Effect of size-dependent properties on electromechanical behavior of composite structures

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Abstract

Due to its electromechanical applications in the form of nanodevices such as distributors, actuators, and sensors, the electromechanical behavior of piezocomposite structures becomes a new avenue for research. This article presents the derivation of an exact analytical solution of the composite plate based on theory of Kirchhoff's plate and extended theory of piezoelectricity. The electromechanical behavior of piezocomposite structures accounting the influence of size-dependent properties such as piezoelectric and surface effect is investigated. In addition to this, the parametric analysis is carried out using the different parameters such as aspect ratio and thickness on the electromechanical response of composite structures. The consequences of the present study explore that the influence of size-dependent properties on the electromechanical behavior of composite structures is noteworthy with respect to the size of structures and can be ignored at bulk sizes. The electromechanical behavior including dynamic response (resonant frequency) of composite plates shows significant enhancement as compared to the conventional composite plate. This current study offers pathways for developing novel composite materials with enhanced control authority and offer guideline for the application and design of nanodevices in energy harvesting. It also highlights the opportunity to evolve high-performance and lightweight micro/nano-electro-mechanical system (M-/NEMS).

Keywords: Piezoelectricity; Surface effect; Composites.

1. Introduction

The quest for exploiting such an extraordinary mechanical property of graphene and their low density as well as high specific surface area led to the opening of an evolving area of research for developing of graphene-based hybrid nanocomposites. In the recent advances, the piezoelectric phenomenon has received ample interest from application as well as fundamental point of view with purpose of evolving MEMS applications such as energy harvesters, distributors, transistors, nano-generators, sensors, actuators and electric switches. To predict overall properties of reinforced composite, Naskar and co-authors [1], [2] studied a new micromechanics model named "stochastic representative volume element (SRVE)" by consideration of spatial distribution. Kundalwal et al.[3] studied the stress transfer characteristics and mechanical properties of composites including nano- and microscale reinforcements via micromechanical pull-out model and molecular dynamic (MD) simulations. Most recently, Shingare and Kundalwal [4] explored the electromechanical behavior of carbon-based material such as graphene-reinforced nanocomposite (GRNC) and composite plate structures by introducing piezoelectric nanoscale graphene fiber in a non-piezoelectric polyimide matrix.

From open scientific literature, it is clearly observed that the conventional fiber-reinforced composite with consideration of graphene nanofillers specifies that graphene is the utmost fascinating nanofillers, immensely considered in the recent period. However, until now, to the best knowledge of the present authors', no single study is proposed for examining modal analysis of a hybrid piezocomposite plate by accounting the surface effect, that may offer many prospects for emerging next generation MEMS application. The aim of present research effort is to further extend the presence of homogeneous electric field and strain in structural plate element. Because of electromechanical coupling of piezoelectric materials and consideration of surface parameters, in-plane displacements generated in the piezoelectric plate depends on the in-plane boundary conditions (BCs). Still, the surface-stress-induced relaxation concept was not reported in earlier studies of nanoplates considering the effect of surface parameters. Hence, dissimilar in-plane BCs are considered in current study to investigate the piezoelectric and surface effects.

2. Problem descriptions

The dynamic analysis of a rectangular plate having thickness h, width b, and length a as demonstrated in Fig. 1 is carried out in the present article. The GHPC plate is described by Cartesian coordinate system (x, y, z) with plate thickness along the z-direction. The top and bottom surfaces of GHPC plate are subjected to an electric voltage V. Due to large in-plane dimension (length and width) of plate as compared to thickness, one can neglect the components of in-plane electric field when the plate is under electromechanical loading along the thickness direction. Hence, in current study, the electric field is considered to be present only across the thickness and is described in terms of the electric potential (voltage V),

$$\mathrm{E_z} = -\emptyset_{,z} \tag{1}$$

The electric BCs ($\emptyset(h/2) = \mathrm{V}$ (volt) and $\emptyset(-\mathrm{h}/2) = 0$) are applied on GHPC plate shown in Fig. 1.

