FACULTY OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

Master of Philosophy

THE BEHAVIOUR OF A FIVE PHASE INDUCTION MOTOR SUPPLIED BY AN INVERTOR

by Helmut Härer

CONTENTS

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H. Härer

LIST OF SYMBOLS

B	induction $(\frac{Vs}{m^2})$
C	capacitance (F)
e	voltage (V)
E_D	voltage between busbar and centre-tap of the d.c. supply
e_{ds}, e_{qs}	two-axes stator voltages
f, f_o	fundamental frequency and reference frequency ($^c/_s$)
$f_{w\nu}$	winding factor of order ν
G	conductance $(\frac{1}{\Omega})$
g	an integer
i	current (A)
i_{ds}, i_{qs}	two-axes stator currents
i_{dr}, i_{qr}	two-axes rotor currents
J	inertia (kg m^2)
k	common order of field harmonics; $k = \frac{\mu}{\nu}$
L	inductance (H)
L_s, L_r	stator and rotor inductances
ℓ_s, ℓ_r	stator and rotor leakage inductances
M	mutual inductance
m	number of phases
n	denotes the nth stator or rotor phase
n_k	speed of rotation of the field of order k
P	number of pole pairs
p	differential operator; $p = \frac{d}{dt}$. Laplace operator

Q	torque (m kg)
R	resistance (Ω)
R_s, R_r	stator and rotor resistance
s	slip
t	time (s)
v_o	voltage between machine star-point and supply star-point (V)
V_1	fundamental stator phase voltage, rms
α	angle on stator or rotor (rad.)
θ, $\dot{\theta}$	rotor-to-stator angle and speed, electrical (rad. and rad./s)
μ	order of current harmonics
ν	order of space harmonics
τ	transformed time; $\tau = \frac{\omega}{\omega_o} \cdot t$
ϕ	flux (Vs)
ψ	flux linkage (Vs)
ω, ω_o	fundamental angular frequency and reference frequency $(\frac{1}{s})$

Other symbols are defined as and when they occur.

Introduction

The introduction of the thyristor into power engineering has given rise to a new generation of drives for industry and traction: the induction motor supplied at variable frequency by invertors. In recent years a vast amount of work has been done both in industry and universities studying the design of invertors, control problems, and the effect of non-sinusoidal supplies on the induction motor.

A review of the publications shows that there are three categories of invertors: cycloconvertors, series invertors and chopper invertors. These types of invertors have in common that they provide a set of waveforms which produces a rotating field in the air-gap of an induction motor. However, their approach to the problem is different. Cycloconvertors convert the a.c. voltage of the mains into an a.c. voltage of lower frequency. Their main disadvantages are their limit in frequency and the high content of harmonics in their output voltage.

Series invertors consist of electronic switches which connect the motor terminals to the positive or negative busbar of a d.c. system thus producing a set of square-wave voltages. They are simple and reliable. The d.c. voltage should be variable.

Chopper invertors also use electronic switches but their switching frequency is much higher. They work on the principle of pulse-width-modulation thus providing voltage fundamentals of correct frequency and amplitude. The high switching frequency requires elaborate commutation and control equipment.

Non-sinusoidal supply voltages of an induction motor lead to non-sinusoidal currents and fluctuations of the torque. A thorough investigation of three-phase induction motors fed by square-wave voltages has been made by other authors (Ref. 8, 9). These investigations show that this kind of supply leads to considerable fluctuations in torque, the fundamental frequency of which is six times the switching frequency. If these torque fluctuations of relatively low frequency are not tolerable for the required drive the simple three-phase series invertor cannot be applied.

The basic idea of the work described in this thesis is to improve the quality of the drive not by improving the invertor but by increasing the number of phases of induction motor and series invertor. An electronic analogue of the induction motor has been set up for convenient investigation under steady-state conditions. A five phase invertor and motor have been made and investigated and the instantaneous steady-state torque has been measured. A comparison of 3 phase and 5 phase machines is given where possible.

It will be seen that, since the available time was rather limited, some problems have not been followed up to a great depth.

CHAPTER I

CHOICE OF NUMBER OF PHASES AND PROPERTIES OF FIVE PHASE MACHINES COMPARED WITH THREE PHASE MACHINES

I.1 Choice of the number of phases

It is assumed that there is a set of $m > 2$ symmetrical alternating voltages each of which is delayed by

$$\zeta = \frac{2\pi}{m} \tag{1.1}$$

with respect to the voltage preceding it. The lag of voltage n with respect to voltage 1 is therefore

$$\zeta_n = \frac{2\pi}{m}(n-1) \qquad n = 1....m \tag{1.2}$$

If m is even and $n = \frac{m}{2} + 1$,

$$\zeta_{\frac{m}{2}+1} = \pi \tag{1.3}$$

This set of voltages shall act on a motor winding which has m phases symmetrically distributed on the circumference, and P pairs of poles. The first pole of each phase shall be called a 'north' pole, the next pole a 'south' pole and so forth.

Fig. 1.1 shows the arrangement for P = 1. The north poles of phase 1 are located on the circumference at angles

$$\alpha_{N1,g} = \frac{2\pi}{P} g \qquad g = 0, 1, ... (P-1) \tag{1.4}$$

starting from the first pole of phase 1, and the south poles are at angles

$$\alpha_{S1,g} = \frac{2\pi}{P} g + \frac{\pi}{P} \tag{1.5}$$

The north poles of phase n are to be found at

$$\alpha_{Nn,g} = \frac{2\pi}{P} g + \frac{2\pi}{P\ m} (n-1) \tag{1.6}$$

and the south poles at

$$\alpha_{Sn,g} = \frac{2\pi}{P} g + \frac{2\pi}{P\ m} (n-1) + \frac{\pi}{P} \tag{1.7}$$

The south poles of phase 1 and the north poles of phase n coincide if

$$\alpha_{S1,g} = \alpha_{Nn,g}$$

$$\frac{2\pi}{P} g + \frac{\pi}{P} = \frac{2\pi}{P} g + \frac{2\pi}{P\ m} (n-1) \tag{1.8}$$

$$n - 1 = \frac{m}{2}$$

Since n and m are integers, Eqn. (1.8) can only be fulfilled if m is even. The coinciding poles will be excited by voltages, and therefore currents, which are in phase opposition as shown by Eqn. (1.3) and their fluxes will add up. The machine will behave like a machine with half the number of phases.

A genuine increase in the number of phases can only be achieved if m is odd. For the present investigation $m = 5$ has been chosen. It can generally be said that every increase in the number of phases improves the quality of the machine as far as parasitic fields and torques are concerned, as will appear later. However, the set of m voltages is provided by an invertor which becomes bigger and more expensive as m increases. For a given output N the power per invertor phase will be $\frac{N}{m}$ and it is therefore possible to employ smaller main thyristors but the exciter unit will become more complicated and expensive as m increases and the commutation equipment will increase depending on the kind of invertor. It was therefore felt that m = 5 is a good compromise.

I.2 Field harmonics in an m-phase machine

I.2.1 Basic considerations

At first, one machine phase shall be considered which consists of a single-slot coil. If this coil is excited by an alternating current an alternating field will result. Fig. 1.2 shows the arrangement and the airgap induction at a given instant.

The Fourier analysis of the airgap induction waveform gives the components

$$| B_\nu | = \frac{4}{\pi} | B | \frac{1}{\nu} \qquad (\nu = 1,3,5....) \qquad (1.9)$$

If instead the single coil is replaced by q coils which have only $\frac{1}{q}$ of the conductors of the original coil (Fig. 1.3) the field harmonics have to be modified by the winding factors:

$$| B_\nu | = \frac{4}{\pi} | B | \frac{f_{w\nu}}{\nu} \qquad (1.10)$$

$$f_{w\nu} = \frac{\sin (\nu \, q \, \frac{\alpha}{2})}{q \, \sin (\nu \, \frac{\alpha}{2})}$$

α is the angle by which the coils are displaced with respect to each other.

In an m-phase machine m sets of coils will be arranged on the stator circumference. Each set of coils provides an alternating flux which closes across the airgaps on either side. The alternating fields set up in each pole pitch can be represented by vectors. Fig. 1.4 shows these vectors for the fundamental field and the 5th harmonic of a 3 phase system and a machine with P = 1 pole pairs. For the fundamental the mutual displacement of the vectors is $\frac{2\pi}{2m}$ but for the harmonics it will be $\nu \frac{2\pi}{2m}$.

The coils are excited by a 2 m-phase current system. The currents will contain harmonics of the order μ and μ shall be odd. The mutual phase shift of the currents is $\frac{2\pi}{2m}$ for the fundamental and $\mu \frac{2\pi}{2m}$ for the harmonics.

If the vectors representing the alternating fields of the coils are added up, the result may be a field rotating in either direction, an alternating field, or mutual extinction. This will be investigated by the following considerations.

The vectors $\overline{B}_\nu$ represent the alternating fields produced by each set of coils. The coils, in turn, are excited by currents of order μ. The result of the vectorial summation is a vector of common order $k = \frac{\mu}{\nu}$

$$\frac{\overline{B_k}}{B_\nu} = \sum_{i=1}^{2m} e^{j(\beta + \frac{\pi}{m} i)\nu} \cos\left[(\zeta + \omega t + \frac{\pi}{m} i)\mu\right] \qquad .(1.11)$$

B_ν is the amplitude of the individual field vectors. β is the arbitrary position of the first coil and ζ the arbitrary phase position of the first current. The cosines represent the magnitude of the fields at the time t.

Various algebraic operations (Ref. 7) lead to the final result

$$\frac{\overline{B}_k}{2m\ B_\nu} = f_{m1}\ e^{j\left[(\zeta + \omega t)\ \mu + \frac{\pi}{2m}\ (2m + 1)\ (\nu + \mu)\right]}$$

$$+\ f_{m2}\ e^{j\left[-(\zeta + \omega t)\ \mu + \frac{\pi}{2m}(2m + 1)\ (\nu - \mu)\right]}$$

$$\overline{B}_k = \overline{B}_{k1} + \overline{B}_{k2} \tag{1.12}$$

$\overline{B}_{k1}$ represents a field which rotates in positive direction and $\overline{B}_{k2}$ a field rotating in negative direction. Their speed of rotation can be obtained from the following considerations: let μ be unity. The individual field of order ν has ν poles as can be seen from Fig. 1.3, and similarly the rotating field has ν P poles if the fundamental field has P poles; its speed of rotation is therefore only $\frac{1}{\nu}$ of the speed of the fundamental field. If n_1 is the speed of rotation of the fundamental field, the field of order ν excited by a current of order μ will rotate at

$$n_k = \pm\ \frac{\mu}{\nu}\ n_1 \tag{1.13}$$

(- for $\overline{B}_{k1}$ and + for $\overline{B}_{k2}$).

However, the existence of these fields depends on the existence of the phase factors

$$f_{m1} = \frac{\sin\ (\nu + \mu)\ \pi}{2m\ \sin\ (\nu + \mu)\ \frac{\pi}{2m}}$$

$$f_{m2} = \frac{\sin\ (\nu - \mu)\ \pi}{2m\ \sin\ (\nu - \mu)\ \frac{\pi}{2m}} \tag{1.14}$$

It must be remembered that ν and μ are integers; the nominators of f_{m1} and f_{m2} will therefore be zero. If, however, the denominators disappear too, f_{m1} and f_{m2} may exist as will be shown for f_{m1}. $\nu + \mu$ is replaced by a common integer g and a small addition x is considered:

$$f_{m1} = \lim_{x \to 0} \frac{\sin (g + x) \pi}{2m \sin (g + x) \frac{\pi}{2m}} \tag{1.15}$$

If now $g = 2m k_1 \quad (k_1 = 1,2....)$

$$f_{m1} = \lim_{x \to 0} \frac{\sin (x \pi)}{\pm 2m \sin (x \frac{\pi}{2m})}$$

$$= \pm \frac{x \pi}{2m x \frac{\pi}{2m}} = \pm 1 \tag{1.16}$$

(- for k_1 = 1,3,5.... and + for k_1 = 2,4,6....)

The phase factors will exist and be either + 1 or - 1 if

$$\nu + \mu = 2m \; g_1 \tag{1.17}$$

and

$$\nu - \mu = 2m \; g_2 \tag{1.18}$$

g_1 and g_2 are integers and can be positive, zero and negative. Eqn. (1.17) indicates fields which rotate in opposite direction to the main field and Eqn. (1.18) fields rotating in the same direction as the main field which is given by $\nu = \mu = 1$. If both Eqn. (1.17) and (1.18) are fulfilled for a given ν and μ an alternating field will be the result.

So far, stator fields have been considered only. These fields excite currents in the bars of a squirrel-cage rotor of an induction motor, which in turn set up rotor fields of the same order and speed relative to the stator as the stator fields and in addition fields of high order due to the distribution of the rotor bars and the slotting of the rotor. There are, furthermore, stator fields due to slotting. The additional rotor fields can interact with stator fields to produce the undesirable synchronous torques which can lead to cogging and synchronous crawling. However, these effects can be avoided by a proper choice of the number of slots of stator and rotor. (Ref. 4, 5, 6).

For the present investigation, only such rotor fields shall be considered which are of the same order and speed as the stator fields.

The results obtained above can be expressed in the following way:

There are stator fields of the order

$$k_s = \frac{\mu_s}{\nu_s} \qquad (1.19)$$

which rotate with the speed

$$n_s = \pm \frac{\mu_s}{\nu_s} n_1 = \pm \frac{f}{P} \frac{\mu_s}{\nu_s} \qquad (1.20)$$

(n_s is positive if Eqn. (1.18) is fulfilled and negative if Eqn. (1.17) is fulfilled).

Their number of pairs of poles is

$$P^* = \nu_s P \qquad (1.21)$$

There are also rotor fields of the order

$$k_r = \frac{\mu_r}{\nu_r} \qquad (1.22)$$

Their speed is

$$n_r = \pm \frac{f}{P} \frac{\mu_r}{\nu_r} \qquad (1.23)$$

and their number of pole pairs

$$P^* = \nu_r \; P \tag{1.24}$$

Only such ν and μ are eligible as fulfill Eqn. (1.17) or (1.18) but not both Eqns. simultaneously.

Generally, torque can only be produced if the interacting stator and rotor fields have the same number of pole pairs:

$$\nu_r = \nu_s \tag{1.25}$$

If the speeds of the interacting fields are equal, the torque is constant in time. If this is not fulfilled, the torque is fluctuating and the frequency of the fluctuations is

$$f_Q = P^* \left| \left[n_s - n_r \right] \right| \tag{1.26}$$

These results shall be used to throw some light on the following problems:

1) Which current harmonics can produce harmonic fields in a 3 phase and 5 phase machine with ideally distributed windings ($\nu = 1$) and what parasitic torques result from this?

2) Which space harmonics exist in a 3 phase and 5 phase machine on sinusoidal supply ($\mu = 1$) and what parasitic torques do they produce?

3) What field harmonics and parasitic torques result from the interaction of current harmonics and space harmonics?

It is assumed that both machines are star-connected and that their star-point is not connected to the star-point of the supply. No currents of order $\mu = g\,m$ ($g = 1, 3, 5 \ldots$) will be able to flow. If the current harmonic of order μ in the first phase can be expressed as

$$i_{1\mu} = \hat{\imath} \cos \mu\omega t \tag{1.27}$$

the current in phase n is

$$i_{n\mu} = \hat{\imath} \cos (\mu\omega t - (n-1) \mu \frac{2\pi}{m}) \tag{1.28}$$

If now $\mu = g m$

$$i_{n\mu} = \hat{\imath} \cos (gm\omega t - (n-1) g 2\pi)$$

$$= \hat{\imath} \cos gm\omega t \tag{1.29}$$

All of the n currents would be in phase and, since the sum of the currents flowing into the star-point must be zero, these currents cannot flow. The phase-voltages of a star-connected machine do not contain harmonics of the order g.m; these harmonics are eliminated by the star-point voltage of the machine with respect to the star-point of the supply.

A consequence of this is that Eqns. (1.17) and (1.18) can never be fulfilled simultaneously which would be possible for $\mu = g m$ and $\nu = \mu$. There are no alternating fields in a star-connected 3 phase or 5 phase machine with unconnected star-points.

I.2.2 Harmonic fields due to non-sinusoidal supply in 3 phase and 5 phase machines with ideally distributed windings: $\nu = 1$

In a 3 phase machine, there are stator and rotor fields of order and speeds

k	1	5	7	11	13 ....
$\frac{n_k}{n_1}$	1	-5	7	-11	13 ....

and in a 5 phase machine,

k	1	9	11	19	21 ....
$\frac{n_k}{n_1}$	1	-9	11	-19	21 ....

The harmonic currents of order μ = 3, 7, 13, 17 .... do not produce rotating fields in a 5 phase machine with ideally distributed windings.

The rotating fields produce two different kinds of parasitic torque:

1) $\underline{n_r = n_s}$

The torques are constant in time and there are torqe-speed-curves which resemble the basic torque-speed-curve. Figures 1.5 and 1.6 show some of these torque-speed-curves for 3 phases and 5 phases. In the important range of the fundamental torque-speed-curve these parasitic torques can be expected to be quite small.

2) $\underline{n_r \neq n_s}$

For a 3 phase machine, Eqns. (1.20), (1.21) and (1.26) indicate torque fluctuations of the frequencies

$$f_Q = 6\ f,\quad 12\ f,\quad 18\ f\ \ldots.$$

In particular, the fundamental stator field and the rotor fields of order 5 and 7 lead to fluctuations of 6 f.

For a 5 phase machine, there are torque fluctuations of the frequencies

$$f_Q = 10\ f,\quad 20\ f,\quad 30\ f\ \ldots.$$

and here the fundamental field and the harmonics of order 9 and 11 produce the most prominent torque fluctuations of 10 f.

In the case of square-wave supply, the r.m.s. value of the voltage harmonic of order μ is $\frac{V_1}{\mu}$ if V_1 is the r.m.s. value of the voltage fundamental. The currents of order μ are above all determined by the leakage impedance except at very low frequencies. The leakage impedance is $\mu \omega \ell_\ell$ (ℓ_ℓ being the total leakage inductance). The magnitude of the harmonic currents decreases rapidly as μ increases. Since the order of the current harmonics which produce parasitic torques is much higher in the case of 5 phases, it can be expected that the torque fluctuations of a 5 phase machine are considerably smaller than those of a 3 phase machine. The frequency of the torque fluctuations is higher for a 5 phase machine which may make it easier to avoid mechanical resonances of the drive.

The field harmonics discussed above are of the same nature as the fundamental fields. It is therefore not possible to suppress them by design measures like chording without impairing the main field as well. A high leakage impedance would decrease them, and thus the torque fluctuations, but high leakage would have disadvantages as far as the running characteristics and the losses of the machine are concerned.

I.2.3 Field harmonics of 3 phase and 5 phase machines on sinusoidal supply (μ = 1)

These field harmonics are due to the imperfect distribution of the windings and are referred to as space harmonics.

In a 3 phase machine, there are fields of order and speeds

k	1	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{11}$	$\frac{1}{13}$ ...
$\frac{n_k}{n_1}$	1	$-\frac{1}{5}$	$\frac{1}{7}$	$-\frac{1}{11}$	$\frac{1}{13}$ ...

and in a 5 phase machine,

k	1	$\frac{1}{9}$	$\frac{1}{11}$	$\frac{1}{19}$	$\frac{1}{21}$ ...
$\frac{n_k}{n_1}$	1	$-\frac{1}{9}$	$\frac{1}{11}$	$-\frac{1}{19}$	$\frac{1}{21}$ ...

No torque fluctuations occur since only stator and rotor fields of the same order and speed can interact. These fields produce the well-known parasitic asynchronous torques. In particular, the 7th space harmonic of a 3 phase machine produces a dip in the torque-speed-curve around $\frac{1}{7}$ of the synchronous speed and the 11th space harmonic of a 5 phase machine is responsible for a similar effect at $\frac{1}{11}$ of the synchronous speed.

Ref. 5 gives the following ratio for the pullout torque Q_ν due to the space harmonics of order ν and the starting torque Q_s due to the main field alone

$$\frac{Q_\nu}{Q_s} \simeq \frac{2}{\nu} \frac{\omega L_r}{R_r} \frac{f_{w\nu}^2}{f_{w1}^2} \tag{1.30}$$

The parasitic asynchronous torques of a 5 phase machine can be expected to be much smaller than those of a 3 phase machine. Fig. 1.7 and 1.8 show the torque-speed-curves of a 3 phase and a 5 phase machine at different supply frequencies.

Eqn. (1.30) shows that the asynchronous torques become unimportant at low frequencies. If the machine is supplied by an invertor it is started at a low supply frequency and then run at low slip frequencies and the range where these torques are of influence is always avoided.

Disturbing space harmonics can be substantially reduced by chording which entails more complicated two-layer windings and a certain loss in the magnitude of the main field. Most commercial 3 phase motors which are used for variable frequency drives are wound in this way. If a 5 phase motor for variable-frequency operation has to be designed there is no need for such measures and simple single-layer windings can be used.

I.2.4 Interaction of current harmonics and space harmonics

In a 3 phase machine, there are additional fields of order and speeds

k	$\frac{5}{7}$	$\frac{7}{5}$	$\frac{5}{13}$	$\frac{13}{5}$ ...	$\frac{5}{5}$	$\frac{7}{7}$ ...	$\frac{7}{13}$	$\frac{13}{7}$	$\frac{11}{17}$	$\frac{17}{11}$ ...
$\frac{n_k}{n_1}$	$-\frac{5}{7}$	$-\frac{7}{5}$	$-\frac{5}{13}$	$-\frac{13}{5}$ ...	1	1 ...	$\frac{7}{13}$	$\frac{13}{7}$	$\frac{11}{17}$	$\frac{17}{11}$ ...

and for a 5 phase machine one obtains

k	$\frac{3}{7}$	$\frac{7}{3}$	$\frac{3}{17}$	$\frac{17}{3}$ ...	$\frac{3}{3}$	$\frac{7}{7}$ ...	$\frac{3}{13}$	$\frac{13}{3}$	$\frac{7}{17}$	$\frac{17}{7}$ ...
$\frac{n_k}{n_1}$	$-\frac{3}{7}$	$-\frac{7}{3}$	$-\frac{3}{17}$	$-\frac{17}{3}$ ...	1	1 ...	$\frac{3}{13}$	$\frac{13}{3}$	$\frac{7}{17}$	$\frac{17}{7}$ ...

These fields lead again to parasitic asynchronous and fluctuating torques. The asynchronous torques in the motoring range

$(0 < \frac{n_k}{n_1} < 1)$ can be expected to be quite small since either μ or ν or both are of high order.

Fluctuating torques occur if there are different μ possible for a given ν. In the case of 3 phases, there are torque fluctuations of the frequency

$$f_Q = 6\,f, \quad 12\,f, \quad 18\,f \;\ldots.$$

and for 5 phases one obtains again

$$f_Q = 10\,f, \quad 20\,f, \quad 30\,f \;\ldots.$$

These torque fluctuations can be expected to be small compared with the fluctuations discussed in Section I.2.2 since the main field does not take part in producing them.

I.2.5 Conclusions

It appears that a 5 phase machine behaves considerably better than a 3 phase machine as far as parasitic fields and torques are concerned.

In particular, torque fluctuations can be expected to be of smaller magnitude and their frequency is higher. There seems to be no need for suppressing certain space harmonics by chording and single-layer windings can be used.

Generally, the frequency of the most important torque fluctuations is

$$f_Q = 2\,m\,f \qquad (1.31)$$

and the order of the current harmonics causing them is

$$\mu = 2\,m \pm 1 \qquad (1.32)$$

Every further increase of m would improve the quality of the drive in this respect.

I.3 Characteristics of the experimental 5 phase machine and voltage-frequency-relation for variable speed.

A 5 phase induction motor has been designed and built. Its parameters are given in Appendix I. The machine has been tested on sinusoidal supply converting the 3 phase system of the mains into a 5 phase system by adding up appropriate voltages. Fig. 1.9 shows the principle.

