The interrelationship between the carbon market and the green bonds market: Evidence from wavelet quantile-on-quantile method

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Highlights

- Relationship between ECX EUA carbon futures prices and the S&P green bonds index.
- Use the MODWT method to form short-, medium-, and long-term perspectives.
- Carbon price Granger causes green bonds revealed from the quantile Granger test.
- The asymmetric influence of the carbon price revealed by the QQ regression.
- The carbon price mostly positively affects green bonds.
- The effects are more pronounced when both markets are in similar extreme conditions.

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Abstract

The 26th edition of the United Nations climate change conference (COP26) underlines the importance of financial products and markets related to “carbon” (e.g., carbon and green bond markets). We, to our knowledge, are the first to construct a framework based on multiple time scales and market conditions to quantify the interrelationship between the carbon futures and green bond markets. Specifically, we estimate it from short-, medium-, and long-term perspectives and different market conditions by combining the maximum overlap discrete wavelet transform (MODWT) and two quantile methods to decompose the sequences into various frequencies and quantiles. We find that the carbon futures price unilaterally Granger causes the green bond index and empirically analyzes the asymmetric impact of the carbon futures with a two-dimensional quantile model constructed by the quantile-on-quantile (QQ) regression approach. We find positive effects of the carbon futures in the medium to long term and erratic performance in the short term. The effects are more pronounced when both markets are in an extreme state. Our findings enrich the research related to eco-economy and carbon finance, providing a more comprehensive and detailed research framework and helping others optimize investment portfolios and policy arrangements.

Keywords: Carbon futures; Green bond; Wavelet analysis; Quantile Granger causality test; Quantile-on-quantile regression

1. Introduction

The United Nations Climate Change Conference hosted in Glasgow, from 31 October to 12 November 2021, marks the 26th former conference of the United Nations (UN) Framework on Climate Change (COP26). One crucial implication of COP26 is to deepen the understanding of climate-related risks, products and markets for both policymakers and academics. The gradual increase in climate-related risks has impelled countries and regions to set up numerous carbon-related trading platforms or markets to balance economic development and carbon emissions (Zhou and Li, 2019; Crecente et al., 2021). Financial products and markets related to “carbon” (e.g., carbon and green bond markets) have had the most apparent and far-reaching effects (Dong et al., 2020; Ren et al., 2021; Arif et al., 2021). The establishment of the carbon futures market is mainly to hedge the risks brought by carbon trading, while the green bond market is a market to provide transitional funds to promote carbon emission reduction (Lucia et al., 2015; Banga, 2019; Rubtsov et al., 2021).

According to the purpose and content of the transactions in the two markets, they share the same intention of reducing greenhouse gas emissions (e.g., carbon dioxide) and realizing environmentally friendly economic development (Tolliver et al., 2020; Flammer, 2021). In some countries and regions, such as Europe and China, various carbon trading and green bond markets have started to develop rapidly at similar times. In the relevant policy arrangements for low-carbon development, these two markets are also frequently concerned together. Green financing represented by green bonds can also provide financial support for various carbon trading markets in many cases. A comprehensive grasp of their features is of great significance to the correlative arrangement for economic activities and low-carbon transformation. Therefore, do these two markets do have some connections? Or is there a coordinated comovement as they develop? Unfortunately, information on these issues still needs to be further explored, motivating us to analyze their relationship in depth.

Against the background of global low-carbon development and economic integration, many unique characteristics of these two markets have been extensively reported, but inadequate attention has been paid to the interrelationship between them (Rannou, 2019; Banga, 2019). The carbon market is susceptible to external economic factors (Zhang and Wei, 2010; Ren et al., 2022a) and can effectively reduce the cost of carbon emission reduction and standardize relevant mechanisms (Cui et al., 2014; Zhu et al., 2020). It becomes a mature and vital financial market with continuous development and improvements (Wen et al., 2020b). There are some deficiencies and potential risks associated with the green bond market compared with the carbon market. For example, a lack of uniform standards, long project acceptance, and “greenwashing” behaviors (Karpf and Mandel, 2018; Flammer, 2021), making its links with other markets relatively less prominent. Nevertheless, the green bond market has also developed rapidly due to its advantages of low-cost issuance, improvement of environmental performance, flexible project scheduling, avoidance of supervision over financial institutions, and so on (Wood and Grace, 2011; Tolliver et al., 2020; Cao et al., 2021). Although both markets have been among the fastest-growing players in the yield of carbon finance in recent years, few studies have investigated the interrelationship between them.

We fill this research gap by studying the interrelationship between the carbon and green bond markets and incorporating more realistic factors into our research framework (i.e. time scales and market conditions). We choose the ECX EUA (European Climate Exchange EU allowances) carbon futures and the S&P (Standard & Poor’s) green bond index as the basic sequences since they are typical and widely used indicators of the carbon and green bond markets (Dhamija et al., 2018). We control the economic policy uncertainty (EPU) to avoid some interference factors caused by economic fluctuations, which could significantly affect both markets (Zhang and Yan, 2020; Adams et al., 2020; Pham and Nguyen, 2021; Ye., 2022). We use the maximum overlap discrete wavelet transform (MODWT) method to divide the sequences into several frequencies corresponding to different time scales. The wavelet decomposition method has more flexibility than the traditional time series analysis method, since the time scales can be adjusted according to the content of the analysis (Kumah and Mensah, 2020). We apply the quantile Granger test and quantile-on-quantile (QQ) regression to further reflect on these two markets' interrelationships and investigate the potential causal relationship and asymmetric effects on these two-dimensional levels for different time scales. These two quantile-based approaches can reflect marginal effects from multiple market conditions, making the empirical process more comprehensive (Lin and Su, 2020; Ren et al., 2022c). We find that the green bond market is influenced unilaterally by the carbon futures market, and the role of carbon futures varies in different situations.

We contribute to the existing literature in at least two aspects. Firstly, this study is the first to focus on the specific interrelation between the European carbon futures market and the global green bond market from a time and frequency view through the MODWT wavelet decomposition. Closely related studies include Rannou et al. (2020) and Jin et al. (2020), which provide somewhat mixed evidence on the specific connection between carbon and green bond markets. Rannou et al. (2020) find a two-way transmission effect between the European carbon market and the green bond market, but there is no significant two-way spillover effect. Meanwhile, Jin et al. (2020) find that the correlation between the carbon futures and the green bond index is the highest among four major market indices (market volatility, commodity, energy, and green bonds), and the green bond index is the best hedging instrument for carbon futures. Unlike their research, this paper tries to concretize the relationship between these two carbon-related financial markets. Different from the literature (e.g., Jin et al., 2020; Rannou et al., 2020; Fang et al., 2020; Gozgor et al., 2019), this paper provides a new perspective on the relationship between carbon and green bond markets for scholars and investors to refer to.

Secondly, we conduct a detailed analysis from short-term, medium-term, and long-term perspectives by decomposing the data into sequences of multiple frequencies, thereby simultaneously reducing the impact of special shocks, such as the COVID-19 pandemic. The causality direction and marginal effects between these two markets are tested by combining the MODWT approach with the quantile Granger and QQ regression methods. This combination constructs short-, medium- and long-term scenarios with various quantiles that reflect their market conditions. We obtain the unilateral Granger causality of the carbon futures market on the green bond market across different quantiles and time scales, providing new evidence for the hedging function of green bonds. Apart from this, we quantify the overall positive role of the carbon futures market in the medium to long term and the negative impact on the green bond market with a bear market condition in the short term, which could reveal strategies for investment optimization and policymaking.

The remainder of this paper is as follows: Section 2 reviews the relevant literature. Section 3 introduces our methods and data. Section 4 presents the results of the empirical analyses and robustness tests. Finally, Section 5 concludes.

2. Literature review

The "carbon market", which refers to the "carbon trading market" in most cases, has unique advantages. The carbon trading market is subject to carbon dioxide emissions or emission rights, and the carbon futures market is one of the core markets of carbon trading, which is to settle or deliver these subject matters in the future. Investors can invest or speculate in carbon futures. A large amount of market supply and demand information about carbon is concentrated in the carbon market, and it plays an increasingly important role. On the one hand, the formation of the carbon market has reduced carbon emissions and has become an essential boost to the development of the environmental economy (Fan et al., 2017; Wen et al., 2020a). On the other hand, the carbon market has become an important market for global investment, risk aversion, and financial planning (Zhang and Huang, 2015; Ren et al., 2022b).

The fossil energy markets are most closely connected with the carbon market, and the relationship between them is also one of the most well-studied areas of research. Energy consumption is the primary source of carbon emissions (Zhang and Sun, 2016; Semeyutin et al., 2021). Moreover, changes in the energy market brought about by economic development will also promote the development of carbon trading and the carbon market (Nazifi and Milunovich, 2010; Cheng et al., 2021). Based on these findings, it is not uncommon to link the carbon market with the energy market. For instance, Mansanet-Bataller and Soriano (2012) find a two-way wave transmission between the oil and carbon trading market, while Reboredo (2014) finds no spillover effect between them by proposing a multivariate conditional autoregressive range model to capture the interrelationship between the oil market and the carbon trading market. Recently, Wang and Guo (2018) use the spillover index and find an asymmetric volatility spillover effect between the EUA carbon market and the WTI oil, Brent oil, and EU natural gas prices. Ji et al. (2018) consider the interrelationship between electricity price and the carbon market and believe that the electricity price is the central receiver of information transmission. Chen et al. (2019) consider oil, natural gas, and coal in their research and verify the volatility spillover effect and the dynamic interrelationship between carbon emission quota and energy prices using an asymmetric model. The comparison shows a relatively stable positive interrelationship between the carbon emission quota and crude oil and natural gas prices. However, the interrelationship between the carbon emission quota price and coal is weaker and less stable.

As many studies have shown a significant correlation with energy markets, the interrelationship between carbon and other financial markets is becoming more powerful. Interestingly, energy markets were more likely to drive changes in the carbon market than financial assets before the financial turmoil caused by the subprime crisis in 2008. After the economic crisis, the carbon market became more sensitive to financial factors, such as stock prices. The carbon market is affected by financial factors and the economic environment. For example, financial development will inevitably bring about a substantial increase in carbon emissions, especially in emerging financial markets and developing countries (Mol, 2012). Furthermore, economic factors could lead to the emergence and expansion of the carbon market. The development of financial services can improve the structure of the carbon market and enhance the liquidity of carbon-related transactions. The activity of financial institutions and investors has also provided an indispensable impetus to the prosperity of the carbon market (Bosetti et al., 2011; Hintermann, 2017).

Using the Copula model, Yuan and Yang (2020) find that the uncertainties in the financial market and the crude oil market both have significant asymmetric risk spillovers in the carbon market. However, when a systemic risk occurs, the uncertainty in the stock market will transfer this risk to the carbon market more effectively than in the crude oil market. Tan et al. (2020) quantitatively analyze the interrelationship between the European carbon market and information from other markets. They find that the carbon market is closely related to the stock and non-energy commodity markets, in which financial risk-based macroeconomic factors also have a huge impact. Still, the correlation with the bond market is insufficient.

Like the carbon market, the green bond market was also set up to mitigate climate change. These two markets were set up with similar intentions to a certain degree and are the backbone of emerging markets that cannot be ignored in recent years. Research on green bonds has mainly focused on their relationship with other markets and policy factors (both macro and micro), while research focusing on its interrelationship with carbon markets is scarce. In most situations, the green bond market is generally considered a recipient of information or shocks due to the market's late start and insufficient maturity. Reboredo (2018) studies the interrelationship between green bonds and the stock, energy, and bond markets and finds that their correlation is weak. Therefore, green bonds can be considered a diversification tool for investment. At the same time, other papers confirm that green bonds have a stronger relationship with the traditional bond market and other fixed-income markets (such as the US treasury bond market) when compared with the clean energy market and other green financial derivatives markets (Baruník and Křehlík, 2018; Broadstock and Cheng, 2019). Recently, Pham (2021) uses a quantile approach similar to that used in this paper to construct a research model of the relationship between the green bond market and the green stock market. The results show that the dependence between green bonds and green stocks is relatively small under normal market conditions. In extreme market movements, green bonds and green stocks are more closely linked. However, all the spillover effects between green bonds and green equity are in the short term and dissipate within the medium- and long-term investment scope.

