

## DEVELOPING STUDENT SPATIAL ABILITY WITH 3D SOFTWARE APPLICATIONS

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*This paper reports on the design of a library of software applications for the teaching and learning of spatial geometry and visual thinking. The core objective of these applications is the development of a set of dynamic microworlds, which enables (i) students to construct, observe and manipulate configurations in space, (ii) students to study different solids and relates them to their corresponding nets, and (iii) students to promote their visualization skills through the process of constructing dynamic visual images. During the developmental process of software applications the key elements of spatial ability and visualization (mental images, external representations, processes, and abilities of visualization) are carefully taken into consideration.*

### INTRODUCTION

There has been much excitement about the potential of new dynamic visualizations for improving education and training. This is perhaps not surprising, because the same claims have been made about every new technology developed in the last century. Our society is becoming increasingly reliant on visual stimulus with new technologies pushing the boundaries between “real life” environments and two-dimensional (2D) representations of three-dimensional (3D) space (Lowrie, 2002). These new technologies are placing more emphasis upon visual and spatial skills with the user required to inspect, encode, transform, and construct information from visual displays (Kirby & Boulter, 1999).

The Principles and Standards of the National Council of Teachers of Mathematics (2000) recommend that 2-D and 3-D spatial visualization and reasoning are core skills that all students should develop. For example, students in grades 3-5 “should become experienced in using a variety of representations for three-dimensional shapes” (p. 168), such as isometric drawings, a set of views (e.g., top, front, and right), and building plans. The field of 3-dimensional geometry is a particularly fertile one for visualization and imagery research. In ascertaining geometric properties of 3-dimensional figures, visual perception plays a major role, and in recalling and describing 3-dimensional figures, image formation is a key ingredient.

In recent years, much disquiet has been expressed about the limitations of traditional approaches to the teaching and learning of geometry and specifically of spatial abilities. Most current curriculum materials and computer-based systems address the investigation of 3D geometry by requiring students to operate within two dimensional

representations of 3D geometry. This is not a natural way of investigating human 3D spatial ability while the limitations of these representations probably hinder the development of many of the 3D spatial abilities identified in the research literature. The main emphasis is put on 2D-geometry, while 3D-geometry rather stays in the background. On the one hand, the teaching of 3D-shapes and drawing in 3D-geometry have been neglected for decades. On the other hand, stereometrical calculations dominate 3D-geometric activities. Spatial-visual skills are frequently and extensively avoided.

The purpose of this paper is to present software applications developed under the DALEST<sup>1</sup> project (Developing an Active Learning Environment for the Learning of Stereometry) aiming at improving elementary and lower secondary students' spatial visualization skills. In this paper we focus on presenting applications' design and capabilities and possible teaching potential with emphasis on dynamic visualization. In the theoretical background below, we delineate fundamental theoretical spatial ability considerations upon which the design of the software applications were based and in the following sections provide a description of the capabilities of the software for the teaching of spatial geometry.

## **THEORETICAL CONSIDERATIONS**

Linn and Petersen (1985) define spatial ability as the mental process used to perceive, store, recall, create, edit, and communicate spatial images. Most researchers define the concept of spatial ability through the use of factors resulted from analytic studies. Lohman (1988) proposes a three factor model for spatial ability, including "spatial visualization", "spatial orientation", and "spatial relations". "Spatial visualization" is the ability to comprehend imaginary movements in a three-dimensional space or the ability to manipulate objects in imagination. "Spatial orientation" is defined as a measure of one's ability to remain unconfused by the changes in the orientation of visual stimuli that requires only a mental rotation of configuration. "Spatial relation" is defined by the speed in manipulating simple visual patterns such as mental rotations and describes the ability to mentally rotate a spatial object fast and correctly.

Gutierrez (1996) considers 'visualization' in mathematics as the kind of reasoning activity based on the use of visual or spatial elements, either mental or physical, performed to solve problems or prove properties. Visualization is integrated by four main elements: Mental images, external representations, processes of visualization, and abilities of visualization. Students in 3D geometry should acquire and improve a set of "abilities" of visualization to perform the necessary processes with specific mental images for a given 3D problem. Depending on the characteristics of the mathematics problem to be solved and the images created, students should be able to choose among several visual abilities which may have quite different foundations.

