Analysis of Hysteric Behaviour in Nonlinear Resonance of Silicon Nanoelectromechanical Resonators

Fang Ben[a], James Fernando[a], Jun-Yu Ou[b], Yoshishige Tsuchiya[a]
[a] School of Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ, UK
[b] Optoelectronics Research Centre, University of Southampton, Southampton, SO17 1BJ, UK
e-mail: F.Ben@soton.ac.uk

The rapid downscaling of Micro-/Nano Electro-Mechanical Systems (M/NEMS) brings challenges with nonlinear characteristics at higher frequency ranges. Already showing superior performance at the nanoscale, development of NEMS has become even more attractive because of innovative applications of nonlinearity being identified, such as neuromorphic computing [1-5]. We previously reported a novel model that can systematically fit the experimental nonlinear resonance with respect to the dependence on RF power and actuation voltage [6]. One challenge is how to treat the hysteric behaviour associated with nonlinear characteristics. In this paper, we introduce a threshold equation in our model to quantitatively investigate the onset of hysteric behaviour. The structure of a doubly-clamped NEMS resonator is schematically shown in Figure 1(a). A heavily doped n-type silicon beam is surrounded by SiO_2 and the out-of-plane flexural mode is actuated by the measurement system, shown in Figure 1(b) [7,8]. Equation.1 presents a model based on the universal expression for a Duffing oscillator.

$$m_b \ddot{x}(t) + \frac{\omega_0}{Q}\dot{x}(t) + \left[\omega_0^2 - \frac{2\varepsilon_0 S V_{dc}(V_{dc}+V_{ac})}{m_b {g_0}^3}\right]x(t) + \left[\beta_m \omega_0^2 - \frac{4\varepsilon_0 S V_{dc}(V_{dc}+V_{ac})}{m_b {g_0}^5}\right]x^3(t) = F_{elec} \quad (1)$$

where $x(t)$, β_m, m_b, Q, ω_0, g_0, S, and F_{elec} are time-domain displacement, intrinsic mechanical nonlinearity, effective mass of the beam, quality factor, resonance frequency, gap from gate to beam, vertical cross-section area and electrostatic force, respectively [6]. The coefficient of $x^3(t)$ includes intrinsic mechanical and electrical stiffness allowing us to investigate the nonlinear voltage effect in a quantitative way. By fitting the voltage-dependent result, as shown in Figure 2, the value of β_m can be extracted, which is -2.823x10^{-5} m^{-2} for a 1-µm-long beam sample [6]. At nonlinear resonance, hysteresis starts to appear when the frequency shift due to the nonlinearity becomes larger than the bandwidth of the original resonance peak [2]. By applying this criterion to the model represented by Equation 1, a threshold determination equation is derived as,

$$V_{dc}(V_{dc}+V_{ac}) \geq \left[\beta_m \omega_0^2 + \frac{2\omega_0^2}{Qx(\omega)^2}\right]\frac{C_{g_0}{g_0}^5}{4} \quad (2)$$

where $x(\omega)$, and C_{g_0} are frequency-domain displacement and gate-induced capacitance, respectively. Under this condition, the model gives multiple solutions representing bifurcation states in nonlinear dynamic theory. Among them, two stable solutions can be extracted to describe the hysteresis phenomenon. By substituting the relevant parameters obtained from the fitting for the 1-µm-long sample as aforementioned and 2-µm-long sample mentioned in Ref.[6] , the critical voltages of the bifurcation state are respectively calculated to be 3.598 V and 1.512 V. Figure 3(a) and (b) show the fitting results for the voltage dependence of nonlinear resonance peaks for both samples. Figure 3(c) and (d) summarise how the peak frequency and in-phase current value are changed with respect to the DC voltage, which are consistent with typical nonlinear resonance characteristics.

We have shown the model developed in [6] can be extended to fit nonlinear resonance of a 1-µm-long beam with the frequency range of up to 221 MHz and also can accommodate the hysteric behaviour along with nonlinear resonance consistently. The model will be useful for design and simulation of large-scale nonlinear NEMS resonator arrays to be developed in future, where a resonance frequency range of hundreds of megahertz is taken into account.

Acknowledgements: The authors would like to thank F. Arab Hassani and H. Mizuta for their device design and support, and E. Ollier and C. Dupre for fabrication of the devices. This work has been partially supported by FP7 EU project NEMSIC (Grant 224525).

References: [1] J. F. Rhoads et al., J. Dynamic Sys. Meas. Cont, 132, (2010),034001. [2] M. V. Andres et al., Elect. Lett, 23, (1987), 952-954. [3] M. Sansa et al., in 29th IEEE MEMS, (2016). [4] F. M. Alsaleem et al., J. Microelecmech. Sys, 27, (2018) 780-789. [5] S. Tiwari et al., J. Micromech. Microeng, 29, (2019),083002. [6] F. Ben et al., in 34th IEEE MEMS (2021). [7] Y. Tsuchiya et al., in 43rd MNE (2017). [8] Y. Tsuchiya et al., in 31st IEEE MEMS (2018)