Research on the specific relationship between these two markets is also emerging. Rannou et al. (2020) point out that Europe is the first region to establish a carbon trading market and a green bond market. The price trajectories of these two markets in the six years from 2014 to 2019 suggest that they have similarities and some complementarities. Rannou et al. (2020) find out a two-way transmission effect between the European carbon market and the green bond market, but there is no significant two-way spillover effect between them. Therefore, the European green bond market can hedge the risk of the carbon market. Meanwhile, Jin et al. (2020) examine the relationship between carbon futures returns and the four major market indices (indexes of the market volatility, commodity, energy, and green bonds) based on the dynamic hedging ratios and the OLS (ordinary least square) method. The correlation between the carbon futures and the green bond index is the highest, and the green bond index is the best hedging instrument for carbon futures, even during crises.

In summary, the carbon market and the green bond market share the same goals of environmental protection, growth speed, and bright prospects. The carbon market has gradually become a crucial part of the global economic system. At the same time, there is no doubt about the trend of green bonds toward prosperity under the macro background of low-carbon development. The literature on these two markets and other markets is growing, forming a relatively comprehensive view of the global market network structure. However, the evidence on the causal relationship between them is inadequate, motivating us to explore the relationship between these two markets comprehensively.

3. Methodology and data

We study the interrelationship between the ECX EUA carbon futures price and the S&P green bond index using multiple time scales and quantiles based on a framework constructed by the wavelet quantile-on-quantile regression methods. We test the quantile causal relationship between the carbon futures price and the green bond index based on the MODWT method to decompose the carbon futures price and green bond index into several frequencies. Then, starting with the quantile causality results, we investigate specific effects using a quantile-on-quantile test and further analyze the interrelationship between these two series under different market conditions.

3.1 Maximum overlap discrete wavelet transform

Following Percival and Walden (2000) and Das and Kannadhasan (2018), we chose the wavelet decomposition method to process the sample data, generating the foundational sequences of research. The basis of wavelet analysis is to construct a pair of special functions. The father wavelet mainly captures the low frequency and stationary part of the sequence, and the mother wavelet mainly captures the high frequency and particular part of the sequence. The integrals of both over the entire time range are 1 and 0, respectively. The specific expressions are as follows:

$$\varphi_{jk} = -2^{-\frac{j}{2}}\varphi\left(\frac{t-2^{jk}}{2^j}\right), \int \varphi(t)dt = 1, \quad (1)$$

$$\psi_{jk} = -2^{-\frac{j}{2}}\psi\left(\frac{t-2^{jk}}{2^j}\right), \int \psi(t)dt = 0, \quad (2)$$

where $j = 1, ..., J$ indexes the scale, and $k = 1, ..., K$ indexes the translation.

The father wavelet smooth coefficients and mother wavelet detail coefficients are set as follows:

$$S_{J,K} = \int f(t)\varphi_{j,k}, \quad (3)$$

$$d_{J,K} = \int f(t)\psi_{j,k}. \quad (4)$$

The mathematical form and simplified form of f (.) above are:

$$f(t) = \sum_k S_{J,k}\,\varphi_{J,k}(t) + \sum_k d_{J,k}\,\psi_{J,k}(t) \ldots + \sum_k d_{j,k}\,\psi_{j,k}(t) \ldots + \sum_k d_{1,k}\,\psi_{1,k}(t), \quad (5)$$

$$f(t) = S_J + D_J + D_{J-1} + \cdots + D_j + \cdots + D_1, \tag{6}$$

with orthogonal components defined as follows:

$$S_j = \sum_k S_{j,k}\ \varphi_{j,k(t)}, \tag{7}$$

$$D_j = \sum_k d_{j,k}\ \psi_{j,k(t)}.\ \ j = 1,2,\dots,J. \tag{8}$$

We rely on the maximum overlap discrete wavelet transform (MODWT) due to its superior flexibility to other wavelet forms (Percival and Walden, 2000). Less stringent sample size requirements and more flexible conversions make MODWT more amenable to economic data analysis. The first step of MODWT is to set the filter. For sequences $X = \{X_t; t = 0, \dots, N-1\}$ with N observations, we define the wavelet filter $\tilde{W}_l = W_1/\sqrt{2}$ and the scale filter $\tilde{G}_l = \frac{G_1}{\sqrt{2}} = (-1)^{l+1}\tilde{G}_{L-1-t}$, which have properties as follows:

$$\sum_{l=0}^{L-1} \tilde{W}_l = 0, \sum_{l=0}^{L-1} \tilde{W}_l^2 = \frac{1}{2}, \sum_{l=0}^{L-1} \tilde{W}_l \tilde{W}_{l+2n} = 0, \tag{9}$$

$$\sum_{l=0}^{L-1} \tilde{G}_l = 1, \sum_{l=0}^{L-1} \tilde{G}_l^2 = \frac{1}{2}, \sum_{l=-\infty}^{\infty} \tilde{G}_l \tilde{G}_{l+2n} = 0, \tag{10}$$

$$\sum_{l=-\infty}^{\infty} \tilde{G}_l \tilde{W}_{l+2n} = 0. \tag{11}$$

Secondly, we clear the wavelet coefficients and scale coefficients as follows:

$$\tilde{H}_{1,t} = \sum_{l=0}^{L-1} \tilde{W}_l X_{t-lmodN},$$

(12)

$$\tilde{V}_{1,t} = \sum_{l=0}^{L-1} \tilde{G}_l X_{t-lmodN}, t = 0,1,\cdots,N-1,$$

(13)

where $\tilde{H}_{1,t}$ and $\tilde{V}_{1,t}$ are the wavelet and scale coefficients of the first layer. mod represents the process of "congruence modulo"[1]. The coefficients of the jth layer are $\tilde{H}_{j,t}$ and $\tilde{V}_{j,t}$, respectively, and the respective equations are:

$$\tilde{H}_{j,t} = \sum_{l=0}^{L-1} \tilde{W}_{j,l} X_{t-lmodN},$$

(14)

$$\tilde{V}_{j,t} = \sum_{l=0}^{L-1} \tilde{F}_{j,l} X_{t-\ lmodN}\ , t = 0,1,\cdots,N-1,$$

(15)

$$\tilde{W}_{j,l} = \frac{W_{j,l}}{2^{\frac{j}{2}}}, \tilde{F}_{j,l} = \frac{F_{j.l}}{2^{\frac{j}{2}}},$$

(16)

where $\tilde{W}_{j,l}$ and $\tilde{G}_{j,l}$ are the wavelet filter and scale filter in layer j, and the width is $L_j = \left(2^j - 1\right)(L-1) + 1$. Following Kumah and Mensah (2020), the periods of 2—4, 4–8, 8–16, 16–32, 32–64, and 64–128 days are represented by wavelet scales D_1, D_2, D_3, D_4, D_5, and D_6, respectively. Furthermore, D_1, D_4, and D_6 correspond to short-term, medium-term, and long-term time scales, respectively. Using the wavelet decomposition method, we can extract the "stable trend" under different frequencies

[1] When two integers are divided by the same positive integer, if the remainder is the same, the two integers are congruent.

and reduce the interference of "noise", such as some special events. What's more, it enables researchers to customize the research frequency according to different research purposes.

3.2 Quantile Granger causality test

This subsection presents the quantile method used to test the causality between the carbon futures market and the green bond market. In short, Granger causality dictates that X_T does not Granger-cause Y_T if it can not predict Y_T. The time *T* can be adjusted according to the research objectives. We introduce the method in this section by taking X_t, Y_t(at the same period *t*) as an example. Mathematically, an explanatory vector $I_t \stackrel{\text{def}}{=} (I_t^Y, I_t^X)' \in R^d,\ d = s + q$. I_t^X is the past information set of X_t, $I_t^X \coloneqq (X_{t-1}, \ldots, X_{t-q})' \in R^q$. The null hypothesis of Granger non-causality is defined as below:

$$H_0: F_Y(y|I_t^Y, I_t^X) = F_Y(y|I_t^Y). \ \ \forall\, y \in R. \tag{17}$$

Here, $F_Y(y|.)$ represents the conditional distribution of given (I_t^Y, I_t^X). X_t does not Granger-cause Y_t in mean if:

$$E(Y_t|I_t^Y, I_t^X) = E(Y|I_t^Y), a.s. \tag{18}$$

where $E(Y_t|I_t^Y, I_t^X)$ and $E(Y|I_t^Y)$ are the mean values of (I_t^Y, I_t^X) and $(Y|I_t^Y)$, respectively. However, the Granger test results for the means do not reflect the effects on different quantiles and may be affected by various factors. Therefore, Jeong et al. (2012) proposed Granger causality in quantiles. If we define $Q_T^{Y,X}(.|I_t^Y, I_t^X)$ as the τ-quantile of $F_Y(.|I_t^Y, I_t^X)$, we obtain the value of $Q_T^Y(.|I_t^Y)$.

We rewrite the null hypothesis as the following (where *T* refers to the compact set and $T \in [0,1]$):

$$H_O: Q_\tau^{Y,X}(Y_t|I_t^Y, I_t^X) = Q_\tau^Y(Y_t|I_t^Y), a.s. \forall\, \tau \in T\ . \tag{19}$$

The conditional τ-quantile of Y_t satisfies the following restrictions:

$$Pr\{Y_t \leq Q_T^Y(Y_t|I_t^Y)|I_t^Y\} \coloneqq \tau, a.s. \forall\, \tau \in T, \tag{20}$$

$$Pr\left\{Y_t \leq Q_T^{Y,X}(Y_t|I_t^Y, I_t^X)\middle|I_t^Y, I_t^X\right\} \coloneqq \tau, a.s. \forall\, \tau \in T, \tag{21}$$

Given the independent variable I_t, the probability $Pr\{Y_t \leq Q_T(Y_t|I_t)|I_t\} = E\{1[Y_t \leq Q_T(Y_t|I_t)]|I_t\}$. Here an event is denoted by an indicator function $1[Y_t \leq Y]$. Hence, the Granger non-causality null hypothesis can be rewritten as follows:

$$E\left\{1\left[Y_t \leq Q_T^{Y,X}(Y_t|I_t^Y, I_t^X)\right]\middle|I_t^Y, I_t^X\right\} = E\{1[Y_t \leq Q_T^Y(Y_t|I_t^Y)]|I_t^Y\}, a.s. \forall\, \tau \in T. \tag{22}$$

Assuming that $Q_T(.|I_t)$ is appropriately specified through a parametric model that refers to a family of functions defined by $M = \{m(.|\theta(\tau))|\theta(.): \tau \to \theta(\tau) \in \Theta \subset R^p$, then the Granger non-causality relationship is such that:

$$H_O: E\left\{1\left[Y_t \leq m\left(I_t^Y, \theta_0(\tau)\right)\right]\middle|I_t^Y, I_t^X\right\} = \tau, a.s. \forall\, \tau \in T. \tag{23}$$

where $m\left(I_t^Y, \theta_0(\tau)\right)$ is the actual conditional quantile for $Q_T^Y(.|I_t^Y)$. We now rewrite the null hypothesis based on the sequence of moment restrictions that are unconditional as given below:

$$E\left\{1\left[Y_t - m\left(I_t^Y, \theta_0(\tau)\right) \leq 0\right] - \tau\right\} exp(i\omega' I_t)\} = 0. \tag{24}$$

Applying the test statistic as proposed by Troster (2018), we get:

$$P_T \coloneqq \int_\tau \int_Z |v_T(\omega, \tau)|^2 dF_\omega(\omega) dF_\tau(\tau), \tag{25}$$

$$v_T(\omega, \tau) \coloneqq \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \left\{1\left[Y_t - m\left(I_t^Y, \theta_0(\tau)\right) \leq 0\right] - \tau\right\} \exp(i\omega' I_t)\}. \tag{26}$$

Let $\varphi_{\tau_j}(.)$ be the function such that $\varphi_{\tau_j}(\varepsilon) \coloneqq 1(\varepsilon \leq 0) - \tau_j$, and applying the test statistic, we obtain the estimation of test statistics:

$$P_T = \frac{1}{Tn}\sum_{j=1}^{n}\left|\vartheta_j' Z \vartheta_j\right|, \tag{27}$$

where Z is defined as the TxT matrix and ϑ_j is the jth column of φ. Troster (2018) showed the subsampling procedure to estimate the critical values of P_T. Although Granger causality test does not indicate that there is a strong causal relationship, we first conduct the Granger causality test to verify whether there is a certain correlation between the two and whether the relationship is unidirectional or bidirectional. The quantile Granger causality test we used showed more predictive power on joint distribution, and the test results also provided a more scientific basis for our subsequent quantile-on-quantile regression.

