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# Gain Equalization for Few-Mode Erbium-Doped Fiber Amplifiers via Strong Mode Coupling

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**Featured Application: Optical fiber transmission.**

**Abstract:** Few-mode erbium-doped fiber amplifiers (FM-EDFAs) are one of the most important optical subsystems for successful space division multiplexed transmission systems. In this paper, we propose a new FM-EDFA design to achieve significantly reduced differential modal gain (DMG) via strong mode coupling. Using a new numerical model based on fiber transfer matrix, the DMGs of FM-EDFAs are systematically investigated and two different types of 6-mode fiber amplifiers are analyzed as exemplar demonstrations. In a uniformly-doped step-index fiber, the DMG can be reduced from 9.3 to 1.1 dB (i.e. 8.2 dB reduction) but it can be further reduced to 0.5 dB in a dual-layer doping structure.

**Keywords:** space division multiplexing; few-mode fibers; erbium-doped fiber amplifiers; strong mode coupling

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## 1. Introduction

With the increasing demand for optical fiber transmission capacity, space division multiplexing (SDM) has been explored in depth as an effective way to overcome the capacity limit of conventional single-mode fiber transmission systems by using spatially multiplexed channels in few-mode fibers (FMFs) and/or multi-core fibers (MCFs) [1-3]. Their matching optical fiber amplifiers are the key building blocks in long-haul SDM transmission systems and various types of few-mode erbium-doped fiber amplifiers (FM-EDFAs) [4-10] and multi-core EDFAs [11,12] have been developed over the past 10 years in order to simultaneously amplify all the spatial channels. These SDM amplifiers can bring significant advantages in cost/space saving and efficient pump utilization compared to multiple single-mode EDFAs [4].

One of the most important aspect of SDM amplifier design is to equalize the amplifier gain between all the spatial channels. In principle, the differential modal gain (DMG) is dependent on the spatial overlap between signal, pump intensity profile and erbium-dopant distribution and it can be effectively improved by optimizing these three aspects. Various advanced amplifier fiber designs (e.g. confinement doping or multi-layer doping) have been investigated to minimize the DMG [5-7] and it has been reported that the DMGs of FM-EDFAs supporting 6, 10 and 21 spatial modes can be reduced to 2–3 dB by adopting these approaches [8-10]. However, it is still very challenging to achieve very low DMG (less than 1 dB) and the fiber design complexity gradually increases with the increase of number of spatial modes. It is also important to point out that conventional single-mode EDFAs need gain equalization with respect to wavelength, i.e., in one dimension, while

FM-EDFAs require two-dimensional gain equalization for both multiple wavelengths and spatial channels. This causes tremendously increased complexity of fiber design and fabrication for FM-EDFAs, as the number of modes increases.

Intentionally enhancing the mode coupling (MC) within a FM-EDFA can be considered as an effective way to further reduce DMG. It has obvious advantages in two main aspects: 1) reduced DMG via completely mixing the signal modes and 2) reduced fiber design complexity with a large number of modes. Please note that strong MC scheme has been already employed in long-haul FMF transmission systems and regarded as an effective method to greatly reduce the complexity of MIMO signal processing [13-16]. Therefore, the use of strong coupling onto FM-EDFAs can naturally match with the transmission fiber links with improved DMG performance. Recently, mode scramblers have been investigated as a means to reduce the DMG in FM-EDFAs and a 1.8-dB DMG reduction was successfully achieved in a 10-mode fiber amplifier [17]. Therefore, it would be highly desirable to systematically investigate the effect of strong mode coupling inside the FM-EDFAs and explore the feasibility of realization.

In this paper, we propose a strongly coupled FM-EDFA for the first time with greatly reduced DMG, which can be wisely controlled by choosing appropriate coupling strength (CS) and the number of coupling points (NCPs). In this way, the challenge of two-dimensional gain equalization can be reduced to a one-dimensional task. Using a fiber transfer matrix method model, we have analyzed two exemplary 6-mode EDFAs and found that strong mode coupling has a great effect on DMG mitigation in both FM-EDFAs. An 8.2-dB DMG reduction is achieved in a uniformly doped FM-EDFA via strong coupling and it can be further reduced to 0.5 dB with an optimized fiber doping profile. The proposed strongly coupled FM-EDFAs are very tolerant to the mode-dependent loss in SDM transmission system and the spectral gain flatness and differential noise figure (NF) could also be maintained to be low.

