# Education, borrowing constraints and growth: A note

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**Abstract**

Kitaura (2012) shows an inverse U-shape relationship between balanced growth and the tightness of educational borrowing constraints and argues that a loosening of constraints need not be Pareto-improving even if growth increases. We provide a careful analysis of the transition, showing that an unanticipated loosening of credit constraints is welfare-improving for initial generations, but may be detrimental to (some) subsequent generations when growth increases. Thus, we argue that governments concerned with re-election may support a loosening of credit at the expense of future generations.

## 1 Introduction

The high growth rates of some developing economies raise important questions for neoclassical growth models. Are these high growth rates primarily the result of physical capital deepening, starting from a low base, as predicted by the Solow model? Or are there other fundamental factors that matter for growth, such as institutions, credit market development and human capital accumulation? Following the seminal paper of DeGregorio (1996), several papers have studied the impact of educational borrowing constraints on growth.

Whereas DeGregorio's model indicates that educational borrowing constraints impede human capital accumulation and lower growth, later models were inconclusive (e.g. de la Croix and Michel, 2007) and empirical work suggests a non-monotonic relationship between credit provision and growth (see Arcand et al., 2015). Motivated by these observations, Kitaura (2012) presents a model with an inverse U-shape relationship between tightness of educational borrowing constraints and balanced growth, and argues that a loosening of credit constraints need not be Pareto-improving even if the long run growth rate increases.

In this note, we revisit the latter result. We provide a careful treatment of the transition dynamics, showing that an unanticipated loosening of credit constraints is welfare-improving for initial generations, but may be detrimental to future generations even if the growth rate increases. This result arises because an expansion of education loans 'crowds out' physical capital in favour of human capital and induces a fall in the savings rate.

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The above result raises an important political economy perspective: a reform making educational loans more available may be supported by governments seeking re-election but harm future generations. Note that this trade-off is absent in Kitaura (2012), because it is assumed that the new steady-state ratio of physical to human capital is reached immediately.

# 2 Model

Individuals have three-period lives and choose education, $e_{t-1}$, when young and savings, $s_t$, when middle-aged to maximize lifetime utility $U_t = \log(c_t) + \rho\log(d_{t+1})$. The young borrow in order to finance education but are constrained by $e_{t-1} \leq \psi w_t h_t / R_t$, where $0 < \psi < 1$. Budget constraints are $c_t = w_t h_t - R_t e_{t-1} - s_t$ (middle aged) and $d_{t+1} = R_{t+1} s_t$ (old).

Given optimization and market-clearing, the key equations are:[1]

$$h_t = \theta e_{t-1}^{\eta} h_{t-1}^{1-\eta} \tag{1}$$

$$e_{t-1} = \psi \frac{w_t}{R_t} h_t \tag{2}$$

$$k_{t+1} = \frac{\rho}{1+\rho}(1-\psi)(1-\alpha)A k_t^{\alpha} h_t^{1-\alpha} - e_t \tag{3}$$

Eq. (1) says that human capital is produced using educational investment, $e_{t-1}$, and parents' human capital, $h_{t-1}$. Eq. (2) is optimal education. We assume $\eta > \psi$ such that the borrowing constraint binds. Note that the expression for optimal education in Kitaura (2012, Eq. 4), $e_{t-1} = [\psi\theta w_t/R_t]^{1/(1-\eta)} h_{t-1}$, is obtained when (1) is used in (2). Eq. (3) is optimal saving. The savings rate, $\frac{\rho(1-\psi)(1-\alpha)}{1+\rho}$, corresponds to a fraction of wage income net of debt repayment. Saving by the middle-aged finances investment in physical capital and education loans, so market clearing is $s_t = k_{t+1} + e_t$. Given Cobb-Douglas production $y_t = Ak_t^{\alpha} h_t^{1-\alpha}$, the factor price ratio in (2) is $\frac{w_t}{R_t} = \frac{(1-\alpha)}{\alpha}\frac{k_t}{h_t}$, implying that $e_{t-1} = \psi\frac{(1-\alpha)}{\alpha} k_t$.

