Experiments on portfolio selection: A comparison between quantile preferences and expected utility decision models

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Abstract

This paper conducts a laboratory experiment to assess the optimal portfolio allocation under quantile preferences (QP) and compares the model predictions with those of a mean-variance (MV) utility function. We estimate the risk aversion coefficients associated to the individuals' empirical portfolio choices under the QP and MV theories, and evaluate the relative predictive performance of each theory. The experiment assesses individuals' preferences through a portfolio choice experiment constructed from two assets that may include a risk-free asset. The results of the experiment confirm the suitability of both theories to predict individuals' optimal choices. Furthermore, the aggregation of results by individual choices offers support to the MV theory. However, the aggregation of results by task, which is more informative, provides more support to the QP theory. The overall message that emerges from this experiment is that individuals' behavior is better predicted by the MV model when it is difficult to assess the differences in the lotteries' payoff distributions but better described as QP maximizers, otherwise.

Keywords: Optimal Asset Allocation, Quantile Preferences, Portfolio Theory, Risk Attitude, Predictive Ability Tests

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1 Introduction

Portfolio selection is a fundamental topic in economics and finance and one of the leading applications of decision theory under uncertainty. Modern portfolio theory derives its main results on diversification and risk under the important paradigm of the expected utility (EU) theory; see, for instance, Cochrane (2005) and Campbell (2017). Nevertheless, the EU framework has been subjected to a number of criticisms, mostly arising from experimental evidence.[1] Investors may also exhibit a preference for positive skewness of returns that is not captured by EU. In response to these and other critiques, the EU model has been successfully generalized to accommodate a variety of behavioral phenomena. Two of the more well-known generalizations are the inclusion of regret (Bell, 1982) and ambiguity in beliefs about probabilities (Gilboa and Schmeidler, 1989). Garlappi et al. (2007) develop a portfolio selection model for an investor with multiple priors and aversion to ambiguity.

Recently, de Castro et al. (2021a) studied the portfolio selection problem in a model with individuals exhibiting quantile preferences (QP). This alternative specification of individuals' preferences has been characterized in early work by Manski (1988), who studied properties of a quantile model for individual's behavior. More recently, QP have been formally axiomatized by Chambers (2009), Rostek (2010), and de Castro and Galvao (2020). Mendelson (1987) introduced the concept of quantile-preserving spread, which is a notion of risk aversion for the quantile model that establishes a parallelism with mean-preserving spreads in the standard EU framework. Giovannetti (2013) modeled a two-period economy with one risky and one risk-free asset, where the agent has QP. de Castro and Galvao (2019) developed a dynamic model of rational behavior under uncertainty, in which the agent maximizes a stream of the future quantile utilities. de Castro et al. (2021b) is one of the few studies that employ an experimental study in which individuals make pairwise choices between risky lotteries to assess the importance of QP and find evidence of behavior compatible with the presence of QP for a share of the population between 30 and 50%.

There exists a literature on optimal portfolio allocation using laboratory experiments. Bossaerts et al. (2007) and Gubaydullina and Spiwoks (2009) study portfolio choices allowing for individuals' heterogeneity with EU. Charness and Gneezy (2010) and Baltussen and Post (2011) investigate diversification in portfolio choice decisions through an experiment. Ahn et al. (2014) show through experiments how individuals have heterogeneity and different risk aversion attitudes but also have pessimism or optimism when selecting a portfolio using EU. Most of the experimental evidence on portfolio allocation has considered the conventional

[1] Rabin (2000) criticized EU theory arguing that EU would require unreasonably large levels of risk aversion to explain the data from some small-stakes laboratory experiments. See also Simon (1979), Tversky and Kahneman (1981), Payne et al. (1992) and Baltussen and Post (2011) as examples providing experimental evidence on the failure of the EU paradigm. Some studies suggesting that individuals do not always employ objective probabilities resulted in, among others, Prospect Theory (Kahneman and Tversky, 1979), Rank-Dependent Expected Utility Theory (Quiggin, 1982), and Cumulative Prospect Theory (Tversky and Kahneman, 1992).

EU framework as the baseline model. Within this framework, studies such as Andreoni and Sprenger (2012), Brandtner (2013) and Gardner (2019), used the mean-variance (MV) utility function for analyzing individuals' rationality. However, we are not aware of experimental studies on optimal portfolio allocation focusing specifically on the role of QP and comparing its predictive ability against other competing behavioral models such as the EU framework.

In this paper we depart from the EU framework and investigate through an experimental exercise the optimal portfolio allocation of individuals endowed with QP. We build on the theoretical results on optimal portfolio allocation under QP derived in de Castro et al. (2021a). These authors explore general conditions under which diversification is optimal, and also provide results for specific families of distribution functions such as the Normal, the Uniform and Chi-square distributions. In contrast to the EU framework, the portfolio allocation with QP is usually characterized by two differentiated regimes. The risk aversion regime given by values of τ smaller than a threshold τ_0 entails diversification between the lotteries comprising the portfolio. On the other hand, the optimal allocation for large values of τ is usually characterized by a corner solution that entails full allocation into one of the assets in the portfolio, usually the asset with highest risk and payoff. This unique feature of the portfolio allocation problem under QP can be used to motivate the presence of under-diversification found in many portfolio choice problems, see Mitton and Vorkink (2007) and references therein.

A second feature of the portfolio selection problem under QP that differs from the standard EU framework is the optimal allocation under the presence of a risk-free asset. Whereas the mutual fund separation theorem of Tobin (1958) obtained under a MV utility function and, more generally the EU framework, predicts a convex combination of the risk-free and the risky asset, the optimal allocation under QP predicts full allocation to the risk-free asset for high levels of risk aversion and full allocation to the risky asset for low levels of risk aversion. This theoretical result confirms the lack of diversification predicted by the QP model.

The main objective of the current study is to assess through a laboratory experiment the portfolio selection insights from the viewpoint of the QP model relative to the MV. In particular, we study the similarities and differences between the optimal portfolio choices of MV and QP individuals. We consider the MV utility function, but comparisons developed in this paper could be extended to other utility functions within the EU framework and beyond, as for example, the prospect theory. Nevertheless, for simplicity and ease of tractability we restrict to the MV utility function that summarizes individuals' utility as a linear function of mean and variance. To compare the optimal portfolio choices across theories, we estimate risk aversion coefficients associated with the individuals' empirical portfolio choices under the QP and MV theories using minimum distance estimators. We employ these methods to construct statistics for model classification, and also adapt the Diebold and Mariano (1995) test – originally introduced for evaluating predictive accuracy – to our setting for statistically comparing the suitability of the QP and MV models.[2]

Our experiment simulates a simple portfolio decision exercise and is formed of 90 tasks. These tasks were answered by 71 subjects. Each task has two assets, either two risky assets or one risk-free and one risky asset, that comprise an investment portfolio. The experiment requires individuals to assign weights $w_1, w_2 \in [0, 1]$ to these assets, with $w_1 + w_2 = 1$, to optimize their investment strategy. This strategy is not reported by participants as part of the experiment. The investment strategy may imply maximizing the expected value of their utility function, the expected portfolio payoff or some other moment of its distribution. One of the objectives of the experimental study is to infer which theory (i.e. QP vs. MV) predicts better individuals' responses. Subjects for the experiment were recruited from undergrads and graduates belonging to the Experiment Science Laboratory (ESL) at the University of Arizona. Due to ongoing Covid-19 pandemic, the experiments were implemented online using Qualtrics over the period December 2020 to February 2021. The actual payment to participants is the sum of the payoff of one of the 90 tasks plus a show-up fee, \$5, for participating in the experiment. The choice of the actual task for payment is done randomly as in similar experiments, see Gubaydullina and Spiwoks (2009), Ahn et al. (2014), and Gardner (2019).

Another important aspect of the experimental exercise is the choice of the distribution function for the risky assets. Previous experiments have relied on binary lotteries characterized by Bernoulli distributions to model the distribution of the payoffs, see Andreoni and Sprenger (2012) and Brandtner (2013). Ahn et al. (2014) is one of the few experimental papers that go beyond the Bernoulli distribution to characterize the payoffs. In this paper, we explore the Uniform distribution function for several reasons. First, it is very intuitive and easy to understand by individuals not familiar with advanced probability theory concepts. Second, it is analytically tractable. In particular, we derive the optimal allocation to each asset under both QP and MV theories under the assumption that both lotteries follow independent Uniform distributions. Third, we avoid the presence of unbounded tails that yield infinite payoffs with some strictly positive probability. Fourth, we can easily consider the whole spectrum of combinations between pairs of lotteries. These combinations reflect first and second order stochastic dominance as particular examples but can also accommodate distributions with overlapping payoffs and no stochastic dominance order.

The data gathered by this experiment are individuals' portfolio weights under a variety of portfolio combinations within the family of Uniform distributions. These data allow us to identify and estimate the QP and MV parameters for each individual and also to implement the Diebold and Mariano (1995) test as a model selection mechanism. The first important finding of our experimental study is the suitability of both theories to predict individuals' choices. There are differences across individuals and tasks but, in general, both models accurately predict the optimal responses of individuals to the tasks. We obtain two different conclusions

[2] In contrast to the original work of Diebold and Mariano (1995), we use the test as a model selection mechanism rather than as a test of predictive accuracy.

depending on whether we aggregate the results by individual or by task. The aggregation of results by individual offers support to the MV theory suggesting that the overall behavior of the individuals participating in the experiment is more aligned with the MV than with the QP portfolio theory. The aggregation of results by task provides richer information on the behavior of individuals when confronted with specific portfolio problems. The evaluation of the results by task shows, in general, more support to the QP theory than the MV theory although the results depend on the specific task under study.

