



# New relaxed stability and stabilization conditions for both discrete and differential linear repetitive processes

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## Abstract

The paper develops new results on stability analysis and stabilization of linear repetitive processes. Repetitive processes are a distinct subclass of two-dimensional (2D) systems, whose origins are in the modeling for control of mining and metal rolling operations. The reported systems theory for them has been applied in other areas such iterative learning control, where, uniquely among 2D systems based designs, experimental validation results have been reported. This paper uses a version of the Kalman–Yakubovich–Popov Lemma to develop new less conservative conditions for stability in terms of linear matrix inequalities, with an extension to control law design. Differential and discrete dynamics are analysed in an unified manner, and supporting numerical examples are given.

**Keywords** Linear repetitive processes · Kalman–Yakubovich–Popov lemma · Stability along the pass · Frequency partitioning

## 1 Introduction

Repetitive processes have their origins in the modeling and control of long-wall coal cutting and material rolling operations, see, e.g., Rogers et al. (2007), which, in turn, cited the original work. Their characteristic is illustrated by the case of a material or workpiece, processed by a

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series of sweeps of a processing tool, which is of finite duration. In the literature, each sweep is often termed a pass and the finite duration is known as the pass length.

An industrial example that highlights the characteristics of repetitive processes is metal rolling operations see, e.g., Rogers et al. (2015), where, in essence, deformation of the workpiece takes place by passing the workpiece between two rolls multiple times, i.e., passes in repetitive process terminology. The objective is to reduce the thickness to a predefined value, where each pass produces a pass profile  $y_k(t)$ ,  $0 \leq t \leq \alpha$ . In this notation  $y$  is the vector or scalar-valued variable under consideration, the non-negative integer  $k$  is the pass number and  $\alpha < \infty$  denotes the pass length. Moreover, the pass profile on any pass explicitly contributes to the profile produced on the next pass and so on.

In general, the control problem for repetitive processes is that the sequence of pass profiles  $\{y_k\}_k$  can contain oscillations that increase in amplitude from pass-to-pass, i.e., in  $k$ . Moreover, this undesirable feature cannot be removed by standard control action, e.g., joining successive passes end-to-end to convert the dynamics to those of a standard system, see Rogers et al. (2007) where the deficiencies of this approach are explained. Instead, control design has to be based on regulating the 2-D dynamics that arise from information propagation along the passes and from pass-to-pass.

A stability theory for these processes has been developed (Rogers et al. 2007) that is of the bounded-input bounded-output form based on the pass profile. In particular, a bounded initial pass profile is required to produce a bounded sequence,  $\{y_k\}_k$ , of pass profiles, where boundedness is defined in terms of the norm on the underlying function space. This theory is based on a general model that includes all linear constant pass length examples as special cases. It allows specific treatment of the boundary conditions at the start of each pass, which must have a very particular and restrictive form for analysis based on the alternative above to produce correct results.

Aside from their use in the analysis and control of physical examples, the repetitive process setting has been used as a basis for solving problems in other areas of control theory, such as iterative learning control (ILC) law design, see, e.g., Rogers et al. (2015) and Paszke et al. (2016)) and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle (Roberts 2002). In this latter case, a more general form of pass initial conditions is required and the alternative setting discussed above is not applicable. Moreover, in the ILC application designs based on repetitive process theory have been followed through to experimental validation, as in Paszke et al. (2016).

For linear dynamics, the stability theory has been developed the stage where the conditions for this property can be tested using well known tests from standard linear systems theory. However, these tests do not form a general basis for control law design and this has led to the development of Linear Matrix Inequality (LMI) based tests that extend naturally to control law design, where this approach is used in Paszke et al. (2016) with supporting experimental validation. These conditions are sufficient but not necessary and hence there is a level of conservativeness associated with their use.

The main objective of this paper is to develop new stability conditions in terms of LMIs by applying a frequency-discretization technique for differential and discrete linear repetitive processes, i.e., processes where the dynamics along a pass are governed, respectively, by a linear differential or difference equation. The outcome is relaxed stability conditions in terms of LMIs, which can be easily solved via standard numerical software. The main idea supporting the new result in this paper is a significant reduction of the conservatism of the resulting stability condition by dividing the entire frequency domain into sub-intervals and applying the Generalized Kalman–Yakubovich–Popov (GKYP) Lemma to each of them. As a result, some matrix variables are not required to be fixed over the entire frequency range.

Also, this property enables control performance objectives to be imposed over specified finite frequency ranges. Hence a reduction in conservatism is possible by avoiding over-design using the entire frequency range. Additionally, using the Elimination Lemma, standard analysis and design criteria are extended by introducing slack matrix variables. Hence, such conditions are more suitable for stabilizing controllers as products between matrix variables and those describing the process state-space model matrices are eliminated.

Preliminary results on this approach for discrete (Rogers et al. 2016; Li et al. 2015) and differential (Boski et al. 2018; Wang et al. 2017) repetitive processes have been reported. In this paper an approach is developed that enables differential and discrete dynamics to be analysed in unified manner by applying the GKYP and Elimination (Projection) Lemmas. Moreover, the new approach allows for a reduction in conservatism by introducing additional decision variables into final stability conditions in LMI form. Next, the stability conditions are extended to allow control law design and more general cases where the process matrices contain uncertain parameters. Finally, the effectiveness and advantages of the new conditions are demonstrated by two examples.

Throughout this paper the null and identity matrices, respectively, with compatible dimensions are denoted by  $0$  and  $I$ . For a square matrix  $M$ ,  $\text{sym}\{M\}$  denotes  $M + M^T$  and  $\rho(\cdot)$  the spectral radius of its matrix arguments. Furthermore,  $M \succ 0$  ( $M \prec 0$ ) means that the symmetric matrix  $M$  is positive definite (negative definite). Also, two specific regions of the complex plane are defined:  $\mathbb{C}_{hp} = \{\lambda \in \mathbb{C} \mid \text{Re}(\lambda) < 0\}$  (open left half plane) and  $\mathbb{C}_{uc} = \{\lambda \in \mathbb{C} \mid |\lambda| < 1\}$  (an open disc centered at the origin and of unit radius). Finally,  $\otimes$  denotes the Kronecker product,  $(*)$  block entries in symmetric matrices and the superscript  $*$  denotes the complex conjugate transpose operation (for both scalars and matrices).

The new results in this paper are formulated in terms of LMIs and hence the following lemmas are useful in transforming non-LMI formulations into a problem subject to LMI constraints, where the first is the Elimination Lemma and the second is a version of the GKYP lemma.

**Lemma 1** (Gahinet and Apkarian 1994) *Consider compatibly dimensioned matrices  $\Gamma$ ,  $\Lambda$  and  $\Sigma$ , where  $\Gamma = \Gamma^T$ . Then there exists a matrix  $W$  such that*

$$\Gamma + \text{sym}\{\Lambda^T W \Sigma\} \prec 0 \quad (1)$$

*if and only if*

$$\Lambda_{\perp}^T \Gamma \Lambda_{\perp} \prec 0 \text{ and } \Sigma_{\perp}^T \Gamma \Sigma_{\perp} \prec 0, \quad (2)$$

*where  $\Lambda_{\perp}$  and  $\Sigma_{\perp}$  are arbitrary matrices whose columns, respectively, form a basis for the nullspaces of  $\Lambda$  and  $\Sigma$ . Hence  $\Lambda \Lambda_{\perp} = 0$  and  $\Sigma \Sigma_{\perp} = 0$ .*

Suppose that  $\Lambda(\Phi, \Psi)$  is a set of complex numbers that represents a certain class of curves in the complex plane defined as

$$\Lambda(\Phi, \Psi) := \left\{ \lambda \in \mathbb{C} : \begin{bmatrix} \lambda \\ 1 \end{bmatrix}^* \Phi \begin{bmatrix} \lambda \\ 1 \end{bmatrix} = 0, \begin{bmatrix} \lambda \\ 1 \end{bmatrix}^* \Psi \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \geq 0 \right\}, \quad (3)$$

where  $\Phi$  and  $\Psi$  are given matrices of dimension  $2 \times 2$ . As detailed in Iwasaki and Hara (2005), the imaginary axis and the unit circle are the particular examples of (3) with  $\Psi = 0$  and  $\Phi$  as

$$\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{or} \quad \Phi = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (4)$$

**Table 1** The values of  $\Psi$  for differential dynamics

	LF	MF	HF
$\Omega$	$ \omega  < \omega_l$	$\omega_1 \leq  \omega  \leq \omega_2$	$ \omega  > \omega_h$
$\Psi$	$\begin{bmatrix} -1 & 0 \\ 0 & \omega_l^2 \end{bmatrix}$	$\begin{bmatrix} -1 & j\omega_c \\ -j\omega_c & -\omega_1\omega_2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -\omega_h^2 \end{bmatrix}$

Where  $\omega_c = \frac{(\omega_1 + \omega_2)}{2}$  and LF, MF and HF denote, respectively, the low, middle and high frequency ranges

**Table 2** The values of  $\Psi$  for discrete dynamics

	LF	MF	HF
$\Theta$	$ \theta  < \theta_l$	$\theta_1 \leq  \theta  \leq \theta_2$	$ \theta  > \theta_h$
$\Psi$	$\begin{bmatrix} 0 & 1 \\ 1 & -2 \cos(\theta_l) \end{bmatrix}$	$\begin{bmatrix} 0 & e^{j\theta_c} \\ e^{-j\theta_c} & -2 \cos(\theta_d) \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 2 \cos(\theta_h) \end{bmatrix}$

Where  $\theta_d = \frac{\theta_2 - \theta_1}{2}$ ,  $\theta_c = \frac{\theta_1 + \theta_2}{2}$

for the imaginary axis, respectively, the unit circle. Furthermore, by appropriately selecting  $\Psi$ , the frequency variable  $\lambda \in \Lambda(\Phi, \Psi)$  can be restricted to a certain range of frequencies (finite or semi-finite) and the following result can be established.