## 2.1 Growth on the balanced growth path

Kitaura (2012) shows that the balanced growth rate with the constraint binding is

$$g_{\psi} = \Omega \left( \frac{\psi^{(1-\alpha)}(1-\psi)}{\alpha + \psi(1-\alpha)} \right)^{\frac{\eta}{1-\alpha(1-\eta)}} \tag{4}$$

where $\Omega = \left(\frac{\rho}{1+\rho}\alpha(1-\alpha)A\theta^{\frac{1-\alpha}{\eta}}\right)^{\frac{\eta}{1-\alpha(1-\eta)}} \left(\frac{1-\alpha}{\alpha}\right)^{\frac{\eta(1-\alpha)}{1-\alpha(1-\eta)}}$.

Kitaura's Proposition 1 (p. 576) states that if the elasticity of human capital to education expenditure ($\eta$) is large enough, the relationship between the tightness of the borrowing constraint ($\psi$) and the growth rate is inverted U-shaped, with a maximum at threshold $\overline{\psi}$.

[1] Parameters satisfy $\alpha, \eta, \rho, \psi \in (0,1)$, $A, \theta > 0$; population is fixed. See Kitaura (2012) for more details.

If $\psi < \overline{\psi}$, long run growth is below the maximum, and a marginal relaxation of credit raises growth. On the other hand, if $\psi > \overline{\psi}$, growth is lower on the new balanced growth path.

Kitaura argues informally that when $g_{\psi'} > g_{\psi}$, a Pareto improvement need not result because welfare of transitional generations may fall (p. 577). That analysis, however, relies on the simplifying assumption that the new steady-state capital ratio and balanced growth are reached *immediately*, and hence ignores the transitional dynamics:

$$\frac{k_{t+1}}{h_{t+1}} = \frac{\Phi k_t^{\alpha} h_t^{1-\alpha}}{\Gamma \Phi^{\eta} k_t^{\alpha\eta} h_t^{1-\alpha\eta}} \quad \Rightarrow \quad x_{t+1} = \frac{\Phi^{1-\eta}}{\Gamma} x_t^{\alpha(1-\eta)} \tag{5}$$

where $x_{t+1} = k_{t+1}/h_{t+1}$, $\Phi = \frac{\rho\alpha(1-\alpha)A(1-\psi)}{(1+\rho)[\alpha+\psi(1-\alpha)]}$ and $\Gamma = \theta(\frac{\alpha}{1-\alpha}\psi)^{\eta}$.

As Eq. (5) makes clear, if we start on the original balanced growth path and there is a relaxation of the credit constraint, the new steady-state capital ratio obtains only as $t \to \infty$. We now show that a careful treatment of the transition dynamics yields useful insights.

## 2.2 Welfare effect on initial generations

Consider an unanticipated loosening of the borrowing constraint, $\psi' \in (\psi, \eta)$, at date $T$. Initial utilities are $U_t = \log(c_t) + \rho\log(d_{t+1})$ for $t = T-1$ (old) and $t = T$ (middle aged). Given $d_T = R_T s_{T-1}$ with $R_T$ predetermined, utility of the initial old is unchanged. For the middle-aged, consumption in old age is $d_{T+1} = R_{T+1}s_T$ and in mid-age $c_T = (1-\alpha)y_T - R_T e_{T-1} - s_T$, where $R_T e_{T-1} = \psi(1-\alpha)y_T$ by (2) and $s_T = \frac{\rho}{1+\rho}(1-\alpha)(1-\psi)y_T$ by (3).[2]

Since $c_T = \rho^{-1}s_T = \frac{(1-\alpha)(1-\psi)}{1+\rho}y_T$, where $y_T = Ak_T^{\alpha}h_T^{1-\alpha}$ is predetermined, middle-age consumption is *unchanged*. The reason is that, given log utility, the middle-aged save a fraction of income that is independent of the expected return on saving. Hence, at date $T$, only the *composition* of saving between between capital and education loans changes. Since $c_T$ is unchanged, utility of middle-aged, $U_T = \log(c_T) + \rho\log(d_{T+1})$, hinges on $d_{T+1} = R_{T+1}s_T$, where interest rates $R_{T+1}$ vary inversely with the capital ratio, $x_{T+1} = k_{T+1}/h_{T+1}$.

