An integrated macroprudential stress test of bank liquidity and solvency

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ABSTRACT

We propose a new measure of systemic financial distress that incorporates idiosyncratic and systemic risks in the financial system network. Using this measure, we develop an integrated stress test of bank liquidity and solvency risks based on the dynamics of financial distress within the banking system network. We apply this stress test framework to the US banking system and identify systemic vulnerability of individual banks as well as the resilience of the system as a whole to an economic shock. The framework helps us identify and monitor systemic interdependencies between banks. The proposed stress testing framework is useful for practical macroprudential monitoring and is informative for policy making.

1. Introduction

The global financial crisis of 2007–2008 has revealed the need for a better macroprudential policy in order to enhance financial stability and to limit the propagation of systemic risk. The crisis has shown that impairment of financial stability can impose significant costs on the real economy in terms of economic growth and social welfare. It is widely accepted in the literature that systemic risk is the main threat to financial stability. Thus, to protect the real economy from a significant volatility in the financial system, it is necessary to detect and to gauge potential sources of systemic risk that emerge at the system level. Meanwhile, to protect the financial system from the growth variations of the real economy, it is necessary to assess the robustness of the response mechanism of the financial system to macroeconomic shocks. To this end, macroprudential stress tests have been considered as the main tool of macroprudential policy (Tarullo, 2016).

The current practice of macroprudential stress testing has improved in the aftermath of the financial crisis. However, the underlying techniques and models that have been developed prior to the crisis have remained broadly the same and there are still some limitations that need to be addressed (Borio et al., 2014). In particular, a recent report from the Basel Committee on Banking Supervision (BCBS) highlights two main limitations; namely, considering liquidity and solvency interactions and considering systemic risk (BCBS, 2015). The International Monetary Fund (IMF) makes a similar recommendation in its 2014 Review of the Financial Sector Assessment Program (FSAP). The review stresses the need to strengthen the systemic focus of the financial stability assessment and to deepen the analytical treatment of interconnectedness (IMF, 2014).

In this paper, we develop and illustrate with an empirical application, an integrated macroprudential stress test of bank liquidity and solvency risk. The proposed approach employs a network theory to introduce a new measure of systemic distress that incorporates microprudential as well as macroprudential risks in the banking system network. Our approach integrates liquidity risk and solvency risk and provides a convenient method to identify the point at which liquidity risk becomes solvency risk. In addition, the proposed stress testing framework is flexible as it allows the stress tester to further use different stress scenarios to assess the impact of liquidity shocks on solvency, and vice versa. The framework also provides a variety of output metrics that capture idiosyncratic as well as systemic economic

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risks at the individual bank level and the overall banking sector as a whole. Yet, the framework is tractable enough to be useful for practical macroprudential monitoring and informative for policy-making.

An important strength of our approach is that it explicitly links liquidity risk and solvency risk in order to incorporate their interactions in the stress testing framework. These interactions have often been neglected in existing stress-testing methodologies. We create this link by estimating both the probability of a bank becoming illiquid and the probability of a bank becoming insolvent based on the same factor; namely, the bank’s distress level. We estimate these probabilities using a Merton-type model that is based on the seminal work of Black and Scholes (1973) and Merton (1974). In so doing, we assume that bank distress is a continuous state with varying levels that depend on both idiosyncratic and systemic risks of each bank, whereas illiquidity and insolvency occur at specific points of highly elevated distress. The higher the distress level of a bank the closer it gets to its illiquidity and insolvency points.

Another important strength of our approach is the way in which we incorporate interconnectedness between banks into the stress test design. Given that our purpose is to assess the vulnerability of banks, it is more appropriate to focus on a bank’s systemic distress rather than its systemic importance. We approximate a bank’s systemic distress using a novel measure named DistressRank. This measure fully incorporates the interbank network topology. It is based on the notion that the distress level of a bank is a function of its idiosyncratic risk as well as its systemic risk stemming from being connected to counterparties through interbank assets or liabilities. DistressRank also captures the dynamics on the network as it changes with the change in banks’ distress levels.

We construct stress scenarios in two different ways, where the first is designed to assess the resilience of the banking system to macroeconomic shocks, and the second is designed to assess the possibility of amplifying endogenous shocks within the banking system and transmitting them to the macroeconomy. The empirical application of the stress test framework to the US banking system shows how it can be effectively used to identify the systemic vulnerability of individual banks and the resilience of the system as a whole to economic risks. It also shows how the proposed approach can be effective for monitoring and assessing systemic interdependencies among banks. The proposed approach, thus, provides a tool for the banking system supervisors to analyse the current state of the system stability and to monitor the evolution of contagion and systemic risk within the system.

Our findings point out the importance of considering interconnectedness in designing macroprudential stress tests. At the system level, the systemic loss due to feedback loops is shown to be significant compared to the direct loss that results from the initial shock to the system. Ignoring these feedback effects may lead to a significant understimation of systemic loss. At the bank level, the results confirm that interlinkages play a significant role in identifying individual banks’ vulnerability. On this premise, we use DistressRank as a measure of a bank’s systemic distress. The results show that a bank’s DistressRank is associated with its systemic feedback loss.

Our findings also lend insights into the possibilities of distress propagation within the system. Applying the proposed framework to the US banking system enables us to identify banks that are most vulnerable to system-wide shocks. In addition, we identify the liquidity distress dependence and solvency default dependence between banks in the system. A striking finding is that banks that are not directly connected through interbank assets or liabilities are still subject to distress from each other through common counterparties.

The remainder of this paper is organised as follows. Section 2 provides an overview of macroprudential stress testing in the literature. Section 3 develops an initial model for illiquidity distress propagation. Section 4 extends the model to link illiquidity to insolvency. Section 5 introduces a framework for an integrated macroprudential stress test of liquidity and solvency risk. Section 6 provides an overview of the data used in this paper and presents various results with regard to the stress test. Section 7 concludes the paper.

2. Related literature

We develop our stress testing framework based on a balance sheet setting which is the natural approach to macroprudential stress testing. This approach has been shown to be generally informative and unbiased with the ability to predict banks’ losses and equity returns due to change in macroeconomic factors (Philippon et al., 2017). It is specially useful in cases of limited or poor market data availability, as the main data required for the test is extracted from banks’ balance sheets (Ong and Čihák, 2014). Early models under this approach provide a framework for an aggregate stress test of the financial system (e.g., Blaschke et al., 2001; Bunn et al., 2005), however, they are fundamentally financial simulations with no formal links to the macroeconomy (Buncic and Melecky, 2013). More recent models attempt to establish this link by using satellite models to link the macroeconomic variables to bank’s asset quality (Čihák, 2007). A more sophisticated, yet tractable, accounting-based stress test is introduced by Drehmann et al. (2010), in which they model assets and liabilities simultaneously. This model integrates credit and interest rate risk in the banking book and provides a framework to assess the impact of different investment strategies on the bank’s profitability. Also, Gourieroux et al. (2012) provide a stress testing framework based on balance sheets but with a focus on bank solvency risk. Another example is the model provided by Greenwood et al. (2015) in which fire sales due to deleveraging cause shocks across bank balance sheets leading to spillover among banks. Nevertheless, it is worth noting that the quality of any analysis that follows a balance sheet approach to stress testing depends on the granularity and availability of the data. Some models attempt to overcome this limitation and provide sophisticated techniques to perform stress testing in cases of limited data (e.g., Segoviano and Padilla, 2006; Ong et al., 2010).

In theory, liquidity and solvency risks interact and can cause each other through banks’ interactions (Diamond and Rajan, 2005). However, empirical evidence on the nexus between liquidity and solvency risks is scarce. Some studies have attempted to establish the link between liquidity and solvency in order to incorporate it into macroprudential stress tests. In particular, Schmitz et al. (2017) suggest that bank funding costs are correlated with bank capital as a result of the interconnections between funding costs and market expectations about bank solvency. Other studies suggest a significant impact of solvency on bank funding costs (Hasan et al., 2016), which appears to be nonlinear with higher sensitivity of funding cost at lower levels of bank solvency (Aymanns et al., 2016). The relationship seems to be intuitive when we consider the interactions between liquidity and solvency. When a bank faces a liquidity shortage, it might be forced to sell its less liquid assets. If other banks with similar conditions adopt the same approach of selling less liquid assets, the initial liquidity shortages may lead to fire sales and consequently declines in asset prices, hence, causing solvency problems (Lee, 2013). Similarly, concerns about bank insolvency can cause liquidity shortages. Increased expectations about a bank insolvency (e.g. declines in credit rating) can increase deposit withdrawal and interbank funding costs as depositors and interbank counterparties, respectively, become less confident about the bank creditworthiness, hence causing a liquidity shortage for the bank (Pierret, 2015). Thus, propagation channels between liquidity and solvency are common and, for macroprudential purposes, they should be integrated within a unified stress testing framework. However, the focus of macroprudential stress testing frameworks has usually been on solvency risk, while liquidity risk is assessed using satellite models on a stand-alone basis.

