Multimode effects in nonlinear fibre optics: From telecommunications to high-harmonic generation

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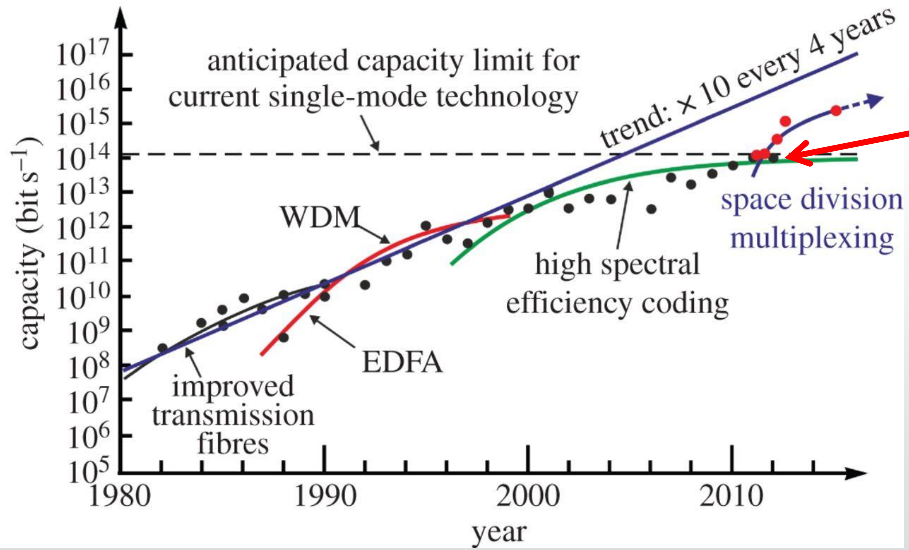
Strasbourg, 6 April 2022

Motivation:
Why multimode nonlinear fibre optics?

Why multimode fibres?

1) Optical telecommunications: Capacity crunch

- Current fibre technologies have reached fibre capacity limits
- One possible solution: use multiple modes, need switching between modes



OSA/OFC/NFOEC 2011

PDPB5.pdf

101.7-Tb/s (370×294-Gb/s) PDM-128QAM-OFDM Transmission over 3×55-km SSMF using Pilot-based Phase Noise Mitigation

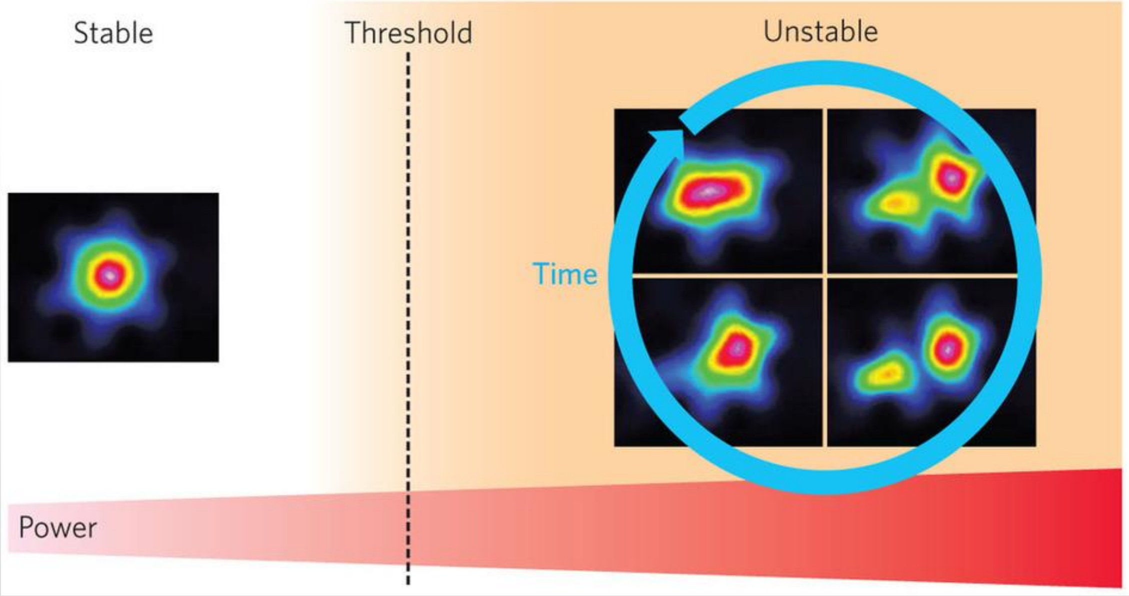
Dayou Qian, Ming-Fang Huang, Ezra Ip, Yue-Kai Huang, Yin Shao, Junqiang Hu, Ting Wang
NEC Laboratories America, Inc., 4 Independence Way, Princeton NJ 08540 USA
dqian@nec-labs.com

DJ Richardson,
Phil. Trans. R. Soc. A **374**, 20140441 (2016)

Why multimode fibres?

2) High power fibre lasers: Mode instability

- High power fibre lasers → large fibre cores to avoid damage → mode instability: laser becomes unstable beyond a certain threshold



- Need to understand and control mode interactions

C Jauregui et al,
Nature Photon. **7**, 861 (2013)

Outline

Theory:

- Multimode generalised nonlinear Schrödinger equation (GNLSE)

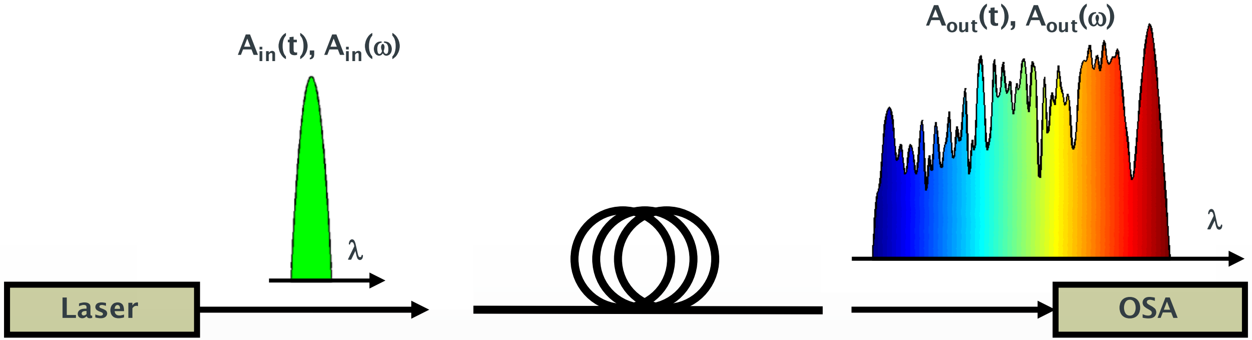
Applications (in order of increasing nonlinear complexity):

- Effectively one-mode operation: Supercontinuum generation in multimode waveguides
- Intermodal cross-phase modulation: Saturable absorber in ring laser
- Few-mode four-wave mixing: Wavelength conversion for optical telecommunications
- Ultrashort pulses in extreme nonlinear regime: High-harmonic generation

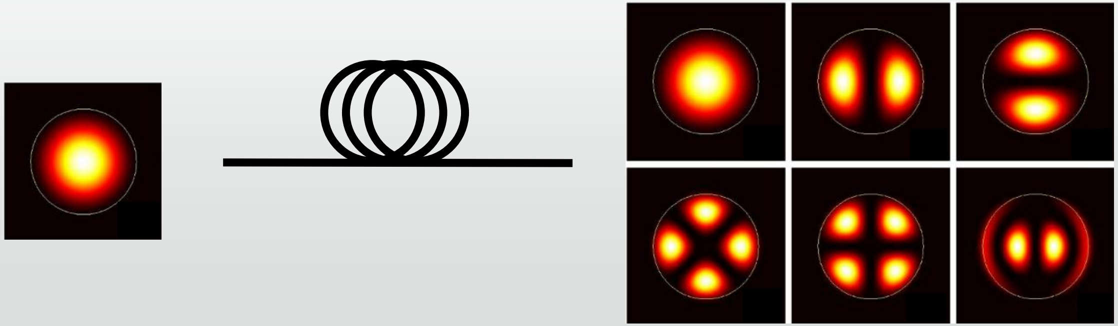
Theoretical & numerical model: Multimode generalised nonlinear Schrödinger equation (GNLSE)