The impact on $x_{T+1}$ can be seen from (5). Since $k_T, h_T$ predetermined, the impact depends on the ratio $\Phi^{1-\eta}/\Gamma$, where $\Phi = \frac{\rho\alpha(1-\alpha)A(1-\psi)}{(1+\rho)[\alpha+\psi'(1-\alpha)]}$ and $\Gamma = \theta(\frac{\alpha}{1-\alpha}\psi')^{\eta}$. Given $\psi' > \psi$, $x_{T+1}$ falls; accordingly, interest rates $R_{T+1}$ increase, implying a welfare gain: $U_T > U_T^{orig}$.[3]

In short, the middle-aged gain from the positive externality of higher returns triggered by a reallocation between capital and education loans. Accordingly, policies making credit more available may be favoured by governments seeking re-election by pleasing current voters.[4]

[2]Note the $\psi$ in Eq. (3) is *unchanged* as it relates to repayment of the past loan, $e_{T-1} = \psi w_T h_T / R_T$.

[3]Another way to see $x_{T+1}$ falls is $e_T = \psi'\left(\frac{1-\alpha}{\alpha}\right)k_{T+1}$ by (2) and using in (3), $k_{T+1} = \frac{\rho\alpha(1-\alpha)(1-\psi)}{(1+\rho)(\alpha+\psi'(1-\alpha))}y_T$, which is decreasing in $\psi'$. We also find $e_T$ is increasing in $\psi'$, which implies that $h_{T+1}$ increases by (1).

[4]It does not seem to matter whether the increase in $\psi$ is unanticipated or anticipated, as optimal savings are not forward-looking (see (3)) and $e_{T-1}$ depends on the credit restriction $\psi$ in place at that time.

## 2.3 Growth and welfare effects on later generations

The long run growth effect is given by Proposition 1 in Kitaura (2012). The initial growth effects need not have the same sign. This is intuitive, since we saw that physical and human capital move in opposite directions in period $T+1$. The initial impact on growth thus depends on the net effect, which is more likely to be negative the smaller the elasticity of human capital to education, $\eta$.[5] Moreover, there is a further growth effect in $T+2$ since savings adjust to a permanent lower fraction of output: $\forall t \geq T+1$, $s_t = \frac{\rho}{1+\rho}(1-\psi')(1-\alpha)y_t$ (see (3)). Intuitively, this is because from period $T+1$ onwards, the middle-aged repay a larger education loan, having borrowed up to the new limit $\psi'$ in their youth.

For welfare, Eqs. (2)–(3), $U_t = \log(c_t) + \rho\log(d_{t+1})$ and the budget constraints in middle and old age imply $\frac{\partial U_t}{\partial \psi} = (1+\rho)\frac{\partial \log(s_t)}{\partial \psi} + \rho\frac{\partial \log(R_{t+1})}{\partial \psi}$, and hence using (5) we have $\forall t \geq T+1$,

$$\frac{\partial U_t}{\partial \psi} = \left[\rho(1-\alpha)\frac{\partial \log(\frac{\Gamma}{\Phi^{1-\eta}})}{\partial \psi} - \frac{1+\rho}{1-\psi}\right] + \alpha\left[1+\tilde{\rho}\right]\frac{\partial \log(x_t)}{\partial \psi} + (1+\rho)\frac{\partial \log(h_t)}{\partial \psi} \tag{6}$$

where $\tilde{\rho} = \rho(1-(1-\alpha)(1-\eta))$.

The first term in (6) has ambiguous sign due to competing effects.[6] The first effect is the stronger (weaker) incentive to accumulate human (physical) capital after borrowing constraints are relaxed – i.e. the 'crowding out' of physical capital – which raises returns. The second effect, which lowers welfare, arises because all generations after the initial ones save a lower fraction of income after borrowing constraints are relaxed.

The middle term in (6) reflects the transition dynamics of the capital ratio $x_t = k_t/h_t$, which are absent in the welfare analysis of Kitaura (2012, Sec. 5). We saw that the capital ratio falls in $T+1$, and by (5) it falls further in period $T+2$ when the saving rate drops. This term is always negative, reflecting convergence to a lower steady-state capital ratio.

Finally, the last term in (6) is linked to growth in human capital that drives growing incomes per-person. Whereas the first two terms in (6) are constant on a balanced growth path, the last term is non-stationary and will come to dominate as time increases. Hence, if growth is higher on the new balanced growth path, distant generations will gain in welfare terms; the interesting question is what happens to *intermediate* generations.

As we shall see, several generations following the initial beneficiaries may experience welfare losses after a relaxation of credit constraints.