If an induction motor is supplied by voltages of variable frequency, the supply voltage must be variable too. The 'classical' one-phase equivalent circuit (Fig. 1.10) may be used to throw some light on the problem of voltage-frequency-relation on sinusoidal supply. The flux linkage between stator and rotor refe red to the stator is given by the magnetizing current $I_m = I_s + I_r$:

$$\psi_m = M I_m = \frac{V_m}{\omega} \tag{1.33}$$

It is desirable to keep this quantity constant at all frequencies. If the stator impedance $R_s + j\,\omega\,\ell_s$ is small compared to $j\,\omega\,M_1$ V_1 will be close to V_m. If ψ_m is to remain constant at different frequencies, V_1 should - in first approximation - vary in proportion with the frequency:

$$V_1 = V_{10} \frac{\omega}{\omega_o} \tag{1.34}$$

(V_{10} = rated voltage at rated angular frequency ω_o)
The pullout torque is proportional to the square of the flux linkage (Ref. 4) and will remain constant as well.

However, the assumption that the stator impedance is small compared to the mutual inductance cannot be maintained at low frequencies. R_s will cause a more and more important voltage drop as the frequency becomes lower, and at zero frequency it would determine the stator current entirely. Consequently, the pullout torque falls off considerably as the frequency decreases. Fig. (1.11) shows some torque-speed-curves at different frequencies the voltage being proportional to the frequency. It follows that the voltage must be higher at frequencies $\omega < \omega_o$ than indicated by Eqn. (1.34) if the pullout torque shall remain constant. The torque-speed-curves of Fig. (1.11) may help to find the proper frequency-voltage-relation. The pullout torque of an induction motor is proportional to the square of the supply voltage. From there, the following approximations may be established for this particular machine:

$$V_1 = V_{10} \left(0{\cdot}15 + 0{\cdot}85 \frac{\omega}{\omega_o}\right) \qquad (1.35)$$

The machine will be supplied by a set of square-wave voltages, i.e. its terminals will be switched to the positive or negative busbar of a d.c. supply in turn. The fundamental of the square-waves should be identical with the peak value of V_1:

$$\sqrt{2}\, V_1 = \frac{4}{\pi} E_D$$

$$E_D = 1{\cdot}11\, V_1 \qquad (1.36)$$

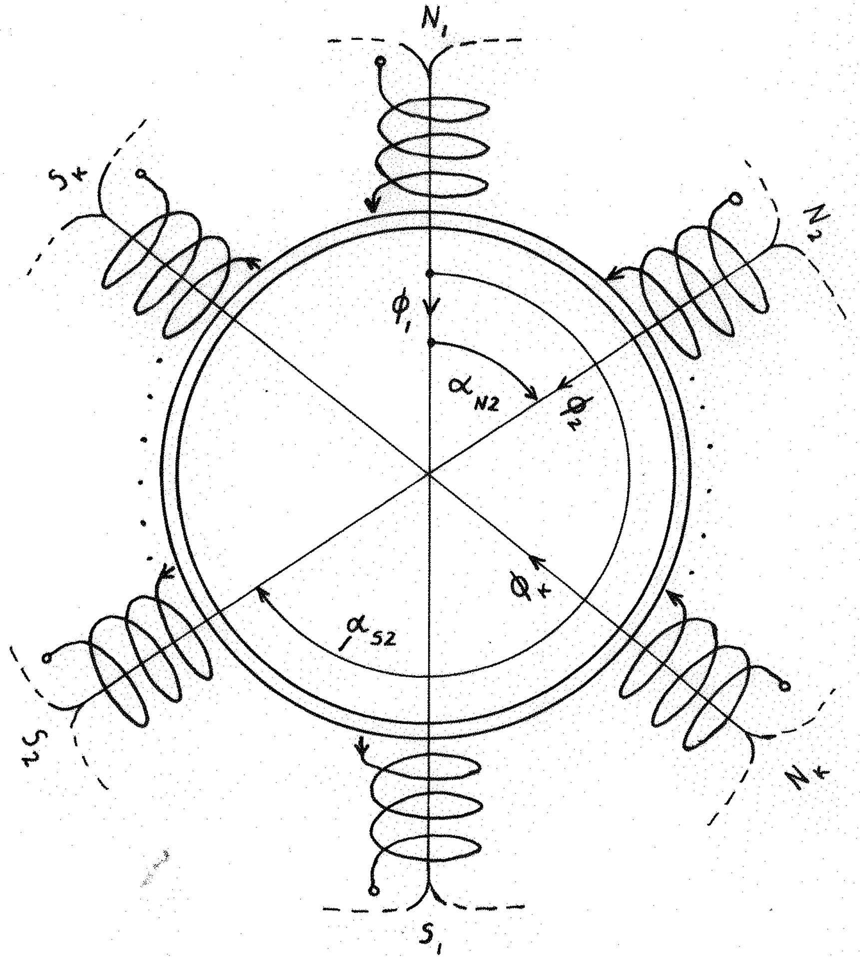


Fig. 1.1

'North' poles and 'south' poles of a machine

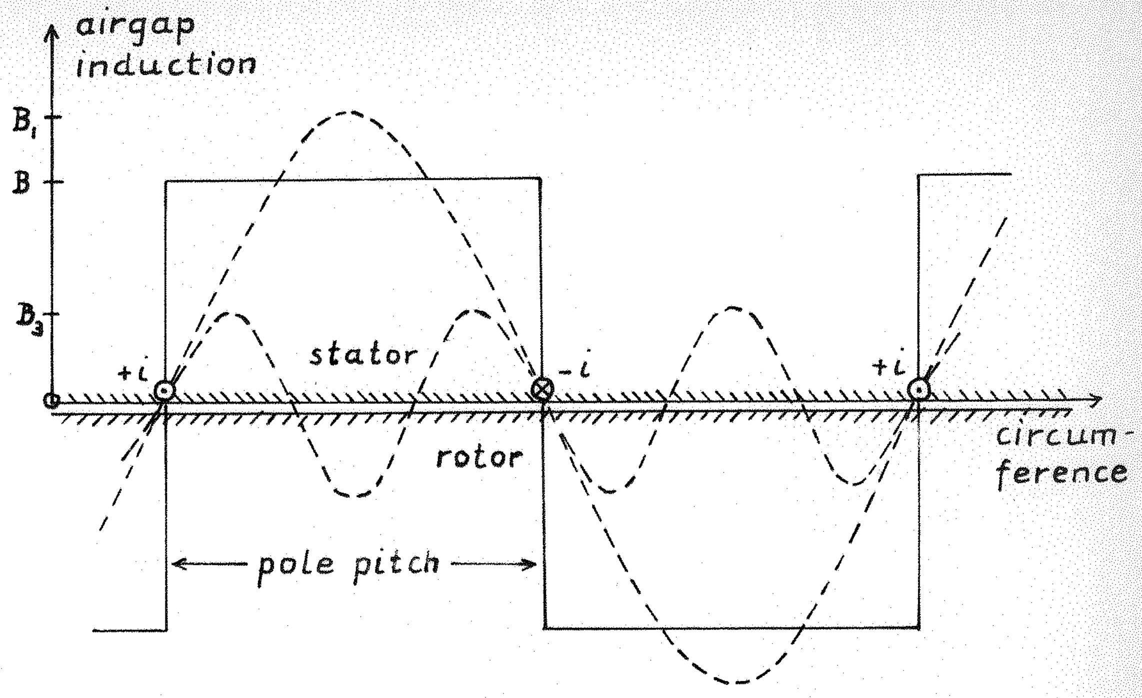


Fig. 1.2

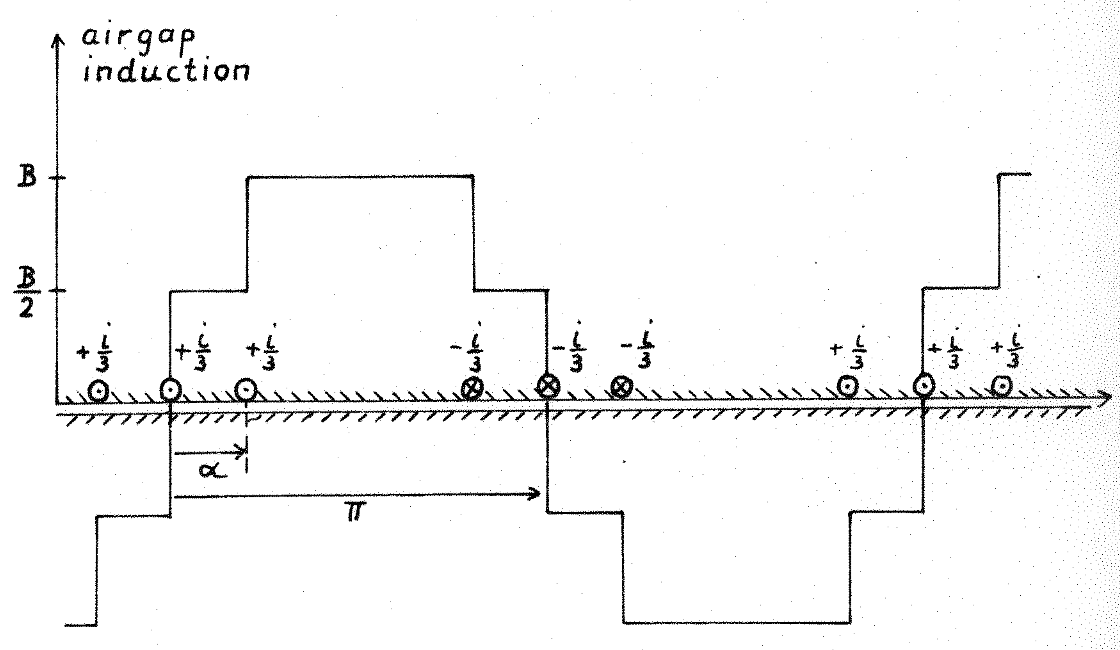


Fig. 1.3

Airgap induction due to stator phase

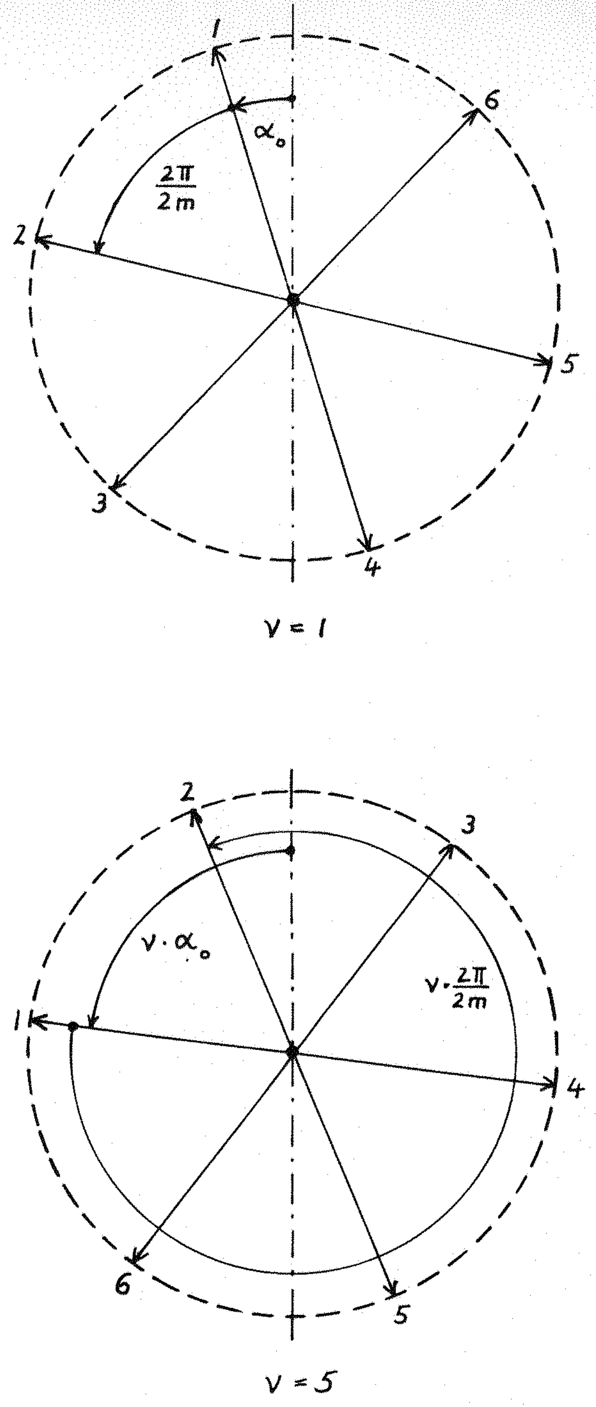


Fig. 1.4

Field vectors of a 3 phase machine

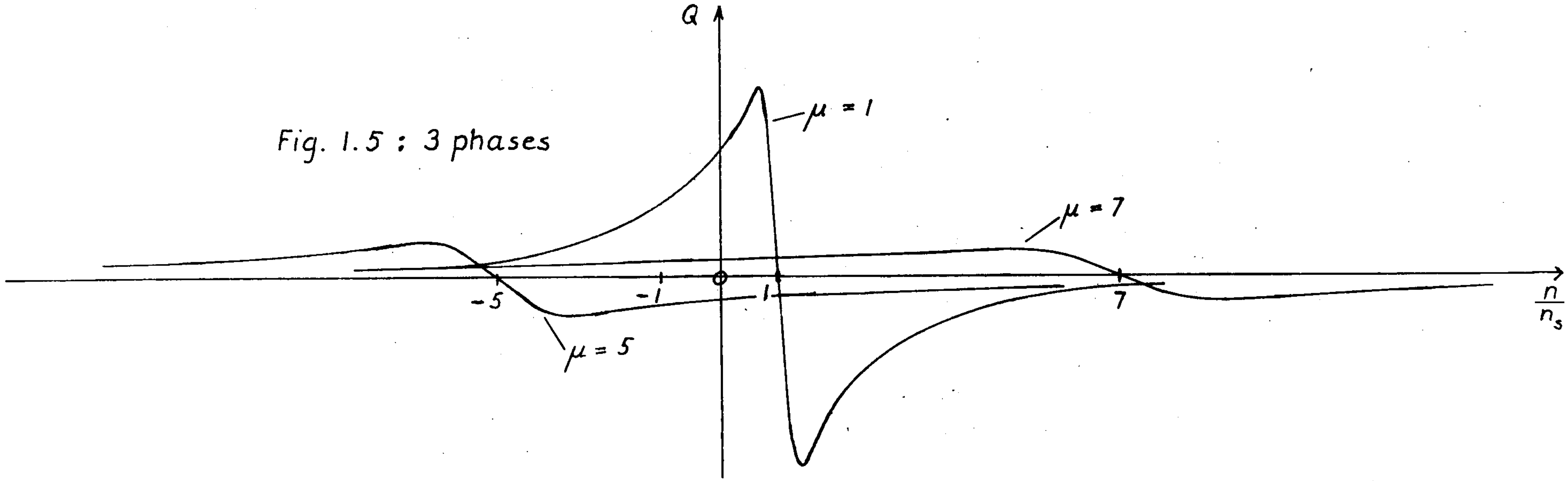


Fig. 1.5 : 3 phases

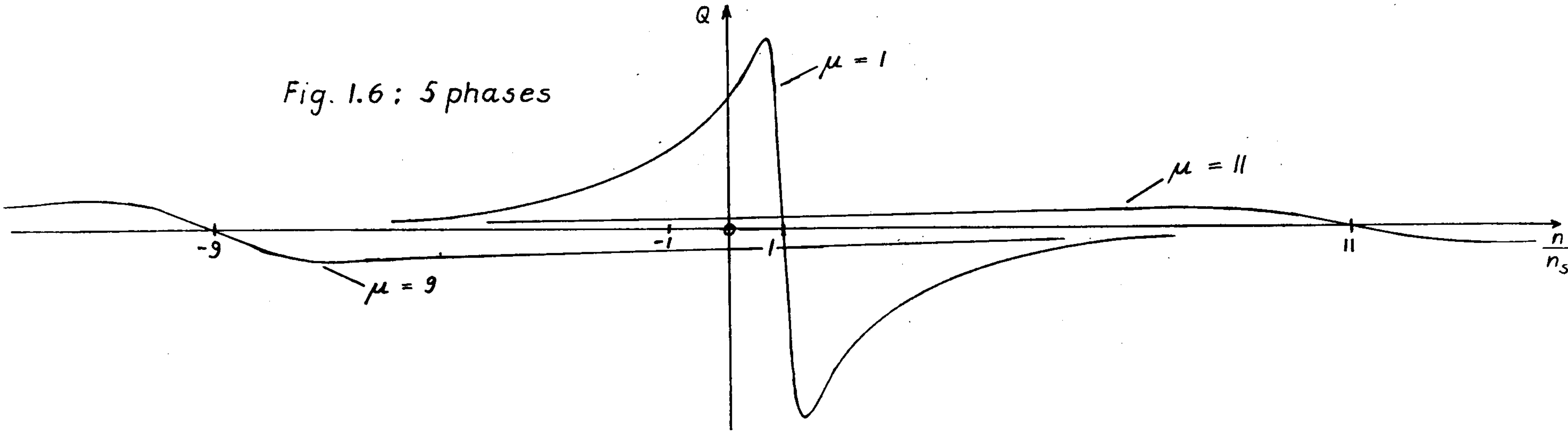


Fig. 1.6 : 5 phases

Asynchronous torque-speed curves due to current harmonics

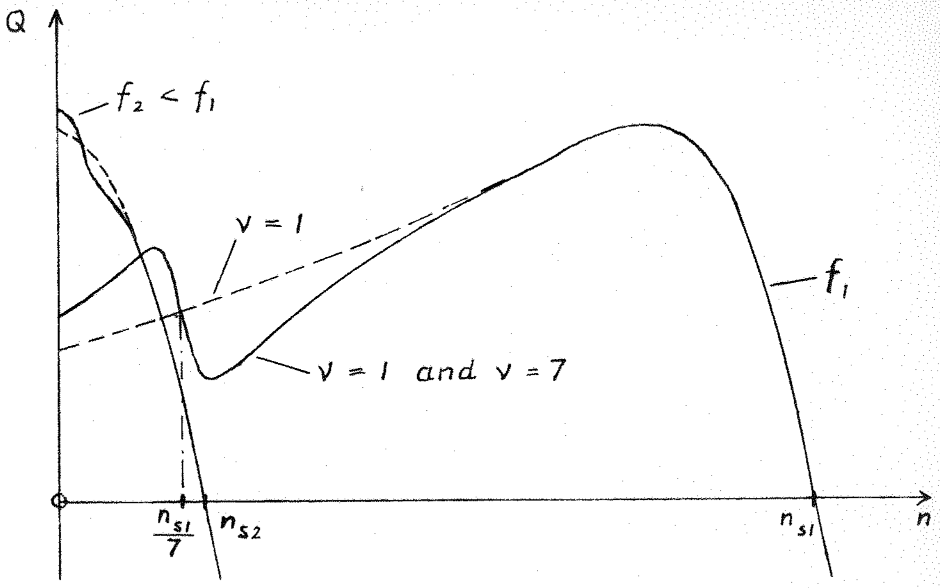


Fig. 1.7 : 3 phases

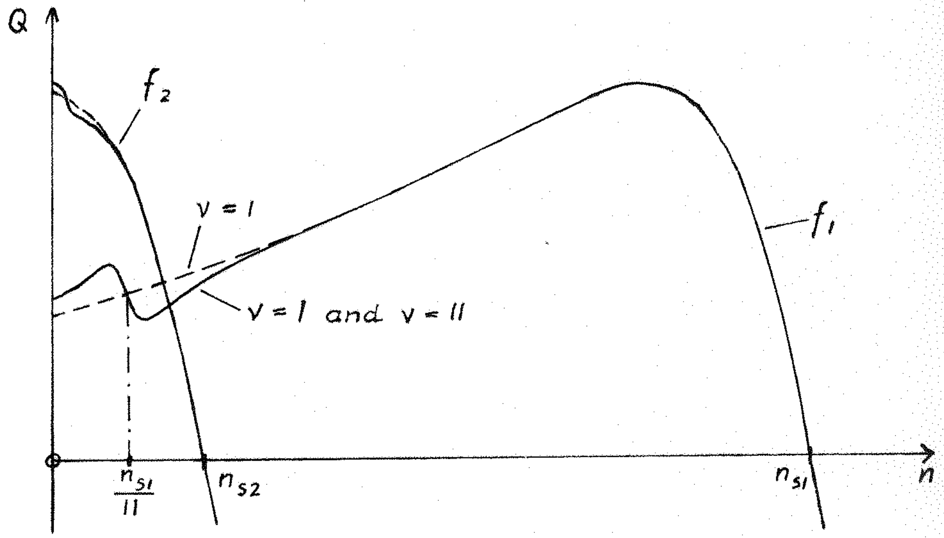
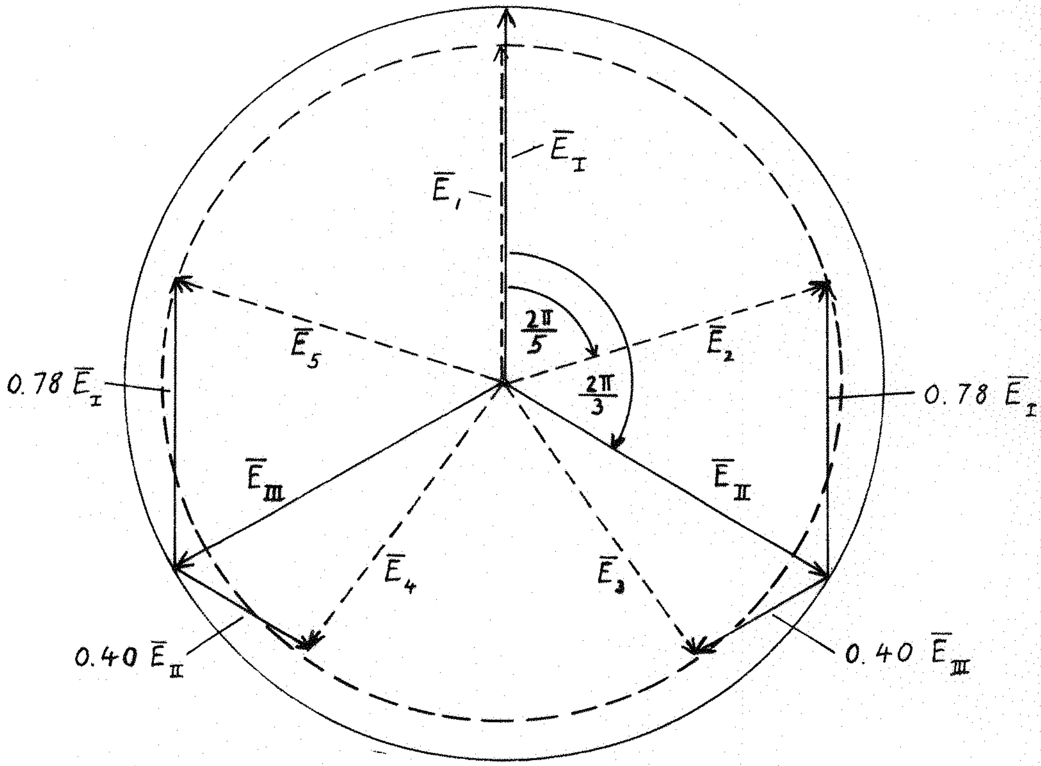