## 2. 2. Numerical Modeling of Strongly Coupled FM-EDFA

To characterize a FM-EDFA with strong MC, we cannot simply evaluate its modal gain (MG) by launching and receiving each mode one-by-one as in an uncoupled or weakly coupled situation, because each input would excite all the other guided modes. There is an accurate way to measure MG, namely, calculating singular value decomposition (SVD) of the fiber transfer matrix (TM) [18].

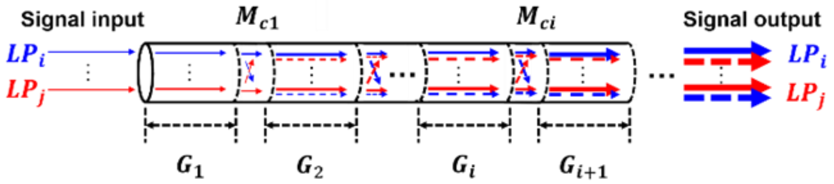

**Figure 1.** Multi-segment model of a strongly coupled FM-EDFA.

In our model, an erbium-doped fiber (EDF) comprises multiple gain segments and coupling matrices, as shown in Fig. 1. Assuming that there are K independent sections, each of which is modeled as an individual amplification gain matrix ($G_1, G_2, \ldots, G_K$). Distributed MC can be viewed as a large number of random matrices ($M_1, M_2, \ldots, M_{K-1}$), randomly located before and after each independent section. The TM of the proposed FM-EDFA is given by

$$M = G_K \cdots G_{i+1} M_{ci} G_i \cdots G_2 M_{c1} G_1 \tag{1}$$

where $M$ is the TM, $G_i$ is the gain matrix in the $i$-th section, and $M_{ci}$ is the random mode-coupling matrix that occurs between two amplification processes (i.e. $G_i$ and $G_{i+1}$).

Then we can obtain the overall MG and DMG from M by SVD technique at each frequency bin [18], which closely resembled the evaluation of polarization dependent loss [19]. *M* can be decomposed into *N* spatial channels:

$$M = V \cdot S \cdot U^* \tag{2}$$

$$S = \begin{pmatrix} e^{\frac{1}{2}g_1} & & 0 \\ & \ddots & \\ 0 & & e^{\frac{1}{2}g_N} \end{pmatrix} \tag{3}$$

where *U* and *V* represent the equivalent mode mixing at the input and output ends, respectively. We obtain $g = (g_1, g_2, \ldots, g_N)$ whose elements are the logarithms of the eigenvalues of $M \cdot M^*$ [20], i.e., the gain of N spatial channels. Thus, the overall DMG can be written as [3]

$$DMG = max\{g_n\} - min\{g_n\} \tag{4}$$

where $max\{g_n\}$ and $min\{g_n\}$ are the maximum and minimum gains in dB.

**Table 1.** Fiber Parameters of the two different types of EDFs.

| Type | Core Radius (μm) | Cladding Radius (μm) | Numerical Aperture | Erbium doping distribution |
|---|---|---|---|---|
| Type A | 8 | 62.5 | 0.15 | Uniform doping ($Er^+$ = 100 ppm) |
| Type B | 8 | 62.5 | 0.15 | Dual-layer doping ($r<b$: $Er^+$ = 65 ppm, $r>c$: $Er^+$ = 115 ppm) |

