An efficient evaluation method for wing fuel mass variations effect on transonic aeroelasticity

Journal:	*Aircraft Engineering and Aerospace Technology*
Manuscript ID	AEAT-08-2021-0227.R1
Manuscript Type:	Research Paper
Keywords:	Efficient aeroelastic evaluation method, CFD-based POD/ROM, Wing fuel mass variations, Transonic aeroelasticity

An efficient evaluation method for wing fuel mass variations effect on transonic aeroelasticity

Abstract

Purpose – This paper aims to develop an efficient evaluation method to more intuitively and effectively investigate the influence of the wing fuel mass variations due to fuel burn on transonic aeroelasticity.

Design/methodology/approach – The proposed efficient aeroelastic evaluation method is developed by extending the standard CFD-based POD/ROM.

Findings – The results show that the proposed aeroelastic efficient evaluation method can accurately and efficiently predict the aeroelastic response and flutter boundary when the wing fuel mass variations due to fuel burn. It also show that the wing fuel mass variations have a significant effect on transonic aeroelasticy, the flutter speed increases as the wing fuel mass decreases. Without rebuilding an expensive, time-consuming CFD-based POD/ROM for each wing fuel mass variation, the computational cost of the proposed method is reduced obviously. It also show that the computational efficiency improvement is grows linearly with the number of model cases.

Practical implications – The work presents a potential powerful tool to more intuitively and effectively investigate the influence of the wing fuel mass variation on transonic aeroelasticity and the results are a theoretical and methodological basis for further research.

Originality/value – The proposed evaluation method make it a reality to apply the efficient standard CFD-based POD/ROM to investigate the influence of the wing fuel mass variation due to fuel burn on transonic aeroelasticity. The proposed efficient aeroelastic evaluation method, therefore, is ideally suited to deal with the investigation of the influence of wing fuel mass variations on transonic aeroelasticity, and may have the potential to reduce of overall cost of aircraft design.

Keywords Efficient aeroelastic evaluation method; CFD-based POD/ROM; Wing fuel mass variations; Transonic aeroelasticity.

Paper type Research paper

Introduction

Aeroelasticity is a critical aspect of modern aircraft design. For aircraft design, the influences of all any possible aerodynamic and structural variations on aeroelasticity have to be assessed. The wing fuel mass account for a large part of the total mass of the wing (Elham, 2010). The wing mass will change greatly with the consumption of fuel during the flight of the aircraft (Balis Crema *et al.*, 1996), which produces a variation of the structural mode shapes and frequencies, and may affect the aeroelasticity of the aircraft (Sewall, 1957). Therefore, it is necessary to develop an efficient evaluation method that presents the ability to investigate the influence of wing fuel mass variations due to fuel burn on the aeroelasticity (Bhatia *et al.*, 2010).

In the past few years, many scholars have done some research on the influence of wing fuel mass variations on the aeroelasticity. Heinze (Heinze, 2007) used μ-κ method to assess the influence of wing fuel variations on the flutter boundary in subsonic regime. It was shown that the variation of the structural mode shapes due to fuel mass variation play a significant role in flutter speed of the aircraft. Wildschek (Wildschek *et al.*, 2014) proposed a Multi-Input Multi-Output (MIMO) adaptive feed-forward controller for alleviation of turbulence-induced rigid body motions and structural vibrations on aircraft in subsonic regime, that the controller takes into account the wing fuel mass variations during flight. Allen (Allen *et al.*, 2015) applied linear doublet lattice model to compute aerodynamic forces described the Finite Element Method (FEM) based aeroelastic analysis of the full scale aircraft in subsonic regime, and five different fuel mass configurations were included in the flutter analysis. Matas (Garcia Matas *et al.*, 2019) investigated the influence of structural non-linearities on worst-case gust load predictions. The fuel tank was included in the model with a lumped mass in a central node of the wing. Voß (Voß, 2019) presented the results of dynamic "one-minus-cosine" gust load simulations for a flying wing configuration in subsonic regime. Nine different mass configurations ranging from no fuel to full fuel are used to reflect different phases of flight during the mission. The fuel mass is seen as several lumped masses distributed on the wing. However, these studies only consider the influence of wing fuel mass variations due to fuel burn on subsonic aeroelasticity, and ignore the influence on transonic aeroelasticity.

To achieve high efficiency and take advantage of the high-performance modern aircrafts, modern aircrafts often cruise in transonic regime. Due to the presence of flow nonlinearities (e.g. shocks, separation) of transonic flow, high-efficiency linear aerodynamic theories fail. Computational fluid dynamics (CFD) is a feasible alternative method to model these flow nonlinearities (Alder, 2015), but the aeroelastic system requires a number of heavy CFD calculations. The use of CFD-based reduced order models (ROMs) to extract key data of the fluid systems to model a low dimensional system in aeroelasticity therefore receives high interest from the research community within recent years. It can be maintained similar accuracy of the full order model, at the same time, the computational cost is significantly reduced (Z. Chen *et al.*, 2019; Kou *et al.*, 2019). System identification (Cowan *et al.*, 1999; Torii *et al.*, 2001; Zhang *et al.*, 2019), proper orthogonal decomposition (POD) (Dowell *et al.*, 2001; Lucia *et al.*, 2004) and harmonic balance method (Dimitriadis, 2008; Thomas *et al.*, 2004; Woodgate *et al.*, 2009) are among the most popular ROMs to modeling nonlinear aeroelastic system. The POD method, in particular, has been successfully applied to transonic aeroelastic analysis (Zhou *et al.*, 2014), active aeroelastic control (G. Chen *et al.*, 2014), LCO control (G. Chen *et al.*, 2010) and gust response analysis (Bekemeyer *et al.*, 2017).

Although CFD-based ROMs have obvious advantages, it have one serious shortcoming. Most of the studies CFD-based ROMs cannot account for any changes in the system configuration and only suitable for a frozen flight conditions (e.g. Mach number, angle of attack) or model configuration (e.g. mass, geometry). During flight, the wing mass variation due to fuel burn, the wing structural model should be updated and recalculated. In CFD-based ROMs aeroelastic analysis, the influence of variation in the structural will also affect the solution of fluid model (Hayes *et al.*, 2014), thus, it is required to construct a new CFD-based ROM (Xie *et al.*, 2019). Therefore, for every wing fuel mass variation of the structural model configuration, CFD-based ROM should be reconstructed, which destroy the computational efficiency of the CFD-based ROM. Combining all possible variations, the number of configurations increases rapidly and it requires a great number of computational cost. To overcome the related computational expense about reconstructing a new CFD-based ROM, some researchers have been trying to make up the shortcoming of traditional CFD-based ROMs. Zhang (Zhang *et al.*, 2015) and Winter (Winter *et al.*, 2017) demonstrated a method based on original modal shapes to get new mode shapes using radial basis function (RBF) that replace the CFD solver used existing CFD-based ROM when the local boundary condition and lumped mass changed. Li (Li *et al.*, 2019) presented an unsteady aerodynamic model based on LSTM (long short-term memory) network from deep learning theory, and the model can accurately capture the dynamic characteristics of aerodynamic and aeroelastic systems for varying flow and structural parameters. However, these studies only consider structural modification at local level such as local boundary condition and lumped mass, and neglect structural modification at global level, such as wing fuel mass variations due to fuel burn. Therefore, it is difficult to intuitively reflect the influence of wing fuel mass variations on transonic aeroelasticity.

With the previous paragraphs as background, in order to more intuitively and effectively investigate the influence of the wing fuel mass variations due to fuel burn on transonic aeroelasticity. In this paper, we propose an efficient aeroelastic evaluation method by extending standard CFD-based POD/ROM. The evaluation method builds on various elements, which are described and validated throughout the manuscript. The method consists of: a) a proper orthogonal decomposition of the linearized Euler equations, calculated around a mean flow solution (equilibrium position), in turn, dependent upon the structural model; b) a structural dynamic reanalysis method that allows calculating the modal characteristics of the modified structure for each wing fuel mass variation, instead of performing full structural modal analysis; and c) a efficient aeroelastic evaluation method without need for calculating an eigenvalue problem of the modified structure for each wing fuel mass variation, or a new set of CFD-based POD/ROM. The proposed method has the potential to be a more powerful and more intuitive tool to reflect the influence of wing fuel mass variations on transonic aeroelasticity and the results are a theoretical and methodological basis for further research, i.e., active control law design considering wing fuel mass variations. And may have the potential to reduce of overall cost of aircraft design.

Efficient aeroelastic evaluation method

Details of the proposed efficient aeroelastic evaluation method for wing fuel mass variations effect on transonic aeroelasticity is described in this section.

Flow and structural solvers

The nonlinear transonic aeroelastic system is formulated using the two-field arbitrary Lagrangian-Eulerian (ALE) approach. The governing equations are

$$\begin{cases} \dfrac{d(\mathbf{A}\cdot\mathbf{w})}{dt} + \mathbf{F}(\mathbf{w}) = \mathbf{0} \\ \mathbf{M}\ddot{\mathbf{d}} + \mathbf{D}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{f} \end{cases} \tag{1}$$

It represents a finite volume discretization of the ALE non-dimensional conservative form of the Euler equations and a finite element discretization of the structural dynamic equations. Here, **A** is a diagonal matrix containing the cell volumes, **w** is the vector of conservative flow variables and **F** is the nonlinear numerical flux function. **M**, **D**, and **K** are the mass, damping, and stiffness matrices, respectively. The vector of structural displacements is denoted by **d**. **f** is the vector of aerodynamic loads calculated at the structural grid points. The CFD solver employs Cartesian grids, using a multi-block structured cell-centered finite volume discretisation, and the second-order Van Leer scheme (Van Leer, 1979) is used for the spatial discretization. The dual time-stepping (Pulliam, 1993) and Lower-Upper Symmetric Gauss-Seidel (LU-SGS) implicit method (R. Chen *et al.*, 2000) are used for time integration.

CFD-based POD/ROM for Aeroelastic System

The POD is a method that can provide a compact description of large scale computational models such as CFD-based aeroelastic analysis (Hall *et al.*, 2000; Lee *et al.*, 2019). Here, POD is employed to construct a CFD-based ROM for Euler equations of transonic aeroelastic analysis, attention is limited to inviscid compressible flows. However, this method is also applicable to the Navier-Stokes equations. The unsteady flow equations are linearized around a mean solution (equilibrium). The vector of structural generalized displacements is denoted by **u**. $\Delta\mathbf{w}$, $\Delta\mathbf{u}$, $\Delta\dot{\mathbf{u}}$ are small perturbations around the equilibrium $\mathbf{w}_0$, $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, respectively. One obtains the linearized flow equations:

$$\mathbf{A}_0\dot{\mathbf{w}} + \mathbf{H}\mathbf{w} + (\mathbf{E}+\mathbf{C})\dot{\mathbf{u}} + \mathbf{G}\mathbf{u} = \mathbf{0} \tag{2}$$

where

$$\mathbf{H} = \frac{\partial\mathbf{F}}{\partial\mathbf{w}}(\mathbf{w}_0,\mathbf{u}_0,\mathbf{u}_{c0}) \qquad \mathbf{G} = \frac{\partial\mathbf{F}}{\partial\mathbf{u}}(\mathbf{w}_0,\mathbf{u}_0,\mathbf{u}_{c0})$$

$$\mathbf{E} = \frac{\partial\mathbf{A}}{\partial\mathbf{u}}\mathbf{w}_0 \qquad \mathbf{C} = \frac{\partial\mathbf{F}}{\partial\dot{\mathbf{u}}}(\mathbf{w}_0,\mathbf{u}_0,\mathbf{u}_{c0})$$

Here, Matrix **H** is the gradient of the numerical flux function with respect to the vector of fluid variables. Matrices **G** and **C** are the gradients of the flux function with respect to the generalized coordinates and their velocities, respectively. Matrix **E** indicates the gradient of the cell volumes with respect to the generalized coordinates. Note that the matrices **G**, **E** and **C** need to be updated if the structural parameters are changed.