Fig. 1.8 : 5 phases

Torque-speed curves on sinusoïdal supply
at different supply frequencies



$\overline{E}_I \ldots \overline{E}_{III}$ 3 phase system

$\overline{E}_1 \ldots \overline{E}_5$ 5 phase system

$E_1 = 0.87\ E_I\ ;\ E_2 = E_5 = 0.88\ E_I\ ;\ E_3 = E_4 = 0.86\ E_I$

Fig. 1.9

5 phase system out of 3 phase system

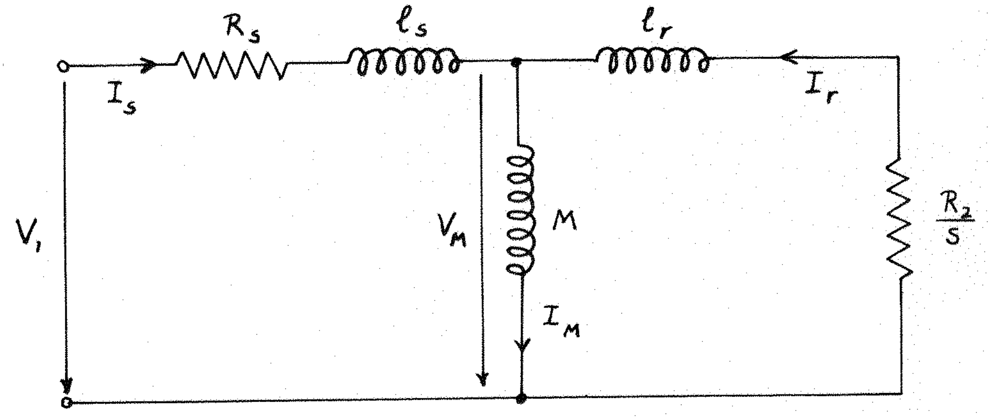


Fig. 1.10

'Classical' one-phase equivalent circuit

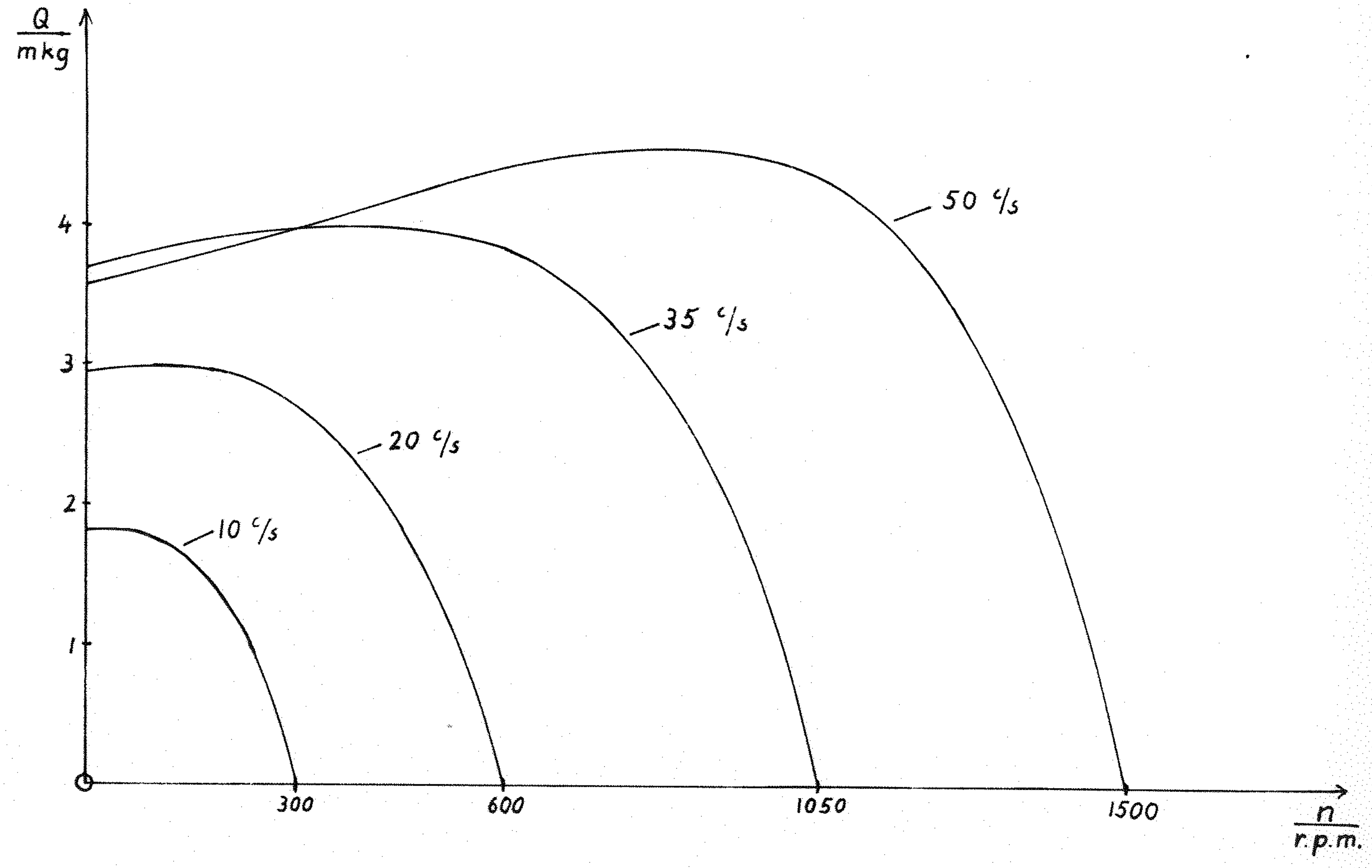


Fig. 1.11

Torque-speed curves ; voltage proportional to frequency

CHAPTER II

EXAMINATION OF THE STEADY-STATE BEHAVIOUR OF 3 PHASE AND 5 PHASE MACHINES USING THE GENERALIZED MACHINE THEORY

II.1 Short presentation of the generalized machine theory

The analysis of the polyphase induction motor supplied by non-sinusoidal voltages has been built on the foundations laid by Park and Stanley and is a theory of the interaction of linear circuits. Generally, the following assumptions are made:

1) The magnetic circuit is linear.
2) Mutual inductances vary as cosine functions of the shaft angle.
3) Self inductances are constant and resistances are not affected by skin effect and temperature.
4) Windings are symmetrical and distributed and no space harmonics are present.
5) The air-gap is uniform.
6) Speed is constant.

The machine is reduced to a two-phase machine by appropriate transformations and possesses two axes at right angle. For the present investigations the two axes will be fixed in space. Rotor quantities are referred to the stator.

II.1.1 Transformations for 3 phase machines

A 3 phase machine can be described (Ref. 2, 3) by

$$\begin{bmatrix} e_{ds} \\ e_{dr} \\ e_{qr} \\ e_{qs} \end{bmatrix} = \begin{bmatrix} R_s + pL_s & pM & 0 & 0 \\ pM & R_r + pL_r & \dot{\theta} L_r & \dot{\theta} M \\ -\dot{\theta} M & -\dot{\theta} L_r & R_r + pL_r & pM \\ 0 & 0 & pM & R_s + pL_s \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \\ i_{qs} \end{bmatrix} \qquad (2.1)$$

Fig. 2.1 shows the equivalent circuit for this set of linear differential equations.

For a squirrel-cage induction motor,

$$e_{dr} = e_{qr} = 0 \qquad (2.2)$$

Under sinusoidal conditions,

$$\begin{aligned} E_{qs} &= -j\,E_{ds} \\ I_{qs} &= -j\,I_{ds} \\ I_{qr} &= -j\,I_{dr} \end{aligned} \qquad (2.3)$$

which leads to the well-known single phase equivalent circuit.

The axes voltages are obtained from the phase voltages by the transformation

$$\begin{bmatrix} e_{ds} \\ e_{qs} \\ e_o \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \cos\frac{2\pi}{3} & \cos\frac{4\pi}{3} \\ 0 & \sin\frac{2\pi}{3} & \sin\frac{4\pi}{3} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} e_{1s} \\ e_{2s} \\ e_{3s} \end{bmatrix} \qquad (2.4)$$

The 'zero sequence' e_o vanishes for a star-connected machine whose star-point is not connected to the star-point of the supply. The phase voltages may in this case be replaced by the potentials of the terminals with respect to the star-point of the supply for obtaining e_{ds} and e_{qs}. If

$$v_o = \frac{1}{3} (e_{1t} + e_{2t} + e_{3t}) \tag{2.5}$$

is the voltage between machine star-point and supply star-point (e_{1t}, e_{2t}, e_{3t} being the potentials of the terminals), the phase voltages are

$$e_{ns} = e_{nt} - v_o \qquad (n = 1....3) \tag{2.6}$$

and

$$v_o \sum_{n=1}^{3} \cos (n-1) \frac{2\pi}{3} = v_o \sum_{n=1}^{3} \sin (n-1) \frac{2\pi}{3} = 0$$

Fig. 2.2 shows the voltage transformation for a 3 phase square-wave system.

The axes currents can be obtained from the phase currents by transformations similar to (2.3).

The reverse transformation for the stator currents is given by

$$\begin{bmatrix} i_{1s} \\ i_{2s} \\ i_{3s} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 1 \\ \cos \frac{2\pi}{3} & - \sin \frac{2\pi}{3} & 1 \\ \cos \frac{4\pi}{3} & - \sin \frac{4\pi}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{os} \end{bmatrix} \tag{2.7}$$

(i_{os} = 0 for star-connection with unconnected star-point), and the phase rotor currents are obtained accordingly from i_{dr} and i_{qr}.

The instantaneous torque of the machine is

$$Q = P M (i_{dr} i_{qs} - i_{qr} i_{ds}) \tag{2.8}$$

II.1.2 Transformations for 5 phase machines

A short derivation of the two-axes transformation for 5 phase machines is given in Appendix II.

The machine can be represented by two sets of equations:

$$\begin{bmatrix} e_{ds} \\ 0 \\ 0 \\ e_{qs} \end{bmatrix} = \begin{bmatrix} R_s + pL_s & pM & 0 & 0 \\ pM & R_r + pL_r & \dot{\theta} L_r & \dot{\theta} M \\ -\dot{\theta} M & -\dot{\theta} L_r & R_r + pL_r & pM \\ 0 & 0 & pM & R_s + pL_s \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \\ i_{qs} \end{bmatrix} \qquad (2.9)$$

and

$$\begin{bmatrix} e'_{ds} \\ 0 \\ 0 \\ e'_{qs} \end{bmatrix} = \begin{bmatrix} R_s + p\ell_s & 0 & 0 & 0 \\ 0 & R_r + p\ell_r & 0 & 0 \\ 0 & 0 & R_r + p\ell_r & 0 \\ 0 & 0 & 0 & R_s + p\ell_s \end{bmatrix} \cdot \begin{bmatrix} i'_{ds} \\ i'_{dr} \\ i'_{qr} \\ i'_{qs} \end{bmatrix} \qquad (2.10)$$

The equations of (2.9) are independent from each other and $i'_{dr} = i'_{qr} = 0$.

The torque is obtained from Eqn. (2.8) and

$$Q = P M (i_{dr} i_{qs} - i_{qr} i_{ds}) \qquad (2.11)$$

The transformation for the voltages is

$$\begin{bmatrix} e_{ds} \\ e_{qs} \\ e'_{ds} \\ e'_{qs} \\ e_o \end{bmatrix} = \sqrt{\frac{2}{5}} \begin{bmatrix} 1 & \cos\alpha & \cos 2\alpha & \cos 3\alpha & \cos 4\alpha \\ 0 & \sin\alpha & \sin 2\alpha & \sin 3\alpha & \sin 4\alpha \\ 1 & \cos 2\alpha & \cos 4\alpha & \cos 6\alpha & \cos 8\alpha \\ 0 & \sin 2\alpha & \sin 4\alpha & \sin 6\alpha & \sin 8\alpha \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} e_{1s} \\ e_{2s} \\ e_{3s} \\ e_{4s} \\ e_{5s} \end{bmatrix} \qquad (2.12)$$

where $\alpha = \frac{2\pi}{5}$. Again, $e_o = 0$ for a star-connected machine. The other four voltages can be obtained by replacing the phase voltages by the potential of the terminals with respect to the star-point of the supply e_{nt}:

$$\begin{aligned} e_{ns} &= e_{nt} - v_o \\ v_o &= \frac{1}{5} \sum_{n=1}^{5} e_{nt} \end{aligned} \qquad (2.13)$$

Fig 2.3 shows these voltages for a 5 phase square-wave system.

The reverse transformation for the stator currents is given by

$$\begin{bmatrix} i_{1s} \\ i_{2s} \\ i_{3s} \\ i_{4s} \\ i_{5s} \end{bmatrix} = \sqrt{\frac{2}{5}} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ \cos\alpha & -\sin\alpha & \cos 2\alpha & -\sin 2\alpha & 1 \\ \cos 2\alpha & -\sin 2\alpha & \cos 4\alpha & -\sin 4\alpha & 1 \\ \cos 3\alpha & -\sin 3\alpha & \cos 6\alpha & -\sin 6\alpha & 1 \\ \cos 4\alpha & -\sin 4\alpha & \cos 8\alpha & -\sin 8\alpha & 1 \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \\ i'_{ds} \\ i'_{qs} \\ i_{os} \end{bmatrix} \qquad (2.14)$$

and $i_{os} = 0$ for star-connection. In particular, the first phase current is

$$i_{1s} = \sqrt{\frac{2}{5}}\ (i_{ds} + i_{ds}') \tag{2.15}$$

A similar transformation gives the rotor phase currents but here $i_{dr}' = i_{qr}' = 0$ and

$$i_{1r} = \sqrt{\frac{2}{5}}\ i_{dr} \tag{2.16}$$

II.2 A dual analogue

It is desirable for convenient investigation to build an analogue for the sets of differential equations representing the induction motors.

One way of doing this would be to embody the circuit of Fig. 2.1. Iron-cored inductors could be used to represent the inductances and fractional-horsepower machines to generate the back-e.m.f.'s. This, however, presents the following difficulties: it is not possible for the rotors to cut the whole of the flux which links with the circuit of the other rotor. Iron-cored inductors will not be strictly linear and changing the parameters of the analogue to represent another motor would possibly involve the re-winding of the inductors.

A more useful and simpler analogue for Eqns. (2.1) and (2.9) is the dual of the circuit of Fig. 2.1. Inductors are replaced by capacitors. Series connections change into parallel connections, voltages into currents and vice versa. Impressed voltages become impressed currents. The analogue can be built up with solid-state electronics.

Eqns. (2.1) and (2.9) can be represented by Eqn. (2.18) if the following substitutions are made:

Axes quantities :	e_{ds}	e_{qs}	i_{ds}	i_{dr}	i_{qr}	i_{qs}	
Dual analogue quantities:	i_1	i_2	e_1	e_3	e_4	e_2	
Axes quantities :	R_s	R_r	L_s	L_r	M	ℓ_s	ℓ_r
Dual analogue quantities:	G_1	G_2	C_1	C_2	C_3	C_4	C_5

Since

$$\begin{aligned} L_s &= M + \ell_s \\ L_r &= M + \ell_r \\ C_1 &= C_3 + C_4 \\ C_2 &= C_3 + C_5 \end{aligned} \tag{2.17}$$

$$\begin{bmatrix} i_1 \\ 0 \\ 0 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_1 + pC_1 & pC_3 & 0 & 0 \\ pC_3 & G_2 + pC_2 & \dot{\theta}C_2 & \dot{\theta}C_3 \\ -\dot{\theta}C_3 & -\dot{\theta}C_2 & G_2 + pC_2 & pC_3 \\ 0 & 0 & pC_3 & G_1 + pC_1 \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_3 \\ e_4 \\ e_2 \end{bmatrix} \tag{2.18}$$

Similarly Eqn. (2.10) can be represented by

$$\begin{bmatrix} i_1' \\ i_2' \end{bmatrix} = \begin{bmatrix} G_1 + pC_4 & 0 \\ 0 & G_1 + pC_4 \end{bmatrix} \cdot \begin{bmatrix} e_1' \\ e_2' \end{bmatrix} \tag{2.19}$$

if i_1' is substituted for e_{ds}', i_2' for e_{qs}', e_1' for i_{ds}', and e_2' for i_{qs}'.

The values of the conductances and capacitances representing the resistances and inductances of the motor can be obtained by the following considerations:

e_1 is proportional to i_{ds}, i.e.

$$e_1 = R_o \; i_{ds} \qquad (2.20)$$

and accordingly for the other analogue voltages and two-axes currents. The analogue currents and two-axes voltages are linked by a scale resistance; for example

$$e_{ds} = R \; i_1 \qquad (2.21)$$

Substituting this into Eqn. (2.1) gives:

$$\begin{bmatrix} e_{ds} \\ 0 \\ 0 \\ e_{qs} \end{bmatrix} = R_o \, R \begin{bmatrix} G_1 + pC_1 & pC_3 & 0 & 0 \\ pC_3 & G_2 + pC_2 & \dot{\theta}C_2 & \dot{\theta}C_3 \\ -\dot{\theta}C_3 & -\dot{\theta}C_2 & G_2 + pC_2 & pC_3 \\ 0 & 0 & pC_3 & G_1 + pC_1 \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \\ i_{qs} \end{bmatrix} \qquad (2.22)$$

The analogue parameters can now be obtained by comparing Eqns. (2.1) and (2.22):

$$G_1 = R_s \frac{1}{R_o R} \qquad C_1 = L_s \frac{1}{R_o R}$$

$$G_2 = R_r \frac{1}{R_o R} \qquad C_2 = L_r \frac{1}{R_o R}$$

$$C_3 = M \frac{1}{R_o R} \qquad (2.23)$$

$$C_4 = \ell_s \frac{1}{R_o R}$$

$$C_5 = \ell_r \frac{1}{R_o R}$$

The voltage of an induction motor and therefore the axes-voltages should in first approximation vary in proportion with the frequency. The magnitude of the analogue current i_1 at a given instant t is

$$i_1 \; (t) = \frac{\omega}{\omega_o} \; i_{10} \; (\tau) \qquad (2.24)$$

ω_o = reference fundamental angular frequency

i_{10} = current at reference frequency and at the instant

$$\tau = \frac{\omega}{\omega_o} \; t$$

Similar equations apply to i_2, i_1', and i_2'. They are introduced into Eqn. (2.18) and 2(19) and the time transformation

$$\tau = \frac{\omega}{\omega_o} \; t \qquad (2.25)$$

is made. If both sides are multiplied by $\frac{\omega_o}{\omega}$, the result is

$$\begin{bmatrix} i_{10} \\ 0 \\ 0 \\ i_{20} \end{bmatrix} = \begin{bmatrix} \frac{\omega_o}{\omega} G_1+p'C_1 & p'C_3 & 0 & 0 \\ p'C_3 & \frac{\omega_o}{\omega} G_2+p'C_2 & \frac{\dot{\Theta}}{\omega} \omega_o C_2 & \frac{\dot{\Theta}}{\omega} \omega_o C_3 \\ -\frac{\dot{\Theta}}{\omega} \omega_o C_3 & -\frac{\dot{\Theta}}{\omega} \omega_o C_2 & \frac{\omega_o}{\omega} G_2+p'C_2 & p'C_3 \\ 0 & 0 & p'C_3 & \frac{\omega_o}{\omega} G_1+p'C_1 \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_3 \\ e_4 \\ e_2 \end{bmatrix} \qquad (2.26)$$

and

$$\begin{bmatrix} i_{10}' \\ i_{20}' \end{bmatrix} = \begin{bmatrix} \frac{\omega_o}{\omega} G_1+p'C_4 & 0 \\ 0 & \frac{\omega_o}{\omega} G_1+p'C_4 \end{bmatrix} \cdot \begin{bmatrix} e_1' \\ e_2' \end{bmatrix} \qquad (2.27)$$

where $p' = \frac{d}{d\tau}$.

The angular supply frequency of the motor only appears on the right hand side of these equations in connection with the conductances G_1 and G_2 and in the ratio $\frac{\dot{\Theta}}{\omega}$. $\frac{\dot{\Theta}}{\omega}$ indicates the slip of the motor:

$$\frac{\dot{\Theta}}{\omega} = 1 - s \qquad (2.28)$$

The frequency of the analogue currents can be kept constant and the chosen value is $f_o = 50\ ^c/_s$. Adjusting the analogue for a motor frequency is done by setting potentiometers representing the conductances to their proper values:

$$\begin{aligned} R_1\ (\omega) &= \frac{1}{G_1} \cdot \frac{\omega}{\omega_o} \\ R_2\ (\omega) &= \frac{1}{G_2} \cdot \frac{\omega}{\omega_o} \end{aligned} \qquad (2.29)$$

Fig. 2.4 shows the dual analogue. The speed of the motor can be set with a ganged precision potentiometer:

$$\frac{\dot{\Theta}}{\omega} = \frac{2x}{\ell} \qquad (2.30)$$

$0 < \frac{x}{\ell} < \frac{1}{2}$ means motoring, $\frac{x}{\ell} > \frac{1}{2}$ generating, and $\frac{x}{\ell} < 0$ breaking.

The transistor circuit of Fig. 2.5 provides an impressed current out of an impressed voltage. Its input impedance is very high. For the two current sources in the feedback circuits the ratio of input voltage and output current must be

$$\frac{v}{i} = \frac{1}{\omega_o C_3} \qquad (2.31)$$

If the two switches S are open the analogue can be used for obtaining the voltages e_1' and e_2' in the case of a 5 phase machine.

The torque of the machine can be obtained by multiplying e_2, e_3 and e_1, e_4 considering Eqns. (2.8) and (2.20):

$$Q = \frac{P\ M}{R_o^{\ 2}} \left(e_2 \ e_3 - e_1 \ e_4 \right) \qquad (2.32)$$

Accurate and relatively simple electronic multiplication is offered by Hall multipliers. The output voltage of a Hall multiplier is proportional to the product of plate current and field current. The voltages $e_1 \ldots . e_4$ have to be turned into proportional impressed currents which can again be done by the circuit of Fig. 2.5. Fig. 2.6 shows the arrangement for the torque computation. Since the outputs of the Hall multipliers must be floating the signals must be fed into d.c. amplifiers in differential mode before they can be added up.

The output voltages of the Hall multipliers are

$$\begin{aligned} u_{H1} &= K \ i_{p2} \ i_{c3} \\ u_{H2} &+ K \ i_{p1} \ i_{c4} \end{aligned} \qquad (2.33)$$

(K = Hall constant) and the plate and field currents are linked with the voltages $e_1 \ldots . e_4$ by resistances:

$$\begin{aligned} i_{p2} &= \frac{e_2}{R_p} \ ; \quad i_{p1} = \frac{e_1}{R_p} \\ i_{c3} &= \frac{e_3}{R_c} \ ; \quad i_{c4} = \frac{e_4}{R_c} \end{aligned} \qquad (2.34)$$

Substituting Eqn. (2.34) into (2.33) and taking into consideration the combined gain of the amplifiers $A = A_1 \; A_2$, one obtains:

$$v_Q = \frac{A\ K}{R_p\ R_c} (e_2\ e_3 - e_1\ e_4) \qquad (2.35)$$

Eqns. (2.32) and (2.35) give the proportionality factor of output voltage v_Q and torque Q:

$$\frac{v_Q}{Q} = \frac{P\ M\ R_p\ R_c}{R_o^2\ A\ K} \qquad (2.36)$$

This analogue can be used to compute the axes currents and the torque of 3 phase and 5 phase machines in the steady state and under the condition that the motor voltage is proportional to the frequency. For the computation the supply frequency and the speed of the motor can be set with potentiometers. The inputs of the two current sources giving i_1 and i_2 or i_1' and i_2' have to be supplied by voltages which are patterns of the axes-voltages e_{ds} and e_{qs} or e_{ds}' and e_{qs}' and whose frequencies and amplitudes are constant. The time is transformed into

$\tau = \frac{\omega}{\omega_o} t$. If $R_o = 1\Omega$,

$$\frac{e_1(\tau)}{[V]} = \frac{i_{ds}(t)}{[amps]} \qquad (2.37)$$

etc. The scale resistor R gives the axes voltages from the analogue currents:

$$e_{dso} = i_{10}\ R \qquad (2.38)$$

etc. (e_{dso} = axis voltage at $\omega = \omega_o = 2\pi \cdot 50 \frac{1}{s}$) and

$$e_{ds}(t) = \frac{\omega}{\omega_o} e_{dso}(\tau) \qquad (2.39)$$

II.3 Computations of axes-currents and torque for 3 phases and 5 phases and square-wave supply

The dual analogue has been used to examine the currents and torques of a 3 phase machine and the experimental 5 phase machine on square-wave supply. To achieve some comparison, the computations for the 3 phase machine have been carried out for a machine which has the same parameters as the experimental 5 phase machine.