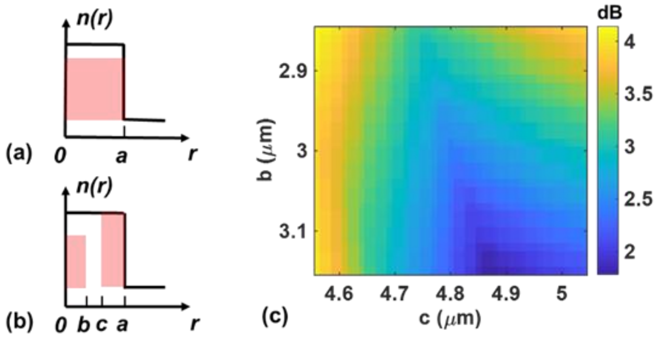

**Figure 2.** Refractive index profile and Er-dopant distribution of (a) the Type-A and (b) Type-B 6-mode EDFs (shaded region represents Er ion doping); (c) DMG of Type-B 6-mode EDFA as a function of *b* and *c*.

Here, EDFs are equally divided into thousands of fiber segments, each of which is modeled using 4th-order Runge-Kutta method [21]. MC randomly happens between two fiber segments. Both the number of coupling points and the coupling strength per point determine the degree of mode coupling. Under a 980-nm pump light ($LP_{01}$), the linearly polarized signal modes ($LP_{01}$, $LP_{11}$, $LP_{21}$, $LP_{02}$) are launched with an equal power of 0.1 mW at the end of the fiber input. In this work, EDFs with two different Er-ion distributions are taken into account: Type-A with a uniform doping and Type-B with a dual-layered doping, as shown in Table 1 and Fig. 2. The two typical Er-ion distributions can be obtained by the Modified Chemical Vapor Deposition (MCVD) combined with solution-doping, and some other complex doping profiles can also be achieved experimentally [22,23].

## 3. Results and discussion

To study the role of mode mixing in reducing DMG of FM-EDFAs, we compare the DMG of a Type-A 6-mode EDFA with or without strong MC. Figure 3 shows MG as a function of EDF length at a pump power of 1 W, which is much higher than the signal power of 0.1 mW mentioned in section 2. In an uncoupled case (Fig. 3a), MG gradually increases until saturated at ~12 m and the overall DMG of the amplifier is ~9.3 dB. In a strong mode coupling regime (Fig. 3b-3d), however, a 6-mode EDFA can get much lower DMG by increasing the degree of the MC, equivalent to increasing the CS or NCP. For example, as shown in Fig. 3(d), DMG is reduced to 3.4 dB when NCP and CS are 600 and -10 dB/pt respectively, which is 5.9-dB lower than that in Fig. 3(a).

Then we focus on the effect of NCP and CS on DMG reduction in strongly coupled FM-EDFAs. Figure 4 shows the calculated DMD results of both Type-A and Type-B 6-mode EDFAs. NCP and CS are in the range of 10 to 600 and -25 to -1 dB/pt, respectively. It should be noted that there are little variations in calculated DMG values at fixed NCP and CS, resulting from the randomness of the coupling matrix and the location of each point, and we have calculated the DMG at each NCP and CS with an average of 100 times. In the uncoupled case, DMGs of Type-A and Type-B fiber amplifiers are 9.3 dB and 2.4 dB respectively, but it is greatly improved by strong mode mixing. Particularly for the Type-A EDFA, 8.2-dB DMG reduction (i.e. from 9.3 to 1.1 dB) could be achieved via strong mode mixing and the DMG of Type-B EDFA is reduced to 0.5 dB with the aid of optimized Er-dopant distribution. Therefore, we can say that the proposed approach is very powerful for high DMG amplifiers but it is also very useful to further mitigate it in moderate-DMG amplifiers.