**Lemma 2** (Iwasaki and Hara 2005) Consider dimensioned matrices  $A, C, \Phi, \Psi$  and  $\Xi$ . Then if  $\det(\lambda I - A) \neq 0$  for all  $\lambda \in \Lambda(\Phi, \Psi)$  the following conditions are equivalent:

(i) The frequency domain inequality

$$\begin{bmatrix} (\lambda I - A^T)^{-1} C^T \\ I \end{bmatrix}^* \Xi \begin{bmatrix} (\lambda I - A^T)^{-1} C^T \\ I \end{bmatrix} < 0 \quad (5)$$

holds  $\forall \omega \in \Omega$  (differential dynamics and  $\lambda = j\omega$ ) or  $\forall \theta \in \Theta$  (discrete dynamics and  $\lambda = e^{j\theta}$ ) where  $\Omega$  and  $\Theta$  are, respectively the frequency ranges given in Tables 1 and 2.

(ii) There exist matrices  $Q \succ 0$  and a symmetric matrix  $P_1$  such that

$$\begin{bmatrix} A & I \\ C & 0 \end{bmatrix} (\Psi^* \otimes Q + \Phi^* \otimes P_1) \begin{bmatrix} A & I \\ C & 0 \end{bmatrix}^T + \Xi < 0. \quad (6)$$

Appropriate choices for the matrix  $\Xi$  in Lemma 2 allows the analysis of the various system properties. Included in these is the bounded realness property, which is related to the stability and finite gain of a linear system. Hence, the matrix  $\Xi$  is fixed as

$$\Xi = \begin{bmatrix} B_0 & 0 \\ D_0 & I \end{bmatrix} (\Pi \otimes P_2) \begin{bmatrix} B_0 & 0 \\ D_0 & I \end{bmatrix}^T, \quad (7)$$

where  $P_2 \succ 0$ ,  $\Pi = \text{diag}\{1, -\gamma^2\}$  and  $\gamma$  is a given scalar satisfying  $0 < \gamma \leq 1$  and allows the specification of a process gain bound. Also, Lemma 2 provides a dual version of the GKYP lemma, which is exploited in the analysis that follows.

## 2 Stability of linear repetitive processes

The linear repetitive processes considered in this paper are of the following form

$$\begin{aligned} \eta x_{k+1}(t) &= A x_{k+1}(t) + B_0 y_k(t) + B u_{k+1}(t), \\ y_{k+1}(t) &= C x_{k+1}(t) + D_0 y_k(t) + D u_{k+1}(t) \end{aligned} \quad (8)$$

with  $\eta$  representing the differential operator (in case of differential repetitive processes) or one step forward shift operator (in case of discrete repetitive processes). When a differential repetitive process is considered then  $t$  is the continuous variable such that  $t \in [0, \alpha]$ , where  $\alpha$  is a strictly positive real number, while in case of discrete repetitive process  $t$  is a discrete variable such that  $t \in [0, \alpha - 1]$  and the domain of  $\alpha$  is limited to positive integers. Also,  $x_k(t) \in \mathbb{R}^n$ ,  $u_k(t) \in \mathbb{R}^m$  and  $y_k(t) \in \mathbb{R}^p$  represent the process state, input and output vectors at time instant  $t$  on pass  $k$ .

The boundary conditions for these processes are the state initial vector on each pass and the initial pass profile. In the analysis of this paper, no loss of generality arises from assuming (for differential processes and with an equivalent statement for discrete processes) that  $x_{k+1}(0) = 0$ ,  $\forall k \geq 0$ , and  $y_0(t) = f(t)$ ,  $0 \leq t \leq \alpha$ , where the entries in the vector  $f(t) \in \mathbb{R}^p$  are known functions of  $t$ .

In repetitive processes described by (8) and the assumed boundary conditions, the influence of the previous pass profile on the dynamics of the next pass arise from the term  $B_0 y_k(t)$  for the state dynamics and  $D_0 y_k(t)$  for the pass profile. The stability theory can be applied over the finite and fixed pass length or for all possible pass lengths, where this last case can be analyzed mathematically by considering the case when  $\alpha \rightarrow \infty$ . The first property is termed asymptotic stability and the latter stability along the pass. Moreover, stability along the pass requires the boundedness property to hold for all possible values of the pass length. In contrast, asymptotic stability requires this property for the pass length of the process and is, therefore, a necessary condition for stability along the pass.

Asymptotic stability guarantees that the sequence of pass profiles  $\{y_k\}_k$  converges as  $k \rightarrow \infty$  to dynamics described by a standard linear systems state-space model but not that this system is stable. The reason for this fact is the finite pass length, over which duration even an unstable system can only produce a bounded output. Stability along the pass removes this difficulty but there are cases, such as the optimal control application (Roberts 2002) where asymptotic stability suffices (or is all that can be achieved).

The structure of the boundary conditions is area where critical differences exist between repetitive processes and other classes of 2D systems dynamics. In some applications for repetitive processes the initial conditions at the start of each pass are explicit functions of points along the previous pass profile and the structure of these initial conditions play a critical role in stability analysis (Rogers et al. 2007). This form of boundary conditions cannot arise in Roesser and Fornasini Marchesini state-space models for quarter plane causal 2D linear systems and hence no exchange of, say, stability tests is possible (a detailed treatment of this issue is given in Rogers et al. (2007), which also cites the references for these models). In cases where the initial state vector on each pass does not depend on the previous pass profile, the dynamics of a linear repetitive process can be written as a Roesser model. Then, the stability tests and procedures available for the design of stabilizing control laws for the Roesser model can be applied. In particular, Bachelier et al. (2019) interprets the stability along the pass property of a linear repetitive process as the structural stability of the equivalent Roesser model and develops weakly conservative stability and stabilization results based on the polynomial solution parameter dependent LMIs. However, these results are for stabilizability and do not enable design to meet performance specifications over finite frequency ranges.

Suppose that the matrix  $\Phi$  and the region of the complex plane  $\mathbb{C}_A$  are chosen as in Table 3.

Then the lemma given next establishes conditions for stability along the pass of linear repetitive processes.

**Lemma 3** (Rogers et al. 2007) *A linear repetitive process described by (8) is stable along the pass if and only if*

**Table 3** Characterization of  $\Phi$  and  $\mathbb{C}_A$ 

	Differential process case	Discrete process case
$\mathbb{C}_A$	$\mathbb{C}_{hp}$	$\mathbb{C}_{uc}$
$\Phi$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

- (i)  $\text{eig}(D_0) \subset \mathbb{C}_{uc}$ ,
- (ii)  $\text{eig}(A) \subset \mathbb{C}_A$ ,
- (iii)  $\text{eig}(G(\lambda)) \subset \mathbb{C}_{uc}$ ,  $\forall \lambda \in \Lambda(\Phi, 0)$ , where  $G(\lambda) = C(\lambda I - A)^{-1}B_0 + D_0$ .

The first two conditions in Lemma 3 pose no computational difficulties but the third requires computations for the entire frequency range, i.e., for all points on the imaginary axis of the complex plane for differential processes and the unit circle for discrete processes. Also, the first condition is the necessary and sufficient condition for asymptotic stability. Moreover, the second condition, which governs the dynamics along a pass, is also only a necessary condition for stability along the pass and this property also requires the third condition, i.e., frequency attenuation of the previous pass profile dynamics over the complete frequency range and therefore as  $k \rightarrow \infty$

$$\|(G(\lambda))^k\| \rightarrow 0, \quad \forall \lambda \in \Lambda(\Phi, 0),$$

where (in the single-input single-output case for ease of presentation)

$$\|G(\lambda)\| = \sup_{\lambda \in \Lambda(\Phi, 0)} |G(\lambda)|.$$

In the remainder of this section the results of Lemmas 1 and 2 are used to develop new LMI based conditions for stability along the pass. In particular, based on the preliminary analysis in Paszke et al. (2013) and Paszke and Bachelier (2013), the following result can be established.