The sets of square-waves are produced by bistable ringcounters; a 5 phase ringcounter is shown in Fig. 2.7. The square-waves are added up with operational amplifiers according to Eqns. (2.4) and (2.12) and the resulting voltages are fed into the analogue.

If both machines shall produce the same average torque at a given frequency and speed, the phase voltage of the 3 phase machine should be

$$V_{3\ \text{phases}} = \sqrt{\frac{5}{3}}\ V_{5\ \text{phases}} \qquad (2.40)$$

This can be taken into account by appropriate feedback resistors of the summation amplifiers.

Figures 2.8 .... 2.14 show the d-axis voltages, the axes currents, and the instantaneous torque for both machines at different frequencies and speeds. The voltage is proportional to the frequency. Examination and comparison of the axes currents and the torque lead to the following results:

1. The torque fluctuations of both machines at normal running speeds do not depend to a great extent on the frequency or the speed. The fluctuations are added to the mean value of the torque which depends on the slip.

2. For a 3 phase machine, the frequency of the fundamental of the torque fluctuations is 6 f. For a 5 phase machine, it is 10 f.

3. The torque fluctuations of a 5 phase machine are considerably smaller than those of a 3 phase machine. If both machines have the same parameters, the peak-to-peak value of the torque fluctuations for a 5 phase machine at a given frequency and speed is reduced to about $\frac{1}{3}$ of the value for a 3 phase machine.

4. The stator current of a 5 phase machine consists of two components, $\sqrt{\frac{2}{5}}\, i_{ds}$ and $\sqrt{\frac{2}{5}}\, i_{ds}'$. The current i_{ds}' does not contribute to the torque and is limited by the stator resistance and leakage impedance. It can be seen that at higher frequencies and therefore voltages this current becomes very big and that it reaches a peak value when the invertor commutation occurs. This means considerable additional losses in the stator windings, and the invertor must be able to commutate this high current.

An inspection of the current i'_{ds} shows that its fundamental frequency is three times the fundamental frequency of the supply voltages. This current is mainly due to the 3rd harmonic of the supply voltage. If the motor is fed by voltages which do not contain a 3rd harmonic, this difficulty could be overcome.