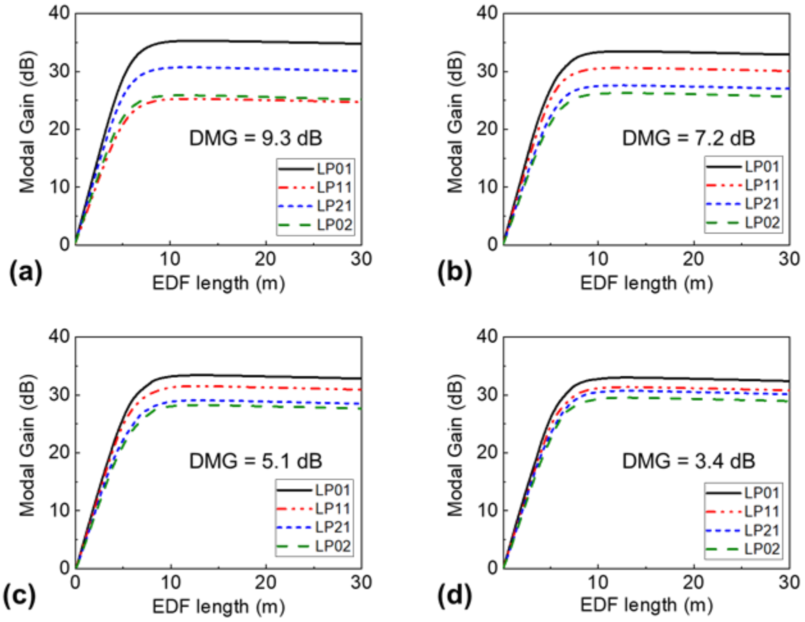

**Figure 3.** MG as a function of EDF length in (a) uncoupled or strongly coupled 6-mode EDFAs at (b) NCP = 300, CS = -15 dB/pt; (c) NCP = 300, CS = -10 dB/pt; (d) NCP = 600, CS = -10 dB/pt.

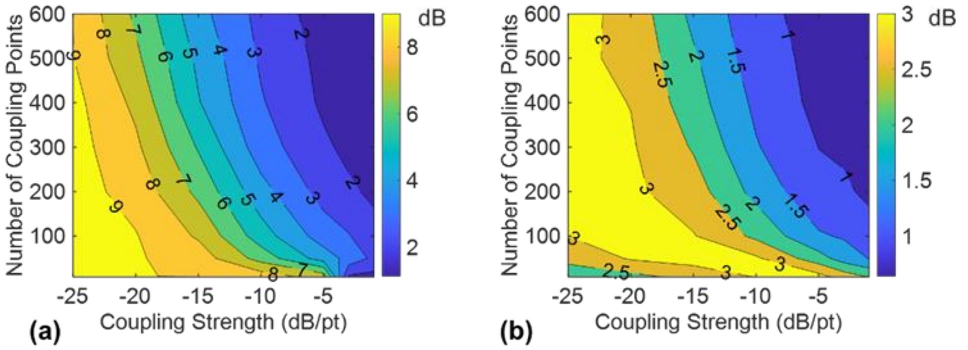

**Figure 4.** DMG of (a) Type-A and (b) Type-B 6-mode EDFAs as a function of NCP and CS.

As mentioned before, NCP and CS are the key factors in reducing DMG. Considering the randomness of coupling matrix and the location of each point, we then evaluate DMG fluctuation of strongly coupled FM-EDFAs after 100 repeated calculations. In Fig. 4, we noticed that the DMGs of the two types of FM-EDFAs follow the similar trend as a function of NCP and CS and we chose Type-A fiber amplifier (i.e. uniform doping), which is the most common and easiest to manufacture, for the following analysis. Figure 5 shows DMG and its corresponding standard deviation as a function of NCP and CS. As shown in Fig. 5(a), the DMG gradually decreases with CS and we can achieve 7.8-dB reduction from 9.3 ± 0.3 to 1.5 ± 0.3 dB when NCP equals 300, while 8.2-dB reduction when NCP equals 600. The DMG also decreases with NCP in Fig. 5(b) from 8.6 ± 0.3 to 4.4 ± 0.7 dB when CS = -15 dB/pt and from 8.4 ± 0.5 to 2.7 ± 0.4 dB for CS = -10 dB/pt. It is worth noting that the standard deviation of DMG increases at the beginning due to the small number of NCP but it decreases with NCP.