Our proposed methodology is closely related to the Macrofinancial Risk Assessment Framework (MFRAF) that has been developed by the Bank of Canada and integrates solvency and liquidity risk (Gauthier et al., 2012b). In the framework, solvency risk is triggered by a macro shock, whereas liquidity risk arises as a result of solvency concerns or deterioration in liquidity position. We use a similar framework that considers potential market liquidity risk and interbank counterparty credit
risk through a network model. Another macroprudential stress testing model that integrates solvency and liquidity risk has been developed by the Hong Kong Monetary Authority (Wong and Hui, 2009). Our methodology shares some characteristics with this model with regard to combining elements of balance sheet-based and market price-based approaches to stress testing. In this model, solvency risk of an individual bank depends on the market value of its total assets, calculated through a Merton-type model. In contrast, we estimate solvency and liquidity risks based on the volatility of liquid assets instead of total assets. In this model, liquidity risk is assessed by introducing an exogenous shock to asset prices which leads to increases in the bank's solvency risk and deposit outflow and reduction in its liquidity generation capability.

Our approach rests on the insight of the Merton-type models of default risk that are based on the seminal work of Black and Scholes (1973) and Merton (1974). In these models, equity of the firm can be viewed as a call option held by owners on its total assets, where the strike price is equal to the outstanding debt owed to creditors at maturity. In this context, a market-implied probability of default can be estimated as the probability that the market value of the firm’s assets falls below the book value of its liabilities (Bohn and Cronbic, 2003). An example of applying this approach to systemic risk is the theoretical model of Tasca et al. (2014) who use the Merton framework to quantify joint default probability of individual banks in order to study the impact of leveraging and diversification on systemic risk. We use the same logic to estimate two types of probability for each individual bank; namely, the probability of illiquidity and the probability of insolvency. We deviate, however, from the standard approach in that we base the estimation of both probabilities on liquid assets only instead of total assets. Our rationale is that, in the short run, the variability in total assets is derived mainly from the variability of liquid assets. This twist enables us to link liquidity and solvency risks directly as both are estimated based on the same factor.

Our methodology is also related to the Contingent Claims Analysis (CCA) that relies on a Merton-type framework to construct a risk-adjusted balance sheet of individual banks (Gray and Malone, 2008; Gray and Jobst, 2010). The CCA model can be used for macroprudential stress testing by applying a macroeconomic shock to the risk-adjusted balance sheet of individual banks and then estimating the change in banks’ market value of equity and probability of default. However, the CCA model limits its focus to solvency risk and lacks the systemic view as it does not provide a method to measure aggregated risk at the system level. Our methodology also shares some characteristics with the distress dependence model of Segoviano and Goodhart (2009) who investigate the effect of macroeconomic variables on bank losses where the joint probability distribution of banks is constructed with Copulas. They model the financial system as a portfolio of banks and use non-parametric statistical techniques to construct a multivariate density function for the financial system. Then, they estimate a joint probability of default and a banking stability index of the whole banking system. We base our stress test on a network model that provides a convenient way to incorporate systemic risk and interconnectedness into the stress testing framework. Early studies of financial contagion suggest that financial networks can provide a better way to study the linkages among financial institutions (e.g. Allen and Gale, 2000; Freixas et al., 2000). More recent studies support the same notion and emphasise that financial networks can provide a better way to study the linkages among financial institutions (e.g. Allen and Gale, 2000; Freixas et al., 2015; Glasserman and Young, 2014; Elliott et al., 2014; Acemoglu et al., 2015; Glasserman and Young, 2016). In this spirit, we depict the relationships within the banking sector as a network in which banks represent the nodes and financial exposures represent the edges between these nodes. This approach to studying the financial markets, in general, enables us to better understand the interconnectedness and the propagation of distress. This is also the approach followed by some regulatory (e.g. Gauthier et al., 2012b; Wong and Hui, 2009; Sole and Espinosa-Vega, 2010) and academic (e.g. Gauthier et al., 2012a; Levy-Carciente et al., 2015) stress testing frameworks. Sole and Espinosa-Vega (2010) use a network setting to simulate the impact of credit and funding shocks on a set of connected banking systems. Levy-Carciente et al. (2015) use a bipartite bank-asset network to design a solvency stress test of the Venezuelan banking system. Gauthier et al. (2012a) use a network model to estimate a bank’s macroprudential capital requirements as a function of its contribution to the system-wide risk.

3. Illiquidity distress

This section provides a simple model of liquidity risk where we use a balance sheet approach to derive a measure of systemic illiquidity distress of a bank in a financial system.

3.1. A system of networked balance sheets

We model an interbank market that consists of a number of banks \( N \in \{1, \ldots, N\} \). The assets of each bank are divided into liquid assets and illiquid assets denoted as \( A^L \) and \( A^S \), respectively. In addition, the liabilities of each bank consist of short-term obligations denoted as \( L^S \), and long-term obligations denoted as \( L^F \). The net worth of bank \( i \) is \( E_i \) and is equal to the difference between its total assets and its total liabilities. Thus, the balance sheet identity of bank \( i \) can be represented as

\[
A^L_i + A^F_i = L^S_i + L^F_i + E_i
\]  
(1)

Furthermore, we differentiate between two sources of liquid assets; namely, interbank liquid assets and other liquid assets denoted as \( A^S_i \) and \( A^O_i \) respectively, where \( A^L_i = A^S_i + A^O_i \). Similarly, the short-term obligations are divided into interbank short-term obligations, \( L^S_i \), and other short-term obligations, \( L^O_i \), where \( L^S_i = L^S_i + L^O_i \). The interbank liquid assets and short-term obligations represent assets and liabilities originating from the interbank market (e.g. interbank repo or derivatives transactions), whereas other liquid assets and other short-term obligations are not related to the interbank market and might include cash or short-term securities.

The separation of interbank liquid assets and short-term obligations enables us to model the liquidity interlinkages across banks in the interbank market as a weighted directed graph whose vertices represent banks and edges represent interbank assets and liabilities. The assets, in monetary units, of bank \( i \) with bank \( j \) are denoted by \( A_{ij} \) where \( i \neq j \), which represents the amount that bank \( i \) should receive from bank \( j \) as a result of some financial transaction (e.g. a derivatives contract). Similarly, the interbank liabilities of bank \( i \) to bank \( j \) is denoted by \( L_{ij} \) where \( i \neq j \), which represents the amount that bank \( i \) should pay to bank \( j \). It then follows that, the interbank liquid assets of bank \( i \) are given by \( A^S_i = \sum_{j=1}^{N} A_{ij} \), whereas the interbank liabilities of bank \( i \) are given by \( L^S_i = \sum_{j=1}^{N} L_{ij} \).

3.2. Liquidity coverage matrix

Banks use their stock of liquid assets to cover their own liquidity requirements. Thus, we can define a liquidity coverage ratio for a bank \( i \) as

\[
\ell^L_i = \frac{A^L_i}{L^S_i}
\]  
(2)

which measures the ability of bank \( i \) to meet its short-term obligations, whether within the interbank market or to outside counterparties. The higher the liquid assets as compared to short-term obligations, the higher the liquidity coverage ratio, and the more liquid the bank is.

Furthermore, in order to measure the ability of bank \( i \) to cover its obligation to another counterparty \( j \) within the interbank market, we
introduce $\epsilon_{ij}$ as the bank $i$’s relative liquidity coverage ratio to bank $j$, where

$$
\epsilon_{ij} = \frac{[A_i^L - L_i^L] - A_{ij} + L_{ij}}{L_{ij}}
$$

(3)

This ratio represents the ability of bank $i$ to cover its interbank obligations to bank $j$ using its net liquidity ($A_i^L - L_i^L$), after paying all other obligations and before exchanging any liquidity with bank $j$. This is why we adjust the net liquidity stock of bank $i$ in the numerator by subtracting the liquidity exposure that is owed to bank $i$ by bank $j$ and adding back the liquidity exposure owed to bank $j$ by bank $i$, to reflect a case before exchanging liquidity.