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The core visual abilities that should be taken into account for the developing of 3D dynamic geometry software are (Gutiérrez, 1996): (a) “Perceptual constancy”, i.e., the ability to recognize that some properties of an object are independent of size, colour, texture, or position, and to remain unconfused when an object or picture is perceived in different orientations, (b) “Mental rotation”, the ability to produce dynamic mental images and to visualize a configuration in movement, (c) “Perception of spatial positions”, the ability to relate an object, picture, or mental image to oneself, (d) “Perception of spatial relationships”, the ability to relate several objects, pictures, and/or mental images to each other, or simultaneously to oneself and (e) “Visual discrimination”, the ability to compare several objects, pictures, and/or mental images to identify similarities and differences among them.

A 3D dynamic geometry software should provide the learner a variety and richness of spatial images. Yakimanskaya (1991) claims that the creation of images is possible because of the accumulation of representations that serves as the starting point. The richer and more diverse the store of spatial representations, the easier is to use images in solving problems. Dynamic visualization can be a very powerful tool to gain a greater understanding of many mathematical concepts or it can be a resource to solve mathematical problems. (Harel & Sowder, 1998; Presmeg, 1986). Dynamic representations of mathematical processes may enable “the mind to manipulate them in a far more fruitful way than could ever be achieved starting from static text and pictures in a book. Dynamic spatial abilities are those that are required to reason about moving stimuli. The advent of computer testing has led to renewed interest in the ability to reason about motion and the ability to integrate spatial information over time. (Hegarty & Waller, 2005). Dynamic visualization is also such an act, but in this case this act constitutes moving pictures in the mind, or on some external medium ‘which the individual identifies with object(s) or process(es) in her or his mind, while Presmeg (1986) defined dynamic images as images with movement in the mind. In other words, the peculiar property of dynamic visualization is that individuals who possess this ability can reason about the essential properties of moving, shrinking, and rotating figures, which appear on the screen or, in their mind (Harel & Sowder, 1998; Presmeg, 1986).

Computer-based learning environments commonly comprise symbolic as well as static and dynamic pictorial representations, frequently combined with the possibility of modifying them interactively. While multiple, dynamic and interactive external representations have the potential to improve learning in various ways, they also place specific demands on learners, such as the need to process and relate different representations, to control and evaluate interactions with these representations, and to construct coherent mental representations. Computer software for the teaching of 3D geometry should allow students to see a solid represented in several possible ways on the screen and to transform it, helping them to acquire and develop abilities of visualization in the context of 3D geometry. Gutiérrez (1996) asserts that when a person handles a real 3-dimensional solid and rotates it, the rotations made with the

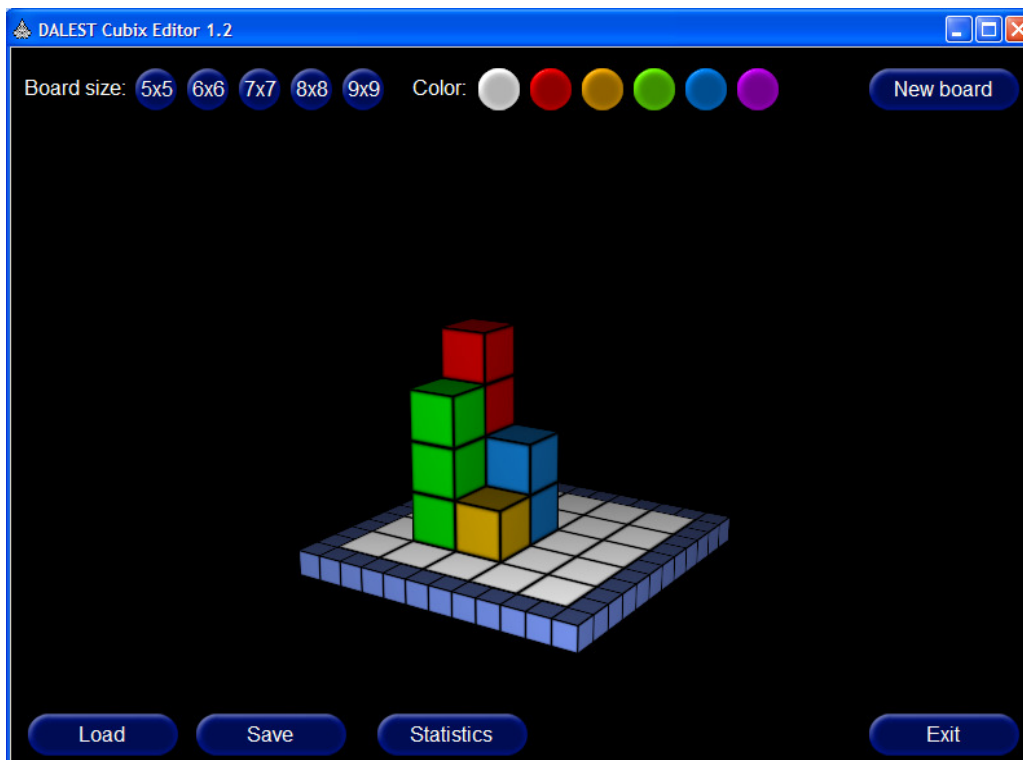
hands are so fast, unconscious, and accurate that one can hardly reflect on such actions. However, 3D software can limit the directions of rotation and force students to devise strategies of movement and to anticipate the result of a given turn.