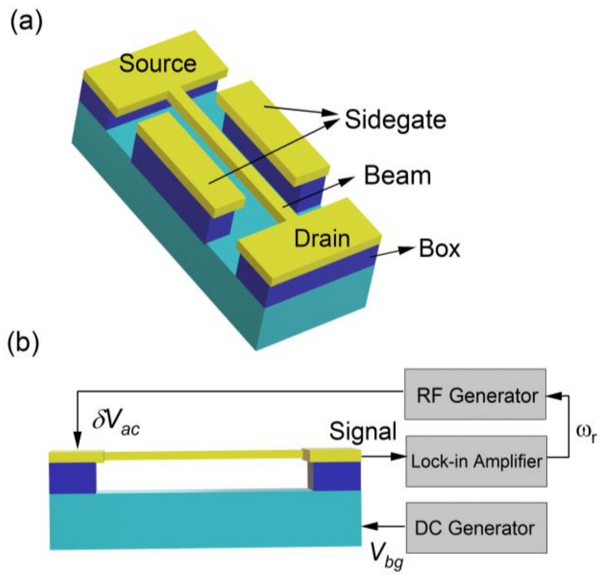

Figure 1. (a)A Schematic of a doubly-clamped silicon NEMS resonator. The side-gates are grounded in this study. (b) A schematic diagram of FM detection

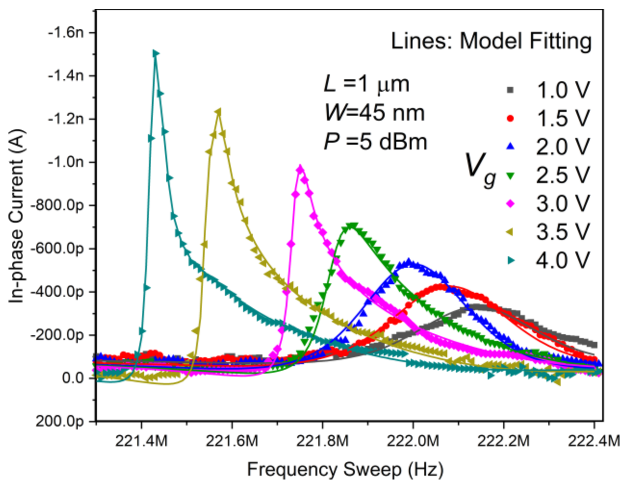

Figure 2. Experimental results and model fitting of backgate voltage (V_g) dependence of the resonance for a 1-µm-long beam sample are plotted. V_g not only tunes the resonance frequency, but also changes the shape of the curve

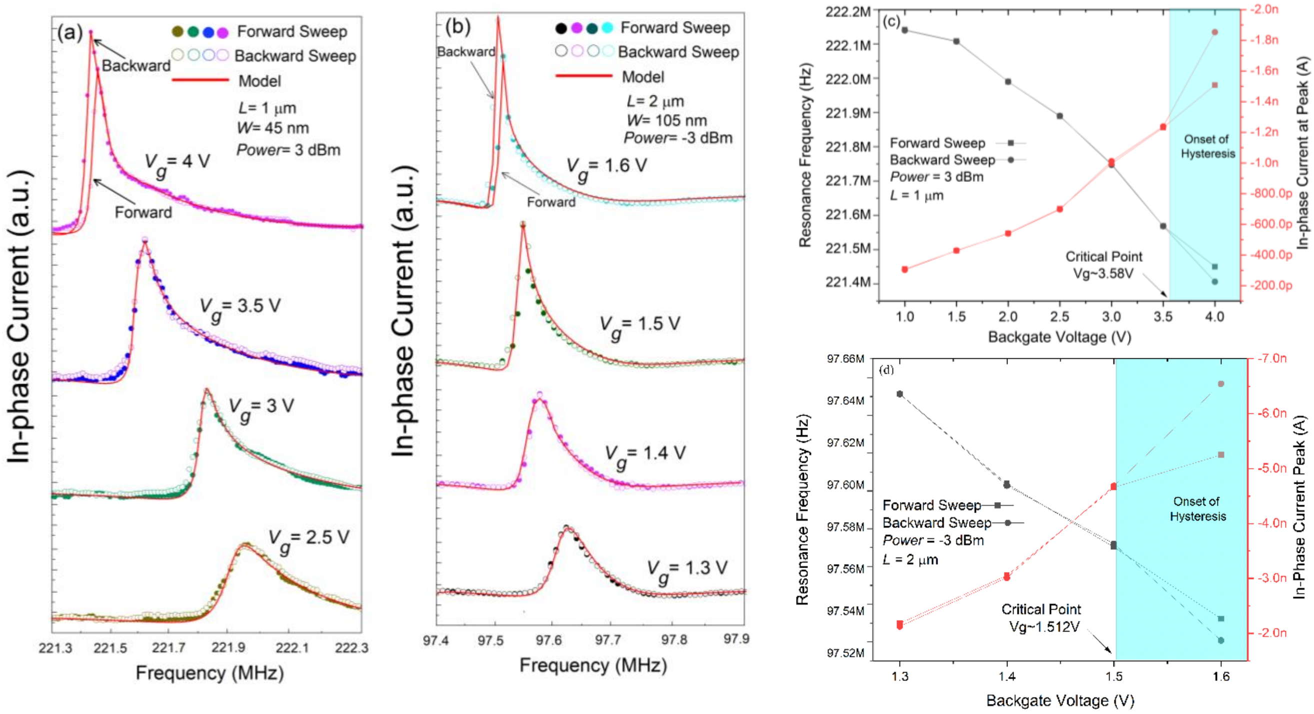

Figure 3. Emergence of hysteresis in nonlinear resonance are displayed for (a) a sample with L=1 µm and W=45 nm and (b) with L=2 µm and W=105 nm. Hysteresis appears at V_g=4 V for the 1-µm-long sample, and at V_g=1.6 V for the 2-µm-long sample. Resonance frequencies and peak amplitude changes are plotted with respect to the back-gate voltage for (c) the sample with L=1 µm and W=45 nm, and (d) with L=2 µm and W=105 nm.