3.3. Illiquidity distress matrix

It is clear from Eqs. (2) and (3) that the better the liquidity position of a bank as measured by its liquidity coverage ratios the lower the threat of illiquidity distress that the bank is exposed to. It is also worth noting that $\epsilon_{ij}$ provides a proxy to the relative vulnerability of bank $j$ to the liquidity distress that might arise at bank $i$. In other words, the lower this ratio is, the higher the probability that bank $i$ will fail to honour its obligation to bank $j$, and the higher the vulnerability of bank $j$.

We use this notion to develop an illiquidity distress matrix defined as $D = [d_{ij}]$, where an element $d_{ij}$ represents the relative vulnerability of bank $i$ to the illiquidity distress of bank $j$; in other words, the contribution of bank $j$ to the vulnerability of bank $i$. We then define $d_{ij}$ as

$$
d_{ij} = \frac{a_{ij}}{\lambda_j}
$$

(4)

where $a_{ij}$ is the respective element from the adjacency matrix $A$ of interbank network which is defined as $A = [a_{ij}]$, where $a_{ij} = 1$ if banks $i$ and $j$ are connected and $a_{ij} = 0$ otherwise.

3.4. DistressRank: A measure of systemic distress

The network literature suggests that the centrality of a node in a given network is a function of its interconnection with its neighbours. One method to quantify this centrality is a measure called eigenvector-centrality, which is based on the notion that the centrality of a node is proportional to the sum of centralities of its neighbours (Newman, 2010). Applying this notion to our financial network results in

$$
c_i = \frac{1}{\lambda} \sum_{j=1}^{N} d_{ij} \epsilon_j
$$

(5)

where $c_i$ is the eigenvector centrality of bank $i$ and $\lambda \neq 0$ is a constant. Thus, the eigenvector centrality can provide a relative ranking of banks. One advantage of this method is that it bases the ranking on both local information related to direct neighbours and global information of the network given that the ranking of neighbours is based on the ranking of their neighbours, and so on Scott (2017).

However, eigenvector centrality is a purely topological measure that is solely based on the adjacency matrix $A$. This limitation renders it subject to two main disadvantages when it comes to ranking banks in a financial network. First, it assumes equal contribution of all exposures in the network in determining the centrality of a given bank. This assumption is not valid as it ignores the state of the bank’s counterparty, i.e. its distress level. A bank is more vulnerable to banks with high distress levels compared to other banks. Second, eigenvector centrality ignores the dynamics in the network as it is based on the mere existence of an exposure between two banks rather than the weight of this exposure. Hence, it is time-independent as it does not change in response to changes in the weights of exposure or the states of banks.

Therefore, we propose DistressRank as an improvement on the standard eigenvector centrality to overcome the disadvantages mentioned above. To this end, we estimate DistressRank based on the distress matrix $D$, which was introduced in Eq. (4). Let $\rho_i$ be the DistressRank of bank $i$, which can be defined as

$$
\rho_i = \frac{1}{\lambda} \sum_{j=1}^{N} d_{ij} \rho_j
$$

(6)

where $\lambda \neq 0$ is a constant. With some rearrangements, Eq. (6) can be rewritten in matrix notation as

$$
D \cdot \rho = \lambda \cdot \rho
$$

(7)

which is a standard eigenvector–eigenvalues problem where $\lambda$ is an eigenvalue and $\rho$ is its corresponding $1 \times N$ vector. Given that the matrix $D$ is non-negative and according to the Perron–Frobenius theorem (Meyer, 2000), the above eigenvector–eigenvalues problem has a unique solution at $\lambda = \lambda_{\max}$. In other words, only the largest eigenvalue $\lambda_{\max}$ results in the desired non-negative eigenvector $\rho$ which represents the DistressRank vector of banks where the $i$th entry corresponds to the DistressRank of the $i$th bank. Eq. (7) can be solved iteratively using the power iteration method (Newman, 2010).

DistressRank is more suitable as a measure of systemic distress of a bank in a financial network because it assigns a rank to each bank in the network based on the distress of its counterparties. Thus, it is more suited for use with dynamic networks where the states of banks and the weights of exposures change during a distress propagation process. Here, we use DistressRank as one of the main metrics in our macroprudential stress test that is introduced in Section 5.

4. Illiquidity and insolvency

Assessing at what point liquidity risk becomes solvency risk is, at best, difficult. In this section we attempt to disentangle these two risks, and show how to express solvency risk in terms of liquidity risk.

4.1. From illiquidity to insolvency

Typically, a bank $i$ is considered illiquid when $A_i^L \leq L_i^S$, in other words, when the market value of its liquid assets is less than the face value of its short-term obligations. The same logic can be extended to insolvency. A bank is considered insolvent when the market value of its assets falls below the face value of its obligations, where $E_i \leq 0$. Fig. 1 illustrates the relation between illiquidity and insolvency. We would expect a bank to be liquid and solvent as shown by the white area in this figure. Nevertheless, a bank might become illiquid while still being solvent as shown by the grey area. However, if the bank’s illiquidity problem is severe enough, it can lead to insolvency as shown by the black area in the same figure.

Another way to consider insolvency is by limiting the focus to liquid assets and short-term liabilities. Insolvency occurs when the decline in liquid assets is severe enough to exceed the value of equity. In other words, the bank becomes insolvent if the market value of its liquid assets deteriorates to the extent that the net change in its liquidity at a given time is larger than its equity. That said, we can introduce a new condition for insolvency in terms of liquid assets by which a bank is considered insolvent if

$$
E_i + \Delta A_i^L \leq 0
$$

(8)

where $\Delta A_i^L$ is the net change in the bank’s liquidity position assuming that short-term liabilities are valued at face value.

Thus, one might argue that, in the short-run, both illiquidity and insolvency can be measured in terms of the change in liquid assets, assuming that the change in illiquid assets is trivially small and liabilities are valued at book value. That said, the illiquidity point for a bank is defined to be the point at which $A_i^L = L_i^S$, as mentioned above. At this
point, we can estimate a bank’s liquidity coverage ratio, denoted as $\ell^L_i$, that corresponds to its illiquidity point as

$$
\ell^L_i = \frac{A^L_i}{L^i_S} \tag{9}
$$

Applying the same logic to insolvency, the insolvency point for a bank can be defined as the point at which $E_i = -\Delta A^L_i$, as shown by Eq. (8). At this point, we can also estimate a bank’s liquidity coverage ratio, denoted as $\ell^S_i$, that corresponds to its insolvency point as

$$
\ell^S_i = \frac{A^L_i - E_i}{L^i_S} \tag{10}
$$

where $E_i$ represents the amount of liquid assets that, if depleted, the bank is considered to have reached the insolvency point.

4.2. From insolvency to illiquidity

One way to measure insolvency risk is to determine how far away a bank is from insolvency. This approach is called distance-to-default, which is developed based on the structural model of corporate debt introduced by Black and Scholes (1973). On this premise, we drive a measure of insolvency risk for individual banks in our system. We call this measure distance-to-default ($\delta^S$) which is completely analogous to and based on the distance-to-default measure in the Moody’s KMV model (see Bohn and Crosbie, 2003). The main difference in our approach is that we estimate the distance-to-default using liquid assets and short-term liabilities only, instead of total assets and total liabilities in the distance-to-default model, assuming that the change in illiquid assets is trivially small and liabilities are valued at book value.

However, liquid assets of banks are not directly observed which make it difficult to assign a specific dynamic process to the bank liquid assets. Therefore, in the same vein of the structural model of Merton (1974), we assume that the liquid asset $A^L$ of a given bank follows a geometric Brownian motion such that $dA^L_t = \mu A^L_t d\tau + \sigma A^L_t dW_t$, $A^L_0 > 0$, where $\mu A^L$ is the mean rate of return on the liquid assets and $\sigma A^L$ is the liquid asset volatility. We further assume that there are no bankruptcy charges where the liquidation value equals the bank value, and that the debt and equity of the bank are frictionless tradeable assets. Further, unlike the Merton model, we consider only one part of the bank’s debt represented in its outstanding short-term liabilities with a face value of $L^S$ and maturity $T$. Thus, at maturity $T$, if the total value of liquid assets is less than the short-term liabilities, the bank is considered illiquid. To quantify the bank’s insolvency, the problem then reduces to identifying how deep into illiquidity a bank can be before the condition in Eq. (8) is satisfied and the bank becomes insolvent.