### 3D SOFTWARE DESIGN

In the framework of DALEST project several software applications have been developed aiming at enhancing middle school students' 3D geometry understanding and spatial visualization skills by utilizing dynamic visualization images. The dragging capability of the software applications enable students to rotate and move 3D objects creating dynamic images. In this article, the following applications are presented and possible didactical situations are discussed: Cubix Editor, Cubix Shadow, Potter's Wheel and Origami Nets.

#### Cubix Editor

An editor which can be used by students to create 3D structures built of unit-sized cubes. The Cubix Editor allows the students to save their structures and reload them, thus making a library of various figures (see Figure 1). The user can also recolor the cubes and get measures for the volume and surface area of the solids constructed. A very useful characteristic of the application is the rotation of the whole platform of the construction, procedure which creates dynamic images and gives to the user the capability to develop dynamic visualizations of the front, side and top view of the object (see Figure 2).



**Figure 1: The Environment of Cubix Editor**

The visual ability ‘Perception of spatial relationships’ can also be enhanced by constructing an object based only on the image of its top, side or front view. The dragging capability of the application is a valuable tool that can be used to develop didactical situations enhancing visual abilities, such as “Perceptual constancy” and “Mental rotation” by producing dynamic mental images and visualizing a solid in movement, and investigating mathematical properties. For example, in the following didactical situations students investigate how the surface area of an object remains stable when its volume changes. To do so, students should visualize the surface area of the object when a cube is removed.

*Didactical Situation:* (a) Is it possible to remove ten unit cubes from a 3 by 3 by 3 cube made from 27 unit cubes so that the surface area of the remaining solid is the same as the surface area of the original 3 by 3 by 3 cube? In how many different ways can this be done? (b) Could you rearrange the unit cubes of a 3 by 3 by 3 cube made from 27 unit cubes so that the surface area of the new solid is ten square units larger than the surface area of the original 3 by 3 by 3 cube? (c) Arrange 9 red cubes, 9 blue cubes and 9 yellow cubes into a large 3 by 3 cube. No row or column of cubes must contain two cubes of the same color.



**Figure 2: The Top, Side and Front view of an Object**

### **Potter’s Wheel**

When a simple 2D object (e.g. a segment, a circle, a square, a triangle, a sinoidal curve or a free shape curve) is rotated in 3D around a vertical axis it can generate a lot of various 3D rotational objects. This application provides a design tool where users can experiment with various cases. They use only the five simple 2D objects, but varying their position and orientation relative to the axis, it is possible to generate different 3D shapes.