The linearization of the structural dynamic equation around the equilibrium state can be written as follows:

$$\bar{\mathbf{M}}\ddot{\mathbf{u}} + \bar{\mathbf{D}}_0\dot{\mathbf{u}} + \bar{\mathbf{K}}_s\mathbf{u} = \mathbf{P}_0\mathbf{w} \tag{3}$$

where

$$\bar{\mathbf{K}}_0 = \frac{\partial \mathbf{f}^{int}}{\partial \mathbf{u}}(\mathbf{u}_0, \dot{\mathbf{u}}_0) \qquad \bar{\mathbf{K}}_s = \bar{\mathbf{K}}_0 - \frac{\partial \mathbf{f}^{ext}}{\partial \mathbf{u}}(\mathbf{w}_0, \mathbf{u}_0)$$

$$\bar{\mathbf{D}}_0 = \frac{\partial \mathbf{f}^{int}}{\partial \dot{\mathbf{u}}}(\mathbf{u}_0, \dot{\mathbf{u}}_0) \qquad \mathbf{P}_0 = \frac{\partial \mathbf{f}^{ext}}{\partial \mathbf{w}}(\mathbf{w}_0, \mathbf{u}_0)$$

For the analysis of the system stability, the terms $\frac{\partial \mathbf{f}^{ext}}{\partial \mathbf{u}}$ and $\bar{\mathbf{D}}_0$ can be neglected, which is studied as an eigenvalue problem:

$$\bar{\mathbf{M}}\ddot{\mathbf{u}} + \bar{\mathbf{K}}_0 \mathbf{u} = \mathbf{P}_0 \mathbf{w} \tag{4}$$

A ROM of the unsteady flow is considered in this work, based on the POD technique. Denote $\{\mathbf{x}^k\}$, $k = 1,2,3\ldots m$, a set of date, with $\mathbf{x}^k$ is the n-dimensional space $\mathbf{\Omega} \in \mathbf{R}^{n\times n}$, and m is the number of snapshots. The POD method searches an m-dimensional proper $\{\mathbf{x}^k\}$ orthogonal subspace, $\mathbf{\Psi} \in \mathbf{R}^{n\times m}$, to minimize the mapping errors from $\mathbf{\Psi}$:

$$\mathbf{G} = \min_{\mathbf{\Omega}} \sum_{k=1}^{m} \left\| \mathbf{x}^k - \mathbf{\Omega}\mathbf{\Omega}^T \mathbf{x}^k \right\| = \sum_{k=1}^{m} \left\| \mathbf{x}^k - \mathbf{\Psi}\mathbf{\Psi}^T \mathbf{x}^k \right\| \tag{5}$$

The minimization problem is equivalent to:

$$\mathbf{H} = \max_{\mathbf{\Omega}} \sum_{k=1}^{m} \frac{\left\langle \left(\mathbf{x}^k, \mathbf{\Omega}\right)^2 \right\rangle}{\left\|\mathbf{\Omega}\right\|^2} = \sum_{k=1}^{m} \frac{\left\langle \left(\mathbf{x}^k, \mathbf{\Psi}\right)^2 \right\rangle}{\left\|\mathbf{\Psi}\right\|^2}, \quad \mathbf{\Omega}^T\mathbf{\Omega} = \mathbf{I} \tag{6}$$

The constraint optimization problem in Eq. (6) is transformed into the following Lagrange equation

$$J(\mathbf{\Omega}) = \sum_{k=1}^{m} \left(\mathbf{x}^k, \mathbf{\Omega}\right)^2 - \xi\left(\left\|\mathbf{\Omega}\right\|^2 - 1\right) \tag{7}$$

Solving the partial derivative of the objective function $J(\mathbf{\Omega})$ with respect to $\mathbf{\Omega}$ gives

$$\frac{d}{d\mathbf{\Omega}} J(\mathbf{\Omega}) = 2\mathbf{X}\mathbf{X}^H \mathbf{\Omega} - 2\xi\mathbf{\Omega} \tag{8}$$

where $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots \mathbf{x}_m\} \in \mathbf{R}^{n\times m}$ is a matrix containing m snapshots as columns. By setting Eq. (8) to zero; thus, the following equation is obtained:

$$\left(\mathbf{X}\mathbf{X}^T - \xi\mathbf{I}\right)\mathbf{\Psi} = 0 \tag{9}$$

The problem is transformed into solving the eigenvalue problem of the POD kernel, $\mathbf{X}\mathbf{X}^T$. The eigenvalue problem has very large size, as $\mathbf{X}\mathbf{X}^T \in \mathbf{R}^{n\times n}$. Because $\mathbf{X}\mathbf{X}^T$ and $\mathbf{X}^T\mathbf{X}$ have the same eigenvalues, so we can obtain $\mathbf{\Psi}$ as following equation:

$$\begin{cases} \mathbf{X}^T\mathbf{X}\mathbf{V} = \mathbf{V}\mathbf{\Lambda} \\ \mathbf{\Psi} = \mathbf{X}\mathbf{V}\mathbf{\Lambda}^{-1/2} \end{cases} \tag{10}$$

where $\mathbf{\Psi} = [\mathbf{\psi}_1, \mathbf{\psi}_2, \ldots, \mathbf{\psi}_m]$, $\mathbf{\Lambda} = diag(\xi_1, \xi_2 \cdots \xi_m)$, $\xi_1 \geq \xi_2 \geq, \ldots, \geq \xi_m$. The value of ξ_i represents the contribution of the *i*-th snapshot to the original system. To build an aeroelastic ROM, it is typically possible to retain the first *r*-order POD modes $\mathbf{\Psi} = [\mathbf{\psi}_1, \mathbf{\psi}_2, \ldots, \mathbf{\psi}_m]$ while retaining most of the energy of the original system. By projecting the full-order series $\mathbf{x}^{n\times 1}$ on the r-order POD modes $\mathbf{\Psi}=[\boldsymbol{\psi}_1,\boldsymbol{\psi}_2,\ldots,\boldsymbol{\psi}_m]$, we can reduced the full order system to a reduced r-order system

$$\begin{cases}\dot{\mathbf{x}}_r=\mathbf{\Psi}_r^T\mathbf{A}\mathbf{\Psi}_r\mathbf{x}_r+\mathbf{\Psi}_r^T\mathbf{B}\mathbf{y}\\ \mathbf{f}^{ext}=\mathbf{C}\mathbf{\Psi}_r\mathbf{x}_r\end{cases} \tag{11}$$

where $\mathbf{A}=-\mathbf{A}_0^{-1}\mathbf{H}$, $\mathbf{B}=-\mathbf{A}_0^{-1}\left[\mathbf{E}+\mathbf{C}\ \ \mathbf{G}\right]$, $\mathbf{y}=\left[\dot{\mathbf{u}}\ \mathbf{u}\right]^T$, $\mathbf{C}=\mathbf{P}$. Here, **A**, **B**, and **y** can be considered as the system inputs, and the $\mathbf{f}^{ext}$ is the system output. The above steps outline the process of CFD-based POD/ROM for a frozen aeroelastic model configuration. The resulting aeroelastic model is obtained by coupling the structural dynamic equations, Eq. (4), with Eq. (11) for the ROM of the fluid.

When wing fuel mass variations due to fuel burn, the mode shapes and frequencies may also be changed. Hence, in order to produce an accurate result, both structure model and CFD-based POD/ROM have to be recalculated, which is called exact model herewith. Furthermore, these reconstructions have to be repeated every time whenever wing fuel mass variation, which destroy the computational efficiency of the CFD-based POD/ROM and could not be applied directly in aeroelastic evaluation process.

Structural dynamic reanalysis method

In many structural optimization problems, it is important to repeatedly calculate the modal characteristics of the structure. For large scale problems, the multiple repeated modal analysis usually require considerable time and effort. Then, several structural dynamic reanalysis methods were proposed to reduce the computational cost and effort of the modal reanalysis (Song *et al.*, 2014). Generally, these reanalysis methods are classified into two categories: direct methods (Huang *et al.*, 2016) and approximate methods (Kirsch *et al.*, 2007). Direct methods are applicable for large but local (or low-rank) modifications, and not suitable for global modifications. Direct methods also suffer from high computational costs. On the contrary, the advantage of approximate methods is a significant reduction of the computational costs, and the applicability is extended for global (or high-rank) modifications of the structures (Song et al., 2014), in which the mode shapes of modified structure are approximated as a linear combination of basic mode shapes. The extended Kirsch combined method (S. Chen *et al.*, 2000) is an efficient approach for the case of large modifications of the structural parameters. More details relevant to this work are given in the remainder of this section.

Consider an original structure with stiffness matrix $\mathbf{K}_0$ and mass matrix $\mathbf{M}_0$. The corresponding mode shapes $\boldsymbol{\phi}_0^i$ and modal frequencies λ_0^i are calculated by solving the set of initial analysis equations:

$$\mathbf{K}_0\boldsymbol{\phi}_0^i=\lambda_0^i\mathbf{M}_0\boldsymbol{\phi}_0^i \quad i=1,\ 2\ldots\ldots\ number\ of\ modes \tag{12}$$

In the investigation of the influence of wing fuel mass variation due to fuel burn on the transonic aeroelasticity, the wing fuel mass variations have no effect on the wing stiffness. When wing fuel mass variations due to fuel burn, $\mathbf{M}_0$ is perturbed into the form $\mathbf{M}_0 + \Delta\mathbf{M}$, in which $\Delta\mathbf{M}$ is the perturbations to the original mass matrices. The eigenvalue problem becomes:

$$\mathbf{K}_0\boldsymbol{\phi}^i = \lambda^i \mathbf{M}\boldsymbol{\phi}^i \tag{13}$$

where λ^i and $\boldsymbol{\phi}^i$ are i-th eigenvalue and eigenvector, respectively.

Extended Kirsch combined method use the second-order eigenvector terms as the basis vectors in the following modal shape reduced basis:

$$\boldsymbol{\phi}^i = \boldsymbol{\phi}_B^i \mathbf{z}^i, \quad \mathbf{z}^i = (\mathrm{z}_0^i,\ \mathrm{z}_1^i,\ \mathrm{z}_2^i)^T \in \mathbf{R}^{3\times 1} \tag{14}$$

where

$$\boldsymbol{\phi}_B^i = \left[\boldsymbol{\phi}_0^i,\ \boldsymbol{\phi}_1^i,\ \boldsymbol{\phi}_2^i\right] \tag{15}$$

$$\lambda_1^i = -\lambda_0^i(\boldsymbol{\phi}_0^i)^T \Delta\mathbf{M}\boldsymbol{\phi}_0^i \tag{16}$$

$$\boldsymbol{\phi}_1^i = \sum_{s=1, s\neq i}^{n} \frac{1}{\lambda_0^i - \lambda_0^s}[-\lambda_0^i(\boldsymbol{\phi}_0^s)^T \Delta\mathbf{M}\boldsymbol{\phi}_0^i]\boldsymbol{\phi}_0^s - \frac{1}{2}[(\boldsymbol{\phi}_0^i)^T \Delta\mathbf{M}\boldsymbol{\phi}_0^i]\boldsymbol{\phi}_0^i = \boldsymbol{\phi}_0\mathbf{Z}_1^i \tag{17}$$

$$\lambda_2^i = -\lambda_0^i(\boldsymbol{\phi}_0^i)^T \Delta\mathbf{M}\boldsymbol{\phi}_1^i - \lambda_1^i(\boldsymbol{\phi}_0^i)^T \mathbf{M}_0\boldsymbol{\phi}_1^i - \lambda_1^i(\boldsymbol{\phi}_0^i)^T \Delta\mathbf{M}\boldsymbol{\phi}_0^i \tag{18}$$

$$\begin{aligned}\boldsymbol{\phi}_2^i = &\sum_{s=1, s\neq i}^{n} \frac{1}{\lambda_0^i - \lambda_0^s}[-\lambda_0^i(\boldsymbol{\phi}_0^s)^T \Delta\mathbf{M}\boldsymbol{\phi}_1^i - \lambda_1^i(\boldsymbol{\phi}_0^s)^T(\mathbf{M}_0\boldsymbol{\phi}_1^i + \Delta\mathbf{M}\boldsymbol{\phi}_0^i)]\boldsymbol{\phi}_0^s \\ &-\frac{1}{2}[(\boldsymbol{\phi}_0^i)^T \Delta\mathbf{M}\boldsymbol{\phi}_1^i + (\boldsymbol{\phi}_1^i)^T(\mathbf{M}_0\boldsymbol{\phi}_1^i + \Delta\mathbf{M}\boldsymbol{\phi}_0^i)]\boldsymbol{\phi}_0^i = \boldsymbol{\phi}_0\mathbf{Z}_2^i\end{aligned} \tag{19}$$

And λ_1^i and λ_2^i are the i-th first-order and second-order eigenvalue of the modified structure, $\boldsymbol{\phi}_1^i$ and $\boldsymbol{\phi}_2^i$ are the i-th first-order and second-order eigenvector of the modified structure, respectively.