Based on the above assumptions and knowing a bank’s insolvency point as derived from Eq. (10) above, the distance-to-insolvency of bank $i$, denoted as $\delta^S_i$, can be defined as

$$
\delta^S_i = \frac{\ln \left( \frac{A^L_i}{\chi^S_i L^i_S} \right) + \left( \mu A^L_i - \frac{1}{2} \sigma^2 A^L_i \right) T}{\sigma A^L_i \sqrt{T}} \tag{11}
$$

where $\mu A^L$ and $\sigma A^L$ are the mean and volatility of return on liquid assets and $T$ is the time to maturity. It is worth noting from Eq. (11) that distance-to-insolvency is simply the number of standard deviations that the bank is away from insolvency. Furthermore, following the assumption in Black and Scholes (1973) that the random component of a firm’s asset returns is normally distributed, we can define the probability of insolvency of a specific bank as

$$
\chi^S = N \left[-\delta^S \right] \tag{12}
$$

where $N(x)$ is the cumulative distribution function (CDF) of the standard normal distribution $N(0,1)$. Notice also that $\chi^S$ is similar to the probability of default in standard credit risk models.

We now turn to estimating two measures of illiquidity risk; namely distance-to-illiquidity and probability of illiquidity. Needless to say, these two measures are analogous to those measures that we introduced above to measure insolvency risk. Thus, in order not to repeat ourselves, we just extend the same logic we used with insolvency. In so doing, we argue that illiquidity can be viewed as a special case of insolvency in the short-run, assuming that the change in illiquid assets is trivially small and liabilities are valued at book value. Thus, knowing the illiquidity point $\ell^L_i$ of a given bank $i$ as derived from Eq. (9), we can estimate the distance-to-illiquidity of this bank, denoted as $\delta^L_i$, as follows

$$
\delta^L_i = \frac{\ln \left( \frac{A^L_i}{\chi^L_i L^i_S} \right) + \left( \mu A^L_i - \frac{1}{2} \sigma^2 A^L_i \right) T}{\sigma A^L_i \sqrt{T}} \tag{13}
$$

Similar to the distance to insolvency, we can interpret the distance to illiquidity as the number of standard deviations that the bank is away from illiquidity. Finally, let $\chi^L$ be the probability of insolvency for bank $i$. It follows that:

$$
\chi^L_i = N \left[-\delta^L_i \right] \tag{14}
$$

where $N(x)$ is the cumulative distribution function (CDF) of the standard normal distribution $N(0,1)$.

The relationship between the illiquidity and insolvency measures that we derive above can best be illustrated by Fig. 2. Let us assume a bank $i$ that operates in a system with a system-wide illiquidity point ($\ell^L$) and insolvency point ($\ell^S$) of 100% and 50%, respectively. In panel (a), we estimate distance to insolvency ($\delta^S$) and distance to illiquidity ($\delta^L$), and in panel (b) we estimate the corresponding probabilities of insolvency ($\chi^S$) and illiquidity ($\chi^L$) over a range of liquidity coverage ratios ($\ell^L$) from zero to 300%. The figure shows that as the liquidity coverage ratio decreases, both $\delta^S$ and $\delta^L$ decrease, while $\chi^S$ and $\chi^L$ increase in parallel. When $\ell^L$ reaches the insolvency point of 100%, $\delta^L$ becomes zero, $\chi^L$ reaches 1, and the bank is considered to be illiquid. However, at the illiquidity point, the bank is still solvent as $\delta^S$ is still higher than zero and $\chi^S$ is still lower than 1. As the bank sinks more into illiquidity, its $\delta^S$ moves towards the insolvency point and its $\chi^S$ converges to 1. At the insolvency point of 50%, $\delta^S$ becomes zero, $\chi^S$ reaches 1, and the bank is considered to be insolvent.

5. A macroprudential stress testing framework

In this section we provide a framework for a macroprudential stress test based on the measures that we introduced in Sections 3 and 4. This framework is illustrated in Fig. 3. Also, in the subsections below, we outline this framework in terms of its inputs (distress scenario), process (distress propagation process), and outputs (DistressRank, Distress Dependence Matrix, Default Dependence Matrix, and Systemic Risk Matrix).
Fig. 2. The relationship between insolvency measures and illiquidity measures of a hypothetical bank \(i\) whose liquidity coverage ratio is denoted by \(\ell_i\). \(\delta_L^i\) is the distance to illiquidity, and \(\delta_S^i\) is the distance to insolvency. \(\chi_L^i\) is the probability of illiquidity, and \(\chi_S^i\) is the probability of insolvency. The system-wide illiquidity point (\(L^S\)) and insolvency point (\(S^S\)) are 100% and 50%, respectively.

5.1. Inputs: Distress scenario

The distress scenario in our framework refers to the set of shocks applied to individual banks, specific groups of banks, or all banks in the system with the aim to examine the systemic impact and vulnerability of individual banks and the stability of the system as a whole. The framework is flexible to include any plausible set of shock events. However, we limit the analysis to two types of shock, with each one designed to examine specific aspects of the stability of the system.

A- The first scenario involves applying a uniform shock to all banks in the system. The immediate effect of this shock is a proportional reduction in all banks’ interbank assets leading to a reduction in liquidity positions. This scenario is also flexible to investigate the impact of a vector of heterogeneous shocks where each bank is affected differently.

B- The second scenario involves shocking banks sequentially. In each round a specific bank loses a given amount of its liquid assets and therefore becomes illiquid. The immediate effect of this shock is that the respective bank cross-defaults in all its interbank liabilities which leads to the write-off of the interbank assets of its counterparties. This scenario is flexible to include a group of banks instead of a single bank.

The feedback round effects and final results of each scenario are explained in more detail in Sections 5.2 and 5.4, respectively.

5.2. Distress propagation process

The distress scenario that is developed in our stress test is assumed to unroll in two rounds:

A- During the first round, the initial effects of shocks to banks’ liquidity positions are estimated by applying the shock to the respective bank or banks. The total initial impact of the shock is equal to the sum of the liquidity loss of all banks affected by the initial shock.
B- During the feedback round, the effects of the distress feedback loops within the system are estimated. The change in liquidity positions of individual banks leads to a change in their liquidity risk profiles. In other words, it leads to a change in each bank’s liquidity coverage ratio as estimated by Eq. (2) and the relative liquidity coverage matrix as estimated by Eq. (3). As the liquidity risk of each bank changes, so does its ability to repay its obligations to its counterparties. This ability is translated into the relative distress matrix as estimated by Eq. (4). The market values of the interbank assets are re-estimated based on the expected value to be collected from counterparties. We estimate this expected value using a distress propagation factor that is directly derived from the relative distress matrix as follows

$$A^B_i(t) = \max \left[ 0, A^B_i(t-1) \left( \frac{d_{ij}(t)}{d_{ij}(t-1)} \right) \right]$$

where $A^B_i(t)$ and $A^B_i(t-1)$ are the interbank assets of bank $i$ with bank $j$ at time steps $t$ and $t-1$ of the distress propagation process, respectively; whereas $d_{ij}(t)$ and $d_{ij}(t-1)$ are the distress of bank $i$ relative to bank $j$ at time steps $t$ and $t-1$ of the distress propagation process, respectively. The idea is that, when the distress of bank $j$ increases, bank $i$’s exposure to bank $j$ deteriorates proportionally, and if bank $j$ becomes insolvent, bank $i$ loses its assets with bank $j$. In fact, Eq. (15) assumes a zero recovery rate, an assumption that is widely followed in the financial contagion literature (see Gai and Kapadia, 2010; Markose et al., 2012). The mark-to-market process is represented by the dashed lines in Fig. 3. The change in the interbank assets matrix leads to repeating the same sequence of distress propagation in the system. This process continues until the initial shock to the system decays when no further significant changes in the system are expected.

After the second round of distress propagation concludes, the system arrives at a new steady state. We then estimate a few metrics to examine the stability of this system which we outline in Section 5.4.

5.3. Default propagation process

The stress test framework that we provide is capable of bridging the space between illiquidity and insolvency. This is possible due to the fact that we model the evolution of insolvency in terms of illiquidity as explained in detail in Section 4 and outlined by the far left column in

Fig. 3. A framework for an integrated macroprudential stress test of liquidity and solvency.