**Theorem 1** Let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$  and  $\Phi$  chosen as in Table 3. Then a linear repetitive process described by (8) is stable along the pass if there exist compatibly dimensioned  $P_1 > 0$ ,  $P_2 > 0$  such that the LMI

$$\begin{bmatrix} A & I \\ C & 0 \end{bmatrix} (\Phi^* \otimes P_1) \begin{bmatrix} A & I \\ C & 0 \end{bmatrix}^T + \begin{bmatrix} B_0 & 0 \\ D_0 & I \end{bmatrix} (\Pi \otimes P_2) \begin{bmatrix} B_0 & 0 \\ D_0 & I \end{bmatrix}^T < 0, \quad (9)$$

where  $\Pi = \text{diag}\{1, -\gamma^2\}$  is feasible.

**Proof** Assume that (9) is feasible for some  $P_1 > 0$ ,  $P_2 > 0$ . Applying directly the result of the KYP lemma (see Theorems 1 and 2 in Rantzer 1996) gives

$$\begin{bmatrix} G^*(\lambda) \\ I \end{bmatrix}^* (\Pi \otimes P_2) \begin{bmatrix} G^*(\lambda) \\ I \end{bmatrix} < 0, \quad (10)$$

where  $G(\lambda)$  is defined by *iii* in Lemma 3. Moreover, the inequality (10) can be rewritten as

$$G(\lambda)P_2G^*(\lambda) - \gamma^2P_2 < 0,$$

where the existence of  $P_2 > 0$  directly implies that  $\rho(G(\lambda)) < 1$ ,  $\forall \lambda \in \Lambda(\Phi, 0)$ , i.e., feasibility of (9) guarantees that condition *iii* of Lemma 3 holds. Furthermore, since  $P_1 > 0$  and  $P_2 > 0$ , then feasibility of (9) yields

$$\begin{bmatrix} D_0 & I \end{bmatrix} (\Pi \otimes P_2) \begin{bmatrix} D_0^T \\ I \end{bmatrix} < 0, \quad \begin{bmatrix} A & I \end{bmatrix} (\Phi^* \otimes P_1) \begin{bmatrix} A^T \\ I \end{bmatrix} < 0.$$

Equivalently, the conditions *i*) and *ii*) of Lemma 3, respectively, hold and the proof is complete.  $\square$

**Remark 1** In contrast to other classes of 2-D linear systems, the conditions on the matrices  $D_0$  and  $A$  in Lemma 3 have physical meanings. The former requires that the initial pass profile sequence  $\{y_k(0)\}_k$  does not become unbounded with  $k$  and the latter that the contribution of the current pass state vector to the pass profile vector is stable. Hence for a given numerical example they should be tested before proceeding to the LMI of (9).

## 2.1 New LMI-based characterization of stability along the pass

Alternative LMI characterizations of the last two results can be obtained by introducing auxiliary slack matrix variables. These in turn, can be used to overcome non-convexity arising in control law design or reduce the level of conservativeness in many control problems for repetitive processes, such as robust control. To proceed, partition the matrix  $\Phi$  (see (4)) into scalar elements as

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 \\ (\star) & \phi_3 \end{bmatrix}$$

and introduce the following matrices of compatible dimensions

$$\mathbb{A} = \begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix}, \gamma_1 = \begin{bmatrix} \phi_1 P_1 & 0 \\ 0 & 0 \end{bmatrix}, \gamma_2 = \begin{bmatrix} \phi_3 P_1 & 0 \\ 0 & -\gamma^2 P_2 \end{bmatrix}, \gamma_3 = \begin{bmatrix} \phi_2 P_1 & F_1 \\ 0 & F_2 \end{bmatrix}, F_{12} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, F_{30} = \begin{bmatrix} 0 & F_3 \end{bmatrix}, \quad (11)$$

where  $F_1$ ,  $F_2$  and  $F_3$  are additional compatibly dimensioned slack matrix variables. Then the theorem established next gives new LMI-based conditions for stability along the pass of linear repetitive processes.

**Theorem 2** Let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$  and  $\Phi$  chosen as in Table 3. Then a linear repetitive process described by (8) is stable along the pass if there exist compatibly dimensioned matrices  $P_1 \succ 0$ ,  $P_2 \succ 0$ ,  $W_1$ ,  $W_2$ ,  $W_3$ ,  $F_1$ ,  $F_2$ ,  $F_3$  such that the LMI

$$\begin{bmatrix} \gamma_1 - \text{sym}\{W_1\} & (\star) & (\star) \\ \gamma_3 + \mathbb{A} W_1^T - W_2 & \gamma_2 + \text{sym}\{W_2 \mathbb{A}^T\} & (\star) \\ F_{30} - W_3 & -F_{12}^T + W_3 \mathbb{A}^T & P_2 - \text{sym}\{F_3\} \end{bmatrix} < 0 \quad (12)$$

is feasible.

**Proof** Assume that the LMI defined in (12) has a feasible solution. Then this LMI can be rewritten as (1) with

$$\Gamma = \begin{bmatrix} \gamma_1 & \gamma_3^* & F_{30}^T \\ \gamma_3 & \gamma_2 & -F_{12} \\ F_{30} & -F_{12}^T & P_2 - \text{sym}\{F_3\} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}, \quad \Sigma = [-I \quad \mathbb{A}^T \quad 0]. \quad (13)$$

Then application of Lemma 1 enables (12) to be equivalently rewritten as (2). Also since  $\Lambda_{\perp} = 0$ , the LMI in (12) reduces to the second inequality in (2), i.e.,

$$\Sigma_{\perp}^T \Gamma \Sigma_{\perp} < 0, \quad (14)$$

where by construction the matrix  $\Sigma_{\perp}$  is

$$\Sigma_{\perp} = \begin{bmatrix} \mathbb{A}^T & 0 \\ I & 0 \\ 0 & I \end{bmatrix}.$$

Furthermore, after some routine matrix manipulations the inequality (14) can be rewritten as

$$\Gamma_1 + \text{sym} \left\{ \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \begin{bmatrix} B_0^T & D_0^T & -I \end{bmatrix} \right\} < 0, \quad (15)$$

where

$$\Gamma_1 = \begin{bmatrix} \phi_1 A P_1 A^T + \phi_3 P_1 + \text{sym} \{ \phi_2 P_1 A^T \} & \phi_1 A P_1 C^T + \phi_2 P_1 C^T & 0 \\ \phi_1 C P_1 A^T + \phi_2 C P_1 & \phi_1 C P_1 C_1^T - \gamma^2 P_2 & 0 \\ 0 & 0 & P_2 \end{bmatrix}$$

and by Lemma 1, feasibility of (15) implies that the inequality

$$\Sigma_{1\perp}^T \Gamma_1 \Sigma_{1\perp} < 0$$

must hold where

$$\Sigma_{1\perp} = \begin{bmatrix} I & 0 \\ 0 & I \\ B_0^T & D_0^T \end{bmatrix}.$$

Finally, this last inequality is equivalent to (9) and by Theorem 1 stability along the pass is ensured.  $\square$

**Remark 2** The design conditions of Theorem 2 are LMIs that can be easily and effectively solved via numerical software. In addition, the scalar parameter  $\gamma$  can be minimized to compute the minimal process gain.

**Remark 3** The LMI variables in (12) could be selected real symmetric instead of complex Hermitian without introducing conservatism, see Pipeleers and Vandenberghe (2011) for further details.

**Remark 4** Revisiting the conditions in Theorem 2, it is found that Lemma 1 can again be invoked to reduce the number of slack matrix variables without adding conservatism to the solution. Specifically, the slack matrix variables can be constrained as  $W_2 = \beta_1 W_1$  where  $\beta_1$  is an arbitrary positive scalar. In particular, in (13) set

$$\left[ \Lambda = \begin{bmatrix} I & \beta_1 I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad W = \begin{bmatrix} W_1 \\ W_3 \end{bmatrix} \right].$$

Then choose  $\Lambda_{\perp}^T = [\beta_1 I \quad -I \quad 0]$  and check if the first inequality in (2) holds. In particular, under the above choice of  $\Lambda$  and  $W$ , the first inequality in (2) becomes

$$\begin{bmatrix} \beta_1^2 \phi_1 P_1 - 2\beta_1 \phi_2 P_1 + \phi_3 P_1 & 0 \\ 0 & -\gamma^2 P_2 \end{bmatrix} + \text{sym} \left\{ \begin{bmatrix} 0 & -\beta_1 I \end{bmatrix} \begin{bmatrix} F_1^T & F_2^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \right\} < 0. \quad (16)$$

Next, assign

$$\Gamma \leftarrow \begin{bmatrix} -2\beta_1 P_1 & 0 \\ 0 & -\gamma^2 P_2 \end{bmatrix}, \quad \Lambda^T \leftarrow \begin{bmatrix} 0 \\ -\beta_1 I \end{bmatrix}, \quad \Gamma \leftarrow \begin{bmatrix} \beta_1^2 \phi_1 P_1 - 2\beta_1 \phi_2 P_1 + \phi_3 P_1 & 0 \\ 0 & -\gamma^2 P_2 \end{bmatrix}$$

and by Lemma 1 the inequality (16) is also equivalent to

$$\begin{bmatrix} I & 0 \end{bmatrix}^T \begin{bmatrix} \beta_1^2 \phi_1 P_1 - 2\beta_1 \phi_2 P_1 + \phi_3 P_1 & 0 \\ 0 & -\gamma^2 P_2 \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} < 0.$$

Finally, the above inequality can be written as  $-2\beta_1 P_1 < 0$  for differential process case and holds for any  $\beta_1 > 0$  and  $P_1 > 0$  or as  $(\beta_1^2 - 1)P_1 < 0$  for discrete process case and holds for any  $|\beta_1| < 1$  and  $P_1 > 0$ .