The main message that emerges from this experimental study is that individuals' behavior is better predicted by the MV model when it is difficult to assess the differences in the lotteries' payoff distributions. Diversification may act as a decision mechanism that individuals use when it is not clear how to assess the relative gains/losses of one strategy over the other as, for example, when the lotteries' payoff distributions overlap. In these cases, MV preferences seem a safer choice as the optimal outcome of these policies usually yield to diversification. In contrast, when individuals are able to clearly assess the differences in the lotteries' payoff distributions their portfolio choices are closer to the optimal decision of a QP maximizer than of a MV maximizer. Individuals are able to maximize over the distribution of the portfolio rather than trading expected return for variance.

The empirical results show evidence that, for pairs of lotteries with very different supports and entailing a first order stochastic dominance relationship between them, individuals' portfolio choices are closer to the predictions of the QP model (full allocation to the dominant lottery) than the MV model. In contrast, when the supports of the lotteries are similar but the stochastic dominance relationship still holds, individuals' choices are closer to the MV strategy, that usually corresponds to a diversified portfolio. Illustrative examples of this scenario are $\mathsf{A} : \mathsf{U}(0, 2)$ vs. $\mathsf{B} : \mathsf{U}(0, 20)$ and $\mathsf{A} : \mathsf{U}(2, 20)$ vs. $\mathsf{B} : \mathsf{U}(0, 20)$ for the former case, with U denoting the Uniform distribution, and $\mathsf{A} : \mathsf{U}(0, 16)$ vs. $\mathsf{B} : \mathsf{U}(0, 20)$ and $\mathsf{A} : \mathsf{U}(12, 20)$ vs. $\mathsf{B} : \mathsf{U}(0, 20)$ for the latter case. These tasks exercises reveal that individuals' choices cannot be fully rationalized by a single theory. Instead, QP predictions are better suited to explain individuals' decisions when the differences in support suggest a clear stochastic dominance relationship between lotteries. In contrast, as these *distributional* differences vanish, individuals smoothly change their objective function and trade expected return for variance despite the fact the stochastic dominance relationship between the lotteries still holds.

We also consider portfolios of lotteries with overlapping supports, such as $\mathsf{A} : \mathsf{U}(2, 22)$ vs. $\mathsf{B} : \mathsf{U}(0, 20)$ and $\mathsf{A} : \mathsf{U}(18, 38)$ vs. $\mathsf{B} : \mathsf{U}(0, 20)$. Lottery A stochastically dominates lottery B in first order in both cases, which corresponds to the optimal portfolio decision under QP and moderate levels of risk aversion. The MV theory predicts more diversification than what we observe in the realized individuals' choices. The distribution of the empirical weights seems more in line with the optimal portfolio allocation obtained under the QP theory. On the other hand, whereas the QP theory predicts full allocation to lottery A the empirical weights show some non-negligible allocation to lottery $\mathtt{B}$ too. This phenomenon is more apparent as the supports of the Uniform distributions corresponding to each lottery are more separated, as in the second example above.

The last set of experiments considers combinations of a risk-free and a risky asset. In this case both theories predict full allocation to the risk-free asset for high levels of risk aversion, however, as the payoff of the risk-free asset and the degree of risk aversion decrease, the MV theory predicts full diversification whereas the QP theory predicts a complete shift to the risky asset (full under-diversification). These differences are reflected in individuals' responses across tasks in this category. For example, for $\mathtt{A} : 2$ vs. $\mathtt{B} : \mathtt{U}[0, 20]$, we find that most individuals allocate some weight to the risk-free asset in the range $(0, 0.25)$, which is more in line with the QP theory than with the MV theory. However, as the payoff of the risk-free asset increases, the predictions of the QP model imply full allocation to the risk-free asset, which may be too drastic from an investor's point of view. In these cases, we do observe a positive shift of the empirical distribution of weights towards the risk-free asset but this increase is smoother than under the QP theory. MV predictions are better able to explain individuals' choices than QP predictions.

Given these empirical results, the overall message that emerges from this analysis is that individuals' behavior is better predicted by the MV model when it is difficult to assess the differences in the payoff distribution of the lotteries comprising the portfolio. Individuals behave as QP maximizers, otherwise. This result suggests that diversification may act sometimes as a decision mechanism that individuals use when it is not clear how to assess the relative gains/losses of one strategy over the other as, for example, when the lotteries' payoff distributions overlap. In these cases, MV preferences seem a safer choice as the optimal outcome of these policies usually yield to diversification. This outcome involves fewer exposures to single assets than the QP theory even if the latter might lead to superior monetary rewards. In contrast, when individuals are able to clearly assess the differences in the distribution of payoffs between lotteries their portfolio choices are closer to the optimal decision of a QP maximizer than of a MV maximizer. In these (simpler) cases, individuals are able to maximize over the distribution of the portfolio rather than trading expected return for variance.

The remainder of the paper is laid out as follows. Section 2 illustrates the portfolio selection problem under QP and shows how individuals can optimize portfolio allocations under this theory. Section 3 sets up our experiment design to obtain individuals' portfolio allocations. Section 4 develops the econometric methodology necessary to estimate the risk aversion coefficients and test the underlying theories explaining individuals' behavior. Section 5 discusses the empirical results of the experiment and explains the results of the tests across individuals and tasks. Section 6 concludes. A separate Online Appendix presents the instructions of the experiment, detailed summary statistics of the experiments, and the payoffs of all the portfolio combinations under QP and MV.

2 Optimal portfolio choice problem

This section studies theoretically the optimal portfolio allocation problem for individuals with quantile preferences under different assumptions on the distribution of the assets' payoffs. We start by formally describing the portfolio selection problem under QP. Let

$$S_w = \sum_{i=1}^{n} w_i r_i,$$

be an investment portfolio comprised by n assets with payoffs (returns) given by r_i. The fraction of wealth allocated to each asset in the portfolio is denoted by $w \equiv (w_1, ..., w_n)$, with $\sum_{i=1}^{n} w_i = 1$. For illustration purposes, we consider portfolios that do not allow short-selling, that is, we further assume $w \in [0,1]^n$, but the model can be extended to relax this restriction.

To be consistent with the literature on optimal portfolio theory under EU preferences, we assume that individuals are endowed with a utility function $u(S_w)$, where $u : \mathbb{R} \to \mathbb{R}$, for describing individual's preferences on wealth. Then, for a given risk attitude $\tau \in (0,1)$, the portfolio choice problem under QP is

$$\max_{w \in [0,1]^n} Q_\tau \left[u\left(S_w\right)\right], \text{ s.t. } \sum_{i=1}^{n} w_i = 1. \tag{1}$$

A well-known and important property of quantiles is its invariance with respect to monotonic transformations. More formally, if $u : \mathbb{R} \to \mathbb{R}$ is continuous and strictly increasing, then

$$Q_\tau[u(S_w)] = u\left(Q_\tau[S_w]\right). \tag{2}$$

It is also important to notice that quantile preferences are in fact independent of the utility function. Indeed, for any continuous and strictly increasing $u : \mathbb{R} \to \mathbb{R}$, from (2),

$$X \succeq Y \iff Q_\tau[u(X)] \geqslant Q_\tau[u(Y)] \iff u(Q_\tau[X]) \geqslant u(Q_\tau[Y]) \iff Q_\tau[X] \geqslant Q_\tau[Y], \tag{3}$$

with $\succeq$ denoting a τ-quantile preference. This result shows that the utility function plays absolutely no role in defining the preference. We can use (2) to make any transformation of u; therefore, we could transform a concave utility function into a convex one without changing the preference. Hence the quantile optimization problem (1) using a given utility is equivalent to maximizing the quantile obtained directly from the distribution of the random variable such that the problem of interest becomes

$$\max_{w \in [0,1]^n} Q_\tau \left[S_w\right], \text{ s.t. } \sum_{i=1}^{n} w_i = 1. \tag{4}$$

Our aim is to uncover the optimal portfolio choices under QP and assess the similarities and differences with those under the MV paradigm. To illustrate these differences, and for simplicity, in what follows, we restrict the analysis to a portfolio of two assets. First, we consider the case of two risky assets and, second, we study the optimal allocation between a risk-free and a risky asset. Let the portfolio be defined as

$$S_w \equiv wX + (1-w)Y, \tag{5}$$

with X and Y continuous random variables, and $0 \leqslant w \leqslant 1$ the portfolio weight. First, we present the optimal allocation between X and Y for the case of two Uniform random variables.

2.1 S_w is a mixture of two Uniform random variables

The case of two Uniform distribution functions is analytically more cumbersome than the choice of lotteries with discrete payoff distributions or following a Normal distribution. However, this choice may be more intuitive for describing the probability law of the payoffs of each random variable for someone without knowledge on financial markets. Second, it is analytically tractable. In particular, we present the optimal allocation to each asset under both QP and MV theories under the assumption that both lotteries follow independent Uniform distributions. Third, we consider lotteries defined by continuous random variables. In this way, we extend most of the literature on portfolio choice experiments that considers lotteries with binary payoffs. Fourth, we avoid the presence of unbounded tails that yield infinite payoffs with strictly positive probability. Finally, we can easily entertain a large spectrum of investment scenarios by considering different combinations of pairs of Uniform random variables.