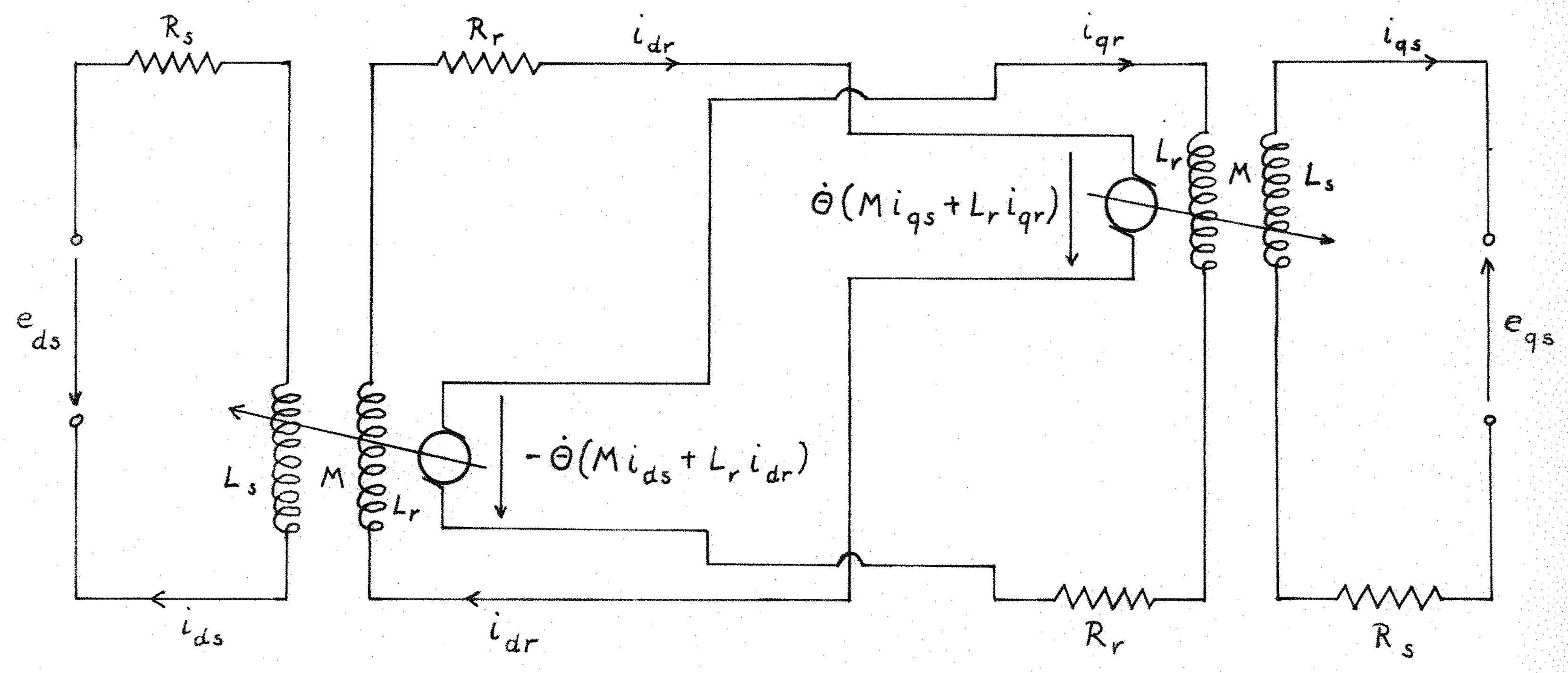


Fig. 2.1

Equivalent circuit

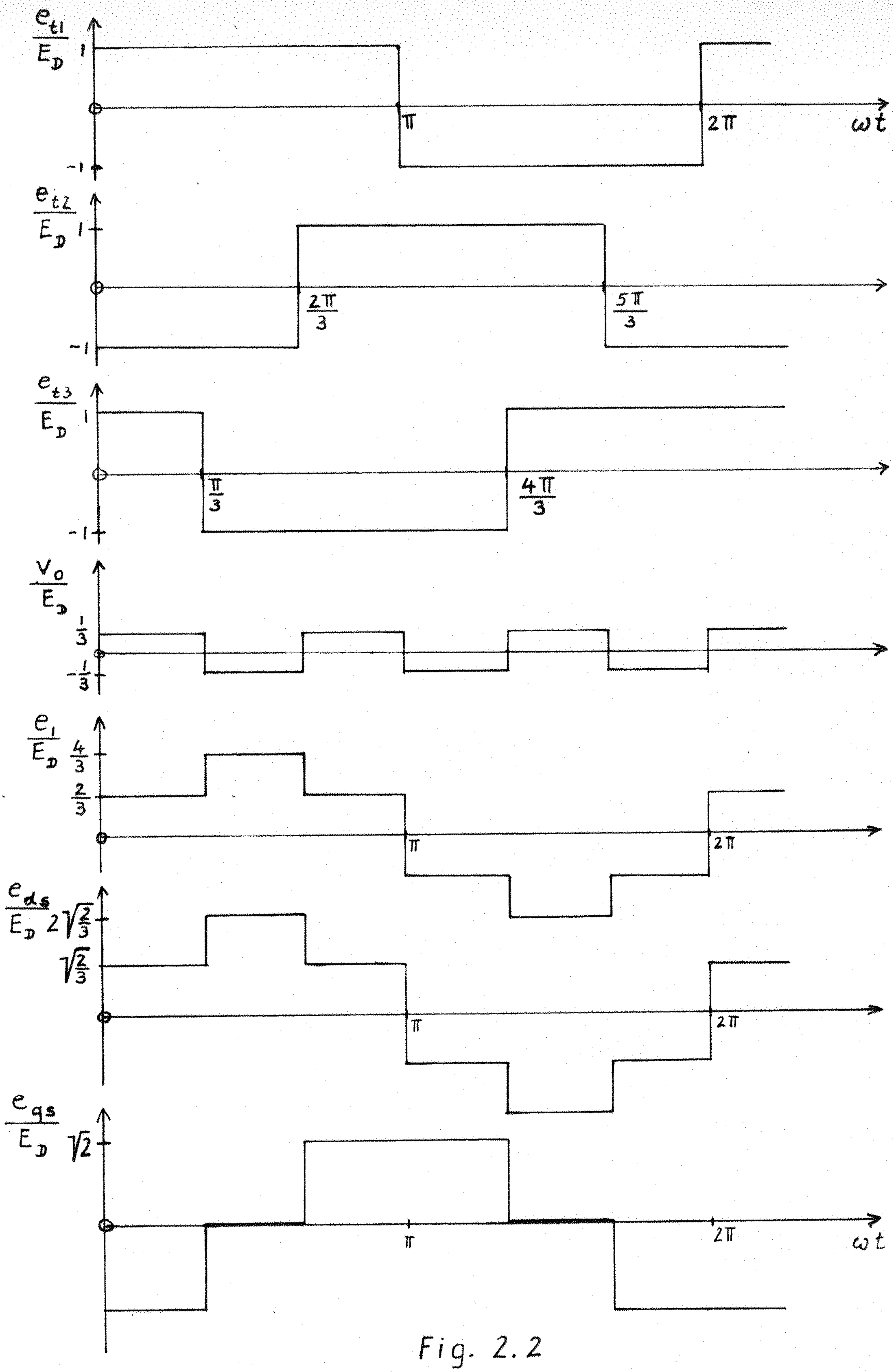


Fig. 2.2

Three phase square-wave system

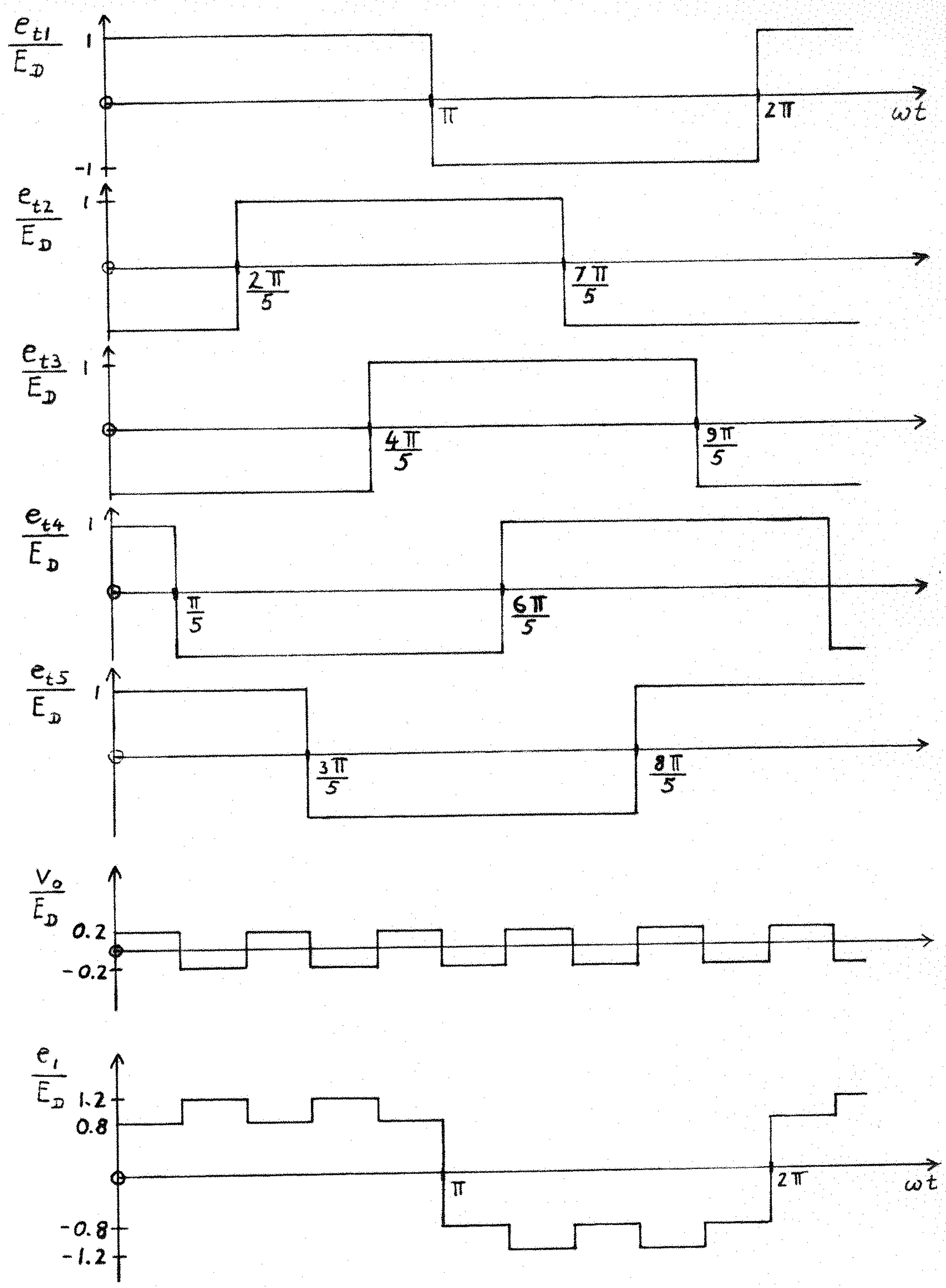


Fig. 2.3

Five phase square-wave system

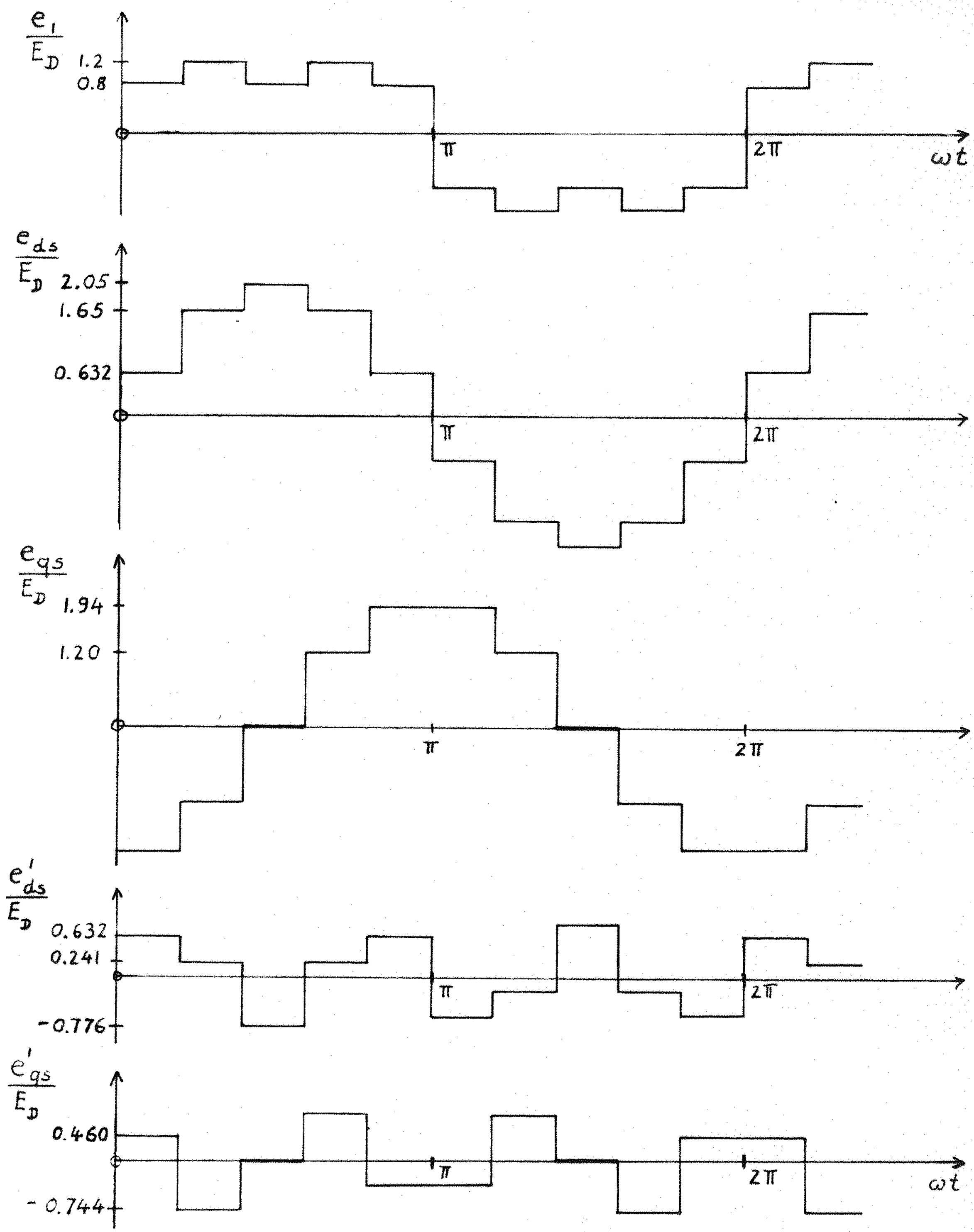


Fig. 2.3

Five phase square-wave system

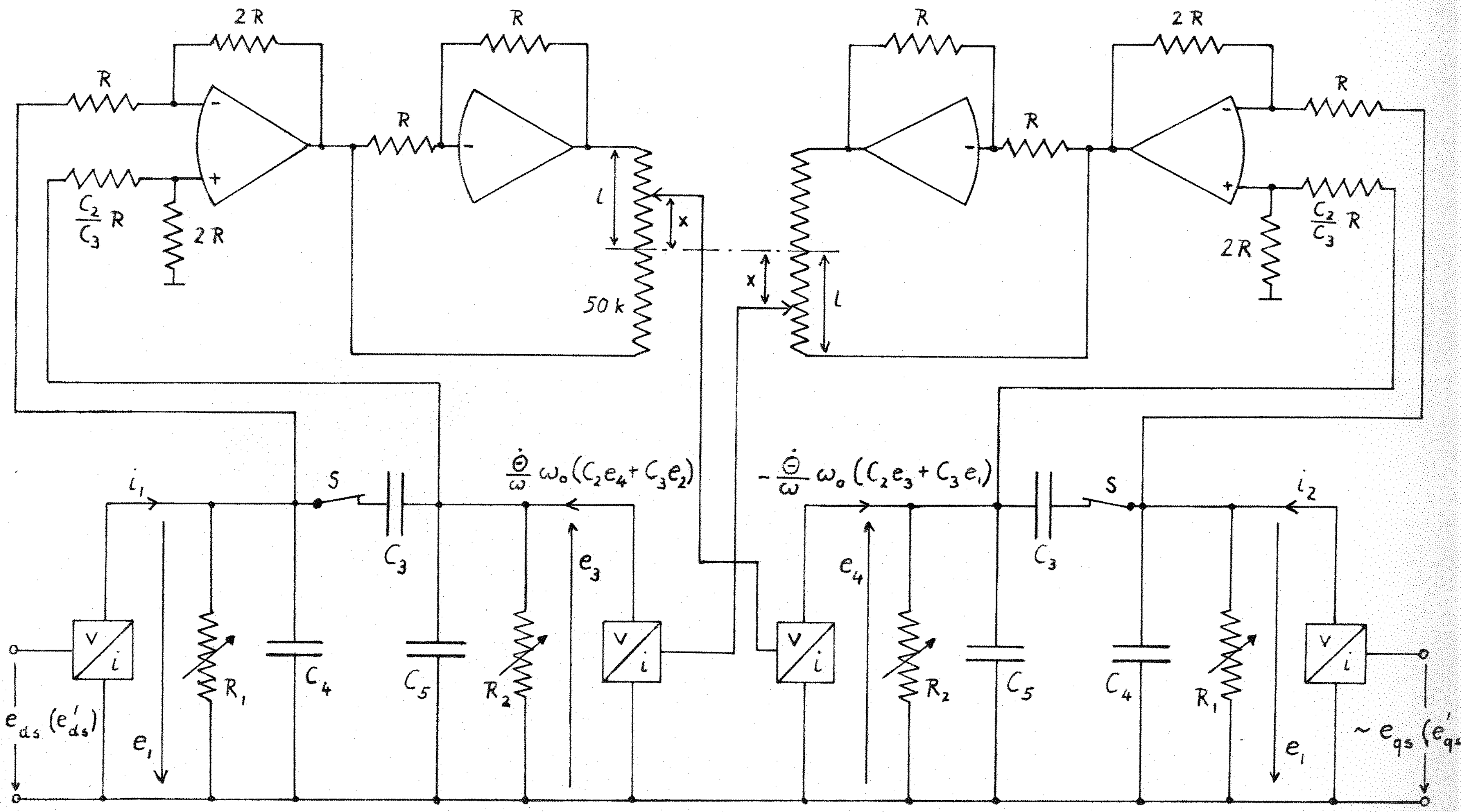


Fig. 2.4

Dual analogue of the induction motor

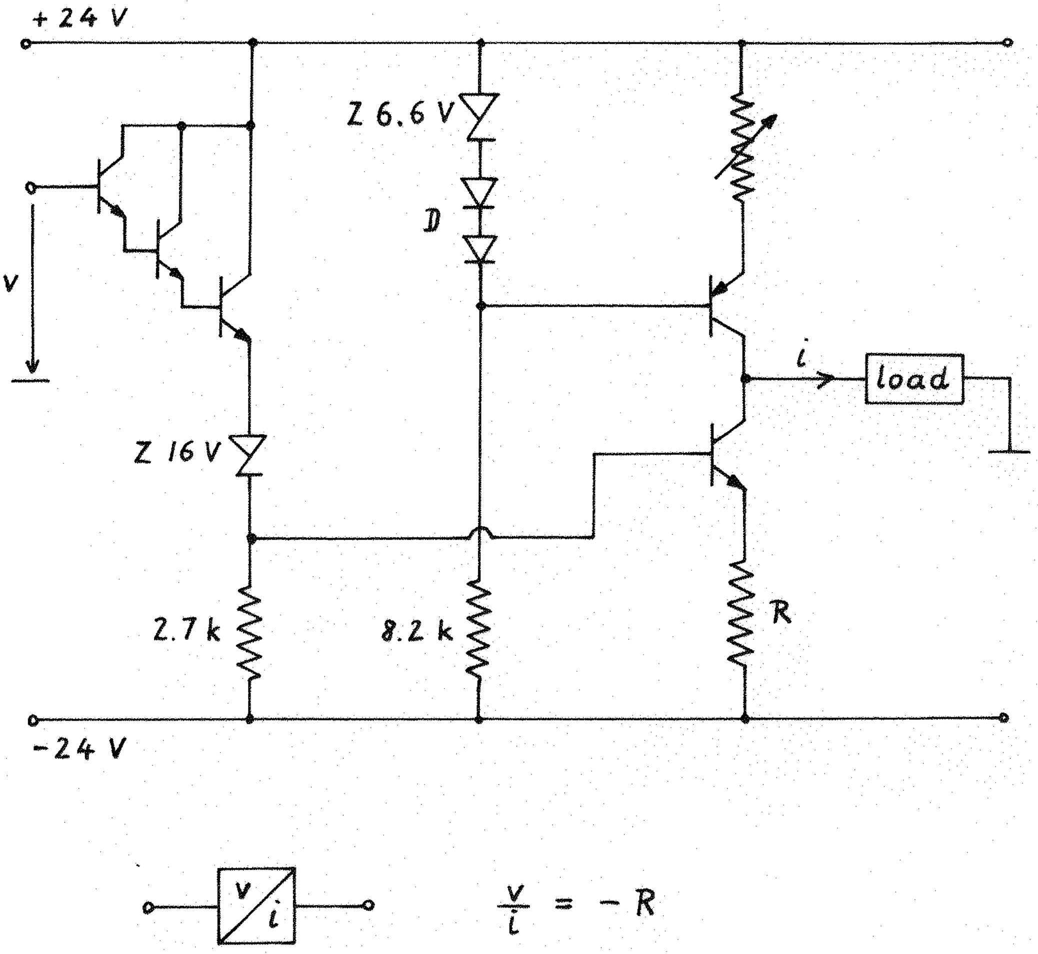


Fig. 2.5

Current source

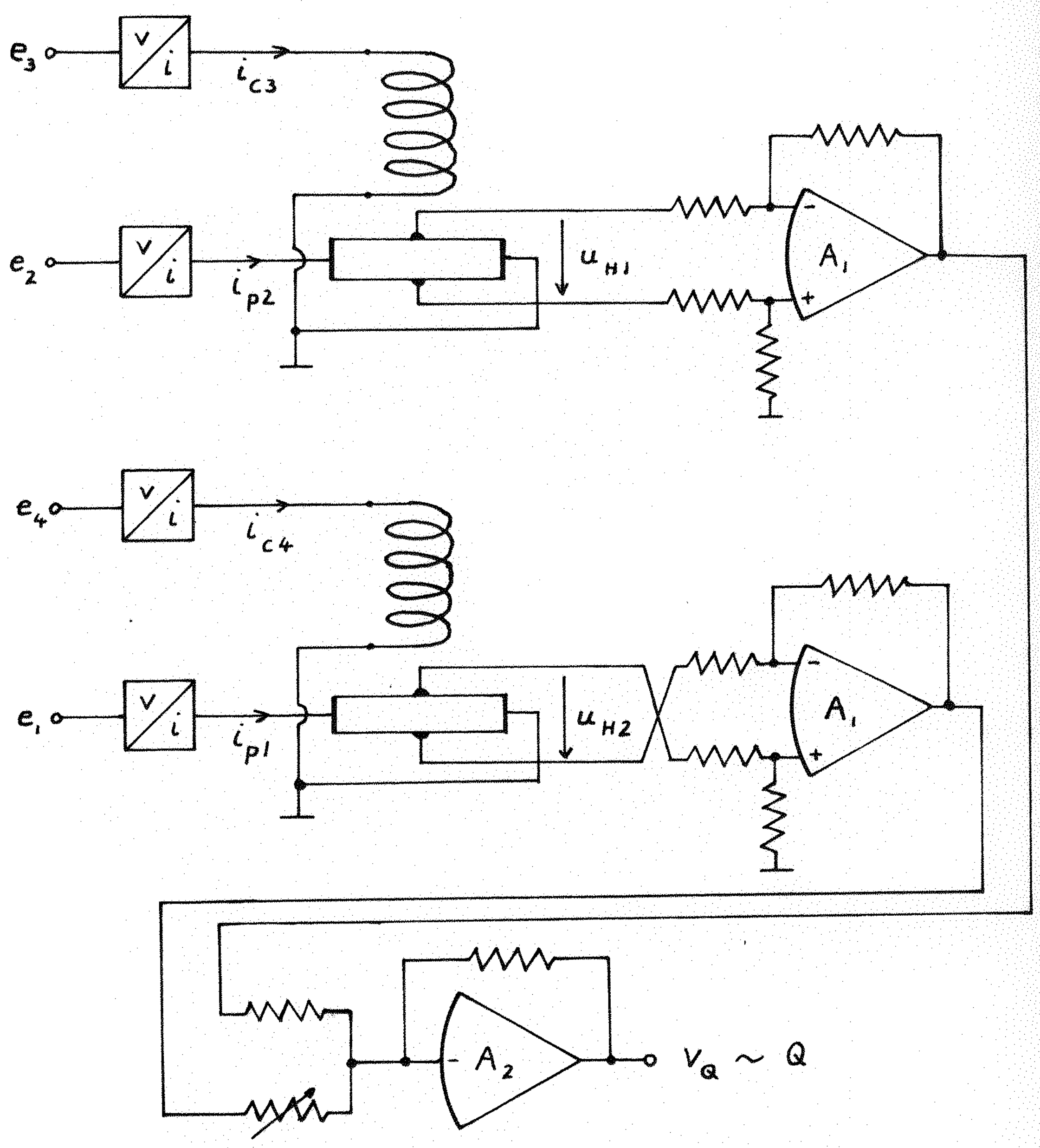


Fig. 2.6

Torque computation arrangement

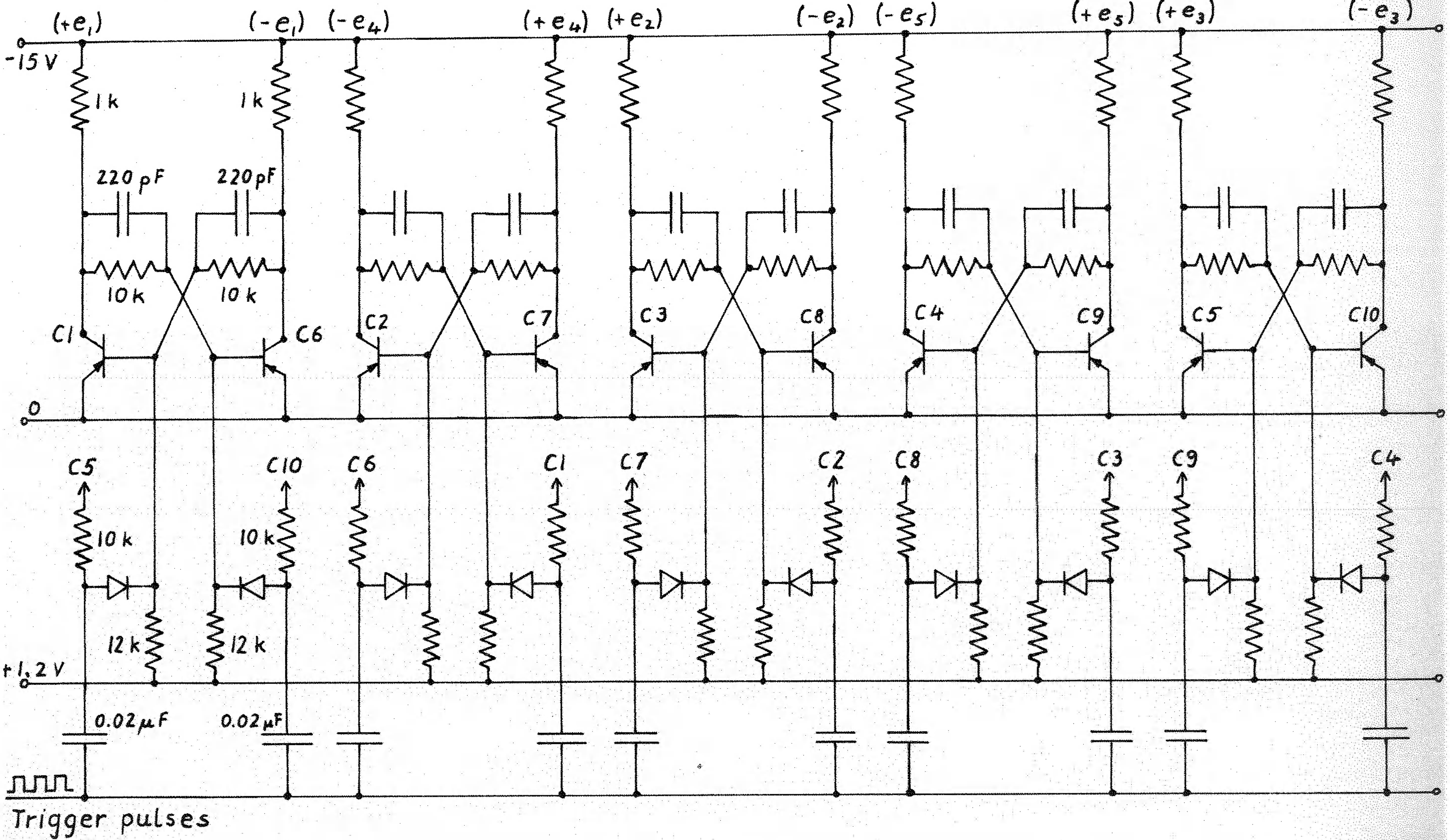
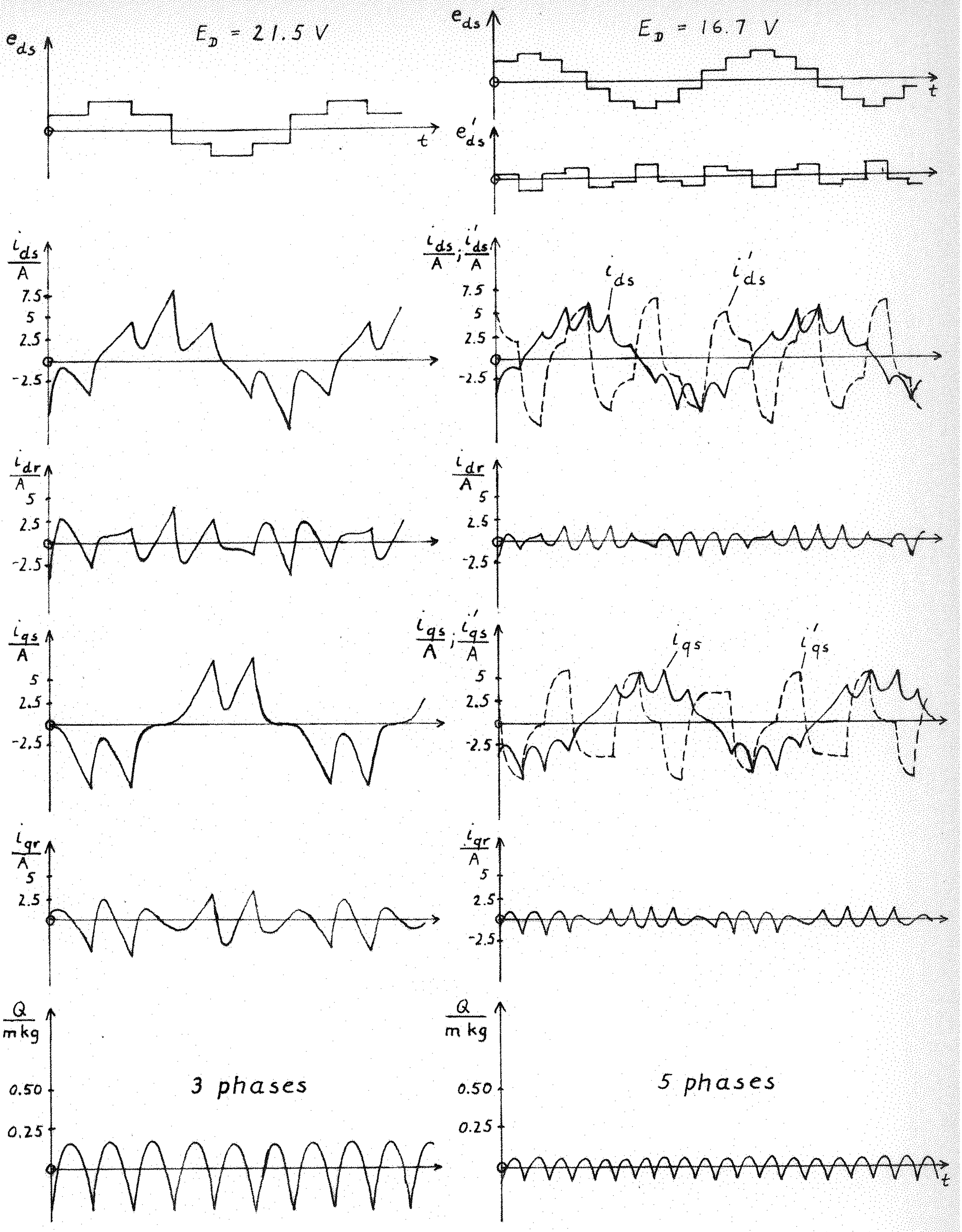


