simplicity, we show Figure 6.2 as the effectiveness of fuel tax on reducing emissions as given in Figure 2 of Corbett et al. (2009). Here option 1 means that we don't consider to meet the COA requirement of annually quantity. Then it is clear that adding harbour time would change the effectiveness of fuel tax under repeated journey assumption (i.e., $i \to \infty$). To be more specified, harbour time will make the fuel tax more effective on fuel consumption reduction (e.g., from 13.3% to 17.4% with 50% tax based on the baseline fuel price) and hence the airborne emissions.

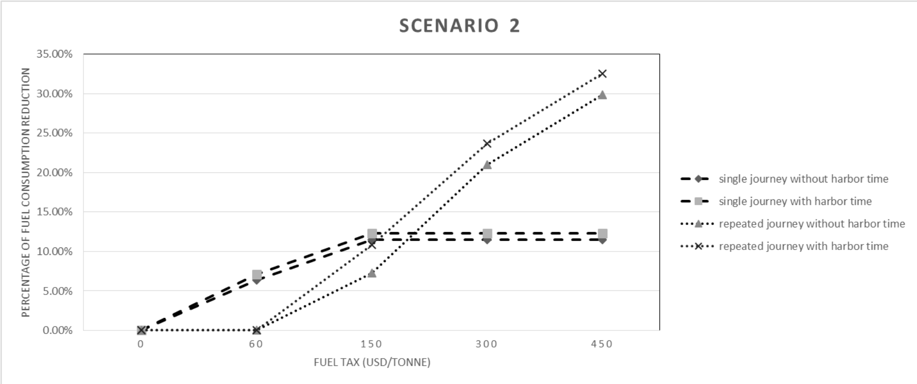


Figure 6.3: Fuel consumption reduction with and without harbour time for option 2

Similarly, we look at the second option that additional ships are needed to fulfill the annually quantity with speed reduction in Figure 6.3. Now adding harbour time would change the power of fuel tax on fuel consumption reduction (e.g., from 11.5% to 12.3% with a 50% tax based on the baseline fuel price) even when $i = 1$. When the journey is repeated, such impact will be enlarged.

Our findings of harbour time effect are in line with Theorem 6.1. In practice, adding harbour time can actually encourage speed reduction (see Psaraftis and Kontovas (2009, 2015)), even for the case where more ship is added for the compensation of annual demand. However, in literature, it is assumed that the total journey time (i.e., travel time plus harbour time) is fixed, see e.g., Johnson and Styhre (2015). Such assumption may only be applicable to liner shipping, where a fixed schedule is preferred. And if one plugs the actual cash-flows into the objective function, we argue here the better strategy can be even to change the fixed schedule (Theorem 6.1). Thus we are facing a dilemma that improving harbour efficiency cannot contribute towards both economic and environmental benefit based on the consideration of profit-seeking behaviour, which is a question left by Johnson and Styhre (2015). In another word, without the help of additional policy measures (e.g., to raise the cost and encourage the ship to slow down), it is unlikely to observe the implementation of this "cost-effective" method for emission controls as discussed in literature.

In summary, this example shows the importance of taking harbour time into consideration for any maritime models. For studies of emission, we show the harbour time can affect the fuel consumption, and thus emission, especially if one decides to add more ships in compensation of the speed reduction or repeat the logistics tasks multiple times. Indeed, the integral optimisation of ship speed and harbour time can bring in more potential collaboration works between port and ship operators. It can also help the government or related society to better understand the effectiveness of different regulations, for example on emissions, as well.

6.3 The Impact of Revenue Sharing Contract

While the traditional contract often specifies the fixed charter hire during the horizon, another choice is to adopt the revenue sharing contract that has been widely applied in supply chain management (Niederhoff & Kouvelis, 2019; Yao, Leung, & Lai, 2008). The idea is to share the risk of charter market fluctuation between ship charterer and ship owner by agreeing a ceiling and a floor in advance according to the Baltic Index. The charterer will always pay at least the amount of charter hire equivalent to the floor when the charter market is low. The maximum payment is limited by the ceiling, and the extra amount of charter hire exceeding the ceiling could be split by both party as agreed.

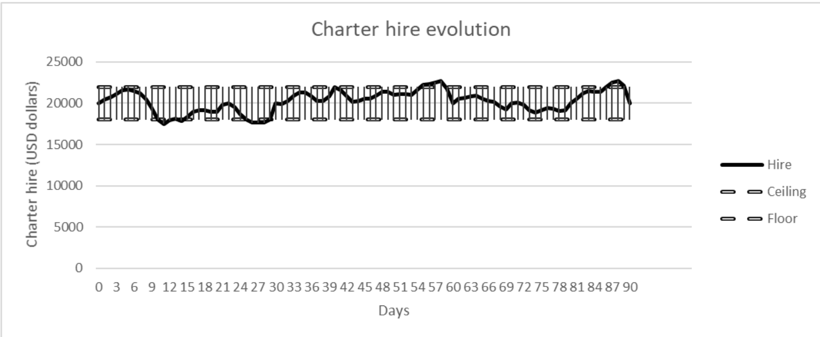


Figure 6.4: Charter hire evolution

The benefit of revenue sharing is to guarantee a rather stable charter hire against the fact that managers do not know the future evolution in real world. What they have is a picture of the trends in a short-term or long-term perspective along with the expected mean value. So this provides a bound to the charter hire that would potentially prevent a larger loss of profitability due to wrong predictions of the future. However, this is not yet investigated in the context of maritime speed optimisation to the knowledge of the authors.

In this section, we will adopt the evolution as shown in Figure 6.4 to first analyse the impact of revenue sharing contract on the optimal speeds and NPVs. The ceiling is set to be $22,000$ USD and the floor is set to be $18,000$ USD. For the sake of simplicity, the extra amount of which exceeds the ceiling will be equally split by both party. The mathematical expression could be written as follows.

$$f^{RS} = \begin{cases} ceiling + 0.5 * (f^{RS} - ceiling) & \text{if } f^{RS} > ceiling; \\ f^{RS} & \text{if } floor \leq f^{RS} \leq ceiling; \\ floor & \text{if } f^{RS} < floor. \end{cases} \tag{6.6}$$

The benchmark scenario is to assume fixed parameters as shown in Section 5.4. We calculate the corresponding fixed charter hire based on NPV by assuming that the fixed charter hire equivalent is a continuous cash-flow of a fixed rate paid within the duration.

$$f^{TCH}(\text{NPV mean}) = \frac{\alpha NPV(f^{RS})}{1 - exp^{-\alpha T}}. \tag{6.7}$$

where T is the total duration in each case and α is the opportunity cost of capital. The comparison is then to substitute the optimal speeds obtained from benchmark model into our dynamic charter hire scenarios.

6.3.1 Mathematical analysis

6.3.1.1 The impact of current charter hire cost

In this section, we consider model (4.6) but again omit the index of repetition i and denote the number of legs considered to be n instead for the ease of analysis. Since the change of charter hire structure only impacts the charter hire cost, we now consider G_1 and taking its derivative with respect to T_n:

$$\frac{\partial \int_0^{T_n} f_n^{TCH} e^{-\alpha t} dt}{\partial T_n} = \frac{\partial \frac{f_n^{TCH}}{\alpha}(1 - e^{-\alpha T_n})}{\partial T_n} = f_n^{TCH} e^{-\alpha T_n}. \tag{6.8}$$

Note that (6.8) is positive definite with positive charter hire and is irrelevant to the current speed decision. It simply suggests that the charter hire cost will accumulate faster with higher daily hire for fixed time duration, which agrees with the common sense. It gives the intuition that higher charter hire would urge the ship to finish the contract earlier if possible. On the other hand, when the charter hire f_n^{TCH} is low, such incentive would be smaller. However the charter hire itself will not suggest the ship to travel slower.

6.3.1.2 The impact of future charter hire cost

Now if we look at G_2 and write down the expression of the aggregated charter cost by taking its derivative with respect to T_{n-1}, we have:

$$\frac{\partial e^{-\alpha T_{n-1}} \int_0^{T_n} f_n^{TCH} e^{-\alpha t} dt}{\partial T_{n-1}} = \frac{\partial e^{-\alpha T_{n-1}} \frac{f_n^{TCH}}{\alpha}(1 - e^{-\alpha T_n})}{\partial T_{n-1}} = -f_n^{TCH}(1 - e^{-\alpha T_n}) e^{-\alpha T_{n-1}}. \tag{6.9}$$

(6.9) is negative definite as long as the charter hire f^{TCH} and future travel time T_n are positive. The mathematics now suggest that with the longer future travel time T_n and the larger charter hire f_n^{TCH}, the accumulated charter cost would decline faster with the increase of current travel time T_{n-1}. In another word, if there exists a large sum of charter hire cost in future (e.g., either high charter hire cost or long duration), the optimal strategy is to postpone such cost by extending the travel time of current journey.

6.3.1.3 The mixed impact

However, the incentive from future aggregation is opposite to the incentive from present, which means one of them will be neutralised by the other. We can actually formulate the sum of them as follows:

$$(f_{n-1}^{TCH} - f_n^{TCH}(1 - e^{-\alpha T_n}))e^{-\alpha T_{n-1}}. \tag{6.10}$$

Under fixed charter hire contract ($f_n^{TCH} = f_{n-1}^{TCH} = \cdots = f_1^{TCH}$), by decreasing the current travel time T_{n-1}, the charterer can reduce the total charter cost as (6.10) is now positive definite. For $T_n \geq 0$, the larger T_n, the smaller impact one can achieve by changing T_{n-1}.

By comparing (6.10) and (6.8), it is easy to see $f_{n-1}^{TCH} - f_n^{TCH}(1 - e^{-\alpha T_n}) < f_n^{TCH}$ for fixed charter hire ($f_n^{TCH} = f_{n-1}^{TCH}$). This actually implies that, when subjected to a long term contract, e.g., in liner shipping or industry shipping, the incentive of charter hire cost is unlikely to change the optimal speeds for earlier journeys. On the other hand, the last journey is the most likely one to get influenced by the marginal effects of charter hire cost. This statement can then be extended to infinite time horizon, and thus the optimal speeds at stationary state is independent of the charter hire cost (as we number the index of legs backwards).

One might argue that since the daily charter hire is fixed, it can be classified as fixed costs and thus have no impact on the optimal strategy. However, the total charter cost is the product of daily charter hire and the contract duration. Though the daily charter hire is fixed, the charterer could still change his optimal speed choices and thus update the contract duration (if possible). Hence, if and only if the charter duration is fixed, we can ignore the impact of charter hire.

6.3.1.4 The revenue sharing contract

In the context of revenue sharing contract, the split and ceiling will decide how high the daily charter hire could be, while the floor describes how low it could be. Hence we expect the ceiling and split to have similar impacts on the optimal speed choices. In the framework of revenue sharing contract, the daily charter hire could be varying over time. For the simplicity of analysis, we denote f_i^{TCH} as the average daily charter hire paid for the i-th leg, $i \in 1, 2, \ldots, n$. And in calculation, one can still plug in the actual daily numbers easily.

By comparing (6.10) and (6.8), we have:

$$f_{n-1}^{TCH} e^{-\alpha T_{n-1}} - (e^{-\alpha T_{n-1}} + e^{-\alpha T_n} - e^{-\alpha(T_{n-1}+T_n)}) f_n^{TCH}, \tag{6.11}$$

Since the travel time T cannot be too large, and the opportunity cost of capital α is extremely small on the daily basis, we can take the linear approximation of (6.11) by using the Taylor expansion. Then numerically, (6.11) can be viewed as:

$$f_{n-1}^{TCH}(1 - \alpha T_{n-1}) - f_n^{TCH}. \tag{6.12}$$

This implies that, unless the charter hire is higher on the next journey, we will always observe a stronger signal of fast steaming on the current journey than on the next one. In another word, the revolution, e.g., trend, of charter hire must be considered as well. This statement can be easily extended from G_1 to G_n.

6.3.2 General example

In this section, a general $A-B-A-B$ scenario is used to examine the effect of revenue sharing contract against traditional fixed charter hire. The distance between each port is assumed to be 4,000 nautical miles and the cargo to be transported from A to B is 150,000 tonnage. The fixed port cost at each port is 250,000 USD/visit. In the benchmark model, the freight rate is fixed to be 17 USD/ton and the bunker price is 659 USD/ton.

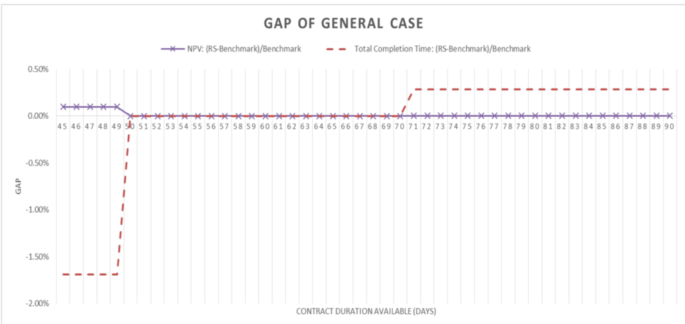


Figure 6.5: The gap of NPVs and Optimal Completion Time: Revenue Sharing

Figure 6.5 is calculated via $\frac{RS-Benchmark}{Benchmark}$, where the fixed charter hire equivalent (6.7) is adopted for each possible contract duration in benchmark model.

If we look at the optimal completion time, the first flat segment of duration suggests that the ship would like to spend around 1.6% time shorter on sea with revenue sharing contract. This is due to the fact that, for charter hire evolution, the market performance, in the early stage, starts from low and then increases sharply (see Figure 6.4). Thus revenue sharing contract would suggest the ship to finish all the journey earlier before that bounce, while the traditional daily charter hire cannot capture such volatility.

With the increase of total charter duration available, the gap falls into 0. This is to say, other parameters, e.g., available time horizon, are playing more important roles, that the ship will use up all the time available to complete the whole journey.

The last segment refers to the situation that the contract length is sufficient. The revenue sharing contract would suggest the ship to spend around 0.25% time longer, as the charter hire evolution is low during that time. The ship thus enjoys the maximum period of low charter hire before it rises up on day 71.

Note that, when subjected to slow steaming (the last segment part), revenue sharing contract suggests to travel slower on ballast leg rather than on laden leg. However, when it comes to fast steaming (the first segment part), the ship would travel faster on both laden and ballast leg compared to the benchmark model. Such asymmetric impact on laden & ballast leg is likely to be caused by the fact that the fuel consumption cost is normally smaller on ballast leg (due to the deadweight tonnage carried on board).

However, the gap of NPVs is trivial for the whole scenario. This indicates the usage of a good approximated mean value can lead to a sufficiently good result of optimal speed choices and profitability and reduce the computational costs. Still, this statement needs to be further validated as we don't include clear trend in Figure 6.4.

6.3.3 Two special cases

The above result suggests that we can replace the revenue sharing contract model by a fixed charter hire model with appropriately estimated NPV equivalent mean for computational simplicity. However, it also shows that the fixed charter hire model cannot capture the trend of either a decrease or increase of charter hire evolution. From the cash-flow point of view, different evolution of charter hire will definitely leads to different NPVs. Hence in this section, we will introduce two special cases of different charter hire evolutions with identical mean value. We are thus curious if the trend of evolution could impact the optimal speeds and if so, how?