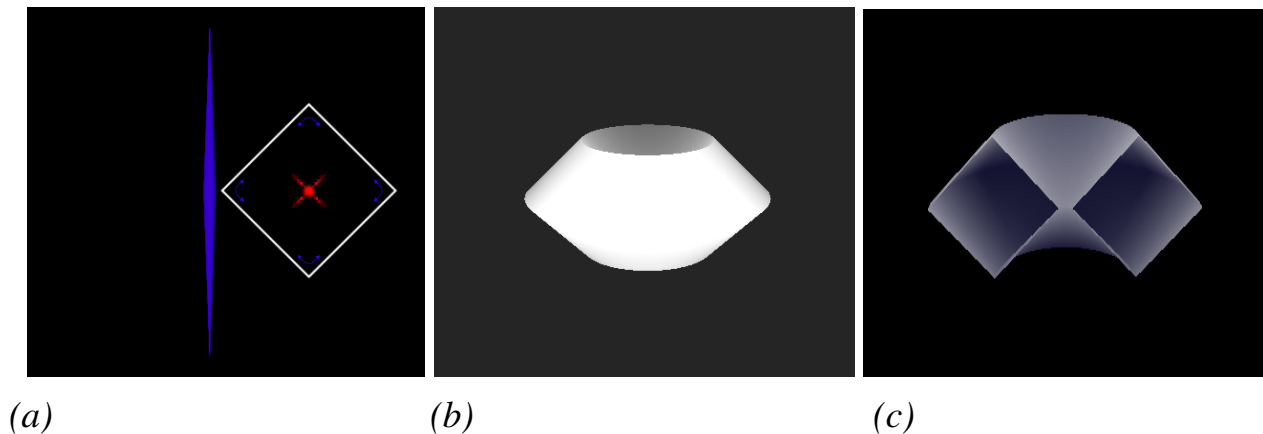


Figure 3: Potter Wheel construction

*Didactical situation:* The rectangle shown in Fig. 3(a) is rotated around the blue axe to generate a solid. Draw in your paper the produced solid.

Figure 3(c) can be given as an intermediated step to help students “perceptually construct” the solid. This intermediate step facilitates students to visualize the revolution procedure by identifying the fundamental 2D shape. Alternatively, this didactical situation can be given vise versa; students can be given a constructed solid (see Fig. 3(b)) and asked to figure out the 3D object used.

In an interesting dimension of this application, students are asked to use the free shape curve tool to construct different bottles for a soda water can and justify their solutions, by documenting how their bottles are related to real world (Mousoulides, Pittalis, Christou, Pitta & Boytchev, 2007).

## Origami Nets

This application makes it possible to build nets using triangles, squares, rectangles, and regular polygon, and then define folding angles. In this way it is possible to experiment with different nets of various 3D solid. The application allows the smooth transition from 2D flat net to 3D folded net. Adding new elements to the net is done interactively. It is also possible to create chain of rectangles or triangles which fold into a cylinder or a cone (see Figure 4). The application is extremely powerful because it builds up an open, dynamic and interactive environment where the user can construct a net without any restrictions. The construction of the net is not a static procedure, but the building net can be rotated helping the user to visualize how the solid will change if a side is put on an edge of the net. Another dynamic feature of the application is the fact that the user can change between the 2D and 3D view of the net and as a result he/she can fold the net before it is completely constructed and get an immediate feedback (see Figure 5). For example, in the following didactical situation the user should visualize the number of the required foldings while he/she constructs the cube net.

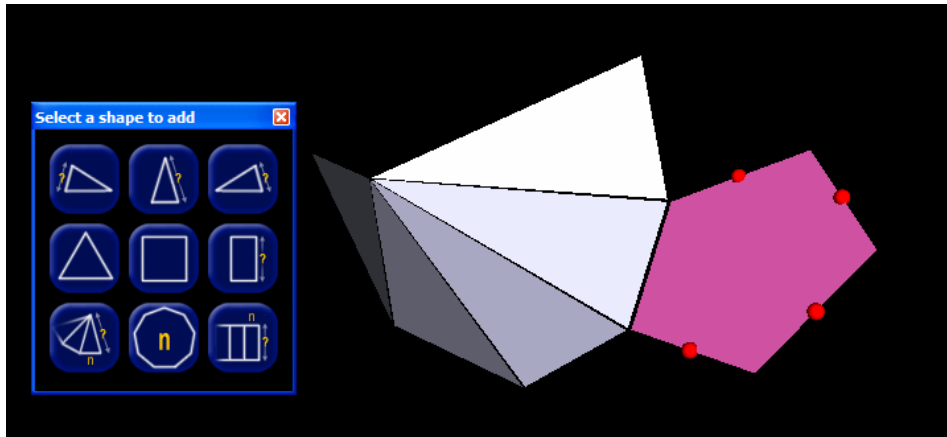


Figure 4: Construction of a Pyramid Net

*Didactical Situation:* Discover which cube net requires the largest number of foldings. This situation requires students to visualize while building the net the possible foldings of the cube sides. This activity enhances students' spatial visualization ability (Vz).

