THE DESIGN AND USE OF GAUGES IN LIFE TESTING

by

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-1986-

To my dearest,

Father : J. S. Batti

&

Mother : A. D. Mantong

CONTENTS

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF MATHEMATICAL STUDIES

OPERATIONAL RESEARCH

Master of Philosophy

THE DESIGN AND USE OF GAUGES IN LIFE TESTING

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Gauges define classes and the use of gauges leads to the observation of frequencies in the defined classes. The advantages of gauges over exact measurement are simplicity, speed of observation, and the possibility of automa tion.

The use of gauges in industrial life testing of items is explored with the Weibull distribution particularly in mind.

The issue of the time value of a gauge is discussed for the case of one and two gauges. The context is the need to make decisions about the goodness of a large batch of items. Single and Double Acceptance Sampling plans for making the necessary decisions are discussed.

The choice of one or two gauges and the type of sampling plan is essentially an economic issue. Appropriate cost functions can help in the quest for good solution.

The progress made in this study can act as a foundation for further work. Some suggestion are made for further work.

Acknowledgments

It is my deep pleasure to record my debt to my supervisor Dr. A.K. Shahani for the help and guidence he has given me during the course of this research.

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Chapter 1.

INTRODUCTION.

The useful ness of a gauge in some applications statistics has long been realized. It can be used for any measurable quantity that can be arranged in an order. The use of a gauge leads to a classification of the observations. Hence, the observations are frequency counts in each class rather than exact measurements. We use these observed frequencies in the analysis. Since the method is based on frequency counts we can have a simple statistic and we would expect it to retain most of the robustness of the test statistic. These two attractive features motivated us to study of the use of gauges.

As a simple example, we consider a single dimension of a manufactured item. We suppose that the length of this item, X, is important and because of the inherent variability, X has to be regarded as a random variable. In practice the form of the probability density function of X will be known, or it will be assumed. Suppose $f(x;\theta)$ is the density function and the parameter,(or parameters), θ is unknown. Observations $x_1,x_2,\ldots\ldots\ldots,x_n$ will be made to make inferences about θ and the concern could be with estimating θ or with testing hypotheses about θ.

With exact measurements $x_1,x_2,\ldots\ldots\ldots,x_n$ we would work with a suitable test statistic in order to make inferences about θ.

With gauges we have the possibility of one or more gauges. With one gauge, set at length L, we would simply note the number of obsevations that are less than or equal to L. Thus in the sample of n, we would have an observation on the random variable Y which is the number of X values$\leq$ L. The following Figure illustrate one gauge for a single dimension.

No. of values $\leq$L is R_1 | No. of values > L is $(n-R_1)$

length.

L

It will be readily appreciated that observing R_1 is much simpler than collecting the measurements $x_1, x_2, \ldots\ldots\ldots\ldots x_n$.

A more general example is the case of k gauges in several dimensions. A sample of n is obtained on a random vector $\underline{X}$ and the k gauges result in the observation of the frequencies in defined classes. For a two dimensional vector with two gauges for each of the dimension we get classes as indicated in Figure 1.1.

Several statistical theories based on the gauge method have been developed. Steven (1948) has used this method for estimating the mean and the standard deviation of a normal distribution. He investigated the use of both symmetrical and asymmetrical gauges. Shahani (1969) has used gauges for testing hypotheses about correlation coefficient in a bivariate normal case. He showed that the test which is based on the frequency counts is a substantial improvement over the medial test. Taj Hirji and Shahani (1978) have used the technique for testing hypotheses about the mean of a