Nonlinear multimode pulse propagation

- Laser pulse launched into fibre → shape, spectrum change during propagation



- Multimode (spatial) effects: modes may couple → phase changes, power transfer



Multimode Generalised Nonlinear Schrödinger Equation (MM-GNLSE)

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- Evolution of pulse amplitude $A_p(z,t)$ in mode p (neglecting self steepening term):

$$\frac{\partial A_p(z,t)}{\partial z} = i(\beta_0^{(p)} - \beta_0)A_p(z,t) - (\beta_1^{(p)} - \beta_1)\frac{\partial A_p(z,t)}{\partial t} + i\sum_{n\geq 2}\frac{\beta_n^{(p)}}{n!}\left(i\frac{\partial}{\partial t}\right)^n A_p(z,t)$$

$$+i\frac{n_2\omega_0}{c}\sum_{l,m,n}\left\{(1-f_R)Q^K_{plmn}A_lA_mA_n^* + f_RQ^R_{plmn}A_l[h*(A_mA_n^*)]\right\}$$

with

- Mode overlap factors of electric fields: (incl. polarisation)

$$Q^R_{plmn} \sim \int \mathrm{d}x\,\mathrm{d}y\,[\mathbf{F}_p^*\cdot\mathbf{F}_l][\mathbf{F}_m\cdot\mathbf{F}_n^*]$$

$$Q^K_{plmn} \sim \frac{2}{3}Q^R_{plmn} + \frac{1}{3}\int \mathrm{d}x\,\mathrm{d}y\,[\mathbf{F}_p^*\cdot\mathbf{F}_n^*][\mathbf{F}_m\cdot\mathbf{F}_l]$$

- Convolution with Raman response function:

$$[h*(A_mA_n^*)](z,t) = \int \mathrm{d}\tau\, h(\tau)A_m(z,t-\tau)A_n^*(z,t-\tau)$$

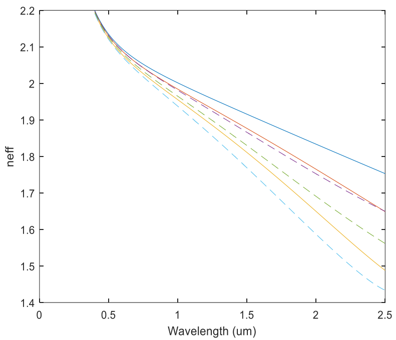
F Poletti and PH, J. Opt. Soc. Am. B **25**, 1645 (2008)

MM-GNLSE

Mode dispersion:

Propagation constant $\beta_p(\lambda)$

Effective mode index $n_{eff,p}(\lambda)$



$$\frac{\partial A_p(z,t)}{\partial z} = i(\beta_0^{(p)} - \beta_0)A_p(z,t) - (\beta_1^{(p)} - \beta_1)\frac{\partial A_p(z,t)}{\partial t} + i\sum_{n\geq 2}\frac{\beta_n^{(p)}}{n!}\left(i\frac{\partial}{\partial t}\right)^n A_p(z,t)$$

$$+i\frac{n_2\omega_0}{c}\sum_{l,m,n}\left\{\underbrace{(1-f_R)Q^K_{plmn}A_lA_mA_n^*}_{\text{Kerr nonlinearity}} + \underbrace{f_RQ^R_{plmn}A_l[h*(A_mA_n^*)]}_{\text{Raman nonlinearity}}\right\}$$

Self-phase modulation $\quad \partial_z A_p \sim A_p|A_p|^2$

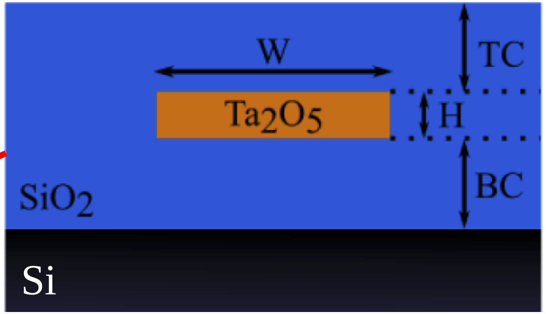
Cross-phase modulation $\quad \partial_z A_p \sim A_p|A_n|^2$

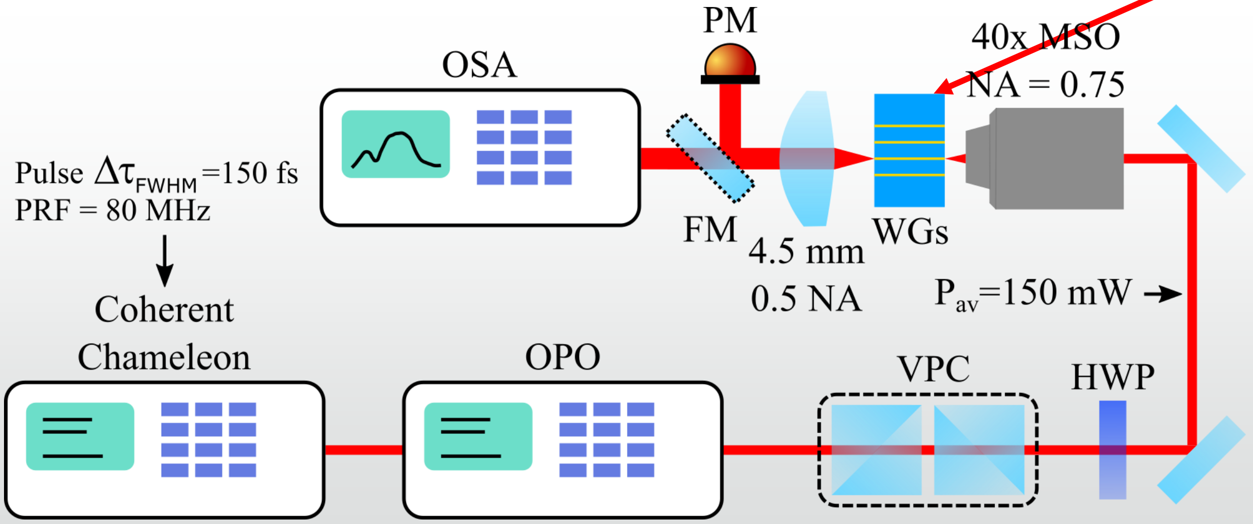
Intermodal four-wave mixing $\quad \partial_z A_p \sim A_lA_mA_n^*$

Effectively one-mode operation: Supercontinuum generation in multimode waveguides

Experimental setup

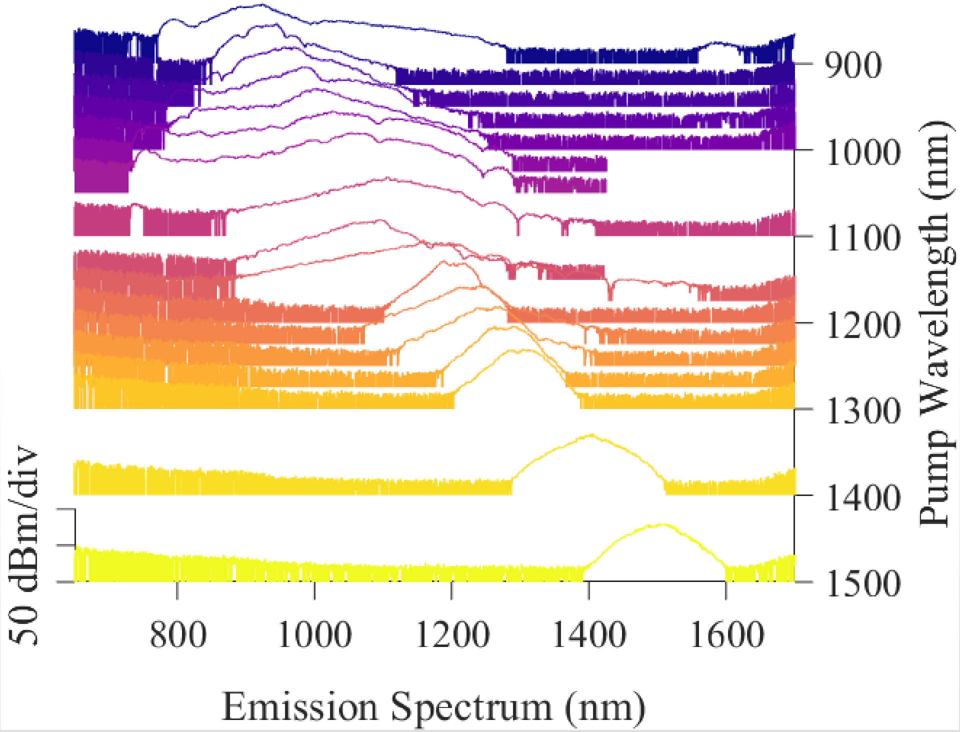
- Widely tunable (750-1500 nm) 150 fs, ~100 mW pulses
- Tantalum pentoxide waveguides in silica cladding, 700x3200 nm, n≈2.08 @ 1 μm, 1.4-4 dB/cm loss, 18 mm