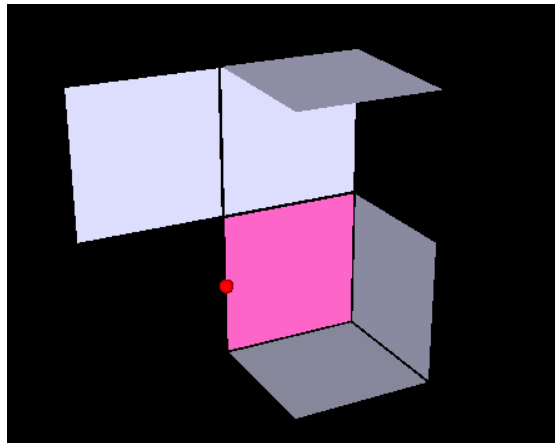


Figure 5: Construction of a Cube Net

## Slider

An invisible 3D solid (a cube, sphere, cone, etc.) can be cut through with a magic plane. Only the intersection becomes visible. By moving and rotating the cutting plane students are supposed to analyze the changes in the intersection and find out the solid. Solids can have various orientations and can be made visible (see Figure 6). The use can switch the axis of view to have different representations of the intersection of the solid and be able to combine them to a unified visual image.

*Didactical Situation:* (a) You have been given three shapes made out of sponge; a sphere, a cylinder and a cone. You are going to make some shapes for printing out of these sponges. How would you cut the sphere to make the largest circle for printing? (b) How could you make the largest possible circle from the cylinder and the cone? (c) If you cut the shapes in different ways, what other shapes for printing could you make?

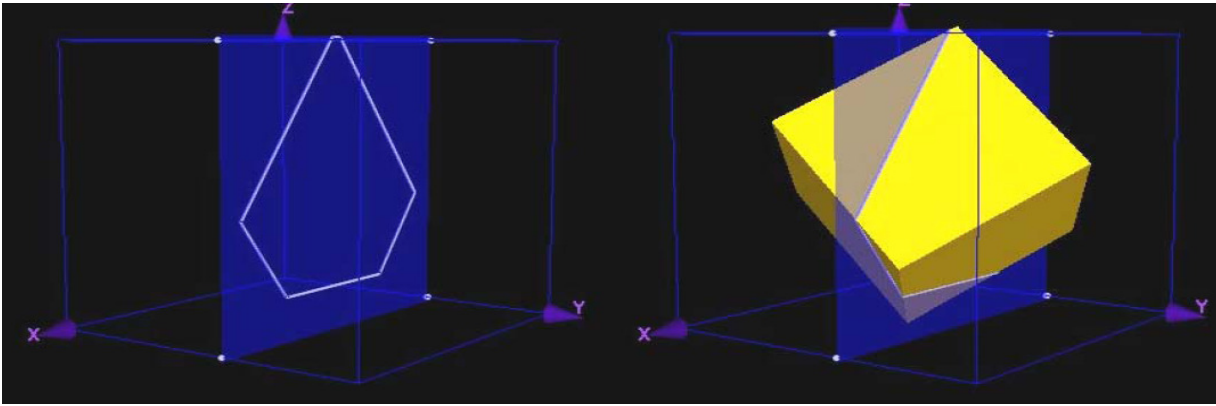
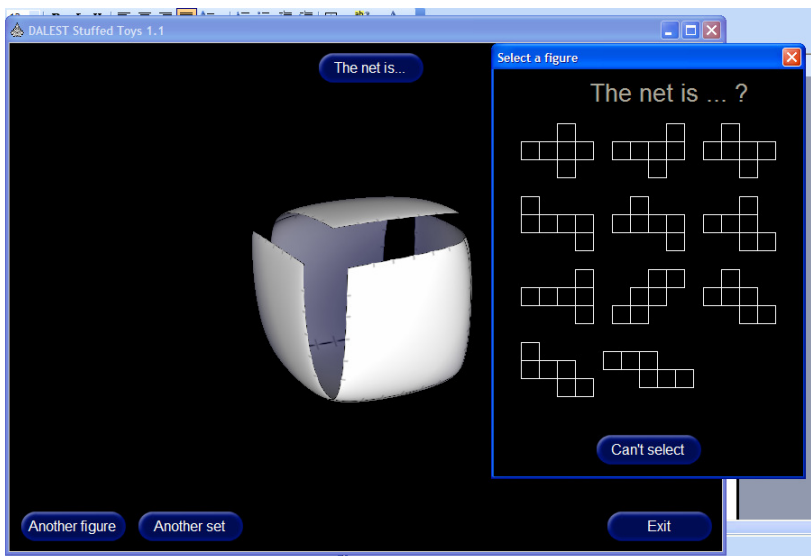