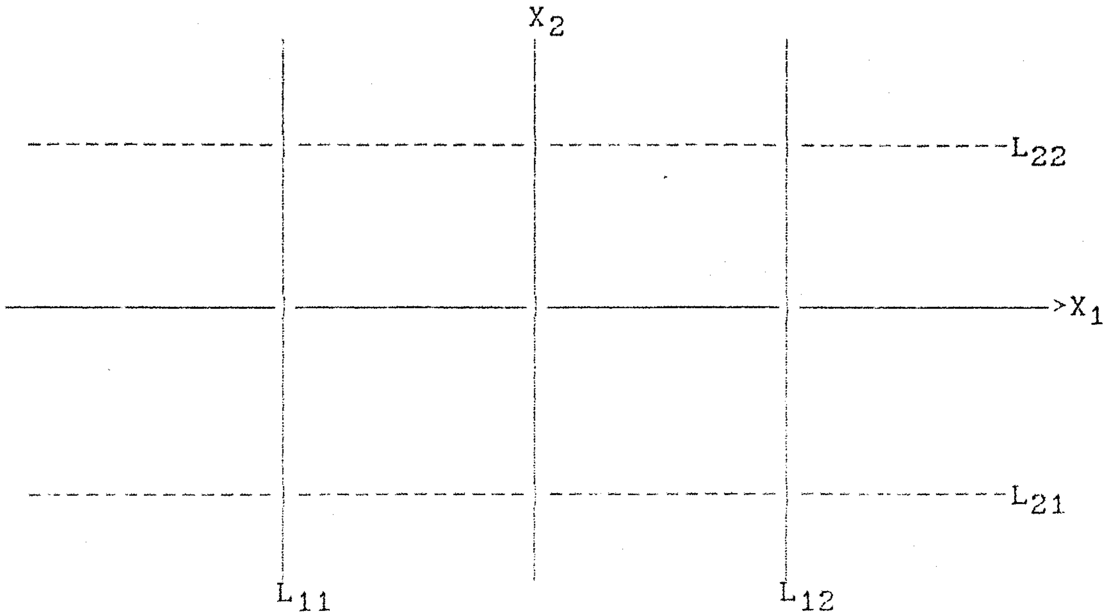

Fig.1.1 Gauges for a two dimensional random vector. For X_1, the gauges are set at L_{11} and L_{12}. For X_2, the gauges are set at L_{21} and L_{22}.

normal distribution. With an extensive numerical investigation they suggest that for a symmetrical position of two gauges about the population mean and sample size $n \geq 20$, should be adequate for the use of a normal approximation. Taj Hirji (1979) pointed out that working with the exact probability distribution of the statistic based on gauges may not be a practical proposition. He also considered the use of gauges in the sequential tests for the mean of a normal distribution and testing hypotheses about means of two related variables.

In quality control where it is easy to collect large a number of samples but difficult to obtain accurate measurement the use of gauges has obvious application. Where the absolutely accurate measurement of an item is not required a gauge method should be suitable. In a factory for example, it may be easier to operate a gauge by mechanical devices without the intervention of an operator. It is possible to construct a device that will take measurements on a series of items and record the results. It would be even easier to construct a machine which has only to classify and record the items into pre-defined classes. Therefore we can apply the technique to a fully automatic quality control system to replace nonproductive inspection work.

Steven (1948) suggest a wide variety of a gauge that can be used on anything whose values can be arranged in a serial order, even if it is not measureable. He also used the two gauges to construct the control chart. Tippett (1944) pointed out that the efficiency of a gauge method depends very much on the setting of the gauge. He pointed out that if we desired to control only the average diameter, a one gauge may be made so that 50% of the items have a large diameter (defective). Then if the frequency distribution of the diameter is approxmately normal, a control chart of the fraction defective based on a sample of about 160 items gives as good a control chart of actual mean based on measurements of sample 100 items. He also pointed out that it is more economical to gauge 160 items than to measure and calculate the mean for 100 items.

In this thesis, we consider the design and use of gauges in the life time tests. The design of gauges is concerned with the setting of gauges to the optimal position where the criteria is the maximisation of the power of test. The value of gauge is the fixed time at which the optimal posistion of gauge occurs. However, for practical purposes we prepare to use the probability term rather than a fixed time to express the position of gauges. We also consider the application of gauges in an acceptance sampling plan.

In Chapter 3, we consider the design and use of one gauge and two gauges for testing hypotheses about mean life time T of a Weibull distribution. Since the exponential distribution is a special case of the Weibull distribution it will be covered in this study. Numerical investigation for various values of shape parameter suggests that we can probably take values of about $p=0.65$ and $p_1=p_3=0.30$ as values for one gauge and two gauges, respectively. However in the practical purposes, since the power of test is quite pretty flat around the optimal position then there is probably room for compromise in setting the gauges. Furthermore, when the power of test is large enough,(say greater or equal to 0.999) we may be able to reduce either the lengthtime of observation or the sample size. Alternatively, we may be able to reduce both of them at the same time.