For each lottery the experimenter induces a monetary payoff associated with the probability of the outcome. Hence, for each quantile τ, we can calculate the optimal theoretical portfolio allocation as

$$w^*(\tau) = \underset{w \in [0,1]}{\arg\max}\; Q_\tau\left(S_w\right).$$

Consider now the MV case. The optimization problem also has a single preference parameter $\gamma \in \Gamma \subset \mathbb{R}_+$. Then,

$$w^\dagger(\gamma) = \underset{w \in [0,1]}{\arg\max}\; \mathcal{U}_\gamma(S_w) = \underset{w \in [0,1]}{\arg\max}\; \left(E\left(S_w\right) - \frac{\gamma}{2} Var\left(S_w\right)\right),$$

where $\mathcal{U}_\gamma$ is the mean-variance representation with parameter γ.[3] The optimal portfolio allocation for $X : U[a, b]$ and $Y : U[c, d]$ two independent random lotteries with Uniform distributions

[3]For a CARA utility function, and normally distributed assets, MV is a special case of expected utility preference. See, for example, Sargent (1987, p.154-155).

is given by

$$w^{\dagger}(\gamma) = \begin{cases} \widetilde{w}(\gamma) & \text{when } 0 < \widetilde{w}(\gamma) < 1, \\ 0 & \text{when } \widetilde{w}(\gamma) \leqslant 0, \\ 1 & \text{when } \widetilde{w}(\gamma) \geqslant 1. \end{cases} \tag{6}$$

where

$$\widetilde{w}(\gamma) = \frac{6[(a+b)-(c+d)] + \gamma(d-c)^2}{\gamma[(b-a)^2 + (d-c)^2]}.$$

The following paragraphs illustrate these theoretical results on optimal portfolio allocation for Uniform distributions with different examples. Thus the case of two standard uniform distributions, $X : U(0,1)$ and $Y : U(0,1)$, is reported in Example 2.1.[4]

Example 2.1. *Consider* $X : U(0,1)$ *and* $Y : U(0,1)$*, independent. The optimal allocation to* X *under QP is*

$$w^*(\tau) = \begin{cases} 0.5, & \text{if } \tau \in \left(0, \frac{1}{2}\right] \\ 1, & \text{if } \tau \in \left(\frac{1}{2}, 1\right). \end{cases}$$

In contrast, the optimal portfolio allocation to X *under MV is* $w^{\dagger}$ *in* (6) *with* $\widetilde{w}(\gamma) = 0.5$*, for all* $\gamma \in \Gamma$*.*

Example 2.2. *Consider* $X : U(0,2)$ *and* $Y : U(0,1)$ *independent. The optimal allocation to* X *under QP is*

$$w^*(\tau) = \begin{cases} 0.5, & \text{if } \tau \in \left(0, \frac{1}{4}\right] \\ 1, & \text{if } \tau \in \left(\frac{1}{4}, 1\right). \end{cases}$$

In contrast, the optimal portfolio allocation to X *under MV is* $w^{\dagger}$ *in* (6) *with* $\widetilde{w}(\gamma) = \frac{6+\gamma}{5\gamma}$*, for all* $\gamma \in \Gamma$*.*

In this case, even though $U(0,2)$ stochastically dominates $U(0,1)$, the optimal portfolio weight w is interior, $w^* = 0.5$, implying that diversification under quantile preferences is optimal for $\tau \leqslant 1/4$. This is an interesting result, because despite the fact that X first order stochastically dominates Y, there exists a convex combination S_w that dominates both random variables X and Y for low quantiles. Notice, however, that this feature is desirable, because the independence of X and Y makes a convex combination of the two less risky than any of them. For the MV case, the optimal portfolio allocation is a function of γ such that for large levels of risk aversion ($\gamma \to \infty$), the optimal allocation to X is 0.2.

Another interesting scenario is the absence of diversification with different lower ends of the distributions of X and Y. For the QP case, de Castro et al. (2021a) show that the optimal choice is $w^* = 1$ for all τ, provided that the difference between the two distributions at the

[4]The numerical methods to solve Examples 2.1–2.5 are described in de Castro et al. (2021a).

left end point is sufficiently large. Example 2.3 illustrates this result for the pair $\mathsf{X} : \mathsf{U}(0.5, 1)$ and $\mathsf{Y} : \mathsf{U}(0, 1)$.

Example 2.3. *Consider* $\mathsf{X} : \mathsf{U}(0.5, 1)$ *and* $\mathsf{Y} : \mathsf{U}(0, 1)$ *independent. Then, the optimal allocation to* X *under QP is* $w^* = 1$ *for all* $\tau \in (0, 1)$. *In contrast, the optimal portfolio allocation to* X *under MV is* $w^\dagger$ *in* (6) *with* $\widetilde{w}(\gamma) = \frac{3+\gamma}{1.25\gamma}$, *for all* $\gamma \in \Gamma$.

The optimal portfolio allocation under the MV case is similar to the QP case. Thus, for large levels of risk aversion ($\gamma \to \infty$), we obtain $\widetilde{w}(\gamma) = 0.8$. Similarly, for low levels of risk aversion ($\gamma \to 0$), the MV theory predicts $\widetilde{w}(\gamma) = 1$.

Nevertheless, the behavior with different lower end points can be complex under the QP theory. For instance, it may be the case that the optimal choice is $w^* \in \{0, 1\}$ for small τ, it becomes interior for intermediate values of τ and then becomes $w^* \in \{0, 1\}$ again for large τ's. The following example illustrates this scenario further.

Example 2.4. *Consider* $\mathsf{X} : \mathsf{U}(0.25, 1.25)$ *and* $\mathsf{Y} : \mathsf{U}(0, 1)$ *independent. The optimal allocation to* X *under QP is* $w^* \in (0, 1)$ *for* $\tau \in (0, 0.25)$ *and* $w^* = 1$ *for* $\tau > 0.25$. *In contrast, the optimal portfolio allocation under MV is* $w^\dagger$ *in* (6) *with* $\widetilde{w}(\gamma) = \frac{3+\gamma}{2\gamma}$, *for all* $\gamma \in \Gamma$.

The last example considers the remaining possible combination between X and Y. In this case, the support of the random variable X is inside the support of Y. Whereas the previous examples represent lotteries exhibiting first order stochastic dominance, the latter example does not.

Example 2.5. *Consider* $\mathsf{X} : \mathsf{U}(0.25, 0.75)$ *and* $\mathsf{Y} : \mathsf{U}(0, 1)$ *independent. The optimal allocation to* X *under QP is* $w^* \in (0, 1)$ *for* $\tau \in \left(0, \frac{1}{2}\right)$ *and* $w^* = 0$ *for* $\tau \in \left(\frac{1}{2}, 1\right)$. *In contrast, the optimal portfolio allocation under MV is* $w^\dagger$ *in* (6) *with* $\widetilde{w}(\gamma) = 0.8$, *for all* $\gamma \in \Gamma$.

Figure 1 plots $w^*(\tau)$ under the QP paradigm for different examples.

2.2 Optimal portfolio allocation when there is a risk-free asset

Manski (1988) derives the preferences of a quantile maximizer between two outcomes X and Y when one of the outcome measures is degenerate, and finds a complete separation in preferences between the degenerate and risky outcome. The deterministic choice is the preferred strategy for low quantiles. In contrast, for high quantiles, the risky outcome is the preferred strategy.

In this section, we provide further formality to the example in Manski (1988) and frame it in an optimal asset allocation context. We assume there is a riskless security that pays a rate

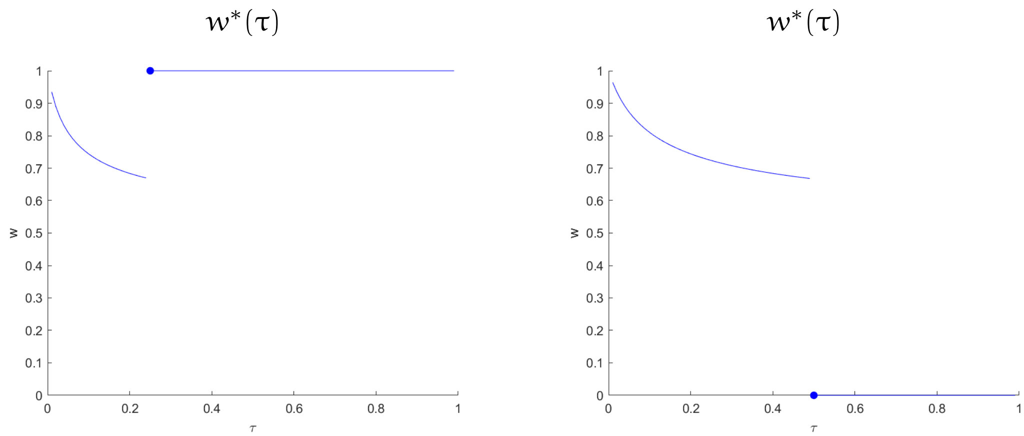

Figure 1: Illustration of Example 2.4 (left panel) and Example 2.5 (right panel).

of return equal to $R_f = \bar{r}$, and just one risky security that pays a stochastic rate of return equal to R with distribution function F_R. The portfolio return is defined by the convex combination

$$R_p = w\bar{r} + (1-w)R = \bar{r} + (1-w)(R-\bar{r}),$$

and the investor's maximization problem (4) for a specific quantile τ is $\arg\max_w Q_\tau[u(\bar{r}+(1-w)(R-\bar{r}))]$. Using the monotonicity of the quantile process, for a continuous and increasing utility function, the investor's problem simplifies to

$$\arg\max_w (1-w)Q_\tau[R] + w\bar{r}.$$

Simple algebra shows that the individual portfolio choice w is then given by the following:

$$w^* = \begin{cases} 1 & \text{when } Q_\tau[R] < \bar{r} \\ 0 & \text{when } Q_\tau[R] > \bar{r} \\ \text{any } w \in [0,1] & \text{when } Q_\tau[R] = \bar{r}. \end{cases}$$

The intuition of this solution is simple. For small values of τ the individual's optimal portfolio choice is $w^* = 1$ and corresponds to full investment on the risk-free asset. This is so because $\bar{r} > Q_\tau[R]$ for any combination R_p characterized by $0 < w < 1$. For larger values of τ, such that $Q_\tau[R] > \bar{r}$, the optimal portfolio decision reverses and yields $w^* = 0$. For $Q_\tau[R] = \bar{r}$, the QP maximizer is indifferent between the risk-free and the risky asset for any $w \in [0,1]$ defining the portfolio return.