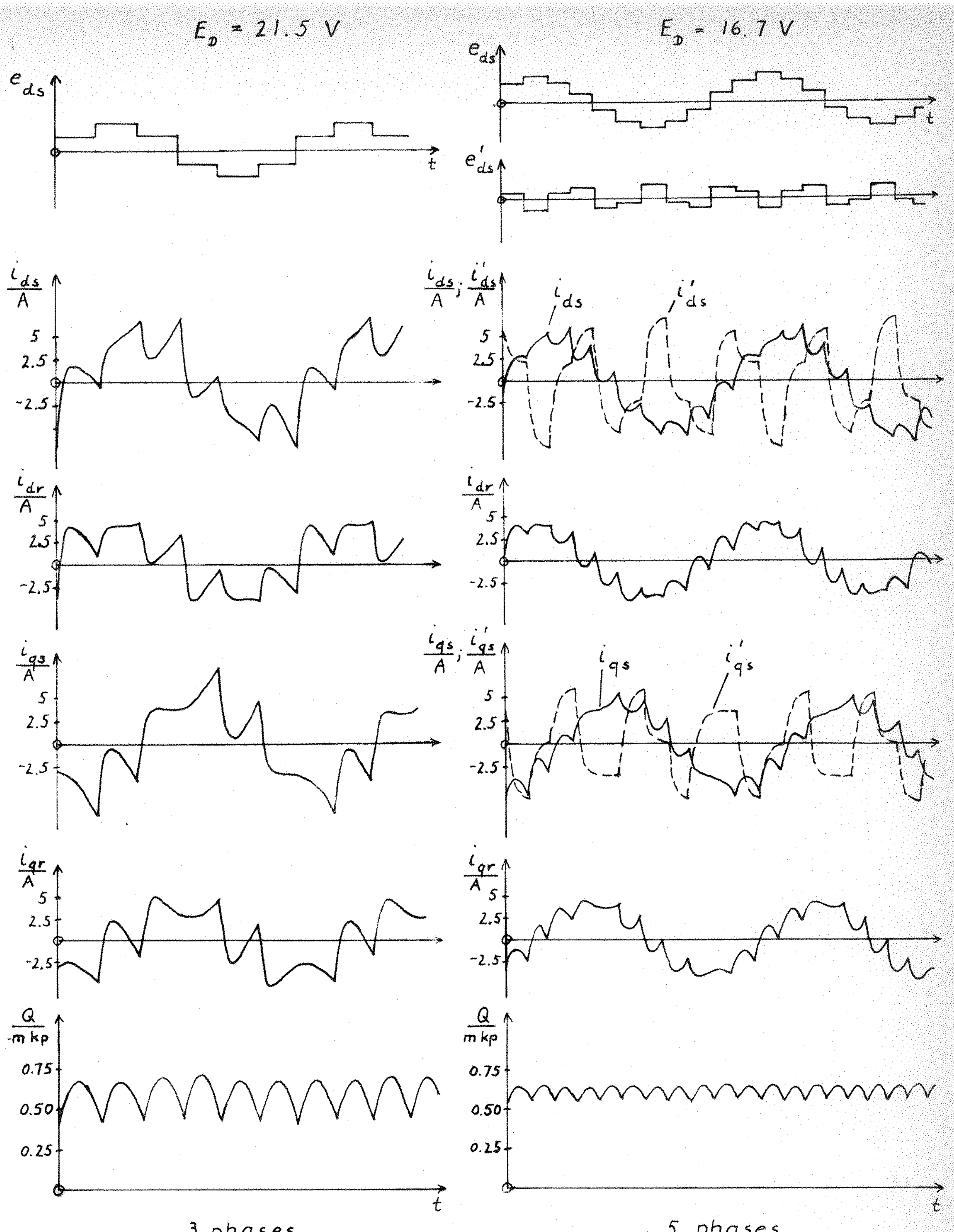
Fig. 2.7

Five phase ringcounter



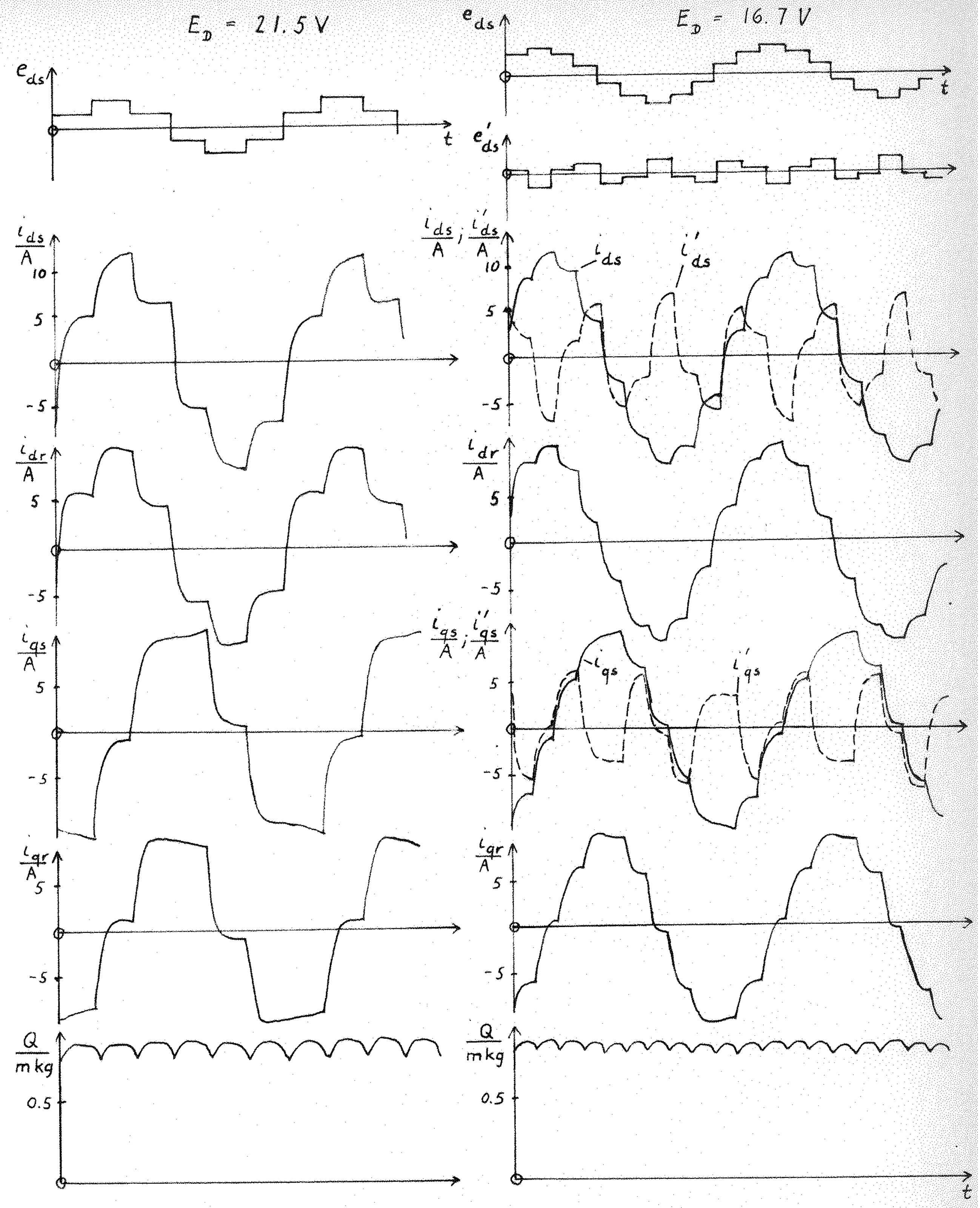
7.5 c/s , synchronism

Fig. 2.8 a



7.5 c/s , 20 % slip

Fig. 2.9 a

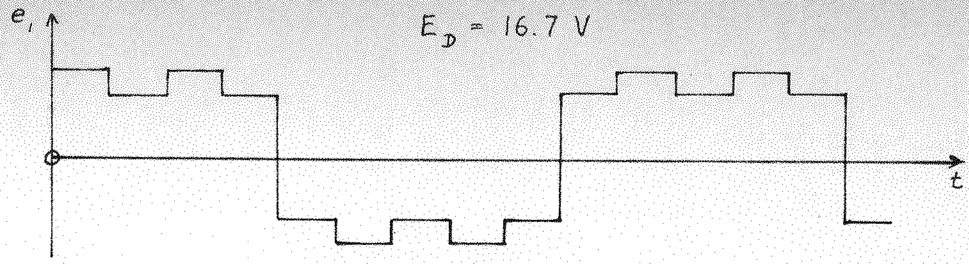


3 phases

5 phases

7.5 c/s, standstill

Fig. 2.10 a



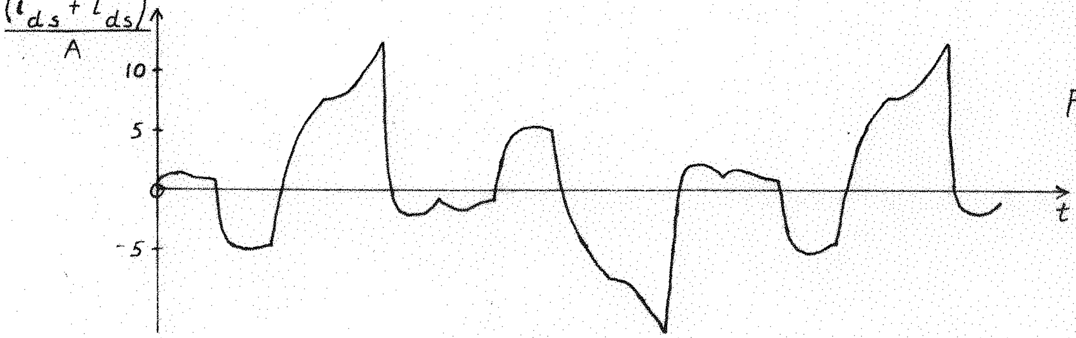


Fig. 2.8 b

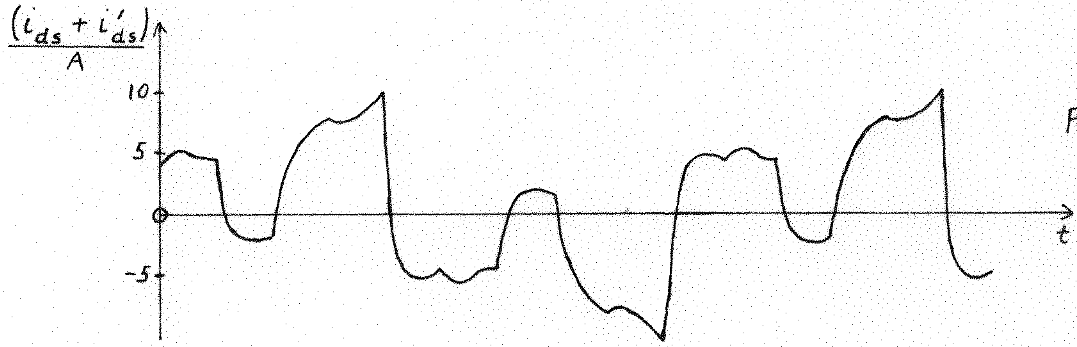


Fig. 2.9 b

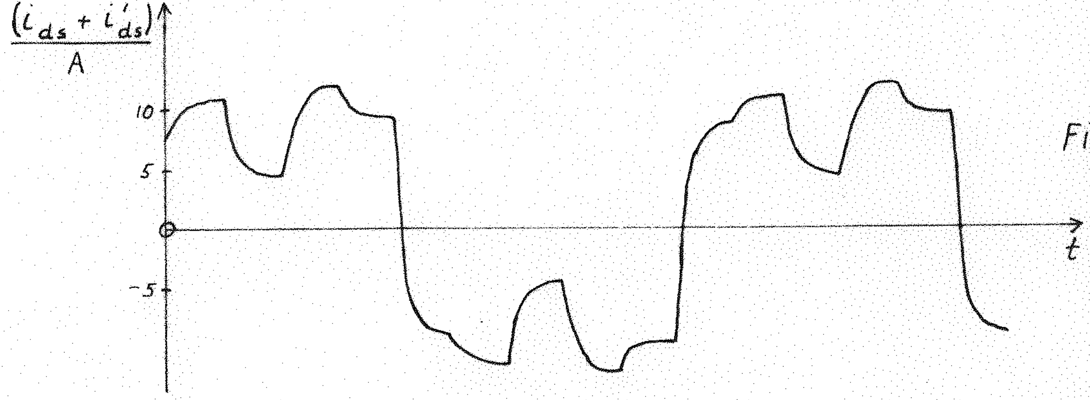


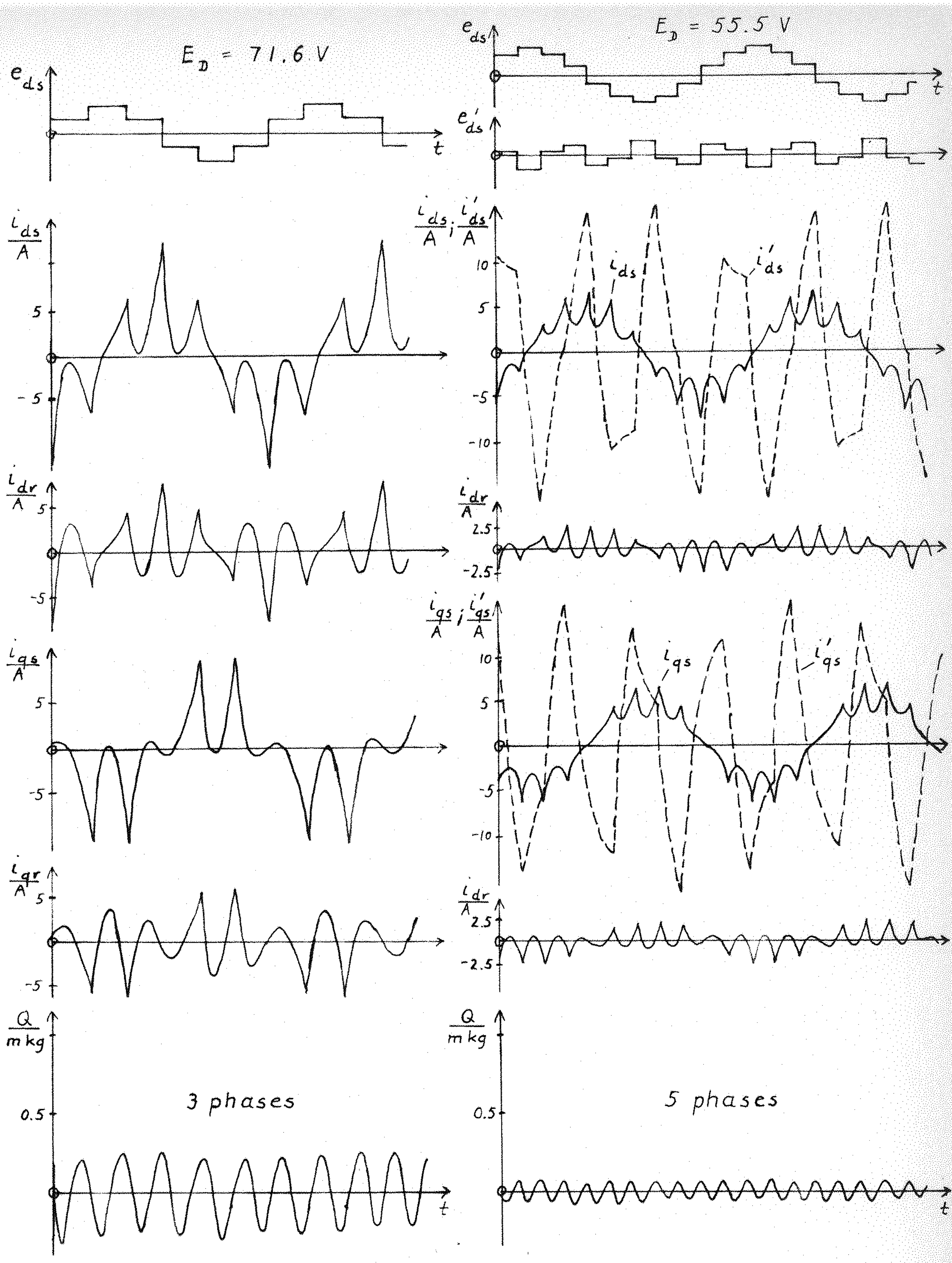
Fig. 2.10b

Fig. 2. 8 b : synchronism

Fig. 2. 9 b : 20 % slip

Fig. 2. 10 b : standstill

Five phase currents: $(i_{ds} + i'_{ds})$ at 7.5 c/s



25 c/s , synchronism

Fig. 2.11

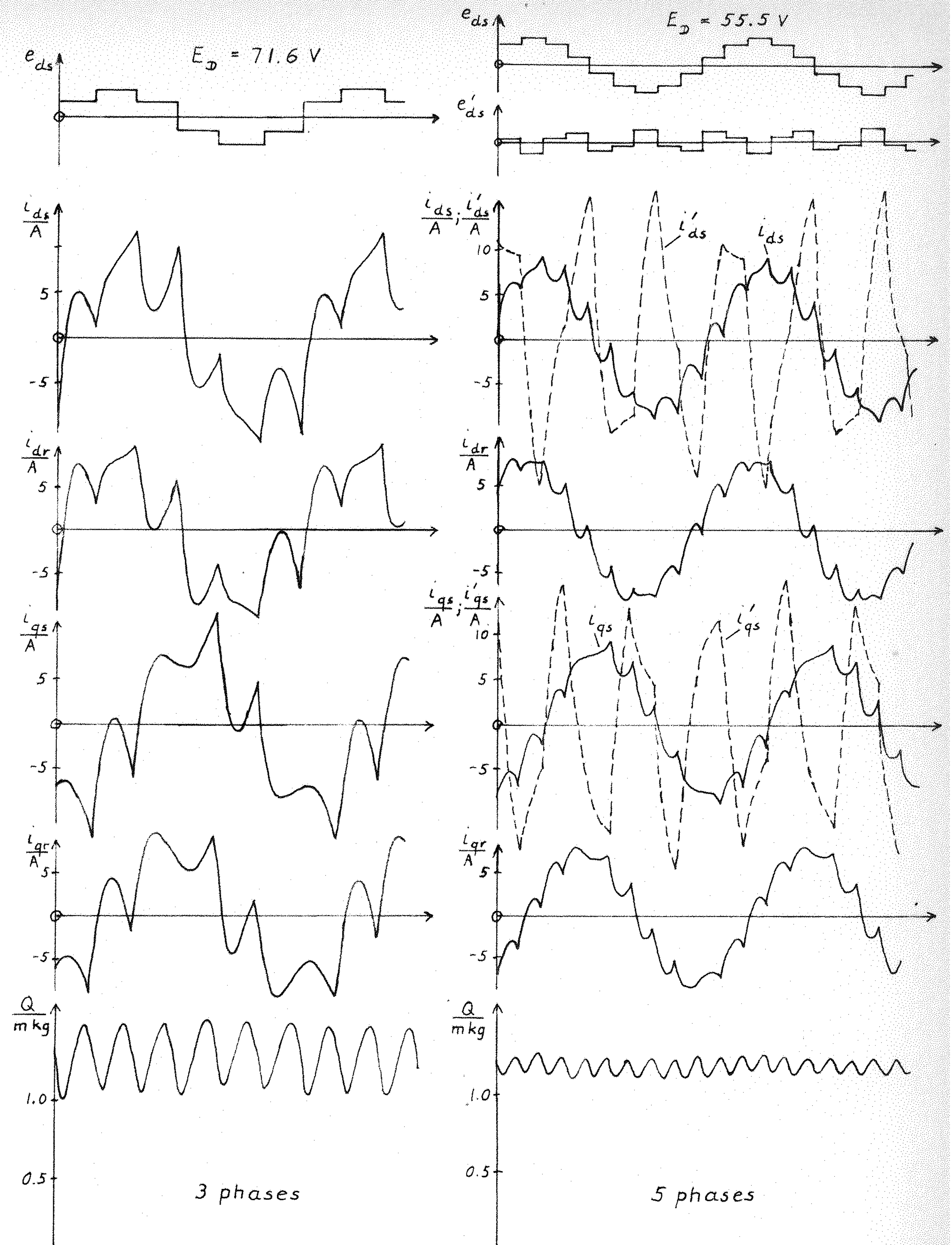
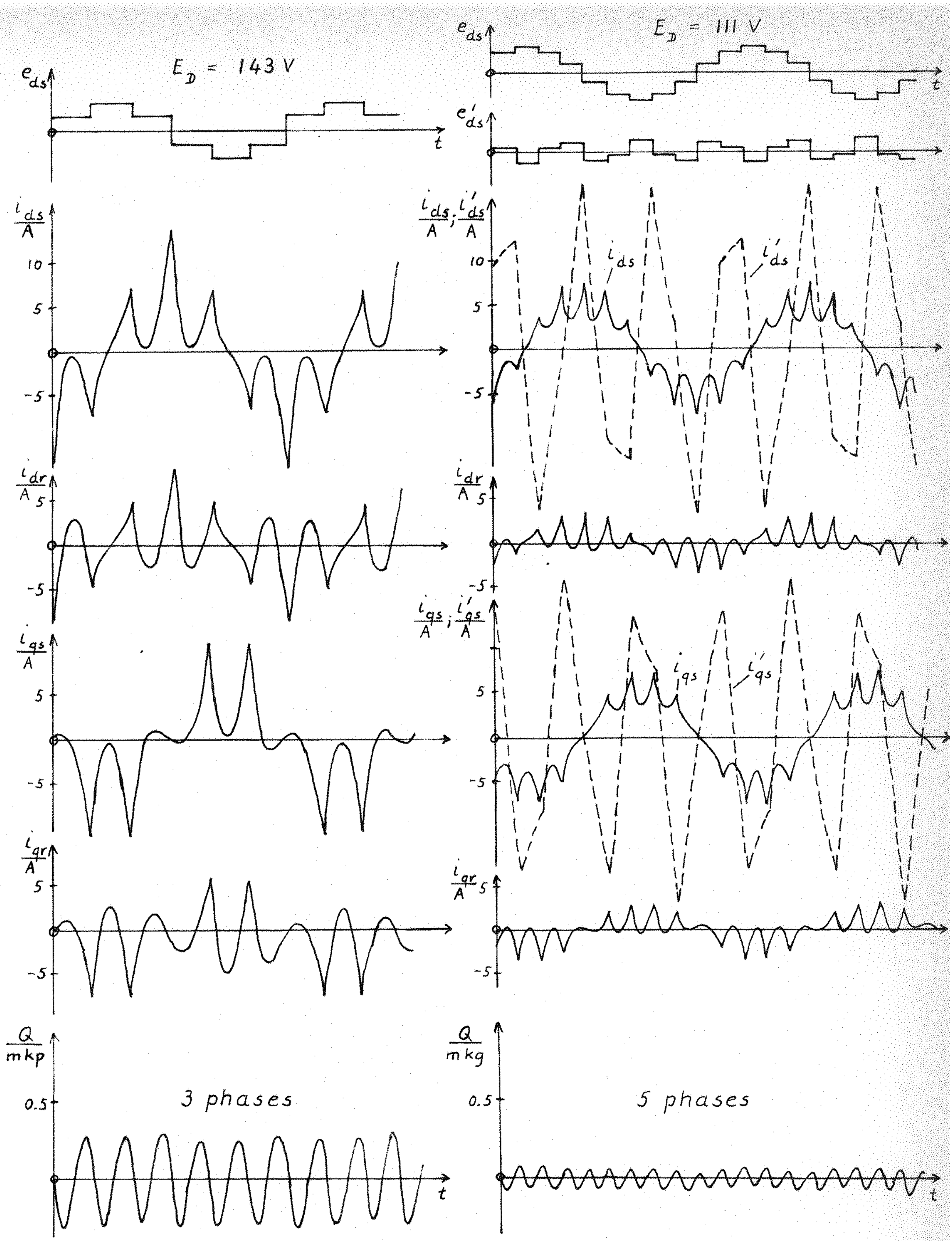


Fig. 2.12 : 25 c/s , 12 % slip



50 c/s, synchronism

Fig. 2.13

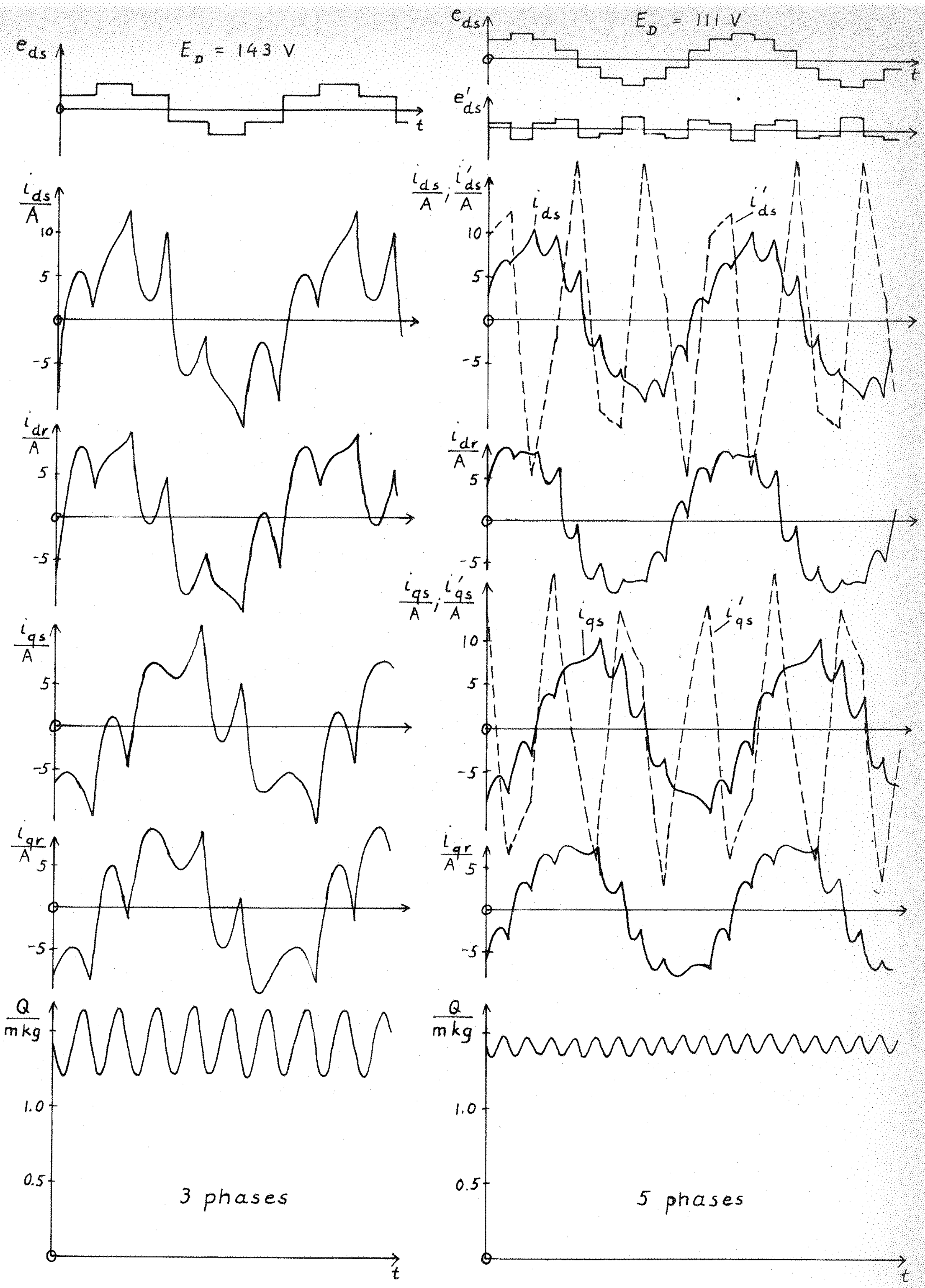


Fig. 2.14 : 50 c/s , 6 % slip

CHAPTER III

EXPERIMENTAL EXAMINATION OF A 5 PHASE INDUCTION MOTOR ON SQUARE-WAVE SUPPLY

III.1 The invertor

III.1.1 Simple series invertor with individual commutation and its disadvantages

At first, experiments have been carried out with an invertor one phase of which is given in Fig. 3.1. The commutation relies on the autotransformer action of the centre-tapped choke. A detailed investigation into the action of this circuit is given in Ref. 9.

It has been found that this invertor is unsuitable at low frequencies and for an induction motor which has a high mutual inductance and a low leakage. In order to explain this, a brief description of the events in this circuit during one half-cycle will be given here.

It is assumed that at the beginning thyristor T1 is conducting and that the load current shall be transferred to the negative busbar. If T1 conducts, the potential of the output terminal will be near the potential of the positive busbar taking into account the resistive and inductive voltage drop across the upper half of the choke. The capacitor C2 is charged up to

$$e_{C2} = 2k_v E_D \qquad (3.1)$$

$k_v < 1$ allows for the voltage drop across the choke.

Now the thyristor T2 is fired. The capacitor voltage e_{C2} appears across the lower half and, due to the autotransformer action, also across the upper half of the choke. At the first instant of the commutation, the voltage across T1 is

$$e_{T1} = -2(2k_v-1) \quad E_D \tag{3.2}$$

A short reverse current through T1 turns the thyristor off. The magnetizing current of the choke has been transferred to the lower half of the choke. It is now assumed that during the relatively short commutation time the choke magnetizing current is held constant by a sufficiently large inductance and that the load current does not change appreciably either, due to the motor inductance. The capacitors C1 = C2 = C are therefore charged by a constant current. The capacitors are virtually in parallel and their current equals $2i_L$ (i_L being the load current at the instant of commutation). Their voltage changes linearly and so does the voltage across T1:

$$e_{T1}(t) = -2(2k_v-1) \quad E_D + 2 \frac{i_L}{C} t \tag{3.3}$$

When e_{T1} reaches zero, the thyristor must be able to block a forward voltage again. The time of negative bias Δt must be longer than the turnoff time of the thyristor and include a safety margin. Eqn. (3.3) yields

$$\Delta t = \frac{C}{i_L} (2k_v-1) \quad E_D \tag{3.4}$$

The capacitance required to commutate a given current at a given voltage is

$$C = \frac{\Delta t}{\frac{E_D}{i_L}(2k_v - 1)} \tag{3.5}$$

If a given current is to be commutated at low frequencies and therefore low voltage, C will become very big. On the other hand, Δt is proportional to the voltage and would be unnecessarily long at high frequencies.

The charging of the capacitors continues until the diode D2 starts to conduct. The load current now flows through D2 and so does the magnetizing current of the choke. The magnetizing current decays with the time constant $\frac{L}{R}$ and the load current changes its polarity in due time. During the conduction time of D2, the output is clamped to the negative busbar and C1 is charged up to nearly $2E_D$. When the now reversed load current equals the circulating magnetizing current, the diode ceases to conduct. The load current flows through the choke and causes a voltage drop of

$$e = R\, i_L + L \frac{di_L}{dt} \tag{3.6}$$

The voltage across C1 is

$$e_{C1} = 2E_D - \Delta e = k_v\, 2E_D \tag{3.7}$$

The choke inductance should be big enough to maintain the magnetizing current during the commutation within certain limits. Ref. 9 gives the relation

$$L = \frac{2C}{a}\left(\frac{k_v\, E_D}{i_L}\right)^2 \tag{3.8}$$

a is the permitted change in current during commutation.

If L is smaller, the charging of the capacitors will not be linear and Δt will be shorter than indicated in Eqn. (3.4).

At low frequencies, the diode will have ceased to conduct before the next commutation occurs. As appears from the recordings of motor currents on square-wave supply, the load current always increases in magnitude shortly before commutation thus making $\Delta e > 0$. If the motor has a high inductance and a low leakage, this increase is more pronounced. Since L must not be too small, a considerable loss in commutation voltage e_{C1} can be expected. If k_v becomes as low as 0·5, commutation is impossible. If it is a little higher, say 0·6, a huge capacitance would be required and L should be accordingly large. However, a big inductance decreases k_v again. It has been found impossible to run the experimental motor on an invertor of this kind at low frequencies. At higher frequencies, the diodes would be conducting during the full half-cycle thus giving a reliable commutation voltage.

III.1.2 Invertor with auxiliary commutation voltage and common commutation of all phases

It follows from section III.1.1 that it is desirable to have an invertor whose commutation voltage does not depend entirely on the voltage of the d.c. busbars, i.e. to introduce an auxiliary commutation voltage.

In Ref. 10 a relatively simple 3 phase invertor with an auxiliary commutation voltage and common commutation of all main thyristors adjacent to the positive or negative busbar is given and its operation is described in some detail. This design has been adopted for 5 phases and a 5 phase invertor is shown in Fig. 3.2.

Since the phases are no longer independent, a detailed analysis of the operation of this circuit is rather difficult to achieve and no attempt is made here to do so. The commutation voltage, i.e. the negative voltage impressed across the thyristors during the first instant of commutation always equals the auxiliary voltage V_r. The auxiliary thyristor T_A is fired whenever one of the upper main thyristors is to be turned off at the end of its half-cycle, and the auxiliary thyristor T_B is used to turn off the lower main thyristors. The other upper or lower main thyristors which are conducting when the firing of T_A or T_B occurs are turned off as well, but they are fired again at the end of the commutation since their firing pulses last for their full half-cycle.

If, for example, T_A is fired, the currents of the upper main thyristors are transferred to the upper diodes and the capacitor C_A is charged until the diode D_- starts to conduct. At the same time C_B is charged through V_r until $e_B = V_r$ and this voltage is available for the commutation of the lower main thyristors. Thus, the commutation voltage always equals the auxiliary voltage V_r irrespective of the voltage of the d.c. busbars. There are circulating currents in all five centre-tapped chokes which speed up the charging of the capacitors. The values of the components in Fig. 3.2 have been found experimentally.

As has been mentioned above, the firing pulses of the main thyristors have to be sustained for the full half-cycle. To produce these pulses, saturable transformers have been used. They have a control winding which, when carrying a sufficient current, eliminates the magnetic coupling between the primary and secondary winding and thus suppresses the firing pulse (for details see Ref. 9). The output voltage is full-wave rectified. The control windings of the ten firing transformers are supplied by a 5 phase low power transistor ringcounter. The output voltages of the ringcounter which are a low-power pattern of the output voltages of the invertor are also used to determine the firings of the auxiliary thyristors. Fig. 3.3 shows the block diagram of the excitation.

III.2 Experimental set-up for measuring the instantaneous torque

For the measurement of the instantaneous torque of the 5 phase machine in the steady state, an inductive torque transducer (Type TG/2 of Vibrometer AG, Fribourg, Switzerland) has been used. The torsion of its shaft is used to put an inductive bridge out of balance. The output signal is amplified and is proportional to the angular twist of the transducer shaft.

The load torque of the motor is provided by a separately excited d.c. generator which works on a resistive load. In the steady state, the average torque of the motor and the average torque of the generator are equal. If the rotor of the motor is assumed to be rigid, the part of the motor torque which is not used up internally to overcome the air friction and the friction of the bearings results in a torsion of the transducer shaft and is measured correctly. The motor friction torques cannot be measured with this method and are assumed to be small.

Now, the measurement of motor torque fluctuations ΔQ_M in the steady state shall be considered. The idealised mechanical system consists of the rigid motor rotor, the shaft of the torque transducer which is rigidly coupled to the motor and generator, and the rotor of the generator. The frame of the rig is also assumed to be rigid.

With these assumptions, the following equation for the motor torque fluctuations in the Laplace transformed form can be written down:

$$J_M \; p \; \Delta\omega_M + \frac{k_s}{p} (\Delta\omega_M - \Delta\omega_G) = \Delta Q_M \qquad (3.9)$$

J_M = inertia of the motor rotor

k_s = torsional elasticity constant of the shaft

$\Delta\omega_M$, $\Delta\omega_G$ = small speed variations of motor and generator.