The coefficient vector, $\mathbf{z}^i$, contains three unknowns (for a second-order perturbation). Substituting Eq. (17) and Eq. (19) into Eq. (15), $\boldsymbol{\Phi}$ can be written as

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\phi}_0^1 \ldots \boldsymbol{\phi}_0^i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\phi}_0^1 \ldots \boldsymbol{\phi}_0^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\phi}_0^1 \ldots \boldsymbol{\phi}_0^i \end{bmatrix} \begin{bmatrix} \mathbf{I}_0^1 \ldots \mathbf{I}_0^i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_1^1 \ldots \mathbf{Z}_1^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_2^1 \ldots \mathbf{Z}_2^i \end{bmatrix}^T \begin{bmatrix} \mathbf{z}^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{z}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{z}^3 \end{bmatrix} = \boldsymbol{\Phi}_0\mathbf{Z} \tag{20}$$

where

$$\mathbf{Z} = \begin{bmatrix} \mathbf{I}_0^1 \ldots \mathbf{I}_0^i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_1^1 \ldots \mathbf{Z}_1^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_2^1 \ldots \mathbf{Z}_2^i \end{bmatrix}^T \begin{bmatrix} \mathbf{z}^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{z}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{z}^3 \end{bmatrix} \tag{21}$$

Substituting Eq. (14) into the modified analysis equations Eq. (13), and premultiplying by $(\boldsymbol{\phi}_B^i)^T$, one obtains:

$$(\boldsymbol{\phi}_B^i)^T \mathbf{K}_0\boldsymbol{\phi}_B^i\mathbf{z}^i = \lambda^i(\boldsymbol{\phi}_B^i)^T(\mathbf{M}_0 + \Delta\mathbf{M})\boldsymbol{\phi}_B^i\mathbf{z}^i \tag{22}$$

Introducing the notation

$$\mathbf{K}_R^i = (\boldsymbol{\phi}_B^i)^T \mathbf{K}_0 \boldsymbol{\phi}_B^i \tag{23}$$

$$\mathbf{M}_R^i = (\boldsymbol{\phi}_B^i)^T (\mathbf{M}_0 + \Delta\mathbf{M}) \boldsymbol{\phi}_B^i \tag{24}$$

and substituting Eq. (23) and (24) into Eq. (22), we can obtain a set of (3×3) matrix equation

$$\mathbf{K}_R^i \mathbf{z}^i = \lambda^i \mathbf{M}_R^i \mathbf{z}^i \tag{25}$$

Thus, the coefficient vector $\mathbf{z}^i$ is evaluated from Eq. (25). The *i*-th eigenvector of the modified structure is obtained by substituting $\mathbf{z}^i$ into Eq. (14) and **Z** is obtained by substituting $\mathbf{z}^i$ into Eq. (21). It needs to be noted that the first-order eigenvector of Eq. (25) is only used (S. Chen et al., 2000).

Finally, the *i*-th eigenvalue of the modified structure, λ_K^i, is computed using Rayleigh quotient:

$$\lambda_K^i = \frac{(\boldsymbol{\phi}^i)^T \mathbf{K}_0 \boldsymbol{\phi}^i}{(\boldsymbol{\phi}^i)^T (\mathbf{M}_0 + \Delta\mathbf{M}) \boldsymbol{\phi}^i} \tag{26}$$

Proposed efficient aeroelastic evaluation method

The mode shapes $\boldsymbol{\Phi}_0$ of the original structure is taken as the basic mode shapes for basic CFD-baded POD/ROM construction. For the wing structure when wing fuel mass variations due to fuel burn, the physical displacement of the wing can be written as

$$\mathbf{d} = \boldsymbol{\Phi}\mathbf{u} \tag{27}$$

Substituting Eq. (20) into Eq. (27), the physical displacement of the wing can also be written as

$$\mathbf{d} = \boldsymbol{\Phi}_0 (\mathbf{Z}\mathbf{u}) \tag{28}$$

And $\mathbf{u}_b = \mathbf{Z}\mathbf{u}$, $\mathbf{u}_b$ is artificially defined as basic generalized displacements.

A change of the wing structure due to wing fuel mass variations affects the matrices **G**, **E** and **C** of the linearized flow solver, Eq. (2). Substituting the relation $\mathbf{u}_b = \mathbf{Z}\mathbf{u}$, the matrices may be rewritten in terms of the vector of basic generalized displacements:

$$\begin{aligned}
\mathbf{G} &= \frac{\partial \mathbf{F}}{\partial \mathbf{u}}(\mathbf{w}_0, \mathbf{u}_0, \dot{\mathbf{u}}_0) = \frac{\partial \mathbf{F}}{\mathbf{Z}^{-1}\partial \mathbf{u}_b}(\mathbf{w}_0, \mathbf{u}_0, \dot{\mathbf{u}}_0) = \mathbf{Z}\mathbf{G}_b \\
\mathbf{E} &= \frac{\partial \mathbf{A}}{\partial \mathbf{u}}\mathbf{w}_0 = \frac{\partial \mathbf{A}}{\mathbf{Z}^{-1}\partial \mathbf{u}_b}\mathbf{w}_0 = \mathbf{Z}\mathbf{E}_b \\
\mathbf{C} &= \frac{\partial \mathbf{F}}{\partial \dot{\mathbf{u}}}(\mathbf{w}_0, \mathbf{u}_0, \dot{\mathbf{u}}_0) = \frac{\partial \mathbf{F}}{\mathbf{Z}^{-1}\partial \dot{\mathbf{u}}_b}(\mathbf{w}_0, \mathbf{u}_0, \dot{\mathbf{u}}_0) = \mathbf{Z}\mathbf{C}_b
\end{aligned} \tag{29}$$

where $\mathbf{G}_b$, $\mathbf{E}_b$, $\mathbf{C}_b$ are the first order terms in a Taylor series expansion of the basis reduced *r*-order aeroelastic system. Now, the reduced fluid model when wing fuel mass variations due to fuel burn is written as

$$\begin{cases}
\dot{\mathbf{x}}_r = \boldsymbol{\Psi}_r^T \mathbf{A} \boldsymbol{\Psi}_r \mathbf{x}_r + \boldsymbol{\Psi}_r^T \mathbf{Z}\mathbf{B}_b \mathbf{y} \\
\mathbf{f}^{ext} = \mathbf{Z}\mathbf{C}_b \boldsymbol{\Psi}_r \mathbf{x}_r
\end{cases} \tag{30}$$

where $\mathbf{A} = -\mathbf{A}_0^{-1}\mathbf{H}$, $\mathbf{B}_b = -\mathbf{A}_0^{-1}\left[\mathbf{E}_b + \mathbf{C}_b \;\; \mathbf{G}_b\right]$, $\mathbf{y} = \left[\dot{\mathbf{u}} \;\; \mathbf{u}\right]^T$, $\mathbf{C}_b = \mathbf{P}_b$.

The structural dynamic equations of the wing structure when wing mass variations due to fuel burn is written as

$$\bar{\mathbf{M}}\ddot{\mathbf{u}}+\bar{\mathbf{K}}\mathbf{u}=\mathbf{f}^{ext} \tag{31}$$

here, $\overline{\mathbf{M}}=\mathbf{Z}^T\mathbf{\Phi}_0^T(\mathbf{M}_0+\Delta\mathbf{M})\mathbf{\Phi}_0\mathbf{Z}$, $\overline{\mathbf{K}}=\mathbf{Z}^T\mathbf{\Phi}_0^T\mathbf{K}_0\mathbf{\Phi}_0\mathbf{Z}$. In addition, when $\Delta\mathbf{M}=\mathbf{0}$, $\mathbf{Z}$ is the identity matrix, Eq. (30) and Eq. (31) are equivalent to Eq. (11) and Eq. (4), respectively.

For the research on the influence of wing fuel mass variations due to fuel burn on transonic aeroelasticity, different from the standard CFD-based POD/ROM, without expensive, time-consuming reconstruction a new set of POD/ROM, the aeroelastic response can be rapidly calculated by coupling Eq. (30) and Eq. (31). When the wing fuel mass variation due to fuel burn, using structural dynamic reanalysis method to get matrix **Z**, and transform generalized displacements **u** to the basis generalized displacements $\mathbf{u}_b$. Then, the aeroelastic response can ben rapidly calculated. For every wing fuel mass variation, the proposed efficient aeroelastic evaluation method can rapidly predict and evaluate aeroelastic characteristics without rebuilding an expensive, time-consuming CFD-based POD/ROM. Because without reconstructing a new set of POD/ROM for every wing fuel mass variation, the proposed efficient method can significantly reduce the computational cost. And may have the potential to reduce of overall cost of aircraft design.

Numerical results and discussion

The accuracy and efficiency of the proposed efficient evaluation method for wing fuel mass variations effect on transonic aeroelasticity will be demonstrated and evaluated in this section.

CFD-based POD/ROM solver validation

Before demonstrating the accuracy and efficiency of the whole proposed efficient evaluation method. Firstly, the accuracy of CFD-based POD/ROM solver is validated using the AGARD 445.6 aeroelastic wing model (Yates, 1987). The AGARD 445.6 wing model is with a 45 deg quarter-chord sweep angle, and the panel aspect ratio and taper ratio are 1.6525 and 0.6576, respectively. The symmetrical NACA65A004 airfoil section is used to control the thickness distribution of the wings in the span-wise direction. The density of the wing material is 381.98 $\mathrm{Kg/m^3}$. The elastic modulus in the wingspan and chord directions are 3.151GPa and 0.416GPa, respectively, and the shear modulus is 0.4392GPa. The Poisson's ratio is 0.31 (Zhong *et al.*, 2016). The structural model, shown in **Figure 1(a)**, consists of 231 nodes and 200 elements. A multi-block structured mesh was employed for the flow predictions. Zhou (Zhou *et al.*, 2016) had analyzed the spatial convergence of the CFD grid, and the results show that the medium and fine grids are in good agreement. The total number of grid points on the medium grid, herein used, is 223,146 ($99\times49\times46$), consisting of 61 computational nodes around each airfoil section and 20 nodes along the wing semi-span, as shown in

Figure 1(b). The total number of DoFs (Degree of Freedom) for the full-order model is about one million.

The flutter speed computed by our in-house CFD/CSD solver and the present CFD-based POD/ROM solver, as shown in **Figure 2**. For comparison, experimental data from Ref (Yates, 1987) are also included. The agreement between the present CFD-based POD/ROM solver and our in-house CFD/CSD solver is good for all Mach numbers considered (0.499 to 1.141), including the well-known transonic dip of the flutter speed. The accuracy of CFD-based POD/ROM solver has been evaluated over the years in a number of aeroelastic studies in Ref (Zhou et al., 2016).

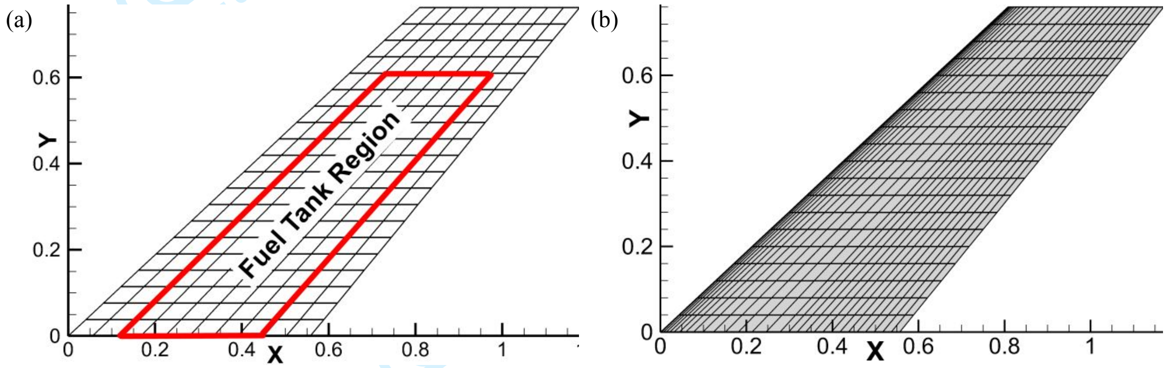


Figure 1 AGARD 445.6 wing: (a) structural model, and (b) surface CFD mesh.