3.3 Quantile-on-quantile regression approach

We further rely on the derivative method of quantile regression, the quantile-on-quantile regression method proposed by Sim and Zhou (2015). This method is robust to outliers and non-normality in actual data, and as a nonparametric local linear regression method, it can reflect the conditional distribution and reveal potential structural mutations. The QQ regression method can comprehensively test the specific marginal influence between variables under each quantile, in contrast with the OLS method and the quantile regression method (Ren et al., 2019; Duan et al., 2021). We take the impact of the carbon futures market on the green bond market as an example to explain the process of the QQ regression method more intuitively. Our operation steps are as follows:

Firstly, we define the nonparametric quantile regression equation for the green bond index return (G_t) as a function of carbon futures return shocks (C_{t-1}) and EPU (E_t)

$$G_t = \beta^\theta(C_{t-1}) + \alpha^\theta E_t + \varepsilon_t^\theta, \tag{28}$$

where C_{t-1} represents carbon futures price return at the time t-1, θ stands for the θ-quantile of green bond index return (G_t), and α^θ is the effect of the θ-quantile of E_t at the time t. $\beta^\theta(\cdot)$ represents the impact of C_{t-1} on G_t, which is the function we want to test.

To examine the impact (represented by C^τ) of the τ-quantile of C_{t-1} shocks on the θ-quantile of G_t, we expand $\beta^\theta(\cdot)$ by making a first-order Taylor expansion around C^τ:

$$\beta^\theta(C_{t-1}) \approx \beta^\theta(C^\tau) + \dot{\beta}^\theta(C^\tau)(G_{t-1} - C^\tau) \equiv b_0(\theta,\tau) + b_1'(\theta,\tau)(C_{t-1} - C^\tau), \tag{29}$$

Combining Eq. (28) and Eq. (29), we obtain:

$$G_t = \beta^\theta(C^\tau) + \dot{\beta}^\theta(C^\tau)(C_{t-1} - C^\tau) + \alpha^\theta E_t + \varepsilon_t^\theta, \tag{30}$$

Then, we solve Eq. (30) by considering

$$\begin{pmatrix} \widehat{b}_0(\theta,\tau) \\ \widehat{b_1}(\theta,\tau) \\ \widehat{\alpha^\theta}(\tau) \end{pmatrix} = arg \min_{b_0, b_1, \alpha^\theta} \sum_{t=1}^{T} \rho_\theta\left[G_t - b_0 - b_1(C_{t-1} - C^\tau) - \alpha^\theta E_t\right] K\left(\frac{F(C_{t-1}) - \tau}{h}\right). \tag{31}$$

where $\rho_\theta(y) = y(\theta - I_{\{y<0\}})$ and I_A is the function of the set A, K is a Gaussian kernel function on R, and $h > 0$ is the bandwidth. The empirical distribution function is $F(C_{t-1}) = \frac{1}{T}\sum_{k=1}^{T} I(O_k < O_{T-1})$. We use the following to obtain the optimal $\widehat{\alpha^\theta}$:

$$\tilde{\alpha}^\theta = \frac{1}{n}\sum_{i=1}^{n} \hat{\alpha}^\theta(\tau_i). \tag{32}$$

Lastly, we use the cross-validation (CV) method to set the optimal *h,* following Duan et al. (2021), and strengthen the robustness of the estimates of the QQ regression method.

3.4 Data

We collect the daily closing prices of the S&P green bond index and the ECX EUA carbon futures for the data analysis in this paper. The original ECX EUA carbon futures price data is from the Intercontinental Exchange, while the daily price of the S&P green bond index is from Bloomberg[2]. Our sample period is from January 08, 2013, to March 10, 2021. In addition, all sequences in our study are processed into the return series. This processing can enhance the stability of the data and reduce errors in the research process.

The time series plots of these three variables above are shown in Fig.1. We cannot observe apparent consistency in the changing trend of these three sequences over our sample period, and need more detailed empirical analyses to investigate the interrelationships among them. Table 1 and Fig. 2. show the descriptive statistics and the density plots of our data, respectively. The standard deviation of the S&P green bond index is the smallest (0.0014), and that of the carbon futures is slightly greater (0.0330). This may be due to the fact that the green bond market belongs to the fixed income securities markets, while the transactions of the carbon futures occur more frequently. Meanwhile, the fluctuation of economic policy uncertainty is the largest among these three (the standard deviation is 0.2196), which is in line with the fact that economic policies could change rapidly.

From the time series plots and the decomposed signal diagrams, we can see that there is no obvious synergistic effect among the three variables. Notably, in the first half of 2020, the three sequences all showed large fluctuations, most likely due to the sudden outbreak of COVID-19 (Elsayed et al., 2022). This phenomenon indicates that our decomposition results can be consistent with the actual situation, which proves the accuracy of our method. However, the shock of COVID-19 is not an individual case for

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[2] The daily data of the EPU of US are obtained from http://www.policyuncertainty.com/index.html. The uncertainty of global economic policy is monthly. In order to maintain the consistency of data, we choose the uncertainty of American economic policy with available daily data. The EPU of U.S. can be used as a representative indicator of global economic policy fluctuations in many cases.

each of the series throughout the sample period and may not even cause the most violent fluctuations. For example, in 2013, carbon futures prices and EPU both experienced periods of severe turbulence. Despite various ups and downs, with the increase of time scale, the sequence changes become gentler after wavelet decomposition, indicating that extreme data and noise are greatly reduced after MODWT processing.

The positive kurtosis values show the fat tail distribution for all sequences. Both the Jarque-Bera (JB) test and the Augmented Dickey-Fuller (ADF) test reject the null hypothesis significantly, which indicates that our data are non-normally distributed and stable. These two characteristics illustrate the necessity and correctness of the quantile method because the traditional approach cannot capture the asymmetry of the sequences in this paper.

[Insert Figs. 1 and 2 about here]

[Insert Table 1 about here]

4. Empirical results and robustness

4.1 Maximum overlap discrete wavelet transform analysis

We use the MODWT to decompose the daily price returns of the ECX EUA carbon futures price, the S&P green bond index, and the EPU data into six frequencies to better understand the interrelationship between the European carbon futures market and the global green bond market at different time scales. The six wavelet signals (i.e., *d1, d2, d3, d4, d5,* and *d6*) represent 2–4, 4–8, 8–16, 16–32, 32–64, and 64–128 days, respectively, and *d1* represents the short term, which is 2–4 days. Meanwhile, *d4* represents the medium term, which is 16–32 trading days with a corresponding period of approximately 3–6 weeks, and *d6* represents the long term, corresponding to 64–128 trading days with a period of approximately 3–6 months (Das and Kannadhasan, 2018; Kumah and Mensah, 2020).

The signals after the decomposition of these three sequences (i.e., the S&P green bond index return, the ECX EUA carbon futures price return, and the US EPU) are shown in Figs. 3, 4, and 5, respectively. According to these three figures, the synergy and regularity of their shifts cannot be extracted directly. However, these pictures display an overview of the performance of the series: the noise in the signal lessens, while their signal curves are smoother from short-term to long-term. The decomposition better captures data characteristics in different periods and reduces the error caused by some abnormal conditions, making the uncovering of the relationship between the carbon futures and the green bond flexible.

[Insert Figs. 3–5 about here]

4.2 Quantile Granger causality analysis

In this subsection, we rely on quantile Granger causality tests on the decomposed sequences to further clarify the Granger causal relationship between the ECX EUA carbon futures price and the S&P green bond index. Fig. 6 and Fig. 7 present these results, respectively. These two charts show the nonparametric mean Granger causality under every quantile (from $q = 0.05$ to $q = 0.95$) over each time scale (the curve above the red line represents the Granger causality at the 5% significance level). The position of the quantile of the return series reflects the performance of the market conditions and is roughly divided into the bear market ($q = 0.05$ to 0.45), the normal market (0.5 positions), and the bull market ($q = 0.55$ to 0.95), as suggested by Mensi et al. (2016), Selmi et al. (2018), and Kumah and Mensah (2020). In addition, we define extreme market conditions (where q is less than 0.05 or greater than 0.95).

First, we focus on the Granger causal test of the carbon futures market on the green bond market (Fig. 6). It can be directly summarized that the resulting curves of the Granger causality test all have an unsmooth inverted U shape, which indicates that the Granger causality between the carbon futures market and the green bond market is most apparent in the quantile of the middle segment (near $q = 0.5$). Changes in the carbon futures market will have the most substantial influence on green bonds when they are in non-extreme market conditions.

Comparing these three results in Fig. 6, we find that the quantiles where the carbon futures market plays a role in green bonds are also increasingly prominent with the expansion of the time scale. In the short term, the quantiles of significant effect range from above 0.25 to below 0.75. In contrast, in the long term, they expand from approximately 0.15 to 0.85. This can also be seen from the gradual expansion of the area enclosed by the resulting curve of the Granger causality test and the horizontal red line. This shows that the carbon futures price will exert less influence on the green bond index in the short term, especially under the unusual market conditions of green bonds. However, as time goes on, the influence of the carbon futures market on the green bond market gradually becomes apparent; even if the green bond is in a “bull market” or “bear market” conditions near the extreme will be affected.

According to Fig. 7, statistically, green bonds do not Granger-cause the carbon futures market (the outcome curve did not exceed the horizontal red line in all scenarios). In other words, green bonds do not predict the development of the carbon futures market, regardless of time scale. Therefore, consistent with the existing conclusions about the green bond market, the green bond market is more of a net price-spillover recipient than an exporter (Reboredo and Ugolini, 2020). Our results are slightly different from the results of a study by Rannou et al. (2020). They find that the European carbon market weakly correlates with the European green bonds market but has little correlation with the global green bonds market. In this paper, we focus on the relationship between the largest carbon futures market (the European market) and the global green bond market (the S&P green bond index is designed to measure the performance of green bonds globally). According to these results, the ECX EUA carbon futures price has a significant effect on the S&P green bond index. Our findings offer new evidence of market correlation in previous studies and demonstrate the prevalence of linkages between individual markets in the global economic system, even if it is only a one-way influence.

Our subsequent empirical analysis will focus on the effect of the carbon futures market on the green bond market with a more detailed and concrete deal, since the influence of S&P green bonds on ECX EUA carbon futures is statistically insignificant.

[Insert Figs. 6 and 7 about here]

4.3 Quantile-on-quantile regression estimates

Applying the QQ regression method, we obtain the empirical results of the influence of each quantile of the carbon futures market on the green bond market at each time scale. These results are shown in Figs. 8, 9, and 10. We mainly analyze the estimation of coefficients $b_0(\theta,\tau)$ and $b_1(\theta,\tau)$. The former $b_0(\theta,\tau)$ represents the constants of the regression analyses while $b_1(\theta,\tau)$ stands for the effect of the τ-th quantile of the ECX EUA carbon futures price on the θ-th quantile of the S&P green bond index.

According to the short-term results (as displayed in Fig. 8), the negative effect of carbon futures on green bonds (the dark blue region in Fig. 8) is mainly concentrated at the lower quantiles (the adjacent areas with θ = 0.1 and θ = 0.3). At the same time, the impact rises rapidly to the highest point ($b_1 = 0.0074$) when both θ and τ are at extremely high quantiles (greater than 0.9). When the return of the ECX EUA carbon futures price increases by 1 unit, the return of the S&P green bond index will increase by 0.0074 units. This indicates that when both the green bonds and carbon futures markets are highly active, the carbon futures have the most apparent positive effect on the green bonds. However, this result may not be very significant in the short term. On the one hand, these two markets are less likely to concurrently stay in extreme conditions. On the other hand, when the green bond market state is excessive, the causal relationship between the two markets will be weaker, as mentioned in section 4.2. In most conditions, the immediate effect of the carbon futures market on the green bonds market is mild and positive.