The application of gauges in acceptance sampling plans is considered in Chapter 4. It is shown that for a given producer's risk α and consumer's risk β we may have a number of plans; i.e pairs of n and T, to satisfy the required conditions. The minimum of the sample size is achieved when the gauges are set around the optimal position. It is also shown that in order to determine the plan that satisfies the required conditions in a double sampling case then we can set the gauges as in a single sampling plan. It will probably be easier to predict the parameters required for a double sampling plan by reference to the solution in a single sampling plan and use a random walk diagram. It is shown that the first rejection number C_2 will depend on the chosen value of n_1 proportional to n and the total sample n_t will depend on the second acceptance number C_3.

The results in Chapter 5 suggests that it seems quite reasonable to expect that the efficiency of test would further improve if more gauges are used. With reference to the number required it shown that the efficiency of gauging relative to exact measurement is over 70%. It is also shown that the R test when based on two gauges, is of relatively high efficiency and more robust than when based on the one gauge. However, in some cases it will probably be more economical to use one gauge. Perhaps when the probability distribution is exactly known or well predicted then it might be a practical proposition to use one gauge, particularly when the cost per unit time is quite large.

Further developments of the methods based on gauging are suggested in Chapter 6.

Chapter 2.

BASIC PROBABILITY FUNCTIONS

2.1. General Comments.

In general, the lengthlife or life time of an item, a device, or a system is a random variable. If we use the symbol T to denote the life time, then like any other random variable, t has a probability density function. In practice, the form of the probability density function is often assumed and the parameters involved are estimated from the appropriate data.

Also, the distribution of lifetime, can be described by the other functions such as the survivors function (S) and the the failure rate function (h). In practice, the survivor function gives the proportion of items surviving longer then time t, and failure rate function gives the proportion of items surviving in an interval per unit time, given that they have survived at the beginning of the interval.

There are many physical causes which influence the life time of items. However, it is very difficult to isolate these physical causes, hence choosing a theoretical distribution to approximate the distribution of life data is a difficult process. It may be that some of the conditions of the experiment are simply unknown or cannot be controlled. For example, two light bulbs may have been manufactured by the same process and used under the same general conditions but still fail at different times. In this case the phenomena can only described in probability terms.

Several theoretical distributions have been widely used to describe the survival time phenomena. Amongst the most important distributions are the Weibull and Ramberg distributions. These distributions are characterized by three and four parameters, respectively.In general, those parameters are known as location, scale and shape parameters. Since it is always possible to choose many different values for those parameters, a wide variety of curve shapes is possible with these distributions. For example when the Weibull shape parameter approximates 3.25, Makino (1984), then the Weibull density function is quite similar to the normal density function. When the shape parameter of Weibull distribution equal to 1 and 2, then the resulting distributions are known as Exponential and Rayleigh distributions, respectively. Similarly, Ramberg and Schimeiser have shown that Ramberg distribution can also provide good approximations to other well known distributions. For example Ramberg distribution can be consider as a normal distribution when location parameter is zero, scale parameter is 0.1975 and shape parameter is 0.1349.

In the following sections we study of the effect of the shape parameter on the shape of the density function and the failure rate function.

2.2. Weibull Distribution.

This distribution was sugggested by Weibull (1951) and it has been used in a wide variety of applications.The probability density function of a random variable T having three-parameter Weibull distribution given by:

$$f(t)=\beta/\theta\{(t-\delta)/\theta\}^{\beta-1}\exp[-\{(t-\delta)/\theta\}^{\beta} \;;\; 0\leq\delta\leq t,\ 0<\theta,\beta$$

where, the parameters θ, β and δ are referred to as scale, shape and location parameter, respectively. The survivor function and the failure-rate function are, respectively :

$$S(t)=\exp[-\{(t-\delta)/\theta\}^{\beta}]$$

$$h(t)=\beta/\theta\ \{(t-\delta)/\theta\}^{\beta-1}$$

And, the cum ulative distribution function is given by:

$$F(t)=1-\exp[-\{(t-\delta)/\theta\}^{\beta}]$$

As we have mentioned in the beginning of this chapter, we are interested in exploring the relationship between shape parameter and failure rate function. For the sake of simplicity, we take the particular case where the Weibull location parameter δ has been assumed to be zero. However, when δ has a non-zero value, all that is necessary is to substract the value of δ from the value of t, in order to get the correct value for t.