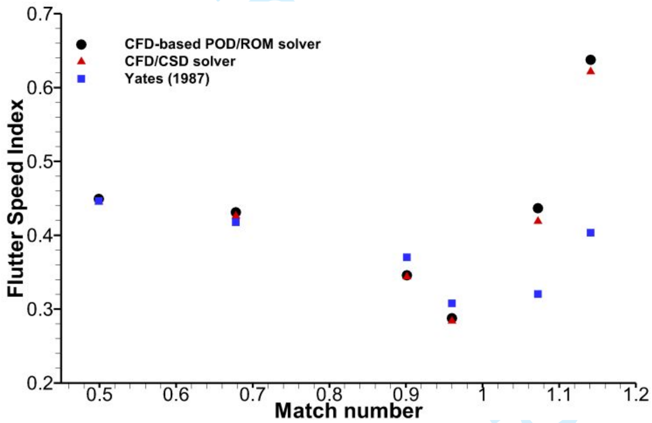


Figure 2 AGARD 445.6 wing flutter boundary; experimental data from Ref (Yates, 1987)

Structural Dynamic Reanalysis Method

An improved AGARD 445.6 wing model, that including a fuel tank, was employed to demonstrate and evaluate the accuracy and efficiency of the proposed aeroelastic efficient evaluation method. The position of the fuel tank on the wing, as shown in **Figure 2(a)**. Some assumptions have been made to allow variation of wing fuel mass due to fuel burn. To obtain the variation of wing fuel mass due to fuel burn, the seven case studies have been chosen as shown in **Table I**, and it is assumed that the wing fuel mass of the original model is twice that of the AGARD 445.6 wing structure. For the seven cases, it display the variation of wing fuel mass during the flight of the aircraft, and may produce a variation of the structural mode shapes and frequencies, and will affect the aeroelastic response and flutter boundary. It is worth observing that these assumptions may not be completely accurate in physical terms, but for the demonstration purposes of this research, a well define and simple model is completely essential.

Table I Seven case.

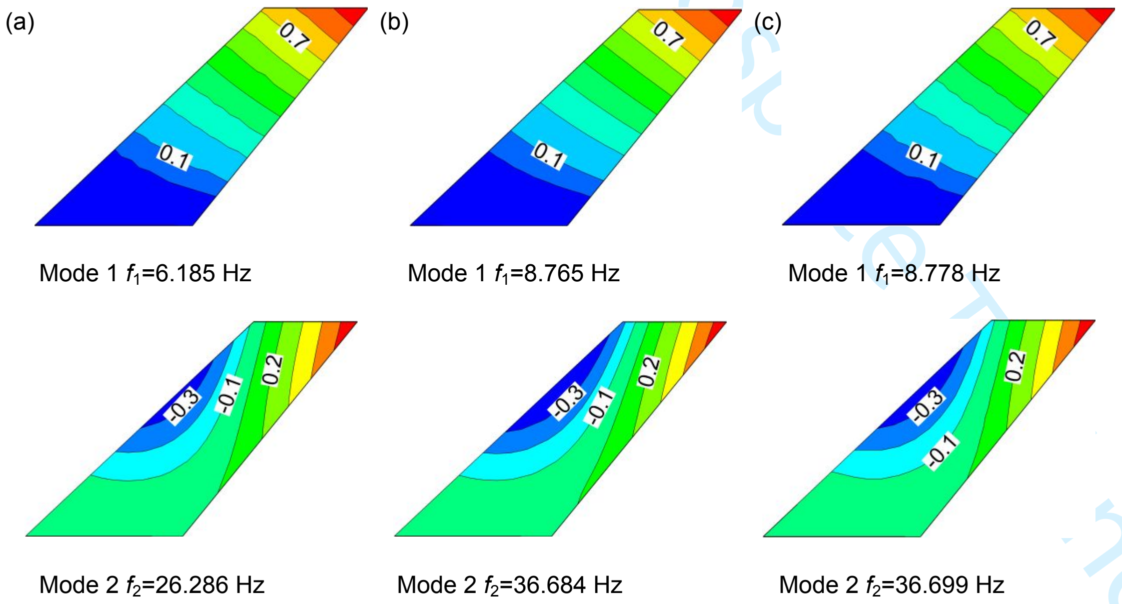
Original	Model A	Model B	Model C	Model D	Model E	Model F	Model G
2wing	7/4wing	6/4wing	5/4wing	4/4wing	3/4wing	2/4wing	1/4wing

To assess the accuracy of the extended Kirsch combined method in reanalyzing the mode shapes of modified structure, the Modal Assurance Criterion (MAC) is introduced, defined as:

$$\mathrm{MAC}(\mathbf{\Phi}_A, \mathbf{\Phi}_E) = \frac{\left|\mathbf{\Phi}_E^T \mathbf{\Phi}_A\right|^2}{(\mathbf{\Phi}_A^T \mathbf{\Phi}_A)(\mathbf{\Phi}_E^T \mathbf{\Phi}_E)} \tag{32}$$

where $\mathbf{\Phi}_E$ represents the exact mode shapes (direct full modal analysis), and $\mathbf{\Phi}_A$ represents the approximate mode shapes (extended Kirsch combined method). For a perfect match between the exact and approximate mode shapes, the MAC is 1.

For the sake of brevity, **Figure 3** only shows the exact and approximate modal characteristics of model G (the largest structural modification case). For comparison, the original model are also included. As can be seen, the mode shapes predicted by extended Kirsch combined method and direct full modal analysis agree fairly well. The other cases have the similar conclusions. Quantitatively, the frequencies errors and the MAC of mode shapes for these seven models are shown in **Table II**. As can be seen from **Table II**, the maximum frequencies error does not exceed 3%, and value of MAC is very close to 1. The results show that the extended Kirsch combined method can accurately obtain the modal data when the wing fuel mass variations due to fuel burn.



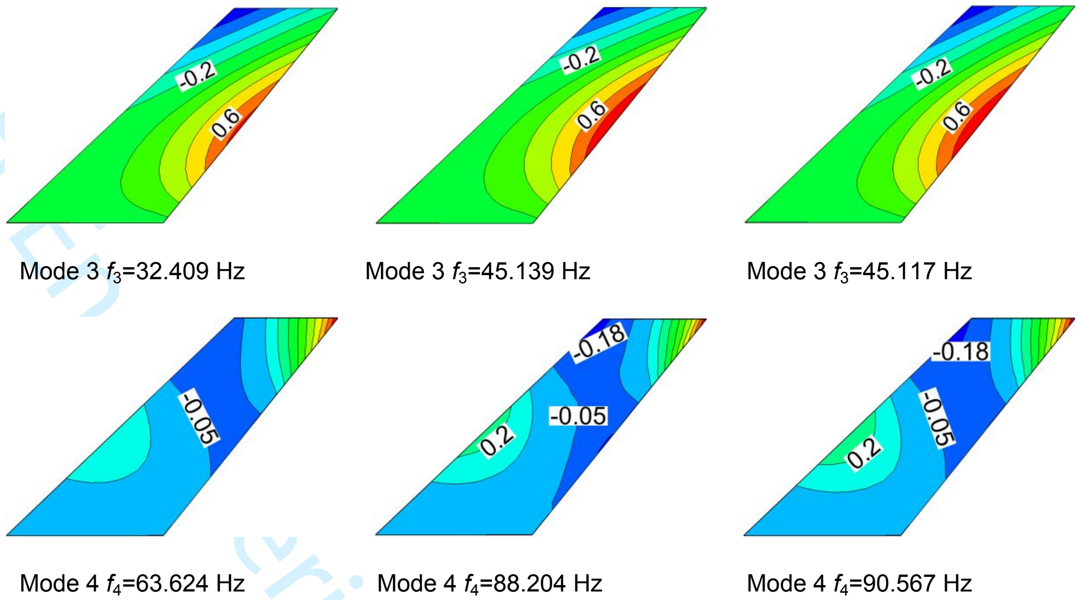


Figure 3 First four mode shapes: (a) original structural model, (b) exact solution of Model G, (c) approximate solution of Model G.

Table II Frequencies errors and MAC values of the improved AGARD 445.6 wing.

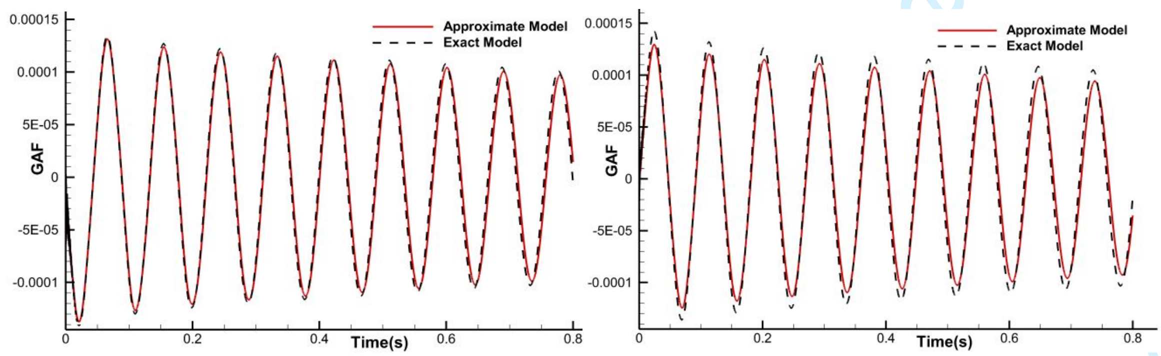
Cases	Mode	Frequency [Hz]			MAC
		Exact	Approximate	Error [%]	
Model A	1	6.421	6.426	0.079	1.0000
	2	27.211	27.213	0.006	0.9999
	3	33.517	33.520	0.009	0.9999
	4	65.772	65.783	0.018	0.9997
Model B	1	6.686	6.691	0.080	1.0000
	2	28.250	28.253	0.011	0.9999
	3	34.761	34.770	0.028	0.9999
	4	68.202	68.258	0.082	0.9986
Model C	1	6.986	6.992	0.081	1.0000
	2	29.426	29.435	0.022	0.9999
	3	36.175	36.197	0.062	0.9997
	4	70.987	71.139	0.215	0.9962
Model D	1	7.330	7.336	0.084	0.9999
	2	30.788	30.800	0.037	0.9998
	3	37.800	37.850	0.114	0.9994
	4	74.222	74.556	0.450	0.9916
Model E	1	7.729	7.736	0.090	0.9999
	2	32.385	32.402	0.052	0.9994
	3	39.736	39.804	0.171	0.9988
	4	78.038	78.701	0.850	0.9832

Model F	1	8.200	8.209	0.105	0.9996
	2	34.306	34.326	0.560	0.9981
	3	42.093	42.170	0.180	0.9991
	4	82.617	83.875	1.522	0.9885
Model G	1	8.765	8.778	0.140	0.9998
	2	36.684	36.700	0.395	0.9922
	3	45.139	45.117	-0.050	0.9938
	4	88.204	90.567	2.679	0.9408

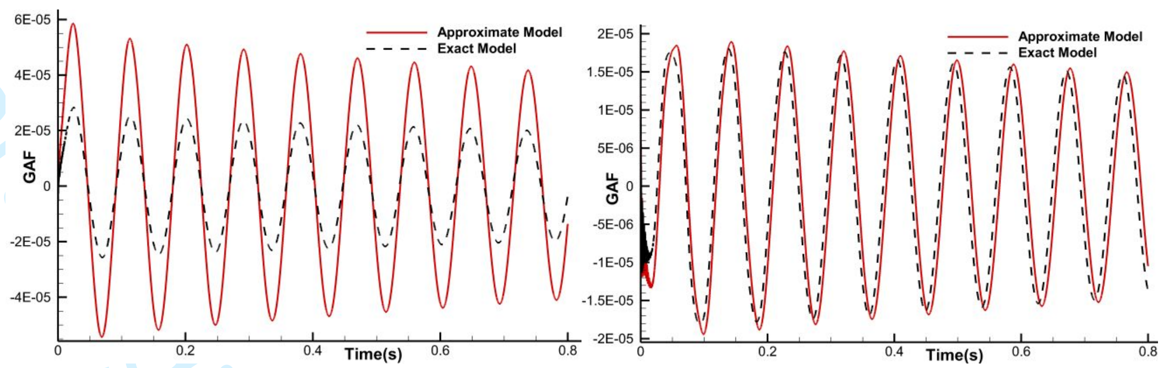
Accuracy evaluation of the proposed efficient aeroelastic evaluation method

After successfully evaluating the accuracy of the CFD-based POD/ROM solver and the structural dynamic reanalysis model, which are both the key parts of the proposed efficient aeroelastic evaluation method, the accuracy of the whole method will be evaluated in this section.

