

Charges, conserved quantities and fluxes in de Sitter spacetime

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We discuss the definition of conserved quantities in asymptotically locally de Sitter spacetimes. One may define an analogue of holographic charges at future and past infinity and at other Cauchy surfaces I_t as integrals over the intersection of timelike surfaces C and the Cauchy surface I_t . In general, the charges Q^t defined on the Cauchy surface I_t depend on C , but if gravitational flux is absent the charges are independent of C . The quantity $\Delta Q^t(C_1, C_2) = Q^t(C_1) - Q^t(C_2)$ is zero and thus independent of I_t in the absence of flux, defining a *conserved quantity* in spacetime. On the other hand, if there is a net gravitational flux entering or leaving the spacetime region bounded by C_1, C_2 and two Cauchy surfaces then $\Delta Q^t(C_1, C_2)$ changes by the same amount.

Introduction Isolated systems with no dynamical gravity have conserved energy and momentum originating from translational invariance. These charges may be obtained from a conserved local energy-momentum tensor via appropriate integrals over a Cauchy surface. If a system is open there may be flux of energy and momentum through its boundary, which is again encoded by the energy-momentum tensor. A prime example is electromagnetic radiation originating from a localised source and propagating out to infinity. When gravity is dynamical the corresponding symmetries are local and as such the corresponding charges would be zero if there are no boundaries or asymptotic regions.

The study of (conserved) charges in gravitational theories and of gravitational radiation has a long history going back to (at least) [1–5]. Diffeomorphism invariance implies that the charges should be defined by surface integrals at infinity, but due to the infinite volume of spacetime any such definition requires a subtraction, potentially making the charges ambiguous. Most of the initial work concerned asymptotically flat gravity, and in this context charges were defined relative to flat spacetime. With such asymptotics one may also define outgoing gravitational radiation through null infinity, as illustrated in Fig. 1. In the presence of a cosmological constant the asymptotic structure changes and new issues arise.

With negative cosmological constant, the anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence suggests that one should be able to formulate the problem in exactly the same way as when gravity is non-dynamical. Indeed, one can show that such spacetimes are always equipped with a covariantly conserved symmetric tensor which may be used to construct conserved charges. This tensor is identified with the (expectation value of the) energy-momentum tensor of the dual CFT in the AdS/CFT correspondence [6]. Asymptotically locally AdS (AlAdS) spacetimes have a timelike conformal boundary and an associated boundary conformal structure. In the neighborhood of the conformal boundary the

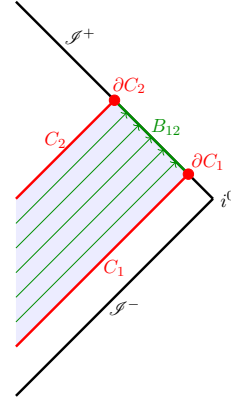


FIG. 1. Penrose diagram illustrating the flux of gravitational radiation through null infinity, \mathcal{I}^+ , in an asymptotically flat spacetime. The hypersurfaces C_1, C_2 are taken to be null (“instants of retarded time”) although they could also have been chosen to be spacelike.

metric admits the following Fefferman-Graham form [7],

$$ds^2 = \frac{l_{AdS}^2}{\rho^2} (d\rho^2 + g_{ab}(\rho, x) dx^a dx^b) \quad (1)$$

where l_{AdS} is the curvature radius and in four dimensions:

$$g_{ab} = g_{(0)ab} + \rho^2 g_{(2)ab} + \rho^3 g_{(3)ab} + \dots \quad (2)$$

Throughout this Letter for concreteness we specialise to four dimensions. Most of the discussion generalises straightforwardly to other dimensions, modulo issues associated with the holographic conformal anomaly in odd dimensions [8]. Here $g_{(0)}$ is the representative of the boundary conformal structure and the holographic energy momentum tensor is given by

$$T_{ab} = \frac{3l_{AdS}}{16\pi G} g_{(3)ab}, \quad (3)$$

where G is Newton’s constant. If the boundary conformal structure admits conformal Killing vectors ζ^a then the energy-momentum tensor may be used to construct

associated conserved charges via a surface integral over a spacelike surface at infinity,

$$Q_\xi = \int_{\partial C} ds_a T_b^a \zeta^b, \quad (4)$$

where ∂C is the intersection of the bulk spacelike surface C with the conformal boundary. It was shown in [9] using Noether's method and covariant phase space methods that (4) are the bulk gravitational charges. These charges are well-defined due to the boundary counterterms introduced by holographic renormalisation [6, 8, 10, 11]. In fact the counterterms are also required for well-posedness of the variational problem when the boundary conformal structure is fixed [9].