Considering the medium-term results (as displayed in Fig. 9) shows that the positive effect of the carbon futures market on the green bond market is relatively stable

(b_1 floating between 0.0031 and 0.0151). In the medium term, when the carbon futures price returns increase by 1 unit, the corresponding increase in green bond index returns fluctuates between 0.0031 and 0.0151. It is particularly noteworthy that when θ is in the lower quantiles (lower than 0.1), the carbon futures market will exert the most significant impact. It will also have an extremely positive effect if τ is simultaneously in the lower quantiles (the combination of market conditions for this highly optimistic impact is the opposite of that in the short term in Fig. 8). When θ gradually increases (in other words, when the green bond market has slowly stabilized from the downturn), this effect becomes smaller until θ is approximately 0.4; then, there is a relatively strong positive effect area. The fluctuation among the other quantiles is not evident, indicating that the positive effect in the medium term is generally mild and not significantly different from that in the short term.

Finally, we analyze the results of the long-term QQ regression estimation (as displayed in Fig. 10). In the long-term scenario, the impact of τ (which represents the state of the carbon futures market) is negligible, and the value of θ (the quantiles of the green bond returns, meaning the market conditions) affects the shape and trend of the graph. When θ is less than 0.2, the influence of the carbon futures on the green bond synchronously increases with it, and a short peak period of b_1 occurs when θ reaches approximately 0.2 to 0.3. Then until θ equals 0.8, the positive impact of carbon futures on green bonds oscillates downward as the value of θ increases. Finally, when θ exceeds 0.8, the coefficient b_1 ushers in a continuous rising stage and the highest peak value of 0.0091 is attained. This characteristic of b_1 represents the impact of the carbon futures price return on the green bond index return. In the long run, this effect does not have extreme points similar to that in the first two frequencies scenarios, and the shift is relatively gentle and does not change abruptly. Furthermore, the market condition of green bonds plays a decisive role at this time scale.

Combined with the above results, the green bond market may be negatively impacted by the carbon futures price when it is in a relatively depressed state (i.e., a bear market) in the short term. Apart from this situation, the influence of the carbon futures market on the green bond market is almost entirely positive at each time scale and quantile level. Still, when these two markets are in abnormal extreme market conditions, it is easy to cause extreme shocks. These results are obtained even when controlling for the uncertainty of economic policies, which increases the credibility of these results, and this further indicates that the carbon futures market has a significant effect on the green bond market. As the time scale increases, the role of the green bond market condition becomes increasingly important, which directly affects the extent of the effect. Arif et al. (2021) also use a quantile-based approach from three frequencies to study the relationship between the green bond index and other financial products. They confirm that the green bond market is becoming increasingly essential and can be used as a hedge market for equity investment and other financial strategies in the medium and long term. Our results also show the degree of price information acceptance of the green bond market to another market. Still, slightly different from their study, our decomposition of time is based on wavelet transform rather than the lag method.

Moreover, our QQ regression approach demonstrates the relationship between two variables and the changing trend more comprehensively than the partial quantile method. We compare our results with those from the OLS method and quantile regression method to more comprehensively show the advantages of the QQ method, and Table 2 provides these results. The OLS method and quantile regression can also verify the overall impact of the carbon futures market on green bonds, but these results cannot conveniently reflect the asymmetric effect of different time scales and market conditions. For example, the area of positive influence in the short term (Fig. 8) cannot be displayed in the results of these two methods. The short-term regression results of the OLS and quantile methods are negative and are the opposite of the short-term outcomes of the QQ method. It is possible that these two methods are based on the regression of the mean value of the series and cannot capture fluctuations and extreme data. In contrast, the QQ regression method can show the relationship between the carbon futures market and the green bond market at different joint quantiles, making it more suitable for complex and changeable practical problems.

Additionally, from the comparison of the results, we can also analyze the influence of EPU on the green bond market more clearly. Economic policy uncertainty also has dramatic results under three different frequency scenarios. In the short term, EPU has no significant impact on the green bond market index. Still, its effect is pronounced in the medium and long term, showing positive and negative, respectively. As for the quantiles, when the green bond index sequence is in the lower quantiles, it will be more affected by the EPU. In other words, the green bond market is more vulnerable to economic policy uncertainty when it is in a downturn. Finally, the significant effect of economic policy uncertainty proves the appropriateness of using it as a control variable in our research.

[Insert Figs. 8–10 about here]

[Insert Table 2 about here]

4.4 Robustness

In this subsection, we test the robustness and accuracy of the QQ regression results by comparing them with those obtained using the quantile regression (QR) method. We have chosen to comparatively analyze the estimated OR parameters with the τ-averaged QQ regression parameters. The equation is as follows:

$$\gamma_0(\theta) \equiv \bar{\bar{b}}_0(\theta) = \frac{1}{D}\sum_{\tau} \hat{b}_0(\theta, \tau)\,, \tag{33}$$

$$\gamma_1(\theta) \equiv \bar{\bar{b}}_1(\theta) = \frac{1}{D}\sum_{\tau} \hat{b}_1(\theta, \tau)\,. \tag{34}$$

where D is the points number of the grid of τ , and Figs. 11, 12, and 13 are the test results. In terms of the overall trend, the results for the constants and influence coefficients obtained by the QR method (represented by the solid green lines) and the

QQ regression method (represented by the dashed red lines) are not very different, regardless of the time scales. However, in the short-term (Fig. 11) and medium-term (Fig. 12) scenarios, the estimation results of the impact coefficient of the carbon futures price on the green bond index have some minor deviations, and the approximate curve trend is consistent. In the short term, the coefficient estimate of the OO method is less than the value measured by the QR approach (the dotted red line is lower than the solid green line in all quantiles), while in the medium term, the result is reversed. However, in the long run (Fig. 13), the resulting curves of these two methods almost coincide.

The occurrence of partial errors indicates that there may be some noise caused by the impact of short-term events, which results in some minor deviations in the estimates of these two methods (the QQ regression considers the average effect on the τ-quantile in this test). In the medium term, the sequence becomes more stable, and the QQ method may better capture the impact of the carbon futures market, so the value of the impact coefficient will be slightly larger. However, our results remain qualitatively robust, regardless of the intercept estimation or the influence coefficients assessment.

[Insert Figs. 11–13 about here]

5. Conclusion

Motivated by the importance and implications of COP26, we study the interrelationship between two derivative financial markets with the same function of environmental protection (i.e., the carbon market and the green bond market) under different time frequencies and market conditions. We combine the wavelet transform and quantile methods. First, we decompose the ECX EUA carbon futures price, the S&P green bond index, and the essential control variable, economic policy uncertainty, into different time scales sequences. Through the quantile Granger test, we find that the global green bond market does not statistically Granger-cause the European ECX EUA carbon futures market. Meanwhile, the carbon futures market significantly impacts the green bond market, regardless of frequency or market conditions. This result shows that there is indeed a one-way rather than two-way relationship between the two markets. It reflects the European carbon futures market's ability to predict the global green bond market, and proves that the current influence of the green bond market may be relatively weak once again.

Then, we use the quantile-on-quantile regression method, an improved quantile regression method, to explore the specific function of the carbon futures price on green bonds. We find that the carbon futures market will have certain adverse effects in the short term when the green bond market is in a relatively low state (i.e., a bear market condition). Otherwise, the impact is positive for most time frames and market conditions. Furthermore, there are some synergies between these two markets. When both markets are in recession or prosperity, the effect of the carbon futures price on the green bond index is more likely to be small or sharply positive. This shows that the effect of the carbon futures on the green bonds is undoubted and positive in most situations, indicating that there may be a particular channel between these two markets, which leads to an inevitable interrelationship. As the time scale increases, the influence of the condition of the green bond market is more critical than that of the carbon futures market. The QQ regression method can be used to examine the influence of the carbon futures market on the green bond market more comprehensively by comparing the OLS and QR methods, which could help elucidate the specific relationship between these two in multiple dimensions. Our results survive several robustness tests. In addition, we also confirmed that economic policy uncertainty does have a significant impact on the green bond market. In particular, the effects of the EPU obtained by quantile regression are slightly different from those of OLS in the long run. Our results indicate that empirical analysis methods may perform differently under numerous scenarios, suggesting the necessity of our research framework in different quantiles and time ranges with wavelet decomposition and the quantile-on-quantile way.

This research has supplemented the relevant literature on carbon trading and green bond markets and confirmed a one-way correlation with new empirical evidence. Our results at different frequencies and market conditions help different types of investors related to these two markets to obtain corresponding information, presenting a picture with more details. It is beneficial for investors to make more reasonable or scientific investment decisions. For example, the synergies we found between the two markets can help investors predict the possible situation when the two markets are extremely active and irrational investments in some extreme market scenarios may be avoided.

Our findings also carry other important implications. For example, regulators can better grasp the interrelationship between the carbon futures market and the green bond market from our analysis. The results could help them improve the supervision and management measures for these two markets through policy adjustment, enabling these two markets to jointly play their role in environmental protection and forming an effective network for low-carbon transformation. Although many regulators have consciously incorporated a carbon trading market and green financial products such as green bonds into their future policy planning, they often seem to have only parallel relations without in-depth exchanges. Our analysis can help regulators pay attention to the differences in the links between the two under different conditions and make targeted policy arrangements. In addition, the role of the green bond market in the carbon trading market is not significant enough. Regulators should reasonably strengthen the financial support role of the green bond market in the carbon market and promote the integration and innovation of the two markets. Finally, we verify the existence and specific performance of the relationship between these two markets, but their influence channels and other aspects have not been investigated, leaving ample space for other researchers to improve or expand our analysis.

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Tables

Table 1. Descriptive statistics of the return series of sample sequences

Table 2. Results of the OLS and quantile regression methods.

Table 1. Descriptive statistics of the return series of sample sequences

	Green bond	Carbon futures	EPU
Minimum	-0.0105	-0.3526	-1.3673
Maximum	0.0087	0.2703	1.3170
25th Quartile	-0.0007	-0.0146	-0.1647
75th Quartile	0.0008	0.0177	0.0897
Mean	0.0000	0.0014	-0.0312
Std.dev	0.0014	0.0330	0.2196
Skewness	-0.6644	-0.2937	-0.0500
Kurtosis	6.7369	11.3247	2.4779
JB test	4146.7174***	11305.9036***	541.6598***
ADF test	-13.8141***	-14.4755***	-13.0639***

Note: (i) This table is the descriptive statistics of the return series of ECX EUA Carbon futures price, S&P green bond index, and the EPU index of the united states. (ii) The time is from Jan 08, 2013, to Mar 10, 2021. (iii) * represents the 10% significance level; ** corresponds to the 5% significance level; *** corresponds to the 1% significance level.

Table 2. Results of the OLS and quantile regression methods

Panel A: OLS and quantile regression results (short-term)				
	OLS	Quantile regression		
		0.1	0.5	0.9
Intercept	0.0000	-0.0249***	-0.0003	0.0253***
	(0.9841)	(0.0000)	(0.5863)	(0.0000)
Carbon	-1.1389**	-2.7378***	-1.2823**	-0.4223
	(0.0378)	(0.0008)	(0.0219)	(0.6947)
EPU	-0.0002	-0.0001	-0.0002	0.0001
	(0.1748)	(0.6403)	(0.2464)	(0.7502)
Panel B: OLS and quantile regression results (medium-term)				
	OLS	Quantile regression		
		0.1	0.5	0.9
Intercept	0.0000	-0.0004***	0.0000	0.0004***
	(0.9817)	(0.0000)	(0.4612)	(0.0000)
Carbon	0.0096***	0.0052***	0.0029***	0.0061***
	(0.0000)	(0.0003)	(0.0026)	(0.0003)
EPU	0.0013***	0.0016***	0.0009***	0.0014***
	(0.0000)	(0.0000)	(0.0013)	(0.0001)
Panel C: OLS and quantile regression results (long-term)				
	OLS	Quantile regression		
		0.1	0.5	0.9
Intercept	0.0000	-0.0002***	0.0000**	0.0002***
	(-0.9808)	(0.0000)	(-0.0213)	(0.0000)
Carbon	0.0047***	0.0025	0.0029**	0.0044***
	(0.0000)	(-0.2382)	(-0.0234)	(-0.0005)
EPU	-0.0020***	-0.0027***	-0.0019***	-0.0023***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Note: (i) This table reports estimations of the ordinary least squares regression (OLS) and quantile regression on different quantiles (i.e., α = 0.1, α = 0.5 and α = 0.9) regarding impacts of the ECX EUA (Carbon) futures price return and EPU index of US (EPU) on the S&P green bond index return. (ii) P values are in parentheses. * denotes the 10% significance level; ** denotes the 5% significance level; *** denotes the 1% significance level.