The CFD-based POD/ROM was generated in two steps. Through the POD/ROM method, a first ROM with 600 DoFs in state space based on CFD is obtained. As the size is still too large to investigate the influence of the wing fuel mass variations due to fuel burn on transonic aeroelasticity, the BT (balanced truncation) method (Gang *et al.*, 2012) was then used to reduce the size of the fluid ROM to 50 DoFs. For simplifying description, only the time histories of the (generalized aerodynamic forces) GAFs predicted by the proposed efficient evaluation method for the largest structural modification (Model G) at Ma=0.960 (air density is 0.06341Kg/m3) and AOA=0deg are shown in **Figure 4**. Reference data are representative of the exact model, where the CFD-based POD/ROM and the structural model reconstructed for every wing fuel mass variation. The proposed efficient evaluation method is essentially an approximate method, so it is called approximate model. As it can be seen, the GAFs predicted by both models agree fairly well. A similar agreement was found for the smaller cases which are not shown herein again for brevity. The well agreements indicate that the proposed evaluation method can accurately capture the generalized unsteady aerodynamic responses of wing structure when the wing fuel mass variations.



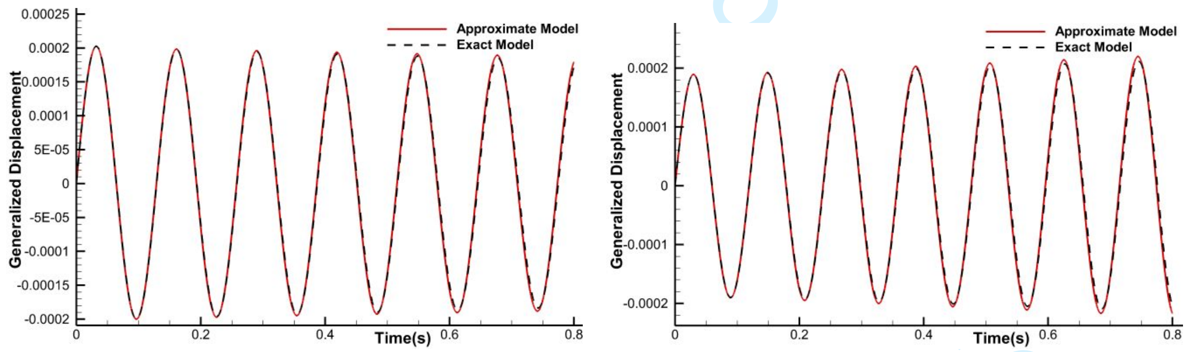
(a) GAF, first bending mode (b) GAF, first torsional mode



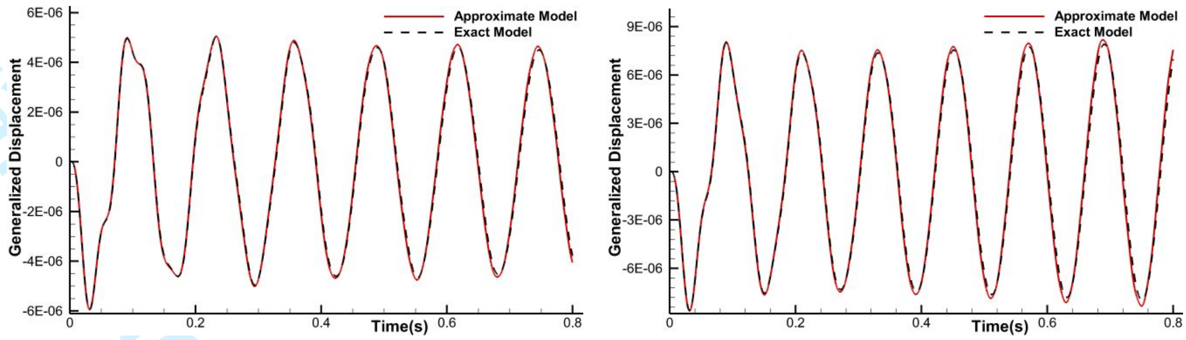
(c) GAF, second bending mode (d) GAF, second torsional mode

Figure 4 Generalized aerodynamic forces for the largest structural modification (Model G) at *Ma*=0.960, *AOA*=0*deg*.

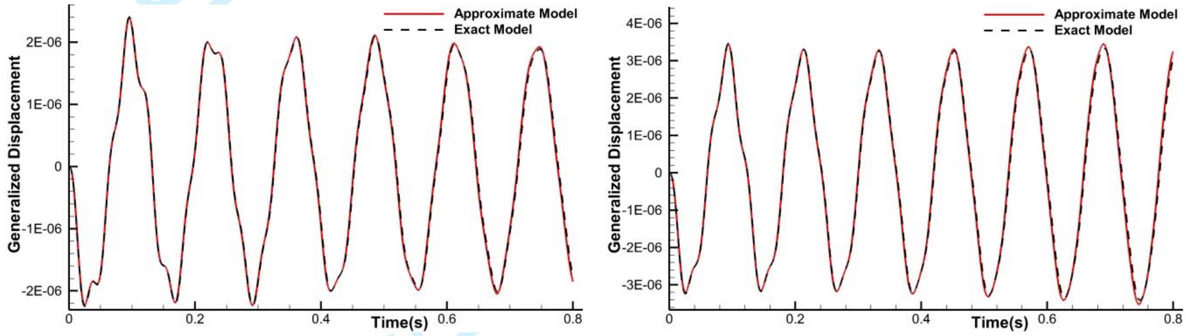
In order to further prove the predictive capability of the proposed efficient aeroelastic evaluation method, two typical aeroelastic responses (decaying and diverging) for the three representative cases (Model A, Model D and Model G) under different free stream dynamic pressures are compared as shown in **Figures 5-7**. It was found again that the aeroelastic responses predicted by both models agree fairly well. As it can also be seen, the discrepancy of aeroelastic responses between the two methods increase for increasing level of the wing fuel mass variations. And, the aeroelastic response error and the modal data error are not strictly proportional, but are positively correlated. The well agreements initially indicate again that the proposed efficient evaluation method has good accuracy for aeroelastic response prediction of wing structure when the wing fuel mass variations due to fuel burn, rather than reconstruct a new set of an expensive, time-consuming CFD-based POD/ROM.



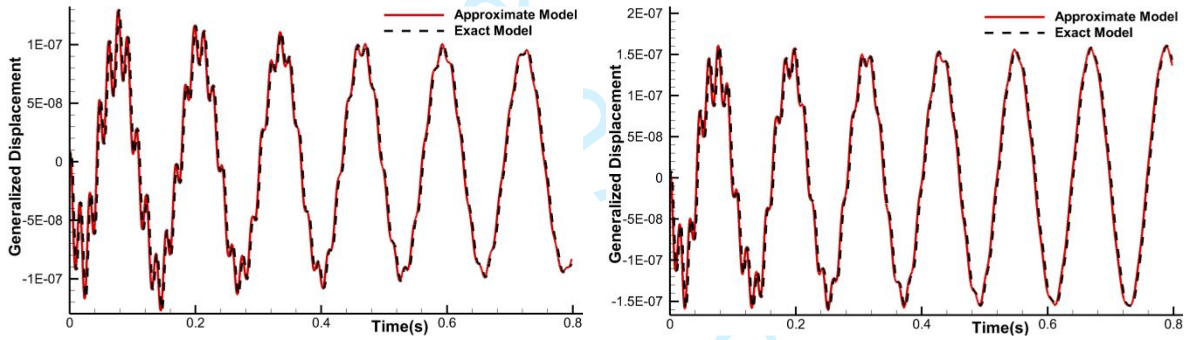
(a) Generalized displacements, first bending mode (left: V_∞=222m/s, right: V_∞=262m/s)



(b) Generalized displacements, first torsional mode (left: V_∞=222m/s, right: V_∞=262m/s)

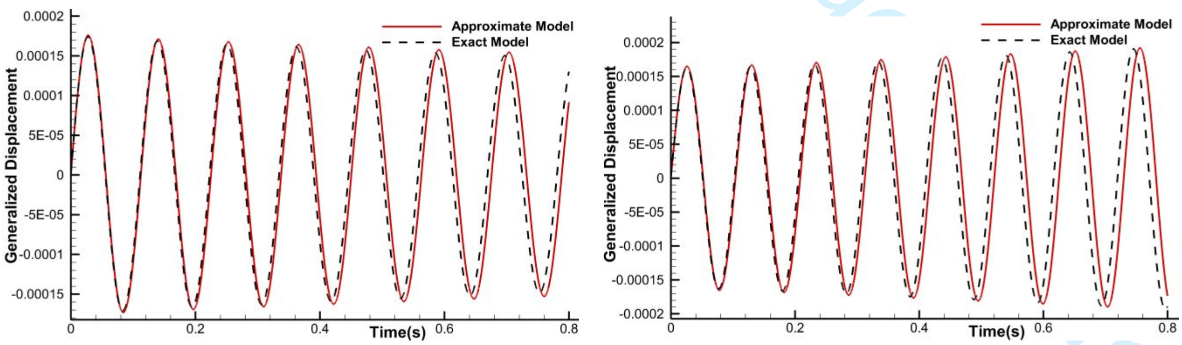


(c) Generalized displacements, second bending mode (left: V_∞=222m/s, right: V_∞=262m/s)

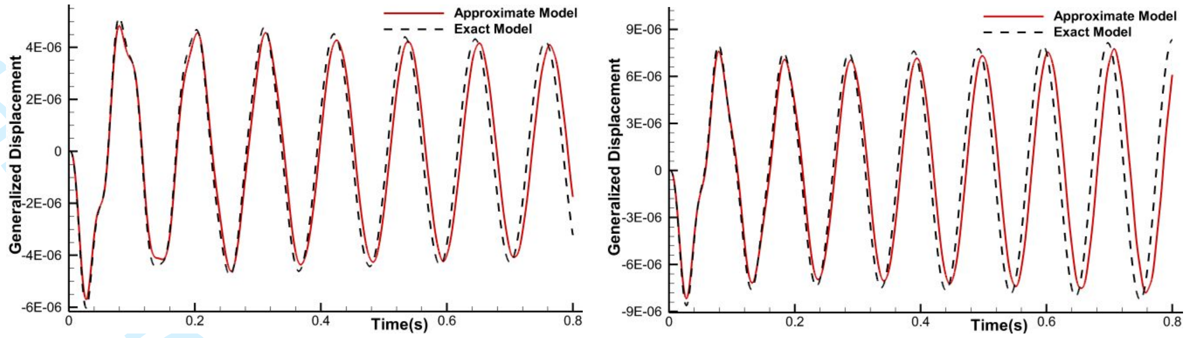


(d) Generalized displacements, second torsional mode (left: V_∞=222m/s, right: V_∞=262m/s)

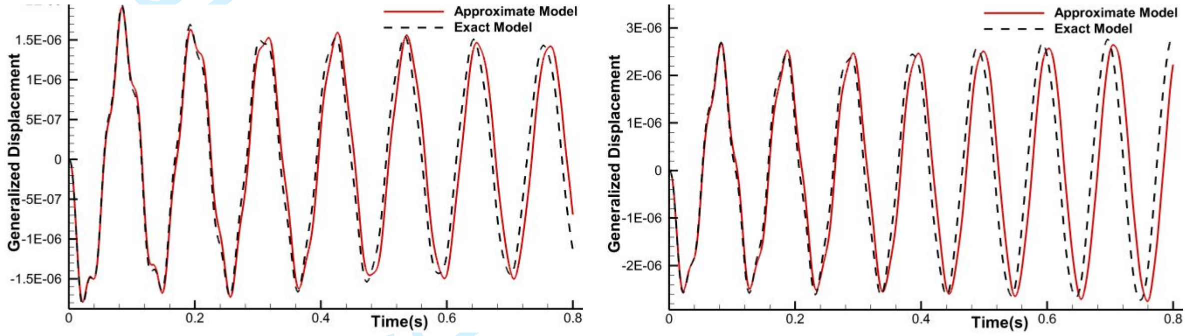
Figure 5 Generalized displacements for the smallest structural modification (Model A) at two freestream speeds; flow conditions: *Ma*=0.960, *AOA*=0*deg*.