In particular, for the example of Uniform distributions discussed above, we can consider $X = a$, with a a fixed payoff, and $Y : U(c,d)$, where $c \leqslant a$. The optimal portfolio allocation under QP is

$$w^* = \begin{cases} 1 & \text{when } Q_\tau[R] < \frac{a-c}{d-c} \\ 0 & \text{when } Q_\tau[R] > \frac{a-c}{d-c} \\ \text{any } w \in [0,1] & \text{when } Q_\tau[R] = \frac{a-c}{d-c}. \end{cases}$$

The corresponding optimal portfolio allocation for mean-variance investors yields

$$w^\dagger(\gamma) = \begin{cases} \widetilde{w}(\gamma) & \text{when } 0 < \widetilde{w}(\gamma) < 1, \\ 0 & \text{when } \widetilde{w}(\gamma) \leqslant 0, \\ 1 & \text{when } \widetilde{w}(\gamma) \geqslant 1, \end{cases} \tag{7}$$

where

$$\widetilde{w}(\gamma) = 1 - \frac{6(c + d - 2a)}{\gamma(d - c)^2}.$$

The outcome of the latter optimal asset allocation problem is known in the literature as the mutual fund separation theorem, see Tobin (1958). In this setting, there is an interior solution to the portfolio allocation problem that is given by a convex combination of a risk-free and a risky asset. The allocation to the risky asset depends on the degree of risk aversion γ. In contrast, under quantile preferences, the investor does not diversify at all. The optimal portfolio specializes in the risk-free asset for quantiles below the magnitude of the standarized risk-free rate and on the risky asset, otherwise.

3 Experiment design

In Section 2, we discussed analytically the optimal allocation between two assets in a portfolio under QP preferences and provide illustrative examples. In the following sections, we present an experiment to assess with real data individuals' optimal portfolio choices, and compare those choices with the predictions of the QP and MV models. This section explains the experiment design. Each experiment session has 90 independent decision-making tasks. Each of these 90 tasks corresponds to an optimal allocation of tokens between two lotteries. The list of tasks is explained in detail in Appendix A. Tasks are divided into five categories and each category considers a different type of relationship between two independent Uniform distributions, each corresponding to a different lottery A and B. Section 3 in the Online Appendix also presents graphs with the optimal portfolio allocation w^* to lottery A. Left panels plot the optimal allocation under the MV framework and the right panels plot the optimal allocation under the QP framework. For the MV case, the x-axis is given by $1/\gamma$, with γ the risk aversion coefficient, and for the QP case ,the x-axis is given by τ.

Table 1 summarizes the composition of the lotteries, stochastic dominance relationship between them, and the optimal allocation to lottery A under quantile preferences. The optimal allocation under MV is given in expression (6). The following paragraphs summarize the cases under investigation.

The first class of experiments in tasks 1 to 20 presents lotteries A and B. These lotteries share the same left end point of the distribution. In this scenario, lottery B first order stochastically dominates lottery A. Example 2.2 above is in this family of lotteries. The optimal allocation to A is given by full diversification for risk averse individuals (characterized by a small τ). For individuals with higher tolerance to risk, the optimal allocation under QP is determined by the FSD relationship.

The second class of experiments in tasks 21 to 35 considers lotteries that share the same right end point of the Uniform distributions. In this scenario, lottery A first order stochastically dominates lottery B. Example 2.3 above is in this family of lotteries. The optimal allocation to A is determined by the FSD relationship if $a < b/2$, otherwise, the QP optimal portfolio decision differs from the FSD relationship and implies an interior solution given by a convex combination of the two lotteries.

The third class of experiments in tasks 36 to 55 considers two lotteries with overlapping supports. In these examples, lottery A first order stochastically dominates lottery B. Example 2.4 above is in this family of lotteries. The optimal allocation to A is given by full diversification for risk averse individuals (characterized by a small τ). For individuals with higher tolerance to risk, the optimal allocation under QP is determined by the FSD relationship.

In the fourth class represented by tasks 56 to 69, there is no stochastic dominance relationship between the variables. Example 2.5 above is in this family of lotteries. The optimal allocation to A is given by an interior solution for risk averse individuals (characterized by values of $\tau \leqslant 0.5$). For individuals with higher risk tolerance ($\tau > 0.5$), the optimal allocation under QP is lottery B.

The last class given by tasks 70 to 90 is given by different combinations of a risk-free asset, represented by a fixed deterministic payoff a, and a risky asset given by a Uniform distribution. In this case, full allocation to the risk-free asset is optimal for values of τ below

Table 1: The 90 tasks in our experiment

Tasks	Composition of lotteries	Stochastic dominance	$w^*(\tau)$ by QP (w^*: optimal allocation)
1 to 20	$A : U(a, b)$ and $B : U(a, d)$ with $b < d$.	B FSD A	0.5 if $\tau \leqq \tau_0$ 0, otherwise
21 to 35	$A : U(a, b)$ and $B : U(0, b)$ with $a > 0$	A FSD B	1 if $a > b/2$ $w^* \in (0.5, 1)$, otherwise
36 to 55	$A : U(a, b)$ and $B : U(c, d)$ with $a > c$ and $b > d$	A FSD B	0.5 if $\tau \leqq \tau_0$ 1, otherwise
56 to 69	$A : U(a, b)$ and $B : U(c, d)$ with $a > c$ and $b < d$	There is no FSD between A and B	$w^* \in (0, 1)$ if $\tau \leqq \tau_0$ 0, otherwise
70 to 90	A: a (constant number) and B: U(c,d)	-	1 if $\tau \leqq \tau_0$ 0, otherwise

Table 2: Descriptive data

	Total	Male	Female	Non-binary
Subjects	71	35	35	1
Earnings ($)	21.16	22.62	19.93	13.27
	(5.94)	(6.19)	(5.27)	(-)
Duration of time for experiment (min)	34.00	33.51	33.80	58.46
	(9.34)	(8.59)	(9.25)	(-)
Quiz score (out of 3)	2.54	2.8	2.26	3
	(0.84)	(0.40)	(1.05)	(-)

*Numbers in parentheses indicate standard deviation.

τ_0, with $\tau_0 = \frac{a-c}{d-c}$. Otherwise, the optimal allocation is the risky asset with payoffs driven by the Uniform distribution.

Our online experiments were programmed in Qualtrics. To create a setting similar to that of a lab-experiment, we used a Zoom meeting room with cameras and informed students they were being observed. We did this because student concentration dropped drastically in the pilot tests when cameras were off. In addition to observing concentration levels, we used the Zoom setting to give students instructions regarding the rules of the experiment. The currency used in the experiment was USD, and subject earnings were paid after each experiment by Venmo or online transfer. We obtained 71 subjects for the experiment. Individuals were a mix of undergraduate and graduate students recruited from the Experiment Science Laboratory (ESL) at the University of Arizona. Due to Covid 19, the experiments were implemented online from December 2020 to February 2021. The experiment consisted of the consent of participating in the experiment (3-5min), introduction for uniform distributions (5-10 min), quiz (5-10min), reading instruction (5-10 min), the main experiment (average of 34min), and payment (3-5min). Because some of the subjects did not have much background in probability theory or mathematics, we explained the basic concept of Uniform distributions and then checked subjects' understanding with three simple questions. This was done prior to the experiment and took about 15 minutes. The instructions shown to subjects in the experiment are found in Section 1 of the Online Appendix. Each session of our experiments lasted about one hour and fifteen minutes on average, and the main experiment took an average of 34 minutes. There was no time limit to complete the experiment; a few individuals took almost an hour to complete it. The standard deviation of the time for the experiment, shown in Table 2, was 8.59 minutes for males and 9.25 minutes for females.

The experiment proceeded as follows: students were asked three questions before the experiment to demonstrate their understanding of Uniform distributions. Subjects then received a bonus depending on how many of these questions they got correct. If they got 3 out of 3, then $2 were offered. If 2 out of 3, then $1 was offered, otherwise they received no bonus. The average overall score for these questions was 2.54 out of 3 (84%). See Table 2 for summary statistics on earnings, duration of experiment, and quiz score. After the quiz, subjects started the main experiment. The investigator gave a short instruction about the experiment, and then subjects reread the instructions to reinforce their understanding about the lotteries in each task, how to assign the 100 tokens, and how their own earnings would be decided as a result of the experiment. As mentioned earlier, subjects completed 90 tasks during the experiment. These tasks are randomly shown in the screen to avoid distractions or a monotonic choice such as (100,0) or (0,100) across tasks. The questions are presented on 6 pages with 15 tasks per page. In each task, 100 tokens are given to the subject. These tokens can be viewed as the weights that subjects allocate to each lottery. They assign between 0 to 100 tokens to each lottery such that the sum of the weights allocated to the two lotteries is 100. If the sum of weights between lotteries within a task is different from 100, the system reports an error message and the subject has to reintroduce the allocation of weights to the lotteries.

Individuals' earnings from taking part in the experiment are given by the sum of the bonus from taking the initial quiz, the fixed show-up fee of \$5, and the earnings from one randomly selected task out of the 90 tasks completed as part of the experiment. Including the show-up fee of \$5, subjects obtained an average of \$21.16. As an illustrative example, suppose the payment-determining task for a subject has two lotteries, where lottery $\mathsf{A} = \$6$ and lottery $\mathsf{B} = \mathsf{U}[\$0, \$20]$. Assume the subject assigns 40 tokens to A and 60 tokens to B. Then, the computer picks a random number in the interval $[0, 20]$ (for the sake of example, let this number be 11). Then, the subject's payoff will be $\$6 \times 0.4 + \$11 \times 0.6 = \$9$. Table 2 includes subject earnings and consumed time for the main experiment by gender.