Figures

Fig. 1. Time series plots of the daily return of S&P green bond index, ECX EUA carbon futures prices, and US EPU from 2013-01-08 to 2021-03-10.

Fig. 2. Density plots of the daily return of S&P green bond index, ECX EUA carbon futures prices, and US EPU from 2013-01-08 to 2021-03-10.

Fig. 3. Maximum overlapping discrete wavelet decomposition of the S&P green bond index return.

Fig. 4. Maximum overlapping discrete wavelet decomposition of the ECX EUA carbon futures price return.

Fig. 5. Maximum overlapping discrete wavelet decomposition of the US EPU

Fig. 6. Quantile Granger causality test of the ECX EUA carbon futures price on the S&P green bond index (from left to right, it represents short-, medium- and long-term in turn)

Fig. 7. Quantile Granger causality test of the S&P green bond index on the ECX EUA carbon futures price (from left to right, it represents short-, medium- and long-term in turn)

Fig. 8. QQR estimates for the impacts of the ECX EUA carbon futures price returns on the S&P green bond index returns (short-term)

Fig. 9. QQR estimates for the impacts of the ECX EUA carbon futures price returns on the S&P green bond index returns (medium-term).

Fig. 10. QQR estimates for the impacts of the ECX EUA carbon futures price returns on the S&P green bond index returns (long-term).

Fig. 11. The robustness check: comparisons of the results from the QR and the QQR method (short-term).

Fig. 12. The robustness check: comparisons of the results from the QR and the QQR method (medium-term).

Fig. 13. The robustness check: comparisons of the results from the QR and the QQR method (long-term).

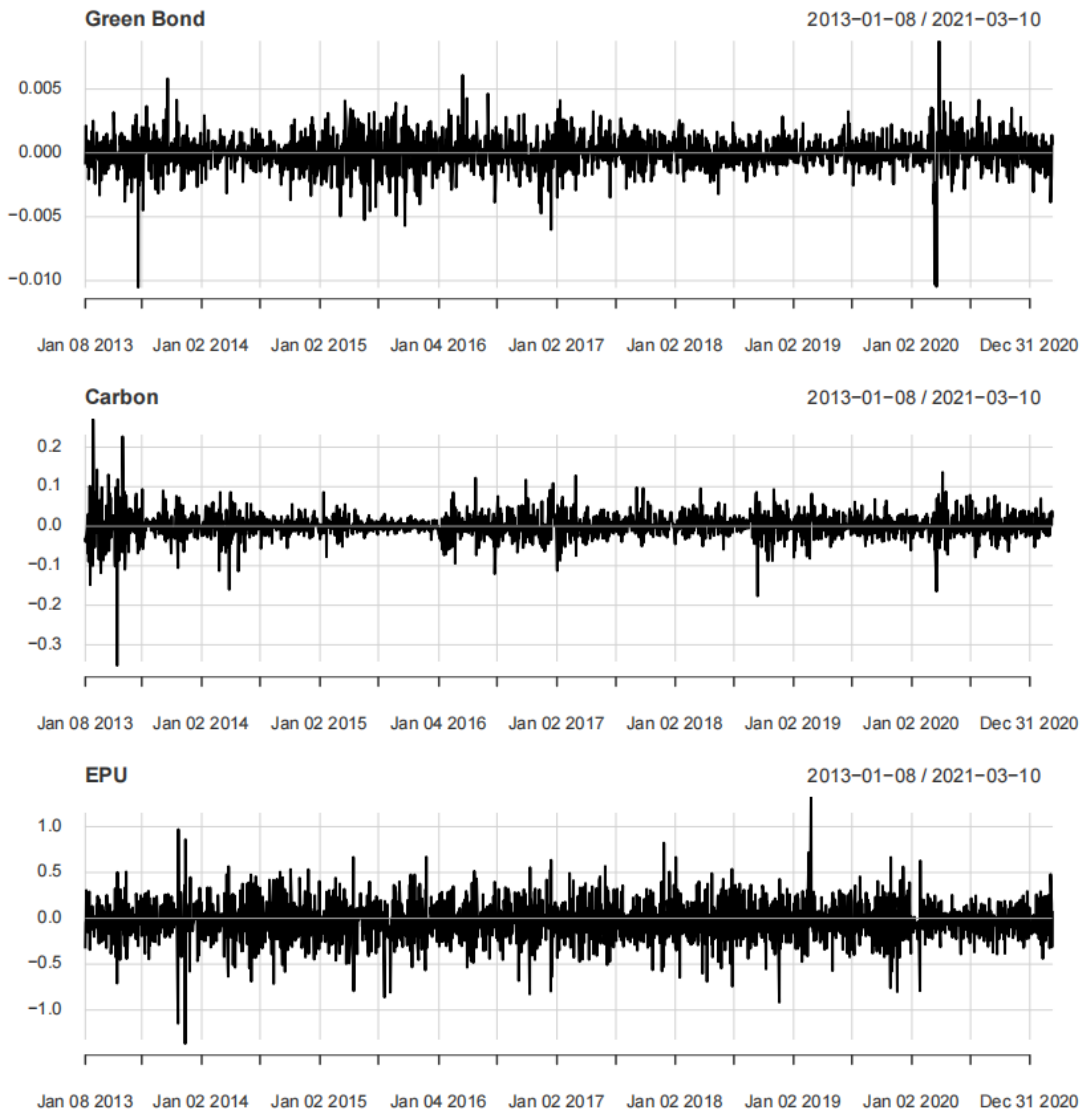


Fig. 1. Time series plots of the daily return of S&P green bond index, ECX EUA carbon futures prices, and US EPU from 2013-01-08 to 2021-03-10.

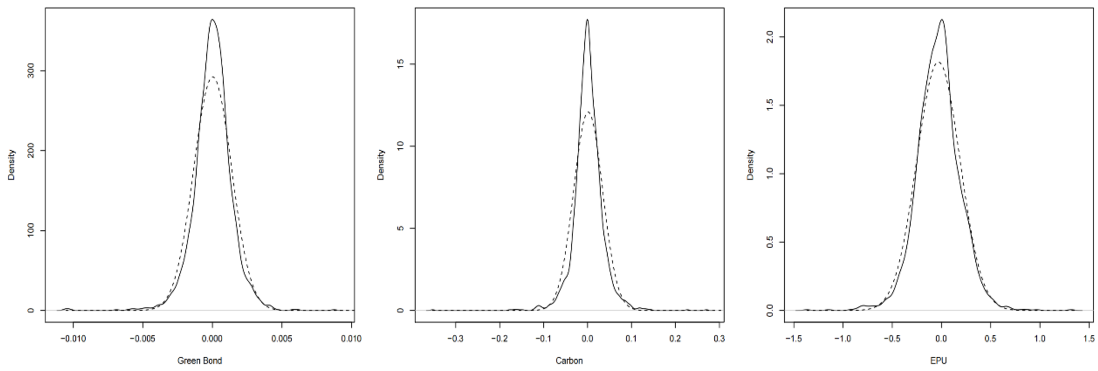


Fig. 2. Density plots of the daily returns of S&P green bond index, ECX EUA carbon futures prices, and US EPU from 2013-01-08 to 2021-03-10.

Note: (i) The dotted black line represents the standard normal distribution, and the solid black line represents the actual distribution of each sequence. (ii) As can be seen from the figure, all three sequences are non-normal distributions.

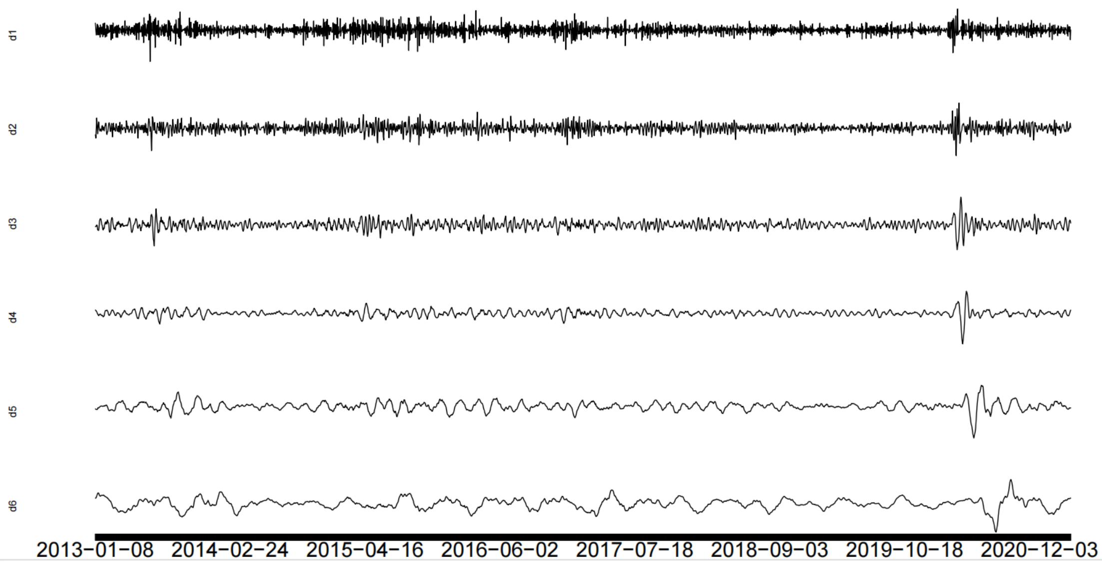


Fig. 3. Maximum overlapping discrete wavelet decomposition of S&P green bond index return

Note : (i) From d1 to d6, the decomposition layers of the MODWT method are getting bigger, and the time range represented is getting longer. d_j corresponds to the time scale: from 2^j to 2^{j+1} trading days. (ii) The larger the time scale, the gentler the change curve is.

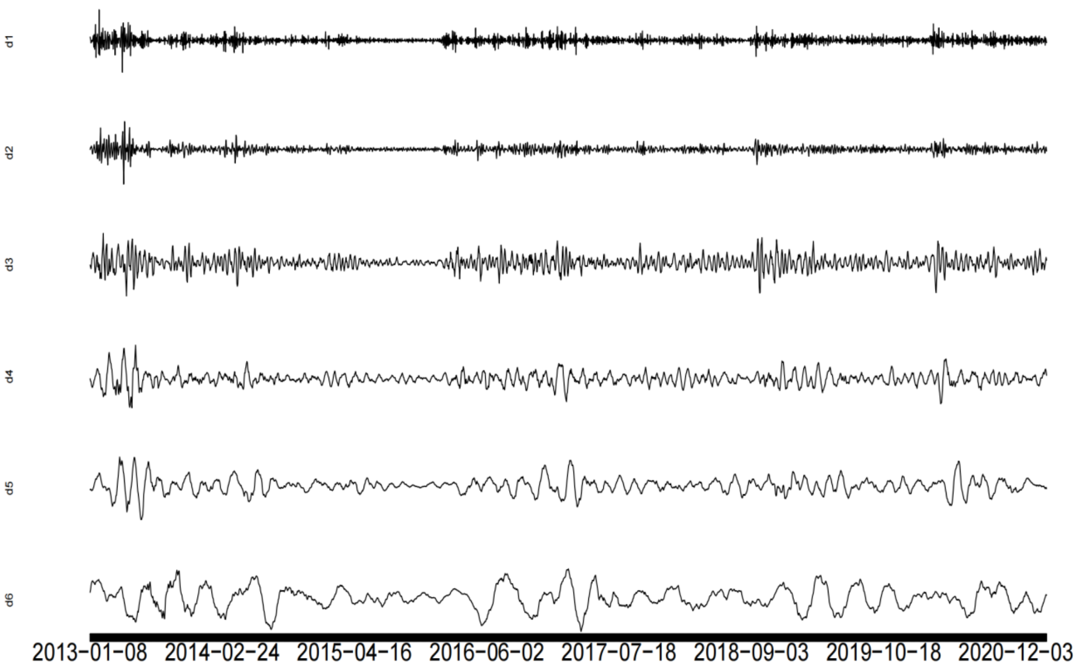


Fig. 4. Maximum overlapping discrete wavelet decomposition of ECX EUA carbon futures price return

Note : (i) From d1 to d6, the decomposition layers of the MODWT method are getting bigger, and the time range represented is getting longer. d_j corresponds to the time scale: from 2^j to 2^{j+1} trading days. (ii) The larger the time scale, the gentler the change curve is.