Figures 2.1a and 2.1b show the probability density function and the falulure rate function with shape parameter $\beta=\{0.2, 0.6, 1, (.5), 2.5\}$. As can be seen from these Figures the density function has no mode and decreases monotonically when $\beta \leq 1$ and the distribution is unimodal when $\beta > 1$. When $\beta = 1$ the failure rate remains constant as time increases and this is the exponential case. The failure rate decreases when $\beta < 1$ and increases when $\beta > 1$ as time t increases. Therefore the Weibull distribution can be used for the lifetime distribution of a population with decreasing, constant, or increasing failure rate.

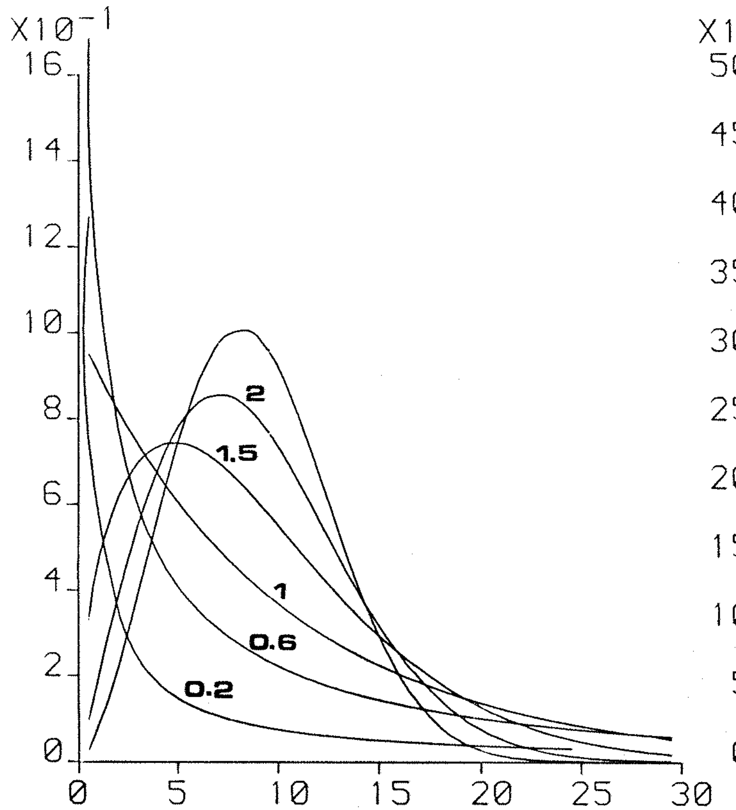

Fig.2.1a.Density Function

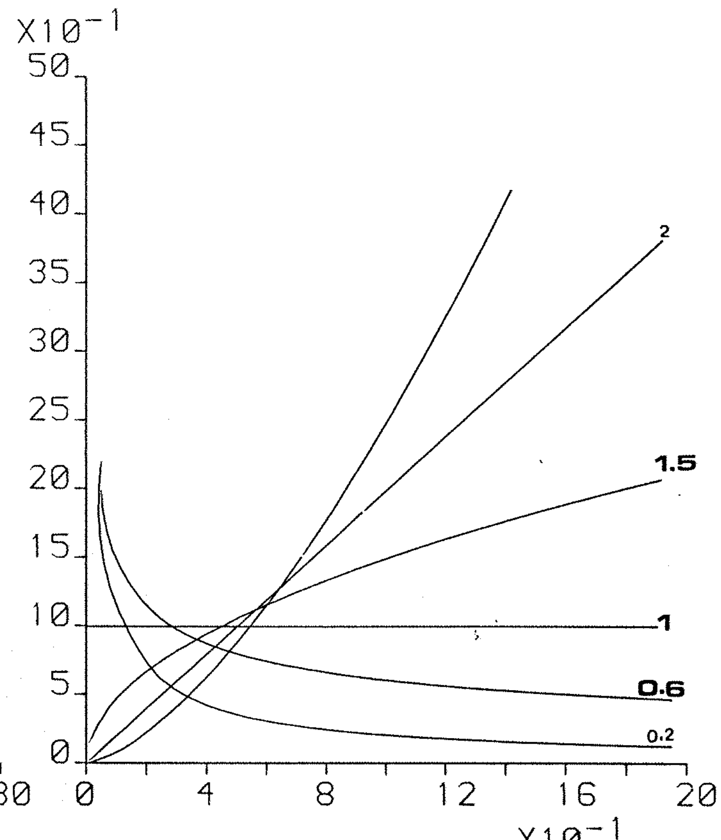

Fig.2.1b.Failure Rate Function

2.3. Ramberg Distribution.

Ramberg distribution is a generalization of Tukey's lambda distribution. It was developed by Ramberg and Schmeiser (1979) to a four-parameter distribution defined by the percentile function. Hence, Ramberg distribution is defined in the terms of the inverse of its distribution function, which is here denoted by I. The inverse function is given by:

$$I(p)= L_1+\{\frac{p^{L_3}-(1-p)^{L_4}}{L_2}\} \quad ; \; 0\leq p\leq 1, L_2\neq 0$$

where L_1 is a location parameter, L_2 is a scale parameter and L_3 and L_4 are shape parameters.

The probability function is given by:

$$f(t)= \frac{L_2}{L_3p^{L_3-1}+L_4(1-p)^{L_4-1}} \qquad ; \; 0\leq p\leq 1$$

hence, I(p)=t. The lower and upper bounds of t are, I(0) and I(1), respectively. The density function can be graphed by letting p take any values between zero and one, plotting f(t) versus I(p). The density function is symmetrical about L_1 when L_3=L_4, hence the mean of Ramberg distribution is equal to L_1, which is not true in unsymmetrical case. In general the mean of Ramberg distribution is given by:

$$\mu=L_1+\left(\frac{1}{L_3+1}-\frac{1}{L_4+1}\right)/L_2 \; ; \; L_2\neq 0$$

As in Weibull distribution, we would be interested in the effect of shape parameter to the shape of the density function and failure function.

Now, consider L_3 and L_4 as coordinates. Figure 2.1 shows the four regions of the shape parameter values. For the reference purposes these regions we numbered as 1,2,3 and 4, repectively. In each region we have indicated where the density function is a valid one in the sense that density function f(t) is nonnegative for all values of t. In the region 1 and 3 the density function is for all values of L_3 and L_4. On the other hand the density function is nonnegative when $L_3<-1$ and $L_4>1$ in the region 2 and when $L_3>1$ and $L_4<-1$ in region 4. The U-shape of distribution is also possible when $1\leq L_3, L_4\leq 2$ and the uniform distribution occure when $L_3=L_4=1$ and 2.

In region 1 the distribution has a negative skewness when $L_3<L_4$ and has a positive skewness when $L_3>L_4$ except the sub-region where given by:

$$\begin{cases} (L_3-1)^2+(L_4-1)^2 \geq 1 \\ 0\leq L_3 \leq 1 \\ 0\leq L_4 \leq 1 \end{cases}$$

In this sub-region the density function has a positive skewness when $L_3<L_4$ and has a negative skewness when $L_3>L_4$.

As an illustration the probability density function of Ramberg distribution has been plotted for several values of L_3 and L_4 with $L_1=1$ and $L_2=0.1$ in region 1 and $L_2=-0.1$ in region 2, 3 and 4.

Figures 2.2a and 2.2b show the Ramberg density function and failure rate function for the values of L_3 and L_4 as shown in Table 2.1.

Table 2.1.Values of L_3 and L_4 for Fig.2.2a and Fig.2.21b.

Curve No.	1	2	3	4	5
L_3	0.05	0.10	0.20	0.30	0.35
L_4	0.35	0.30	0.20	0.10	0.05

Figures 2.3a and 2.3b show the Ramberg density function and failure rate function for the values of L_3 and L_4 as shown in Table 2.2.

Table 2.2.Values of L_3 and L_4 for Fig.2.3a and Fig.2.3b.