JRC Woods et al,
Opt. Express **28**, 32173 (2020)

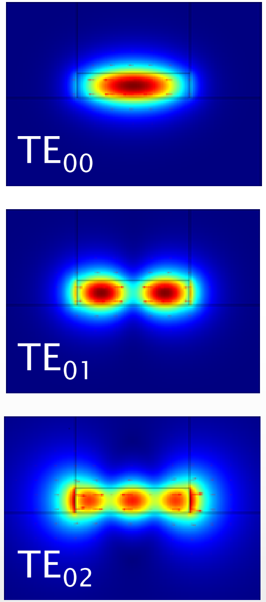
Supercontinuum generation vs pump wavelength

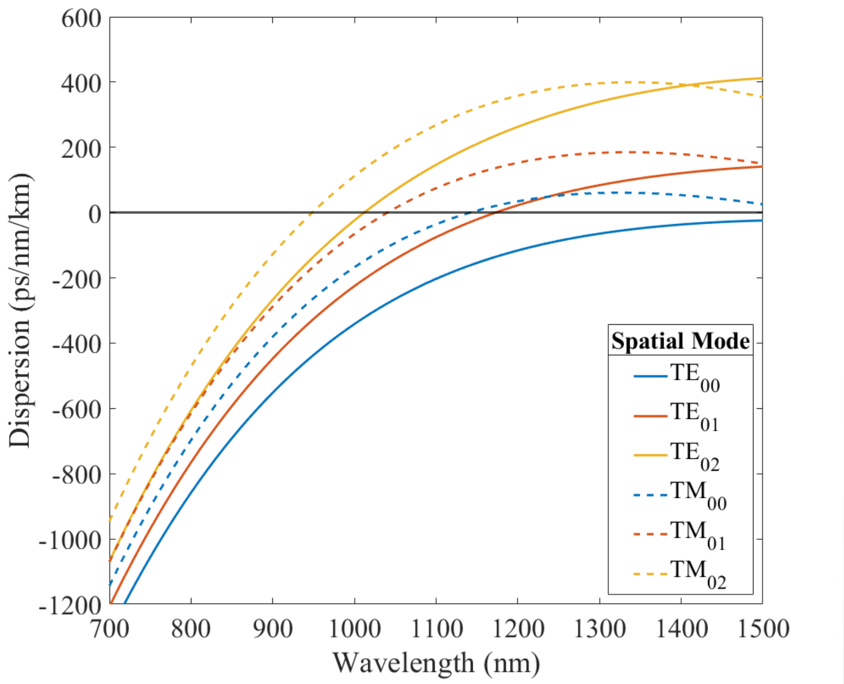


TM polarisation

- No continuous change of supercontinuum spectrum with pump wavelength
- Some SPM dominated spectra
- Some soliton spectra
- Some "standard" supercontinuum spectra for pumping near a ZDW

Mode structure and dispersion

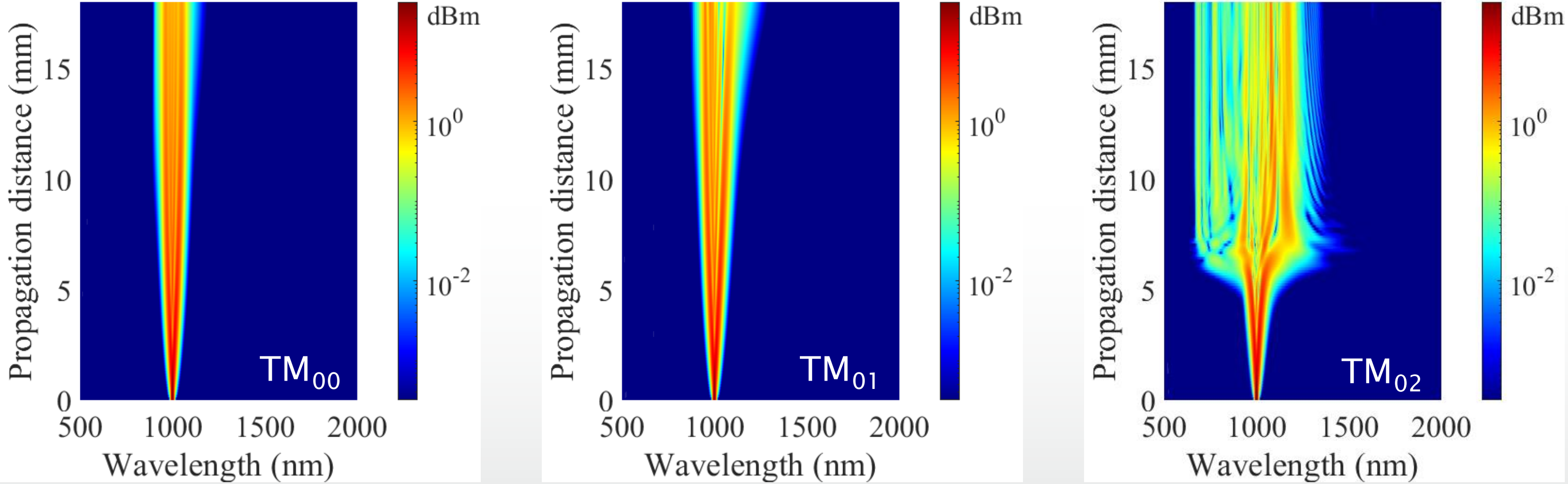




- Many modes, mostly TE_{0n} and TM_{0n}
- Widely different zero-dispersion wavelengths, dispersion curves

GNLSE simulations

- Assume launch of light into individual TM modes, e.g. at λ=1000 nm



- For different pump wavelengths, simulations in different modes agree best with experiments

Comments

- Measurements best described by assuming laser is launched (preferentially) into different modes, depending on laser wavelength

  ⇨ launch conditions (e.g. spot size) change with wavelength

- Spectra consistent with **effectively single-mode simulations**, but assuming **dispersion profiles** of individual modes