Assume the charter hire would evolve from $24,000$ USD/day to $16,000$ USD/day in Case 1, and develop exactly the opposite in Case 2 (see Figure 6.6). A brief of the statistics is given in Table 6.1, where the mean refers to the average charter hire of the whole scenario, the total refers to the total amount of charter hire that needs to be paid without ceiling

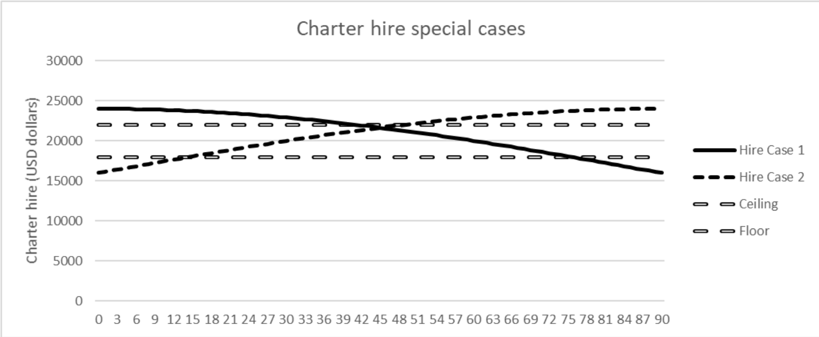


Figure 6.6: Charter hire evolution of two special cases

Table 6.1: The statistics of two special cases

	Case 1	**Case 2**
mean (US dollars)	21,080.8	21,080.8
total (US dollars)	1,897,273.8	1,897,273.8
NPV (US dollars)	307,488.7	228,708.7
RS total (US dollars)	1,905,908.6	1,905,908.6
RS NPV (US dollars)	296,458.0	239,786.8

and floor. If we calculate the NPV of these two cash-flows, apparently Case 1 would bring in more costs since the charter hire is higher in the beginning. The RS total is the total amount of money should be paid according to revenue sharing contract (without discount). Because of the floor, actually the RS total is larger. With opportunity cost of capital $\alpha > 0$, the NPV (discounted) of charter cost with revenue sharing contract in Case 1 is reduced (compared to the original NPV), while the discounted cost in Case 2 has been increased. This is the example that a conventional model would ignore (because the two cases are exactly the same if we only consider the mean, variance and bounds) and cannot be captured by distribution-free models without specified trend term.

Table 6.2: The results of revenue sharing contract in two special cases

Setting	**CASE 1**		**CASE 2**		**Setting**	**CASE 1**		**CASE 2**	
Total Horizon (days)	Optimal Duration (days)	NPV of charterer (USD)	Optimal Duration (days)	NPV of charterer (USD)	Total Horizon (days)	Optimal Duration (days)	NPV of charterer (USD)	Optimal Duration (days)	NPV of charterer (USD)
45	34.5	388,656.28	35.3	528,852.24	68	67.9	834,428.60	67.9	943,070.48
46	34.5	388,656.28	35.3	528,852.24	69	68.9	838,547.57	68.8	943,538.80
47	34.5	388,656.28	35.3	528,852.24	70	69.9	841,460.86	68.8	943,538.80
48	34.5	388,656.28	35.3	528,852.24	71	70.9	843,302.15	68.8	943,538.80
49	34.5	388,656.28	35.3	528,852.24	72	71.5	843,649.78	68.8	943,538.80
50	34.5	388,656.28	49.9	530,945.78	73	71.5	843,649.78	68.8	943,538.80
51	50.9	444,373.55	50.9	591,694.60	74	71.5	843,649.78	68.8	943,538.80
52	51.9	495,194.22	51.9	641,458.75	75	71.5	843,649.78	68.8	943,538.80
53	52.9	539,960.10	52.9	685,018.00	76	71.5	843,649.78	68.8	943,538.80
54	53.9	580,093.44	53.9	723,793.15	77	71.5	843,649.78	68.8	943,538.80
55	54.9	616,203.36	54.9	758,393.56	78	71.5	843,649.78	68.8	943,538.80
56	55.9	648,493.58	55.9	789,021.62	79	71.5	843,649.78	68.8	943,538.80
57	56.9	677,276.86	56.9	815,990.49	80	71.5	843,649.78	68.8	943,538.80
58	57.9	703,014.64	57.9	839,761.30	81	71.5	843,649.78	68.8	943,538.80
59	58.9	725,823.33	58.9	860,449.56	82	71.5	843,649.78	68.8	943,538.80
60	59.9	745,947.97	59.9	878,300.96	83	71.5	843,649.78	68.8	943,538.80
61	60.9	763,732.25	60.9	893,658.48	84	71.5	843,649.78	68.8	943,538.80
62	61.9	779,236.67	61.9	906,582.50	85	71.5	843,649.78	68.8	943,538.80
63	62.9	792,684.74	62.9	917,297.00	86	71.5	843,649.78	68.8	943,538.80
64	63.9	804,282.22	63.9	926,006.91	87	71.5	843,649.78	68.8	943,538.80
65	64.9	814,095.98	64.9	932,779.76	88	71.5	843,649.78	68.8	943,538.80
66	65.9	822,331.18	65.9	937,821.12	89	71.5	843,649.78	68.8	943,538.80
67	66.9	829,077.94	66.9	941,220.18	90	71.5	843,649.78	68.8	943,538.80

Together the results of two special cases shown in Table 6.2 confirm that the impact of trend do exist and it is asymmetry. This, to some degree, can be explained as the setting of revenue sharing contract itself is not symmetry. Only the part exceeding the ceiling is shared by two parities, while the floor is only set for the charterer.

In particular, when the total horizon (the end of contract duration) is between 51 to 68 days, both Case 1 and 2 would reach the same optimal travel time but clearly with different optimal NPVs. And the difference of travel time between two cases is smaller when $H = 45$ days ($|34.5 - 35.3| = 0.8$ days) compared to when $H = 90$ days ($|71.5 - 68.8| = 2.7$ days).

This is actually explained by above mathematical analysis that higher charter hire would encourage the ship to travel faster, while lower charter hire gives smaller incentive to do so. It is also why, in Case 1, the optimal travel time is shorter than in Case 2 when H is small. With the increase of H, there is a period that, in both case, the charterer could squeeze in the second round trip and use up the time available. When the H is sufficiently long, without the incentive of squeezing in the third round trip, now ship will travel much faster in Case 2 due to the high charter hire at the end of contract.

6.3.4 Ship charterer and ship owner

In this section, we will investigate further into the settings of revenue sharing contract. The question remained is whether the change of split, ceiling and floor would impact the optimal strategy and how. In addition, we are also curious about what is the impact on ship owner's profitability.

The profit of the ship owner only comes from the charter hire in this discussion. Thus we can write the owner's objective function of profitability as :

$$G_{owner} = \int_0^{L_n} f^{TCH} e^{-\alpha t} dt \tag{6.13}$$

Here the profitability of the owner solely depends on when does the charterer finish all the logistics tasks and return the ship L_n, which is the sum of all the decision variables T for the charterer. According to the analysis in section 6.3.1, if the charter hire actually paid is changed, such incentive will suggest the charterer to change his speed decisions and thus the L_n. This is to say, the charterer and the ship owner is not playing a zero-sum game, but it is behaving like a Stackelberg Game, where the owner has to decide his best strategy by guessing the reaction of charterer.

We will illustrate and confirm the above statement by presenting the numerical examples. No contract period is pre-determined to prevent the impact of a rather tight time limit on the optimal solutions. The charterer could evaluate the contract with specific conditions and then decide what is the optimal charter length. Settings of the two special cases (introduced in last section) are adopted in this section.

Table 6.3 shows the impact of different splits on the profitability of the two parties with different charter hire evolution scenarios. For ship charterer, apparently the lower split

Table 6.3: The impact of splits on the profitability of charterer and owner

Setting	**CASE 1**			**CASE 2**		
Splits	Optimal Duration (days)	NPV of charterer (USD)	NPV of Owner (USD)	Optimal Duration (days)	NPV of charterer (USD)	NPV of Owner (USD)
0	71.5	871,426.85	1,514,641.45	69.3	951,350.20	1,390,300.09
0.1	71.5	865,871.44	1,520,196.87	69.2	949,748.72	1,389,718.80
0.2	71.5	860,316.02	1,525,752.28	69.0	948,174.72	1,386,912.67
0.3	71.5	854,760.61	1,531,307.69	68.9	946,626.84	1,386,259.94
0.4	71.5	849,205.20	1,536,863.11	68.9	945,081.41	1,387,802.44
0.5	71.5	843,649.78	1,542,418.52	68.8	943,538.80	1,387,108.10
0.6	71.5	838,094.37	1,547,973.93	68.5	942,030.62	1,381,883.93
0.7	71.5	832,538.96	1,553,529.35	68.5	940,541.15	1,383,370.58
0.8	71.5	826,983.54	1,559,084.76	68.5	939,051.68	1,384,857.23
0.9	71.5	821,428.13	1,564,640.18	68.3	937,567.24	1,381,758.05
1	71.5	815,872.72	1,570,195.59	68.1	936,122.13	1,378,602.80

means higher profits. And intuitively, a higher split would bring the ship owner more profits. However, this is not always true.

The profitability actually depends on how would the charterer respond to the split. When the overall performance of charter market is falling (as in Case 1), increasing the split will have no impact on the optimal strategy of the ship operation. This is due to the fact that, the additional costs are paid in the early stage and as long as the logistics tasks are still profitable, he will take it and stick to the original plan. However, if the charter hire market is rocketing and the extra amount of charter hire cost is paid in later stage (as in Case 2), the charterer will now adjust his plan to travel at a faster speed. In this case, both party will lose their profitability.

To conclude it, Only if the charterer keeps his original plan when the split is increased, the owner would gain extra profit.

Table 6.4: The impact of the ceiling on the profitability of charterer and owner

Setting	**CASE 1**			**CASE 2**		
Ceiling (USD)	Optimal Duration (days)	NPV of charterer (USD)	NPV of Owner (USD)	Optimal Duration (days)	NPV of charterer (USD)	NPV of Owner (USD)
20000	71.5	894,997.24	1,491,092.43	69.4	972,977.13	1,370,849.76
20400	71.5	883,306.22	1,502,783.45	69.3	965,616.30	1,376,051.99
20800	71.5	872,284.14	1,513,805.52	69.2	958,935.74	1,380,543.62
21200	71.5	861,978.24	1,524,111.42	68.9	953,016.33	1,379,879.62
21600	71.5	852,415.85	1,533,673.81	68.9	947,877.91	1,385,010.15
22000	71.5	843,649.78	1,542,439.88	68.8	943,538.80	1,387,108.10
22400	71.5	835,742.25	1,550,347.41	68.5	940,114.23	1,383,796.95
22800	71.5	828,790.31	1,557,299.36	68.5	937,660.95	1,386,245.43
23200	71.5	822,925.22	1,563,164.44	68.3	936,300.77	1,383,022.12
23600	71.5	818,390.78	1,567,698.88	68.1	936,122.13	1,378,602.80
24000	71.5	815,872.72	1,570,216.95	68.1	936,122.13	1,378,602.80

Similar results can be observed in Table 6.4. When the extra charter hire cost is to be paid in the early stage, which in other words is unavoidable, the game between the ship owner and the charterer is zero sum.

However, for Case 2, we now see that the incentive for charterer to travel at higher speed is not as strong as in previous example (depicted in Table 6.3). Thus the ship owner does make some extra profit by increasing the ceiling of the revenue sharing contract. Still, whenever the charterer decides to reduce the total charter time, it is a lose-lose situation for both party.

Roughly speaking, a higher ceiling is preferable to the ship owner as the potential loss is relatively small compared to the potential gain. The ship charterer, on the contrast, would definitely want to set a lower ceiling to split the cost of charter hire to the ship owner.

Table 6.5: The impact of the floor on the profitability of charterer and owner

Setting	**CASE 1**			**CASE 2**		
Floor (USD)	Optimal Duration (days)	NPV of charterer (USD)	NPV of Owner (USD)	Optimal Duration (days)	NPV of charterer (USD)	NPV of Owner (USD)
16000	71.5	843,649.78	1,542,409.63	68.8	958,940.35	1,371,706.55
16400	71.5	843,649.78	1,542,409.63	68.8	958,159.10	1,372,487.80
16800	71.5	843,649.78	1,542,409.63	68.8	956,233.97	1,374,412.93
17200	71.5	843,649.78	1,542,409.63	68.8	953,161.17	1,377,485.73
17600	71.5	843,649.78	1,542,409.63	68.8	948,932.89	1,381,714.01
18000	71.5	843,649.78	1,542,409.63	68.8	943,538.80	1,387,108.10
18400	71.5	843,649.78	1,542,409.63	68.8	936,964.93	1,393,681.97
18800	71.4	843,494.36	1,540,724.09	68.8	929,194.24	1,401,452.57
19200	71.2	842,370.42	1,538,108.85	68.8	920,204.97	1,410,440.68
19600	71.1	840,083.48	1,538,495.30	68.8	909,973.22	1,420,670.05
20000	70.9	836,608.35	1,538,091.74	68.8	898,472.18	1,432,167.47

When it comes to increasing the floor, Case 2 in Table 6.5 confirms that when the impact happens in the early stage, the charterer will not change his original plan. But both party could lose profitability when the optimal plan is to shorten the charter duration.

We notice that the impact of floor seems to be not as sensitive as the ceiling or the split. This is due to the fact that the affected periods are different in each cases.

In Case 1, the actual charter hire would fall to $20,000$ US dollars on day 60 while the overall charter duration is around 70 days. Thus the floor is irrelevant for the earlier period. In another word, the ship owner is not guaranteed to get the extra profit as expected from setting a higher floor (as in Case 1). But if we compare the profitability of Case 2 in Table 6.3. 6.4 and 6.5, rising the floor would definitely be the best way to guarantee a good amount of profits when the market is rather low.

In summary, this numerical example indicates that the optimal strategy for charterer is only affected, when the extra payment is made at the later stage of the charter contract, which is consistent to the mathematical analysis iun previous section. Once the charterer would like to shorten the contract, both the ship owner and charterer bear a loss of profitability. Otherwise it could be a zero sum game between them in the context of revenue sharing contract. In another word, when it comes to the negotiation of the settings of revenue sharing contract, the game between ship owner and charterer is a stackelberg game, where the owner is a leader and the charter is the follower.

6.4 Optimal Shipping Decisions under Contract of Affreightment

6.4.1 Introduction

This section concerns a particular contract condition, namely the Contract of Affreightment (COA), of which the particular total amount of cargo is agreed to be transported from specific origin to destination within a given time period. Unlike other contracts, COA often guarantees sustained predictable profits for ship operators over a longer time period, while leaving however so degree of flexibility for them to make logistics decisions, unlike traditional voyage charters. The ship operator, thus, has the option to charter in new ships from the spot market, if he finds alternative job with better rewards to fulfil the logistics requirement of COA (c.f., Williams (1996)).

It, as a consequence, makes the shipping decisions under COA to be rather important, since the operator now needs to decide which alternative job to take while arranging the shipments to complete the required cargo transportation. Actually, this is a scenario other than traditional liner shipping or tramp shipping, but serves as a mix of them. The cargo specified under COA are often required to be transport between fixed ports, and thus it serves like a liner shipping situation. However, if the ship operator decides to undertake other logistics tasks in the meanwhile, such decision is alike to the scenario often occurring in tramp shipping where one needs to maximise his net revenue by selecting the best task(s) available, and thus, possibly, solve the problem of route selection and speed optimisation. For a more comprehensive review of maritime OR literature about COA, the readers are referred to Ronen (1993) and Norstad et al. (2011) and the references therein.

In the discussion in Christiansen et al. (2007), as an extension of their previous work (Christiansen et al., 2004), it is stated that the problem of a shipping company when facing a COA offer is to evaluate if the contract would be profitable. They further state that such an evaluation would highly depend on the assumption of the future market situation. If, for example, the market is expected to be low in profitability for shipping companies, the company may want to extend existing logistics contracts, to cover that period, or to reduce the fleet size.

Psaraftis and Kontovas (2014), in their section 2.6, claim that since the ‘mandatory’ cargoes, to be moved under COA, are fixed and pre-determined, only when the ‘optional’ cargoes are considered, the optimisation problem now makes sense as the ship may want to speed up to accommodate these ‘optional’ cargoes. They state more specifically that the decision for assigning the COA needs to be made *simultaneously* with the consideration of which ‘optional’ cargoes to serve, as it would be risky to assign a larger ship for COA without knowing potential ‘optional’ cargoes financially.

This is obviously the best possible situation. We argue, however, that a re-evaluation of the inclusion of other cargoes within a COA setting may still be needed after the COA has been agreed. This is because optional cargoes may arise after signing the COA, and after a set of mandatory cargoes (for the COA) have already been served.

The increased usage of COA contracts, and the consideration of optional cargoes, is further supported by our talks with a large shipping company.

6.4.2 Model formulation

Without loss of generality, denote the origin port P_0, the destination port P_d and each round trip consists m legs, where d and m are integers and $0 < d < m < +\infty$. The ship thus needs to visit ports in accordance with the sequence $\{P_1, P_2, \ldots, P_d, \ldots, P_{m-1}\}$ and then back to P_0 as dispicted in Figure 6.7. Even though there is no ship specified under COA, once there is a candidate ship, the operator knows its capacity and also the total amount committed, and thus the number of repetition n is known instantly.