## 2.4 Numerical example

We now consider a numerical simulation of the transition. We choose the same parameters values as in Kitaura (2012, Fig. 1): $A = \theta = 1$, $\rho = 0.30$, $\eta = 0.25$ and $\alpha = 0.39$. For the tightness of the borrowing contraint, we consider a relaxation from $\psi = 0.205$ to $\psi' = 0.248$;

[5] Note $g_{T+1}/g_\psi = (k_{T+1}/k_{T+1}^{orig})^\alpha (e_T/e_T^{orig})^{\eta(1-\alpha)}$, where $h_{T+1} = \theta e_T^\eta h_T^{1-\eta}$ is used. By $e_T = \psi'\left(\frac{1-\alpha}{\alpha}\right)k_{T+1}$ and $k_{T+1}$ (see Fn. 3), we may obtain a condition under which growth increases (or decreases).

[6] The term is positive (negative) if $\rho^{-1} < (>)(1-\alpha)\left[\frac{\eta(1-\psi)}{\psi} + \frac{1-\eta}{\alpha+\psi(1-\alpha)}\right] - 1$.

the latter gives (approx.) the maximum long run growth rate when the constraint binds (see (4) and its discussion). We assume the change in $\psi$ takes place at date $T = 1$.

The results are shown in Figure 1.[7] The initial middle-aged are better off, as indicated by our analytical results, and growth in output per person falls at the first adjustment (see both panels). Many subsequent generations have welfare losses due to the 'drag' on utility from the falling capital ratio $x_t$ (see (6)) in conjunction with the lower savings rate. Welfare gains are realized only once enough time has elapsed for growth in human capital to raise saving despite the lower savings rate and the sizeable drop in the capital ratio (right panel).

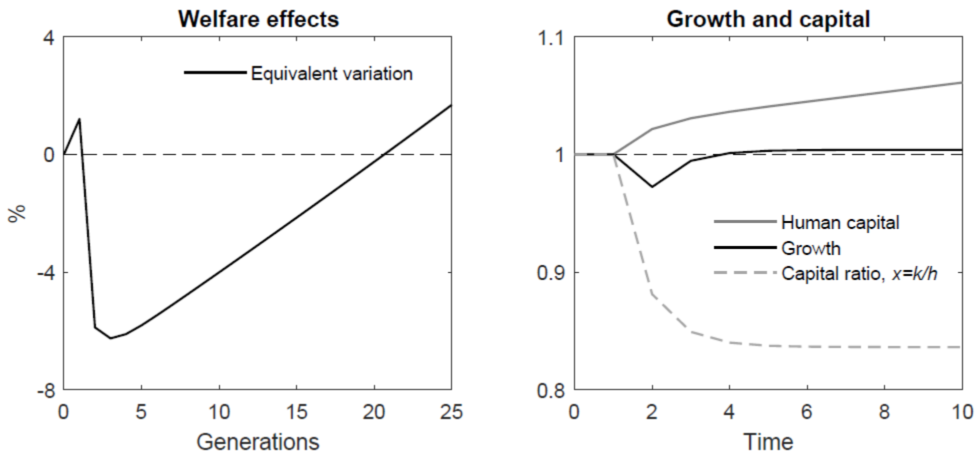


Figure 1: Transitional effects of loosening credit constraints at date $T = 1$

# 3 Conclusion

This note has revisited the results of Kitaura (2012) on the welfare effects of relaxing educational borrowing constraints. We showed that an unanticipated loosening of credit benefits the first transitional generation, but may be detrimental to (some) future generations even if growth increases. This result arises because an expansion of education loans reduces the savings rate and 'crowds out' physical capital, thus raising the return to saving and starting physical and human capital on different initial growth paths.

We argued that governments seeking re-election may support such a loosening of credit; this could be direct by relaxing credit market regulations, or indirect through state support for credit providers. We assumed that the relaxation of credit constraints was unanticipated, but a pre-announced loosening in credit has the same effects. These results raise important questions about government motives for supporting expansions of credit and the consequences for inequalities between current and future generations.

[7]Except for equivalent variations, which are in % of lifetime consumption, all variables are expressed relative to their (counterfactual) values on the original balanced growth path.

## References

Arcand, J.L., Berkes, E., Panizza, U. 2015. Too much finance? J. Econ. Growth 20, 105-148.

DeGregorio, J. 1996. Borrowing constraints, human capital accumulation and growth. J. Mon. Econ. 37, 49-71.

de la Croix, D., Michel, P. 2007. Education and growth with endogenous debt constraints. Econ. Theory 33, 509–530.

Kitaura, K. 2012. Education, borrowing constraints and growth. Econ. Lett. 116, 575-578.