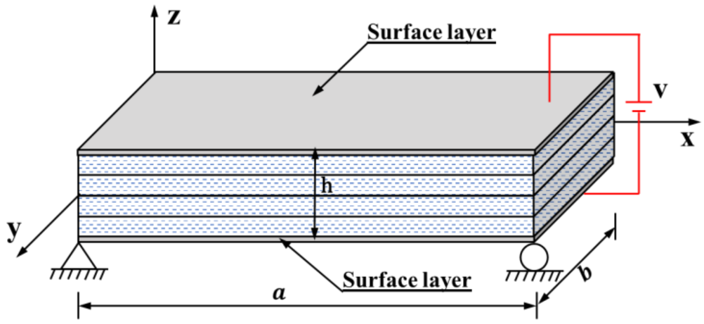

Fig. 1: Schematic of GHPC plate subjected to electric voltage.

The surface piezoelectric model to account the surface effects is implemented here. As per model, the GHPC plate is made of a bulk part and the top and bottom surfaces with insignificant thickness. In case of bulk part, the material follows the equivalent constitutive expressions as that of conservative piezoelectric materials. By considering the plane stress condition, the component of stress in z-direction can be neglected. Thus, the constitutive relations are deduced as:

$$\sigma_{xx} = C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} - e_{31}E_z; \quad \tau_{xy} = C_{66}\gamma_{xy}; \; D_z = e_{31}(\varepsilon_{xx} + \varepsilon_{yy}) + \epsilon_{33} E_z \tag{2}$$

here C, e and $\in$ represent elastic, piezoelectric and dielectric coefficients. The constitutive relations for surface parameters are same as that of bulk parameters but surface residual stress is only additional term.

$$\sigma^s_{xx} = \sigma^0_{xx} + C^s_{11}\varepsilon^s_{xx} + C^s_{12}\varepsilon^s_{yy} - e^s_{31}E_z; \quad \sigma^s_{xy} = \sigma^0_{xy} + C^s_{66}\gamma_{xy}; \quad D^s_x = D^0_x, D^s_\gamma = D^0_\gamma \tag{3}$$

whereas D^s_α and $\sigma^s_{\alpha\beta}$ denote surface electric flux density and stresses; C^s_{11}, C^s_{12}, C^s_{66} denote surface elastic coefficient; e^s_{31} denotes piezoelectric coefficient with surface effect; D^0_α and $\sigma^0_{\alpha\beta}$ denote residual surface electric flux density and surface stress. The presence of the surface stresses generates traction jumps employed on the bulk part. The in-plane displacements of plate are expressed as [5]. The electric flux density must satisfy Gauss's law due to absence of free charges,

$$\nabla D_{x,x} + \nabla D_{y,y} + \nabla D_{z,z} = 0. \tag{4}$$

The vibration response of GHPC plate considering simply supported BCs are expressed [6]:

$$\text{at } x = 0, a; \quad w = 0, M_{xxs} = 0 \quad \text{and} \quad \text{at } y = 0, b; \quad w = 0, M_{yys} = 0 \tag{5}$$

The following two cases are considered.

1. Traction-free condition: $N_{xxs} = N_{xys} = 0$ at $x = 0, a$ and $N_{yys} = N_{xys} = 0$ at $y = 0, b$. Considering $\sigma^0_x = \sigma^0_y = \sigma^0$ and $\sigma^s_{yy} = 0$, Eq. (6) can be re-expressed with $\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4}\right) = W_4$ and $\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = W_2$:

$$F_{11}(W_4) + 2(F_{12} + 2F_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + \rho h \frac{\partial^2 w}{\partial t^2} - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}(W_2) = 0 \tag{6}$$

$$F_{11}=\left(C_{11}+\frac{e_{31}^2}{\epsilon_{33}}\right)\frac{h^3}{12}+\left(C_{11}^s+\frac{e_{31}e_{31}^s}{\epsilon_{33}}\right)\frac{h^2}{2};\ F_{12}=\left(C_{12}+\frac{e_{31}^2}{\epsilon_{33}}\right)\frac{h^3}{12}+\left(C_{12}^s+\frac{e_{31}e_{31}^s}{\epsilon_{33}}\right)\frac{h^2}{2}$$
$$F_{66}=C_{66}\frac{h^3}{12}+C_{66}^s\frac{h^2}{2} \tag{7}$$