Curve No.	1	2	3	4	5
L_3	0.25	0.50	0.75	1.00	1.25
L_4	1.25	1.00	0.75	0.50	0.25

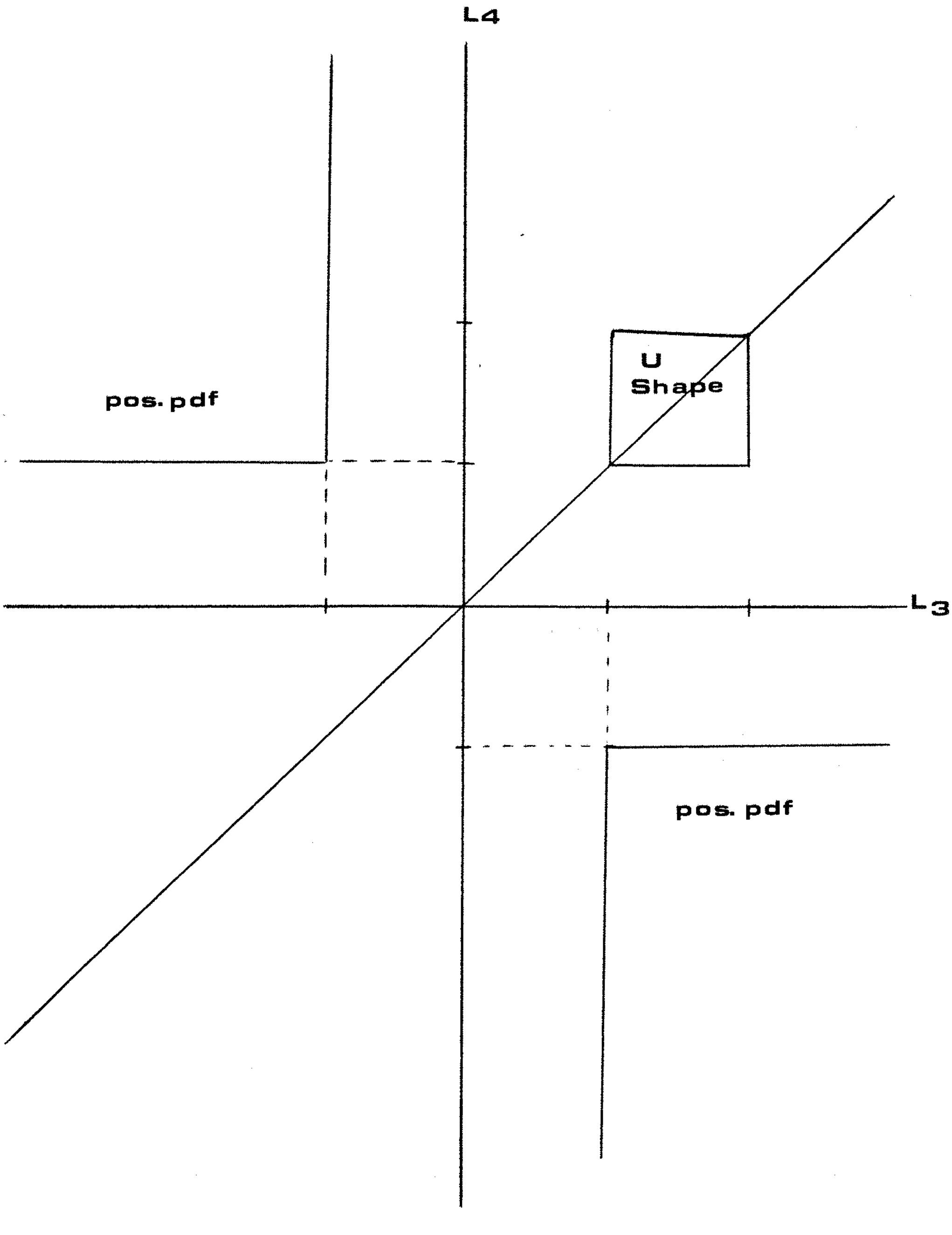

Fig.2.1 The four regions of the shape parameter values