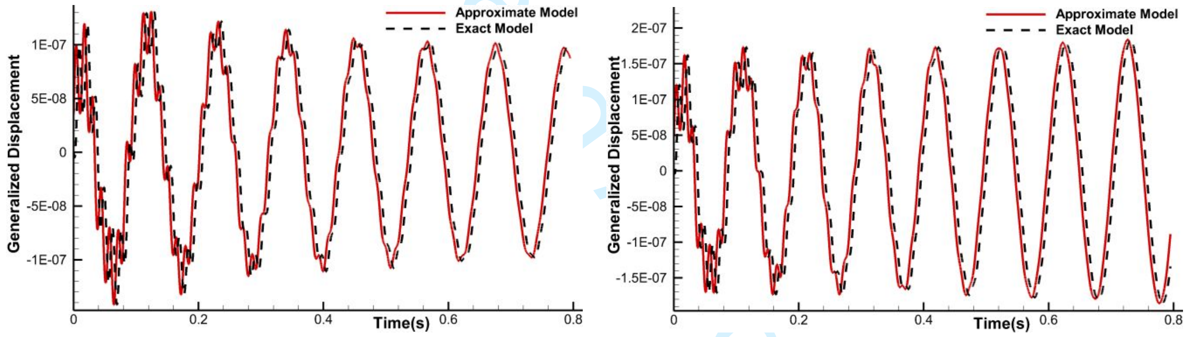
(a) Generalized displacements, first bending mode (left: V_∞=230m/s, right: V_∞=270m/s)



(b) Generalized displacements, first torsional mode (left: V_{∞}=230m/s, right: V_{∞}=270m/s)

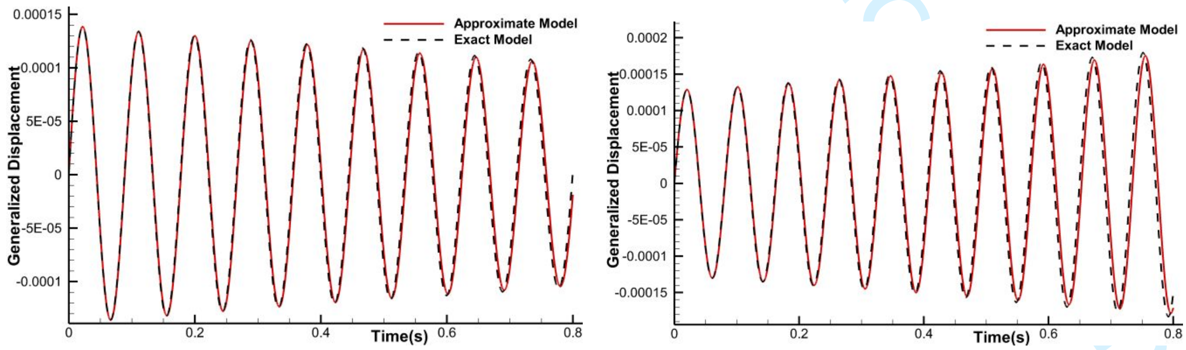


(c) Generalized displacements, second bending mode (left: V_{∞}=230m/s, right: V_{∞}=270m/s)

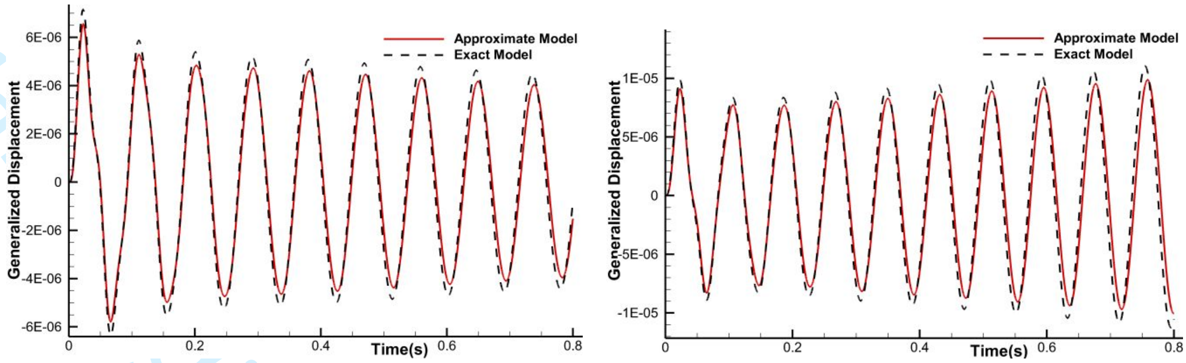


(d) Generalized displacements, second torsional mode (left: V_{∞}=230m/s, right: V_{∞}=270m/s)

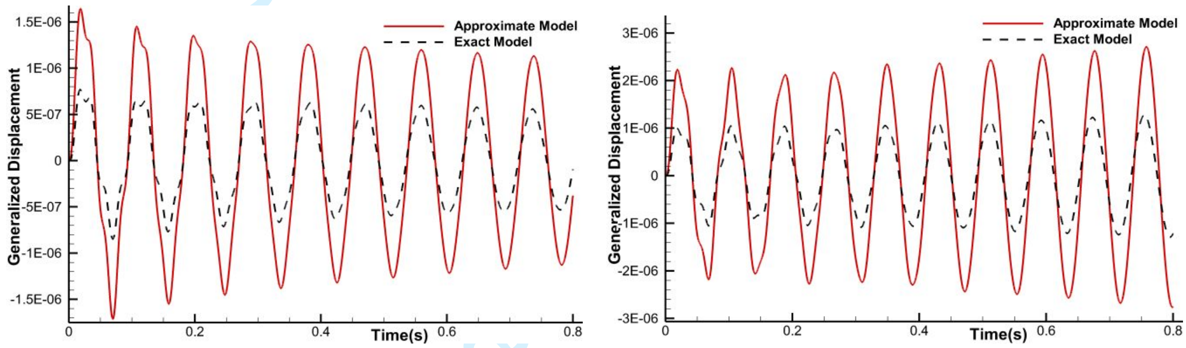
Figure 6 Generalized displacements for the medium structural modification (Model D) at two freestream speeds; flow conditions: *Ma*=0.960, *AOA*=0*deg*.



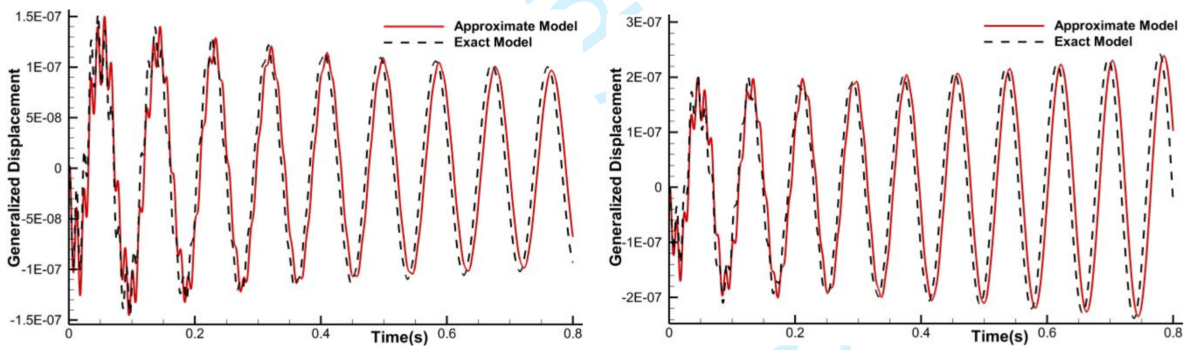
(a) Generalized displacements, first bending mode (left: V_{∞}=255m/s, right: V_{∞}=295m/s)



(b) Generalized displacements, first torsional mode (left: V_∞=255m/s, right: V_∞=295m/s)



(c) Generalized displacements, second bending mode (left: V_∞=255m/s, right: V_∞=295m/s)



(d) Generalized displacements, second torsional mode (left: V_∞=255m/s, right: V_∞=295m/s)

Figure 7 Generalized displacements for the largest structural modification (Model G) at two freestream speeds; flow conditions: *Ma*=0.960, *AOA*=0*deg*.

For these low V_∞, **Figure 5** (left), **Figure 6** (left) and **Figure 7** (left) show that the modes amplitude of generalized displacements decay with time marching. At a higher V_∞, the amplitudes of the modes would diverge with time marching are shown in **Figure 5** (right), **Figure 6** (right) and **Figure 7** (right). Between these two V_∞ conditions, there must have a particular point where the system is neutrally stable. In the given flight conditions (Mach and AOA), these particular points is called flutter speed, are shown in **Table III**. The flutter speed errors for the seven cases predicted by the exact model and proposed efficient evaluation method are also illustrated in **Table III**. Although the discrepancy between the two methods increase for the increasing level of the wing fuel mass variations, the max difference is still less than 3% for the largest modification (Model G) of the structure. All of the above comparison results indicate that the proposed efficient evaluation method can accurately capture the aeroelastic responses of wing structure when the wing fuel mass variations due to fuel burn, including the GAFs and the generalized displacements in different freesteam velocities, and predict the flutter boundary with good accuracy, even for the largest fuel mass variation due to fuel burn.

Table III Flutter speed obtained by exact method and proposed method at *Ma*=0.960.

Cases	Flutter speed (m/s)		Error (%)
	Exact method	Proposed method	
Model A	242.9	244.9	0.823
Model B	244.2	248.1	1.597
Model C	247.8	252.0	1.695
Model D	251.8	257.0	2.065
Model E	257.7	263.4	2.212
Model F	265.5	271.8	2.373
Model G	275.8	282.7	2.502

Efficiency evaluation of the proposed efficient aeroelastic evaluation method

The computational efficiency is one of the most important criteria of the proposed efficient aeroelastic evaluation method. All analyses were performed on a Windows 10 system PC with Intel® Core(TM) i7-9700K CPU (3.60 GHz, 8 cores, but only one core used) and 32GB RAM.

For the exact method, a set of POD modes is generated requiring about 16*h* per configuration. For the seven cases, it required about 112*h*. In contract to this, the set of POD modes is generated only once for the proposed evaluation method. Because, the response in the time domain is inexpensive. The proposed efficient evaluation method only required about 16*h*. With the capability to effectively investigate the influence of wing fuel mass variation on transonic aeroelasticity, the computational advantage of proposed aeroelastic evaluation method becomes more significant when more wing fuel mass variation cases to the structure are considered. Within an aircraft design process, it is reasonable to consider about 100 wing fuel mass variation cases and 20 values of the freestream dynamic pressure to assess the aeroelastic stability. With information reported in **Table** Ⅳ, the exact method would require over 16 thousand CPU hours (this consists of 16*h*×100 and 1.48*s*×20×100, totaling 1600.82*h*), whereas the proposed method just need less than 17 CPU hours (16*h*×1 and 0.65*s*×100+1.72*s*×20×100, totaling 16.97*h*). Without rebuilding an expensive, time-consuming CFD-based POD/ROM for each wing fuel mass variation, the computational cost of the proposed method is reduced obviously, especially after the POD/ROM for original structure was constructed. Obviously, the expected speed-up is grows linearly with the number of wing fuel mass variation cases.