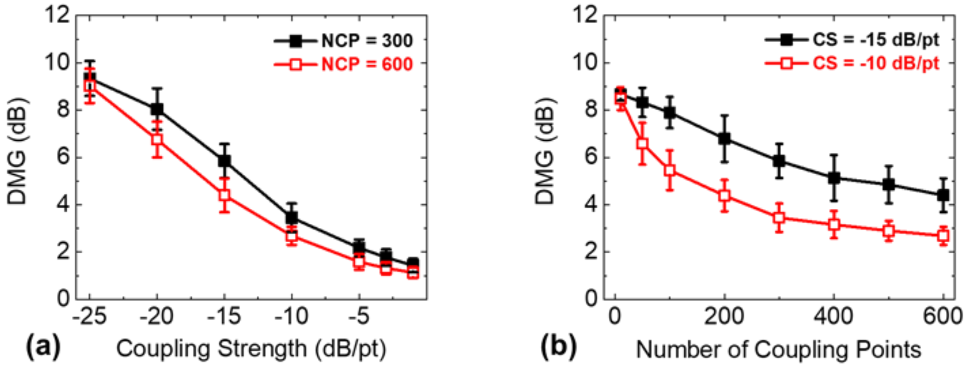

**Figure 5.** DMG and corresponding standard deviation as a function of (a) CS at NCP = 300 and 600; (b) NCP at CS = -15 dB/pt and -10 dB/pt.

Averaged MGs and DMGs of (un)coupled FM-EDFAs have been studied at different pump power. As shown in Fig. 6(a), the averaged MGs in both cases are almost the same and the DMGs remain unchanged for pump power in the range of 200–1000 mW. Apart from low DMG, it is also important to obtain wideband operation and low noise figure (NF) for FM-EDFAs. Figure 6(b) shows the averaged MGs and DMGs over the full C-band. In both cases, the average MG spectra of the amplifiers are flat over the whole C-band and the wavelength dependance of the DMGs is also negligible. Thus, the gain bandwidth and flatness of the amplifier is not sacrificed via strong mode mixing in FM-EDFAs.

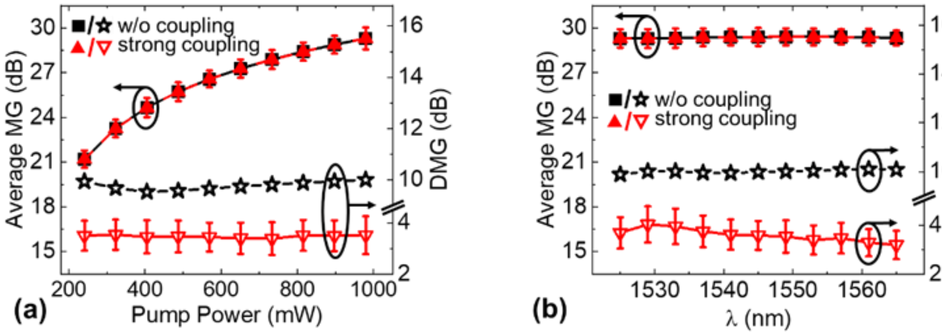

**Figure 6.** (a) Pump power and (b) wavelength dependence of uncoupled and strongly coupled 6-mode EDFAs at NCP =300 and CS = -10 dB/pt.

NF in FM-EDFAs can be written by $NF_i = \frac{P_{ASE\_i}}{h\nu\Delta\nu g_i} + \frac{1}{g_i}$ [24], where $g_i$ and $P_{ASE\_i}$ represent the *i*-th MG and ASE noise power respectively. Here, note that the difference of $g_i$ in the strongly coupled FM-EDFA is greatly reduced compared to uncoupled fiber amplifier and strong mode mixing, making the ASE noise power in different modes consistent. Thus, NF in strongly coupled FM-EDFAs would be lower (or similar at least) than that of un-coupled amplifiers and NF difference between spatial modes can be smaller.