### 3 State feedback based control of linear repetitive processes

In this section, the stability results of the previous section are extended to the case of controlled processes. The 2D system's structure of these processes requires that any control law has structure of feedback action on the current pass (stabilization and/or assignment of the state matrix eigenvalues to satisfy the along the pass specifications) and feedforward action from the previous pass (convergence of the pass-to-pass dynamics). One such control law has the form

$$u_{k+1}(t) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_{k+1}(t) \\ y_k(t) \end{bmatrix} = K_1 x_{k+1}(t) + K_2 y_k(t), \quad (17)$$

where  $K_1$  and  $K_2$  are matrices to be found. Application of this control law results in the controlled process state-space model

$$\begin{aligned} \eta x_{k+1}(t) &= (A + BK_1)x_{k+1}(t) + (B_0 + BK_2)y_k(t), \\ y_{k+1}(t) &= (C + DK_1)x_{k+1}(t) + (D_0 + DK_2)y_k(t), \end{aligned} \quad (18)$$

where the meaning of  $\eta$  is the same as in (8). Furthermore, by defining

$$\mathbb{B} = \begin{bmatrix} B \\ D \end{bmatrix}, \quad \mathbb{K} = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \quad (19)$$

it follows immediately that the results of the previous section can be directly used to develop a condition suitable for computing control law gains  $K_1$  and  $K_2$ . In particular, the following result is obtained by applying Theorem 2 to (18).

**Theorem 3** *Let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$  and  $\Phi$  chosen as in Table 3. Suppose also that the control law (17) is applied to a repetitive process described by (8). Then the resulting controlled process is stable along the pass if there exist matrices  $P_1 > 0$ ,  $P_2 > 0$ ,  $Y$ ,  $W_1$ ,  $F_1$ ,  $F_2$ ,  $F_3$  such that the LMI*

$$\begin{bmatrix} \Upsilon_1 - \text{sym}\{W_1\} & (*) & (*) \\ \Upsilon_3 + \mathbb{A}W_1^T + \mathbb{B}Y - W_1 & \Upsilon_2 + \text{sym}\{\mathbb{A}W_1^T + \mathbb{B}Y\} & (*) \\ F_{30} - [0 \ I]W_1 & -F_{12}^T + [0 \ I](\mathbb{A}W_1^T + \mathbb{B}Y)^T & P_2 - \text{sym}\{F_3\} \end{bmatrix} < 0 \quad (20)$$

is feasible. In addition, if the above LMIs are feasible, the corresponding matrices in the control law (17) are given by

$$\begin{bmatrix} K_1 & K_2 \end{bmatrix} = YW_1^{-T}. \quad (21)$$

**Proof** Assume that the LMI (20) holds. Also, it follows immediately that the feasibility of (20) implies that  $W_1$  is non-singular. Also setting  $Y = KW_1^T$  leads to the interpretation of the condition of Theorem 2 for (18) and hence stability along the pass for the controlled process. This completes the proof.  $\square$

**Remark 5** This last result requires the following constraints on the multipliers

$$W_1 = W_2 \text{ and } W_3 = [0 \ I]W_1$$

and introduces conservativeness but still some additional freedom is introduced by the slack matrix variables  $F_1, F_2, F_3$ . A possible way to reduce conservatism is to apply the constraints

$$W_2 = \beta_2 W_1, \quad W_3 = [0 \ \beta_3 I]W_1,$$

where  $\beta_2$  and  $\beta_3$  are some scalars. Hence, line searches on these scalars are required to make the LMI less conservative (in terms of performance provided by  $\gamma$ ).

## 4 Less conservative conditions for stability along the pass

The results of Theorems 1, 2 and 3 can introduce a significant degree of conservativeness since it has been obtained by keeping  $P_2$  (and some other matrix variables) constant and independent of frequency. This means that  $P_2$  must work for entire imaginary axis or the unit circle, i.e.,  $\forall \lambda \in \Lambda(\Phi, 0)$ . One possible way to overcome this problem is to extend the results developed in previous sections of this paper to allow piecewise constant matrix variables defined over union of segments on the entire imaginary axis or the unit circle in the complex plane.

These segments must be contiguous to ensure continuity of  $\lambda$  over  $\Lambda(\Phi, 0)$  and this can be achieved by dividing the entire frequency range into a number of finite or semi-finite ranges. Also, specific choices of a pair of  $\Phi$  define (semi-) finite frequency ranges as shown in Tables 1 and 2 and correspond to segments of the imaginary axis (differential processes) or unit circle (discrete processes). Moreover, using such frequency partitioning enables specifications to be imposed over frequency ranges relevant to a specific application of repetitive processes. Furthermore, to allow for more flexibility in practical design, frequency specifications in different ranges can be introduced without augmenting the plant with frequency dependent scalings or weights.

To establish the main result of this section, consider the partitioning

$$\begin{aligned} \lambda \in \Lambda(\Phi, 0) &\Leftrightarrow \lambda \in \{\Lambda(\Phi, \Psi_1) \cup \Lambda(\Phi, \Psi_2) \cup \dots \cup \Lambda(\Phi, \Psi_N)\} \\ &\Leftrightarrow \lambda \in \bigcup_{i=1}^N \Lambda(\Phi, \Psi_i), \end{aligned} \quad (22)$$

where  $N \geq 2$ . Then by different choices of  $\Psi$  (i.e.  $\Psi_1, \Psi_2, \dots, \Psi_N$ ) the specific ranges of the complex curves are characterized. Moreover, both continuous-time (frequency variable  $\lambda$  on the imaginary axis) and discrete-time cases ( $\lambda$  on the unit circle) can be treated by a different choice of  $\Phi$ —see (4).

In what follows, the partitioning (22) implies that the frequency variable  $\lambda$  is defined over unions of finite or semi-finite frequency ranges. In particular, for the continuous-time setting

**Table 4** The values of  $\Omega_i$  for differential dynamics

$\Omega_i$	$\Omega_1$	$\Omega_r$	$\Omega_N$
$\omega$	$ \omega  < \omega_1$	$\omega_{r-1} \leq  \omega  \leq \omega_r$	$ \omega  > \omega_{N-1}$
$\Psi$	$\begin{bmatrix} -1 & 0 \\ 0 & \omega_1^2 \end{bmatrix}$	$\begin{bmatrix} -1 & j\omega_{cr} \\ -j\omega_{cr} & -\omega_r\omega_{r-1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -\omega_{N-1}^2 \end{bmatrix}$
$\tau$	-1	-1	1
$\nu$	0	$-j\omega_{cr}$	0
$\varsigma$	$\omega_1^2$	$-\omega_{r-1}\omega_r$	$-\omega_{N-1}^2$

Where  $2 \leq r \leq N-1$  and  $\omega_{cr} = \frac{(\omega_r + \omega_{r-1})}{2}$

**Table 5** The values of  $\Theta_i$  for discrete dynamics

$\Theta_i$	$\Theta_1$	$\Theta_r$	$\Theta_N$
$\theta$	$ \theta  < \theta_1$	$\theta_{r-1} \leq  \theta  \leq \theta_r$	$ \theta  > \theta_{N-1}$
$\Psi$	$\begin{bmatrix} 0 & 1 \\ 1 & -2\cos(\theta_1) \end{bmatrix}$	$\begin{bmatrix} 0 & e^{j\theta_{cr}} \\ e^{-j\theta_{cr}} & -2\cos(\theta_{dr}) \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 2\cos(\theta_{N-1}) \end{bmatrix}$
$\tau$	0	0	0
$\nu$	1	$e^{-j\theta_{cr}}$	-1
$\varsigma$	$-2\cos(\theta_1)$	$-2\cos(\theta_{dr})$	$2\cos(\theta_{N-1})$