5.4. Stress test output

The stress test framework presented here provides a variety of output metrics that aim to depict the individual banks and the system’s stability. These metrics are presented in the bottom row in Fig. 3. We briefly explain these metrics below.

A- DistressRank

DistressRank provides a convenient way to depict the systemic vulnerability of each bank in the system. It is estimated based on the relative distress matrix and thus reflects the relative vulnerability of each bank to the distress of its counterparties. Banks with higher DistressRank measure are more vulnerable to system-wide shocks than otherwise comparable banks. The exact method of estimating DistressRank is explained in more detail in Section 3.4.

B- Distress Dependence Matrix

The distress dependence matrix provides a more detailed method to examine the systemic vulnerability of each bank in the system. In particular, for each pair of banks in the system, we estimate the pairwise conditional probability of illiquidity. The matrix is row-wise, meaning that it shows the probability of illiquidity of the bank specified in the row, given that the bank specified in the column has become illiquid. Thus, it provides an indicator of distress contagion possibilities within the system.

C- Default Dependence Matrix

The default dependence matrix is another way to depict the dependency within the system in detail, while, at the same time linking illiquidity to insolvency. In particular, for each pair of banks in the system, we estimate the probability of insolvency of a given bank conditional on the other bank becoming illiquid. The matrix is also row-wise as it provides the probability of insolvency of the bank specified in the row, given that the bank specified in the column has become illiquid. Thus, it provides an indicator of default contagion possibilities within the system.
D- Systemic Risk Matrix

The previous metrics provide a convenient way to depict the systemic vulnerability and dependencies within the system. This is very important to assess the contagion possibilities in the system. The stress test also provides another way to do this through a systemic risk matrix which lends itself more to economic interpretation. In this matrix we quantify systemic vulnerability and impact in terms of expected economic loss. We explain the constituents of the systemic risk matrix in detail here as it was not introduced elsewhere.

Similar to previous matrices, each row represents the vulnerability of the bank in this row to the distress of other banks. Let $V_{ij}$ be the expected loss of the bank in row $i$ due to the distress of the bank in column $j$. Assuming that bank $j$ has become illiquid, the loss that bank $i$ encounters is equal to $[A_i^T(0) - A_i^T(T)]$ and in percentage terms it is $[(A_i^T(0) - A_i^T(T))/A_i^T(0)]$, where $A_i^T(0)$ and $A_i^T(T)$ are the total interbank liquid assets of bank $i$ at times $t = 0$ and $t = T$, respectively. Then, the amount of expected liquidity loss can be estimated as

$$V_{ij} = (1 - R) \chi^T(0) \left[ \frac{A_i^T(0) - A_i^T(T)}{A_i^T(0)} \right]$$

(16)

where $R$ is the recovery rate and $\chi^T(0)$ is the probability of illiquidity of bank $j$ at $t = 0$. We can then estimate the systemic expected liquidity loss of bank $i$ as

$$V_i = \frac{A_i^T(0)}{\sum_i N_j} \sum_j V_{ij}$$

(17)

This measure of systemic expected loss represents the systemic vulnerability of bank $i$ measured as the probability-weighted expected liquidity loss of bank $i$ due to the distress of other banks in the system. To enable comparability, this measure is weighted by the liquid assets of bank $i$ to the total liquid assets in the system.

Similarly, let $I_{ij}$ be the relative impact of bank $i$ on bank $j$ which represents the expected loss that the distress of bank $i$ can induce to bank $j$. We can estimate this amount as

$$I_{ij} = (1 - R) \chi^T(0) \left[ \frac{A_i^T(0) - A_i^T(T)}{A_i^T(0)} \right]$$

(18)

We can then estimate the systemic impact of bank $i$ as the total expected liquidity loss caused to all other banks due to the distress of bank $i$, as follows

$$I_i = \sum_j A_j^T(0) - I_{ij}$$

(19)

which represents the probability-weighted expected liquidity loss of other banks in the system due to the distress of bank $i$. Also, to enable comparability, this measure is weighted by the liquid assets of each bank $j$ to the total liquid assets in the system. Finally, we provide a measure of the system-wide expected liquidity loss as

$$\Phi = \sum_i I_i = \sum_i V_i$$

(20)

where $\Phi$ is the probability-weighted systemic expected liquidity loss. This measure provides an indication of the system-wide stability. The higher the systemic expected loss, the lower the system stability.

Appendix provides a toy model to illustrate the dynamics of the stress test using simulated data of a simplified banking system.

6. Empirical application

In this section, we provide an overview of the data used and the main results of applying the stress test framework outlined in Section 5 to the US banking system.

6.1. Data and interbank network construction

The data used to implement the stress test is related to the largest 25 bank holding companies in the US based on interbank derivatives activities as of December 31, 2020.\footnote{We use the term holding companies to refer to all types of holding companies under the direct supervision of the Federal Reserve Board including domestic bank holding companies (BHC), savings and loan holding companies (SLHC), U.S intermediate holding companies (IHC) and securities holding companies (SHC).} For each holding company (bank henceforth), we obtain data about balance sheet holdings, liquidity coverage ratios, and derivatives exposures. The balance sheet data is collected from the Consolidated Financial Statements of banks (FR Y-9C reports) provided by the National Information Centre.\footnote{Available at https://www.ffiec.gov/nicpubweb/nicweb/nichome.aspx.} From these reports we extract information about total assets, total liabilities, derivatives assets and liabilities, and equity. We use the Quarterly Report on Bank Derivatives Activities from the Office of the Comptroller of the Currency to obtain data about the interbank exposures of each bank.\footnote{Available at https://www.occ.gov/topics/capital-markets/financial-markets/derivatives/derivatives-quarterly-report.html.} The data about liquidity coverage ratio is hand collected from the annual and quarterly reports of each bank. From these reports we collect the reported amounts of high quality liquid assets, net cash outflows expected over the next 30 days, and the liquidity coverage ratio of each bank.

We use the interbank derivatives exposures as they represent liquidity flows between banks and are included in calculating the liquidity coverage ratio that banks disclose in their reports (BCBS, 2013b). Any change in the amounts of derivatives assets or liabilities leads to changes in the estimated liquidity coverage ratio, and hence can be used as a way to monitor distress propagation within the interbank market. The network of derivatives assets and liabilities within the interbank market can be represented by the following matrix:

$$A^D = \begin{bmatrix} A_{11} & \ldots & A_{1j} & \ldots & A_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ A_{N1} & \ldots & A_{nj} & \ldots & A_{NN} \end{bmatrix}$$

where $A_{ij}$ represents the derivatives assets of bank $i$ with bank $j$ or the derivatives liabilities of bank $j$ to bank $i$. The matrix size is $N \times N$ where $N$ is the number of banks. The sum of a row represents the derivatives assets of the respective bank where $A^D_i = \sum_j A_{ij}$ and the sum of a column represents the derivatives liabilities of the respective bank where $L^D_j = \sum_i A_{ji}$. Unfortunately, the network of interbank exposures is not observable, as banks and supervisors do not provide granular data about bilateral exposures.

Since information on the bilateral interbank exposures is essential for our analysis, we estimate these data to fill in the interbank matrix. To this end, we use the Minimum Density (MD) method suggested by Anand et al. (2015). The idea behind MD is to distribute each bank’s assets and liabilities among the lowest possible number of counterparties. The economic rationale for this is that the interbank network appears to be constructed based on relationships and, as a result, is sparse (Cocco et al., 2009) as banks aim to minimise the costs of establishing and maintaining linkages including the costs of information processing and risk management. This rationale is supported by studies of real-world financial networks of the US (Bech and Atalay, 2010) and Germany (Craig and von Peter, 2014).
6.2. Results of the stress test

The proposed stress test offers a variety of metrics to assess the system resilience. We provide here an overview of the system before applying any stress scenarios. Then, we provide the results of applying the first and second stress scenarios to assess the system stability and systemic interdependencies, respectively.

6.2.1. System profile

As discussed in Section 3, DistressRank provides a relative rank of all banks within a system with regard to their vulnerability to the distress of other banks. In addition, DistressRank can be estimated before applying any stress scenarios which offers the advantage of depicting the stability of the system at any point of time. We use this indicator to provide an overview of the current state of the US banking system, as of 31 December 2020. Fig. 4 shows the interbank market network which comprises the 25 individual banks included in our stress test. On this network, the size of each bank is scaled proportionally to its DistressRank. As illustrated, Citi Group is the most vulnerable bank, followed by JP Morgan Chase and Goldman Sachs, while the two least vulnerable banks are Suntrust Banks and DB USA Corporation. The other banks have comparable ranks. Further illustration of DistressRank is provided in Fig. 5.