## 4. Implementation of strongly coupled FM-EDFAs

The feasibility of our proposed strongly coupled FM-EDFAs is also examined. Various fiber-type mode scramblers can be considered as a strong MC but we have particularly focused on the long-period gratings (LPGs) inscription technique during the fiber drawing process by $CO_2$ laser annealing [25,26] because it can provide highly controllable MC along the entire length of active fiber. According to [27], the grating angle $\Psi_0$ and grating strength $\Delta n_g$ are the most essential parameters for LPG inscription and they directly determine CS. With $\Psi_0 = 0$, the refractive index change is described as $\Delta n_g(x,y,z) = \Delta n_g exp\left[-\rho\left(\sqrt{a^2-y^2}-x\right)\right]\left[1+cos\left(2\pi z/\Lambda\right)\right]$, where $\Lambda$ is the grating period, $\rho$ represents the asymmetric shape of the transverse index profile, and a is the core radius. Specifically, considering the dimension of our fibers, 10% power transfer (i.e., CS = -10 dB) occurs after a 4-cm-long fiber when Δng equals $7.5\times10^{-5}$. For the mentioned 6M-EDFAs, we choose the length of EDF to be 12 m, which can accommodate three hundred 4-cm-long LPGs in total, meaning NCP = 300. Besides, according to [28], the mode-dependent loss (MDL) for the device is assumed to be 4 dB and uniformly distributed along the EDF. Table 2 summarizes the gain characteristics with or without losses for two types of EDFs. Although the averaged MGs was reduced about 1 dB due to the MDL but there was little change in both DMGs and their standard deviations (only 0.5-dB and 0.2-dB changes of DMGs for Type-A and -B EDFs, respectively).

**Table 2.** EDF gain characteristics w or w/o losses at CS = -10 dB/pt and NCP = 300

| | Parameters | Type A | Type B |
|---|---|---|---|
| Without Losses | Averaged MG (dB) | 31.31 | 30.65 |
| | DMG (dB) | 3.40 | 1.59 |
| | Standard deviation of DMG (dB) | 0.65 | 0.24 |
| With Losses | Averaged MG (dB) | 30.10 | 29.57 |
| | DMG (dB) | 3.91 | 1.38 |
| | Standard deviation of DMG (dB) | 0.72 | 0.25 |

Note that conventional LPGs are strongly wavelength-dependent, which may limit the gain bandwidth of the proposed amplifiers. Mode coupling induced by LPGs varies with the wavelength, which will make the DMG also vary with the wavelength based on the analysis in section 3. Therefore, in order to obtain low DMG over a wide band, wideband LPGs are needed. Many research works have been recently reported to improve the bandwidth [29,30] using chirp the grating profiles [31] or cascading several gratings with different periods [32] and the widest bandwidth achieved so far with a LPFG is ~300 nm (10-dB bandwidth) [33]. Therefore, we believe that the proposed strongly coupled FM-EDFAs can be achievable with currently available LPG fabrication techniques.

## 5. Conclusions

We have proposed a new strongly-coupled FM-EDFA design to realize low DMG and developed the amplifier model for the first time. Using this approach, significant DMG reductions could be obtained for both uniformly-doped and dual-layer 6-mode EDFAs and a wideband operation was confirmed across the C-band. We believe that the proposed strongly coupled FM-EDFAs can be further scaled to large number of spatial modes and achievable using currently available long period fiber fabrication technologies.

**Author Contributions:** Y.L. planned and wrote the paper. X.W. contributed to the writing of the fourth part. Z.Y. took the overall responsibility in managing the manuscript. Y.J. revised the whole article and gave some advice. L.Z. supervised the work and provided technical leadership. All co-authors contributed to the final version with suggestions and critical comments. All authors have read and agreed to the published version of the manuscript.