Figure 6.7: Optimal speeds under COA

When ignoring the ‘optional’ cargoes, the ship will be laden before arriving P_d and on ballast afterwards. If, for simplicity, we assume $d = 1$ and $m = 2$, the whole problem then reduces to the laden-ballast scenario.

Given the fact that COA is usually set to be rather long time and the ship operator can freely decide which ship to use, the ship selected for such ‘mandatory’ tasks should always be able to transfer these cargoes in time[2]. Thus we consider the $\mathcal{P}(n, m, G_o)$ model developed in Chapter 4, that is to optimise the recursive equation, viz:

$$G^i_j(T^i_j) = h_j(T^i_J) + G^i_{j+1}(T^i_{j+1})e^{-\alpha T^i_j}, \tag{6.14}$$

with

$$G^i_{m+1}(T^i_{m+1}) = G^{i-1}_1(T^i_1), \tag{6.15}$$

for any leg $j \in \{1, 2, \ldots, m\}$ within the round trip of index $i \in \{1, 2, \ldots, n\}$. The term $h_j(T^i_J)$ represents the net revenue of current leg with a given travel time T^i_J, i.e. the speed decision, discounted to the start time of execution of this leg:

$$h_j(T^i_j) = (R_j - C^u_j)e^{-\alpha T^i_j} - C^l_j(T^i_j) - \int_0^{T^i_j} f^{TCH} e^{-\alpha t} dt, \tag{6.16}$$

[2]Otherwise, if the time constraint is rather tight, a modified version of P_m model designed in Chapter 3 can be used instead.

where R is the revenue of this particular leg, C^u is the costs involved at the unloading port, C^l is the associated costs occurred at the loading port, including the fuel consumption cost, which is a function of travel time T_j^i, and ftc is the fixed charter hire if the ship is chartered, or the fixed maintenance costs for the ship owner to operate the ship.

In particular, for the last leg of the planned journey, (6.14) can be expressed as follows:

$$G_m^1(T_m^1) = h_m(T_m^1) + G_o e^{-\alpha T_m^1}, \tag{6.17}$$

where G_o is the Future Profit Potential (FPP) that represents the decision maker's expectations about the NPV of all cash-flows the ship is deemed to be able to generate within the relevant future, discounted to the moment that the ship completes its n journeys that are currently considered in the optimisation problem.

Thus by using Algorithm 1 in Chapter 4, for any given sequence of ports, we can maximise the Net Present Value (NPV) of such a logistics task by finding the optimal travel time (i.e. speed) for each leg:

$$T_j^{i*} = arg \ \ max \ \ \Big(h_j(T_J^i) + G_{j+1}^{i*}(T_{j+1}^{i*})e^{-\alpha T_j^i}\Big). \tag{6.18}$$

6.4.3 Timing of freight

As suggested by Psaraftis and Kontovas (2014), traditional maritime literature assumes the optimal speeds for COA without 'optional' cargoes are only to minimise the fuel consumption costs or to fulfil certain time windows as identical to liner shipping. However, the importance of incorporating the incentive of revenue term is mentioned by Psaraftis (2017) and also confirmed by our previous chapters. Two main types of standard-form COAs are published by BIMCO: (1) VOLCOA, the Standard Volume Contract of Affreightment for the Transportation of Bulk Dry Cargoes; and (2) INTERCOA 80, the Tanker Contract of Affreightment. They differ mainly in terms of the type of products transported, and we will not focus on the specifics in this dissertation.

Both COA contracts allow the parties to negotiate the freight rate and the timing or method of payments. We are curious to investigate how the timing of payments will impact the profitability of for ship operator, and the optimal ship speeds.

Consider a ship, with data as given in Appendix 8.2.1, entering the COA contract to transport $1,200,000$ barrels of light crude oil from port 2 to port 1 for exactly 10 times. The distance of both laden and ballast leg is 8000 nm. The agreed freight rate is 20 USD/tonne.

In the notation introduced in Chapter 4, this corresponds to a problem in class $\mathcal{P}$ $(10, 2, 0)$ in which a ballast-laden journey is repeated. For simplicity, we have assumed that the FPP $G_o = 0$. This would correspond to the situation in which the ship used is time chartered for the period that the ship operator needs it for executing the COA. Indeed, as seen in Chapter 3, this situation would correspond to model P_1 in which the journey consists of 10 times the same ballast-laden journey. The decision maker does not care about the ship's profitability after completion of the COA. The value of G_o can be expected to affect the results in these experiments about timing of payments, but a further investigation is postponed (see Section 6.4.5).

The possible arrangements of timing of payments we consider here are as follows (see also Table 6.6). In scenario #1, the shipper agrees to pay the ship operator the total sum due at the start of the first ballast leg. In #2, the shipper pays the total sum due at the end of the last laden leg. In # 3, the ship operators is paid at the start of loading each time the ship enters port 2. Finally, in scenario #4, the payment occurs upon unloading at port 1, corresponding to the situation considered in Chapter 4.

Table 6.6: Possible cases of timing of freight

#	**Timing of Payment**	**Payment Structure**
1	start of contract	one lump sum
2	contract completion	one lump sum
3	loading	for each laden leg
4	unloading	for each laden leg

We have solved the optimisation problem for each of these different payment structures. The results, summarised in Table 6.7, clearly show a significant impact on the profitability for the ship operator, as well as on the total execution time of the contract. In general, the later the payment, the lower the profitability, the higher the ship speeds, and the shorter the total contract duration.

These results not only show that the model behaves according to our intuition, but that the impact can also be rather significant. We now present further clarification of these findings by means of a further (mathematical) analysis.

Table 6.7: Optimal results with different timing of payments

#	**Journey Time (days)**	$G_{10}(1)$ **(USD)**	V_1^{10}	V_2^{10}	V_1^1	V_2^1
1	641.7	8028411	11.6	10.1	12.5	10.8
2	590.4	4187774	12.8	11.0	13.7	11.7
3	612.8	6047063	12.9	11.0	12.6	10.8
4	610.1	5828822	12.9	11.1	12.6	10.9

Note that changing the timing of payments results in changing the revenue structure h given in (6.16).

Comparison 1: *# 1 VS. # 2.*

We start the comparison by giving the following question:

If the shipper agrees to pay the freight all at once, would the ship operator prefer # 1 or # 2?

This is an easy problem, as the total amount of cash received in both scenarios will be the same, but in # 1 it is received earlier. This is always preferable if the decision maker uses the NPV criterion when $\alpha > 0$. Indeed, the Net Present Value of a given amount of cash received is higher when received earlier per definition of the NPV, see (2.6).

We now address the following question:

Will # 1 and # 2 lead to different optimisation problems and thus optimal ship speeds?

Let M denote the total revenue received on the COA. This is in # 1 received at the start of the contract. The profit structure in the optimisation of each leg's speed will now only have to account for the costs $-C_j(T_j^i)$. Thus the optimisation problem for # 1 has to operate on the following recursive formula:

$$T_j^{i*} = arg \;\; max \;\; \left(-C_j(T_j^i) + G_{j+1}^{i*}(T_{j+1}^{i*}) e^{-\alpha T_j^i} \right). \tag{6.19}$$

In # 2, the payment M is received when completing the contract, and thus the optimisation problem for # 2 is:

$$T_j^{i*} = arg \;\; max \;\; \left(-C_j(T_j^i) + G_{j+1}^{i*}(T_{j+1}^{i*}) e^{-\alpha T_j^i} + M e^{-\alpha T_j^i + (\sum_{l=j}^{m} T_l^{i*} + \sum_{k=1}^{i-1} \sum_{l=1}^{m} T_l^{k*})} \right). \tag{6.20}$$

By noticing that (6.19) only minimises the costs, which is largely decided by the fuel consumption, it is clear that # 1 corresponds to the assumption made in Psaraftis and Kontovas (2014).

For # 2, however, the revenue term still plays its role in the optimisation problem (6.20). By speeding up, the ship operator can finish the logistics commitment quicker, and thus receive the rewards earlier.

As a consequence, the optimal speeds for ship under # 1 are much slower than under # 2. This can be easily confirmed by numerical results in Table 6.7.

We can also see that this must be so by means of the following reasoning, based on the FPP concept formulated in Chapter 4. The problem of #2 and as formulated in (6.20) can also be interpreted as a problem in which the profits for each leg only consist of the costs incurred, while the FPP takes on the value of M. Thus, the problem in #2 equals that of #1 but where the G_o is changed from a value of 0 to a value of M. From experiments in Chapter 4, we recall that the optimal leg speeds in #2 will be higher than those in #1.

The above reasoning clearly also shows that changing the actual G_o value (in these experiments kept at zero), will therefore also affect the optimisation results in each of the four scenarios. □

If the COA consists a large amount of cargoes, it is unlikely for the shipping company to agree on a scenario as in #2, i.e. bearing a large amount of costs before making any profits. A more reasonable assumption may then be to separate the payments according to the delivery of the individual cargoes, as in #3 and #4.

Comparison 2: *# 3 VS. # 4.*

With a similar analysis as above, we can arrive at the conclusion that the profitability for the ship operator under # 3 is higher than under # 4. Thus the remaining question is:

What is the difference between the optimisation problems of # 3 and # 4?

Note that (6.16) gives the expression of the recursion function associated with # 4. We can re-write it as follows:

$$T_j^{i*} = arg \;\; max \;\; \left(R_j e^{-\alpha T_j^i} - C_j(T_j^i) + G_{j+1}^{i*}(T_{j+1}^{i*}) e^{-\alpha T_j^i} \right), \tag{6.21}$$

with

$$G_j^i(T_j^i) = R_j e^{-\alpha T_j^i} - C_j(T_j^i) + G_{j+1}^i(T_{j+1}^i) e^{-\alpha T_j^i}. \tag{6.22}$$

For # 3, however the corresponding recursive function for the optimisation is:

$$T_j^{i*} = arg \;\; max \;\; \left(R_j - C_j(T_J^i) + G_{j+1}^{i*}(T_{j+1}^{i*}) e^{-\alpha T_j^i} \right), \tag{6.23}$$

with

$$G_j^i(T_j^i) = R_j - C_j(T_j^i J) + G_{j+1}^i(T_{j+1}^i) e^{-\alpha T_j^i}. \tag{6.24}$$

Even though at first glance the optimal speeds do not depend on the revenue terms in (6.23), (6.24) shows that the current speed decision T_j^i is depending on the present value of all the revenue terms henceforth. As a result, the ship is still encouraged to travel faster to collect these future rewards earlier.

If we write down the optimisation problem of the second last leg for # 3, see (6.25), and substitute the term R_m in (6.26) for the last leg of # 4 by the term $R_m - C_m(T_m^{1*})$ in (6.25), then the two optimisation problems are actually equivalent.

$$T_{m-1}^{1*} = arg \;\; max \;\; \Big(R_{m-1} - C_{m-1}(T_{m-1}^1) + [R_m - C_m(T_m^{1*})]e^{-\alpha T_{m-1}^1}\Big), \tag{6.25}$$

$$T_m^{1*} = arg \;\; max \;\; \Big(R_m e^{-\alpha T_m^1} - C_j(T_J^i)\Big). \tag{6.26}$$

The optimisation problems for # 3 and # 4 are thus very similar. The impact of revenue terms are smaller in # 3 than in # 4 in the sense that each leg "sees" a smaller future profit (potential), and thus the ship will travel slower. The differences are not as significant that between #1 and #2 because the differences in payment timing are also much smaller. See also Table 6.7. □

One can clearly see from these experiments that the ship operator always prefers to receive the revenues earlier, as it would bring in higher NPV of profits. In addition, such arrangements of freight payments can lead to slower steaming being part of the ship operator's optimal strategy, and thus help to contribute towards greener shipping.

Practical problems with advance payments, however, may prohibit the adoption of scenarios such as #1. In particular, the shipper may fear that the required shipments may not occur or not occur according to agreed conditions when payments are made too early.

Slower steaming, and thus longer travel times, may affect the ability to incorporate so-called 'optional' cargoes. For example, given a fixed contract period, the ship now has less flexibility and availability to transfer additional cargoes not specified in the COA.

As a way out of such problems, the value of potential additional cargoes should be considered when evaluating any COA proposal, see also Psaraftis and Kontovas (2014). The above analysis illustrated that in principle one may also need to consider the timing of payments in contracts as having a potential impact on the attractiveness of contracts, and the impact on optimal contract lengths and ship speeds.

In the rest of paper, we assume # 4 as a default setting for the timing of revenues received.

6.4.4 'Optional' cargoes

As specified in both VOLCOA and INTERCOA 80 contracts, the delivery of 'mandatory' cargoes is preferred to be evenly spread within the time period as the shipping activity itself servers only a part of the whole supply chain or the manufacture. Arguably, this preference is not always committed to in shipping practice. Without loss of generality, we assume here the ship operator can freely decide when and how to transfer all the cargoes, including 'mandatory' and 'optional' ones, as long as the required quantity of COA is met.

6.4.4.1 A basic scenario: route selection

In this section, we ignore the aspect of the COA and simply consider the situation where all potential cargoes are optional. This is also a common situation occurring in tramp shipping. We consider this problem as a route selection problem which contains speed optimisation as a sub-problem.

We present the route selection problem using the following notation. Let $\mathcal{N} := 1, 2, \ldots, N$ be the set of ports. Assume different rewards R_{ij} of 'optional' cargoes are known for any route from Port i to Port j. Similarly, we can denote the route specified costs as $C_{ij}(T_{ij})$ due to the difference of port charges, bunker price, etc. The optimal route selected at any origin port i with leg index m (analogue to the definition of the index for round trip in Figure 6.7) is thus obtained by solving the following problem:

$$j^* := arg \;\; max_{j \in \mathcal{N}} \;\left(G_m(i) = R_{ij} e^{-\alpha T^*_{ij}} - C_{ij}(T_{ij}) + G^*_{m-1}(j) e^{-\alpha T^*_{ij}} \right), \tag{6.27}$$

where T^*_{ij} is optimised simultaneously for any $j \in \mathcal{N}$, and $G^*_{m-1}(j)$ is the maximum NPV, discounted to the time of completing leg with index m, of all future profits that can be realised from the legs with indices $m-1, ..., 1$, when finishing leg m at port P_j.

Example 1: *Which port to visit next?*

We consider for this and further examples the following case. A ship as described in Appendix 8.2.1 can be used to transport possible cargoes of equal quantity of $1,200,000$ barrels (say Q tonne), as further specified in Table 6.8. If, for example, we are currently in port 1, then the ship could execute a laden leg to port 2 covering a distance of $4,000$ nm and earn $5Q$ USD, or else a laden leg to port 3 covering $8,000$ nm and earning $10Q$.

The ship will be chartered for a fixed daily costs according to a P_1 time charter contract, and the cost of fuel is the same in each harbour. We further adopt the following simplifying assumptions: Each potential cargo has the same quantity, there is no waiting time at ports, and the port costs are all identical.

Table 6.8: Journey characteristics of 3 ports; (*) nautical miles; (**) USD/tonne.

	Distance*			**Freight rate****		
Origin\Destination	**1**	**2**	**3**	**1**	**2**	**3**
1	0	4000	8000	0	5	10
2	4000	0	4000	5	0	5
3	8000	4000	0	10	5	0

Assume the ship is waiting at port 1 and we need to decide which port is next to visit. We can construct optimisation models from the class $\mathcal{P}\ (1, m, 0)$, where the m legs are chosen such as to reflect a particular selected route. Each may end in its own particular final port.

In particular, for the simple case in which $m = 1$, the two possible solutions are a leg to port 2, or one to port 3. The solutions, which also optimise to leg speeds, are summarised in Table 6.9. The second column gives the total NPV at the start of any journey when the ship is in port 1. This is the optimisation criterion of models $\mathcal{P}\ (1, m, 0)$, and thus shows we should prefer route $1 - 3$.

The third column list the daily profit value (USD/day) that can be earned. According to this criterion, one might conclude that there is almost no difference between the two options, and perhaps prefer route $1 - 2$ instead because the daily profits are slightly higher. (In fact, if one ignores the time value of money, the USD/day criterion would in this particular example the make route $1-2$ and $1-3$ have exactly the same daily profit value.) We recall that this criterion is used in e.g. Besbes and Savin (2009); Magirou et al. (2015). In this case, adopting route $1-2$ would not lead to the best decision because the underlying assumption of this approach, as already discussed in Chapter 4, would not be valid here.