The purpose of this Letter is to address the analogous issues for spacetimes with positive cosmological constant. Asymptotically locally de Sitter (AldS) spacetimes share a number of properties with AlAdS spacetimes but there are also important differences. The conformal boundary of AldS spacetimes is spacelike and typically has two causally connected components, one at past infinity and one at future infinity (in contrast the AdS conformal boundary is typically connected¹). Near each component the structure of the spacetime is related by analytic continuation to that of a corresponding AlAdS spacetime [13], with results analogous to (1), (2), (3) (recovering the asymptotic expansion first presented in [14]). AldS spacetimes are spatially compact so one cannot define conserved charges as integrals at spatial infinity. Nevertheless, explicit solutions (for example the Schwarzschild dS solution) have integration constants that traditionally are interpreted as the values of conserved charges (like mass and angular momentum). These charges are associated with conformal Killing vectors of the conformal structure at future infinity and may be computed by integrals of the energy momentum tensor at future infinity. However, they are not conserved in the usual sense, *i.e.* under time evolution. Instead these charges are conserved under spatial translations if there is no gravitational radiation, as we discuss below.

The discussion of gravitational charges in de Sitter spacetimes goes back to at least [15], and there has been renewed interest in this topic since our Universe appears to have positive cosmological constant and gravitational waves have been observed. Recent works include those of Ashtekar *et al.* [16–21] (which builds upon earlier work, [22, 23]) that discuss charges and radiation for linearised fields on dS, and Chruściel *et al.* [24–26] where different approaches to the computation of energy and linear fluxes between null cones in dS are considered, see also [27–31]. A number of other approaches to examining radiative bodies in dS can be found in [32–37]. As outlined

above, there are many similarities between dS and AdS and works that use the similarity to define charges and fluxes include [38–46]. Our work further develops and draws from this set of papers.

Covariant phase space formalism The main tool we use in our analysis is the covariant phase space [47–52] (see also the recent works [53, 54] and references therein, and [55] for an alternative but equivalent formalism). The key object is the $(d-1)$ -form symplectic current ω . Starting from a diffeomorphism covariant Lagrangian d -form $\mathbf{L}(\psi)$, the symplectic current is given by

$$\omega(\psi, \delta_1 \psi, \delta_2 \psi) = \delta_1 \Theta(\psi, \delta_2 \psi) - \delta_2 \Theta(\psi, \delta_1 \psi), \quad (5)$$

where Θ is the total derivative in the onshell variation of $\mathbf{L}(\psi)$, $\delta \mathbf{L} = d\Theta$ (ψ denotes the metric and other fields). A crucial property of ω is that it is closed onshell,

$$d\omega = 0. \quad (6)$$

Consider now onshell configurations ψ whose variations $\delta\psi$ are also onshell. For a diffeomorphism generated by ξ the corresponding Hamiltonian H_ξ is given by

$$\delta H_\xi = \int_C \omega(\psi, \delta\psi, \mathcal{L}_\xi \psi) \quad (7)$$

with C a $(d-1)$ -dimensional slice, which for many applications is a Cauchy surface, and \mathcal{L}_ξ is the Lie derivative. Given such a diffeomorphism we can associate a $(d-1)$ -form Noether current, $\mathbf{J}(\xi) = \Theta(\psi, \mathcal{L}_\xi \psi) - i_\xi \mathbf{L}$, where i_ξ is the interior product. The current \mathbf{J} is closed onshell and is thus locally exact: $\mathbf{J}(\xi) = d\mathbf{Q}(\xi)$ with $(d-2)$ -form Noether charge \mathbf{Q} , the variation of H_ξ may be written as

$$\delta H_\xi = \int_{\partial C} (\delta \mathbf{Q}(\xi) - i_\xi \Theta(\psi, \delta\psi)) \quad (8)$$

where ∂C is the boundary of C .