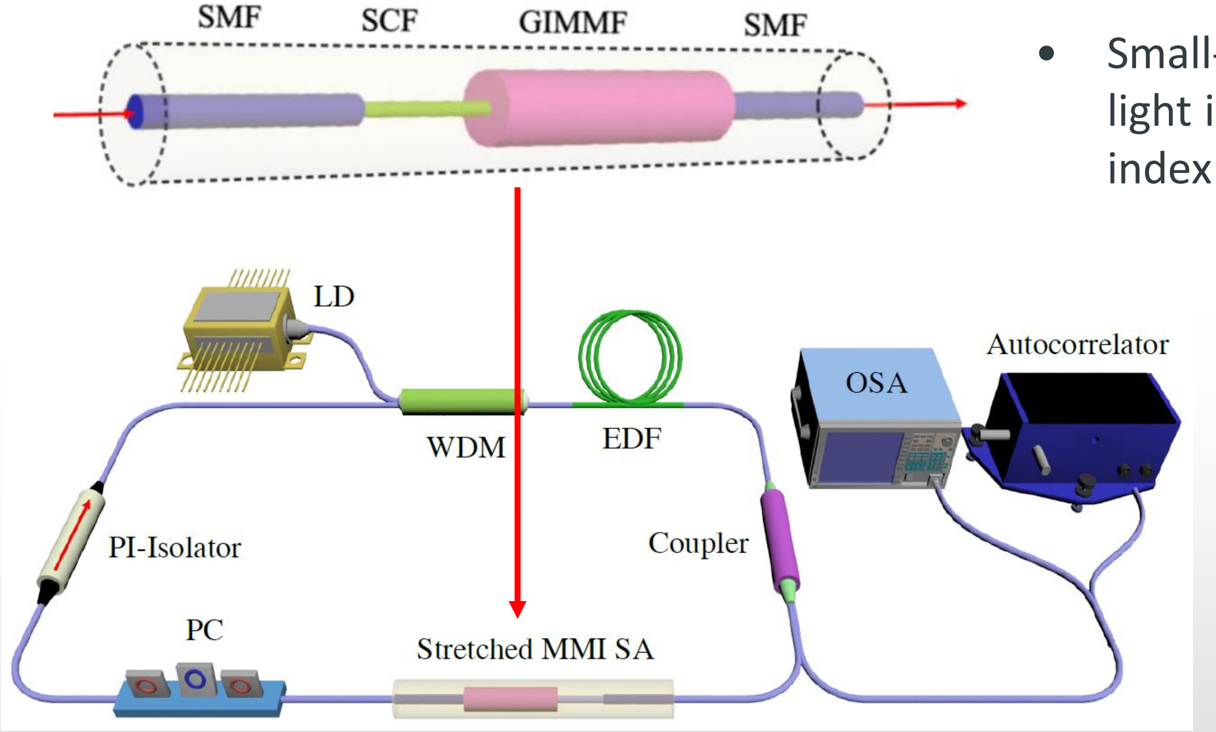
  ⇨ No intermodal interaction, vastly different modal dispersion

$$\frac{\partial A_p(z,t)}{\partial z} = i(\beta_0^{(p)} - \beta_0)A_p(z,t) - (\beta_1^{(p)} - \beta_1)\frac{\partial A_p(z,t)}{\partial t} + i\sum_{n\geq 2}\frac{\beta_n^{(p)}}{n!}\left(i\frac{\partial}{\partial t}\right)^n A_p(z,t)$$
$$+ i\frac{n_2\omega_0}{c}\cancel{\sum_{l,m,n}}\left\{(1-f_R)Q^K_{plmn}A_l A_m A_n^* + f_R Q^R_{plmn}A_l[h*(A_m A_n^*)]\right\}$$

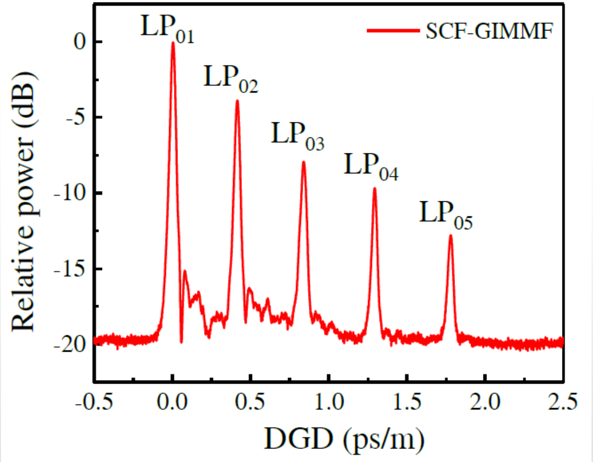
l=m=n=p

Intermodal cross-phase modulation: Saturable absorber in ring laser

Mode-locked laser setup



- Small-core fibre (SCF) used to launch light into many modes of a graded index multimode fibre (GIMMF)

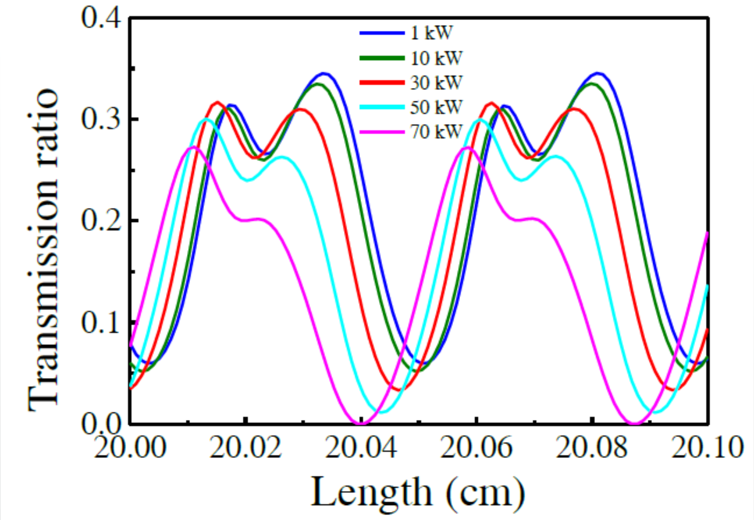


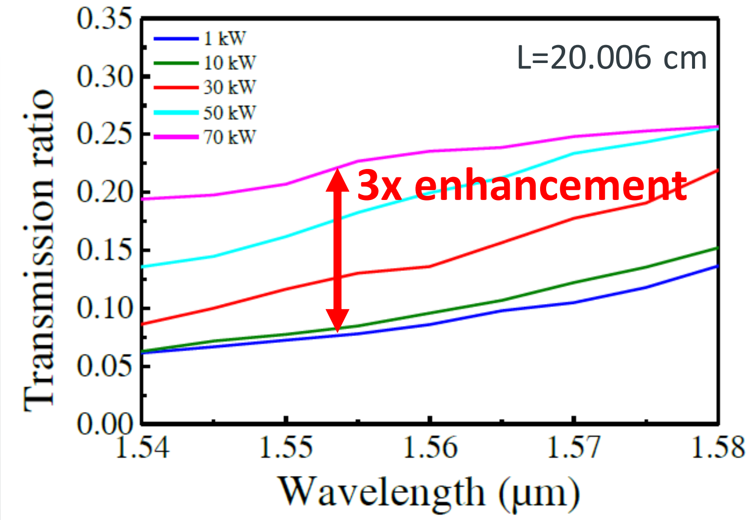
T Wang et al, Appl. Opt. **60**, 9007 (2021)

Passive mode-locking by nonlinear multimode interference as saturable absorber

- Coupling efficiency from GIMMF back into SMF depends on multimode interference
- XPM changes relative phases between modes ⇨ power-dependent transmission

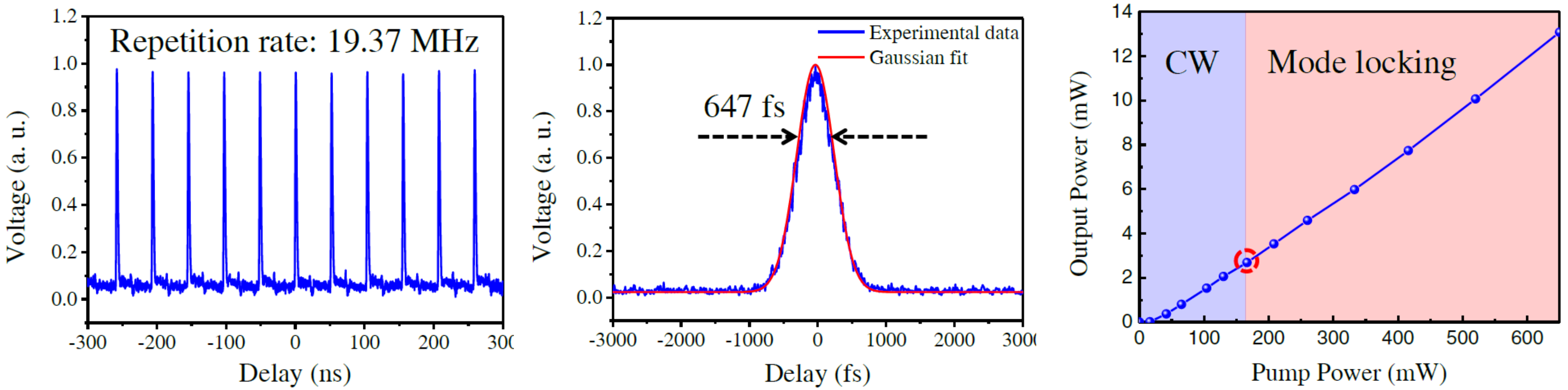
GNLSE simulations:





Mode-locked laser operation

- Experiment shows mode locking above a threshold pump power



Comments

- Multimode nonlinear effects used to create a **saturable absorber** in a structure of small-core fibre → graded index multimode fibre → single-mode fibre
- Mode dispersion creates position-dependent interference
- No intermodal phase matching and power transfer, but **nonlinear phase shifts**

$$\frac{\partial A_p(z,t)}{\partial z} = i(\beta_0^{(p)} - \beta_0)A_p(z,t) - (\beta_1^{(p)} - \beta_1)\frac{\partial A_p(z,t)}{\partial t} + i\sum_{n\geq 2}\frac{\beta_n^{(p)}}{n!}\left(i\frac{\partial}{\partial t}\right)^n A_p(z,t)$$

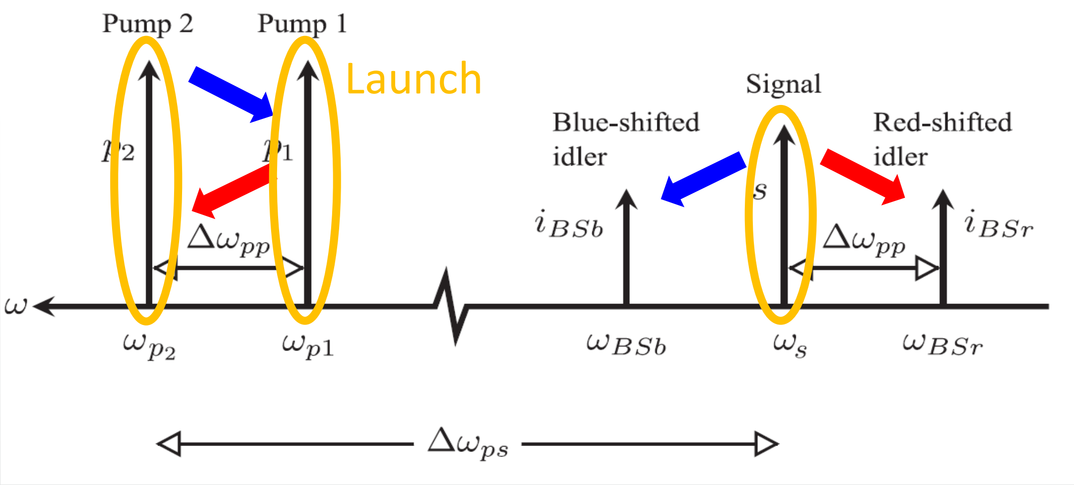
$$+ i\frac{n_2\omega_0}{c}\left\{\underbrace{\sum_l Q_{ppll}A_p|A_l|^2}_{\text{Self-phase modulation, Cross-phase modulation}} + \underbrace{\cancel{\sum_{l,m,n} Q_{plmn}A_l A_m A_n^*}}_{\text{Intermodal four-wave mixing}}\right\}$$

Few-mode four-wave mixing: Wavelength conversion for optical telecommunications

Phase matched four-wave mixing

- For efficient transfer of power between wavelengths we need to fulfil phase matching

Example: nonlinear Bragg scattering



- Blue-shifted idler

$$\omega_{p2} + \omega_s = \omega_{p1} + \omega_b$$

$$\beta_{p2} + \beta_s = \beta_{p1} + \beta_b$$

- Red-shifted idler

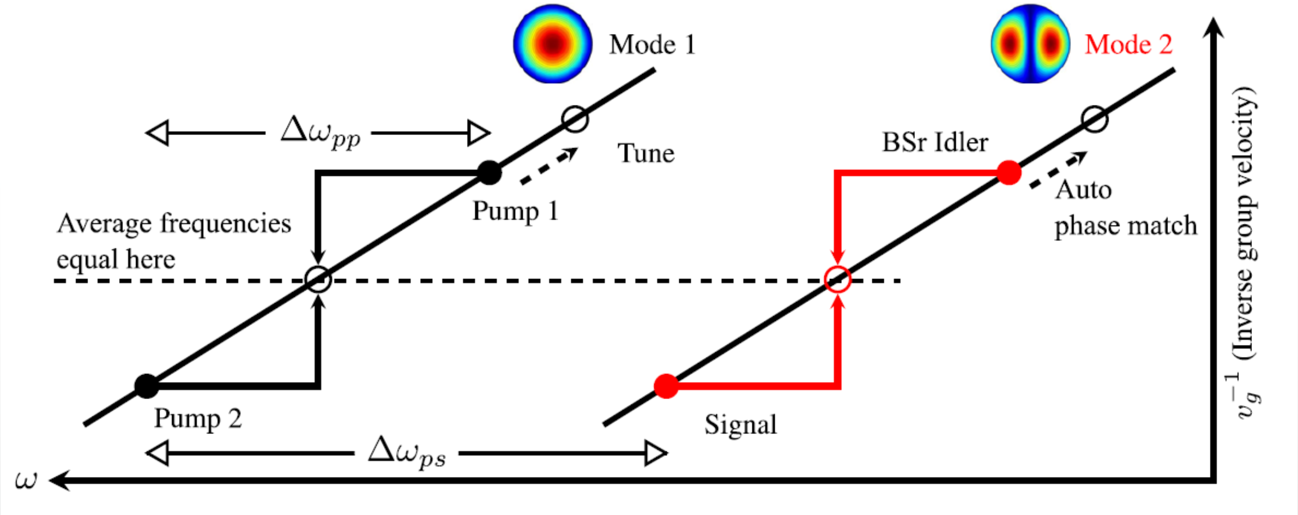
$$\omega_{p1} + \omega_s = \omega_{p2} + \omega_r$$

$$\beta_{p1} + \beta_s = \beta_{p2} + \beta_r$$

Intermodal phase matching

- Pumps in **mode 1**, signal and idler in **mode 2**
- Phase matching ⇔ inverse group velocity (IGV) at the average frequency in each mode is the same

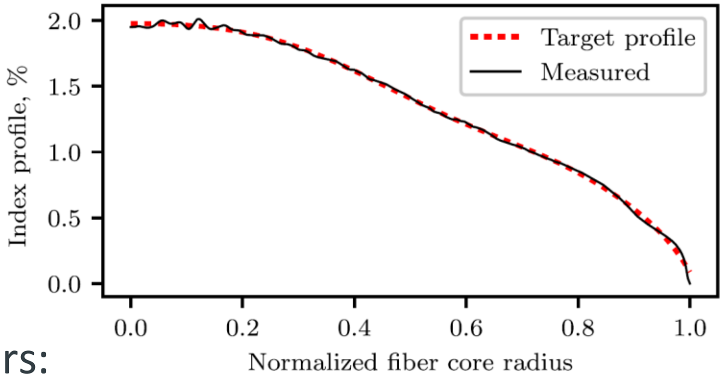
RJ Essiambre et al, PTL **25**, 539 (2013)