Table 6.9: Optimal solutions of route with $m = 1$ leg and $G_o = 0$.

Route	$G_m(1)$ **(USD)**	**Daily Profit (USD/day)**
1-2	247854	16149.49
1-3	491635	16069.09

In a similar fashion as in the examples given in Chapter 4, we can consider values of the FPP different from zero. If different ports have different values, it may affect the route selection decision, and also the ship speed. Adopting an FPP different from zero does correspond to different contractual situations than the time charter P_1 model. The possible contracts this can model were already discussed in Chapter 4. In section 6.4.5 we will present further examples. □

Example 2: *Incorporating a transition port or not?*

Questions like these can be anwered again by constructing optimisation models from the class $\mathcal{P}$ $(1, m, 0)$, where the m legs are chosen such as to reflect a particular selected route.

We consider here the same example as before, but the situation where the ship can either do route 1-3 or route 1-2-3. Results are given in Table 6.10. In this case it is beneficial to include a visit to port 2. This will obviously delay receiving the revenues from the delivery to port 3, and make the total journey time longer.

If one ignores the time value of money, then in this example there would be no difference between route $1-3$ and route $1-2-3$ from either the criterion USD/day or total USD earned. □

Table 6.10: Optimal solutions with of route with $m = \{1, 2\}$ leg(s) and $G_o = 0$.

Route	$G_m(1)$ **(USD)**	**Daily Profit (USD/day)**
1-3	491635	16069.09
1-2-3	494875	16156.76

In conclusion, it may be important in problems of route selection (either in the context of a COA or not) to also consider the time value of money, and thus adopt the NPV criterion. Furthermore, as once more illustrated in the above examples, the model should ideally also reflect the type of contract under which the ship sails.

The value of the FPP adopted in the model must also reflect the type of contract. If a COA will be executed via a dedicated time charter contract (P_1) then the FPP is zero, but if the COA will be executed with e.g. a ship that the operator owns, then the G_o values in the model must reflect the Future Profit Potential at the final port. See further Section 6.4.5.

6.4.4.2 Optional cargoes in a COA.

We now turn to the question of how to evaluate the value of optional cargoes by a ship that will be used for executing a COA. This single-ship problem is a sub-problem encountered in the fleet-mix situations in which this kind of question may present itself in practice. It may also be part of problems in which one also needs to select the particular (type of) ship used for the COA - indeed this may depend on e.g. the characteristics of the typical optional cargoes one may also wish to have the ship accept.

In principle, this can be examined using the $\mathcal{P}$ (n, m, G_o) class of models developed in Chapter 4.

We illustrate this with the following example. Assume we are now at an arbitrary time point during the execution of a COA contract. The contract requires the ship to transport 'mandatory' cargoes from port 1 to port 3 and ballast back to port 1 with an agreed freight rate 20 USD/tonne. Now the decision maker knows that to fulfil the contract, the ship needs to do 3 more times of delivery. The 'optional' tasks are released and all the freight rates are given in Table 6.11.

Table 6.11: All cargoes available; (*) USD/tonne; (**) freight rate of the 'mandatory' cargoes

	Freight rate*		
Origin\Destination	**1**	**2**	**3**
1	0	10	20**
2	0	0	0
3	0	10	0

Table 6.12: Possible route scenarios

Case	**Route**	G_o **(USD)**
1	1-3-1-3-1-3-1	0
2	1-2-1-3-1-3-1-3-1	0
3	1-3-1-3-1-3-1-2-1	0
4	1-3-2-1-3-1-3-1	0
5	1-3-1-3-1-3-2-1	0

Before deciding which 'optional' cargoes to transport, Case 1 is the original route of COA, see Table 6.12, and it servers as the benchmark here. Case 2 is to do the 'optional' task that the ship has to first do a laden-ballast journey between port 1 and port 2. Similarly, Case 3 is the scenario that we decide to delay the 'optional' cargoes after the completion of 'mandatory' cargoes (or we know the job only until then). Last, Case 4 describes the scenario that the ship carries 'optional' cargoes from port 3 to port 2 while sailing back to port 1, and Case 5 describes if we have to do that job later. For the sake of comparison, the ship is required to be returned at port 1 and no future profits are considered here.

Table 6.13 summarised the optimal solutions in each of the five cases examined. Case 4 maximises the NPV (see fifth column).

Table 6.13: Optimal solutions with given route of $m = \{6, 7, 8\}$ legs; (*) Ratio of distance; (**) Ratio of optimal travel time.

Case	**Journey Time (days)**	**Laden/Ballast Ratio***	**Laden/Ballast Ratio****	$G_m(1)$ **(USD)**
1	184.7	100.00%	115.81%	1875315
2	217.7	100.00%	115.85%	1911871
3	217.8	100.00%	115.83%	1923419
4	191.3	140.00%	161.99%	2755188
5	191.0	140.00%	162.27%	2733543

Example 3: *Case 2 and Case 3.*

Both Case 2 and 3 corresponds to selecting the 'optional' job at port 2, and thus a laden plus a ballast leg are added to the route. With the consideration of port related costs, both case is not preferred.

This is because, though it seems that the 'optional' cargoes from port 1 to port 2 has the same 'daily profit' to the 'mandatory' cargoes, i.e., exactly half the freight rate (the revenue) and the distance (the fuel consumption cost), the port related costs are not deducted to the half. In addition, the future usage of the ship, i.e., the re-positioning has to be considered as well. As a consequence, the journey laden from port 1 to port 2 and then ballast back to port 1 is not profitable, even if we only consider the loading and unloading costs. Furthermore, there are other fixed port costs, such as towage fees and other service charges, that cannot be ignored or deducted without agreement. Thus, even though we have the same ratio of laden & ballast distance and even higher ratio of optimal travel time, the result could be lowering the Net Present Value of the whole logistics journey.

Arguably, a well-known rule of thumb in practice is that the ship owner (the ship operator) always wants to cut the waiting time or re-positioning time even at the cost of a discounted freight rate (c.f. Hellenic and International Shipping (2015)). Such criterion, as shown in this example, may not be effective enough for route or cargo selection in this example. For example, Case 2, compared to Case 3, guarantees the ship to have more laden time, but result in lower profits. This is because, if we understand the route 1-2-1 as a bad investment with a negative Net Present Value, executing this route earlier will lead to not only negative cash flow at the end of it, but also delaying the profitable 'mandatory' jobs. □

Example 4: *Case 4 and Case 5.*

For Case 4 and 5, only a laden leg is added to the original route, and to be more specific, part of the ballast leg (from port 3 to port 1) is replaced by a laden leg.

Doing these 'optional' cargoes can be understood as adding extra profits, and since the execution time of these jobs won't delay much of the revenues of 'mandatory' cargoes, the decision maker would always prefer to receive the additional profits earlier.

As a result, we see that both the laden & ballast ratio is much higher than Case 1. as well as the profits. Intuitively, this provides an example of why industry would consider such a criterion. In addition, it is noted that the total journey time is much closer in Case 4 and 5 (compared to Case 1) than in Case 2 and 3. Naturally, this would lead to an intuition that the comparison of different routes (or cargoes) should be made with fairly equal total journey time. However, such statement is doubtful as it fails to reflect also the practice, as claimed by Christiansen et al. (2004), that ship owner may want to have the period covered by logistics tasks as long as possible when he anticipate a future downturn of market, e.g., the FPP can be other than zero, and thus the results in Table 6.13 is no longer valid. □

6.4.5 Impact of FPP on the value and execution of a COA

In this section, we consider a more flexible and practical scenario that the ship operator can view the routing problem depending on the Future Profit Potential (FPP), that is, for different ports, a specific FPP can be assigned to represent varied maritime activities and/or regional market condition.

We consider a similar problem as in the previous section, namely the consideration of optional cargoes in a COA, but extended with additional 'options' which not necessarily include actual cargoes.

An example is, after completing a set of logistics tasks, say in North America, the ship operator now has the option to: (1) send the ship to a particular port for maintenance or lay-up, and consider the further usage of ship afterwards; (2) repeat the logistics tasks in North America as before; or (3) re-position the ship to another port, say in Europe, and serve profitable cargoes there. Clearly, the FPP associated with each option will be different.

Without further complicating the problem, we consider a similar routing problem to the discussion above, but with two more candidature destination ports, e.g., port 4 and 5 (see Table 6.14), apart from port 1, given to be considered at the end of journey.

Table 6.14: Journey characteristics of 5 port; (*) nautical mile.

	Distance*				
Origin\Destination	**1**	**2**	**3**	**4**	**5**
1	0	4000	8000	6000	12000
2	4000	0	4000	2000	8000
3	8000	4000	0	6000	4000
4	6000	2000	6000	0	10000
5	12000	8000	4000	10000	0

In particular, sailing back to port 1 means the the current logistics tasks will be repeated. Port 4 allows the ship to be maintained or lay-up at certain cost. And the ship can also sail to port 5, which is located in another economic and geometric region, to perform other profitable tasks.

If, the freight market is expected to go low, and thus the logistics task between port 1 and port 3 will become not profitable, which is equivalently formulated in Table 6.12, alternatively, we may consider the following scenarios in Table 6.15 to re-position the

Table 6.15: Possible route scenarios

Case	Route	G_o (USD)
1.1	1-3-1-3-1-3-4	-100000
1.2	1-3-1-3-1-3-5	100000
2.1	1-2-1-3-1-3-1-3-4	-100000
2.2	1-2-1-3-1-3-1-3-5	100000
3.1	1-3-1-3-1-3-1-2-4	-100000
3.2	1-3-1-3-1-3-1-2-5	100000
4.1	1-3-2-1-3-1-3-4	-100000
4.2	1-3-2-1-3-1-3-5	100000
5.1	1-3-1-3-1-3-2-4	-100000
5.2	1-3-1-3-1-3-2-5	100000

ship differently. For the fairness of comparison, assume the decision maker only cares about the FPP (after the completion of current journey) for the next two months, which is roughly the time period needed for maintenance, if arrived at port 4.

Table 6.16: Optimal solutions with given route of $m = \{6, 7, 8\}$ legs; (*) Ratio of distance; (**) Ratio of optimal travel time.

Case	Journey Time	Laden/Ballast Ratio	Laden/Ballast Ratio	$G_m(1)$*
1.1	178.0	109.09%	126.33%	1981318
1.2	171.2	120.00%	139.01%	2376172
2.1	211.0	107.69%	124.79%	2017110
2.2	204.2	116.67%	135.21%	2409124
3.1	211.2	107.69%	124.77%	2028440
3.2	231.3	87.50%	101.36%	1618044
4.1	184.6	155.56%	180.02%	2861038
4.2	177.8	175.00%	202.55%	3255326
5.1	184.3	155.56%	180.38%	2839183
5.2	204.4	116.67%	135.18%	2426363

The optimal results in Table 6.16, compared to those in Table 6.13, show that, whenever considering the re-position of ship at port 3, sailing to port 5 for profitable jobs is the best option. On the contrary, if, the ship is at port 2, then it is recommended to visit port 4 for a maintenance, as sailing to port 5 or 1 would be expensive. Consequently, this brings out the importance of re-positioning as the next port to visit may highly depend on where the ship is located.

Table 6.17: Optimal solutions with given route of $m = \{7\}$ legs; (*) $G_o = 500,000$ USD at port 5 and $G_o = -500,000$ USD at port 4.

Case	Journey Time	Laden/Ballast Ratio	Laden/Ballast Ratio	$G_m(1)$*
5	191.0	140.00%	162.27%	2733543
5.1	184.5	155.56%	180.38%	2455028
5.2	204.1	116.67%	135.18%	2808850

It also appears that, say, for example, by comparing both the laden & ballast ratio, it is obvious to find out the best route within Case 5, 5.1 and 5.2. However, the FPP set in this example is rather small, and thus if we change G_o from $(\pm)100,000$ USD to $(\pm)500,000$ USD for instance (see Table 6.17), then the optimal decision is again changed, and against such criterion of laden & ballast ratio.

Recall the discussion before in Chapter 4, the importance of FPP is also confirmed here in the context of route selection. Indeed, the FPP, which is an expected sum of revenues net the costs, may also be time depending, since the freight rate (Kavussanos & Alizadeh-M, 2001) and bunker price (Stefanakos & Schinas, 2014) have clear seasonality. A discussion in Magirou et al. (2015)[Section 7.2] reveals that the ship may re-position to different ports due to such seasonality. As a consequence, the same ship may decide to re-position to different ports when evaluate the scenario, say in the summer or in the winter.

Our model, that solves the route selection problem of which captures the importance of re-positioning of a ship for nice jobs available nearby, thus can be extended to allowing FPP to be not only port specific, but also time dependent. This feature will reflect the decision maker's expectation about future economics at different time point throughout the life of a ship (or fleet). Consequently, For different shipping companies, due to the difference of information and thus the expectations about future, the route selection (or the cargo selection) in the setting of COA or even wider scopes can be optimised differently.

6.5 Optimal Speed Decision under Uncertainty

6.5.1 Introduction

In most (if not all) situations, ship operator is making the decisions under uncertainty. Ship performance (engineering perspective), market fluctuation and weather condition are always changing over time. Hence, it is important for decision maker to have a consistent policy or method to generate such policy to decide the optimal strategy for ship to operate with uncertain conditions.

In this section, we have proposed a markov decision model incorporated with uncertain weather conditions for different risk preference decision makers to achieve the optimal policy.

6.5.2 Literature

In the context of maritime optimisation, stochastic properties of parameters have been investigated mostly in time window constraint (liner shipping optimisation, see Aydin et al. (2017); S. Wang and Meng (2012)), bunker fuel price & consumption (bunker management optimisation, see Besbes and Savin (2009); Sheng, Lee, and Chew (2014)), market conditions such as freight rate and demand (speed optimisation & scheduling, see Magirou et al. (2015); S. Wang et al. (2018); X. Wang, Fagerholt, and Wallace (2018); Wu, Pan, Wang, and Yang (2018)) and weather conditions (routing & engineering, see Kepaptsoglou, Fountas, and Karlaftis (2015)).

Only a few among them have adopted the method of markov decision modelling. Magirou et al. (2015) have designed a semi-markov process model to find an equilibrium with infinite time horizon and stochastic freight rate. S. Wang et al. (2018) have built a markov decision model to detect the optimal waste disposal policy with stochastic amount of waste.

The discussion about the utility theory and the analysis of risk attitude in maritime transportation is limited. Back to 1973, Lorange and Norman (1973) have used different lotteries to interview ship owners and then evaluate their risk attitudes accordingly. Cullinane (1991) has used surveys to assess and measure attitude towards market risk in shipping industry. Another work is done by H. Wang, Meng, and Zhang (2014), where they have assigned the random utility to reflect the attractiveness of liner shipping providers in the context of competition analysis among liner companies.

6.5.3 Mathematical model

In this section, we first introduce some notations. Given a sequence of ports $P = \{P_0, P_1, \ldots, P_n\}$ to be visited, we assume a chartered ship is ready in P_0 and would like to visit the rest ports one by one undertaking some logistics tasks, and finally to be returned to P_n. Without losing generality, P_0 and P_n can be exactly the same port, and hence this is a round-trip situation.

In the stochastic scenario, the first difference here compared to deterministic scenario is that, we don't always have exact information about the future evolution of the market conditions, such as charter hire, fuel price and freight rate. Another thing is that, during the journey, the ship might encounter with extreme weather or under-perform and hence be delayed.

Now the problem is, within the charter duration H, what is the best operation strategy? To be more specified, every time before we are going to load the cargo and leave a random port P_t, what speed shall we adopt to travel until arrive in the next port? Shall we wait for a better freight rate or lower bunker price?

6.5.3.1 Basic markov decision model

To answer the questions above, we first start from building a basic markov decision model to decide the optimal travel speeds for each leg.