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## References

1. Richardson, D. J.; Fini, J. M.; Nelson, L. E. Space-division multiplexing in optical fibres. *Nat. Photonics* **2013**, 7, 354–362.
2. Li, G.; Bai, N.; Zhao, N.; Xia, C. Space-Division Multiplexing: The Next Frontier in Optical Communication. *Adv. Opt. Photon.* **2014**, 6, 413-487.
3. Mizuno, T.; Takara, H.; Shibahara, K.; Sano, A.; Miyamoto, Y. Dense Space Division Multiplexed Transmission Over Multicore and Multimode Fiber for Long-haul Transport Systems. *J. Lightw. Technol.* **2016**, 34, 1484-1493.
4. Bai, N.; Ip, E.; Wang, T.; Li, G. Multimode fiber amplifier with tunable modal gain using a reconfigurable multimode pump. *Opt. Express* **2011**, 19, 16601-16611.
5. Krummrich, P. M. Optical amplifiers for multi-mode/multi-core transmission. Proceedings of Optical Fiber Commun. Conf. (OFC), Los Angeles, United States, 4–8 March 2012.
6. Gaur, A.; Rastogi, V. DMG control in six-mode-group EDFA for Space Division Multiplexing. In International Conference on Fibre Optics and Photonics, Kanpur, India, 4–8 December 2016.
7. Trinel, J.; Cocq, G. L.; Andresen, E. R.; Quiquempois, Y.; Bigot, L. Latest results and future perspectives on few-mode erbium doped fiber amplifiers. *Opt. Fibre Technol.* **2017**, 35, 56-63.
8. Jung, Y.; Kang, Q.; Sahu, J. K.; Corbett, B.; O'Callagham, J.; Poletti, F.; Alam, S.; Richardson, D. J. Reconfigurable modal gain control of a few-mode EDFA supporting six spatial modes. *Photonic Tech. L.* **2014**, 26, 1100-1103.
9. Fontaine, N. K.; Huang, B.; Eznaveh, Z. S.; Chen, H.; Cang, J.; Ercan, B.; Veláquez-Benitez, A.; Chang, S. H., Ryf, R.; Schulzgen, A.; Zaharias, J. C. A.; Sillard, P.; Gonnet, C.; Lopez, J. E. A.; Amezcua-Correa, R. Multi-mode Optical Fiber Amplifier Supporting over 10 Spatial Modes. In Optical Fiber Commun. Conf. (OFC), Anaheim, United States, 20–22 March 2016.
10. Zhang, Z.; Guo, C.; Cui, L.; Mo, Q.; Zhao, N.; Du, C.; Li, X.; Li, G. 21 spatial mode erbium-doped fiber amplifier for mode division multiplexing transmission. *Opt. Lett.* **2018**, 43, 1550-1553.