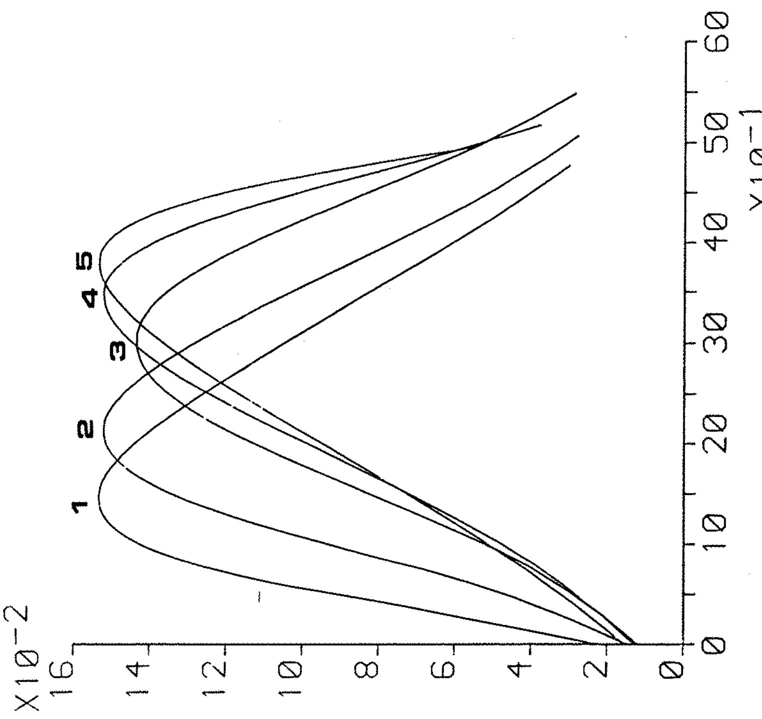

Fig.2.2a.Density function

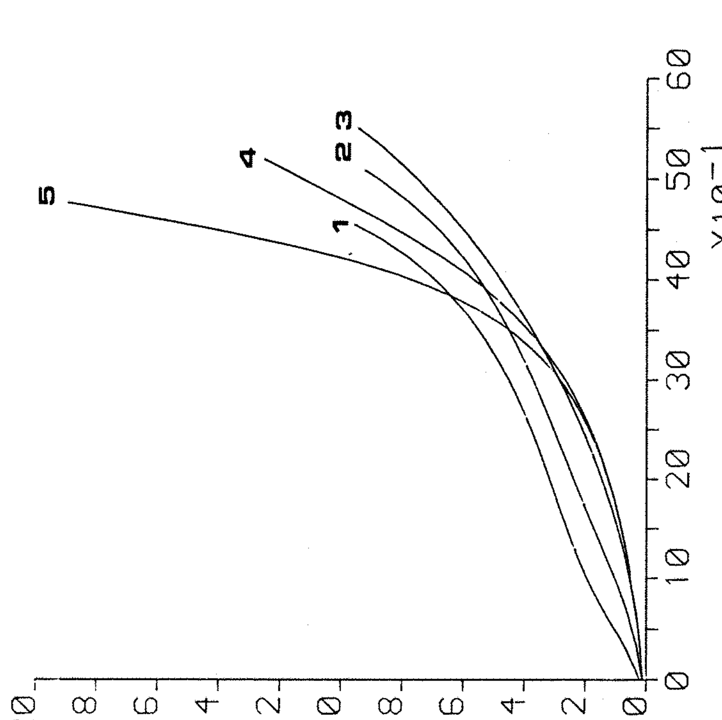

Fig.2.2b.Failure rate function

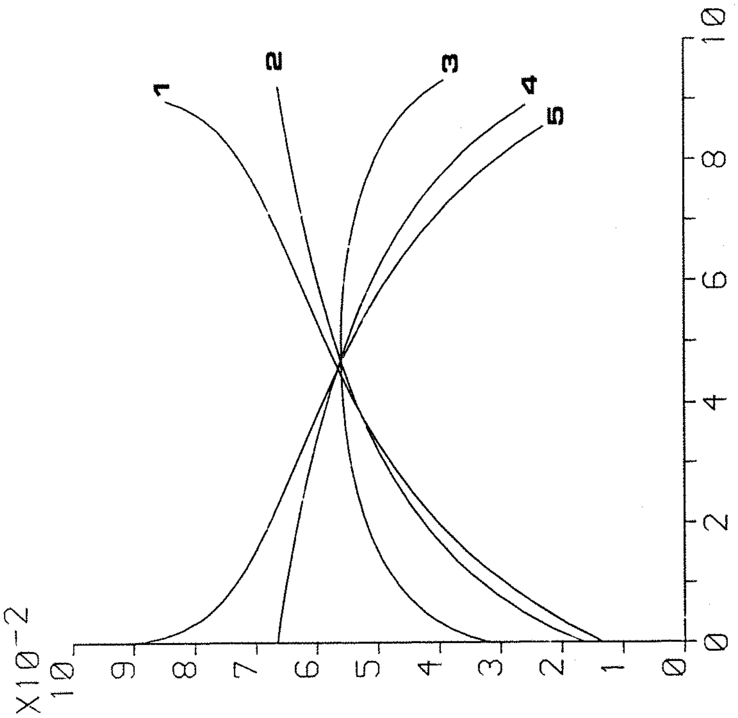