Let $\Omega = \{\Omega_0, \Omega_1, \ldots, \Omega_n\}$ denotes the set of state space, which is in consistency with the time epoch of which the decision maker is going to load at each port. For a ship unloading in Port P_t at time point $j \in \Omega_t$, we have $\sum_1^j (s_t/V^+ + T_{t-1}^l + T_t^u) \leq j \leq \sum_1^j (s_t/V^- + T_{t-1}^l + T_t^u)$ and $\Omega_0 = 0$. Here s_t is the distance between P_{t-1} and P_t, V^+ and V^- are the maximum and minimum possible travel speed limited by the ship's physical characteristics, T_{t-1}^l is the time spent on loading cargo in P_{t-1}, similarly T_t^u is the time spent unloading cargo in P_t. The action space $A = \{a | V^- \leq a \leq V^+\}$ is the possible traveling speed adopted for each leg. With a reward function $r(.)$ and the probability distribution $p_t(j \in \Omega_{t+1} | i \in \Omega_t, a \in A)$ indicating how likely when starting from P_t at time i with speed action a, the ship will arrive P_{t+1} at time j, we can formulate the discounted reward function u_t at decision making epoch t as follows.

$$u_t(i \in \Omega_t) = \max_{a \in A} \sum_{j \in S_{t+1}} (r_t(i, j, a) + u_{t+1}(j) e^{-\alpha(j-i)}) p_t(j | i, a). \tag{6.28}$$

By breaking down the $r_t(i, j, a)$ part, we have:

$$r_t(i, j, a) = R(d_t, i) e^{-\alpha(j-i)} - C(d_t, i, j) - F(ftc, i, j), \tag{6.29}$$

where $R(.)$ is the revenue function which is generated by agreeing a specified freight rate at time i with demand d_t, and received it on the arrival of P_{t+1} at time j. $C(.)$ is the cost function which includes the fixed port dues after leaving P_t, the loading cost of cargo at P_t, bunker cost at P_t that covers the whole journey towards P_{t+1}, and the unloading cost of cargo at P_{t+1}. $F(.)$ is the charter cost function, which is concerned with the agreed charter hire ftc and the duration $(j - i)$.

In addition, the ship has to return the ship empty loaded back to P_n at the end of contract. This means, after arriving at P_n, there is no reward being given. If the ship violates the agreed contract duration H, there is a daily penalty c_H needs to be paid.

$$u_n(i \in S_n) = \begin{cases} 0 & \text{if } i \leq H; \\ -c_H(i - H) & \text{otherwise,} \end{cases} \tag{6.30}$$

Reminding of equation (6.28), we can state the problem as follows:

$$A^* = \{a_0^*, a_1^*, \ldots, a_{n-1}^*\} = arg \;\; max \;\; \{E(u_0), E(u_1), \ldots, E(u_n)\}. \tag{6.31}$$

6.5.3.2 Waiting option

Apart from the basic model, in practice, there is an option for ship to wait in harbour for better deals or to simply avoid making losses. We notice that, The investigation about waiting is rarely discussed in current literature. Because people could argue that the ship can always travel slower on the earlier leg and it is better than to wait in port. However, in a stochastic setting, the market condition is evolving everyday with noises that we can't fully understand. As a consequence, the operator never knows exactly how slow the ship should travel in order to enjoy a better deal.

Denote w_t the waiting time spent in port P_t before signing the freight contract and loading the corresponding cargo. For simplicity, we can assume w_t is bounded by a positive number σ. If w_t is approaching infinity, this would match the scenario of laying up the ship at port forever.

Similar to the basic model, we adopt the same notations here, but some of them need to be updated. For a ship unloading in Port P_t at time point $j' \in \Omega_t$, we have $\sum_0^j (s_t/V^+ + T_{t-1}^l + T_t^u + w_{t-1}) \le j' \le \sum_0^j (s_t/V^- + T_{t-1}^l + T_t^u + w_{t-1})$ and $\Omega_0 = 0$. The action space is $A = \{(a, w) | V^- \le a \le V^+, 0 \le w \le \sigma\}$. So the new discounted reward function u_t is as follows.

$$u_t(i \in S_t,) = \max_{a \in A} \sum_{j' \in S_{t+1}} (r_t(i, j', a, w) + u_{t+1}(j') e^{-\alpha(j'-i-w)}) p_t(j'|i + w, a), \tag{6.32}$$

where the $r_t(.)$ part is:

$$r_t(i, j', a, w) = R(d_t, i + w) e^{-\alpha(j'-i-w)} - C(d_t, i + w, j) e^{-\alpha w} - F(ftc, i, j'). \tag{6.33}$$

The problem is then stated as follows:

$$A^* = \{(a_0^*, w_0^*), (a_1^*, w_1^*), \ldots, (a_{n-1}^*, w_{n-1}^*)\} = \arg\max \ \{E(u_0), E(u_1), \ldots, E(u_n)\}. \tag{6.34}$$

Intuitively, by comparing the equations (6.32) and (6.33) to (6.28) and (6.29), we see that the waiting option only affect the agreed freight rate, bunker price, the delay factor of aggregated future rewards $u_{t+1}(j')$ (Note that $j' = j + w$, if we take the same action a) and the associated probability p_t. The similarity in model structure indicates that we can use the same type of algorithm to solve it.

6.5.4 When to lay up?

In particular, there is also a strategy of laying the ship up in harbour, when the market is low. This normally happens when the global demand of cargo transportation is smaller than the supply. Thus the operator might find it no longer profitable to do any logistics task. In order to save up the costs (e.g., crew cost and fuel consumption cost), they would lay up the ship in a harbour being idle for months.

There are investigations on lay-up option in both economic and maritime sense (Dixit & Pindyck, 1994; Sødal, 2006). However, they all look at the problem from the perspective of investment and thus consider it at a higher level. In particular, they don't explicitly discuss the different types of costs and revenues, but combine them as a return rate of the investment. Here we would like to look further into this option from the operational level to explain the different incentives.

For simplicity, we denote $u_{t+1}(j') = M_1$ and $u_{t+1}(j) = M_2$, and assume the transition probability p_t only depends on the action a, i.e., $p_t(j'|i+w,a) = p_t(j|i,a) = p_t(a)$. By comparing (6.32) and (6.28), we have:

$$D(w) = \sum_{j \in S_{t+1}} [(R(d_t, i+w)e^{-\alpha w} - R(d_t, i))e^{-\alpha(j-i)} \tag{6.35}$$

$$- (C(d_t, i+w, j)e^{-\alpha w} - C(d_t, i, j)) \tag{6.36}$$

$$- (F(ftc, i, j+w) - F(ftc, i, j)) \tag{6.37}$$

$$+ (M_1 e^{-\alpha(j-i+w)} - M_2 e^{-\alpha(j-i)})]p_t(a), \tag{6.38}$$

where $D(w)$ is the reward function of spending w time waiting in harbour compared to the optimal policy without waiting, and thus $D(w) = 0$ when $w = 0$. Since the waiting time cannot be negative, now the task is to find the optimal w^* within given range that maximises $D(w)$. To lay up the ship is equivalently to say that for all the possible waiting time $w_1 > w_2 > 0$, we always find $D(w_1) > D(w_2) > 0$ (i.e., $D'(w) > 0$ for all $w \in [0, d]$, where d is a positive constant given).

To better understand it, we now re-write (6.35) to (6.38) respectively. Here (6.35) is the term of revenue part. The only difference is the agreed freight rate ($r(.)$) and possibly the amount of cargo to transport (d_t). To avoid complicating the analysis, we assume fixed cargo demand here, as one could using the same methodology to incorporate the time dependent demand $d_t(.)$. Thus the difference of revenue term can be reduced to the difference of freight rate $(r(i+w) - r(i))$, multiplied by a fixed positive number d_t and then discounted by the fixed term $e^{-\alpha(j-i)}$.

Without loss of generality, we assume αw is a small term near 0, and thus we have the linear approximation $e^{-\alpha w} = 1 - \alpha w$. If the freight rate function $r(.)$ is n-times differentiable at i, by applying Taylor Expansion, we have $r(i+w) = r(i)+wr'(i)+op(1)$, and thus (6.35) can be expressed as follows:

$$D(w) = \sum_{j \in S_{t+1}} [(wr'(i) - \alpha w(r(i) + wr'(i)))d_t e^{-\alpha(j-i)} \tag{6.39}$$

(6.36) then gives the difference of cost term. Note that the cost can be divided into two parts: (1). the part c_{fix} is independent of time, such as the fixed harbour dues; (2). the part $c_{fuel}(.)$ that depends on time, since the bunker consumption cost is affected by the bunker fuel price that changes over time.

For c_{fix} term, the difference is multiplied by the term $e^{-\alpha w} - 1$ which changes from 0 to -1 monotonically with the increase of w. The $c_{fuel}(.)$ part is more complicated. However, the fuel consumption FC is irrelevant to the start time and only depends on the action a. Thus, if the fuel price function $c_{fuel}(.)$ has n-th order derivative, we can re-write (6.36) as follows:

$$-(-c_{fix}\alpha w + ((c_{fuel}(i) + wc'_{fuel}(i))(1 - \alpha w) - c_{fuel}(i))FC) \tag{6.40}$$

Now the charter hire term is easy to understand that by adding additional waiting time, we have to pay the extra charter hire to cover that duration. If we assume the charter hire is a continuous cash-flows ftc over time, then we can simplify the expression:

$$-\,ftc \quad w \tag{6.41}$$

Similarly, the discounted accumulated future rewards term (6.38) can be simplified as:

$$+\,e^{-\alpha(j-i)}(M_1(1 - \alpha w) - M_2)]p_t(a). \tag{6.42}$$

By observing (6.39) to (6.42), we see that $D(w)$ has no clear monotonic property. For (6.41) and (6.42), they are monotone decreasing with w. But the term of freight rate $r(.)$ is not necessarily monotonic as well as the fuel price c_{fuel}. Hence it is hard to draw an universal conclusion here with regards to the best waiting time.

Consider $D'(w) = \frac{\partial D(w)}{\partial w}$. By denoting the discount factor $e^{-\alpha(j-i)} = \phi(i, j)$, we have:

$$D'(w) = \sum_{j \in S_{t+1}} [((1-2\alpha w)r'(i) - \alpha r(i))d_t\phi(i,j) \tag{6.43}$$
$$+ (\alpha c_{fix} - ((1-2\alpha w)c'_{fuel}(i) + (1-\alpha)c_{fuel}(i))FC) \tag{6.44}$$
$$- ftc \tag{6.45}$$
$$- \alpha M_1 \phi(i,j)]p_t(a), \tag{6.46}$$

where $r'(i)$ and $c'_{fuel}(i)$ are the marginal freight rate and fuel price at time epoch i respectively. They can be understood as the change rate of the market based on the information available at time i. For example, $r'(i) > 0$ means we believe there will be better deals with higher freight rate coming later and $c'_{fuel}(i) < 0$ means the fuel price is gonna drop.

By observing (6.43) to (6.46), we notice that, if $r'(i) > 0$ is large enough, or $c'_{fuel}(i) < 0$ is small enough, we would have $D'(w) > 0$ for all the $w \in (0, d]$ (because all the other terms are fixed here). This is to say, we would like to lay up the ship in harbour if we believe there is a higher freight rate or lower fuel price in the future.

In particular, because αw is small, the term $1 - 2\alpha w > 0$ is positive and monotone decreasing with w, and thus the attractiveness of $r'(i)$ and $c'_{fuel}(i)$ is less in far future than at present. In another word, the ship is not encouraged to wait too long for a better freight rate or fuel price.

However, such statement is not complete. This is because, by saying large or small enough, we have to consider the fixed charter hire ftc and aggregated future rewards term M_1, which are constants here.

When ftc is set too high, waiting in harbour means wasting money doing nothing, and it is unlikely we can make it up by catching up better deals later. Thus it encourages the ship to do any profitable logistics task without considering much about the future.

And if M_1 is too large, for example, if we are talking about the first leg of a long profit generating journey, it is unlikely that we will observe such $r'(i)$ and $c'_{fuel}(i)$ that satisfy the condition. Hence the best waiting strategy cannot be to lay up the ship as long as possible.

In fact, on the contrary, if the M_1 term is small, or even negative (which is equivalent to say that we believe the market is too low and any logistics tasks are no longer profitable), then it is easy to have $D'(w) > 0$ with possibly negative $r'(i)$ and positive $c'_{fuel}(i)$.

Note that the above analysis is based on the assumption of w being small. However, if we decide to wait for a long time in harbour, i.e., lay up the ship, things are a bit different. When $w \to \infty$, we can formulate $D(w)$ as follows:

$$D(w \to \infty) = -\sum_{j \in S_{t+1}} [\frac{ftc}{\alpha} e^{-\alpha i} + R(d_t, i) e^{-\alpha(j-i)} - C(d_t, i, j) - F(ftc, i, j) + M_2 e^{-\alpha(j-i)}] p_t(a), \quad (6.47)$$

This is because when we decide to wait forever in the port, though the revenue and fuel cost would never be received or paid, the charter hire cost still needs to be paid. As a consequence, (6.47) shows that $D(w \to \infty) > 0$ is met if the daily charter hire is relatively cheap, and the optimal total profitability without waiting option is not profitable. Alternatively, the first term on the right hand side, that is the $\frac{ftc}{\alpha} e^{-\alpha i}$, can be interpreted as present value of the total cost of lay up, including maintenance, berth, etc. Thus the conclusion here agrees with the traditional analysis of lay up option in literature.

To conclude it, in the sense of profit seeking optimisation, the trade-off of waiting option is between the long-term market performance (e.g., charter hire & future rewards) and the short-term market fluctuation (e.g., freight rate & fuel price). And we shall use the waiting option (e.g., to lay up the ship) for the benefit of short-term fluctuation only if the long-term rewards are not guaranteed (e.g., the future profitability is small or even negative).

6.5.5 Algorithm and property

With the knowing information we have about u_n and the ultimate goal to find the optimal u_0, it is straightforward to consider a backward algorithm.

Algorithm 5: Backward algorithm

Step 1 Set $t = n$ and

$$u_n^*(i) = r_n(i) \quad \text{for all} \quad i \in S_n$$

Step 2 Substitute $t-1$ for t and compute u_t^* for each $i \in S_t$ by

$$u_t^*(i \in S_t) = \max_{A_t} \sum_{j \in S_{t+1}} (r_t(i, j, a_t) + u_{t+1}^*(j) e^{-\alpha(j-i)}) p_t(j|i, a_t)$$

Set

$$A_{i,t}^* = \arg\max_{A_t} u_t^*(i)$$

Step 3 If $t = 0$, stop. Otherwise return to step 2.

Algorithm 5 still falls into the area of dynamic programming. The optimality of the algorithm is guaranteed due to the nature of dynamic programming (Bertsekas, 1995).

Thus according to *the property of optimality* (Bellman, 1954), we can adjust our optimal speed decision (the optimal policy at each state S_t) whenever the information of parameters are updated. And the new optimal solution will not depend on our past actions. This is to say, we are able to combine our proposed model with other techniques, e.g., the rolling horizon decision making framework (Sethi & Sorger, 1991) that the decision maker can add new information to the model at each time epoch and receive the updated optimal policy from it.

Notice that here both the decision time epoch i in the state space S_t and the action a in the action space A are continuous in real life. In this paper, we treat them as discrete numbers with a fixed interval δ. This is because a small difference of i will not provide us much information. And it is not necessary to say the ship has to spend 2 minutes less on a 14 days journey in the sense of practice. Thus we simplify the model by introducing a gradient δ. Now the discrete-time model can be viewed as an approximation of the continuous model and provide a lower bound for the profit maximisation problem.

However, even if we have simplified the model, according to Bertsekas (1995), the computational cost of $u_t(i \in S_t)$ will explode exponentially with the increase of decision stages n due to the nature of dynamic programming. Another thing to be noted is the 'curse of dimensionality' of the state space S_t. This is to say, if we extend the state space, say, by including the bunker inventory I in S_t to incorporate the refueling problem. the model will be extremely hard to solve.

6.5.5.1 Two types of iteration methods

We notice the proposed problem itself is non-linear and we are unable to get the closed-form of the optimal solution. Thus to make the MDP model solvable, we will introduce two basic but important techniques and the rationale behind them in this section.

- The first technique, namely the *Value Iteration*, is to use a sequence of initial value functions (randomly selected or approximated by heuristic or simulation) as a start point and by refining it recursively, we can find a sequence of value functions that are no worse than the initial one.
- Similarly, the second one, namely the *Policy Iteration*, is to use a sequence of initial policies (randomly selected or approximated by heuristic or simulation) as a start point and by refining it recursively, we can find a sequence of policies that are no worse than the initial one.