There is a striking observation that can be noticed from Fig. 4 about banks’ DistressRank. The asset size of a bank does not entail its systemic vulnerability. For example, Bank of America is the second largest bank measured by total assets; however, its DistressRank is comparable to other smaller banks such as RBC USA Group and Capital One. Moreover, even a bank’s interbank assets or liabilities alone do not completely determine its DistressRank. An example of this is Citi Group which has the second largest interbank assets but ranks third based on DistressRank. In fact, DistressRank is affected by the interconnectedness within the interbank market in addition to the size of both interbank assets and liabilities. This finding has some important implications for the methodology of identifying global systemically important banks (G-SIBs) (BCBS, 2013a). In particular, the methodology should consider systemic distress as well as systemic importance in measuring a bank’s interconnectedness as one of the indicators used to identify G-SIBs.

6.2.2. System stability

We turn now to assess the stability of the banking system following our proposed stress testing framework. To this end, we implement the first stress scenario (as explained in Section 5) in which a uniform shock is applied to all banks in order to assess the resilience of the banking system to macroeconomic shocks. We use a vector of shocks that ranges from 1% to 25% decline in the interbank liquid assets, which are extreme enough, yet plausible. We can think of a shock as resembling a severe change in risk free rates or widening in credit spreads that affect all banks simultaneously. The initial shock leads to a proportional reduction in the interbank assets of all banks leading to reductions in their liquidity positions. Then, the distress propagation process unfolds. The stress testing exercise provides a variety of output metrics which are outlined below.

Fig. 6 shows the number of distressed banks that become illiquid or insolvent following each shock. As expected, both numbers increase with the increase in the shock applied to the system. It is worth noting that the increase in both numbers is not linear. This is due to the fact that whether a distressed bank becomes illiquid or insolvent depends not only on its liquidity position but also on the liquidity position of its counterparties and the severity of the shock. As illustrated, banks are resilient to small shocks up to 4%, while some banks reach the illiquidity point starting from shocks of as low as 5%. The insolvency point is reached much later as the first time a bank becomes insolvent occurs at a shock level of 19%.

Fig. 7 provides a decomposition of systemic loss into first-round loss due to the initial shock and feedback loss occurring during second and upper rounds. Systemic loss is estimated at the system level as the total reduction in the value of banks’ liquid assets. A striking observation that is shown in this figure is that the feedback loss can be as large as the initial loss due to the systemic shock. It can also exceed the initial loss at a high level of shocks. This observation highlights the need to consider the feedback loss due to interconnectedness between banks while designing macroprudential stress tests.

Another way to highlight the role of interconnectedness is to consider the relationship between DistressRank and systemic feedback loss at the bank level. We use DistressRank as a measure of systemic distress that captures interconnectedness, while a bank’s systemic feedback loss is estimated as its share in the total feedback loss at the system level due to a specific shock. We limit the analysis here to a shock size of 10%. The result of this exercise is shown in Fig. 8. As illustrated, there seems to be a positive relationship between the DistressRank of a bank and its systemic feedback loss. To investigate this further, we run a simple regression of systemic feedback loss on DistressRank. The results show a positive slope which is significant at a 95% level with adjusted R² of 67%. This result confirms the importance of considering interconnectedness in designing macroprudential stress tests.

Finally, we illustrate the resilience of the system to shocks by tracing the change in the probability of illiquidity and the probability of insolvency of each bank following a specific shock. Figs. 9 and 10 show, for each bank, the change in probability of illiquidity and the change in probability of insolvency, respectively. Again, we limit the analysis to a shock size of 10% of interbank assets. As can be seen clearly from these figures, both probabilities show remarkable increase with almost all banks having higher probabilities of illiquidity and insolvency following the shock. While some banks become illiquid following the shock, some of them see their probability of insolvency nearly doubled.

6.2.3. Systemic interdependencies

So far, our analysis of stability has focused on the resilience of the system to a system-wide shock that represents a macroeconomic shock. We extend the analysis here to examine the interdependencies within the system. To this end, we implement the second stress scenario which involves shocking banks sequentially (see Section 5 for more details). The results of this exercise are shown below.

A- Systemic Distress Dependence

The distress dependence matrix provides insight into the interlinkages between banks and how vulnerable they are to the distress of each other. In particular, the output shown by this matrix can be viewed as the conditional change in the probability of illiquidity of the bank in the row relative to the bank in the column. In Fig. 11, we present the distress dependence matrix estimated for the group of 25 US banks included in the stress test. In this matrix, each cell represents the percentage change in the probability of illiquidity of the bank in the row given that the bank in the column has become illiquid. The diagonal of this matrix represents the implied percentage change in the probability of illiquidity of a given bank when this bank itself becomes illiquid. For better illustration, we provide the matrix as a heatmap. As can be seen from the matrix, distress dependence is higher among banks that are located at the upper left quadrant of the matrix. Put differently, large changes in the probability of illiquidity are associated with banks that have large interbank exposures with each other. For example, JP Morgan is more vulnerable to the distress of Citi Group, Goldman Sachs, and Bank of America compared to other banks in the sample. Another interesting observation is that the four most vulnerable banks; namely, JP Morgan, Citi Group, Goldman Sachs, and Morgan Stanley, derive their vulnerability from each other. This is explained by the fact that the exposure of these banks to each other represents a large portion of their overall interbank assets. Any distress that arises with one of them will definitely lead to a serious liquidity problem with the others. It is also worth noting that banks in the lower right quadrant seem to be
Fig. 4. The distress network and DistressRank of individual banks in the system. Nodes represent banks and links represent interbank exposures. The size and colour of each node are scaled proportionally to the value of its DistressRank. The width and colour of each link are scaled proportionally to the value of relative distress induced by the bank at the start of the link to the bank at the end of the link. Number of banks is 25. Banks’ names are coded from B1 to B25 where B1: JPMorgan Chase & CO.; B2: Citigroup Inc.; B3: Goldman Sachs Group, Inc.; B4: Morgan Stanley; B5: Bank of America Corporation; B6: Wells Fargo & Company; B7: Mizuho Americas LLC; B8: SMBC Americas Holdings, Inc.; B9: HSBC North America Holdings Inc.; B10: State Street Corporation; B11: Bank of New York Mellon Corporation; B12: US Bancorp; B13: RBC US Group Holdings LLC; B14: Barclays US LLC; B15: PNC Financial Services Group, Inc.; B16: TD Group US Holdings LLC; B17: Northern Trust Corporation; B18: Truist Financial Corporation; B19: Credit Suisse Holdings (USA), Inc.; B20: Capital One Financial Corporation; B21: MUFG Americas Holdings Corporation; B22: Citizens Financial Group, Inc.; B23: BOK Financial Corporation; B24: Fifth Third Bancorp; and B25: Ameriprise Financial, Inc.

resilient to the distress of each other mainly due to the fact that they have limited exposures to each other.

B- Systemic Default Dependence

The stress test output includes another interesting matrix called the default dependence matrix. This matrix examines the possibility that illiquidity distress evolves to become insolvency default. It is also similar to the distress dependence matrix in that it provides insight into the possibility of contagion within the system. The default dependence matrix is illustrated in Fig. 12 where each row represents the percentage change in the probability of insolvency of the bank in the row when the bank in the column becomes illiquid. Again, each cell can be viewed as the conditional change in the probability of insolvency of the bank in the row relative to the bank in the column. The diagonal of this matrix represents the implied percentage change in the probability of insolvency of a given bank when this bank itself becomes illiquid. For better illustration, the matrix is shown as a heatmap.

The same observations on the distress dependence matrix also apply here. The default dependence seems to be higher among banks in the upper left quadrant and lower among banks in the lower right quadrant. Again, this is due to concentration of exposure between big banks and each other or big banks and other smaller banks, while exposures between smaller banks and each other are limited. For example, Bank of America appears to be the most vulnerable bank to shocks from its counterparties and particularly from JP Morgan and Citi Group. If JP Morgan becomes illiquid, the probability of insolvency of Bank of America increases by 185%. Any distress at JP Morgan will definitely lead to a serious liquidity problem with its counterparties.