4 Econometric methodology

In this section we describe the econometric methods to estimate the parameter of interest – risk attitude – for both the QP and MV cases, as well as the procedure used to compare the fit of these models.

4.1 Identification

The identification of the parameters of interest, for both QP and MV cases, is achieved by varying the shape and support of the lotteries presented to the subjects. Consider the QP case. As discussed in Section 2, for a given quantile τ and lotteries X and Y, the economic agent chooses the weight w. Hence, we are able to identify the τ by using different Uniform distribution across tasks, and varying their associated support – see, e.g., Examples 2.1–2.5 above. Thus, for a given preference parameter τ, we induce different choices of portfolio, and identify the parameter from the data. In other words, identification is attained by assuming that individuals are endowed with the same quantile independently of the magnitude of the payoffs involved, and by varying the support of the Uniform distributions in the lotteries. This variation induces different choices for different tasks. An analogous argument is valid for the MV case.[5] The previous section and Appendix A discuss the different combinations of supports used to elicit the parameters of interest.

4.2 Estimation methods

We use minimum distance (MD) estimators to estimate the parameters of interest.[6] The MD estimator is very simple to implement in practice. In particular, it is computed by minimizing the quadratic distance of an observed variable and its theoretical counterpart. In particular, we will minimize the distance using either the quantile or the expected value, for QP and MV respectively.

Consider individuals $i = 1, 2, ..., I$. Let $j = 1, 2, ..., J$ index the J tasks each individual face in the experiment. For the experiments in this empirical exercise we have $I = 71$ and $J = 90$. First, from the experiments, we observe data for the choices $w_{ij} \in (0, 1)$, the portfolio selection each individual i make for task j. Second, from the theoretical results for the portfolio section discussed in Section 2, for each fixed parameter, we are able to calculate the optimal choices. Of course, this optimal choice depends on the underlying preference risk parameter, which is the parameter to be estimated by the MD method. We propose estimators based on MD to estimate τ_i and γ_i for each individual i. For the latter, we will assume that it lies on a compact set Γ, which is assumed the same for all individuals. We consider a mean-squared loss function estimator. For any portfolio w and task j, we can compute the implied objective function, $Q_\tau(S^j_w)$ for QP and $\mathcal{U}_\gamma(S^j_w)$ for MV. Moreover, we can compute the optimal value of the objective function, $Q_\tau\left[S^j_{w^*_j(\tau)}\right]$ for QP and $\mathcal{U}_\gamma\left[S^j_{w^\dagger_j(\gamma)}\right]$ for MV. Thus, for each individual, we define

$$\widehat{\tau}_i = \underset{\tau \in (0,1)}{\arg\min} \sum_{j=1}^{J} \left(Q_\tau\left[S^j_{w_{ij}}\right] - Q_\tau\left[S^j_{w^*_j(\tau)}\right]\right)^2, \tag{8}$$

$$\widehat{\gamma}_i = \underset{\gamma \in \Gamma}{\arg\min} \sum_{j=1}^{J} \left(\mathcal{U}_\gamma\left[S^j_{w_{ij}}\right] - \mathcal{U}_\gamma\left[S^j_{w^\dagger_j(\gamma)}\right]\right)^2, \tag{9}$$

for QP and MV, respectively.

The standard asymptotic properties of MD estimators are established in Newey and McFadden (1994). We omit the details for brevity, but notice that, for these estimators, the

[5] We are assuming that there is no measurement error in the data, as for example, systematic rounding errors.

[6] The minimum distance (MD) estimator dates back to Berkson (1944), Neyman (1949), Taylor (1953), and Ferguson (1958), whose among others, aimed to produce statistically efficient and computationally tractable alternatives to maximum likelihood estimators. Although very simple conceptually, MD estimation has been used by many scholars in statistics and econometrics since Malinvaud (1970) and Rothenberg (1973). The literature on MD is vast, hence we only list a limited set of examples: Amemiya (1976, 1978), Nagaraj and Fuller (1991), Lee (1992), Koenker et al. (1994), Newey and McFadden (1994), Lehmann and Casella (1998), Moon and Schorfheide (2002), and Lee (2010).

validity of asymptotic results assume that $J \to \infty$ and independence across tasks.

To implement the estimators we use a grid search where we consider $\tau \in (0,1)$ and $\gamma \in [0,20]$ with a grid size of 2000.

4.3 Explaining individuals' preferences using experimental data

We divide the exercise into classification and testing of individuals' portfolio choices.

4.3.1 Classification

We consider the following strategy to compare and classify both preferences. The estimators can be compared on the same decision choice, i.e. w. We define a minimum distance indicator using

$$\widehat{d}_{ij} = \mathbf{1}\left[|w_{ij} - w_j^*(\widehat{\tau}_i)| < |w_{ij} - w_j^\dagger(\widehat{\gamma}_i)|\right]. \tag{10}$$

Equation (10) defines an indicator function that, for each individual i and task j, compares the distance of the observed choices w_{ij} to the theoretical choices using the estimated quantile, $w_j^*(\widehat{\tau}_i)$, with the corresponding distance for the MV case. Therefore, the statistic $\widehat{d}_{ij}$ provides a notion of whether the optimal QP weight is closer to the individual's portfolio choice than the MV weight counterpart.

We then define the following statistics for the indicator. For each task j we can compute the proportion of cases where the QP provides a better fit than MV,

$$\bar{d}_j = \frac{1}{I}\sum_{i=1}^{I} \widehat{d}_{ij}. \tag{11}$$

Similarly, for each individual i, we can compute the proportion over the tasks as

$$\bar{d}_i = \frac{1}{J}\sum_{j=1}^{J} \widehat{d}_{ij}. \tag{12}$$

Intuitively, the statistic in (11) provides the proportion of subjects such that the decisions are a better fit for the QP for each task j. Moreover, the statistic in (12) shows the proportion of tasks that are better explained for QP for each individual.

4.3.2 Testing

We also formally test which model has a better fit for the observed choices. To do so, we use a Diebold-Mariano (DM) testing strategy (Diebold and Mariano, 1995). Provided that both estimators can be compared on the same loss function, i.e. comparing actual choice and optimal choice, then we can evaluate goodness-of-fit measures between QP and MV. Define

$$\phi_{ij}(\widehat{\tau}_i) = \left[w_{ij} - w_j^*(\widehat{\tau}_i)\right]^2$$

and

$$\psi_{ij}(\widehat{\gamma}_i) = \left[w_{ij} - w_j^\dagger(\widehat{\gamma}_i)\right]^2,$$

the value of the loss function for QP and MV, respectively.

Consider now the null hypothesis that for a given task $j = 1, ..., J$ and for a sequence of independent individuals $i = 1, 2, ...,$ both models have the same loss

$$H_0^j : \mathsf{E}\left[\phi_{ij}(\tau_i) - \psi_{ij}(\gamma_i)\right] = 0,$$

against one-sided and two-sided alternative hypotheses. The null hypothesis states that both preferences representations give the same expected loss, while the alternatives look for systematic differences across representations. A positive (negative) sign indicates that MV (QP) is better than QP (MV), i.e. QP (MV) has a higher value of the loss function than MV (QP).

Then, define the following test statistic

$$\mathsf{DM}_j := I^{1/2} \frac{\frac{1}{I}\sum_{i=1}^{I}\left[\phi_{ij}(\widehat{\tau}_i) - \psi_{ij}(\widehat{\gamma}_i)\right]}{\sqrt{V_j}}, \tag{13}$$

where $V_j = \frac{1}{I}\sum_{i=1}^{I} \mathsf{Var}\left[\phi_{ij}(\widehat{\tau}_i) - \psi_{ij}(\widehat{\gamma}_i)\right]$. One can show that under the null hypothesis H_0^j,

$$\mathsf{DM}_j \xrightarrow{d} \mathsf{N}(0,1), \quad \text{as} \quad I \to \infty,$$

and hence the critical values are taken from a simple standard normal distribution.

In order to implement the test in (13) one needs a consistent estimator of the variance V_j. We consider the following estimator

$$\widehat{V}_j = \hat{V}_{\phi j} + \hat{V}_{\psi j} - 2\hat{C}_{\phi\psi j}, \tag{14}$$

with

$$\widehat{V}_{\phi j} = \frac{1}{I}\sum_{i=1}^{I}\left(\phi_{ij}(\widehat{\tau}_i) - \bar{\phi}_j\right)^2,$$

$$\widehat{V}_{\psi j} = \frac{1}{I}\sum_{i=1}^{I}\left(\psi_{ij}(\widehat{\gamma}_i) - \bar{\psi}_j\right)^2,$$

$$\widehat{C}_{\phi\psi j} = \frac{1}{I}\sum_{i=1}^{I}\left(\phi_{ij}(\widehat{\tau}_i) - \bar{\phi}_j\right)\left(\psi_{ij}(\widehat{\gamma}_i) - \bar{\psi}_j\right),$$

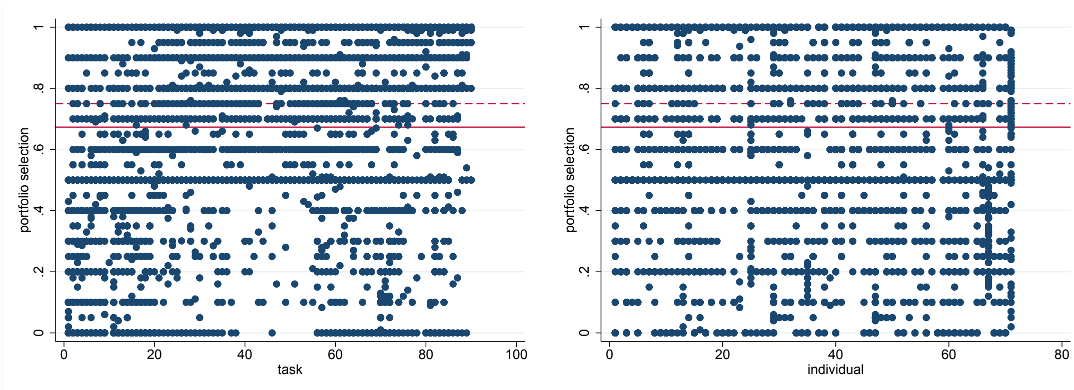

Figure 2: The left panel reports the selected portfolio weights by task, and the right panel the selected portfolio weights by individual. The solid and dashed horizontal lines are the overall mean and median values, respectively.

where $\bar{\phi}_j = \frac{1}{I}\sum_{i=1}^{I}\phi_{ij}(\widehat{\tau}_i)$ and $\bar{\psi}_j = \frac{1}{I}\sum_{i=1}^{I}\psi_{ij}(\widehat{\gamma}_i)$.