To be more specific, Value Iteration is to use the Bellman equation given in Step 2 of Algorithm 5 to update the value function until convergence. Once the maximum of value function is found, the corresponding optimal policy can be solved accordingly.

As to Policy Iteration, the first step is to evaluate the current policy by substituting it into the objective function and calculate the reward value at each stage. Then the next step is to use this value as the future reward in the Bellman equation to find out a new policy that maximises the expected reward of each stage. By repeating these two steps, the optimal policy is found.

Here we summarise the ideas of each of them by giving proposition 6.2 as follows:

Proposition 6.2.

- *The optimality is guaranteed for both Value Iteration and Policy Iteration;*
- *Compared to Value Iteration, Policy iteration takes considerably fewer number of iterations to converge although each iteration is more computationally expensive.*

The proof can be referred to Bellman (1954) as they are both monotone convergence. Generally speaking, in practice, Policy Iteration could be more computational efficient (i.e., requires fewer iterations to converge), when compared to Value Iteration. However, once more decision variables are introduced into the policy space, such as how many bunker fuel to intake at each port in our problem, the computational cost of each iteration for Policy Iteration would increase exponentially. A potential improvement is to use reinforce learning technique as the dynamic programming method itself can be transferred into a reinforce learning algorithm easily.

6.5.6 Weather uncertainty

In this section, we are going to specify what kind of uncertainties are we discussing within this paper, which, in particular, refers to the weather. The impact of weather is always one of the major concerns in the context of shipping. For example, researches and applications of weather routing use satellite data collected and predictions for weather conditions on sea, e.g., wind and wave, to help ship avoiding route with unwanted weather (see for instance the weather routing problems Dickson, Farr, Sear, and Blake (2019); H. Lee, Aydin, Choi, Lekhavat, and Irani (2018); K. Wang et al. (2018)).

In this paper, to better reflect the reality that bad weather would normally slow the ship down, cause delays and bring in additional fuel consumption costs, we assume the ship encountering extreme weather has to get around the affected areas and travel at the cost of additional distances with same speed. The probability of delays could be generated on a per route basis from the archive data. Without of losing generality, here we assume there are two probabilities $p_0^i(a)$ and $p_1^i(a)$ ($p_0^i(a) + p_1^i(a) = 1$) in accordance with good weather and extreme weather on the given route with start time i with travel speed a. Then, we can write down the rewards for each scenarios based on probabilities as a lottery:

$$\begin{pmatrix} p_0^i(a) = p_t(j_0|i,a) & p_1^i(a) = p_t(j_1|i,a) \\ u_0^i & u_1^i \end{pmatrix}$$

By extending the probabilities linked to weather (or varied uncertainties arisen from other interests), the lottery itself can be enriched for each route. This, naturally, transfers the original problem into a set of fixed scenarios, which is simple but effective as shown by Nasri, Bektaş, and Laporte (2018) in a road transportation case.

6.5.7 Risk attitude

The expected value of such a lottery above is $u_t(i)$. This is to say, the markov decision process model has implicitly assumed the decision maker is risk-neutral. Hence the expected profitability $u_t(i) = u_0^i p_0^i + u_1^i p_1^i$ is indifference to the lottery we have here and the alternative investment with certain reward $u_t(i)$. However, if the decision maker, on the contrary, is risk averse or risk loving, this is no longer the case.

In order to investigate the impact of risk attitude, here we will introduce the utility theory developed in game theory by Morgenstern and Von Neumann (1953). The idea is to transfer the profitability with an utility function to incorporate the risk attitude, e.g., $U_t(i) = U(u_0^i)p_0^i + U(u_1^i)p_1^i$. Hence the expected profitability of transferred lottery is weighted by the risk preference. This can be used further in route selection or liner shipping competition, see also H. Wang et al. (2014).

Chapter 7

Conclusions

7.1 Contributions and findings

It is recognised that ship speed plays an important role to determine the overall profitability in maritime shipping as well as reflecting the ship operators' reaction towards market and policy. Classical speed optimisation models however fail to account for many practical issues or provide clear managerial insights that the management in shipping industry faces. This thesis investigates how to look at the maritime speed optimisation problem from a new angle of NPV approach that explicitly accounts the exchange of cash-flows between charter parties and shareholders. The main purpose is to provide a more robust and powerful framework that contributes to not only the practice for management and society but also theories in maritime OR literature by exploring how each payment and associated shipping activity change the optimal speed choice under contractual scenarios, while providing of which practical situations the proposed model can produce more economic benefits or contribute more environmental efforts. In particular, the investigation in this thesis is related to charter contract, voyage-alike contract and specific contractual clauses, e.g., redelivery clause, in the sense of deterministic setting.

The finding in Chapter 3 suggests that traditional speed optimisation models are not suitable for time chartered ship of which to be used for a series of identical journeys, where a fixed time horizon is pre-determined. Based on the cash-flow NPV approach, we propose a P_M model that provides a more general and robust technique to solve such problem. In addition, two conventional speed optimisation models are shown to be equivalent to P_1 and P_∞ models as two special cases in the NPV framework. We also aim to provide the reasoning of different speed choices in practice that each decision maker may view the problem at varied time point, and also have different risk attitude. Thus we hope the findings can contribute to the understanding of speed optimisation not only to literature but also the practice.

In Chapter 4, We extend our investigation to the voyage-alike contract scenario where a general optimisation modelling framework $\mathcal{P}\ (n, m, G_o)$ for a series of (identical) multi-legs journeys. The findings demonstrates that the NPV framework presented exhibits two novel elements in comparison to existing speed optimisation modeling approaches in the literature: (1) When executing a series of identical journeys, optimal ship speeds from one execution of the journey to the next execution are shown to change. We refer to this as the chain effect; (2) The ship's optimal speed is in general highly dependent on the decision maker's view about the ship's future profit potential. The analysis and numerical result compared to classic speed optimisation frameworks clearly show the robustness of our framework and also the importance of underlying assumptions made. Thus the methodology developed in the chapter can further help the study of other maritime optimisation problems.

To study the dynamic nature of market as pointed out in Psaraftis and Kontovas (2014), in Chapter 5 we derive the mathematical analysis of market parameters such as freight rate and fuel price by assuming they are time dependent. This helps to improve the understanding of optimal speed decisions as the reaction to complicated market conditions. Given a specific scenario, namely the end of contract scenario, which is also the situation specified in redelivery clause of a time charter contract. We find the belief of decision maker towards future will impact the speed choice and thus the preference of types of contract. Generally speaking, Shell Time 4 clause may contribute more to pollution reduction and be preferred in continental shipping, while ExxonMobil 2000 clause may provide more flexibility that comes with better economic benefits especially for coastal shipping.

The findings of selected topics in Chapter 6 bring in further managerial insights of different maritime shipping scenarios based on NPV framework. The role and importance of the harbour time, or the time that the ship spends in a port, cannot be ignored in speed optimisation models. By adding the harbour time, future cash-flows are postponed accordingly, and thus change the total profitability with the consideration of NPV. As a consequence, the optimal speeds are changed. This implies that the assumptions made in literature, e.g., Johnson and Styhre (2015), that reducing a harbour time can save up more time for ship to spend on sea, is not always true from the perspective of profit-seeking decision maker. The numerical example, obtained from Corbett et al. (2009), again demonstrates that improving the harbour efficiency will not simultaneously contributes to the emission reduction.

An alternative type of charter contract between ship owner and charterer, namely the revenue sharing contract, is also discussed. This type of contract has been widely applied in supply chain management (Yao et al., 2008). When assuming time varying charter hire, both from the mathematical analysis and numerical results, the revenue sharing contract shows its power to capture the market evolution, and the fixed charter hire contact can be used as an approximation with parameters carefully defined for certain situations as it implicitly assumes no trends of the market development. However, when there is a clear trend, the optimisation with revenue sharing contract will provide a better profitability. Furthermore, we also present the profitability from the perspective of ship owner, and a Stackelberg Game is detected between the ship owner and charterer.

The Contract of Affreightment (COA) is also discussed in the NPV framework. In particular, we demonstrate, from both the mathematical analysis and the numerical experiments, that the timing of payments will lead to different profitability and also the optimal speed decisions. The route selection problem involved to select the 'optimal' cargoes is also investigated. The $\mathcal{P}$ (n, m, G_o) model developed in previous chapter confirms that, with the consideration of time value of money, the optimal route selected may also depend on the Future Profit Potential (FPP). Traditional criterion, such as ratio of laden & ballast and the daily profit (USD/day), may fail to provide useful information for situations such that decision maker may have different FPP for different port.

Finally, in the stochastic setting, we also present the optimal lay-up strategy from the operational level of optimisation. We show that the lay-up option is a useful tool to balance the long-term market performance of which includes charter hire costs and future rewards of logistics activities, and the short-term market fluctuations such as freight rate and fuel price. It is only recommended to use the lay-up option for the benefit of short-term market fluctuations when the long-term rewards are not guaranteed, e.g., future profitability is relatively low.

To conclude it, this thesis has delivered important managerial implications for many practical scenarios and contractual details by deriving the mathematics to find the root of the rationale. The cash-flow approach used in this thesis has built a bridge between the theories and applications that can potentially close of gap between researchers and companies. The encouraging results of adopting NPV framework indicate that the efforts of this thesis should not be limited to ship speed optimisation but can also be extended to all maritime relevant scenarios especially of practical concerns as it provides a robust modelling technique to explicitly incorporate and explain factors with regard to reality as suggested by Psaraftis (2017).

7.2 Implications

7.2.1 General

Unlike traditional research on maritime optimisation, we do not consider classic problems like scheduling and routing or emerging problems like virtual arrival that incorporate with time windows. However, the results of this thesis can serve as the foundation to solve these problems with potentially more insights.

Section 6.4, for instance, presents how routing and scheduling problem in maritime shipping can be better solved and understood in the NPV framework. The existing criterion used for route selection in practice may fail to arrive at the optimal schedule in terms of maximising the NPV of decision maker. The assistance of FPP in the NPV framework would better reflect the reality and thus help improving the decision making.

Other types of shipping problems with time window can be solved within NPV framework as well, as shown in Chapter 5. Even though port time window is arguably an exogenous input instructed by the ship operator or contract, rather than port operator (Psaraftis, 2017), problems like virtual arrival could potentially helps to reduce GHG emissions as many uncertainties may affect the actual execution of the journey and thus the optimal decisions may be updated when needed. In this case, the problem is similar to a rolling horizon decision making framework, with each time the decision maker may have different information available. The dynamic programming algorithms developed in this thesis as well as the NPV models can be directly adopted for this type of decision making process, due to the *property of optimality*.

In short, we hope to demonstrate that this research has built a solid foundation for maritime optimisation with the assist of NPV framework. Extensions of existing findings towards other maritime optimisation problems seem worthwhile to explore.

7.2.2 Environmental

This research can offer a framework to investigate practical implications of various potential initiatives which are being examined at the level of the IMO with the aim to decarbonise shipping and reduce emissions in general. In particular, it can help to evaluate short-term operational measures for speed reduction and speed optimisation.

According to Psaraftis (2019a), ships sailing much slower than their design speed is typically observed in periods of depressed market conditions or high fuel prices, and this is reported in every market. Slow steaming not only saves fuel but also reduces emissions. In the context of ongoing climate change concerns, ship speed has thus become an important consideration as well (Psaraftis & Kontovas, 2014).

As shortly examined in Section 3.6.4, the effect of imposing speed limits can be examined in the context of time charter using our proposed NPV models. With the maximum speed of the ship being reduced from 17 kn to 14 kn, the cargo capacity transported by the ship is thus reduced as well. Such scenario like discussed in Ronen (2011) should therefore be carefully examined as more ships may be introduced in order to fulfil the total demand.

On the other hand, the impacts of bunker levy, as shown in Section 4.5.4, stress that such evaluation requires careful selection of the model and underlying assumption. That is, individual ship operators may react very differently to the bunker levy implemented.

The findings of Section 6.2 in addition answer the question proposed by Johnson and Styhre (2015) that reducing harbour time does not always encouraging emission reduction, and further point out the importance of taking harbour time into consideration for speed optimisation. Existing literature, such as Corbett et al. (2009), may need to be re-visited, in terms of examining the operational options to reduce GHG emissions.

While not explicitly examined, we envisage that the modelling framework can be adapted towards different technologies of propulsion, engine, hull design, and other (retrofit) options. Specifically, the fuel consumption function can be replaced without mathematical difficulty by other (convex) functions of speed that better reflect the particular ship design. The benefit of this modelling framework is that it allows for the economic / environmental evaluation of any new technology within a particular logistics context.

To conclude it, the NPV models proposed in this thesis provide robust and sophisticated framework to examine different options under various shipping scenarios that are considered by IMO and other stakeholders regarding greening the shipping industry. The research thus has the potential to help facilitate the making of maritime policy and regulation in the future.

7.3 Limitations and future work

There are various limitations in this thesis that can be improved in the future. This thesis mainly looks at the operational level of the usage of the chartered ship and thus a view from higher scope, such as tactical level or strategy level can be investigated as well. The type of shipping is limited to tramp (or at least tramp alike) within this thesis. However, the adoption to liner shipping and industrial shipping, potentially from the perspective of a global supply chain and thus including the inventory problem, can be done based on the existing findings.

In particular, the end of contract scenario has been briefly discussed in Chapter 3, as well as the redelivery clause in Chapter 6. However, the decision's risk attitude, as well as the speed decision dynamics can be further extended, especially when considering the whole problem in a stochastic setting. For instance, when having a wrong prediction of future markets, decision makers, subjected to their own risk attitude, may bear a very different loss. As a consequence, the potential investigation on the relationships between decisions' expectations and speed choices can offer more practical insights for the manager broad, especially when dealing with the real life uncertainties.

The usage of $\mathcal{P}(n, m, G_o)$ and the concept of Future Profit Potential (FPP) developed in Chapter 4, can be also extended to wider scope. The discussion of COA contract, as a good example, confirms that our NPV framework can model different maritime scenarios explicitly and thus offers new insights for the management. The application of this modelling framework thus can be also used, for example, to determine when and where to go for maintenance or lay-up. Also, by allowing the FPP to be not only port specific, but also time dependent can help to improve the models, e.g., take into account the 'seasonality' as discussed in literature (Magirou et al., 2015).

Another direction led by the examination of contract related problems in Chapter 5 is the negotiation (game) between ship owner and charterer. The revenue sharing contract, as partly examined in Chapter 6, reveals a Stackelberg game with the owner being the leader and the charterer being the follower. Further study that focus on the contractual details and the potential games within charter party will provide a new angle to understand how to evaluate different types of clauses during the negotiation and what is the best strategy for each stakeholder within the party to choose.

In addition, the stochastic nature of shipping context problems are worthwhile for future investigation as well. Based on the initial findings of time-varying parameters in Chapter 5, allowing the parameters, either already included or not, to be random over time will better reflect the reality of what industry is facing everyday. The NPV framework, as developed within this thesis, enables the analysis of such a complex problem together with other managerial factors, such as risk attitude and information availability. The computational work can be improved as well by introducing the Machine Learning algorithms, e.g., reinforcement learning, that can be easily transformed from the proposed dynamic programming technique.

Chapter 8

Appendix

8.1 Chapter 3

8.1.1 Data of Ship Characteristics

- **Ship characteristics**: $157,800$ tonne dwt capacity (scantling) and $145,900$ tonne dwt (design); $A = 49,000$ tonne lightweight of ship; $k = 3.910^{-6}$, $p = 381$, $g = 3.1$ and $h = 2/3$, corresponding to a fuel consumption of 60.5 tonne bunker fuel at nominal speed 15.2 kn. Auxiliary fuel consumption (in ports) at 5 tonne/day. Maximum speed $v^+ = 17$ kn and minimum speed $v^- = 10$ kn. Cargo discharge rate at 3000 m^3/hour. Ballast stability condition: minimum fill-rate of vessel at 30% of design dwt (ballast tank capacity $54,500$ m^3).

- **Journey characteristics**: Distance between the two ports is $8,293$ nm (symmetric). Waiting time (queueing plus towing to berth) 24 hours in first port and 48 hours in second port; loading rates 3000 m^3 per hour in both ports.

- **Demand characteristics**: parcel of $1,200,000$ barrels of light crude oil at 1.07 m^3/tonne, and 1 barrel equals 0.136 m^3.