Table Ⅳ Computational cost of the exact method and proposed method.

	Process	**CPU time**
Exact method	Construct a set of POD/ROMs	16*h*
	Time histories responses of the generalized displacement for a values of freestream dynamic pressure	1.48s
	Seven model cases	112*h*
	100 model cases	1600.82*h*
Proposed method	Construct the initial set of POD/ROMs	16*h*
	Compute **Z** use extended Kirsch combined method	0.65*s*
	Time histories responses of the generalized displacement for a values of freestream dynamic pressure based	1.72*s*
	Seven model cases	16.07*h*
	100 model cases	16.97*h*

The above comparison results indicate that the proposed aeroelastic evaluation method has high computational efficiency and accuracy, which is very suitable for the investigation the influence of the wing fuel mass variation due to fuel burn on the transonic aeroelasticity.

Conclusions

For aircraft design, the influence of all any possible aerodynamic and structural variations on aeroelasticity have to be assessed, such as wing fuel mass variations due to fuel burn. The standard CFD-based ROM including POD/ROM cannot account for any aerodynamic and structural variations in the aeroelastic system. In this paper, an efficient aeroelastic evaluation method was proposed by extending the standard CFD-based POD/ROM. It provide a potential powerful tool to more intuitively and effectively investigate the influence of the wing fuel mass variation on transonic aeroelasticity, which may have the potential to reduce of overall cost of aircraft design.

The accuracy and efficiency of the proposed efficient aeroelastic evaluation method was demonstrated and evaluated by an improved AGARD 445.6 aeroelastic wing model. Firstly, the accuracy of the CFD-based POD/ROM solver and extended Kirsch combined structural reanalysis method were successfully evaluated. Then, the accuracy of the whole proposed evaluation method was evaluated by comparing the aeroelastic responses including the GAFs and the generalized displacements. Finally, the flutter speeds were also compared. Although the discrepancy of aeroelastic responses and the flutter speed errors increase as the level of the wing fuel mass variations increasing, the max flutter speed error is still less than 3%. The good agreements of the numerical results show that the proposed efficient aeroelastic evaluation method can accurately predict the aeroelastic response and flutter boundary when the wing fuel mass variations due to fuel burn. Through these numerical studies, it also has been found that the flutter speed increases as the wing fuel mass decreases. The computational efficiency of the proposed method, which generates only once the set of POD modes, the computational cost is reduced obviously. It is obvious that the efficiency improvement is grows linearly with the number of wing fuel mass variation cases.

The main advantage of the proposed efficient aeroelastic evaluation method is not only to accurately evaluate the transonic aeroelasticity when the wing fuel mass variations, but also to keep down the computational cost. It is therefore ideally placed for the application of the control law design. All of these research indicate that the proposed evaluation method make it a reality to apply the efficient standard CFD-based POD/ROM to investigate the influence of the wing fuel mass variation due to fuel burn on transonic aeroelasticity. It provides a potential more powerful and more intuitive tool to deal with the investigation of the influence of wing fuel mass variations on transonic aeroelasticity, and may have the potential to reduce of overall cost of aircraft design.

Conflict of interest statement

There is no conflict of inteset.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China (No. 51775518, 11672225, 11511130053 and 11272005), the National Program on Key Research Projects (No. MJ-2015-F-010), the Shannxi Province Natural Science Foundation (No. 2016JM1007), and the founds for the Central Universities (2014XJJ0126).

References

Alder, M. (2015), "Development and validation of a fluid–structure solver for transonic panel flutter", *AIAA Journal,* Vol. 53 No. 12, pp. 3509-3521.

Allen, T., Sexton, B. and Scott, M. J. (2015), "SUGAR truss braced wing full scale aeroelastic analysis and dynamically scaled wind tunnel model development", in *56th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, pp. 1171.

Balis Crema, L., Mastroddi, F., Coppotelli, G., Iazzetta, A. and Pecora, M. (1996), "Influence of Structural Modifications on the Aeroelastic Analysis of Large Transport Aircraft", in *ICAS PROCEEDINGS*, pp. 2081-2089.

Bekemeyer, P. and Timme, S. (2017), "Reduced order transonic aeroelastic gust response simulation of large aircraft", in *35th AIAA Applied Aerodynamics Conference*, pp. 4361.

Bhatia, M., Gur, O., Kapania, R., Mason, W., Schetz, J. and Haftka, R. (2010), "Progress towards multidisciplinary design optimization of truss braced wing aircraft with flutter constraints", in *13th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference*, pp. 9077.

Chen, G., Li, Y. and Yan, G. (2010), "A nonlinear POD reduced order model for limit cycle oscillation prediction", *Science China Physics, Mechanics and Astronomy,* Vol. 53 No. 7, pp. 1325-1332.

Chen, G., Wang, X. and Li, Y. (2014), "Reduced-order-model-based placement optimization of multiple control surfaces for active aeroelastic control", *International Journal of Computational Methods,* Vol. 11 No. 06, pp. 1350081.

Chen, R. and Wang, Z. (2000), "Fast, block lower-upper symmetric Gauss-Seidel scheme for arbitrary grids", *AIAA journal,* Vol. 38 No. 12, pp. 2238-2245.

Chen, S., Yang, X. and Lian, H. (2000), "Comparison of several eigenvalue reanalysis methods for modified structures", *Structural and Multidisciplinary optimization,* Vol. 20 No. 4, pp. 253-259.

Chen, Z., Zhao, Y. and Huang, R. (2019), "Parametric reduced-order modeling of unsteady aerodynamics for hypersonic vehicles", *Aerospace Science and Technology,* Vol. 87, pp. 1-14.

Cowan, T. J., Arena, A. S. and Gupta, K. K. (1999), *Development of a discrete-time aerodynamic model for CFD-based aeroelastic analysis*, Citeseer.

Dimitriadis, G. (2008), "Continuation of higher-order harmonic balance solutions for nonlinear aeroelastic systems", *Journal of aircraft,* Vol. 45 No. 2, pp. 523-537.

Dowell, E. H. and Hall, K. C. (2001), "Modeling of fluid-structure interaction", *Annual review of fluid mechanics,* Vol. 33 No. 1, pp. 445-490.

Elham, A. (2010), "Structural design: weight estimation", *Encyclopedia of Aerospace Engineering*.

Gang, C., Jian, S. and Yueming, L. (2012), "Active flutter suppression control law design method based on balanced proper orthogonal decomposition reduced order model", *Nonlinear Dynamics,* Vol. 70 No. 1, pp. 1-12.

Garcia Matas, L., Haddad Khodaparast, H., Friswell, M. I., Castrichini, A., Coggon, S., Leong, B. and Cooper, J. E. (2019), "Worst-case gust loads prediction with the effects of local structural nonlinearity", in *AIAA Scitech 2019 Forum*, pp. 0760.

Hall, K. C., Thomas, J. P. and Dowell, E. H. (2000), "Proper orthogonal decomposition technique for transonic unsteady aerodynamic flows", *AIAA journal,* Vol. 38 No. 10, pp. 1853-1862.

Hayes, R., Yao, W. and Marques, S. o. P. (2014), "The Influence of Structural Variability on Limit Cycle Oscillation Behaviour", in *15th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, pp. 3276.

Heinze, S. (2007), "Assessment of critical fuel configurations using robust flutter analysis", *Journal of Aircraft,* Vol. 44 No. 6, pp. 2034-2039.

Huang, G., Wang, H. and Li, G. (2016), "An exact reanalysis method for structures with local modifications", *Structural and Multidisciplinary Optimization,* Vol. 54 No. 3, pp. 499-509.

Kirsch, U., Bogomolni, M. and Sheinman, I. (2007), "Efficient dynamic reanalysis of structures", *Journal of structural engineering,* Vol. 133 No. 3, pp. 440-448.

Kou, J. and Zhang, W. (2019), "A hybrid reduced-order framework for complex aeroelastic simulations", *Aerospace science and technology,* Vol. 84, pp. 880-894.

Lee, S.-H., Cho, H., Kim, H. and Shin, S. J. (2019), "Expedited Aeroelastic Analysis Using Proper Orthogonal Decomposition and Reduced Order Modelling", in *AIAA Scitech 2019 Forum*, pp. 1024.

Li, K., Kou, J. and Zhang, W. (2019), "Deep neural network for unsteady aerodynamic and aeroelastic modeling across multiple Mach numbers", *Nonlinear Dynamics,* Vol. 96 No. 3, pp. 2157-2177.

Lucia, D. J., Beran, P. S. and Silva, W. A. (2004), "Reduced-order modeling: new approaches for computational physics", *Progress in aerospace sciences,* Vol. 40 No. 1-2, pp. 51-117.

Pulliam, T. (1993), "Time accuracy and the use of implicit methods", in *11th Computational Fluid Dynamics Conference*, pp. 3360.

Sewall, J. L. (1957), "An experimental and theoretical study of the effect of fuel on pitching-translation flutter", *Technical Note 4166, NACA*.

Song, Q., Chen, P. and Sun, S. (2014), "An exact reanalysis algorithm for local non-topological high-rank structural modifications in finite element analysis", *Computers & Structures,* Vol. 143, pp. 60-72.

Thomas, J. P., Dowell, E. H. and Hall, K. C. (2004), "Modeling viscous transonic limit cycle oscillation behavior using a harmonic balance approach", *Journal of aircraft,* Vol. 41 No. 6, pp. 1266-1274.

Torii, H. and Matsuzaki, Y. (2001), "Flutter margin evaluation for discrete-time systems", *Journal of Aircraft,* Vol. 38 No. 1, pp. 42-47.

Van Leer, B. (1979), "Towards the ultimate conservative difference scheme. V. A second-order sequel to Godunov's method", *Journal of computational Physics,* Vol. 32 No. 1, pp. 101-136.

Voß, A. (2019), "Open and closed loop gust loads analyses for a flying wing configuration with variable longitudinal stability", *Aerospace Science and Technology,* Vol. 89, pp. 1-10.

Wildschek, A., Bartosiewicz, Z. and Mozyrska, D. (2014), "A multi-input multi-output adaptive feed-forward controller for vibration alleviation on a large blended wing body airliner", *Journal of Sound and Vibration,* Vol. 333 No. 17, pp. 3859-3880.

Winter, M., Heckmeier, F. M. and Breitsamter, C. (2017), "CFD-based aeroelastic reduced-order modeling robust to structural parameter variations", *Aerospace Science and Technology,* Vol. 67, pp. 13-30.

Woodgate, M. and Badcock, K. (2009), "Implicit harmonic balance solver for transonic flow with forced motions", *AIAA journal,* Vol. 47 No. 4, pp. 893-901.

Xie, D., Xu, M. and Dai, H. (2019), "Effects of damage parametric changes on the aeroelastic behaviors of a damaged panel", *Nonlinear Dynamics,* Vol. 97 No. 2, pp. 1035-1050.

Yates, E. C. (1987), "AGARD standard aeroelastic configuration for dynamic response, candidate configuration I", *Wing,* Vol. 445, pp. 103-129.

Zhang, W., Chen, K. and Ye, Z. (2015), "Unsteady aerodynamic reduced-order modeling of an aeroelastic wing using arbitrary mode shapes", *Journal of Fluids and Structures,* Vol. 58, pp. 254-270.

Zhang, W., Lv, Z., Diwu, Q. and Zhong, H. (2019), "A flutter prediction method with low cost and low risk from test data", *Aerospace Science and Technology,* Vol. 86, pp. 542-557.

Zhong, J. and Xu, Z. (2016), "Coupled fluid structure analysis for wing 445.6 flutter using a fast dynamic mesh technology", *International Journal of Computational Fluid Dynamics,* Vol. 30 No. 7-10, pp. 531-542.

Zhou, Q., Chen, G. and Li, Y. (2014), "A REDUCED ORDER MODEL BASED ON BLOCK ARNOLDI METHOD FOR AEROELASTIC SYSTEM", *International Journal of Applied Mechanics,* Vol. 06 No. 06, pp. 1450069.

Zhou, Q., Li, D.-f., Da Ronch, A., Chen, G. and Li, Y.-m. (2016), "Computational fluid dynamics-based transonic flutter suppression with control delay", *Journal of Fluids and Structures,* Vol. 66, pp. 183-206.