Where  $2 \leq r \leq N-1$  and  $\theta_{dr} = \frac{\theta_r - \theta_{r-1}}{2}$ ,  $\theta_{cr} = \frac{\theta_r + \theta_{r-1}}{2}$

and  $\lambda = j\omega$ , (22) imposes the following frequency partitioning

$$\forall \omega \in \Omega \Leftrightarrow \omega \in \bigcup_{i=1}^N \Omega_i, \quad (23)$$

where

$$\Omega_1 = \{\omega : \{|\omega| < \omega_1\}\}, \quad \Omega_r = \{\omega : \{\omega_{r-1} \leq |\omega| \leq \omega_r\}\}, \quad \Omega_N = \{\omega : \{|\omega| > \omega_N\}\}$$

and  $2 \leq r \leq N-1$ . In the discrete-time case,  $\lambda = e^{j\theta}$  and the following partitioning results

$$\forall \theta \in \Theta \Leftrightarrow \theta \in \bigcup_{i=1}^N \Theta_i, \quad (24)$$

where

$$\Theta_1 = \{\theta : \{|\theta| < \theta_1\}\}, \quad \Theta_r = \{\theta : \{\theta_{r-1} \leq |\theta| \leq \theta_r\}\}, \quad \Theta_N = \{\theta : \{|\theta| > \theta_N\}\}$$

and  $2 \leq r \leq N-1$ .  $\Omega_i$  and  $\Theta_i$  are the frequency ranges given below in Tables 4 and 5.

Each matrix  $\Omega_i$  or  $\Theta_i$  is of dimension  $2 \times 2$  and consists of three different elements (scalars) which depend on the chosen frequency interval. In particular, the following partitioning of  $\Psi$  is enforced

$$\Psi = \begin{bmatrix} \tau & \nu^* \\ \nu & \varsigma \end{bmatrix}.$$

The values of  $\tau$ ,  $\nu$  and  $\varsigma$  for specified choices of frequency intervals are also shown in Tables 4 and 5. Also, the application of piecewise constant matrices  $P_{1i}$ ,  $P_{2i}$  and  $Q_i \forall i = 1, \dots, N$ ,

is possible and hence the LMI feasibility problem of the next theorem is a novel condition for stability along the pass.

**Theorem 4** Let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$  and  $\Phi$  chosen as in Table 3. Also, suppose that the entire frequency range is arbitrarily divided into  $N$  different frequency intervals as given in (22)–(24) and the matrix  $\Psi$  chosen as in Tables 4 or 5. Then a linear repetitive process described by (8) is stable along the pass if there exist  $P_{1i} > 0$ ,  $P_{2i} > 0$ ,  $Q_i > 0$ ,  $W_{1i}$ ,  $W_{2i}$ ,  $W_{3i}$ ,  $F_{1i}$ ,  $F_{2i}$ ,  $F_{3i}$  such that the LMIs

$$\begin{bmatrix} \gamma_{1i} - \text{sym}\{W_{1i}\} & (\star) & (\star) \\ \gamma_{3i} + \mathbb{A}W_{1i}^T - W_{2i} & \gamma_{2i} + \text{sym}\{W_{2i}\mathbb{A}^T\} & (\star) \\ F_{30i} - W_{3i} & -F_{12i}^T + W_{3i}\mathbb{A}^T & P_{2i} - \text{sym}\{F_{3i}\} \end{bmatrix} < 0 \quad (25)$$

are feasible for  $i = 1, \dots, N$ , where

$$\gamma_{1i} = \begin{bmatrix} \tau Q_i + \phi_1 P_{1i} & 0 \\ 0 & 0 \end{bmatrix}, \quad \gamma_{2i} = \begin{bmatrix} \varsigma Q_i + \phi_3 P_{1i} & 0 \\ 0 & -\gamma^2 P_{2i} \end{bmatrix}, \quad \gamma_{3i} = \begin{bmatrix} \phi_2 P_{1i} + \nu Q_i & F_{1i} \\ 0 & F_{2i} \end{bmatrix} \quad (26)$$

$$F_{12i} = \begin{bmatrix} F_{1i} \\ F_{2i} \end{bmatrix}, \quad F_{30i} = \begin{bmatrix} 0 & F_{3i} \end{bmatrix}.$$

**Proof** Suppose that there exist matrices  $P_{1i} > 0$ ,  $P_{2i} > 0$  and  $Q_i > 0$  for  $i = 1, \dots, N$ , such that the LMIs (25) hold for  $i = 1, \dots, N$ . Then by simple matrix manipulations on each LMI in (25) it follows that

$$\begin{bmatrix} A & I \\ C & 0 \end{bmatrix} (\Phi^* \otimes P_{1i} + \Psi^* \otimes Q_i) \begin{bmatrix} A & I \\ C & 0 \end{bmatrix}^T + \begin{bmatrix} B_0 & 0 \\ D_0 & I \end{bmatrix} (\Pi \otimes P_{2i}) \begin{bmatrix} B_0 & 0 \\ D_0 & I \end{bmatrix}^T < 0. \quad (27)$$

Finally, the proof is completed applying Lemma 2 to each frequency interval with the choice of piecewise constant matrices  $P_{1i} > 0$ ,  $P_{2i} > 0$ ,  $Q_i > 0$ ,  $\forall i = 1, \dots, N$ .  $\square$

By analogy with the derivation of Theorem 3, the above result can be directly extended to the case of controlled processes (18) where the control law has the structure given in (17). Therefore, with the notation used above, the following result extends Theorem 4 to the controlled process.

**Theorem 5** Let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$ ,  $\Phi$  is as in Table 3 and  $\Psi$  chosen as in Tables 4 or 5. Suppose also that the control law (17) is applied to a repetitive process described by (8) and the entire frequency range is arbitrarily divided into  $N$  different frequency intervals as given in (22). Then the controlled process is stable along the pass if there exist matrices  $P_{1i} > 0$ ,  $P_{2i} > 0$ ,  $Q_i > 0$ ,  $F_{1i}$ ,  $F_{2i}$ ,  $F_{3i}$ ,  $W_1$ ,  $Y$  such that the LMIs

$$\begin{bmatrix} \gamma_{1i} - \text{sym}\{W_1\} & (\star) & (\star) \\ \gamma_{3i} + \mathbb{A}W_1^T + \mathbb{B}Y - W_1 & \gamma_{2i} + \text{sym}\{\mathbb{A}W_1^T + \mathbb{B}Y\} & (\star) \\ F_{30i} - [0 \ I]W_1 & -F_{12i}^T + [0 \ I](\mathbb{A}W_1^T + \mathbb{B}Y)^T & P_{2i} - \text{sym}\{F_{3i}\} \end{bmatrix} < 0 \quad (28)$$

are feasible for  $i = 1, \dots, N$ . In addition, if these LMIs are feasible, the required control law matrices  $K_1$  and  $K_2$  of (17) can be calculated using (21).

**Proof** The result follows from straightforward application of the same steps as in proof of Theorem 3 where the matrix variables  $P_1$ ,  $P_2$ ,  $F_1$ ,  $F_2$ ,  $F_3$  are directly replaced by their frequency range dependent form  $P_{1i}$ ,  $P_{2i}$ ,  $F_{1i}$ ,  $F_{2i}$ ,  $F_{3i}$  and hence details are omitted.  $\square$

**Remark 6** These last two results enable different performance specifications to be imposed in different frequency ranges. In particular, it is reasonable from the practical point of view to impose different control performance by choosing different process gains in specified frequency ranges, e.g., to ensure that  $G(\lambda)$  meets the specification

$$\rho(G(\lambda)) < \gamma_i, \forall \lambda \in \bigcup_{i=1}^N \Lambda(\Phi, \Psi_i), \quad (29)$$

where  $\gamma_i \in (0, 1] \forall i = 1, \dots, N$ .

## 5 Robustness

This section further extends the results obtained so far to processes with uncertain model data, i.e., uncertain parameters are present in the process matrices. In the following, we will consider two types of parameter uncertainties: norm-bounded and polytopic uncertainties.

### 5.1 Norm-bounded uncertainty case

It is assumed that the deviation of the process parameters from their nominal values is norm-bounded. Specifically, the uncertainty associated with the state-space model (8) is of the form

$$\begin{bmatrix} A + \Delta A(t) & B_0 + \Delta B_0(t) & B + \Delta B(t) \\ C + \Delta C(t) & D_0 + \Delta D_0(t) & D + \Delta D(t) \end{bmatrix} = \begin{bmatrix} A + E_1 \delta(t) H_1 & B_0 + E_1 \delta(t) H_2 & B + E_1 \delta(t) H_3 \\ C + E_2 \delta(t) H_1 & D_0 + E_2 \delta(t) H_2 & D + E_2 \delta(t) H_3 \end{bmatrix}, \quad (30)$$

where the matrices  $A, B, B_0, C, D$  and  $D_0$  represent the nominal process dynamics of (8) and are assumed to be time and pass number invariant. The matrices  $\Delta A(t), \Delta B(t), \Delta B_0(t), \Delta C(t), \Delta D(t)$  and  $\Delta D_0(t)$  denote time-varying uncertainties where  $E_1, E_2, H_1, H_2$  and  $H_3$  are given real constant matrices with appropriate dimensions and  $\delta(t)$  is an unknown and time-varying perturbation satisfying  $\delta^T(t) \delta(t) \preceq I, \forall t \in [0, \alpha]$ .