- **Revenue structure**: Freight rate at $0,50$ USD per barrel and 1,000 nm. Revenue received 12 hours before start of unloading at destination port.

- **Cost structure**: Total fixed port costs of a journey towards the first port equals $300,000$ USD, while being $500,000$ USD for a journey towards the second port. Unloading and loading charges are at a rate of $4,000$ USD/hour in both ports. Main bunker fuel at 63 USD/barrel and auxiliary fuel at 590 USD/tonne in both ports. Port and fuel costs due within 72 hours after leaving port.

- **Charter party data**: Zero forward start, and TCH at $20,000$ USD/day.

- **Opportunity cost of capital**: $\alpha = 0.08$.

8.1.2 Algorithm

Since the physical limits of ship (maximum and minimum speed) and travel distances between each ports are known in advance, it is possible to discretise the arrival time for each port (i.e., each L_i). The dynamic programming algorithm is then introduced as follow:

Backward Dynamic Programming

Step 1 Define the step size of travel time ζ.

Define a set $L_i := \{L_i^{min}, L_i^{min} + \zeta, L_i^{min} + 2\zeta, \ldots, L_i^{max}\}$ that arrival times at each port i are divided into discrete values in the set, where $\mathrm{L}_i^{min} = \frac{\sum_i S_i}{365*24v_{max}}$ and $L_i^{max} = \frac{\sum_i S_i}{365*24v_{min}}$;

Step 2 For $i = n \to 1$

Find the optimal $T_i^{'=\arg\max G_{n-i+1}(L_{i-1})}$ for each L_{i-1} in the set;

Find the optimal L_{i-1}^* and the associated $T_i^{'}$ (denoted as T_i^*) with the maximum $G_{n-i+1}^*(L_{i-1}^*)$;

Step 3 The new set $T^* := \{T_1^*, T_2^*, \ldots, T_n^*\}$ is the optimal decisions and G_n^* is the maximum NPV of the total journey.

The optimality of such backward algorithm can be proved easily according to Bellman (1954).

8.1.3 Proof of theorem 1

Proof. (I) The NPV of p1 is:

$$\mathrm{NPV}(x, \alpha) = \frac{a(x)}{\alpha}(1 - e^{-\alpha\delta(x)}) \tag{8.1}$$

$$= \frac{a(x)}{\alpha}(1 - \sum_{k=0}^{\infty} \frac{(-\alpha\delta(x))^k}{k!}). \tag{8.2}$$

With $\overline{\mathrm{NPV}}$ representing the linear approximation of NPV, we get:

$$\overline{\mathrm{NPV}}(x, \alpha) = \frac{a(x)}{\alpha}(1 - (1 - \alpha\delta(x))) \tag{8.3}$$

$$= a(x)\delta(x). \tag{8.4}$$

(II) The NPV of p2 is:

$$\mathrm{NPV}(x, \alpha) = \frac{a_1}{\alpha}(1 - e^{-\alpha\delta_1}) + \frac{a_2}{\alpha}(1 - e^{-\alpha\delta_2})e^{-\alpha\delta_1} \tag{8.5}$$

$$= \frac{a_1}{\alpha}(1 - \sum_{k=0}^{\infty} \frac{(-\alpha\delta_1)^k}{k!}) + \frac{a_2}{\alpha}(1 - \sum_{k=0}^{\infty} \frac{(-\alpha\delta_2)^k}{k!}) \sum_{k=0}^{\infty} \frac{(-\alpha\delta_1)^k}{k!}. \tag{8.6}$$

Algebraic manipulation shows that:

$$\overline{\text{NPV}}(x,\alpha) = a_1(x_1)\delta_1(x_1) + a_2(x_2)\delta_2(x_2) + \alpha a_2(x_2)\delta_2(x_2)\Big(\delta_1(x_1) - \frac{\delta_2(x_2)}{2}\Big) \quad (8.7)$$

$$\neq a_1(x_1)\delta_1(x_1) + a_2(x_2)\delta_2(x_2), \quad (8.8)$$

unless $\alpha = 0$ or $\delta_2(x_2) = 2\delta_1(x_1)$. (III) The NPV of p1 is:

$$\text{NPV}(a,\alpha) = \frac{a}{\alpha}(1 - e^{-\alpha\delta}). \quad (8.9)$$

Therefore, if $A = NPV(a, \alpha)$:

$$a = NPV(a,\alpha)\frac{\alpha}{1 - e^{-\alpha\delta}} = NPV(a,\alpha)\sum_{i=0}^{\infty} \alpha e^{-i\alpha\delta} \quad (8.10)$$

$$= \alpha NPV(A,\alpha) = AS(A,\alpha). \quad (8.11)$$

Because both projects have the same $\delta(x)$ ($x \in X$), maximising a of p1 is thus equal to maximising the AS of p3. (IV) The AS of p4, given A_1 and A_2 being the NPV of a_1 and a_2, is:

$$\text{AS} = \Big[\frac{a_1}{\alpha}(1 - e^{-\alpha\delta_1}) + \frac{a_2}{\alpha}(1 - e^{-\alpha\delta_2})e^{-\alpha\delta_1}\Big]\frac{\alpha}{1 - e^{-\alpha(\delta_1+\delta_2)}} \quad (8.12)$$

$$= \frac{a_1(1 - e^{-\alpha\delta_1})}{1 - e^{-\alpha(\delta_1+\delta_2)}} + \frac{a_2(1 - e^{-\alpha\delta_2})e^{-\alpha\delta_1}}{1 - e^{-\alpha(\delta_1+\delta_2)}}. \quad (8.13)$$

Note that:

$$\frac{\alpha}{1 - e^{-\alpha X}} = \frac{1}{X} + \frac{\alpha}{2} + \frac{\alpha^2 X}{12} + O(\alpha^3). \quad (8.14)$$

Therefore, algebraic manipulation shows that:

$$\overline{\text{AS}} = \frac{1}{(\delta_1 + \delta_2)}\Big[a_1\delta_1 + a_2\delta_2 + \frac{\alpha}{2}(a_1(\delta_1)^2 + a_2(\delta_2)^2) - \alpha a_2\delta_1\delta_2\Big] + \frac{\alpha}{2}(a_1\delta_1 + a_2\delta_2) \quad (8.15)$$

$$\neq \frac{a_1\delta_1 + a_2\delta_2}{\delta_1 + \delta_2}, \text{ unless } \alpha = 0. \quad (8.16)$$

(V) (a) follows immediately from the proof of part (IV). For (b) observe from proof (IV) that:

$$\lim_{\alpha\to 0}\overline{\text{AS}} = \frac{a_1\delta_1 + a_2\delta_2}{\delta_1 + \delta_2} \quad (8.17)$$

$$= \frac{a_1\delta_1}{\delta_1 + \delta_2} + \frac{a_2\delta_2}{\delta_1 + \delta_2} \quad (8.18)$$

$$\neq \frac{1}{2}(a_1 + a_2), \text{ unless } \delta_1 = \delta_2. \quad (8.19)$$

The contributions of a_1 and a_2 need to be weighted by their relative duration to the total journey duration. An objective function $a_1 + a_2$ places too much emphasis on the process with the shortest relative duration. This completes the proof. □

8.2 Chapter 4

8.2.1 Data of the example of base case

In the base case example used to examine the models of Ronen (1982), the following data has been used:

- **Ship characteristics**: 157, 800 tonne dwt capacity (scantling) and 145, 900 tonne dwt (design); $A = 49,000$ tonne lightweight of ship; $k = 3.910^{-6}$, $p = 381$, $g = 3.1$ and $a = 2/3$. Auxiliary fuel consumption (in ports) at 5 tonne/day. Maximum speed $v^{+} = 17$ kn and minimum speed $v^{-} = 10$ kn. Cargo discharge rate at 3000 m^3/hour. Ballast stability condition: minimum fill-rate of vessel at 30% of design dwt (ballast tank capacity 54, 500 m^3).
- **Journey characteristics**: Distance for any of the legs is 8, 000 nm. Loading rates 3000 m^3 per hour in any port. The waiting time needed in each port is 24 hours.
- **Demand characteristics**: parcel of 1,200,000 barrels of light crude oil at 1.07 m^3/tonne for $A-B$, 600,000 barrels of light crude oil for $C-D$ and 800,000 barrels of light crude oil for $D-A$. And 1 barrel equals 0.136 m^3.
- **Revenue structure**: Freight rate at 10 USD/tonne for $A-B$ and $D-A$, 8 USD/tonne for $C-D$.
- **Cost structure**: Total fixed port costs of a journey towards the each port equals 300, 000 USD. Unloading and loading charges are at a rate of 4, 000 USD/hour in each port. Main bunker fuel at 498 USD/tonne and auxiliary fuel at 590 USD/-tonne in each port.
- **Charter party data**: Zero forward start, and TCH at 20, 000 USD/day.
- **Opportunity cost of capital**: $\alpha = 0.08$ per year.

8.2.2 Data of the example of literature

In the example of Fagerholt and Psaraftis (2015), the data are as in 8.2.1, except for:

- **Ship characteristics**: 10, 000 tonne dwt capacity; $A = 5,000$ tonne lightweight of ship; $k = 3.910^{-6}$, $p = 381$, $g = 3.1$ and $a = 2/3$. Maximum speed $v^{+} = 21$ kn and minimum speed $v^{-} = 15$ kn.

- **Journey characteristics**: The voyage used in the example is between Antwerp and Halifax with a total distance of 2873 nm (symmetric), while the distance outside the ECA zone is 773 nm and the distance inside ECA zone is 2100 nm. No waiting time at both ports. (This corresponds to data reported in Fagerholt and Psaraftis (2015).)
- **Demand characteristics**: The ship is fully loaded to transport the cargo from the port outside of the ECA zone to the port inside of the ECA zone.
- **Revenue structure**: Freight rate is 45 USD/tonne.
- **Cost structure**: No fixed port cost at both port. Main bunker fuel price of HFO, which is used outside the ECA zone, is 294.5 USD/tonne, This price is used as the baseline in the example. The auxiliary fuel is 500 USD/tonne in both ports.

8.2.3 Necessary conditions for concavity

8.2.3.1 For a single leg h_j

To begin with, we first look at the net profitability of a random leg with index i: $h_j(T_j^i)$. Now in order to find out how to can find its maximum value, we take its first and second order derivative with respect to T_j^i, viz:

$$\frac{\partial h_j(T_j^i)}{\partial T_j^i} = -\alpha(f^{TCH}+R_j-C_j^u)e^{-\alpha T_j^i}-(-2c_j^f k(W_j+A)^h S_j^3/T_j^i+c_j^f kp(W_j+A)^h), \tag{8.20}$$

$$\frac{\partial^2 h_j(T_j^i)}{\partial T_j^{i2}} = \alpha^2(f^{TCH} + R_j - C_j^u)e^{-\alpha T_j^i} - 2c_j^f k(W_j + A)^h S_j^3. \tag{8.21}$$

By observing (8.21), $\frac{\partial^2 h_j(T_j^i)}{\partial T_j^{i2}}$ is decreasing with T_j^i as the travel time can not be negative. In addition, one shall notice that the discount coefficient α^2 is an extremely small multiplier. Thus $f^{TCH} + R_j - C_j^u$ is relatively small for any trip (with $T_j^i > 0$) as otherwise it would be unrealistic in the sense of practice (e.g., the daily charter hire f^{TCH} is normally around $20,000$ to $30,000$ USD per day, and the revenue R_j cannot be at least hundreds of millions for a single leg). This implies that, generally speaking, $\frac{\partial^2 h_j(T_j^i)}{\partial T_j^{i2}} < 0$ holds, and thus by solving $\frac{\partial h_j(T_j^i)}{\partial T_j^i} = 0$, the optimal travel time that maximises the net revenue of a single leg is found as depicted in Figure 4.3.

On the other hand, if, the revenue of a single leg R_j or the daily charter hire cost f^{TCH} is a very large number, e.g., the logistics task is either super profitable or the cost of chartering a ship is too high, we might encounter an extreme case of which $\frac{\partial^2 h_j(T_j^i)}{\partial T_j^{i2}} \geq 0$. Equivalently, we have $\alpha^2(f^{TCH} + R_j - C_j^u)e^{-\alpha T_j^i} \geq 2c_j^f k(W_j + A)^h S_j^3 > 0$. By substituting it into (8.20), the equation can be re-written as:

$$\frac{\partial h_j(T_j^i)}{\partial T_j^i} \leq -2c_j^f k(W_j + A)^h S_j^3/\alpha + 2c_j^f k(W_j + A)^h S_j^3/T_j^i - c_j^f kp(W_j + A)^h. \quad (8.22)$$

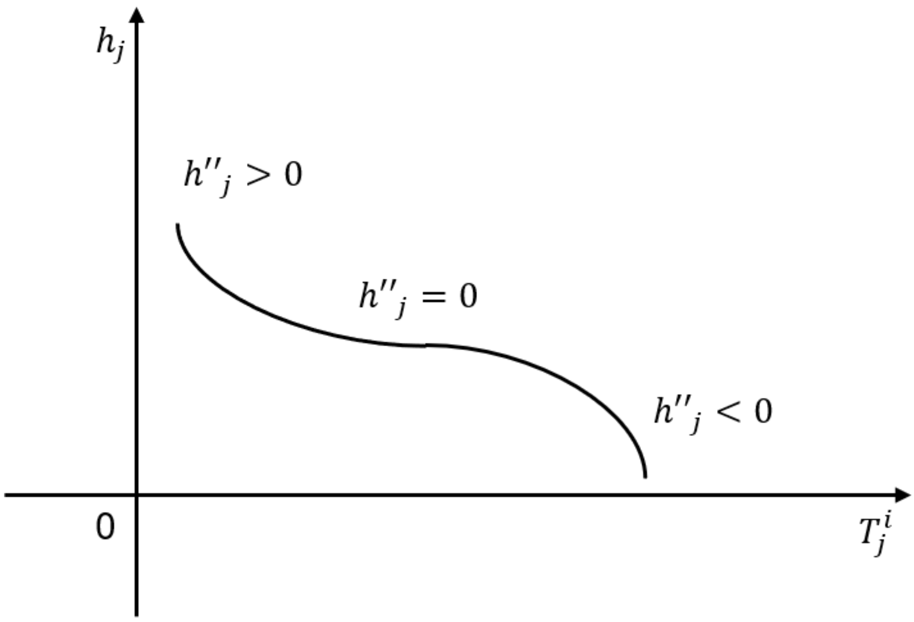


Figure 8.1: an extreme example of h_j

If $\alpha < T_j^i$, which is true in practice, we have $\frac{\partial h_j(T_j^i)}{\partial T_j^i} < 0$. Thus we can draw the graph of h_j as given in Figure 8.1. It suggests that now the optimal plan is to travel as fast as possible when the revenue is too attractive or the daily charter hire is too high.

8.2.3.2 For a journey h with m legs

Back to the discussion of roundtrip, without loss of generality, let the roundtrip journey contains two legs, i.e., $m = 2$. Thus the net profitability of any roundtrip is given:

$$h(\Gamma_i) = h_1(T_1^i) + h_2(T_2^i)e^{-\alpha T_1^i}. \quad (8.23)$$

Reminding Algorithm **??**, the optimal travel time of latter leg, e.g., T_2^i here, is solved first and it is independent of the choices of T_1^i. For simplicity, we denote T_2^{i*} as the optimal travel time for the second leg, and $h_2(T_2^{i*})$ is the corresponding maximum net profitability. Now take the derivatives of $h(\Gamma_i)$ with respect to T_1^i and we have:

$$\frac{\partial h(\Gamma_i)}{\partial T_1^i} = \frac{\partial h_1(T_1^i)}{\partial T_1^i} - \alpha h_2(T_2^{i*})e^{-\alpha T_1^i}, \tag{8.24}$$

$$\frac{\partial^2 h(\Gamma_i)}{\partial T_1^{i2}} = \frac{\partial^2 h_1(T_1^i)}{\partial T_1^{i2}} + \alpha^2 h_2(T_2^{i*})e^{-\alpha T_1^i}. \tag{8.25}$$

If the second leg is a ballast leg, then (8.25) is negative for sure, and thus the profit structure for the journey h is concave. However, when the second leg is laden, there are two possible scenarios.