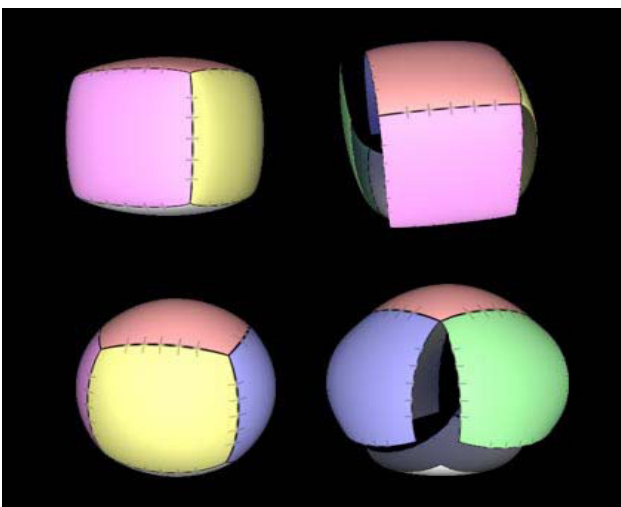
Figure 6: One of the Unexpected Intersections of a Cube

## Stuffed Toys



In this application students are given different nets of cubes, which are presented in a 3D view. Students are able to rotate and observe these 3D cube nets from different viewing angles. Upon completion, students can open a new window, presenting eleven possible nets and select the net they think corresponds to the 3D net presented earlier.

Figure 7: The 3D Cube Net and the Possible Unfolded Nets



In this application students can work with a cube which looks like a stuffed toy – see Figure 7. The main difficulty to the user is the fact that the ripped toy can be rotated, but cannot be unfolded, activating spatial visualization skills. Except for regular stuffed cubes the application can work with other forms, which are topologically equivalent to cubes – they are composed of six faces connected the same way as in cubes.

Figure 8: More Difficult Stuffed Toys



Figure 8 shows several of the stuffed toys in their original and ripped off state. It has been found that the curvature of the faces gives additional difficulty to the recognition of the net, demanding more complex spatial visualization procedures to percept the net.

### Scissors application

An interesting application which is related to Stuffed Toys application is called Scissors. It implements the true reverse problem of cube folding. In this application students are given a target net of a cube and they have to appropriately cut a cube to create the target net after unfolding. An example of this application is given below.

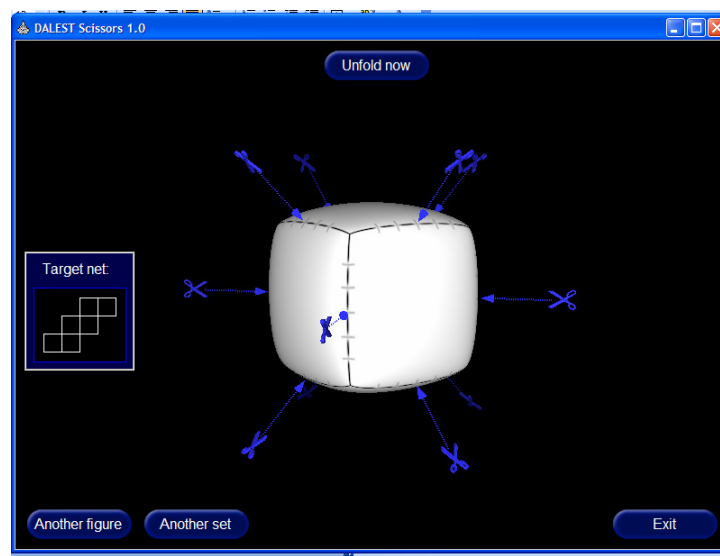


Figure 8: The Scissor Application

Quite interestingly, students face a number of challenges during feedback from the software. If they cut more than the required number of seams, the ripped cube cannot be unfolded completely, because some faces will be entirely disembodied. Again, in some cases, if students cut less seams than needed, the resulting figure could not be unfolded into a *planar* net. And finally, even if the number of ripped off seams is the correct one, the faces could unfold into another net, not the targeted one.

### CONCLUSION

The design of the DALEST applications is in the process of development. As presented earlier, the main purpose of these applications is to enhance students' understanding of visualization and spatial thinking. Thus, during the developmental process the key elements of visualization as defined by Presmeg (1986), Bishop (1980), Clements (1982) and Gutiérrez (1996) (mental images, external representations, processes, and abilities of visualization) are carefully taken into consideration.

We expect that the final library of DALEST applications will constitute a powerful tool in the teaching of spatial geometry. We also expect that applications will enhance students' dynamic visualization ability and enable them to gain a greater understanding of 3D mathematical and spatial concepts.

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