In a similar fashion one can produce DM statistics for a given individual $i = 1, ..., I$ for a sequence of independent tasks $j = 1, 2...$, using the number of tasks to compute the asymptotic results. That is, under the null hypothesis $H_0^i : E\left[\phi_{ij}(\tau_i) - \psi_{ij}(\gamma_i)\right] = 0$, it follows that

$$DM_i := J^{1/2}\frac{\frac{1}{J}\sum_{j=1}^{J}\left[\phi_{ij}(\widehat{\tau}_i) - \psi_{ij}(\widehat{\gamma}_i)\right]}{\sqrt{V_i}} \xrightarrow{d} N(0,1), \quad \text{as} \quad J \to \infty, \tag{15}$$

where $V_i = \frac{1}{J}\sum_{j=1}^{J} Var\left[\phi_{ij}(\widehat{\tau}_i) - \psi_{ij}(\widehat{\gamma}_i)\right]$.

5 Empirical results

5.1 Data description

We have 90 tasks per individual and 71 individuals. The total is thus 6390 selections. Figure 2 reports the selected weights by task and individual, respectively. These scatter plots provide a rough description of the heterogeneity across tasks and individuals. All answers are in the graphs, albeit some are repeated (i.e.one task could have more than one w value or one individual could have selected the same w in different tasks). In both cases we observe that choices are not systematically fixed on a given value of w, but that there is heterogeneity on individuals' and tasks' responses.

Now, we discuss the parameter estimates associated to individuals' preferences. For QP we consider the estimated individual-specific quantile indexes $\{\hat{\tau}_i\}_{i=1}^{71}$ in equation (8). The left panel of Figure 3 presents the histogram of estimated τs. The average $\hat{\tau}$ is about 0.32 while the median is 0.33, and the standard deviation is 0.15. For the MV approach, we consider the estimated individual-specific gamma estimates $\{\hat{\gamma}_i\}_{i=1}^{71}$ obtained from expression (9) in the

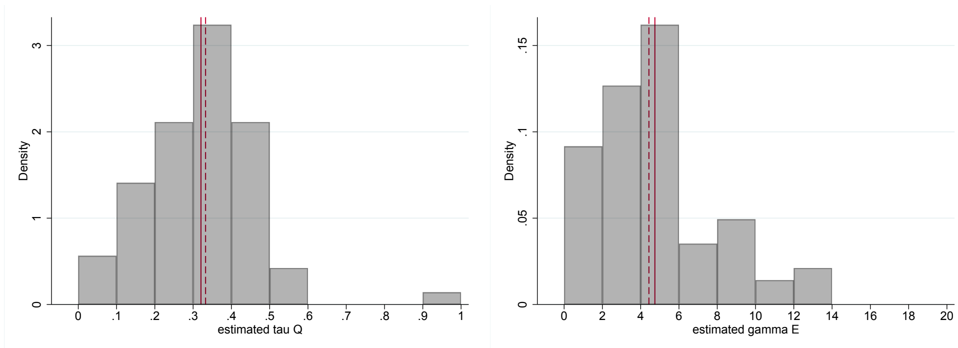

Figure 3: Left panel histogram of the estimated τ for QP preferences by individual. Right panel for the histogram of the estimated γ for MV preferences by individual. The solid and dashed vertical lines are the mean and median values, respectively.

right panel of Figure 3. The average $\hat{\gamma}$ is about 4.74 while the median is 4.43, and the standard deviation is 3.06.

The next exercise compares the estimates of individuals' risk aversion across theories. For each individual i, we report in Figure 4 the pair $(\hat{\gamma}_i, \hat{\tau}_i)$. These estimates are obtained from minimizing the distance between the implied quantiles, for $\hat{\tau}$, and the expected values, for $\hat{\gamma}$. The results show a negative relationship between the two. This result shows that both theories (QP and MV) capture individuals' underlying risk attitude. A high risk aversion given by a large value of γ for the MV model is corresponded by a low τ, the parameter that reflects risk aversion under the QP model. In general, individuals exhibit a risk-averse behavior with just a few individuals characterized by values of τ greater than 0.5 and values of γ smaller than one.

5.2 Discussion of results per individual

In this subsection, we study the allocation to each lottery per individual. To do this, we aggregate by task. The left panel of Figure 5 reports the pairs $\{\bar{d}_i, DM_i\}_{i=1}^{71}$, which corresponds to the estimates of the proportion of tasks that are closer to the QP than to the MV model $(\bar{d}_i)$ and the DM statistics.

The overall analysis of the results suggests that the MV utility function is better able to describe individuals' behavior than the QP approach. This is so because most of the observations are above zero in the x-axis, reflecting a positive DM statistic, and are below 0.5 in the y-axis, reflecting a proportion smaller than 0.5 for the QP theory. The graph also reveals the presence of individuals that can be clearly categorized as QP maximizers using one or the other metric. Note also that in order to gain a better understanding into individuals' behavior we need to study the individuals' choices by task. This is done in the next subsection.

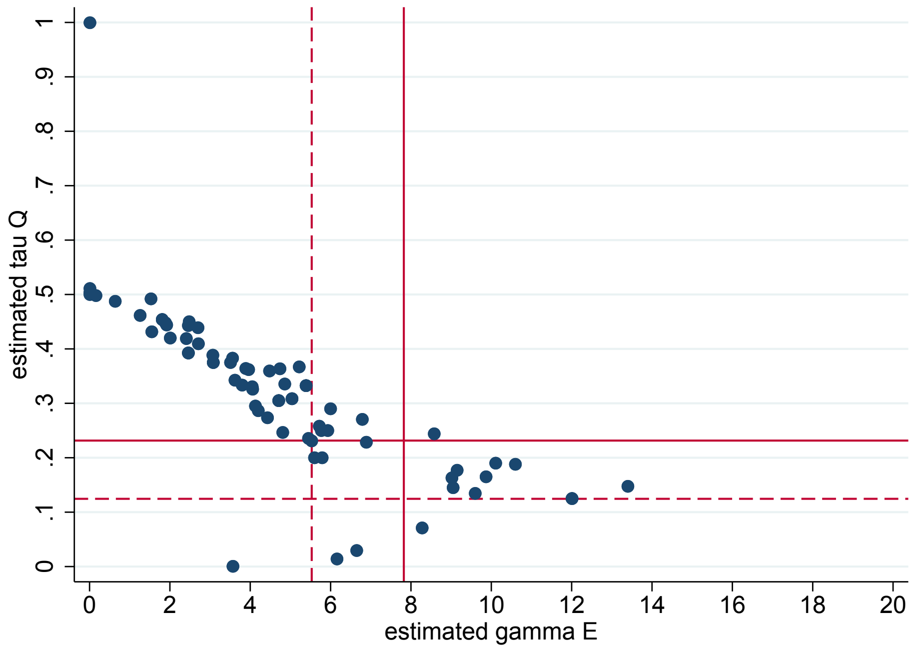

Figure 4: Scatter plot of $(\hat{\gamma}_i, \hat{\tau}_i)$. The solid and dashed vertical and horizontal lines are the mean and median values, respectively.

5.3 Discussion of results per task

The right panel of Figure 5 reports the pairs $\{\bar{d}_j, DM_j\}_{j=1}^{90}$ by task. These statistics allow us to classify the tasks as MV and QP driven. The scatter plot shows a negative correlation between the statistics $\bar{d}_j$ and DM_j, across tasks. This negative correlation shows that large values of the statistic $\bar{d}_j$ are corresponded by small (or negative) values of the DM statistic. The interpretation of this scatter plot is as follows. For a given task, a large value of $\bar{d}_j$ implies that the proportion of individuals with portfolio allocations closer to the quantile model is higher than with the MV theory. This is reflected in the DM statistic, such that if this value is smaller than -1.96, we obtain statistical evidence favoring the model driven by QP. In contrast, for low values of $\bar{d}_j$, the DM statistic usually takes positive values such that if the statistic is greater than 1.96, we obtain statistical evidence favoring the MV model with respect to the QP model. Visual inspection of the graph suggests that there is evidence of both types of results across tasks. In contrast to the analysis per individual, we find a negative DM statistic for many tasks, which suggests that the QP model is better able to explain individuals' choices than the MV counterpart. In some cases, these values are also statistically significant at 5% significance level.

To obtain further insights into the relationship between the tasks and the type of preferences that drive the optimal portfolio choices, we explore these choices for each task separately. Table 3 in Section 2 of the Online Appendix presents summary statistics on portfolio weights by task.