One can integrate this equation to obtain H_ξ iff the right hand side is a total variation, requiring

$$\int_{\partial C} i_\xi \Theta(\psi, \delta\psi) = \delta \int_{\partial C} i_\xi \mathbf{B}(\psi) \quad (9)$$

Note that one can also express the condition for existence of H_ξ in terms of $[\delta_1, \delta_2]H_\xi = 0$:

$$\int_{\partial C} i_\xi \omega(\psi, \delta_1 \psi, \delta_2 \psi) = 0. \quad (10)$$

The difference between Hamiltonians at two different C s is given by

$$\delta H_\xi|_{\partial C_2} - \delta H_\xi|_{\partial C_1} = - \int_{B_{12}} \omega(\psi, \delta\psi, \mathcal{L}_\xi \psi) \quad (11)$$

where B_{12} is a hypersurface with boundary $\partial C_1 \sqcup \partial C_2$. If (10) is satisfied by virtue of $\omega|_{B_{12}} = 0$, then equation (11) tells us that H_ξ is independent of the slice C . We now review the application of this formalism to AlAdS spacetimes, before discussing AldS spacetimes.

¹ See [12] for general results regarding the Euclidean AdS case.

Asymptotically locally anti-de Sitter: This case was fully analysed in [9]. The slice C in (7) is a bulk spacelike surface that ends on the boundary of AIAdS. Considering Dirichlet conditions², where the conformal class $[g_{(0)}]$ is kept fixed, and a $[g_{(0)}]$ that admits conformal Killing vectors, the integrability criterion (10) is automatically satisfied by virtue of $\omega(g_{\mu\nu}, \delta g_{\mu\nu}, \mathcal{L}_\xi g_{\mu\nu})|_{B_{12}} = 0$, where here ξ is an asymptotic conformal Killing vector (ACKV), *i.e.* it approaches the conformal Killing vectors ζ at the boundary, see appendix B of [9] for the specific fall-off conditions) Then the Wald Hamiltonian integrates to (4).

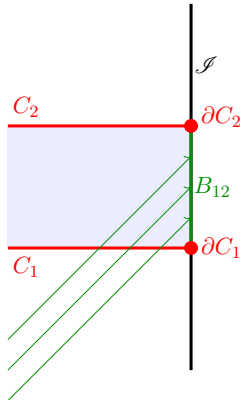


FIG. 2. Illustrating flux in an AIAdS spacetime. If flux arrives in B_{12} , the gravitational charge at C_2 will differ from that in C_1 . The conformal boundary \mathcal{S} is a timelike hypersurface and the C_1, C_2 are spacelike.

When there is gravitational radiation from the interior, it will reach the conformal boundary at finite proper time. As a result, radiative AdS spacetimes have a time-dependent boundary conformal structure, as was confirmed recently in [56]. In such cases there are no conserved charges because the boundary conformal structure does not admit conformal Killing vectors. However, if the gravitational flux arrives in finite time intervals, so that the boundary geometry outside these intervals admits conformal Killing vectors, there will be piece-wise constant gravitational charges, and the difference in their values between two instances of time will account for the gravitational flux arriving at the conformal boundary, see Fig. 2. More generally, if the spacetime admits a limit where conserved quantities exist, then one can discuss the flux of such quantities away from that limit. An example is the Robinson-Trautmann solution which we will discuss below in the context of $\Lambda > 0$.

Asymptotically locally de Sitter: We now consider AdS spacetimes with Dirichlet boundary conditions, *i.e.*

we keep fixed a conformal class at each end. Different boundary conditions may be implemented as discussed in footnote 2. One may adapt straightforwardly all steps of $\Lambda < 0$ case to $\Lambda > 0$; in most formulae this amounts to differences in signs. However, there are important conceptual differences. The analogue of the surface C in (7) we used in the AIAdS case is now timelike, and because the conformal boundary has two components, one at future infinity and another at past infinity, such a C will have two ends. Let us consider the case where both past and future infinity admit conformal Killing vectors and a hypersurface C that extends from future infinity to past infinity. Let ξ be a bulk vector that is an ACKV near both past and future conformal infinity, approaching a boundary conformal Killing vector ζ_\pm^b at \mathcal{S}^\pm . Then

$$H_\xi[C] = Q_\xi^+ - Q_\xi^- \quad (12)$$

where $Q_\xi^\pm[C] = \int_{\partial C^{(\pm)}} ds_a T_{(\pm)b}^a \zeta_\pm^b$; $T_{(\pm)b}^a$ is the holographic energy momentum tensor, and ∂C^\pm are the two ends of C . Now consider two such surfaces C_1 and C_2 , see Fig. 3. Then (11) implies