The proofs of the results in this section make use of the following result.

**Lemma 4** (Petersen 1987) *Given matrices  $\mathcal{X}, \mathcal{Y}, \mathcal{Z} = \mathcal{Z}^T, \delta(t)$  of compatible dimensions, then*

$$\mathcal{Z} + \text{sym}\{\mathcal{X} \delta(t) \mathcal{Y}\} < 0$$

*for all  $\delta(t)$  satisfying  $\delta^T(t) \delta(t) \preceq I$  if, and only if, there exists  $\varepsilon > 0$  such that*

$$\mathcal{Z} + \varepsilon \mathcal{X} \mathcal{X}^T + \varepsilon^{-1} \mathcal{Y}^T \mathcal{Y} < 0.$$

Introduce the notation

$$W_i = \begin{bmatrix} W_{1i} \\ W_{2i} \\ W_{3i} \end{bmatrix}, \quad \gamma_{4i} = \begin{bmatrix} \gamma_{1i} - \text{sym}\{W_{1i}\} & (\star) & (\star) \\ \gamma_{3i} + \mathbb{A}W_{1i}^T - W_{2i} & \gamma_{2i} + \text{sym}\{W_{2i}\mathbb{A}^T\} & (\star) \\ F_{30i} - W_{3i} & -F_{12i}^T + W_{3i}\mathbb{A}^T & P_{2i} - \text{sym}\{F_{3i}\} \end{bmatrix},$$

$$\gamma_{5i} = \begin{bmatrix} \gamma_{1i} - \text{sym}\{W_{1i}\} & (\star) & (\star) \\ \gamma_{3i} + \mathbb{A}W_{1i}^T + \mathbb{B}Y - W_{1i} & \gamma_{2i} + \text{sym}\{\mathbb{A}W_{1i}^T + \mathbb{B}Y\} & (\star) \\ F_{30i} - [0 \ I]W_{1i} & -F_{12i}^T + [0 \ I](\mathbb{A}W_{1i}^T + \mathbb{B}Y)^T & P_{2i} - \text{sym}\{F_{3i}\} \end{bmatrix}.$$

Then, based on Theorem 4, the following results are obtained for stability along the pass in the presence of the parametric uncertainties in the state-space model matrices described by (30).

**Theorem 6** Consider an uncertain linear repetitive process described by the version of (8) with uncertainty described by (30). Also, let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$ ,  $\Phi$  is as in Table 3 and  $\Psi$  chosen as in Tables 4 or 5. Also, suppose that the entire frequency range is arbitrarily divided into  $N$  different frequency intervals as given in (22)–(24). Then a linear repetitive process described by (8) and (30) is robustly stable along the pass if there exist compatibly dimensioned matrices  $P_{1i} > 0$ ,  $P_{2i} > 0$ ,  $Q_i > 0$ ,  $W_{1i}$ ,  $W_{2i}$ ,  $W_{3i}$ ,  $F_{1i}$ ,  $F_{2i}$ ,  $F_{3i}$  and a positive scalar  $\epsilon_1$  such that the following LMIs

$$\begin{bmatrix} \gamma_{4i} & \epsilon_1 \mathcal{E}_1 & W_i \mathcal{H}_1^T \\ \epsilon_1 \mathcal{E}_1^T & -\epsilon_1 I & 0 \\ \mathcal{H}_1 W_i^T & 0 & -\epsilon_1 I \end{bmatrix} < 0 \quad (31)$$

are feasible for  $i = 1, \dots, N$ , where

$$\mathcal{E}_1 = [0 \ E_1^T \ E_2^T \ 0]^T, \quad \mathcal{H}_1 = [H_1 \ H_2].$$

**Proof** Suppose that the LMIs (31) are feasible for  $i = 1, \dots, N$ . Then application of Schur's complement formula to (31) gives

$$\gamma_{4i} + \epsilon_1 \mathcal{E}_1 \mathcal{E}_1^T + \epsilon_1^{-1} W_i \mathcal{H}_1^T \mathcal{H}_1 W_i^T < 0.$$

Next, for  $i = 1, \dots, N$ , assign  $\mathcal{Z} \leftarrow \gamma_{4i}$ ,  $\mathcal{X} \leftarrow \mathcal{E}_1$ ,  $\mathcal{Y} \leftarrow \mathcal{E}_1 \mathcal{H}_1 W_i^T$  and by Lemma 4 the last inequality is feasible if and only if

$$\gamma_{4i} + \text{sym} \left\{ \mathcal{E}_1 \delta(t) \mathcal{H}_1 W_i^T \right\} < 0.$$

The last inequality is (25) applied to the uncertainty case. Moreover, the LMI of (31) can be obtained by employing the same steps used in the absence of uncertainty and hence a version of (4) applied to the plant with uncertainty is obtained. Finally, by the result of Theorem 3, feasibility of (31) ensures that a linear repetitive process of the form (8) and (30) is robustly stable along the pass.  $\square$

The following result extends Theorem 5 to the case when norm bounded uncertainty is present. This is achieved by applying the result of Lemma 4 to Theorem 5 for uncertain processes.

**Theorem 7** Consider an uncertain linear repetitive process described by the version of (8) with uncertainty of the form (30) present and with the control law (17) applied. Also, let  $\gamma$

be a positive scalar satisfying  $0 < \gamma \leq 1$ ,  $\Phi$  is as in Table 3 and  $\Psi$  chosen as in Tables 4 or 5. Suppose that the entire frequency range is arbitrarily divided into  $N$  different frequency intervals as given in (22)–(24). Then, the controlled process is robustly stable along the pass if there exist  $P_{1i} > 0$ ,  $P_{2i} > 0$ ,  $Q_i > 0$ ,  $F_{1i}$ ,  $F_{2i}$ ,  $F_{3i}$ ,  $Y$  and a positive scalar  $\epsilon_2$  such that the following LMIs

$$\begin{bmatrix} \Upsilon_{5i} & \epsilon_2 \mathcal{E}_1 & \mathcal{W}_1 \mathcal{H}_1^T + \mathcal{Y}_1^T H_3^T \\ \epsilon_2 \mathcal{E}_1^T & -\epsilon_2 I & 0 \\ \mathcal{H}_1 \mathcal{W}_1^T + H_3 \mathcal{Y}_1 & 0 & -\epsilon_2 I \end{bmatrix} < 0 \quad (32)$$

are feasible for  $i = 1, \dots, N$ , where  $\mathcal{E}_1$  and  $\mathcal{H}_1$  are as in (31) and

$$\mathcal{W}_1 = \begin{bmatrix} W_1 \\ W_1 \\ [0 \ I] W_1 \end{bmatrix}, \quad \mathcal{Y}_1 = [Y \ Y \ [0 \ I] Y]$$

If these LMIs are feasible, the corresponding matrices in the control law (17) can be computed using (21).

**Proof** The result follows from a straightforward application the same steps as in proof of Theorem 6 for a controlled process and hence the details are omitted.  $\square$

## 5.2 Polytopic uncertainty case

An alternative way of dealing with uncertain processes is to assume that the repetitive process matrices are not exactly known but reside within a given polytope. Specifically, this means that all process matrices are actually dependent on a real parameter vector  $\xi$  with a polytopic dependency:

$$\begin{bmatrix} A & B_0 & B \\ C & D_0 & D \end{bmatrix} \in \mathcal{P} := \left\{ \begin{bmatrix} A(\xi) & B_0(\xi) & B(\xi) \\ C(\xi) & D_0(\xi) & D(\xi) \end{bmatrix} = \sum_{j=1}^M \xi_j \begin{bmatrix} A_j & B_{0j} & B_j \\ C_j & D_{0j} & D_j \end{bmatrix}, \quad \xi \in \mathcal{D} \right\}, \quad (33)$$

where

$$\mathcal{D} = \left\{ \xi = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_M \end{bmatrix} : \xi_j \geq 0, \sum_{j=1}^M \xi_j = 1 \right\}. \quad (34)$$