The inertia of the shaft is neglected.

There may also be small torque fluctuations of the d.c. generator:

$$J_G \; p \; \Delta\omega_G + \frac{k_s}{p} (\Delta\omega_G - \Delta\omega_M) = - \Delta Q_G \qquad (3.10)$$

Eqns. (3.9) and (3.10) yield:

$$\frac{\Delta\omega_M - \Delta\omega_G}{p} = \frac{\Delta Q_M + \Delta Q_G \dfrac{J_M}{J_G}}{J_M \left(p^2 + k_s \dfrac{J_M + J_G}{J_M \, J_G}\right)} \qquad (3.11)$$

$(\Delta\omega_M - \Delta\omega_G)/p$ is the angle $\Delta\alpha$ of torsion of the shaft.

If $J_G >> J_M$ and ΔQ_G is small (as can be expected from a d.c. generator in the steady state), Eqn. (3.11) can be written as

$$\Delta\alpha = \frac{\Delta Q_M}{J_M \, (p^2 + k_s \frac{1}{J_M})} \tag{3.12}$$

The natural frequency of the mechanical system is given by

$$\omega_e{}^2 = k_s \frac{J_M + J_G}{J_M \quad J_G} \approx k_s \frac{1}{J_M} \tag{3.13}$$

The torque transducer measures the torsion angle and

$$\Delta Q = k_s \; \Delta\alpha \tag{3.14}$$

is the measured torque.

Therefore,

$$\frac{\Delta Q}{\Delta Q_M} = \frac{1}{\frac{J_M}{k_s} \; (p^2 + k_s \; \frac{1}{J_M} \,)} \tag{3.15}$$

In the case of sinusoidal excitation, $p = j\omega$ (ω being the angular frequency of the torque fluctuations), and

$$\frac{\Delta Q}{\Delta Q_M} = \frac{1}{\frac{J_M}{k_s} \; (\omega_e{}^2 - \omega^2)} \tag{3.16}$$

If $\omega^2 << \omega_e{}^2$,

$$\frac{\Delta Q}{\Delta Q_M} \rightarrow 1 \tag{3.17}$$

and the torque fluctuations are measured correctly.

For the experiments, the inertia of the generator has been increased with a big flywheel between torque transducer and d.c. generator. The inertia of the flywheel has been estimated to be approximately 8 kg m^2 and the inertia of the motor rotor is appr. $4 \cdot 10^{-2}$ kg m^2.

A crude estimate of the natural frequency gives

$$\omega_e \geqq 10^3 \frac{1}{s}$$

and the corner frequency, i.e. the frequency at which

$$\frac{\Delta Q}{\Delta Q_M} = \sqrt{2}$$

is $$\omega = 0{,}54 \ \omega_e$$

The fundamental frequency of the torque fluctuations is $10\ \omega$ (ω being the fundamental angular frequency of the supply). The arrangement should allow satisfactory torque readings up to a supply frequency of appr. 8•5 c/s.

Experiments with the arrangement have shown that good torque readings could be obtained at supply frequencies of appr. 5-8 c/s. At very low frequencies, vibrations in the frame of the rig could be observed which upset the measurements.

III.3 Experimental results

Experiments with the arrangement have been carried out at low supply frequencies in order to avoid mechanical resonance which could damage the torque transducer.

Figures 3.4 .... 3.7 show recordings of the phase voltage, the phase current, and the instantaneous steady-state torque at various speeds and at a supply frequency of 7•5 c/s. There is a voltage droop at normal running conditions shortly before commutation occurs due to the current peak; the circulating current in the invertor choke has ceased and one half of the choke is in series with the motor phase.

There is a reasonable agreement between the analogue computations and the measured phase current and instantaneous torque taking into consideration the deformation of the phase voltage and other inaccuracies, such as error in torque measurement, error of analogue, and the simplifying assumptions on which the analogue computations are based.

Torque fluctuations are small compared with the nominal torque; at rated torque the peak-to-peak value of the torque fluctuations is about $\frac{1}{6}$ of the rated torque.

The phase current is strongly non-sinusoidal. Measurement of the current harmonics at 7·5 $^{c}/_{s}$ and 215 r.p.m. gave the following result (in % of the fundamental):

3rd harmonic	73%
7th harmonic	34%
9th harmonic	22%
11th harmonic	11%
13th harmonic	12%

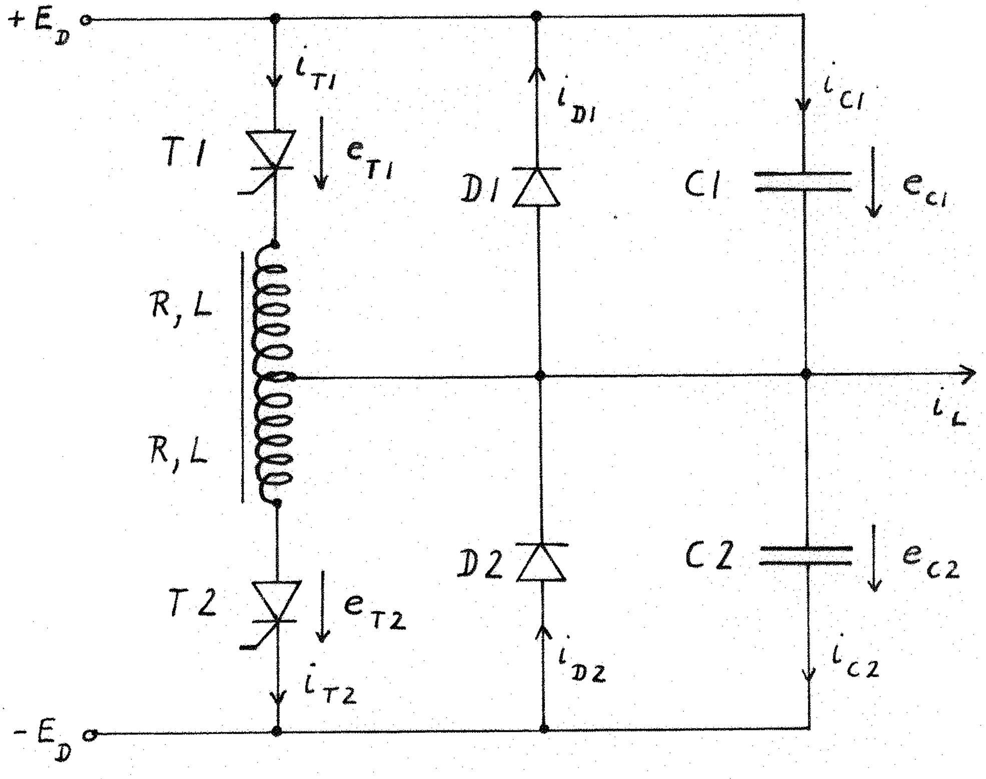


Fig. 3.1

Simple series invertor

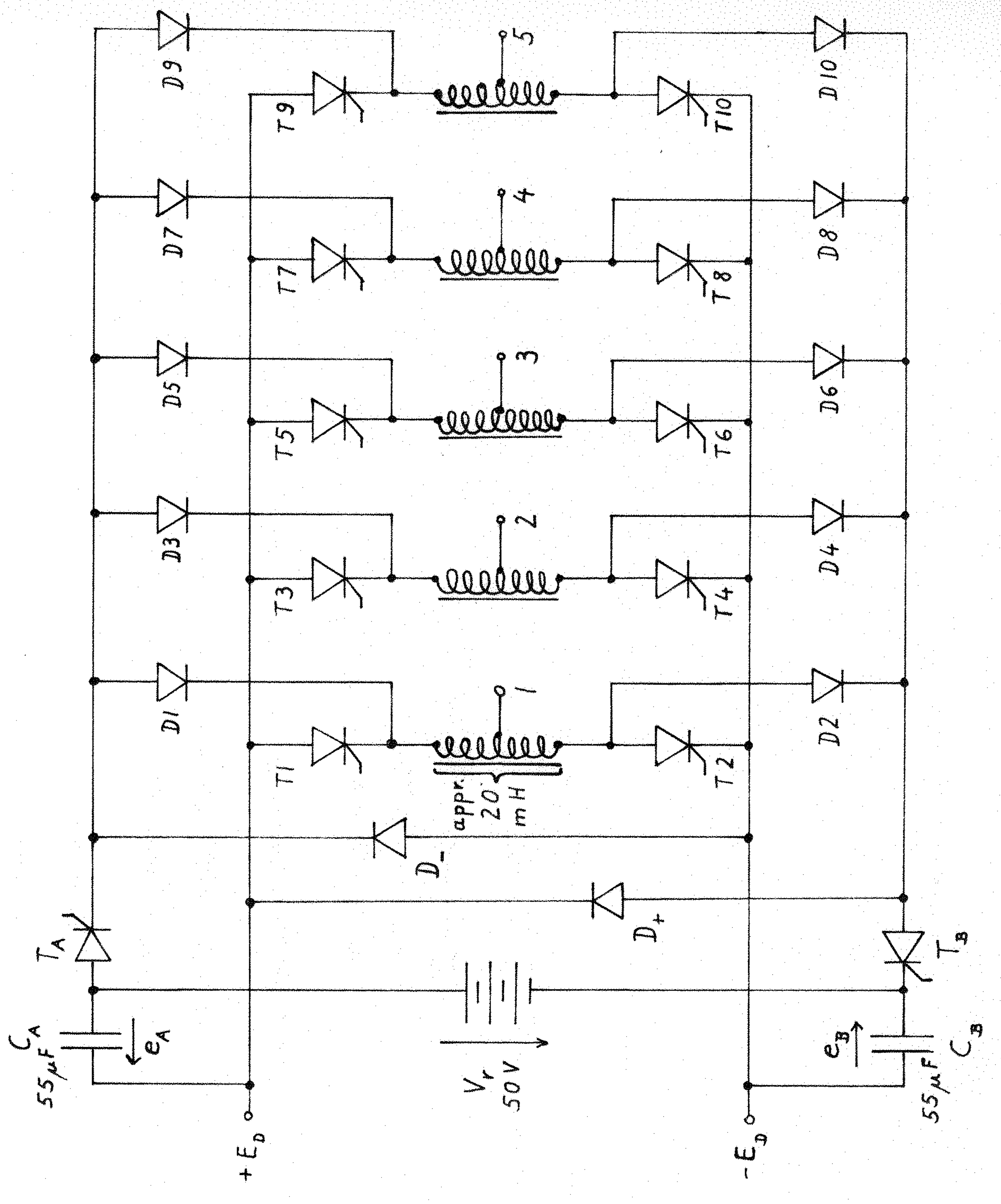


Fig. 3.2

Five phase invertor

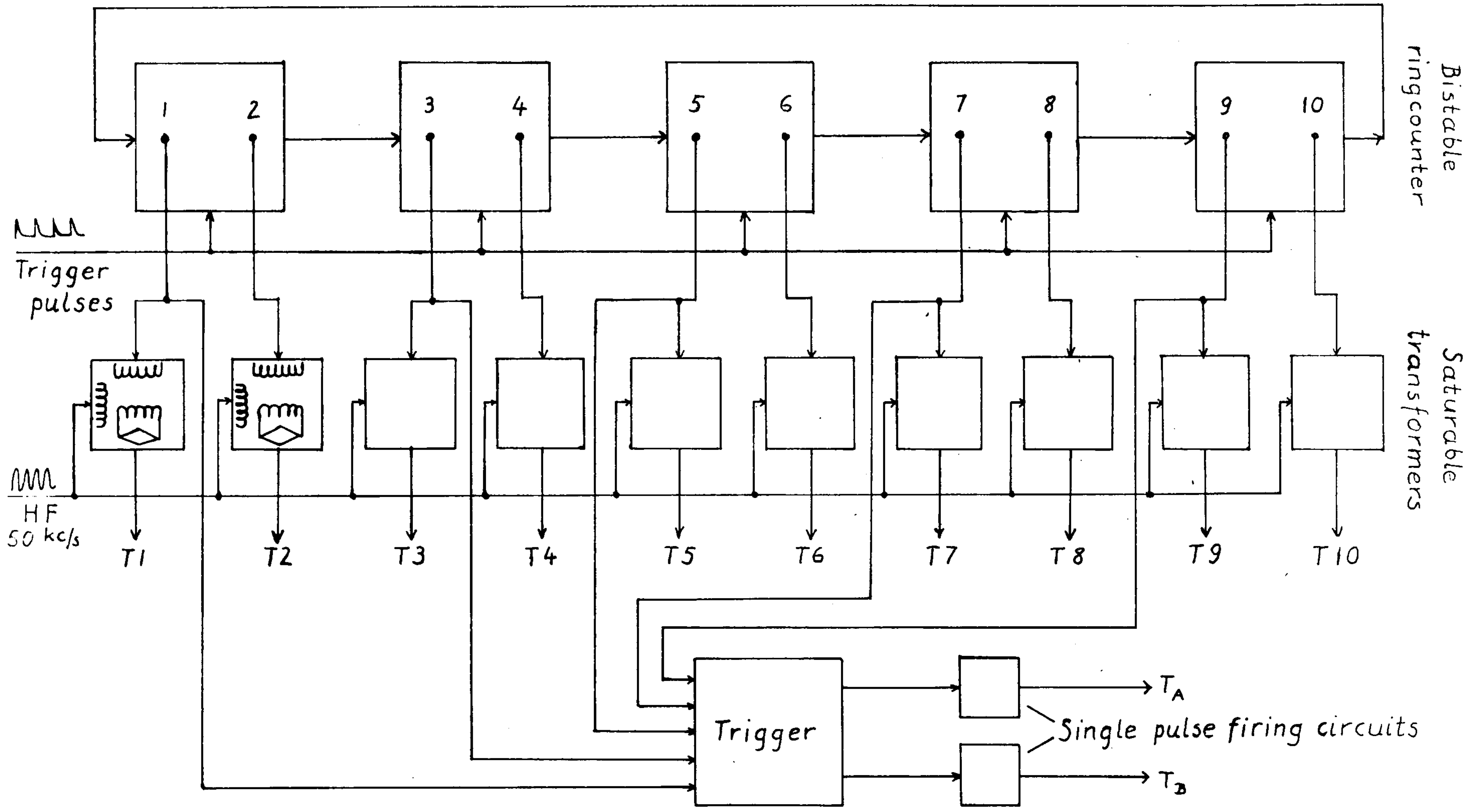


Fig. 3.3

Invertor excitation

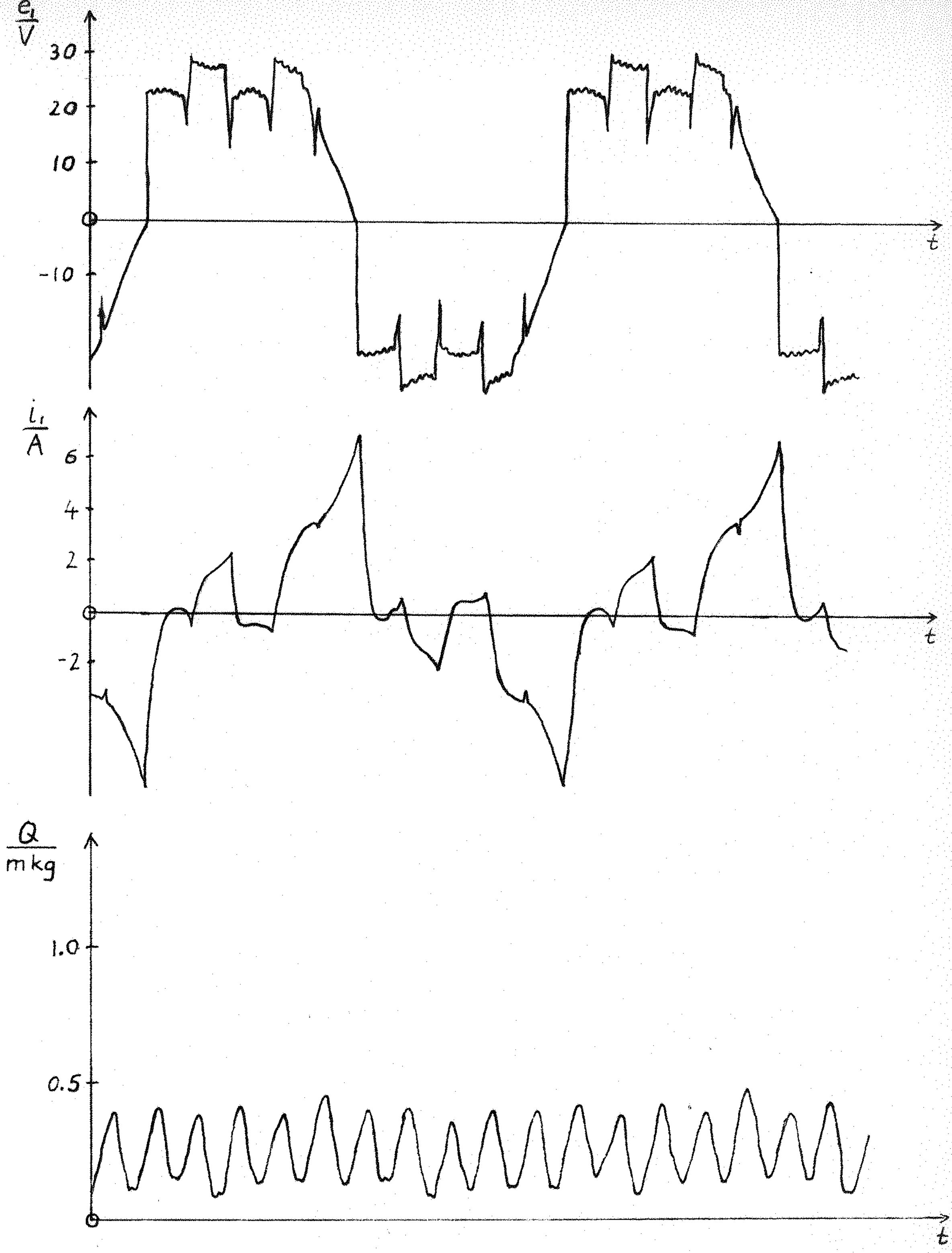


Fig. 3.4

Phase voltage, phase current, and torque

7.5 c/s, 215 r.p.m.

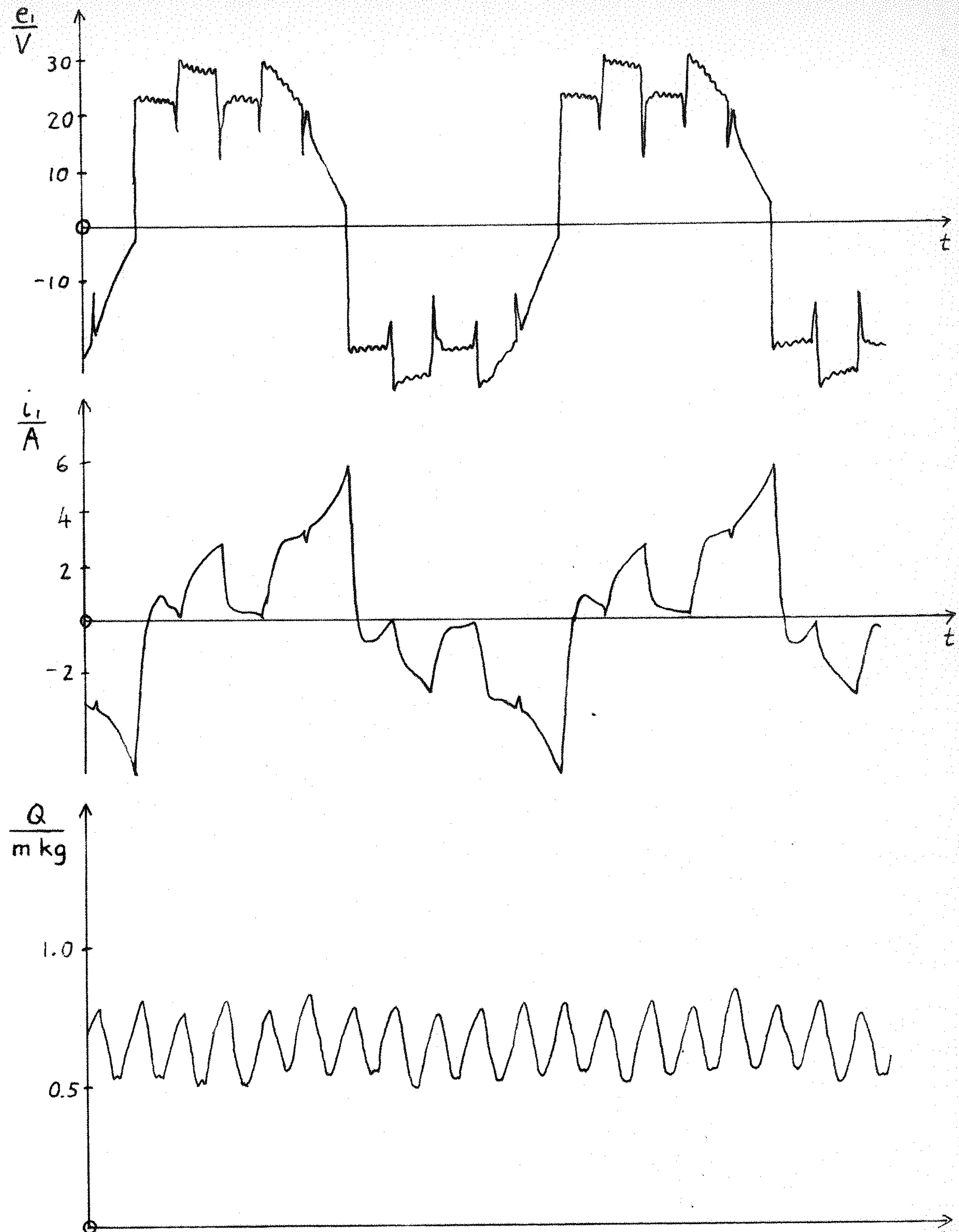


Fig. 3.5

7.5 c/s , 206 r.p.m.