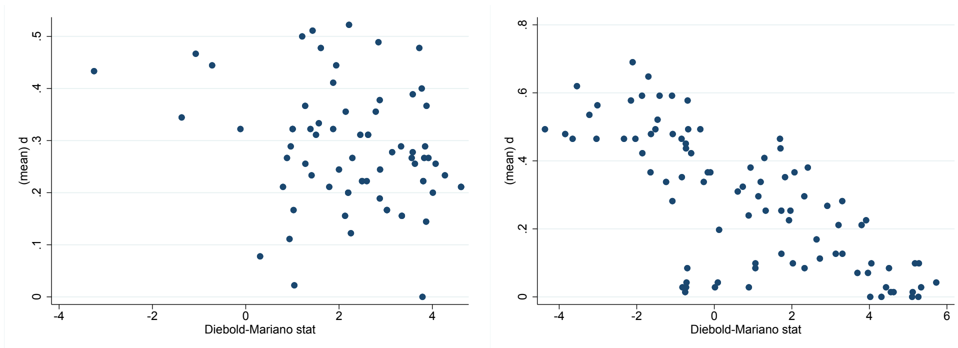

Figure 5: Left panel for the scatter plot for the pairs $\{\bar{d}_i, DM_i\}_{i=1}^{71}$, by individual. Right panel for the scatter plot of pairs $\{\bar{d}_j, DM_j\}_{j=1}^{90}$, by task.

These statistics reflect the empirical distribution of weights across individuals and illustrate the heterogeneity in individuals' responses per task. We compare the empirical results in this table with the optimal allocation to lotteries A and B for each task under the QP and MV theories derived above. To illustrate the optimal portfolio choice under each theory and for all tasks, Figures 1-90 in Section 3 of the Online Appendix report the optimal values of the portfolio weight as the risk aversion coefficient (τ or γ) varies in the intervals $(0, 1)$ and $(0, 4)$, respectively. These graphs show the optimal allocation to lottery A as the risk aversion coefficient decreases.

Tasks 1 - 20:

The specific tasks within this group are found in Appendix A.1. A canonical example of this class of experiments is reported in Example 2.2. This class is characterized by $A : U(a, b)$ vs. $B : U(a, d)$, with $b < d$. Lottery B first order stochastically dominates (FSD) lottery A. Figures 1-20 in the Online Appendix show a decrease in the optimal allocation to lottery A as the risk aversion coefficient decreases. This decrease is more pronounced under QP and depends on the specific value of b. Similarly, for high levels of risk aversion, individuals with MV preferences allocate more wealth to lottery A than QP individuals. The latter type allocates a maximum weight of 0.5 to lottery A. These differences in portfolio allocations are more important for tasks that involve lotteries where b is much smaller than d. More formally, the theoretical optimal weight allocation under QP preferences is $w^*(\tau) = 0.5$ for values of τ below some threshold τ_0 and $w^*(\tau) = 0$, otherwise. The specific value of τ_0 depends on the choice of the payoff b in lottery A.

We compare these theoretical allocations with the distribution of weights across individuals per task reported in Table 3 in the Online Appendix. The empirical weights oscillate between 0 and 0.5 for most tasks in this group, with a median between 0.1 and 0.5 and a 90% percentile in the range $(0.5, 0.8)$. These weights are in line with the optimal allocations obtained under both QP and MV theories. For those tasks that involve significant differences between the supports of lotteries A and B (b is much smaller than d) the empirical weights are closer to the QP theory than to the MV theory. However, for the other tasks, the empirical weights take large values being more in line with the predictions of the MV theory. To provide statistical support to these findings, Table 4 in the Online Appendix reports the values of the pairs $\{\bar{d}_j, DM_j\}_{j=1}^{20}$ computed using the mean square distance and the quantile implied measures developed above. The results confirm the findings described above and uncover those tasks for which individuals behave as QP maximizers and those tasks for which they behave as MV maximizers.

Tasks 21 - 35:

The specific tasks within this group are found in Appendix A.2. These portfolio experiments are represented by lotteries $A : U(a, b)$ vs. $B : U(0, b)$, with $a > 0$ and different values of b. A canonical example is $A : U(0.5, 1)$ vs. $B : U(0, 1)$ in Example 2.3. This family of tasks satisfies that A FSD B and the optimal allocation under QP theory depends on the value of a. Figures 21-35 in Section 3 of the Online Appendix report the optimal allocations to lottery A under QP and MV theories as the risk aversion coefficient decreases. For Lotteries A with large a (beyond the mean of lottery B), we find that $w^*(\tau) = 1$ for all $\tau \in (0, 1)$, however, if a is small then the optimal QP allocation is between 0.5 and 1. Similar results are found for the MV theory. In this case, the optimal allocation under MV theory converges to $w^* = 1$ when a is greater than the average of lottery B. Diversification is only present in this group under high levels of risk aversion and for tasks where the support of A is large enough (small a).

The analysis of the distribution of the weights provides interesting insights, though. The 10^{th} quantile for these tasks is around 0.50, the average oscillates between 0.70 and 0.868, and the median is even higher, which suggests that individuals behave according to these theories. The DM statistic driven by the squared distance between the weights does not yield conclusive evidence and in some cases the DM statistic is NA. This is because the two proportions are exactly the same and it is not possible to calculate the variance and covariance for the DM statistic. In contrast, the statistics based on the implied quantiles provide very informative results. The corresponding DM statistic is negative and statistically significant giving support to the QP theory compared to the MV theory. Individuals when confronted with these tasks seem to follow the first order stochastic dominance rule and diversify less than under the MV theory.

Tasks 36 - 55:

The specific tasks within this group are found in Appendix A.3. These portfolio experiments are represented by overlapping lotteries such that $A : U(a, b)$ vs. $B : U(c, d)$, with $a > c$ and $b > d$. A canonical example is $A : U(0.25, 1.25)$ vs. $B : U(0, 1)$ in Example 2.4. This family of tasks satisfies that A FSD B and the optimal allocation under QP theory is $w^* \in (0, 1)$ for $\tau \leqslant \tau_0$ and $w^* = 1$, otherwise. In contrast, the optimal allocation under MV theory starts at $w^* = 0.5$ for all tasks for high levels of risk aversion and increases towards one as the risk aversion coefficient decreases. These allocations are reported in Figures 36-55 of the Online Appendix.

In contrast to the preceding example, the analysis of the theoretical weights in expression (6) and visual inspection of Figures 36-55, allow us to clearly discriminate between MV and QP. Thus, the empirical distribution of weights in Table 3 (Online Appendix) provides results that align very well with the theoretical predictions of both QP and MV theories. More specifically, the 10^{th} quantile for these tasks is around 0.50 in most cases, the average oscillates between 0.70 and 0.878, and the median is usually higher than the mean. A closer look to the summary statistics suggests, however, that these values are only consistent with the MV case for $\gamma < 1$, otherwise, the MV theory predicts more diversification than what we observe in individuals' choices. The distribution of the empirical weights seems to be more in line with the optimal portfolio allocation obtained under the QP theory. On the other hand, whereas the QP theory predicts full allocation to lottery A, the empirical weights show some non-negligible allocation to lottery B too. These empirical findings are inconclusive so the inspection of the statistics $\{\bar{d}_j, DM_j\}_{j=36}^{55}$ in Table 4 (Online Appendix) may be useful in this case to discriminate between theories. The $\bar{d}_j$ statistic yields values greater than 0.5 for several tasks but values close to zero for many other tasks in this group. In general, the DM statistic under both estimation methods is very positive providing statistical support to the MV case. The overall analysis of the empirical results for this group suggests that the observed weights are closer to the predictions of the MV model for individuals with low levels of risk aversion.

Tasks 56 - 69:

The specific tasks within this group are found in Appendix A.4 and the optimal allocations are reported in Figures 56-69 of the Online Appendix. These portfolio experiments are represented by lotteries such as $A : U(a, b)$ vs. $B : U(c, d)$, with $a > c$ and $b < d$. The support of A is strictly included in the support of B, implying that the variance of the latter lottery is larger than the former. A canonical example is $A : U(0.25, 0.75)$ vs. $B : U(0, 1)$ in Example 2.5. The optimal portfolio allocation is an interior solution for $\tau \leqslant \tau_0$ and $w^* = 0$, otherwise. In contrast, the optimal allocation to lottery A under the MV theory oscillates between 0.5 and 1. It approaches $w^* = 1$ as the variance of lottery A compared to lottery B decreases (the support of A decreases with respect to the support of B). For many of the tasks in this class (those with same mean) this scenario corresponds to A that stochastically dominates lottery B in second order.

The analysis of the distribution of the weights in Table 3 (Online Appendix) provides interesting findings that align very well with both theories. The empirical distribution of weights seems to be in the range $(0.5, 1)$ and the median is also quite high. It is difficult to discriminate between both theories using the empirical weight distribution. However, inspection of the statistics $\{\bar{d}_j, DM_j\}_{j=56}^{69}$ in Table 4 (Online Appendix) sheds further light on the empirical results. This table provides strong statistical support on the superiority of the QP theory compared to MV for this class of tasks. The $\bar{d}_j$ statistic yields values greater than 0.5 for several tasks, which indicates that there is a larger proportion of individuals that can be classified as QP maximizers compared to MV maximizers. Similarly, the DM_j statistic is negative in many cases and provides statistically significant results.

Tasks 70 - 90:

Tasks in this experiment include a risk-free asset. The specific tasks within this group are found in Appendix A.5 and the optimal allocations are reported in Figures 70-90 of the Online Appendix. Both theories predict similar optimal portfolio allocations. The MV allocation is smoother than the QP allocation, nevertheless, both allocations predict full investment on the risk-free asset for high levels of risk aversion. As the level of risk aversion decreases, the QP theory predicts a complete shift to the risky asset. In contrast, the MV theory predicts some diversification. This is only the case if the payoff of the risk-free asset is lower than the mean of the risky asset, otherwise, the optimal allocation is full investment on the risk-free asset independently of the risk aversion coefficient.