The first one is, by noticing that α^2 is extremely small, as long as $h_2(T^{i*}_2)$ is not too large, the term $\alpha^2 h_2(T_2^{i*})e^{-\alpha T_1^i} \approx 0$, and thus the concavity of h is still guaranteed. This case is more likely to be observed in practice.

The other possibility is, when $h_2(T^{i*}_2)$ is a very large positive number, i.e., the future profitability is too attractive, note the profit structure for the journey h is now convex, and thus to maximising h, we would like to have (8.24) to be either a large positive number or small negative number. Given the fact that $h_2(T^{i*}_2) > 0$, now the optimal T^{i*}_1 that maximises h is to force $\frac{\partial h_1(T^{i*}_1)}{\partial T^{i*}_1} > 0$. According to Figure 4.3, this is to say that the ship will travel as fast as possible in the first leg, if the net profitability of later journey is too attractive. Using the same induction, we can extend the number of legs from 2 to m and arrive at the similar conclusion.

8.2.4 Proof of Theorem 1

For a random leg j of the journey with index i, the optimal travel time T_j^{i*} is found by solving the following equation:

$$\frac{\partial G_i}{\partial T_j^i} = \frac{\partial h(\Gamma_i)}{\partial T_j^i} - \alpha G_{i-1}^* e^{-\alpha L_m^i} = 0.$$

If $h > 0$, we have $G_{i-1}^* > 0$ (by definition), and thus $\frac{\partial h(\Gamma_i)}{\partial T_j^i} > 0$ (or $\frac{\partial h(\Gamma_i)}{\partial T_j^i} < 0$, if $h < 0$). By comparing T_j^{i*} and T_j^{i-1*} we have:

$$\begin{aligned}
\frac{\partial h(\Gamma_i)}{\partial T_j^i} - \frac{\partial h(\Gamma_{i-1})}{\partial T_j^{i-1}} &= \alpha(G_{i-1}^* e^{-\alpha L_m^{i}{}^*} - G_{i-2}^* e^{-\alpha L_m^{i-1*}}) \\
&= \alpha e^{-\alpha L_m^{i}{}^*}(G_{i-1}^* - G_{i-2}^* e^{-\alpha(L_m^{i-1*} - L_m^{i}{}^*)}) \\
&= \alpha(1-\varepsilon_1)(G_{i-1}^* - G_{i-2}^*(1-\varepsilon_2)) \\
&= \alpha(G_{i-1}^* - G_{i-2}^*) - \varepsilon.
\end{aligned}$$

In practice, the completion time of a roundtrip ${L_m^i}^*$ cannot be super large, e.g., a few years. As a consequence, numerically the term $\varepsilon \to 0$ can be ignored, unless we assume the profitability of a single journey is also such a small number (smaller than 1). Thus, if $h > 0$, term $G_{i-1}^* - G_{i-2}^* > 0$.

Now we give the expression of $\frac{\partial h(\Gamma_i)}{\partial T_j^i}$ as follows:

$$\frac{\partial h(\Gamma_i)}{\partial T_j^i} = h_j'(T_j^i) - \alpha(h_{j+1}^*({T^{i*}}_{j+1})e^{-\alpha T_j^i} + \cdots + h_m^*({T^{i*}}_m)e^{-\alpha \sum_{k=j}^m T_k^i})$$

Since $h_j''(T_j^i) < 0$, it can be shown that $\frac{\partial h(\Gamma_i)}{\partial T_j^i}$ is decreasing with T_j^i. As a consequence, we have $T_j^i < T_j^{i-1}$, if $h > 0$. Similarly, the others can be proved in the same way. **Q.E.D.** $\square$

8.2.5 Proof of Theorem 2

Denote a mapping F:

$$F(\Phi(n)) = h(\Gamma_n) + e^{-\alpha L_m^n}\Phi(n-1).$$

Such that given any function Φ of n, it is obvious that for any $\Phi > \Phi'$, we always have $F\Phi > F\Phi'$, namely the monotone property. Another property is, for any real number $p \in \mathbb{R}$, we have $F(\Phi + pI)(n) = F\Phi(n) + pe^{-\alpha L_m^n}$ for any n, where $I(n) \equiv 1$ is a unit function. They are easily proved by the definition of F.

Now, if ϕ and φ are two bounded functions, then we can denote a constant $b = \max_n |\phi(n) - \varphi(n)|$. This gives the following inequality:

$$\phi(n) - b \leq \varphi(n) \leq \phi(n) + b,$$

By applying the mapping F to above inequality, we have

$$F(\phi(n) - b) = F\phi(n) - be^{-\alpha L_m^n} \leq F\varphi(n) \leq F(\phi(n) + b) = F\phi(n) + be^{-\alpha L_m^n},$$

which is equivalent to say

$$\max_n |F\phi(n) - F\varphi(n)| \leq be^{-\alpha L_m^n} = e^{-\alpha L_m^n} \max_n |\phi(n) - \varphi(n)|,$$

where $e^{-\alpha L_m^n} \in [0, 1)$ for all $L_m^n > 0$.

Now consider the scalar function $H(n) = \sum_{i=1}^{n} h(\Gamma_i)e^{-\alpha \sum L_m^{i-1}}$ with $L_m^0 \equiv 0$, which is the total discounted net profit across all journeys. Because $T_j^i \in [T_{min}, T_{max}]$ with $0 < T_{min} < T_{max} < +\infty$ (by definition, which is the result of the physical limits of speeds and positive travel distance which cannot be extended to ∞), we have $0 < L_m^i < +\infty$ (because $m < +\infty$). h is bounded by M, then we show $H(n)$ is also bounded in $\mathbb{R}$:

$$\lim_{n\to\infty} |H(n)| = \sum_{i=1}^{\infty} |h(\Gamma_i)|e^{-\alpha \sum L_m^{i-1}} \leq \sum_{i=1}^{\infty} Me^{-i\alpha m T_{min}} = \frac{M}{1 - e^{-\alpha m T_{min}}}.$$

Denote the space $\Omega = [-\frac{M}{1-e^{-\alpha m T_{min}}}, \frac{M}{1-e^{-\alpha m T_{min}}}]$ in $\mathbb{R}$. According to *Heine–Borel theorem*, Ω is compact, since it is a closed and bounded subset of $\mathbb{R}$. By replacing ϕ by $H(n)$, φ by $\hat{H}(n)$ (we can understand it as an approximation of $H(n)$ in the same space Ω), the inequality $\max_n |FH(n) - F\hat{H}(n)| \leq e^{-\alpha T_n} \max_n |H(n) - \hat{H}(n)|$ shows that the mapping $F : \Omega \to \Omega$ is a contraction mapping. According to the *Banach fixed-point theorem*, there exists a unique point H^*, such that $\lim_{n\to\infty} FH(n) = H^*$.

Notice that $FH(n) = h(\Gamma_n) + e^{-\alpha L_m^n} H(n-1)$ is the discounted net profit at stage n, which is equivalent to $G_n = h(\Gamma_n) + e^{-\alpha L_m^n} G_{n-1}$. Thus we show that $\lim_{n\to\infty} G_n = G^*$, which is equivalent to say there exists Γ^*, such that $\Gamma^* := arg\ maxG^* = arg\ max(h(\Gamma_n) + G_{n-1}e^{-\alpha L_m^n})$ with $n \to \infty$. **Q.E.D.** $\square$

8.3 Chapter 6

8.3.1 Proof of Theorem 1

We now show the proofs for each of the statement in sequence.

- We start from giving the derivative of h^s of a single journey with respect to T^p:

$$\frac{\partial h^s}{\partial T^p} = -(Q_2 + f^{TCH})e^{-\alpha L_{s-1}^{i'}} e^{-\alpha T^p}.$$

If $Q_2 + f^{TCH} > 0$, we have $\frac{\partial h^s}{\partial T^p} < 0$ and then reducing harbour time T^p will bring in higher profitability.

Now in a more general sense, if the journey is repeated, similarly we give the derivative of G_i^s with respect to T^p:

$$\frac{\partial G_i^s}{\partial T^p} = \frac{\partial h^s(\Gamma_i', T^p)}{\partial T^p} + \frac{\partial G_{i-1}^s}{\partial T^p} e^{-\alpha(L_m^{i'} + T^p)} - \alpha G_{i-1}^s T^p e^{-\alpha(L_m^{i'} + T^p)}.$$

If $Q_2 + f^{TCH} > 0$, $\frac{\partial h^s(\Gamma'_i,T^p)}{\partial T^p} < 0$. Since we assume the round trip is profitable, now we have $-\alpha G^s_{i-1} T^p e^{-\alpha(L^{i'}_m + T^p)} < 0$.

By observing its structure, the term $\frac{\partial G^s_{i-1}}{\partial T^p}$ is the aggregation of $\frac{\partial h^s(\Gamma'_j,T^p)}{\partial T^p} - \alpha G^s_{j-1} T^p e^{-\alpha(L^{j'}_m + T^p)}$ for $j = i-1, i-2, \ldots, 1$[1], which should be negative as well.

Thus for any repetition, as long as $Q_2 + f^{TCH} > 0$, the statement holds. In addition, The larger i is, the bigger impact should we observe, i.e., $|\frac{\partial G^s_i}{\partial T^p}| > |\frac{\partial G^s_{i-1}}{\partial T^p}|$.

- For the second statement, we now look at h^s. For the first part, we choose a random leg j within Q_2 and take the derivative of $h^s(\Gamma'_i, T^p)$ with respect to $T^{i'}_j$:

$$\frac{\partial h^s(\Gamma'_i, T^p)}{\partial T^{i'}_j} = (h'_j(T^{i'}_j) - \alpha(h_{j+1}(T^{i}_{j+1}{}') e^{-\alpha T^{i'}_j} + h_m(T^{i}_m{}') e^{-\alpha(T^{i'}_j + \ldots T^{i'}_{m-1})}))e^{-\alpha T^p}.$$

Only if $T^p \to +\infty$, term $e^{-\alpha T^p} \to 0$, and thus make the decision variable $T^{i'}_j$ irrelevant here (i.e., for any speed, the change of current value of the net revenue is zero). However, this is the scenario that clearly out of practice. Thus by letting $\frac{\partial h^s(\Gamma'_i,T^p)}{\partial T^{i'}_j} = 0$, T^p shows no impact on the optimal $T^{i'}_j$ here. Because j is chosen randomly, the first part of the statement holds.

- Similarly we choose a random leg k within Q_1, thus we have:

$$\begin{aligned}\frac{\partial h^s(\Gamma'_i, T^p)}{\partial T^{i'}_k} &= h'_k(T^{i'}_k) - \alpha(h_{k+1}(T^{i'}_{k+1}) e^{-\alpha T^{i'}_k} + \cdots + h_m(T^{i'}_m) e^{-\alpha(T^{i'}_k + \cdots + T^{i'}_{m-1})}) \\ &- \alpha(Q_2 e^{-\alpha T^p} - \frac{f^{TCH}(1 - e^{-\alpha T^p})}{\alpha}) e^{-\alpha(L^{i'}_{s-1} - L^{i'}_{k-1})}.\end{aligned}$$

Here the term $Q_2 e^{-\alpha T^p} - \frac{f^{TCH}(1-e^{-\alpha T^p})}{\alpha}$ is fixed for any k, which is the postponed future aggregated net revenues minus the additional charter hire cost. For simplicity, we denote $Q(T^p) = Q_2 e^{-\alpha T^p} - \frac{f^{TCH}(1-e^{-\alpha T^p})}{\alpha}$.

It is easy to see that $Q(T^p)$ is decreasing with T^p, if and only if $Q_2 + f^{TCH} > 0$ (i.e., Q_2 is not a large negative number). If $Q_2 + f^{TCH} = 0$, we have $Q(T^p) < 0$. Since h'_k is decreasing, we have $T^{i'}_k > T^i_k$, i.e., adding harbour time will now urge the ship to travel slower.

On the contrary, if Q_2 is a large negative number such that $Q_2 + f^{TCH} < 0$, though including harbour time will still bring in the incentives as same as discussed above, such incentives are reduced each time with a larger harbour time T^p.

- When $i > 1$, now we choose a random leg j within Q_2 and take the derivative of G^s_i with respect to $T^{i'}_j$, which is the optimal travel time for the i-th repetition:

[1] Notice that $h^s(\Gamma'_1, T^p) = G^s_1$ and $G^s_0 = 0$.

$$\frac{\partial G_i^s}{\partial T_j^{i'}} = (h_j'(T_j^{i'}) - \alpha(h_{j+1}(T_{j+1}^{i'})e^{-\alpha T_j^{i'}} + \cdots + h_m(T_m^{i'})e^{-\alpha(T_j^{i'}+\cdots+T_{m-1}^{i'})}))e^{-\alpha T^p}$$
$$- \alpha(h^s(\Gamma_{i-1}', T^p)e^{-\alpha L_m^{i'}} + \cdots + h^s(\Gamma_1', T^p)e^{-\alpha(L_m^{2'}+\cdots+L_m^{i'})})e^{-\alpha T^p}.$$

We notice the original optimal T_j^i without harbour time should be:

$$\frac{\partial G_i}{\partial T_j^i} = (h_j'(T_j^i) - \alpha(h_{j+1}(T_{j+1}^i)e^{-\alpha T_j^i} + \cdots + h_m(T_m^i)e^{-\alpha(T_j^i+\dots T_{m-1}^i)}))$$
$$- \alpha(h(\Gamma_{i-1})e^{-\alpha L_m^i} + \cdots + h(\Gamma_1)e^{-\alpha(L_m^2+\cdots+L_m^i)}).$$

Denote $Q(h^s) = h^s(\Gamma_{i-1}', T^p)e^{-\alpha L_m^{i'}} + \cdots + h^s(\Gamma_1', T^p)e^{-\alpha(L_m^{2'}+\cdots+L_m^{i'})}$ and $Q(h) = h(\Gamma_{i-1})e^{-\alpha L_m^i} + \cdots + h(\Gamma_1)e^{-\alpha(L_m^2+\cdots+L_m^i)}$.

Let j move to m and make the above two derivatives both equal to zero, we have:

$$h_m'(T_m^{i'}) - h_m'(T_m^i) = \alpha(Q(h^s) - Q(h)).$$

If $Q_2 + f^{TCH} > 0$, then we have $Q(h^s) < Q(h)$. Since h_j' is decreasing, we show that $h_m'(T_m^{i'}) < h_m'(T_m^i)$ and $T_m^{i'} > T_m^i$. By moving j backward to s, for any other $j \in \{s, s+1, \dots, m\}$, we have:

$$h_j'(T_j^{i'}) - h_j'(T_j^i) < 0,$$
$$T_j^{i'} > T_j^i.$$

This is to say, by adding harbour time, the adjusted optimal speeds within Q_2 will be slower than original.

Similarly, we choose a random leg k within Q_1 and thus we have:

$$\frac{\partial G_i^s}{\partial T_k^{i'}} = h_k'(T_k^{i'}) - \alpha(h_{k+1}(T_{k+1}^{i'})e^{-\alpha T_k^{i'}} + \cdots + h_m(T_m^{i'})e^{-\alpha(T_k^{i'}+\cdots+T_m^{i'})})$$
$$- \alpha(Q_2 e^{-\alpha T^p} - \frac{f^{TCH}(1 - e^{-\alpha T^p})}{\alpha})e^{-\alpha(L_{s-1}^{i'} - L_{k-1}^{i'})} - \alpha Q(h^s)e^{-\alpha T^p}.$$

$$\frac{\partial G_i}{\partial T_k^i} = h_k'(T_k^i) - \alpha(h_{k+1}(T_{k+1}^i)e^{-\alpha T_k^i} + h_m(T_m^i)e^{-\alpha(T_k^i + \dots T_m^i)})$$
$$- \alpha \hat{Q}_2 e^{-\alpha(L_{s-1}^i - L_{k-1}^i)} - \alpha Q(h).$$

$\hat{Q}_2$ refers to the original revenue structure of those journeys as in Q_2 and without the impact of harbour time. According to the conclusion obtained above, it is easy to see $\hat{Q}_2 > Q_2$.

If $Q_2 + f^{TCH} > 0$, we notice that $T_k^{i'} > T_k^i$ by letting k move from $s-1$ back to 1 each time. Thus adding harbour time would urge the ship to travel slower for the whole trip.**Q**.**E**.**D**.

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