$$\Delta Q_\xi^+(C_1, C_2) = \Delta Q_\xi^-(C_1, C_2) = 0 \quad (13)$$

where $\Delta Q_\xi^\pm(C_1, C_2) = Q_\xi^\pm[C_1] - Q_\xi^\pm[C_2]$. The first equality in (13) follows from $\omega(g_{\mu\nu}, \delta g_{\mu\nu}, \mathcal{L}_\xi g_{\mu\nu})|_{B_{12}^\pm} = 0$ and the second from applying Stokes' theorem to ΔQ_ξ^\pm independently at B_{12}^\pm , using the explicit formulae for Q_ξ^\pm . It follows that while the values of the charges may change from \mathcal{S}^- to \mathcal{S}^+ , their difference is zero. Recall that \mathcal{S}^\pm are Cauchy surfaces and consider any other Cauchy surface I_t . Then as we now argue, $\Delta Q_\xi^t = \Delta Q_\xi^-$, where $\Delta Q_\xi^t(C_1, C_2) = Q_\xi^t[C_1] - Q_\xi^t[C_2]$, and $Q_\xi^t[C] = \int_{C \cap I_t} ds_a 2\pi_b^a \xi^b$, with π_{ab} the conjugate momentum of the induced metric at I_t (here, for simplicity, we assume that we deal with pure gravity). In other words, ΔQ_ξ^t is conserved under time evolution, *i.e.* independent of the Cauchy surface I_t . To see this, note that since ω is closed, its integral over the boundary of the region enclosed by $C_1, C_2, I_t, \mathcal{S}^-$ is zero, so ΔQ_ξ^- must be equal to ΔQ_ξ^t plus any flux passing through B_{12}^+ . However, there cannot be such flux, since any flux would eventually reach future infinity and as a result the conformal class at \mathcal{S}^+ would not admit conformal Killing vectors. Since we assume that future infinity admits conformal Killing vectors, there cannot be gravitational flux in the interior, and we find $\Delta Q_\xi^- = \Delta Q_\xi^t$. So while Q_ξ^t may depend on t , $\Delta Q_\xi^t = 0$ for all t . One may also check that known solutions such as Kerr-de Sitter have $\Delta Q_\xi^+ = 0$.

In our discussion we consider timelike slices C that extend from the future infinity to past infinity. One may also consider hypersurfaces that start and end at future infinity. Such C may be obtained by using the portion of the hypersurface C_1 from future infinity till I_t , then join

² Other boundary conditions may be obtained by adding finite boundary terms that implement this change of boundary condition and tracking their contributions, see also [45, 46].

B_{12}^t and return to future infinity using (the time-reverse of) C_2 .

Let us now consider the effects of radiation, *i.e.* ξ is no longer an ACKV. In this case the same argument implies that ΔQ_ξ^+ will differ from ΔQ_ξ^t by the amount of net radiation in the region bounded by \mathcal{I}^+ , C_1 , C_2 and I_t . In many previous discussions the contribution from I_t is either ignored or argued to be zero by postulating suitable decay rates in the deep interior. Assuming $\Delta Q_\xi^t = 0$, any non-zero ΔQ_ξ^+ will be due to gravitational radiation at \mathcal{I}^+ .

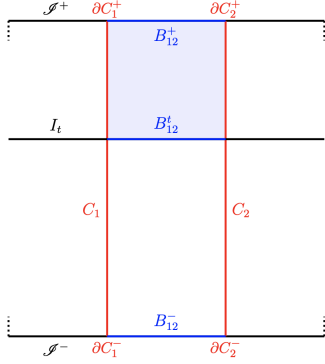


FIG. 3. Timelike C_1, C_2 intersecting spacelike conformal boundaries \mathcal{I}^\pm in AldS spacetime.