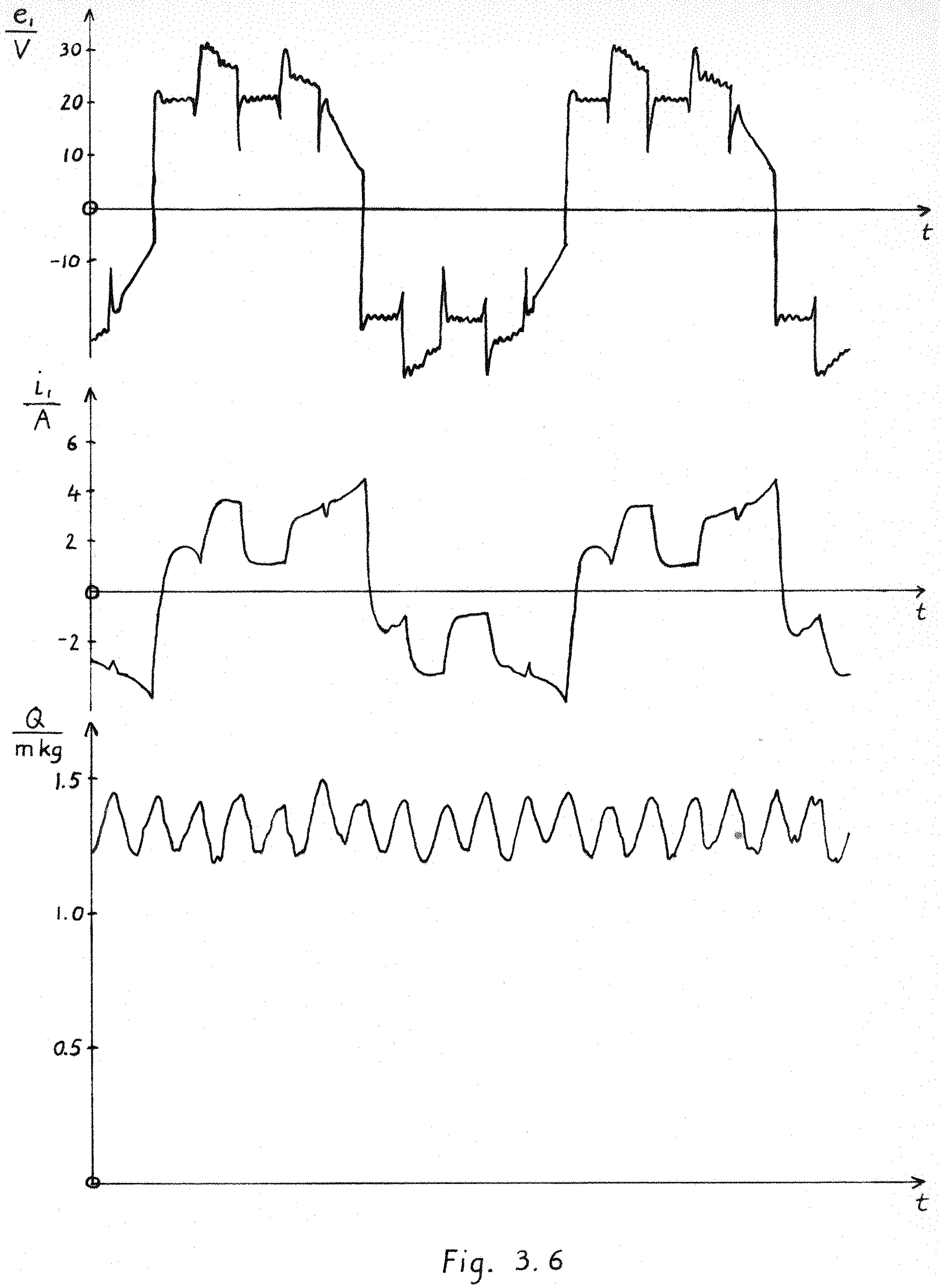


Fig. 3.6

7.5 c/s , 185 r.p.m.

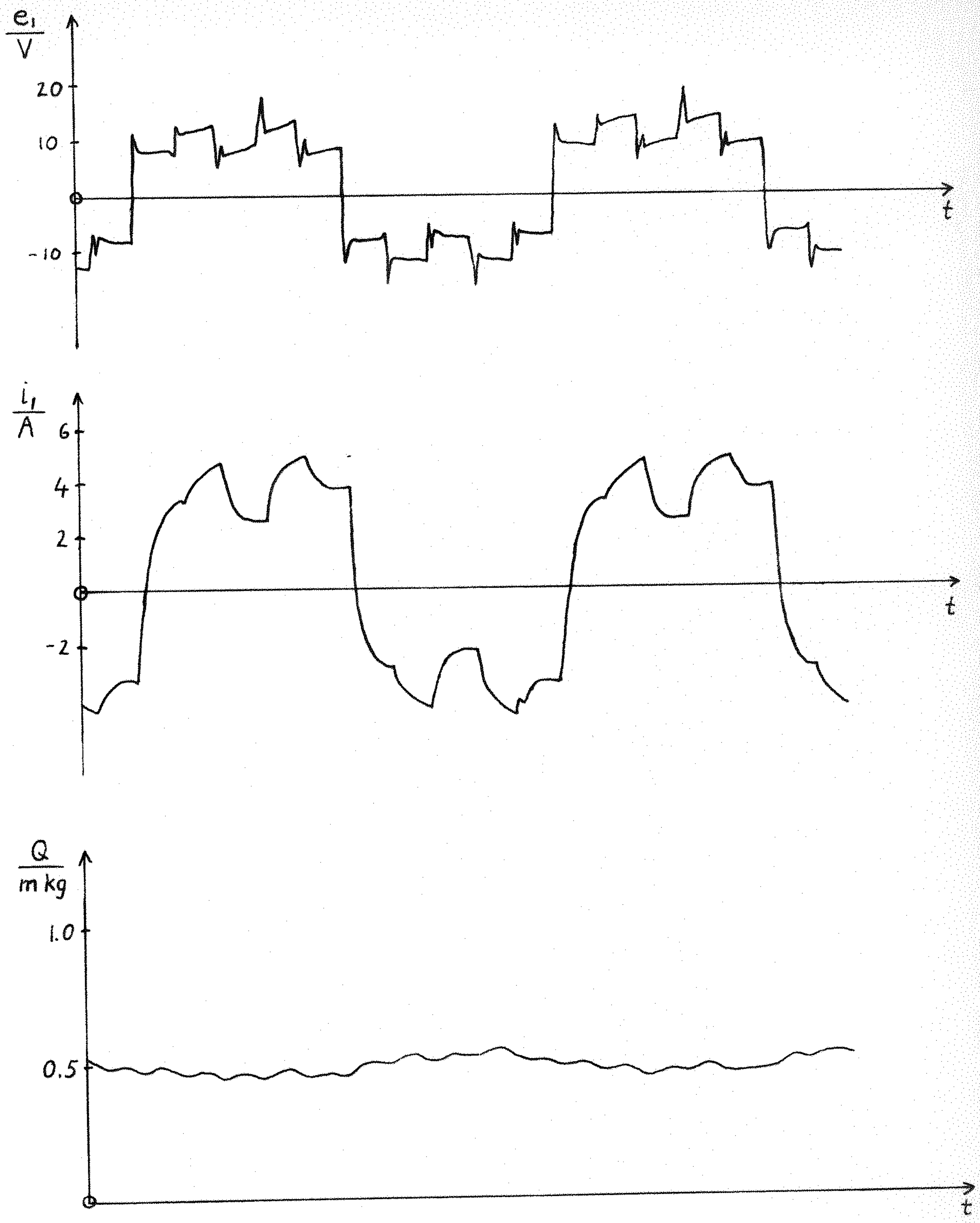


Fig. 3.7

7.5 c/s , standstill

IV. CONCLUSIONS

It has been shown that a 5 phase induction motor on square-wave supply behaves considerably better than a 3 phase machine as far as parasitic fields and torques are concerned. Torque fluctuations in the steady state are substantially reduced and their frequency is higher.

However, the stator current is strongly non-sinusoidal, especially at higher supply frequencies and therefore higher voltages. It has been seen that the stator current can be split up into two components. One component sets up rotating fields in the air-gap. The other component, however, does not lead to rotating fields and is only limited by the stator resistance and leakage impedance. It consists mainly of the 3rd current harmonic which is due to the 3rd voltage harmonic. This current component becomes very big at higher frequencies, and it reaches a peak value when the current is to be commutated. Stator losses are increased and the commutation of the invertor becomes more difficult.

It does not seem practical to run a 5 phase motor on voltages which contain an appreciable 3rd harmonic. It should, however, be possible to design a relatively simple invertor whose output voltages do not contain a 3rd harmonic. An invertor voltage waveform which meets this requirement is given in Fig. 4.1.

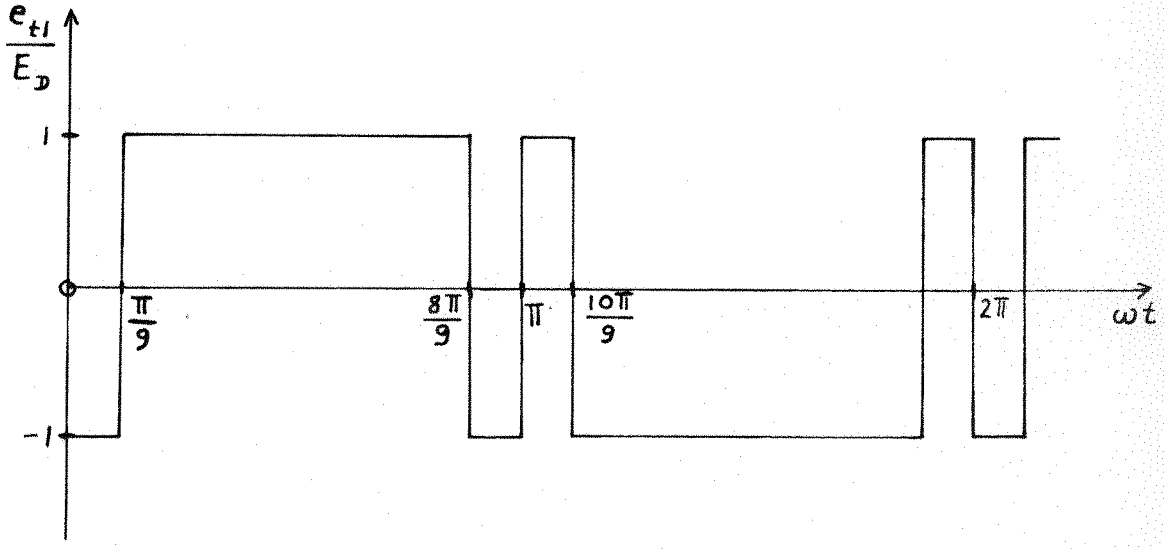


Fig. 4.1

Invertor output voltage not containing a 3^{rd} harmonic

APPENDIX I

Parameters of the experimental 5 phase machine

Connection	Star
Pairs of poles	2
Rated torque	1·4 m kg
Power	2 kW
Rated phase voltage at 50 c/s	100 V rms
Rated slip at 50 c/s	6%
R_s	1·26 Ω
R_r	1·03 Ω
ℓ_s	4·76 mH
ℓ_r	1·70 mH
M	151·5 mH

APPENDIX II

Short derivation of the 5 phase-two axes transformation

The flux vectors along the axes of the phases of a 5 phase stator winding can be added up in the following way:

$$\bar{\phi} = \phi_{1s} + a\,\phi_{2s} + a^2\,\phi_{3s} + a^2\,\phi_{4s} + a^4\,\phi_{5s} \qquad \text{(A1)}$$

The operator $a^{(n-1)} = e^{j\frac{2\pi}{5}(n-1)}$ indicates the direction of the nth phase with reference to the first phase and ϕ_{ns} is the magnitude of the phase fluxes at a given instant.

Now the complex transformation

$$\begin{bmatrix} \phi_{t1} \\ \phi_{t2} \\ \phi_{t3} \\ \phi_{t4} \\ \phi_{t5} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 & a & a^2 & a^3 & a^4 \\ 1 & a^2 & a^4 & a^6 & a^8 \\ 1 & a^3 & a^6 & a^9 & a^{12} \\ 1 & a^4 & a^8 & a^{12} & a^{16} \\ 1 & a^5 & a^{10} & a^{15} & a^{20} \end{bmatrix} \cdot \begin{bmatrix} \phi_{1s} \\ \phi_{2s} \\ \phi_{3s} \\ \phi_{4s} \\ \phi_{5s} \end{bmatrix} \qquad \text{(A2)}$$

is considered where $a = e^{j\frac{2\pi}{5}}$.

$$\left[\phi_{tn}\right] = \frac{1}{5}\,[A]\,\left[\phi_{ns}\right] \qquad \text{(A3)}$$

An inspection of the transformation matrix shows that ϕ_{t4} is the complex conjugate of ϕ_{t1}:

$$\phi_{t4} = \phi_{t1}^{*} \qquad \text{(A4)}$$

and furthermore

$$\phi_{t3} = \phi_{t2}^{*} \qquad \text{(A5)}$$

ϕ_{t5} can be regarded as a 'zero sequence' since $a^{5n} = 1$.

Physical significance can be attached to ϕ_{t1}:

$$\overline{\phi} = 5 \; \phi_{t1} \tag{A6}$$

The complex conjugate of the transpose of $[A]$ is

$$[A]_t^* = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ a^{-1} & a^{-2} & a^{-3} & a^{-4} & 1 \\ a^{-2} & a^{-4} & a^{-6} & a^{-8} & 1 \\ a^{-3} & a^{-6} & a^{-9} & a^{-12} & 1 \\ a^{-4} & a^{-8} & a^{-12} & a^{-16} & 1 \end{bmatrix} \tag{A7}$$

and

$$[A]_t^* \cdot [A] = 5 \; [I] \tag{A8}$$

$[I]$ = unit matrix

The reverse transformation is therefore given by

$$[\phi_{ns}] = [A]_t^* \; [\phi_{tn}] \tag{A9}$$

If the airgap self inductance of each phase is L_s', and the phase currents are i_{ns} ($n = 1 \ldots 5$),

$$[\phi_{ns}] = L_s' \; [i_{ns}] \tag{A10}$$

The currents are transformed as well:

$$[i_{ts}] = \frac{1}{5} \; [A] \; [i_{ns}] \tag{A11}$$

and

$$[\phi_{ts}] = L_s' \; [i_{ts}] \tag{A12}$$

Now the flux linkages can be established starting with the linkage of the stator phases with the stator flux. The component of the stator flux in the direction of the nth stator phase can be written as:

$$\psi_{nss} = \mathrm{Re}\left[\bar{\phi}\ a^{-(n-1)}\right] + \ell_s\ i_{ns} \qquad (A13)$$

$\ell_s\ i_{ns}$ denotes the leakage flux

With Eqns. (A6) and (A4):

$$\begin{aligned}\psi_{nss} &= \mathrm{Re}\left[5\ \phi_{t1}\ a^{-(n-1)}\right] + \ell_s\ i_{ns} \\ &= \frac{5}{2}\left[\phi_{t1}\ a^{-(n-1)} + \phi_{t4}\ a^{(n-1)}\right] + \ell_s\ i_{ns} \\ &= \frac{5}{2}\ L_s'\left[i_{t1}\ a^{-(n-1)} + i_{t4}\ a^{(n-1)}\right] + \ell_s\ i_{ns} \qquad (A14)\end{aligned}$$

Hence,

$$\left[\psi_{nss}\right] = \frac{5}{2} L_s' \cdot \begin{bmatrix} 1 & 1 \\ a^{-1} & a \\ a^{-2} & a^2 \\ a^{-3} & a^3 \\ a^{-4} & a^4 \end{bmatrix} \cdot \begin{bmatrix} i_{t1} \\ i_{t4} \end{bmatrix} + \ell_s \left[i_{ns}\right] \qquad (A15)$$

In the transformed form:

$$\left[\psi_{tnss}\right] = \frac{5}{2}\ L_s' \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{t1} \\ i_{t4} \end{bmatrix} + \ell_s \left[i_{tn}\right] \qquad (A16)$$

The same operation can be carried out for the rotor and the flux linkage of the rotor due to the rotor flux in the transformed form is

$$\left[\psi_{Tnrr}\right] = \frac{5}{2} L_r' \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{T1} \\ i_{T4} \end{bmatrix} + \ell_r \left[i_{Tn}\right] \qquad (A17)$$

(T denotes transformed rotor quantities)

Now the linkage of the rotor phases with the stator flux are considered. The nth phase of the rotor is in the direction $e^{j\Theta} a^{(n-1)}$ with respect to the first stator phase. The flux linkage is

$$\begin{aligned} \psi_{nrs} &= \text{Re}\left[5 \, \phi_{t1} \, e^{-j\Theta} \, a^{-(n-1)}\right] \\ &= \frac{5}{2} M' \left[e^{-j\Theta} \, a^{-(n-1)} \, i_{t1} + e^{j\Theta} \, a^{(n-1)} \, i_{t4}\right] \end{aligned} \qquad (A18)$$

M' = maximum mutual inductance between a stator phase and a rotor phase.

This can be expressed in the transformed form as

$$\left[\psi_{Tnrs}\right] = \frac{5}{2} M' \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{t1} \, e^{-j\Theta} \\ i_{t4} \, e^{j\Theta} \end{bmatrix} \qquad (A19)$$

It should be noted that only i_{t1} and i_{t4} contribute to this flux linkage.

If the rotor is ahead of the stator by the angle Θ, the stator is ahead of the rotor by $-\Theta$. The flux linkage of the stator phases due to the rotor flux can be given as

$$\left[\psi_{tnsr}\right] = \frac{5}{2} M' \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{T1} e^{j\Theta} \\ i_{T4} e^{-j\Theta} \end{bmatrix} \tag{A20}$$

The total flux linkage of the stator due to the stator and rotor is obtained from Eqns. (A16) and (A20) and is

$$\left[\psi_{tns}\right] = \frac{5}{2} \begin{bmatrix} L_s' & 0 & M' & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_s' & 0 & M' \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{t1} \\ i_{t4} \\ i_{T1} e^{j\Theta} \\ i_{T4} e^{-j\Theta} \end{bmatrix} + \ell_s \left[i_{tn}\right] \tag{A21}$$

The flux linkage of the rotor due to rotor and stator is

$$\left|\psi_{Tnr}\right| = \frac{5}{2} \begin{bmatrix} M' & 0 & L_r' & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & M' & 0 & L_r' \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{t1} e^{-j\Theta} \\ i_{t4} e^{j\Theta} \\ i_{T1} \\ i_{T4} \end{bmatrix} + \ell_r \left[i_{Tn}\right] \tag{A22}$$

If the total flux linkage of the nth stator phase is ψ_{ns}, the equation

$$e_{ns} = p\,\psi_{ns} + R_s\, i_{ns} \tag{A23}$$

can be written down.

The voltages are again transformed by

$$\left[e_{tn}\right] = \frac{1}{5}\,[A]\,\left[e_{ns}\right] \tag{A24}$$

and with Eqns. (A21) and (A20) one obtains

$$\left[e_{tn}\right] = p\left[\psi_{tns}\right] + R_s\left[i_{tn}\right] \tag{A25}$$

for the stator and similarly

$$\left[e_{Tn}\right] = p\left[\psi_{Tnr}\right] + R_r\left[i_{Tn}\right] \tag{A26}$$

for the rotor. In the case of a squirrel-cage rotor,

$$e_{nr} = e_{Tn} = 0 \tag{A27}$$

From Eqns. (A21), (A22), (A25) and (A26) the following sets of equations can be extracted:

$$\begin{bmatrix} e_{t1} \\ e_{T1}\,e^{j\theta} \end{bmatrix} = \begin{bmatrix} R_s + p\,L_s & pM \\ M(p - j\dot{\theta}) & R_r + L_r(p - j\dot{\theta}) \end{bmatrix} \cdot \begin{bmatrix} i_{t1} \\ i_{T1}\,e^{j\theta} \end{bmatrix} \tag{A28}$$

$$\begin{bmatrix} e_{t4} \\ e_{T4}\,e^{-j\theta} \end{bmatrix} = \begin{bmatrix} R_s + p\,L_s & pM \\ M(p + j\dot{\theta}) & R_r + L_r(p + j\dot{\theta}) \end{bmatrix} \cdot \begin{bmatrix} i_{t4} \\ i_{T4}\,e^{-j\theta} \end{bmatrix} \tag{A29}$$

$$\begin{bmatrix} e_{t2} \\ e_{T2} \end{bmatrix} = \begin{bmatrix} R_s + p\,\ell_s & 0 \\ 0 & R_r + p\,\ell_r \end{bmatrix} \cdot \begin{bmatrix} i_{t2} \\ i_{T2} \end{bmatrix} \tag{A30}$$

$$\begin{bmatrix} e_{t3} \\ e_{T3} \end{bmatrix} = \begin{bmatrix} R_s + p\,\ell_s & 0 \\ 0 & R_r + p\,\ell_r \end{bmatrix} \cdot \begin{bmatrix} i_{t3} \\ i_{t3} \end{bmatrix} \qquad \text{(A31)}$$

$$\begin{bmatrix} e_{t5} \\ e_{T5} \end{bmatrix} = \begin{bmatrix} R_s + p\,\ell_s & 0 \\ 0 & R_r + p\,\ell_r \end{bmatrix} \cdot \begin{bmatrix} i_{t5} \\ i_{T5} \end{bmatrix} \qquad \text{(A32)}$$

where:

$$\begin{aligned} L_s &= \frac{5}{2}\,L_s' + \ell_s \\ L_r &= \frac{5}{2}\,L_r' + \ell_r \\ M &= \frac{5}{2}\,M' \end{aligned} \qquad \text{(A33)}$$

Since $e_{Tn} = 0$, $i_{T_2} = i_{T_3} = i_{T_5} = 0$.

For a rotor reference frame which is fixed in space and coincides with the stator reference frame, $\Theta = 0$.

It can be seen that Eqn. (A29) is the complex conjugate of (A28) and (A31) is the complex conjugate of (A30) since $e_{t_4} = e_{t1}{}^*$, $i_{t_4} = i_{t1}{}^*$ etc.

Now the real and imaginary quantities can be separated, real quantities corresponding to d-axis quantities and imaginary quantities being q-axis quantities. If this is carried out for Eqns. (A28), (A29) and (A30), (A31) and if the axes quantities

$$\begin{aligned} e_{ds} &= k'\,(\mathrm{Re}\ e_{t1} + \mathrm{Re}\ e_{t4}) \\ e_{qs} &= k'\,(\mathrm{Im}\ e_{t1} - \mathrm{Im}\ e_{t4}) \\ e'_{ds} &= k'\,(\mathrm{Re}\ e_{t2} + \mathrm{Re}\ e_{t3}) \\ e'_{qs} &= k'\,(\mathrm{Im}\ e_{t2} - \mathrm{Im}\ e_{t3}) \\ e_{o} &= k'\,e_{t5} \end{aligned} \tag{A34}$$

(and similarly for the currents) are introduced, Eqns. (2.9) and (2.10) will be the result. The factor $k' = \sqrt{\frac{5}{2}}$ provides invariance of power. The transformation Eqn. (2.12) is obtained from (A24) and (A34). The reverse transformation (2.14) is the result of similar considerations.

Torque is caused by the interaction of flux produced by the stator and linked with the rotor, and the rotor currents. The torque on the nth rotor phase is

$$Q_n = i_{nr} \frac{d}{d\Theta_M} (\psi_{nrs}) \tag{A35}$$

where Θ_M = mechanical angle and $\Theta_M = \frac{\Theta}{P}$. The total torque is the sum of the phase torques and with Eqn. (A13) it can be given by

$$Q = P\,M\,\left[i_{nr}\right]_t \cdot \begin{bmatrix} 1 & 1 \\ a^{-1} & a \\ a^{-2} & a^2 \\ a^{-3} & a^3 \\ a^{-4} & a^4 \end{bmatrix} \cdot \frac{d}{d\Theta} \cdot \begin{bmatrix} i_{t1}\,e^{-j\Theta} \\ i_{t4}\,e^{j\Theta} \end{bmatrix} \tag{A36}$$

$$Q = 5\,P\,M \begin{bmatrix} i_{T4} & i_{T1} \end{bmatrix} \cdot \begin{bmatrix} -j\, i_{t1}\, e^{-j\Theta} \\ j\, i_{t4}\, e^{j\Theta} \end{bmatrix} \tag{A37}$$

and $\Theta = 0$. If the transformed currents are split up in real and imaginary parts, for example

$$\begin{aligned} i_{t1} &= \frac{1}{2k'}\,(i_{ds} + j\, i_{qs}) \\ i_{t4} &= \frac{1}{2k'}\,(i_{ds} - j\, i_{qs}) \end{aligned} \tag{A38}$$

and the multiplication is carried out, Eqn. (2.11) will be the result.

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ABSTRACT

FACULTY OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

Master of Philosophy

THE BEHAVIOUR OF A FIVE PHASE INDUCTION MOTOR SUPPLIED BY AN INVERTOR

by Helmut Härer

If a three phase induction motor is supplied by a simple series invertor which produces a set of square-wave voltages, relatively strong torque fluctuations in the steady state will be the result. The present work is an attempt at reducing the torque fluctuations due to square-wave supply by increasing the number of phases to five.

A method of analysing the steady-state behaviour of three phase and five phase induction motors is given which is based on the generalized machine theory. The two-axes transformation for five phase induction motors is derived in the Appendix. An electronic analogue of the induction motor is described which may be used for convenient investigation. Parasitic fields and torques are discussed and a comparison of three phase and five phase machines is given where possible.

Experiments with a five phase invertor and motor have been carried out and the instantaneous torque in the steady state has been measured.