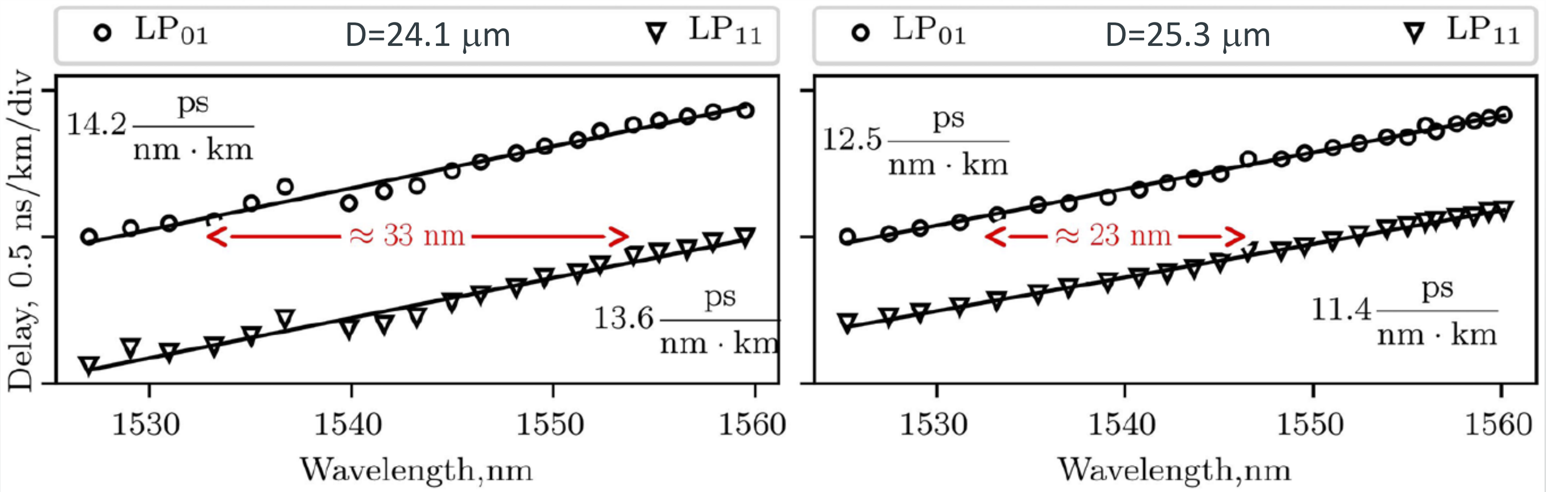
- If the IGV curves of the modes are parallel, phase matching for red idler is maintained when pump wavelength 1 is tuned

Fibre design and characterisation

- **Designed and fabricated** a graded index multimode fibre

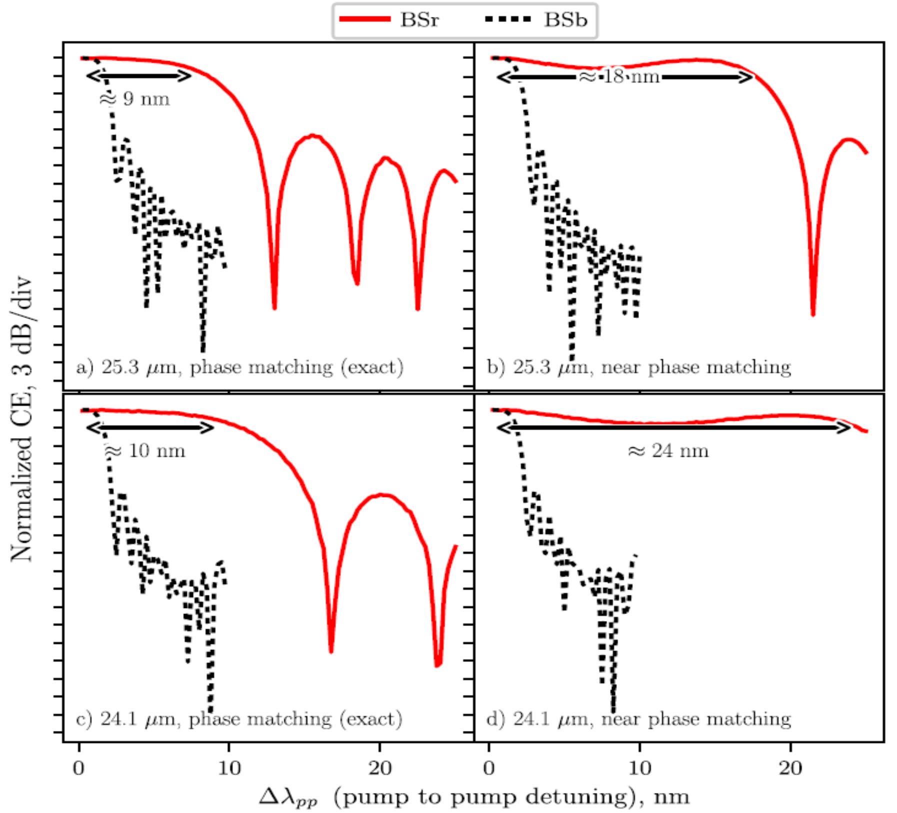


- Measured IGV for fibre with 2 core diameters:

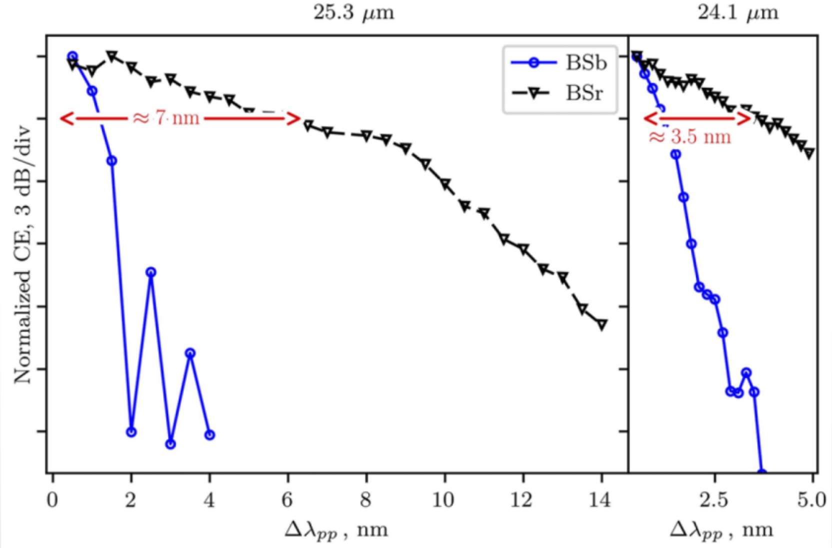


FWM conversion efficiency

GNLSE simulations



Experiment



OF Anjum et al, APL Photonics **4**, 022902 (2019)

Comments

- By carefully designing a fibre, **multimode phase matching** can be achieved over a relevant **bandwidth**

- Here, pumps in one mode, signal and idler in another mode, but many other situations have been investigated for mode and wavelength conversion

$$\frac{\partial A_p(z,t)}{\partial z} = i(\beta_0^{(p)} - \beta_0)A_p(z,t) - (\beta_1^{(p)} - \beta_1)\frac{\partial A_p(z,t)}{\partial t} + i\sum_{n\geq 2}\frac{\beta_n^{(p)}}{n!}\left(i\frac{\partial}{\partial t}\right)^n A_p(z,t)$$

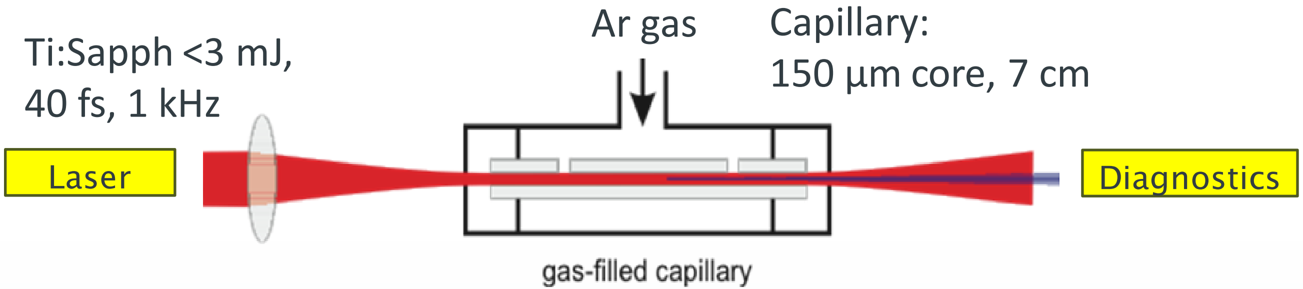
$$+i\frac{n_2\omega_0}{c}\sum_{l,m,n}\underbrace{\left\{(1-f_R)Q^K_{plmn}A_lA_mA_n^* + f_RQ^R_{plmn}A_l[h*(A_mA_n^*)]\right\}}$$

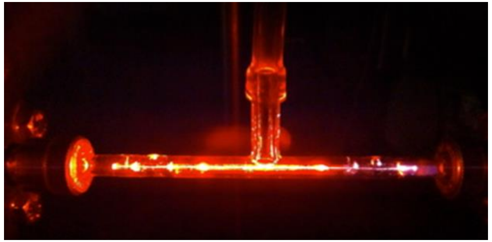
For cw light, Raman and Kerr nonlinearity merge: $Q_{plmn}A_lA_mA_n^*$