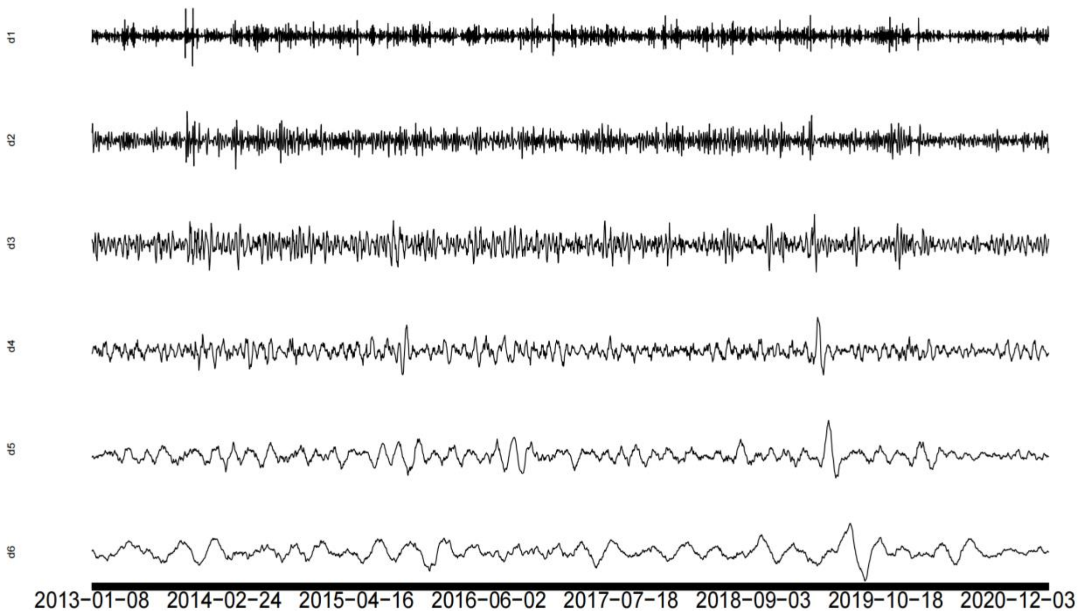


Fig. 5. Maximum overlapping discrete wavelet decomposition of the US EPU

Note : (i) From d1 to d6, the decomposition layers of the MODWT method are getting bigger, and the time range represented is getting longer. d_j corresponds to the time scale: from 2^j to 2^{j+1} trading days. (ii) The larger the time scale, the gentler the change curve is.

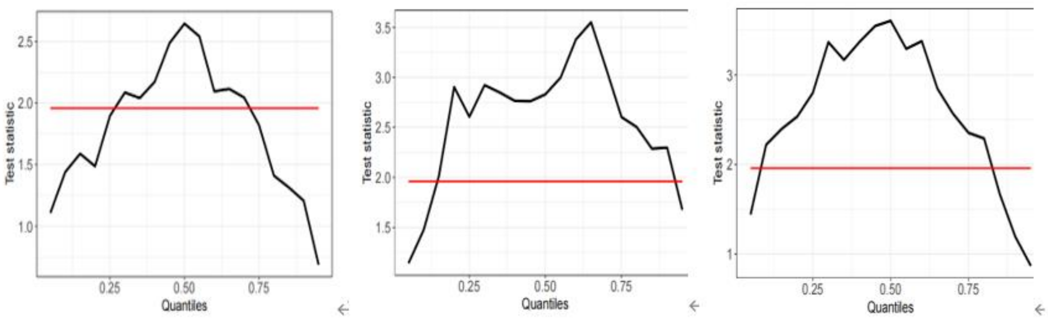


Fig. 6. Quantile Granger causality test of the ECX EUA carbon futures price on the S&P green bond index (from left to right, it represents short-, medium- and long-term in turn)

Note: (i) The horizontal red solid line represents the 5% critical value. (ii) The vertical axis reports test statistics of the null hypothesis of the Granger causality test, and the horizontal axis indicates quantiles.

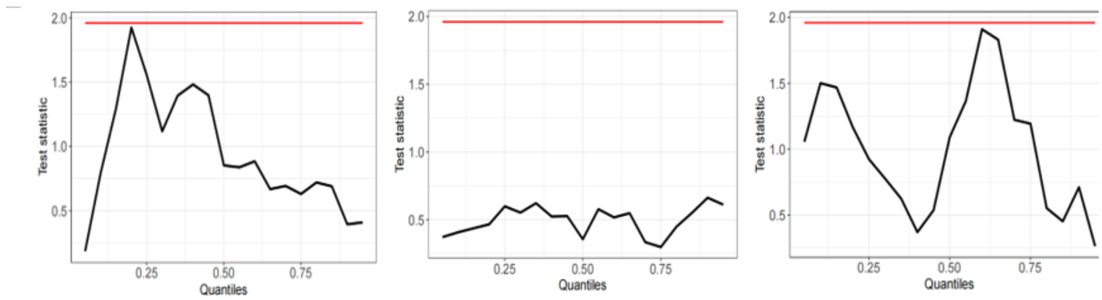
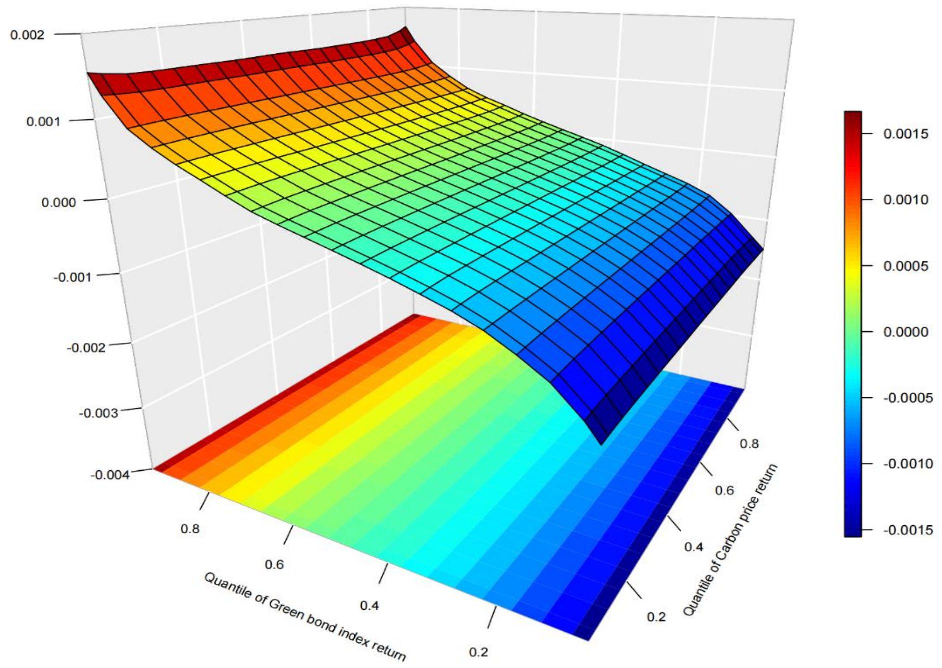
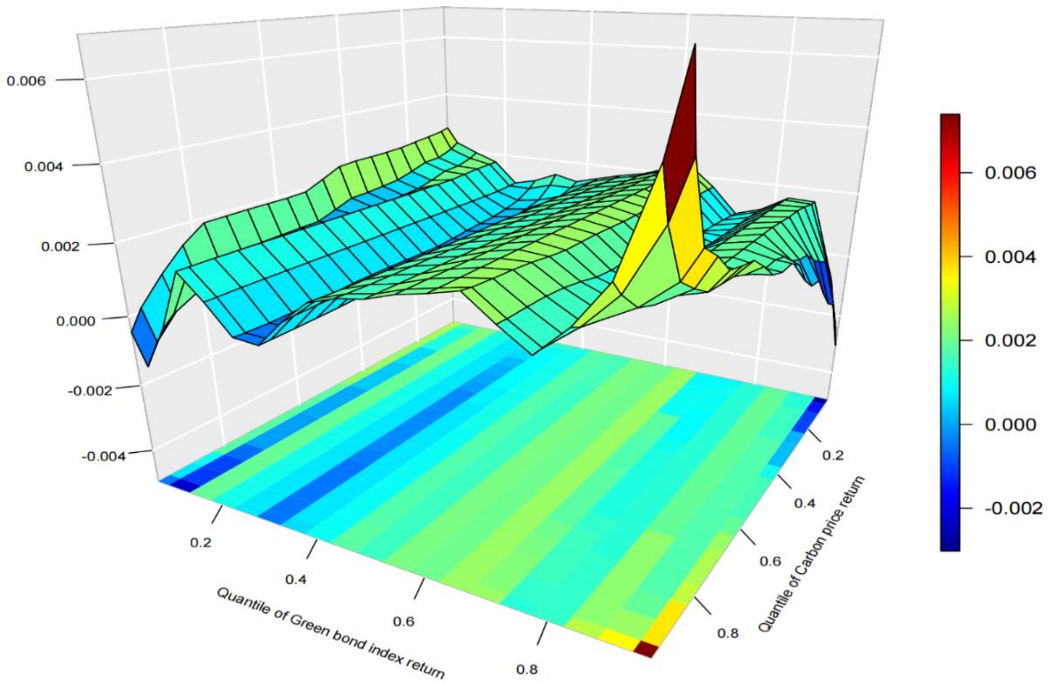


Fig. 7. Quantile Granger causality test of the S&P green bond index on the ECX EUA carbon futures price (from left to right, it represents short-, medium- and long-term in turn)

Note: (i) The horizontal red solid line represents the 5% critical value. (ii) The vertical axis reports test statistics of the null hypothesis of the Granger causality test, and the horizontal axis indicates quantiles.



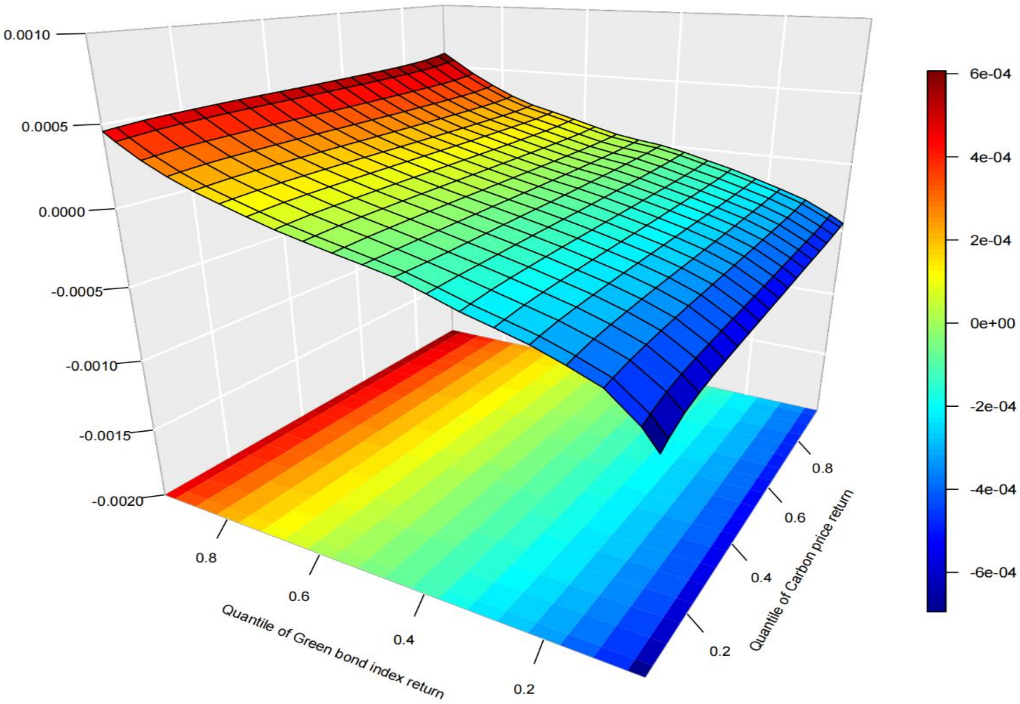
(a)b_0（θ，τ）



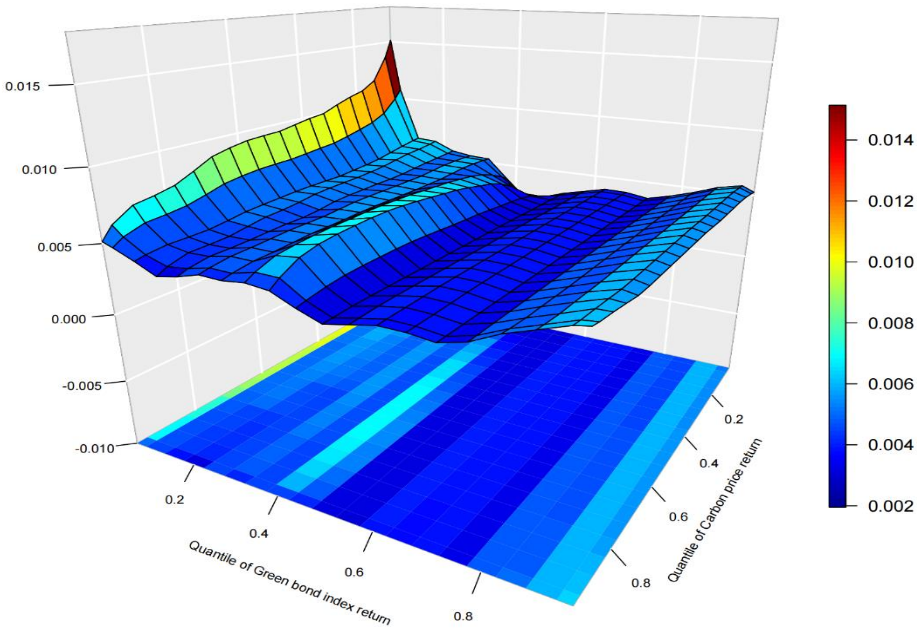
(b)b_1（θ，τ）

Fig. 8. QQR estimated impacts of the ECX EUA carbon futures price returns on the S&P green bond index returns (short-term)