It means that the various matrices  $A_j$ ,  $B_{0j}$ ,  $B_j$ ,  $C_j$ ,  $D_{0j}$ ,  $D_j$  define the vertices of the polytope  $\mathcal{P}$  in which the actual process matrices  $A$ ,  $B_0$ ,  $B$ ,  $C$ ,  $D_0$  and  $D$  lie. As known, this clearly removes the requirement for keeping the same matrix variables over the entire parametric uncertainty domain and given frequency range. In particular, some matrix variables can be made parameter-dependent (linearly dependent on  $\xi$ ) due to their appearance free from products with the repetitive process matrices. Simply, we assume that matrix functions  $P_{1i}(\xi)$ ,  $P_{2i}(\xi)$ ,  $Q_i(\xi)$ ,  $F_{1i}(\xi)$ ,  $F_{2i}(\xi)$ ,  $F_{3i}(\xi)$ , which are to be determined, are expected to have the

following forms

$$\begin{aligned} P_{1i}(\xi) &= \sum_{j=1}^M \xi_j P_{1ij}, \quad P_{2i}(\xi) = \sum_{j=1}^M \xi_j P_{2ij}, \quad Q_i(\xi) = \sum_{j=1}^M \xi_j Q_{ij}, \\ F_{1i}(\xi) &= \sum_{j=1}^M \xi_j F_{1ij}, \quad F_{2i}(\xi) = \sum_{j=1}^M \xi_j F_{2ij}, \quad F_{3i}(\xi) = \sum_{j=1}^M \xi_j F_{3ij}. \end{aligned} \quad (35)$$

According to the linear parameter-dependence of the above matrix variables, matrices  $\Upsilon_{1i}$ ,  $\Upsilon_{2i}$ ,  $\Upsilon_{3i}$ ,  $F_{12i}$  and  $F_{30i}$  defined in (26) have the affine form

$$\begin{aligned} \Upsilon_{1i}(\xi) &= \sum_{j=1}^M \xi_j \Upsilon_{1ij}, \quad \Upsilon_{2i}(\xi) = \sum_{j=1}^M \xi_j \Upsilon_{2ij}, \quad \Upsilon_{3i}(\xi) = \sum_{j=1}^M \xi_j \Upsilon_{3ij}, \\ F_{12i}(\xi) &= \sum_{j=1}^M \xi_j \Upsilon_{3ij}, \quad F_{30i}(\xi) = \sum_{j=1}^M \xi_j \Upsilon_{3ij}. \end{aligned} \quad (36)$$

Also, based on (33) the process matrices are parameter dependent and hence the block matrices  $\mathbb{A}$  and  $\mathbb{B}$  defined in (11) and (19) respectively have the affine form

$$\mathbb{A}(\xi) = \sum_{j=1}^M \xi_j \mathbb{A}_j, \quad \mathbb{B}(\xi) = \sum_{j=1}^M \xi_j \mathbb{B}_j, \quad \text{where } \mathbb{A}_j = \begin{bmatrix} A_j & B_{0j} \\ C_j & D_{0j} \end{bmatrix}, \quad \mathbb{B}_j = \begin{bmatrix} B_j \\ D_j \end{bmatrix}. \quad (37)$$

Along the above notation, we have the following linearly parameter-dependent approach to stability along the pass for processes with polytopic uncertainty.

**Theorem 8** Let  $\gamma$  be a positive scalar satisfying  $0 < \gamma \leq 1$  and  $\Phi$  chosen as in Table 3. Also, suppose that the entire frequency range is arbitrarily divided into  $N$  different frequency intervals as given in (22)–(24) and the matrix  $\Psi$  chosen as in Tables 4 or 5. Then an uncertain linear repetitive process with process model matrices described by (33) and (34) is robustly stable along the pass if there exist  $P_{1ij} \succ 0$ ,  $P_{2ij} \succ 0$ ,  $Q_{ij} \succ 0$ ,  $F_{1ij}$ ,  $F_{2ij}$ ,  $F_{3ij}$ ,  $W_{1i}$ ,  $W_{2i}$  and  $W_{3i}$ , such that, the LMIs

$$\begin{bmatrix} \Upsilon_{1ij} - \text{sym}\{W_{1i}\} & (*) & (*) \\ \Upsilon_{3ij} + \mathbb{A}_j W_{1i}^T - W_{2i} & \Upsilon_{2ij} + \text{sym}\left\{W_{2i} \mathbb{A}_j^T\right\} & (*) \\ F_{30ij} - W_{3i} & -F_{12ij} + W_{3i} \mathbb{A}_j^T & P_{2ij} - \text{sym}\{F_{3ij}\} \end{bmatrix} < 0 \quad (38)$$

are feasible for  $i = 1, \dots, N$ , and  $j = 1, \dots, M$ .

**Proof** Assume that the LMIs (38) are feasible for  $i = 1, \dots, N$ , and  $j = 1, \dots, M$ . Then multiply LMIs in (38) by the uncertain parameter  $\xi$  and sum the result to obtain a version of (25) where the affine parameter dependent matrices of (35)–(37) are used. Then application of Theorem 4 implies that the LMIs (38) are feasible for  $j = 1, \dots, M$  and the proof is complete.  $\square$

The following result is a linearly parameter-dependent approach to controller design for processes with polytopic uncertainty.

$$\begin{bmatrix} \gamma_{1ij} - \text{sym}\{W_1\} & (\star) & (\star) \\ \gamma_{3ij} + \mathbb{A}_j W_1^T + \mathbb{B}_j Y - W_1 & \gamma_{2ij} + \text{sym}\{\mathbb{A}_j W_1^T + \mathbb{B}_j Y\} & (\star) \\ F_{30ij} - [0 \ I] W_1 & -F_{12ij}^T + [0 \ I](\mathbb{A}_j W_1^T + \mathbb{B}_j Y)^T & P_{2ij} - \text{sym}\{F_{3ij}\} \end{bmatrix} < 0 \quad (39)$$

**Proof** The proof can be carried out by employing similar lines to that of the proofs of Theorem 5 and 8. Hence the details are omitted.  $\square$

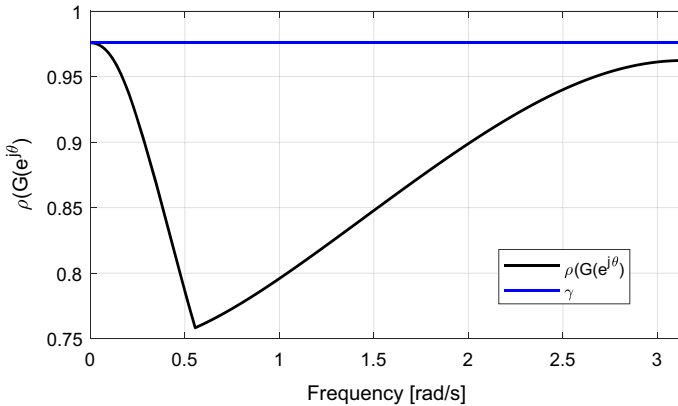
To justify the effectiveness and feasibility of the formulated results, two numerical examples are presented. The first example is formulated to show the reduced conservatism compared with Li et al. (2015) for stability along the pass of discrete repetitive processes. Next, the second example illustrates the results of controller design procedure for differential repetitive processes compared with Rogers et al. (2007) and its extended form in Maniarski et al. (2020).

Let the process matrices in (8) be give as follows (as noted, the below example is adopted from Li et al. (2015))

$$A = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & -0.1 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0.4 & 1.1 \\ 0.6 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} -0.1 & -0.1 \\ -0.2 & 0.6 \end{bmatrix}, \quad D_0 = \begin{bmatrix} -0.5 & -0.5 \\ -0.1 & -0.7 \end{bmatrix}.$$

By testing the stability condition in Li et al. (2015) (formulated in Theorem 1 of Li et al. 2015) over entire frequency range, it follows that the result in Li et al. (2015) does not support feasibility for  $\gamma \leq 1$  and hence fails to decide the stability along the pass of the considered process. Thus, the effectiveness of the proposed condition is clearly demonstrated since our result is less restrictive compared with Li et al. (2015) for entire frequency range. The stability along the pass for the process in Example 1 can be confirmed with Li et al. (2015) only in the cases when the entire frequency range is partitioned uniformly into 4 subintervals or partitioned non-uniformly into 2 subintervals ( $[0, 0.5] \cup [0.5, \pi]$ ). See also Fig. 1.