Effects on Raman nonlinearity on SPM/XPM and FWM: I Begleris and PH, Opt. Express **26**, 21527 (2018)

Ultrashort pulses in extreme nonlinear regime:
High-harmonic generation

High-harmonic generation in a capillary



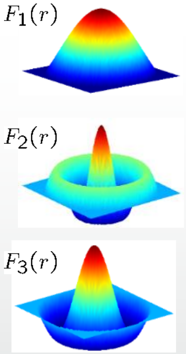


- Laser intensity (~10^{14} W/cm^2) to ionise Ar → additional nonlinearity, loss, dispersion
- Phase matching in low ionisation regime, quasi-phase matching mechanisms in high ionisation regime demonstrated
- Extremely nonlinear regime in highly multimode waveguide

A Rundquist et al, Science **280**, 1412 (1998)

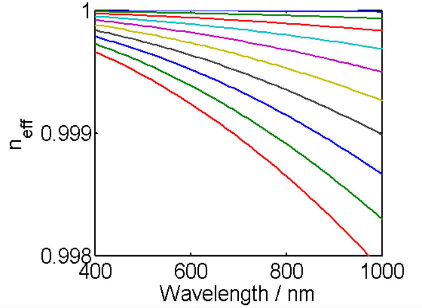
Extension of MM-GNLSE

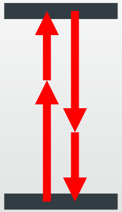
$$\frac{\partial A_n}{\partial z} = i\mathcal{D}_n\{A_n\} + i n_2(z) k_0 \sum_{k,l,m} Q_{nklm} A_k A_l A_m^* + \frac{i}{2} k_0 \int r\,dr\,F_n(r) E(r,z,t)\, n_{pl}^2(r,z,t) - \frac{1}{2} \int r\,dr\,F_n(r) E(r,z,t) \frac{\rho(r,z,t) W(r,z,t) U}{I(r,z,t)}$$



$$E(r,z,t) = \sum_n F_n(r) A_n(z,t)$$

Dispersion

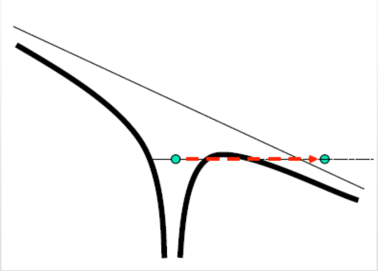




Ar gas nonlinearity:
SPM, XPM, FWM

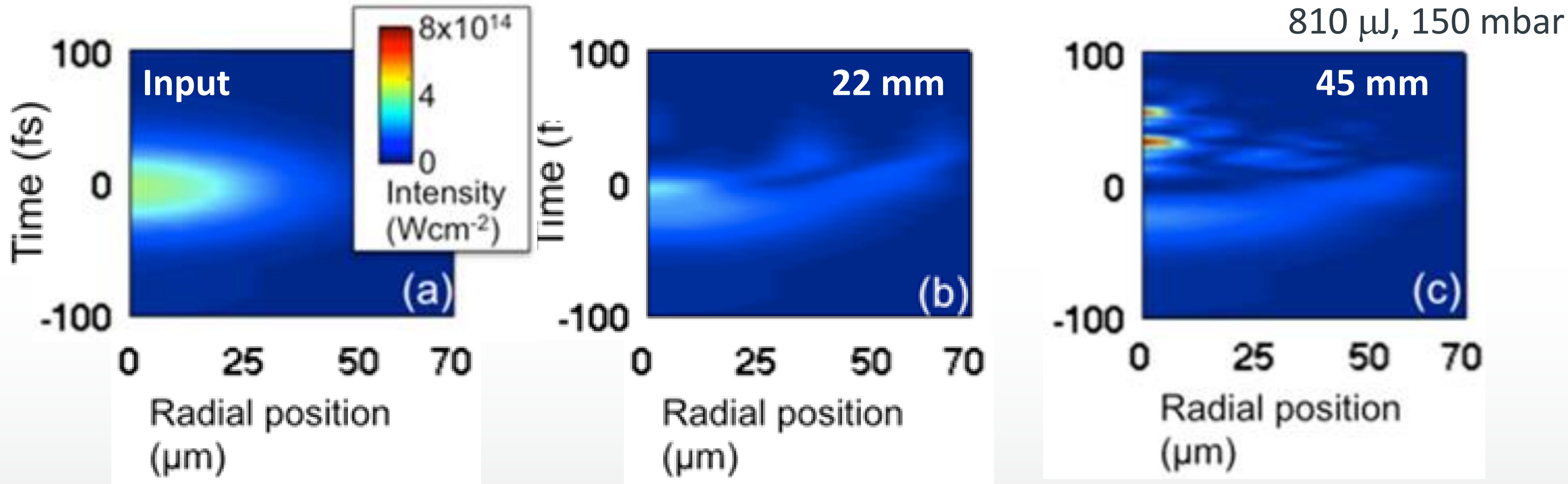
Plasma refractive index,
ionisation losses (Keldysh theory):

$$W \propto E^{-0.358} \exp\left(-\frac{K}{E}\right)$$



RT Chapman et al, Opt. Express **18**, 13279 (2010)

Spatio-temporal pulse dynamics

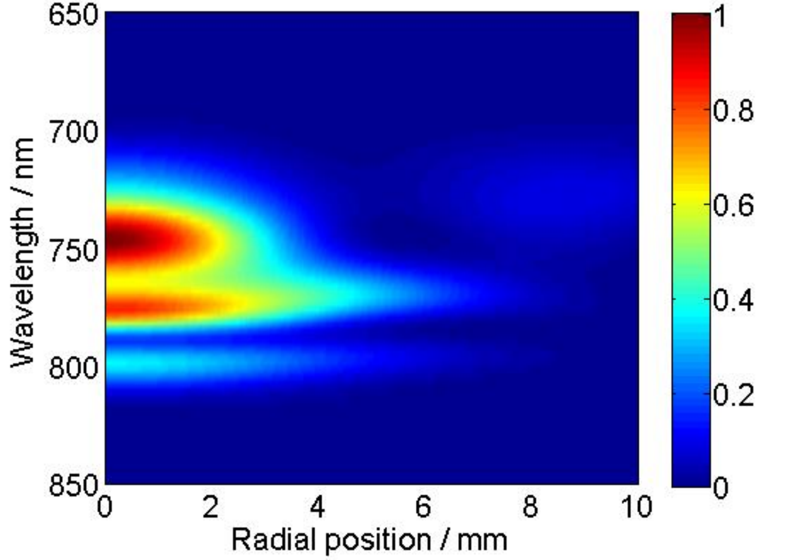


- Complex multimode pulse dynamics induced by ionisation nonlinearity
- Strong pulse compression possible in space and time

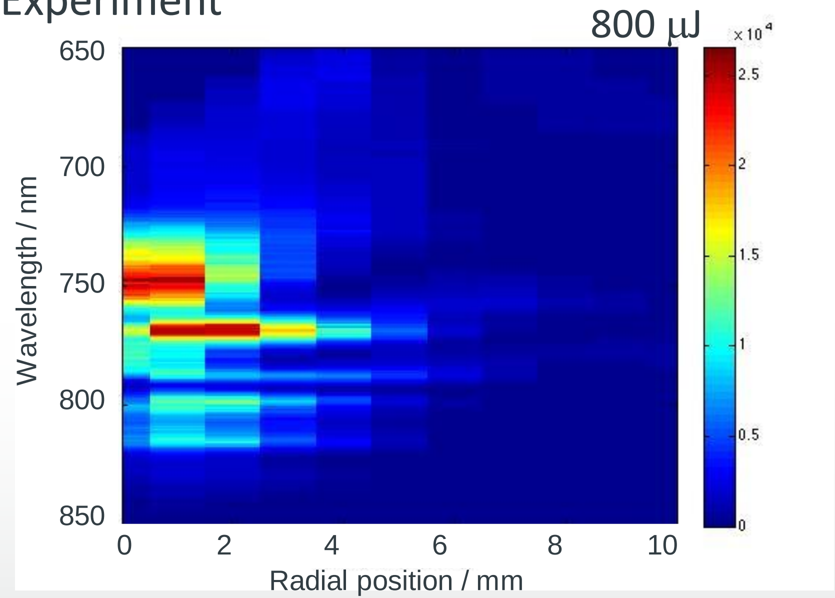
TJ Butcher et al, Phys. Rev. A **87**, 043822 (2013)