C- Systemic Risk Matrix

The systemic risk matrix provides an estimation of systemic expected loss of each bank due to other banks’ distress. It provides another way to study the interdependence among banks by quantifying the systemic expected economic loss due to systemic distress between pairs of banks. Each cell in the matrix represents the expected loss of the bank in the row given that the bank in the column has become illiquid. The diagonal of this matrix represents the implied systemic loss of a given bank when this bank itself becomes illiquid. The matrix is shown as a heatmap. SysImpact stands for systemic impact of the bank in the column and is estimated as in Eq. (19) while SysVul stands for systemic vulnerability of the bank in the row and is estimated as in Eq. (17). The cell in the intersection of SysImpact and SysVul represents the systemic loss which is estimated as in Eq. (20). As explained earlier, to enable comparability, SysVul and SysImpact are estimated as a percent of the total liquid assets in the system.

The systemic risk matrix confirms the same results obtained by analysing distress and default dependence matrices. As would be expected, systemic loss is more concentrated in the upper left quadrant and limited among banks in the lower right quadrant. Counterparties of large banks are more vulnerable to systemic risk compared to...
Fig. 5. The DistressRank of individual banks in the system. Number of banks is 25. Bank names are as in Fig. 4.

Fig. 6. Number of Illiquid and Insolvent Banks due to a specific shock to the interbank assets. Shock size ranges from 1% to 25% of interbank assets.

others. If JP Morgan becomes illiquid, it induces a 1.54% system-wide expected loss which represents its systemic impact on the system. Goldman Sachs has a systemic vulnerability of 1.21% which represents its probability-weighted expected loss from its counterparties. The expected systemic loss is 6.58% which represents a system-wide stability measure. The higher this indicator is the more fragile the system is,
Fig. 7. Decomposition of systemic loss into first-round loss due to the initial shock and feedback loss occurring during second and upper rounds. Shock size ranges from 1% to 25% of interbank assets. Systemic loss is estimated at the system level as the total reduction in the value of banks’ assets.

Fig. 8. DistressRank and systemic feedback loss. A bank’s systemic feedback loss is estimated as its share in the total feedback loss at the system level due to a specific shock. Shock size is 10% of interbank assets. Each circle in the figure represents a bank.
Fig. 9. Change in probability of illiquidity for each bank in the system due to a specific shock. Shock size is 10% of interbank assets. $\chi^L_i(0)$ and $\chi^L_i(T)$ are the probability of illiquidity of bank $i$ before and after applying the shock to the system, respectively. Each circle in the figure represents a bank.

Fig. 10. Change in probability of insolvency for each bank in the system due to a specific shock. Shock size is 10% of interbank assets. $\chi^S_i(0)$ and $\chi^S_i(T)$ are the probability of insolvency of bank $i$ before and after applying the shock to the system, respectively. Each circle in the figure represents a bank.
Fig. 11. Distress dependence matrix. Each cell in the matrix represents the percentage change in the probability of illiquidity of the bank in the row when the bank in the column becomes illiquid. The diagonal represents the implied percentage change in the probability of illiquidity of a given bank when it becomes illiquid. Bank names are as in Fig. 4. The matrix is presented as a heatmap where cells’ colour is scaled from green for low values to red for high values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 12. Default dependence matrix. Each cell in the matrix represents the percentage change in the probability of insolvency of the bank in the row given that the bank in the column has become illiquid. The diagonal represents the implied percentage change in the probability of insolvency of a given bank when it becomes illiquid. Bank names are as in Fig. 4. The matrix is presented as a heatmap where cells’ colour is scaled from green for low values to red for high values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Another interesting finding from the systemic risk matrix is related to the relationship between systemic impact (SysImpact row) and systemic vulnerability (SysVul column) of each bank. We illustrate this relationship in Fig. 14. While most banks seem to be vulnerable to shocks from other banks as measured by their expected loss, not all banks have significant systemic impact as the systemic impact values of smaller banks seem to be negligible. Only big banks in the sample have systemic impact levels that are comparable to their systemic vulnerability levels. In addition, banks do not have the same ranking based on systemic impact and systemic vulnerability indicators. This finding has important implications for designing a macroprudential stress test that aims to consider interconnectedness. In particular, using measures of systemic impact is not sufficient to identify the vulnerabilities within a system. A comprehensive analysis of interconnectedness should consider systemic vulnerability as well as systemic impact of the financial institutions in the system.

7. Conclusion

This paper proposes a macroprudential stress testing approach and illustrates its empirical application on a data set of the US banking system. The innovative features of the proposed macroprudential stress test were inspired by the recent regulatory recommendations to strengthen the systemic focus and to more deeply consider the interactions between liquidity and solvency risks in designing effective macroprudential stress tests. In particular, the proposed approach provides a tool for the banking system supervisors to analyse the current representing more interconnectedness and/or higher probabilities of distress.

Another interesting finding from the systemic risk matrix is related to the relationship between systemic impact (SysImpact row) and systemic vulnerability (SysVul column) of each bank. We illustrate this relationship in Fig. 14. While most banks seem to be vulnerable to shocks from other banks as measured by their expected loss, not all banks have significant systemic impact as the systemic impact values of smaller banks seem to be negligible. Only big banks in the sample have systemic impact levels that are comparable to their systemic vulnerability levels. In addition, banks do not have the same ranking based on systemic impact and systemic vulnerability indicators. This finding has important implications for designing a macroprudential stress test that aims to consider interconnectedness. In particular, using measures of systemic impact is not sufficient to identify the vulnerabilities within a system. A comprehensive analysis of interconnectedness should consider systemic vulnerability as well as systemic impact of the financial institutions in the system.

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This paper proposes a macroprudential stress testing approach and illustrates its empirical application on a data set of the US banking system. The innovative features of the proposed macroprudential stress test were inspired by the recent regulatory recommendations to strengthen the systemic focus and to more deeply consider the interactions between liquidity and solvency risks in designing effective macroprudential stress tests. In particular, the proposed approach provides a tool for the banking system supervisors to analyse the current
M. Bakoush et al.  
Journal of Financial Stability 60 (2022) 101012

Fig. 13. Systemic risk matrix. Each cell in the matrix represents the expected loss of the bank in the row given that the bank in the column has become illiquid. SysImpact stands for systemic impact of the bank in the column. SysVul stands for systemic vulnerability of the bank in the row. The cell in the intersection of SysImpact and SysVul represents the systemic loss. To enable comparability, SysVul and SysImpact are estimated as a percent of the total liquid assets in the system. Bank names are as in Fig. 4. The matrix is presented as a heatmap where cells’ colour is scaled from green for low values to red for high values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 14. The systemic impact and systemic vulnerability of individual banks.

state of the system stability. The empirical application of the stress test shows how it can be effectively used to identify the systemic vulnerability of individual banks as well as the resilience of the system as a whole to economic risks. The findings confirm the need to consider interconnectedness in designing macroprudential stress tests. At the bank level, the results confirm that interlinkages play a significant role in identifying individual banks’ vulnerability. On this premise, we propose DistressRank as a measure of the systemic distress of a bank. The results show that a bank’s DistressRank is associated with its systemic feedback loss. At the system level, the systemic loss due to feedback loops was shown to be significant compared to the direct loss that results from the initial shock to the system. Ignoring these feedback effects may lead to a significant underestimation of systemic loss.

Moreover, the proposed approach provides a tool for the banking system supervisors to monitor the evolution of contagion and systemic
risk within the system due to endogenous or exogenous shocks. Applying the stress test framework to the US banking system shows how it can be effective for monitoring and assessing interdependence among banks. Our findings provide an insight into the possibilities of distress propagation within the system. An important finding that is shown here is that banks that are not directly connected together through interbank assets or liabilities are still subject to distress from each other through common counterparties. These findings can form the basis for intervention by policy makers in case a specific bank has become distressed and there is a need to identify banks that are likely to be affected the most.

Although the framework demonstrated here was applied using a reconstructed network of interbank exposures, these data were sufficient to highlight the merit of the proposed stress test framework. However, we have to acknowledge that using granular supervisory bank data (e.g. FR 2052a daily liquidity report in the US) may change the results related to risks of individual banks in our sample even though we would expect to obtain similar results on systemic vulnerabilities in the banking system. Also, given that this stress test aims to identify the raw systemic impact of the proposed stress scenarios, it does not consider some important risk mitigation techniques in the interbank market. This includes collateralization requirements in interbank repo transactions that could limit the systemic impact of some shocks. Also, the existence of central counterparties could help reduce counterparty credit risk in the interbank market due to the daily initial and variation margins. This stress testing framework also takes the perspective of the banking system supervisor as it aims to help in making the intervention decisions related to which banks to support and by how much. Therefore, it identifies the systemic impact without considering the role of the central bank in providing liquidity to the banks and markets.