As an illustrative example of the new results in this paper consider the linearized differential linear repetitive process model that represents the metal rolling process given in Bochniak



**Fig. 1** Plot of spectral radius of  $G(e^{j\theta})$  and its upper bound

et al. (2008) and where the state-space model matrices are

$$A = \begin{bmatrix} 0 & 1 \\ -a_0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ -b_0 + a_0 b_2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ c_0 \end{bmatrix}, C = [1 \ 0], D_0 = -b_2, D = 0$$

and

$$a_0 = \frac{\lambda_1 \lambda_2}{M(\lambda_1 + \lambda_2)}, b_0 = \frac{-\lambda_1 \lambda_2}{M(\lambda_1 + \lambda_2)}, b_2 = \frac{-\lambda_2}{\lambda_1 + \lambda_2}, c_0 = \frac{-\lambda_1}{M(\lambda_1 + \lambda_2)}.$$

Here  $\lambda_1 = 600$  [N/m] is the stiffness of the adjustment mechanism spring,  $\lambda_2 = 2000$  [N/m] is the hardness of the metal strip and  $M = 100$  [kg] denotes the lumped mass of the roll-gap adjusting mechanism. In this case, the nominal state-space model matrices in (8) are

$$\begin{bmatrix} A & B_0 & B \\ C & D_0 & D \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -4.6154 & 0 & 1.0651 \\ 1 & 0 & 0.7692 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Suppose that the parameter  $M$  is uncertain and possibly time-varying, and its values are assumed to be within the interval  $[90, 112.5]$ . Then this case corresponds to the scenario when the matrices  $E_1, E_2, H_1, H_2$  and  $H_3$  are

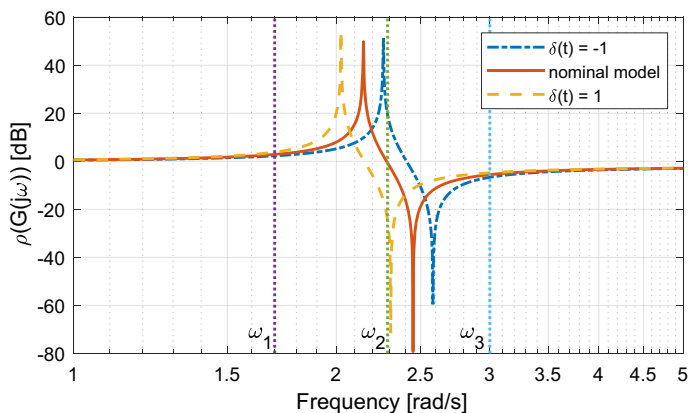
$$E_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, E_2 = 0, H_1 = [0.5128 \ 0], H_2 = -0.1183, H_3 = 0.0003.$$

The plots of  $\rho(G(j\omega))$  given in Fig. 2 show that the nominal and uncertain repetitive processes (for the extremes of the uncertain element ranges, i.e.  $\delta(t) = 1$ ) are unstable along the pass. Clearly, the condition *iii*) of Lemma 3 is not satisfied for these processes and hence the LMIs in (12) (in case of the nominal process) and (31) (in case of the uncertain process) are not feasible for any frequency partitioning.

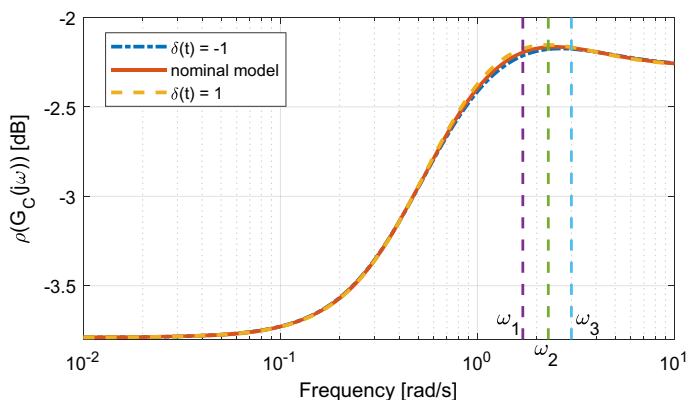
Consider the case when the entire frequency range  $[0, \infty)$  is arbitrarily non-uniformly divided into 4 frequency sub-intervals (given in [rad/s]), which are shown by dashed lines in Figs. 2, 3 and 4

$$[0, 1.7) \cup [1.7, 2.29) \cup [2.29, 3) \cup [3, \infty) = [0, \infty).$$

Hence  $\omega_1 = 1.7$  [rad/s],  $\omega_2 = 2.29$  [rad/s] and  $\omega_3 = 3$  [rad/s]. However, there is no systematic way to determine what is the best choice of the frequency sub-intervals but our intention is to



**Fig. 2** Plot of  $\rho(G(j\omega))$  for the models: nominal and two with the extreme uncertainties



**Fig. 3** Plot of  $\rho(G_C(j\omega))$  for the models: nominal and two with the extreme uncertainties

keep resonance and antiresonance frequencies within separate sub-intervals since different performance (provided by  $\gamma_i$ —see Remark 6) may be achieved in specified frequency ranges (sub-intervals).

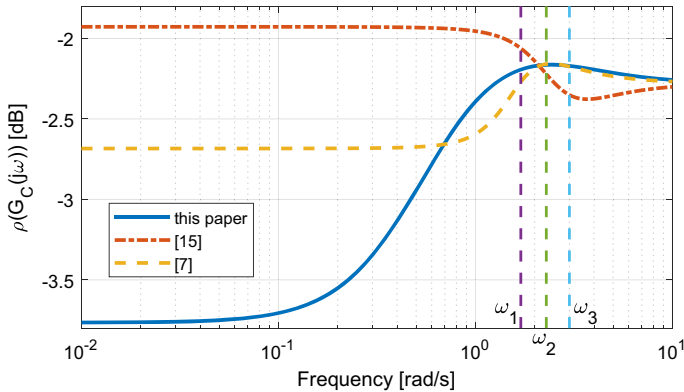
Solving the LMI condition obtained in Theorem 7 for  $\gamma = 1$  and using (21) we obtain the following control law matrices

$$K_1 = 10^3 \cdot [-1.2527 \quad 1.4887], \quad K_2 = 551.9732$$

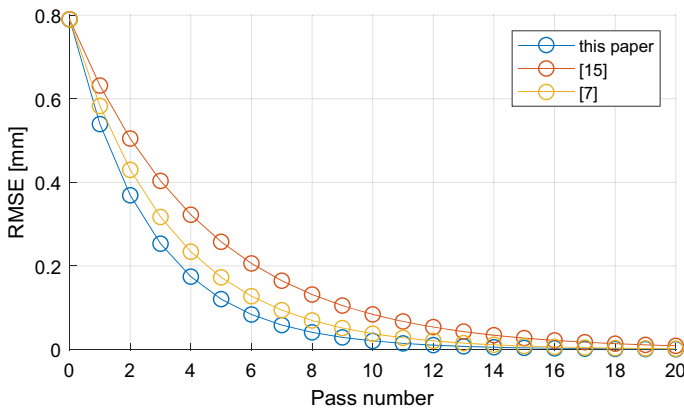
The controlled uncertain process is stable along the pass as seen in Fig. 3 where the spectral radius of the controlled process is below 1 for the entire frequency range, i.e.,  $\rho(G_C(j\omega))$ ,  $\omega \in [0, \infty)$  where

$$G_C(j\omega) = (C + DK_1)(j\omega I - (A + BK_1))^{-1}(B_0 + BK_2) + D_0 + DK_2.$$

Note that the existing results on control law design for uncertain differential linear repetitive process in Rogers et al. (2007) and its extended form Maniarski et al. (2020) are feasible for this example as demonstrated in Fig. 4. However, new design procedure provides better performance for low frequencies.



**Fig. 4** Spectral radius associated with the controlled dynamics for new design procedure compared to those in Rogers et al. (2007) and Maniarski et al. (2020)



**Fig. 5** RMSE values computed from the simulation data

In addition, the controlled dynamics were simulated over 20 passes where the initial pass profile is chosen as  $y_0(t) = \sin(2\pi t/20) + 0.5 \sin(4\pi t/20)$  [mm] for  $0 \leq t \leq 20$ . Also, it was hypothetically assumed that the sheet metal thickness should be finally close to zero, hence the quality of the control over each pass profile can be measured by the RMSE (Root Mean Squared Error — computed along each pass) and shown in Fig. 5. This also confirms robust stability along the pass of the controlled process and better performance obtained by the new stabilization design since the convergence rate is higher than the alternatives.

## 7 Conclusions and future works

This paper has developed new results on the stability and stabilization for linear repetitive processes. Both discrete and differential dynamics of a given repetitive process have been considered and novel conditions for stability along the pass and control law design have been obtained. The major benefit of these new results is that stability along the pass tests can be directly extended to the control law design algorithms for nominal and uncertain models. Also,

the advantage over current results in this last aspect is the avoidance of product terms between the state-space model matrices and some Lyapunov/LMI decision matrices. This decoupling has been achieved through the use of slack matrix variables and therefore a reduction in the conservatism of the analysis and design tests is possible. Finally, the numerical examples to demonstrate the effectiveness of the new results have been given.

Future research should include a detailed investigation into the  $\mathcal{H}_\infty$  robust performance in the presence of external bounded non-repetitive disturbances in both the state and pass profile vectors. Another area is design of the more complex controller structures, e.g. a dynamic controller, and their experimental validation. Moreover, it is worth noting that reducing the conservatism of conditions for stability along the pass recently gained increasing attention, and significant progress has been made based on interpreting the stability along the pass of a linear repetitive process as structural stability of 2D Roesser model. Therefore, it is also promising to develop robust control results by extending the methods of Bachelier et al. (2019).

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