Model validation: far field spectra

MM-GNLSE simulation



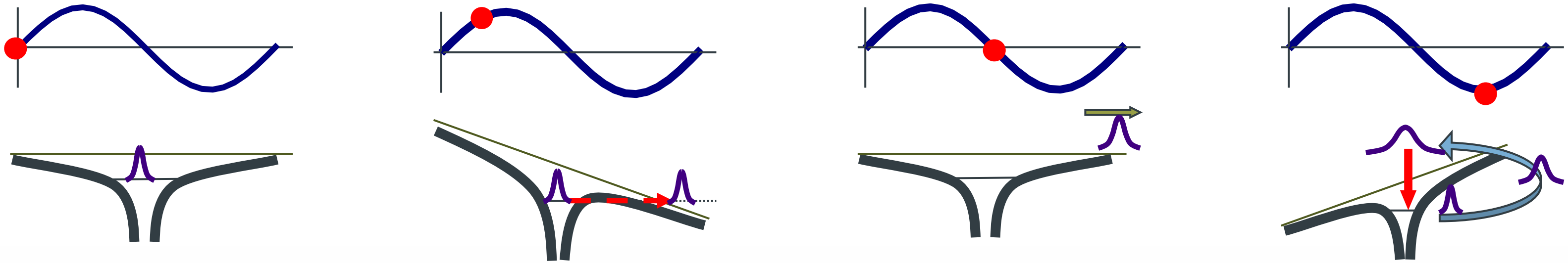
Experiment



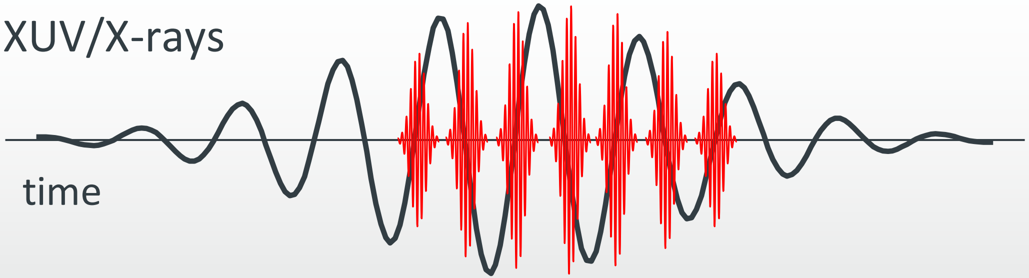
- Spectrum measured in far field through a pinhole, moving the pinhole **changes the observed spectrum**
- **Different wavelengths** exhibit different position dependence
- Experiment in good agreement with simulation results

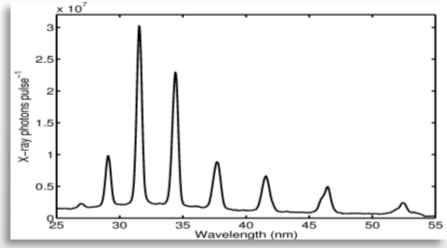
High-harmonic generation: model

- At every point in the capillary, the laser pulse drives high-harmonic generation



- Emission of XUV/X-rays



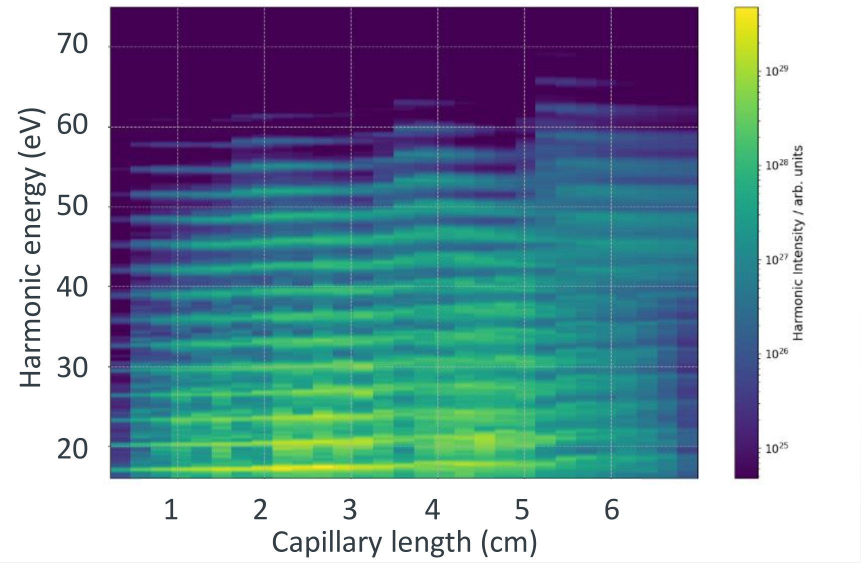


- Numerics: solve time-dependent Schrödinger equation for electron wavepacket

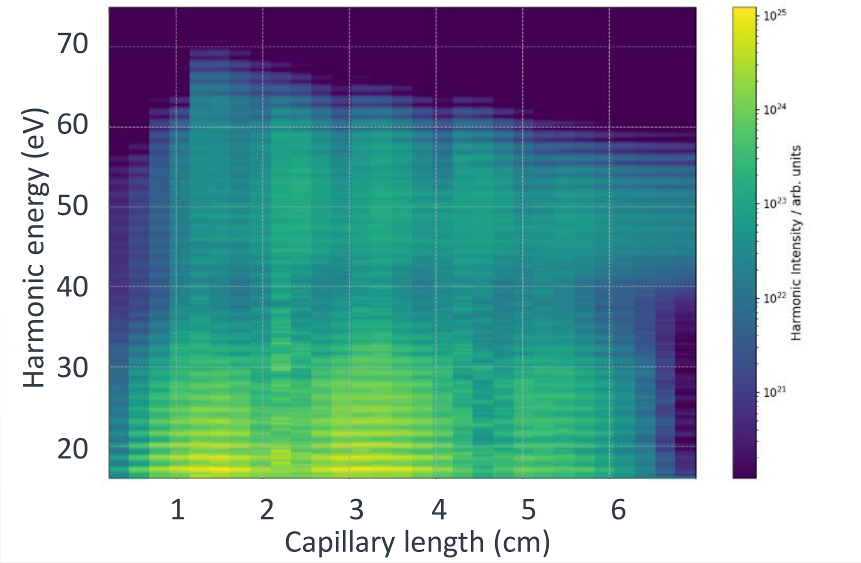
$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}(t)\psi \qquad \hat{H} = -\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2} - \left(\frac{e^2}{4\pi\epsilon_0\sqrt{\alpha + x^2}} + xeE(t)\right)$$

High-harmonic generation: simulations

Ti:Sapph (800nm, 50fs, 1mJ, 1kHz) [our lab]



Mid-IR OPCPA (0.17mJ, 50fs, 1700nm, 100kHz) [UK Central Laser Facility]



- Nonlinear broadening
- Ionisation-induced blue shifting

- Mode dispersion and beating
- Harmonic energy-dependent reabsorption

SM Senior et al, CLEO Europe 2021

Comments

University of Southampton

- Multimode GNLSE extended by **additional nonlinearities**
- Highly nonlinear, highly multimode regime of pulse propagation
- Coupled to a complicated **material response** at every point in space
  - High-harmonic generation
  - Solution of millions of Schrödinger equations on HPC cluster…

Conclusions

- Nonlinear multimode interactions in fibres take many forms, mostly depending on relative mode dispersion:
    - Effectively single-mode (different phase and group velocities)
    - Phase modulation (same group velocity, different phase velocity)
    - Power transfer (phase matching)
- Kerr, Raman, ionisation, plasma nonlinearities, etc.
- Numerical tools for modelling, e.g. multimode GNLSE
- Lots of applications!

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