Note: (i) $b_0(\theta,\tau)$ represents constants of the regression analyses while $b_1(\theta,\tau)$ stands for the effect of the τ-th quantile of ECX EUA carbon futures price on the θ-th quantile of S&P green bond index. (ii) Different colors represent the numerical magnitude and sign (positive or negative) of the coefficients.



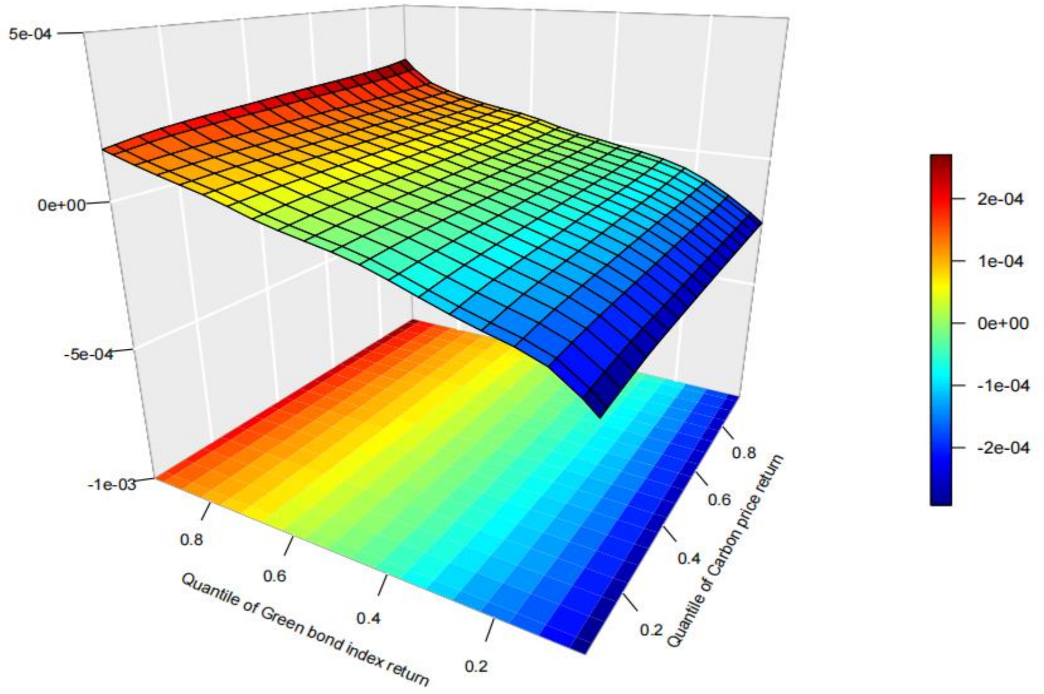
(a)b_0（θ，τ）



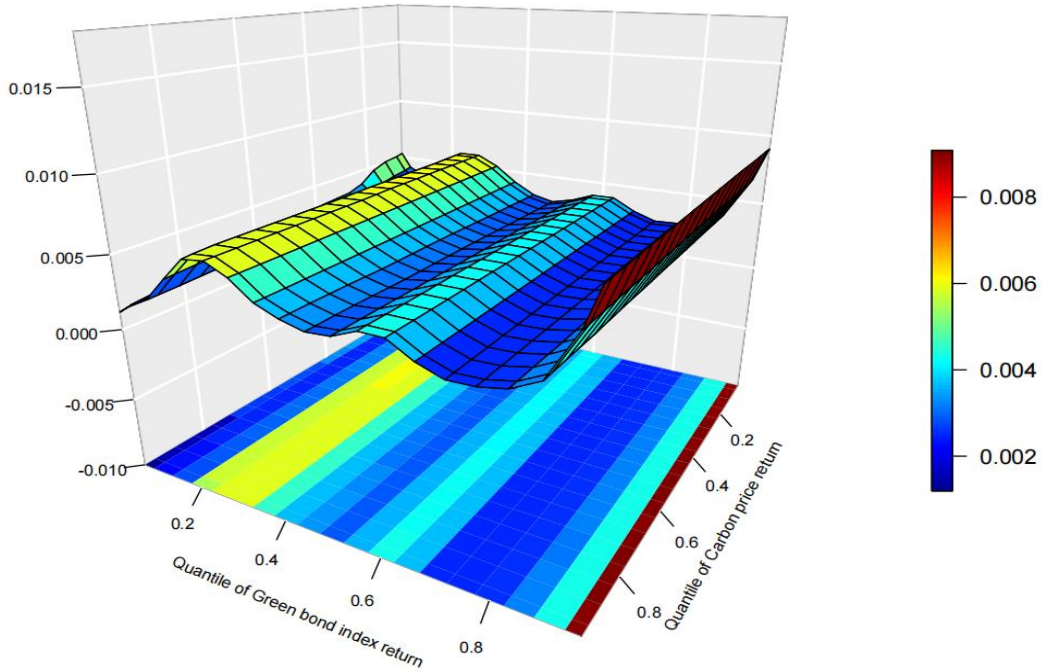
(b)b_1（θ，τ）

Fig. 9. QQR estimated impacts of carbon futures returns on green bond returns (medium-term)

Note: (i) $b_0(\theta,\tau)$ represents constants of the regression analyses while $b_1(\theta,\tau)$ stands for the effect of the τ-th quantile of ECX EUA carbon futures price on the θ-th quantile of S&P green bond index. (ii) Different colors represent the numerical magnitude and sign (positive or negative) of the coefficients.



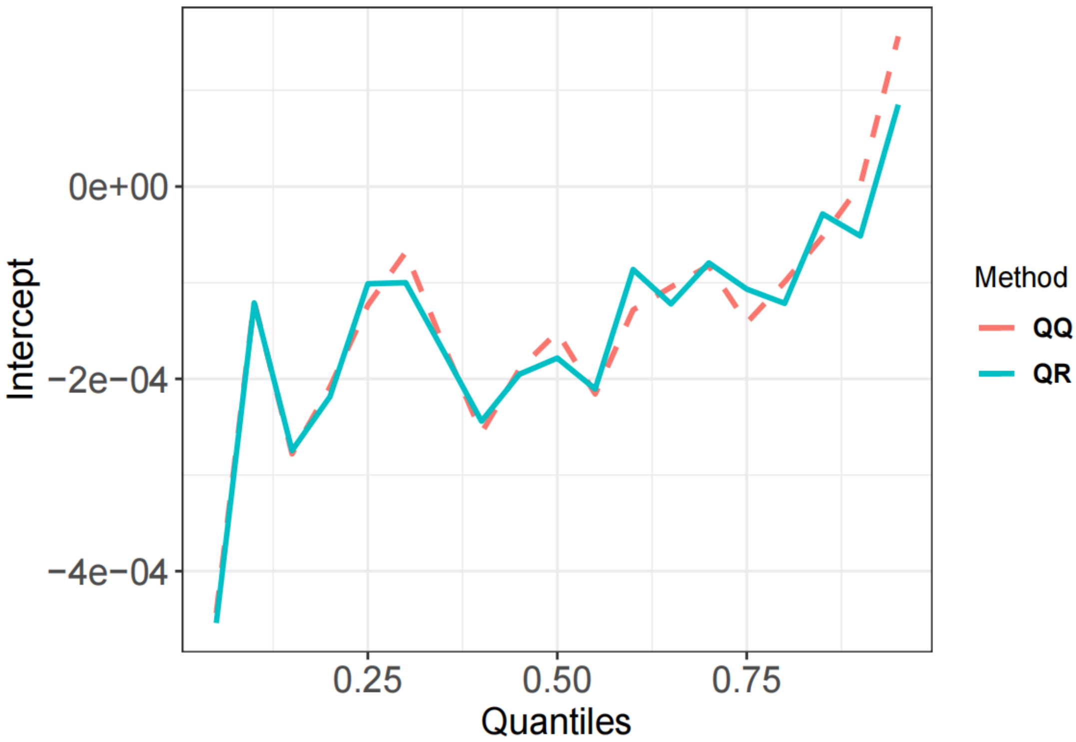
(a)b_0（θ，τ）



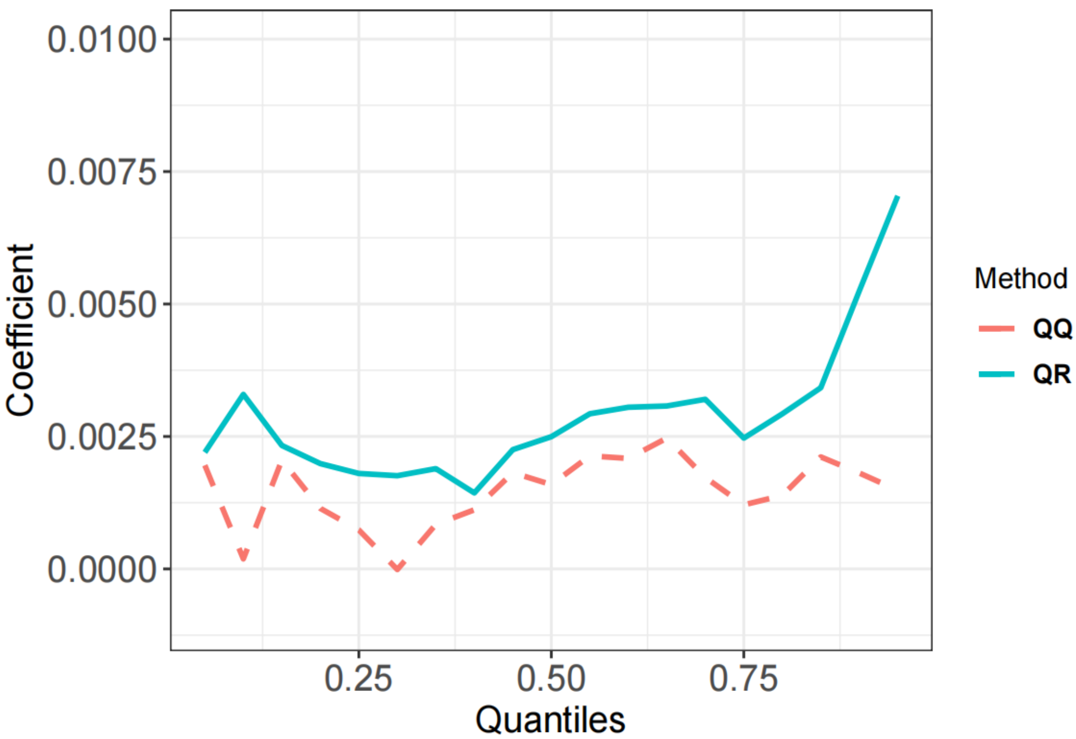
(b)b_1（θ，τ）

Fig. 10. QQR estimated impacts of carbon futures returns on green bond returns (long-term)

Note: (i) $b_0(\theta,\tau)$ represents constants of the regression analyses while $b_1(\theta,\tau)$ stands for the effect of the τ-th quantile of ECX EUA carbon futures price on the θ-th quantile of S&P green bond index. (ii) Different colors represent the numerical magnitude and sign (positive or negative) of the coefficients.