An efficient evaluation method for wing fuel mass variations effect on transonic aeroelasticity

Response to reviewers' comments

Dear Editors and Reviewers:

On behalf of my co-authors, we thank you very much for giving us an opportunity to revise our manuscript. We appreciate editor and reviewers very much for their positive and constructive comments and suggestions on our manuscript entitled "An efficient evaluation method for wing fuel mass variations effect on transonic aeroelasticity" (ID: AEAT-08-2021-0227). Changes have been tracked in dark blue colour in the revised manuscript. Please find below our responses to the reviewers' comments and suggestions on an item by item basis. The main revisions are listed as follows:

Responds to the reviewer's comments:

Reviewer #1

Recommendation: Major Revision

Comments: no other comments

Response: Thank you very much for your kind comments and appreciation.

Additional Questions:

Comment 1. Originality: Does the paper contain new and significant information adequate to justify publication?:

The authors proposed modification of the CFD-based POD / ROM standard method to increase the efficiency of aeroelastic calculations during the aircraft design stage. This allows, for example, to analyse the impact of fuel mass changes in the wings of an aircraft on the aeroelastic properties of the structure in transonic flight conditions. In this sense, the reviewed paper fits very well with contemporary design trends. Fast and accurate methods of aeroelastic analysis allow for effective optimization of the solution in an acceptable time.

Response: Thank you very much for your kind comments and appreciation.

Comment 2. Relationship to Literature: Does the paper demonstrate an adequate understanding of the relevant literature in the field and cite an appropriate range of literature sources? Is any significant work ignored?:

The authors have made a fair review of the literature related to the subject discussed. Most of the significant items directly related to the issues discussed were included in the review of literature sources.

Response: Thank you very much for your kind comments and appreciation.

Comment 3. Methodology: Is the paper's argument built on an appropriate base of theory, concepts, or other ideas? Has the research or equivalent intellectual work on which the paper is based been well designed? Are the methods employed appropriate?:

However, research work requires a proper and detailed presentation of the research methodology used, which should be fully described in the paper. The lack of a detailed description of the research methodology is a significant shortcoming of this work. First of all, a detailed methodological description should contain theoretical assumptions of the conducted work. Apart from the assumptions relating to the models described, there are no other assumptions. The methodology should also include a detailed problem description, which prompts to ask research questions, and allows to formulate the thesis or hypothesis and research goals. Research variables and their indicators were not defined. The methods, techniques and research tools that were used in the research process were not indicated. The research procedures are not described. We can only guess the research procedures based on the paper text. Only the calculation models are described in detail. The analysed cases were also not described in sufficient detail.

If the research methodology is not defined and well described, all work can only be regarded as low-ranking engineering activity, not research work.

Response: Thank you for the reviewer's comment. For the methods, techniques and research tools of this research, it builds on proper orthogonal decomposition of the linearized Euler equations and structural dynamic reanalysis method. This content had already added in Page 3. It has been tracked in dark blue colour in the revised manuscript. Actually, the describe of our in-house CFD solver, CFD/CSD coupled solver and POD/ROM solver have been fully described in our previous publications such as Aerospace Science and Technology, Journal of Fluids and Structures, Journal of Aircraft. The accuracy and efficiency of these solvers have been validated for several kinds of models from 2-d airfoils to 3D wing-body aircarfts. However, according to the comments, this content are brief introduced and had already added in Page 3-4. It has been tracked in dark blue colour in the revised manuscript. For research variables and their indicators, this content had already added in Page 10. It has been tracked in dark blue colour in the revised manuscript.

Comment 4. Results: Are results presented clearly and analysed appropriately? Do the conclusions adequately tie together the other elements of the paper?:

The conclusions refer to the analysed cases in detail. Due to the lack of precise information relating to the analysed scenarios, it is difficult to verify the obtained results. E.g., why there is a big difference between approximate model and exact model for second bending mode is not discussed here.

General conclusions indicate the possibilities of using the developed method, but they do not precisely define the directions of further works.

Response: Thank you for the reviewer's comment. These cases may not be completely accurate in physical terms, but for the demonstration purposes of this research, a well define and simple model is completely essential. This content had already added in Page 11. It has been tracked in dark blue colour in the revised manuscript. For big difference between approximate model and exact model for second bending mode, because this is mainly caused by the structural dynamic reanalysis method, which is not the focus of this manuscript, so the issue is not discussed in detail. The proposed efficient aeroelastic evaluation method have ability to investigate the influence of wing fuel mass variations due to fuel burn on the aeroelasticity. And the results are a theoretical and methodological basis for further research, i.e., active control law design considering wing fuel mass variations. This content had already added in Page 3. It has been tracked in dark blue colour in the revised manuscript.

Comment 5. Implications for research, practice and/or society: Does the paper identify clearly any implications for research, practice and/or society? Does the paper bridge the gap between theory and practice? How can the research be used in practice (economic and commercial impact), in teaching, to influence public policy, in research (contributing to the body of knowledge)? What is the impact upon society (influencing public attitudes, affecting quality of life)? Are these implications consistent with the findings and conclusions of the paper?:

A modern approach to aircraft design requires precise and diverse analyses already at the conceptual design stage. This allows for multidisciplinary analysis and optimization of the designed aircraft. Until now, aeroelastic analysis have rarely been carried out in the preliminary stages due to their complexity and insufficient information about the designed aircraft. However, this type of analysis, especially concerning aircraft flying at higher speeds, is necessary and should be performed at the earliest possible stage. In this sense, the reviewed paper fits very well with contemporary design trends. Fast and accurate methods of aeroelastic analysis allow for effective optimization of the solution in an acceptable time.

The authors indicate that the application of the developed method will allow to reduce the computational effort at the design stage of the aircraft and enable faster aeroelastic analyses without reducing the quality of the obtained results.

Response: Thank you very much for your kind comments and appreciation.

Comment 6. Quality of Communication: Does the paper clearly express its case, measured against the technical language of the field and the expected knowledge of the journal's readership? Has attention been paid to the clarity of expression and readability, such as sentence structure, jargon use, acronyms, etc.:

The paper is written in the comprehensible and correct language, sufficient for the needs of scientific work.

Response: Thank you very much for your kind comments and appreciation.

Reviewer #2

Recommendation: Major Revision

Comments:

The paper could be accepted subject to major revision. The key issue is that sufficient details about computational models and results are not provided. (although the formulations are provided)

Response: Thank you for the reviewer's comment. For the computational models of this research, the research methodology builds on proper orthogonal decomposition of the linearized Euler equations and structural dynamic reanalysis method. This content had already added in Page 3. It has been tracked in dark blue colour in the revised manuscript. And also the solution of uneasy flow had already added in Page 3-4. It has been tracked in dark blue colour in the revised manuscript. For results, it is explained in detail in Page 17. It has been tracked in dark blue colour in the revised manuscript.

Additional Questions:

Comment 1. Originality: Does the paper contain new and significant information adequate to justify publication?:

The paper aims to develop a method in order to investigate the influence of the wing fuel mass variations due to fuel burn more easily. The key claim here is a reduction in computational cost for aeroelasticity analysis. However, the numerical method for the solution of uneasy flow is not covered significantly. Computational setups, convergence etc. are not explained.

Response: Thank you for the reviewer's comment. The details of how the solution of uneasy flow is not the focus of this paper, so it is not described in detail in this paper. Actually, the accuracy of our in-house CFD solver and CFD/CSD coupled solver have been validated for several kinds of models from 2-d airfoils to 3D wing-body aircarfts, which have been used in our previous publications such as Aerospace Science and Technology, Journal of Fluids and Structures, Journal of Aircraft. However, according to the comments, the key algorithms used in the the solution of uneasy flow are brief introduced. This content had already added in Page 3-4. It has been tracked in dark blue colour in the revised manuscript. For convergence etc, the explanation had already added in **Comment 4**, and this content had already added in Page 17. For computational setups, this content had already have been fully described in our previous publications such as Nonlinear Dynamics, Aerospace Science and Technology, Journal of Fluids and Structures, Journal of Aircraft. However, according to the comments, this content had already added in Page 9 and Page 13**.** It has been tracked in dark blue colour in the revised manuscript.

Comment 2. Relationship to Literature: Does the paper demonstrate an adequate understanding of the relevant literature in the field and cite an appropriate range of literature sources? Is any significant work ignored?:

There are many other aeroelasticity methods such as the various versions of harmonic balanced and time methods which have not been covered in the literature review.

Response: Thank you for the reviewer's comment. For the review harmonic balanced and time methods, this content had already added in Page 2. It has been tracked in dark blue colour in the revised manuscript.

Comment 3. Methodology: Is the paper's argument built on an appropriate base of theory, concepts, or other ideas? Has the research or equivalent intellectual work on which the paper is based been well designed? Are the methods employed appropriate?:

The limitations of the method in terms of accuracy should be explained. For example, if viscous effects have not been considered what would be the impacts on the accuracy of the results, sufficient details about all computational models would be helpful.

Response: Thank you for the reviewer's comment. The CFD tool employed in the present work is based on the Euler equations, only inviscid compressible flow was taken into account. Euler equation based solver is also widely used in aeroelastic flutter analysis because in the small angel of attack, the flutter characteristics can be well captured compared with many experimental results. In this paper we compared our predicted flutter results with the wind tunnel experimental results and Lee's numerical results which indicates the choice is ok.

Our solver had the BL model and SA model, the Euler and Navier-Stokes solvers have been implemented and tested in our code. For the coupled computation time reduction, in this paper we just used Euler solver to evaluated our proposed structrual reanalysis aeroelastic prediction method, whose effectiveness is independent on the type of CFD solvers. This content had already added in Page 4. It has been tracked in dark blue colour in the revised manuscript.

Comment 4. Results: Are results presented clearly and analysed appropriately? Do the conclusions adequately tie together the other elements of the paper?:

Only displacement results are provided. No details about flow or damping are presented. More explanation is needed on how to flutter speed is predicted.

Response: Thank you for the reviewer's comment. The research mainly focuses on the aeroelastic response of wing structure as most of the aeroeasltic researchers did in there work.

In general, the damping of the wing can be classified into two categories: aerodynamic damping and structural damping. Due to the structural damping is much smaller than the aerodynamic damping, and structural damping is often difficult to measure accurately. In aeroelastic stable analysis such as flutter prediction, the structural damping is generally neglected because the damping usually only effects on the structural responses itself but not effects the divergent or convergent characteristics of the flutter dynamic pressure. That's why in most of the CFD/CSD coupled flutter prediction solvers, the structural damping usually was neglected or given an constant such as 0.001. But if the structural damping is measured or given such as 0.001, it is very easy to adding into the CFD/CSD coupled solver. In this manuscript, the damping of the wings only includes the aerodynamic damping, and in the same conditions of dynamic pressure, damping is the same. For an given structure model, the initial condition is the same.

For low V_∞, the response of generalized displacements decay with time marching. At a higher V_∞, the response would diverge with time marching. Between these two V_∞ conditions, there must have a particular point where the system is neutrally stable, these particular points is called flutter speeds. This content had already added in Page 17. It has been tracked in dark blue colour in the revised manuscript.

Comment 5. Implications for research, practice and/or society: Does the paper identify clearly any implications for research, practice and/or society? Does the paper bridge the gap between theory and practice? How can the research be used in practice (economic and commercial impact), in teaching, to influence public policy, in research (contributing to the body of knowledge)? What is the impact upon society (influencing public attitudes, affecting quality of life)? Are these implications consistent with the findings and conclusions of the paper?:

The paper has implications for research and practice.

Response: Thank you very much for your kind comments and appreciation.

Comment 6. Quality of Communication: Does the paper clearly express its case, measured against the technical language of the field and the expected knowledge of the journal's readership? Has attention been paid to the clarity of expression and readability, such as sentence structure, jargon use, acronyms, etc.:

The English language has to be improved through the paper for example one of the highlights is "has potential to be a potential more powerful"

Response: Thank you for the valuable comment, we have carefully read the manuscript and modified some language errors and inaccurate statements in this paper. It has been tracked in dark blue colour in the revised manuscript.

We appreciate for Editor/Reviewers' warm work earnestly, and hope that the correction will meet with approval. Once again, thank you very much for your comments and suggestions.