Due to coupling of surface parameters and piezoelectric materials, in-plane strains might be generated, which are deduced considering traction as $\varepsilon=\frac{e_{31}V+2\left(\sigma^0+e_{31}^s\frac{V}{h}\right)}{(C_{11}+C_{12})h+2(C_{11}^s+C_{12}^s)}$.

2. $u_0(x,y)=v_0(x,y)=0$: It is obtained by fixing the boundaries of plate restricting the in-plane movement. The governing Eq. of the transverse vibration of the plate is expressed as

$$F_{11}(W_4)+2(F_{12}+2F_{66})\frac{\partial^4 w}{\partial x^2\,\partial y^2}+\rho h\frac{\partial^2 w}{\partial t^2}-\frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}(W_2)=\left[2\left(\sigma^0+e_{31}^s\frac{V}{h}\right)+e_{31}V\right](W_2). \tag{8}$$

As per BCs, the harmonic solution is described as [4]. Making use of harmonic solution, the resonant frequency (ω_{mn}) is expressed with half wavenumbers (m, n) for case 1 and case 2 as:

$$\omega_{mn}^{(1)}=\frac{(\pi)^2\left(\sqrt{F_{11}\left[\left(\frac{m}{a}\right)^4+\left(\frac{n}{b}\right)^4\right]+2(F_{12}+2F_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2}\right)}{\rho h+\left(\frac{\rho h^3}{12}\right)\left[\left(\frac{m\pi}{a}\right)^4+\left(\frac{n\pi}{b}\right)^4\right]} \tag{9}$$

$$\omega_{mn}^{(2)}=\frac{1}{\rho h+\left(\frac{\rho h^3}{12}\right)\left[\left(\frac{m\pi}{a}\right)^4+\left(\frac{n\pi}{b}\right)^4\right]}\left\{\begin{array}{c}(\pi)^2\left(\sqrt{F_{11}\left[\left(\frac{m}{a}\right)^4+\left(\frac{n}{b}\right)^4\right]+2(F_{12}+2F_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2}\right.\\ \left.+\left[2\left(\sigma^0+e_{31}^s\frac{V}{h}\right)+e_{31}h\right]\left(\frac{m\pi}{a}\right)^4+\left(\frac{n\pi}{b}\right)^4\right)\end{array}\right\} \tag{10}$$

3. Results and discussions

In this Section, the results obtained to explore the electromechanical behavior of piezocomposite plates with the dissimilar in-plane constraints are discussed. The properties of GHPC are chosen: C_{11} = 112 GPa, C_{12} = 3.34 GPa, C_{66} = 2.04 GPa, e_{31} = −6.933 Cm^{-2} and ϵ_{33} = 3.24 × 10^{-9} F/m for the bulk part [7]. The surface elastic moduli and piezoelectric constants for the present model is equal to the elastic moduli and piezoelectric constants of GHPC multiplied by its surface layer thickness, assumed as 1 nm. Moreover, the residual surface stress is taken as 1.0 N/m. First, we considered the effect of the surface elastic moduli, surface piezoelectric coefficient and surface stress on the dynamic analysis of the plate. The dimension of plate is considered as a = b = 20h. Figure 2 demonstrates the normalized resonant frequency $\omega_{11}/\omega_{11}^0$ of mode (1,1) with respect to the plate thickness in case of free vibration whereas ω_{11}^0 denotes the resonant frequency with zero surface effects. Both surface elastic and piezoelectric effect have similar influence on the resonant frequency of the plate with dissimilar in-plane constraints in case 1 and 2 for zero electric voltage. Though, the residual surface stress has no influence on the transverse vibration of the plate with case 1, whereas in-plane constraints will generate in-plane relaxation. In case 1 and 2, the surface effects show significant results for smaller plate thickness, whereas their effect decreases with increase in thickness of plate, as shown from both plots. From this figure, it is also seen that the in-plane constraints have a noteworthy influence on the dynamic response of the plate, i.e., the resonant frequency do not vary with mode numbers and applied electrical load with in-plane traction-free situations.