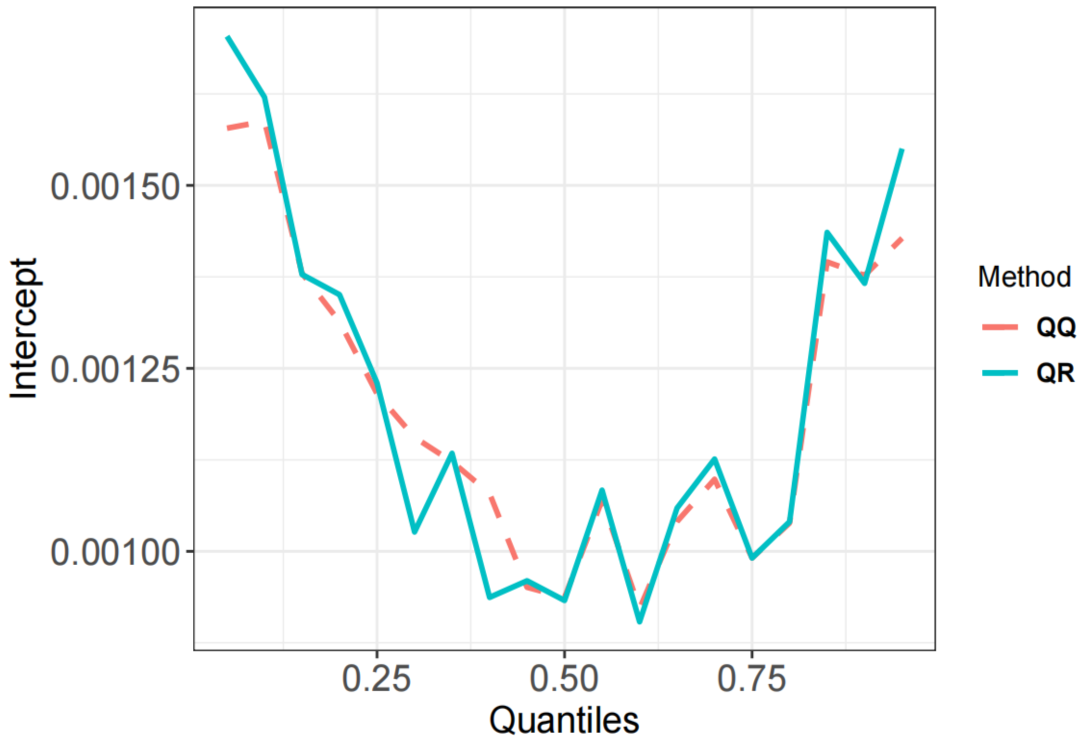
(a) The intercept of the green bond index return



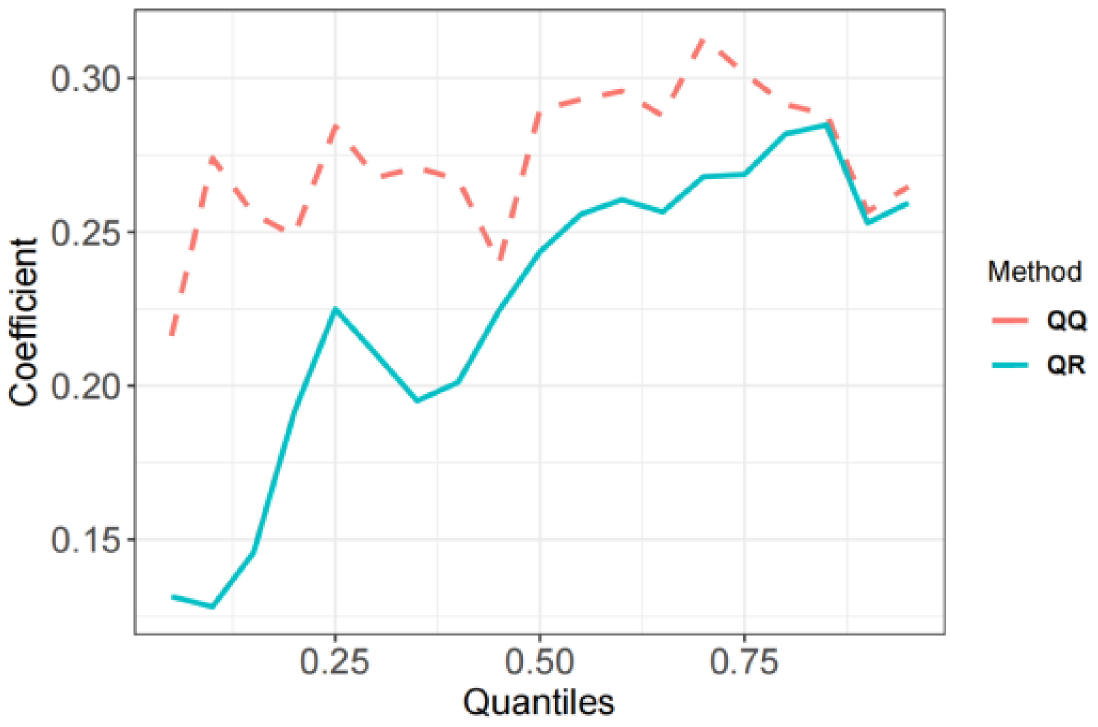
(b) Impact of carbon futures price return on the green bond return

Fig. 11. Robustness: Comparisons of the results from the QR and the QQR estimate(short-term).

Note: (i) The graph plots and compares the estimates of the traditional quantile regression parameters (denoted by QR: continuous green line) and the averaged quantile-on-quantile parameters (represented by QQ: red dotted line). (ii) QQ method regarding averaged impacts of the ECX EUA carbon futures price returns on different S&P green bond index returns quantiles.



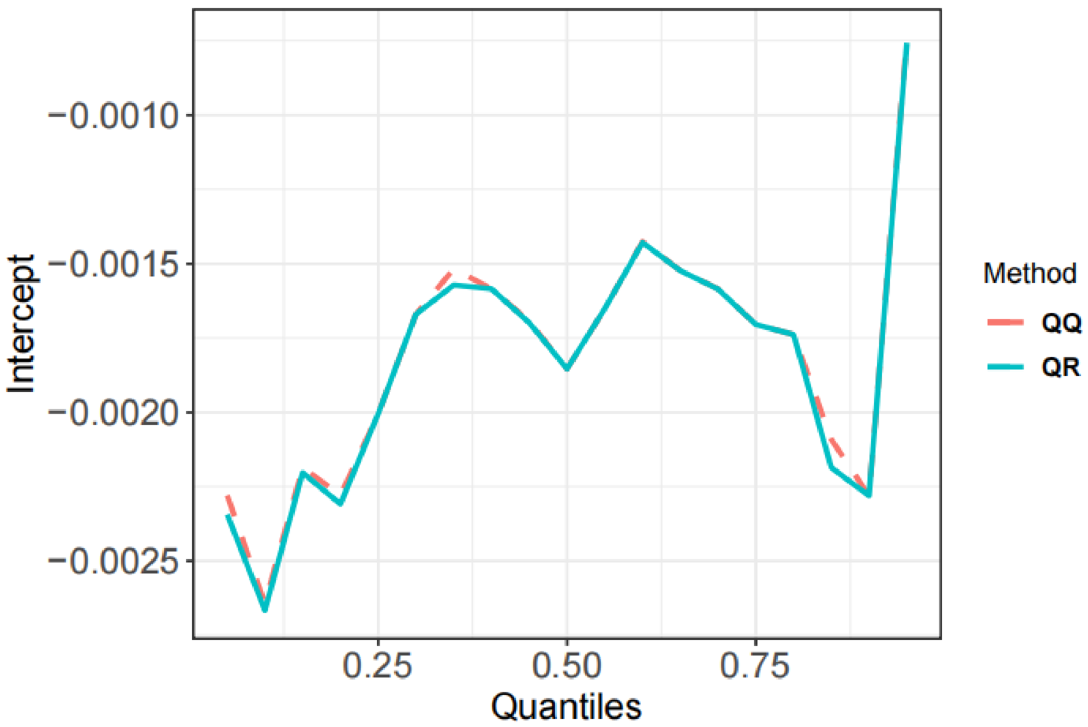
(a) The intercept of the green bond index return



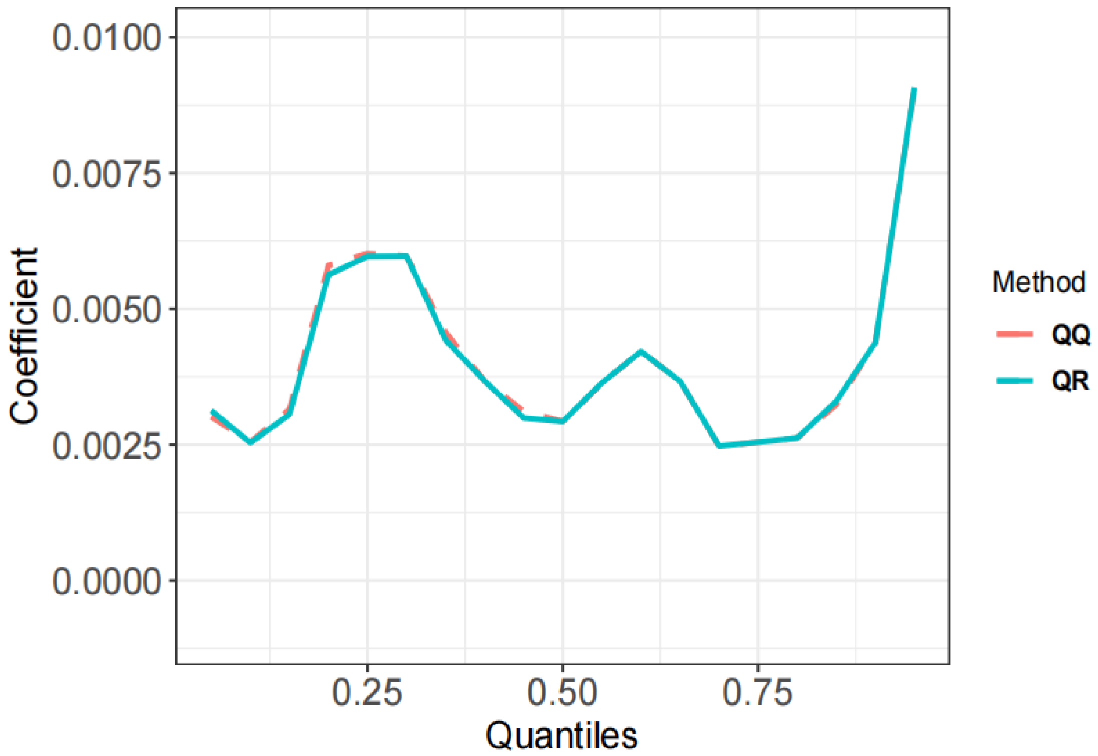
(b) Impact of carbon futures price return on the green bond return

Fig. 12. Robustness: Comparisons of the results from the QR and the QQR estimate (medium-term).

Note: (i) The graph plots and compares the estimates of the traditional quantile regression parameters (denoted by QR: continuous green line) and the averaged quantile-on-quantile parameters (represented by QQ: red dotted line). (ii) QQ method regarding averaged impacts of the ECX EUA carbon futures price returns on different S&P green bond index returns quantiles.



(a) The intercept of the green bond index return



(b) Impact of carbon futures price return on green bond index return

Fig. 13. Robustness: Comparisons of the results from the QR and the QQR estimate (long-term).

Note: (i) The graph plots and compares the estimates of the traditional quantile regression parameters (denoted by QR: continuous green line) and the averaged quantile-on-quantile parameters (represented by QQ: red dotted line). (ii) QQ method regarding averaged impacts of the ECX EUA carbon futures price returns on different S&P green bond index returns quantiles.

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