In conclusion, the proposed macroprudential stress test can reveal the systemic vulnerabilities in a banking system, giving policymakers insights into the system resilience. Extending the analysis to include additional banks would provide a tool for policymakers to more comprehensively monitor and regulate the interdependencies in the banking system and the resilience of the system as a whole. Another avenue for extending the work done here is to consider the reactions of banks to shocks and the possibilities of deleveraging and its impact on the magnitude of systemic loss.

Appendix. Toy model

This appendix provides an illustration of the stress test framework based on simulated data of three banks. Appendix A.1 outlines the banking system profile. Appendices A.2 and A.3 provide the results of the two stress scenarios discussed in Section 5.1.

A.1. System profile

Table A.1 provides information on the balance sheet of three banks that represent the banking system. These banks interact with each other and have a network of interbank assets $A^B$ and liabilities $L^B$. Table A.2 further decomposes the interbank assets and liabilities of banks where the value of each cell represents an asset of the bank in the row and a liability for the bank in the column.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^B$</td>
<td>$A^O$</td>
</tr>
<tr>
<td>$L^B$</td>
<td>$L^O$</td>
</tr>
<tr>
<td>$E$</td>
<td></td>
</tr>
</tbody>
</table>

Table A.2

<table>
<thead>
<tr>
<th>Bank</th>
<th>Bank 2</th>
<th>Bank 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>Bank 2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Bank 3</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Table A.3

<table>
<thead>
<tr>
<th>Liquidity coverage matrix</th>
<th>Liquidity coverage ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>Bank 2</td>
</tr>
<tr>
<td>Bank 1</td>
<td>0.25</td>
</tr>
<tr>
<td>Bank 2</td>
<td>1.00</td>
</tr>
<tr>
<td>Bank 3</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table A.4

<table>
<thead>
<tr>
<th>Distress matrix</th>
<th>DistressRank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>Bank 2</td>
</tr>
<tr>
<td>Bank 1</td>
<td>1.00</td>
</tr>
<tr>
<td>Bank 2</td>
<td>4.00</td>
</tr>
<tr>
<td>Bank 3</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table A.5

<table>
<thead>
<tr>
<th>Illiquidity point</th>
<th>Insolvency point</th>
<th>Probability of illiquidity</th>
<th>Probability of insolvency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>1</td>
<td>0.63</td>
<td>24.02%</td>
</tr>
<tr>
<td>Bank 2</td>
<td>1</td>
<td>0.53</td>
<td>15.25%</td>
</tr>
<tr>
<td>Bank 3</td>
<td>1</td>
<td>0.47</td>
<td>9.95%</td>
</tr>
</tbody>
</table>

Table A.6

| Interbank assets and liabilities matrix following stress scenario 1. |
|---------------------------|--------------------------|
| Bank 1 | Bank 2 | Bank 3 |
| Bank 1 | 18     | 20     | 60     |
| Bank 2 | 22.5   | 15     | 60     |
| Bank 3 | 27     | 20     | 50     |

Table A.7

<p>| Balance sheets of banks following stress scenario 1. |
|---------------------------|--------------------------|</p>
<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^B$</td>
<td>$A^O$</td>
</tr>
<tr>
<td>$L^B$</td>
<td>$L^O$</td>
</tr>
<tr>
<td>$E$</td>
<td></td>
</tr>
<tr>
<td>Bank 1</td>
<td>18</td>
</tr>
<tr>
<td>Bank 2</td>
<td>22.5</td>
</tr>
<tr>
<td>Bank 3</td>
<td>27</td>
</tr>
</tbody>
</table>

Table A.8

| Liquidity coverage matrix following stress scenario 1. |
|---------------------------|--------------------------|
| Bank 1 | Bank 2 | Bank 3 |
| Bank 1 | 0.20   | 1.05   | 1.17   |
| Bank 2 | 0.90   | 0.40   | 1.25   |
| Bank 3 | 0.80   | 1.53   | 1.57   |

Table A.9

| Distress matrix following stress scenario 1. |
|---------------------------|--------------------------|
| Bank 1 | Bank 2 | Bank 3 |
| Bank 1 | 1.11   | 1.25   | 0.66   |
| Bank 2 | 5.00   | 0.65   | 0.91   |
| Bank 3 | 0.95   | 2.50   | 0.76   |
The information from balance sheets are then used to estimate the overall liquidity coverage ratio and relative liquidity coverage matrix as illustrated in Table A.3. Further, the distress matrix is then estimated based on the liquidity coverage matrix and DistressRank is estimated based on Eq. (6) and the results are shown in Table A.4. We then outline the system stability profile in Table A.5 where the illiquidity and insolvency points based on Eqs. (9) and (10), respectively. Also, the probabilities of illiquidity and insolvency are estimated based on Eqs. (12) and (14), respectively. As the system profile shows, while Bank 1 has the highest insolvency point which is associated with the highest probability of illiquidity and insolvency, Bank 3 seems to be the most stable bank with the lowest insolvency point and probability of illiquidity and very low probability of insolvency. Bank 2 comes in between based on these stability measures.

### A.2. Stress scenario 1

Stress scenario 1 aims to study the overall system stability by applying a uniform shock to all banks in the system. We apply this by exposing the three banks to a shock that reduces their interbank assets by 10%. The affected interbank assets and liabilities matrix is shown in Table A.6. This leads to modified balance sheets as shown in Table A.7. We follow the same procedure as in Appendix A.1 above to estimate the liquidity coverage matrix (Table A.8), and the distress matrix together with the DistressRank (Table A.9).

The main results of the first stress scenario are shown in Table A.10. As can be seen, the 10% shock has led to impairment of the stability of all the three banks as shown by the increases in insolvency point, probability of illiquidity and probability of insolvency. However, on a relative basis, it seems that Bank 2 was affected the most by the shock as it has the highest percentage change in its probabilities of illiquidity (+45.15%) and insolvency (+157.75%). The second most affected was Bank 3, and the least affected was Bank 1. These results are very interesting as they follow the same order as the estimates of the systemic distress of the three banks as measured by DistressRank (see Table A.4). This shows that the systemic distress that results from the feedback loop in the interbank assets/liabilities network is the determining factor in the overall impact of uniform shock to the system.

### A.3. Stress scenario 2

Stress scenario 2 aims to evaluate the systemic interdependence in the system. We apply this by sequentially exposing the three banks to a shock that reduces their interbank assets by a sufficient amount to make the bank illiquid. For example, Banks 1 is exposed to a 37% decline in its interbank assets which is sufficient to make this bank illiquid. Similarly, Bank 2 and Bank 3 are exposed to a 40% and 67% decline in their interbank assets, respectively. Similar to stress scenario 1, the affected interbank assets and liabilities matrix, the balance sheets, the liquidity coverage matrix, and the distress matrix are estimated for each bank. In order to avoid repetition and due to limited space, we only show the main results here.

Table A.11 shows the systemic distress dependence matrix where each cell shows the change in the probability of illiquidity of the bank in the row when the bank in the column becomes illiquid under stress scenario 2. Similarly, Table A.12 shows the systemic default dependence matrix where each cell shows the change in the probability of insolvency of the bank in the row when the bank in the column becomes illiquid under stress scenario 2. These two matrices are useful to highlight the interdependence among the banks in the system. For example, the results show that Bank 1 illiquidity affects Bank 3 more than Bank 2, while Bank 3 is affects Bank 2 more than Bank 1. This is again in line with the initial systemic distress profile of the three banks as measured by DistressRank.

Another way to evaluate the systemic interdependence in the system is by estimating the expected loss in liquid assets of banks when a given bank becomes illiquid as shown in Table A.13. The main cells in this table shows the expected loss in liquid assets of the bank in the row when the bank in the column becomes illiquid under stress scenario 2. In addition, the total of each row shows the systemic vulnerability of the bank at the left of this row, while the total of each column shows the systemic impact of the bank at the top of this column. The results are still in line with the initial distress profile of the system. These results complement the estimates of change in probabilities of illiquidity and insolvency by providing a measure of economic loss in value.

### References