In this study, we only presented the results for case 1, but for the sake of brevity, the results for case 2 are not described here. In addition to this, we checked the effect of different aspect ratios on the resonant frequency of plate. From this, it is noticed that different aspect ratios do not affects the resonant frequency for case 1. It can be stated that the electric voltage generates the in-plane strain with zero in-plane traction situation. It is determined that the dynamic response of plates is noticeably influenced because of in-plane BCs. It is noticed that the electric voltage and surface stress have zero influence on the resonant frequency of the plate, while they generate a relaxation strain.

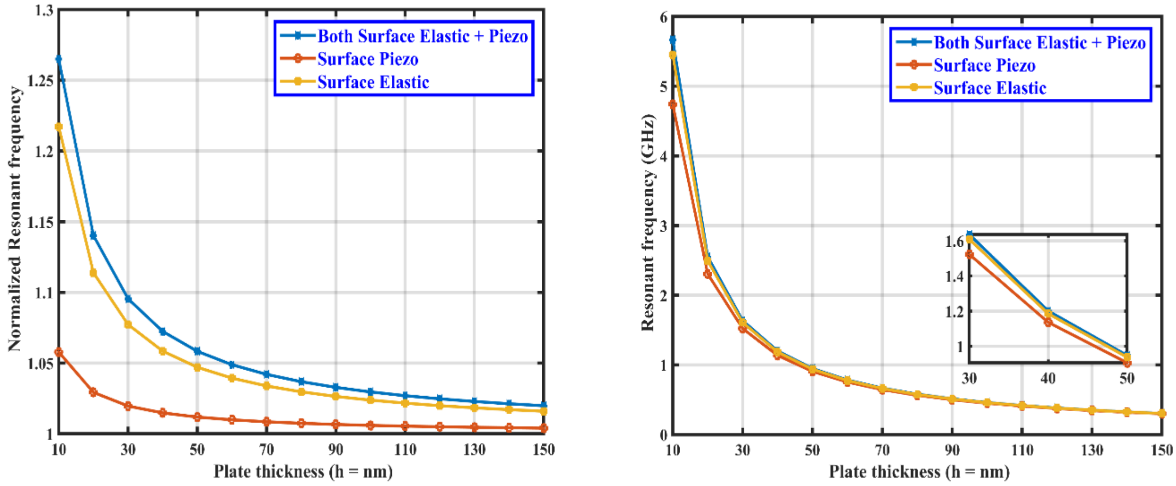

Fig. 2: (a) Normalized resonant frequency and (b) resonant frequency Vs plate thickness.

4. Conclusions

In this article, a hybrid simply supported piezocomposite plate under dissimilar in-plane constraints is considered to study the influence of surface parameters on their resonance frequency based on a modified Kirchhoff plate theory. The effect of surface parameters is considered according to the surface piezoelectric model. Surface effects show significant results for smaller plate thickness, whereas their effect decreases with an increase in thickness of plate. The obtained outcomes demonstrate that the influence of surface parameters on the vibration response of plate depend on in-plane constraints. In case of traction-free case, the electric voltage and surface stress have zero influence on the resonant frequency of the plate, while they generate a relaxation strain. Moreover, the effect of surface parameters on the normalized resonant frequency of the plate do not varies with aspect ratio and mode numbers. This work will help to understand the scale-dependent characteristics of piezo-composite structures and offer pathways for design of NEMS applications.

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