The distribution of weights in Table 3 (Online Appendix) exhibits some heterogeneity across individuals for some tasks. For example, for Task 70 given by $A : 2$ vs. $B : U[0, 20]$, we find that most individuals allocate a weight to the risk-free asset in the range $(0, 0.25)$, which is more in line with the QP theory than with the MV theory. For the latter theory to be consistent with the observed empirical distribution of weights, γ needs to be smaller than 1/3, which corresponds to very low levels of risk aversion. Similar findings are obtained for the analysis of Task 71. However, as the payoff of the risk-free asset increases, we observe a positive shift of the empirical distribution of weights. In these cases, the MV predictions are better able to explain individuals' choices than the QP predictions. This is mainly due to the additional flexibility of the MV case that accommodates some diversification for moderate values of the risk aversion coefficient, in contrast to the QP case. Finally, for those cases when the payoff of the risk-free asset is high both theories yield the same optimal portfolios given by full allocation into the risk-free asset.

The comparison of the theories using the statistics $\{\bar{d}_j, DM_j\}_{j=71}^{90}$ in Table 4 (Online Appendix) also present mixed results. There are, however, more tasks that are better represented by the MV theory than by the QP theory.

6 Conclusion

This paper has studied optimal portfolio allocation under quantile preferences using a laboratory experiment with 71 undergraduate and graduate students from University of Arizona.

The experiment simulates a simple portfolio decision exercise and is formed of 90 tasks. Each task has two assets, either two risky assets or one risk-free and one risky asset, with the risky assets following a Uniform distribution function. We have used this experiment to assess the insights of the quantile preference model with real data. We have studied the suitability of the predictions of the MV and QP theories to explain the data on portfolio weights collected from the experiment. The results of the experiment confirm that both theories help to predict individuals' optimal choices. Subjects in the experiment are clearly risk averse under both specifications of individuals' preferences. The aggregation of results by individual offers partial support to the MV theory whereas the aggregation of results by task is more supportive of the QP theory.

The overall message that emerges from this analysis is that individuals' behavior is better predicted by the MV model when it is difficult to assess the differences in the payoff distribution of the lotteries comprising the portfolio. Individuals behave as QP maximizers, otherwise. This result suggests that diversification may act sometimes as a decision mechanism that individuals use when it is not clear how to assess the relative gains/losses of one strategy over the other as, for example, when the lotteries' payoff distributions overlap. In these cases, MV preferences seem a safer choice as the optimal outcome of these policies usually yield to diversification. This outcome involves fewer exposures to single assets than the QP theory even if the latter might lead to superior monetary rewards. In contrast, when individuals are able to clearly assess the differences in the distribution of payoffs between lotteries their portfolio choices are closer to the optimal decision of a QP maximizer than of a MV maximizer. In these (simpler) cases, individuals are able to maximize over the distribution of the portfolio rather than trading expected return for variance.

A Tasks for experiment

Here we show the optimal portfolio MV allocations and the optimal portfolio QP allocations from all of the tasks. These 90 tasks are divided in five categories from A.1 to A.5. Each category considers a different type of relationship between two Uniform distributions, each corresponding to a different lottery A and B. The section also presents figures with the optimal portfolio allocation w^*, that corresponds to lottery A. Left panels plot the optimal allocation under the MV framework and the right panel plot the optimal allocation under the QP framework. The x-axis captures risk aversion. For the MV case, we report w^* as a function of $1/\gamma$, and for the QP case, we report w^* as a function of τ.

A.1 Experiments replicating Example 2.2:

In these examples, B first order stochastically dominates A. Note that the options in this experiment, described as lotteries, are reversed compared to the example.

[1] $A : U[0, 2]$ $B : U[0, 20]$
[2] $A : U[0, 4]$ $B : U[0, 20]$
[3] $A : U[0, 6]$ $B : U[0, 20]$
[4] $A : U[0, 8]$ $B : U[0, 20]$
[5] $A : U[0, 10]$ $B : U[0, 20]$
[6] $A : U[0, 12]$ $B : U[0, 20]$
[7] $A : U[0, 14]$ $B : U[0, 20]$
[8] $A : U[0, 16]$ $B : U[0, 20]$
[9] $A : U[0, 18]$ $B : U[0, 20]$
[10] $A : U[0, 20]$ $B : U[0, 20]$
[11] $A : U[4, 10]$ $B : U[4, 20]$
[12] $A : U[4, 12]$ $B : U[4, 20]$
[13] $A : U[4, 14]$ $B : U[4, 20]$
[14] $A : U[4, 16]$ $B : U[4, 20]$
[15] $A : U[4, 18]$ $B : U[4, 20]$
[16] $A : U[10, 16]$ $B : U[10, 25]$
[17] $A : U[10, 18]$ $B : U[10, 25]$
[18] $A : U[10, 20]$ $B : U[10, 25]$
[19] $A : U[10, 22]$ $B : U[10, 25]$
[20] $A : U[10, 24]$ $B : U[10, 25]$

A.2 Experiments replicating Example 2.3:

In these examples, lottery A first order stochastically dominates lottery B. Note that there are different lower ends of the distributions.

[21] $A : U[2, 20]$ $B : U[0, 20]$
[22] $A : U[4, 20]$ $B : U[0, 20]$
[23] $A : U[8, 20]$ $B : U[0, 20]$
[24] $A : U[12, 20]$ $B : U[0, 20]$
[25] $A : U[16, 20]$ $B : U[0, 20]$
[26] $A : U[4, 25]$ $B : U[0, 25]$.
[27] $A : U[8, 25]$ $B : U[0, 25]$
[28] $A : U[12, 25]$ $B : U[0, 25]$
[29] $A : U[16, 25]$ $B : U[0, 25]$
[30] $A : U[20, 25]$ $B : U[0, 25]$
[31] $A : U[2, 10]$ $B : U[0, 10]$
[32] $A : U[4, 10]$ $B : U[0, 10]$
[33] $A : U[6, 10]$ $B : U[0, 10]$
[34] $A : U[8, 10]$ $B : U[0, 10]$
[35] $A : U[10, 15]$ $B : U[0, 15]$

A.3 Experiments replicating Example 2.4:

In these examples, lottery A stochastically dominates lottery B. The support of the random variables overlaps.

[36] $A : U[2, 22]$ $B : U[0, 20]$ [37] $A : U[4, 24]$ $B : U[0, 20]$ [38] $A : U[6, 26]$ $B : U[0, 20]$
[39] $A : U[8, 28]$ $B : U[0, 20]$ [40] $A : U[10, 30]$ $B : U[0, 20]$ [41] $A : U[12, 32]$ $B : U[0, 20]$
[42] $A : U[14, 34]$ $B : U[0, 20]$ [43] $A : U[16, 36]$ $B : U[0, 20]$ [44] $A : U[18, 38]$ $B : U[0, 20]$
[45] $A : U[16, 26]$ $B : U[10, 20]$ [46] $A : U[2, 12]$ $B : U[0, 10]$ [47] $A : U[14, 30]$ $B : U[0, 16]$
[48] $A : U[12, 28]$ $B : U[0, 16]$ [49] $A : U[10, 26]$ $B : U[0, 16]$ [50] $A : U[8, 24]$ $B : U[0, 16]$
[51] $A : U[6, 22]$ $B : U[0, 16]$ [52] $A : U[4, 20]$ $B : U[0, 16]$ [53] $A : U[2, 18]$ $B : U[0, 16]$
[54] $A : U[2, 16]$ $B : U[0, 14]$ [55] $A : U[4, 18]$ $B : U[0, 14]$.

A.4 Experiments replicating Example 2.5:

In these examples, there is no stochastic dominance on either side.

[56] $A : U[2, 18]$ $B : U[0, 20]$ [57] $A : U[4, 16]$ $B : U[0, 20]$ [58] $A : U[6, 14]$ $B : U[0, 20]$
[59] $A : U[8, 12]$ $B : U[0, 20]$ [60] $A : U[6, 22]$ $B : U[4, 24]$ [61] $A : U[8, 22]$ $B : U[4, 24]$
[62] $A : U[6, 20]$ $B : U[4, 24]$ [63] $A : U[8, 20]$ $B : U[4, 24]$ [64] $A : U[12, 18]$ $B : U[10, 20]$
[65] $A : U[14, 18]$ $B : U[10, 20]$ [66] $A : U[16, 18]$ $B : U[10, 20]$ [67] $A : U[14, 16]$ $B : U[12, 20]$
[68] $A : U[13, 17]$ $B : U[12, 18]$ [69] $A : U[14, 16]$ $B : U[12, 18]$

A.5 Example with risk-free asset:

In these examples, there is no stochastic dominance on either side. Lottery A corresponds to a risk-free asset with fixed payoff.

[70] $A : 2$ $B : U[0, 20]$ [71] $A : 4$ $B : U[0, 20]$ [72] $A : 6$ $B : U[0, 20]$
[73] $A : 8$ $B : U[0, 20]$ [74] $A : 10$ $B : U[0, 20]$ [75] $A : 12$ $B : U[0, 20]$
[76] $A : 14$ $B : U[0, 20]$ [77] $A : 16$ $B : U[0, 20]$ [78] $A : 18$ $B : U[0, 20]$
[79] $A : 20$ $B : U[0, 20]$ [80] $A : 12$ $B : U[8, 16]$ [81]$A : 9$ $B : U[8, 20]$
[82] $A : 8$ $B : U[4, 24]$ [83] $A : 10$ $B : U[8, 12]$ [84] $A : 6$ $B : U[2, 10]$
[85] $A : 14$ $B : U[4, 24]$ [86] $A : 16$ $B : U[4, 24]$ [87] $A : 18$ $B : U[4, 24]$
[88] $A : 20$ $B : U[4, 24]$ [89] $A : 22$ $B : U[4, 24]$ [90]$A : 24$ $B : U[4, 24]$

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