Robinson-Trautman To conclude we discuss an example of a radiative dS solution: the Robinson-Trautman (RT) solution. RT [57] describes an isolated system which relaxes to equilibrium by radiating excess energy via gravitational waves, becoming Schwarzschild dS. The solution exists for any value of the cosmological constant, and here we discuss the $\Lambda > 0$ case. The metric is

$$ds^2 = -\Phi_\Lambda du^2 - 2dudr + 2r^2 P^{-2} d\zeta d\bar{\zeta}, \quad (14)$$

where $P = P(u, \zeta, \bar{\zeta})$ and

$$\Phi_\Lambda = \Delta \ln P - 2r(\ln P)_{,u} - \frac{2m}{r} - \frac{\Lambda}{3} r^2. \quad (15)$$

$\Delta = 2P^2 \partial^2 / \partial \zeta \partial \bar{\zeta}$ and P satisfies the *Robinson-Trautman* equation:

$$(\ln P)_{,u} + \frac{1}{12M} \Delta \Delta (\ln P) = 0. \quad (16)$$

Using results from [58], a representative of the conformal class at \mathcal{I}^+ is

$$ds_{(0)}^2 = g_{(0)ab} dx^a dx^b = dy^2 + \frac{6}{\Lambda} \hat{P}^{-2} d\zeta d\bar{\zeta} \quad (17)$$

where \hat{P} denotes the boundary value of P . This metric generically possesses no conformal Killing vectors and thus we cannot define charges. However, as $y \rightarrow \infty$ the solution approaches Schwarzschild dS which has an isometry related to mass, and therefore we may discuss its

flux. Let us consider a vector field which in this limit is associated with *Bondi “time” translation* along the boundary (note that it is spacelike). The vector generating these translations is

$$\xi_{(0)}^a \partial_a = \frac{\hat{P}_0}{\hat{P}} \partial_y \quad (18)$$

where $\hat{P}_0 = 1 + \zeta \bar{\zeta} / 2$ is the limit of \hat{P} as $y \rightarrow \infty$. The normalisation of this vector relates to the RTdS metric (14) needing to undergo a coordinate transformation to be expressible in Bondi gauge [59]. Exploiting results from [58], one can show that

$$Q_\xi^+ = \frac{2m}{\kappa^2} \int_{\partial C^+} \left(\frac{P_0}{P} \right)^3 d\mu_0 \quad (19)$$

where $d\mu_0$ is the area element on the unit S^2 . This quantity is analogous to the *Bondi mass*, \mathcal{M}_B , of RT in the asymptotically flat setting [4]. It has been previously observed [58, 60, 61] that the quantity defined in equation (19) is monotonically decreasing regardless of the sign of Λ . Our construction thus illustrates explicitly how the Bondi mass of the asymptotically flat RT solution arises in the current context.

Since we have not used an asymptotic conformal Killing vector to construct \mathcal{M}_B , we should expect to discover additional flux through $B_{12}^+ \subset \mathcal{I}^+$ when comparing the difference in Bondi mass between the hypersurfaces C_1 and C_2 . We find the following equation for the flux through B_{12}^+ :

$$\Delta Q_\xi^+ = \mathcal{M}_B|_{\partial C_1^+} - \mathcal{M}_B|_{\partial C_2^+} = - \int_{B_{12}^+} \mathbf{F}_{\xi(0)} \quad (20)$$

where

$$\mathbf{F}_\xi = \left(-\frac{1}{2} \sqrt{g_{(0)}} T^{ab} \mathcal{L}_\xi g_{(0)ab} \right) \epsilon_3 \quad (21)$$

Applying this to the Robinson-Trautman solution we find

$$\mathbf{F}_{\xi(0)} = -\frac{2m}{\kappa^2} \partial_y \left(\frac{P_0}{P} \right)^3 \sqrt{\hat{\sigma}} \epsilon_3 \quad (22)$$

where $\hat{\sigma}_{AB} dx^A dx^B = 2P_0^{-2} d\zeta d\bar{\zeta}$ is the metric on the unit round S^2 . This flux formula gives

$$\mathcal{M}_B|_{\partial C_1^+} - \mathcal{M}_B|_{\partial C_2^+} \leq 0. \quad (23)$$

by monotonicity of the expression defined in equation (19) [60, 61].

It would be interesting to compute ΔQ_ξ^t and confirm monotonicity in u . For this we would need to know the form of Bondi time translation vector field in the deep interior of the RT spacetime. We leave this computation to future work.

Conclusions We have discussed the definition of charges in asymptotically locally de Sitter spacetimes. Our main result is that the difference of charges $\Delta Q_{\xi}^t(C_1, C_2)$ defined using timelike surfaces C_1 and C_2 at the Cauchy surface I_t is zero if gravitational flux is absent. If there is gravitational flux the corresponding quantities differ by the net amount of flux. It would be interesting to exemplify our discussion with further explicit examples, investigate possible connections with work on gravitational wave memory [62–69], and analyse possible relevance to present and future gravitational wave observations.

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