11. Chen, H.; Fontaine, N. K.; Ryf, R.; Jin, C.; Huang, B.; Shang, K.; Essiambre, R.; Wang, L.; Hayashi, T.; Nagashima, T.; Sasaki, T.; Messaddeq, Y.; LaRochelle, S. Demonstration of cladding-pumped six-core erbium-doped fiber amplifier. *J. Lightw. Technol.* **2016**, 34, 1654-1660.
12. Wada, M.; Sakamoto, T.; Aozasa, S.; Yamamoto, T.; Nakajima, K. L-band randomly-coupled 12 core erbium doped fiber amplifier. In Optical Fiber Commun. Conf. (OFC), San Diego, United States, 3–7 March 2019.
13. Arık, S. Ö.; Ho, K.; Kahn, J. M. Delay spread reduction in mode-division multiplexing: Mode coupling versus delay compensation. *J. Lightw. Technol.* **2015**, 33, 4504-4512.
14. Liu, H.; Wen, H.; Huang, B.; Correa, R. A.; Sillard, P.; Chen, H.; Li, Z.; Li, G. Reducing group delay spread using uniform long-period gratings. *Sci. Rep.* **2018**, 8, 3882.
15. Sakamoto, T.; Mori, T.; Wada, M.; Yamamoto, T.; Yamamoto, F.; Nakajima, K. Strongly-coupled multi-core fiber and its optical characteristics for MIMO transmission systems. *Opt. Fibre Technol.* **2017**, 35, 8-18.
16. Ryf, R.; Fontaine, N. K.; Guan, B.; Essiambre, R.-J.; Randel, S.; Gnauck, A. H.; Chandrasekhar, S.; Adamiecki, A.; Raybon, G.; Ercan, B.; Scott, R. P.; Yoo, S. J. B.; Hayashi, T.; Nagashima, T.; Sasaki, T. 1705-km transmission over coupled-core fibre supporting 6 spatial modes. In Optical Fiber Commun. Conf. (OFC), San Francisco, United States, 9–13 March 2014.
17. Wada, M.; Sakamoto, T.; Yamamoto, T.; Aozasa, S.; Nakajima, K. Low mode dependent gain few-mode EDFA with fiber based mode scrambler. In Proceedings of Optoelectronics and Commun. Conf. (OECC/PSC), Fukuoka, Japan, 7-11 July 2019.
18. Randel, S.; Schmidt, C.; Ryf, R.; Essiambre, R.; Winzer, P. J. MIMO-based signal processing for mode-multiplexed transmission. In Proceedings of Photonics Society Summer Topical Meeting Series, Seattle, United States, 9-11 July 2012.
19. Mecozzi, A.; Shtaif, M. The statistics of polarization-dependent loss in optical communication systems. *IEEE Photonic Tech. L.* **2002**, 14, 313-315.
20. Choutagunta, K.; Arik, S. Ö.; Ho, K.; Kahn, J. M. Characterizing mode-dependent loss and gain in multimode components. *J. Lightw. Technol.* **2018**, 36, 3815-3823.
21. Becker, P. M.; Olsson, A. A.; Simpson, J. R. Erbium-doped fiber amplifiers: fundamentals and technology. Elsevier, 1999.
22. Laurent, B.; Guillaume, L. C.; Yves, Q. Few-mode erbium-doped fiber amplifiers: A review. *J. Lightw. Technol.* **2015**, 33, 588-596.
23. Ip, E. Gain equalization for few-mode fiber amplifiers beyond two propagating mode groups. *Photonic Tech. L.* **2012**, 24, 1933-1936.
24. Becker, P. C.; Olsson, N. A.; Simpson, J. R. Erbium-Doped Fiber Amplifiers Fundamentals and Technology. Elsevier, 1998.
25. Hirose, T.; Saito, K.; Kojima, S.; Yao, B.; Ohsono, K.; Sato, S.; Takada, K.; Ikushima, A.J. Fabrication of long-period fibre grating by $CO_2$ laser-annealing in fibre-drawing process. *Electron. Lett.* **2007**, 43, 443-445.
26. Wang, X.; Yang, Z.; Yu, W.; Liu, Y.; Di, Z.; Li, G.; Zhang, L. DMD Reduction by Strong Mode Coupling in Few-mode Fibers Using Multi-laser Inscribed LPFGs. In Conf. on Lasers and Electro-Optics (CLEO), Washington, United States, 10–15 May 2020.
27. Fang, J.; Li, A.; Shieh, W. Low-DMD few-mode fiber with distributed long-period grating. *Opt. Lett.* **2015**, 40, 3937-3940.
28. Zhao, Y.; Chen, H.; Fontaine, N. K.; Li, J.; Ryf, R.; Liu, Y. Broadband and low-loss mode scramblers using $CO_2$-laser inscribed long-period gratings. *Opt. Lett.* **2018**, 43, 2868-2871.
29. Wang, W.; Wu, J.; Chen, K.; Jin, W.; Chiang, K. S. Ultra-broadband mode converters based on length-apodized long-period waveguide gratings. *Opt. Express* **2017**, 25, 14341-14350.
30. Qiang, L.; Zhengtian, G.; Guan, T. Few-mode tilted long period fiber grating mode filter based on dual-peak resonance near phase-matching turning point. *Optik*, **2020**, 164656.
31. Chen, H.; Kuan, W.; Lin, K. Ultrabroadband Tunable Long-Period Fiber Grating for Stress Sensing and Optical Filtering. In IEEE SENSORS, Glasgow, UK, 29 October – 1 November 2017.
32. Perez-Sanchez, G. G.; Soto-Jasso, M. A.; Hernández-Valdez, G.; Ocampo-Hidalgo, J. J.; Medina-Velázquez, D. Y. Design and manufacturing of cascaded LPFG with CNC $CO_2$ laser machine. In Photonic Fiber and Crystal Devices: Advances in Materials and Innovations in Device Applications XIV, September 2020.
33. Zhao, X.; Liu, Y.; Liu, Z.; Mou, C. All-fiber bandwidth tunable ultra-broadband mode converters based on long-period fiber gratings and helical long-period gratings. *Opt. Express* **2020**, 28, 11990-12000.