Fig.2.3a.Density function

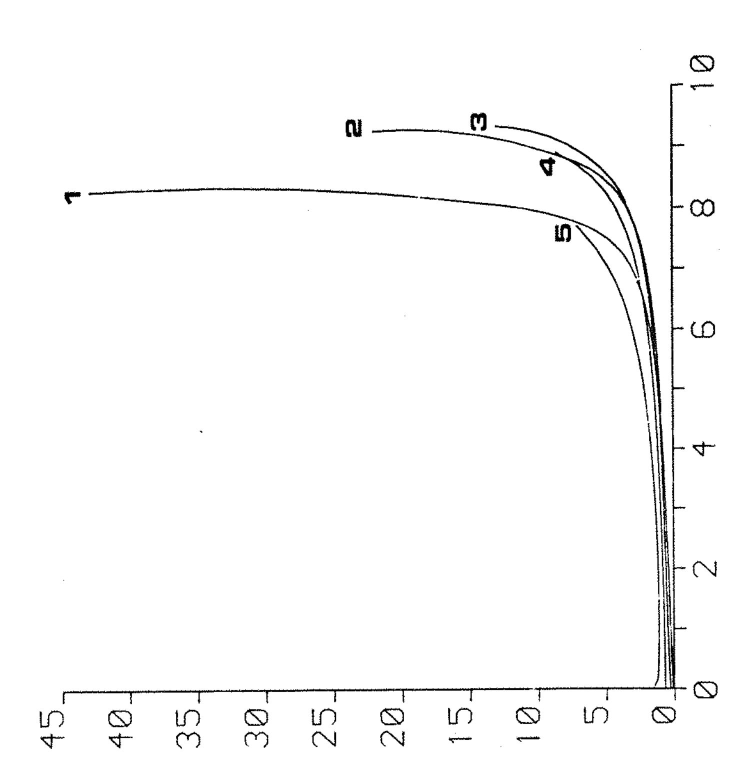

Fig.2.3b.Failure rate function

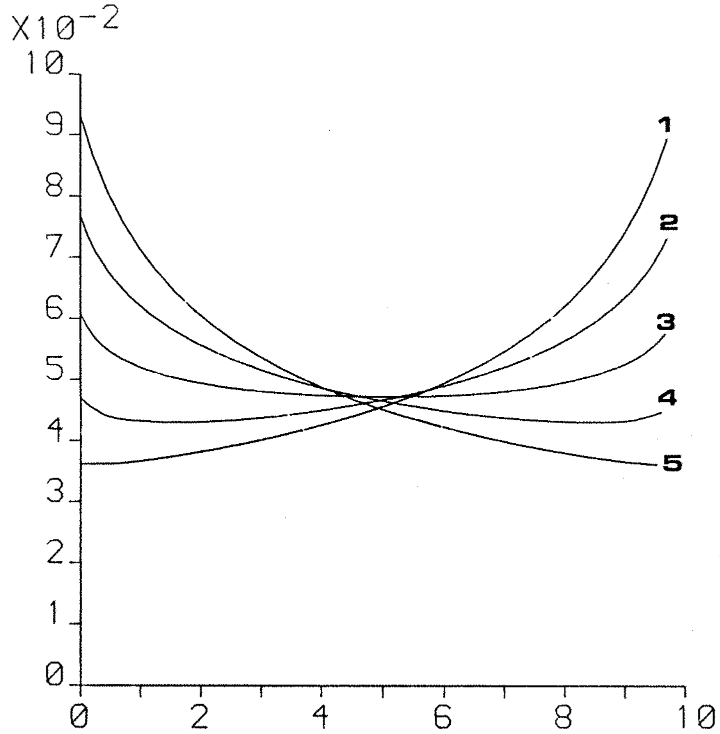

Fig.2.4a.Density function

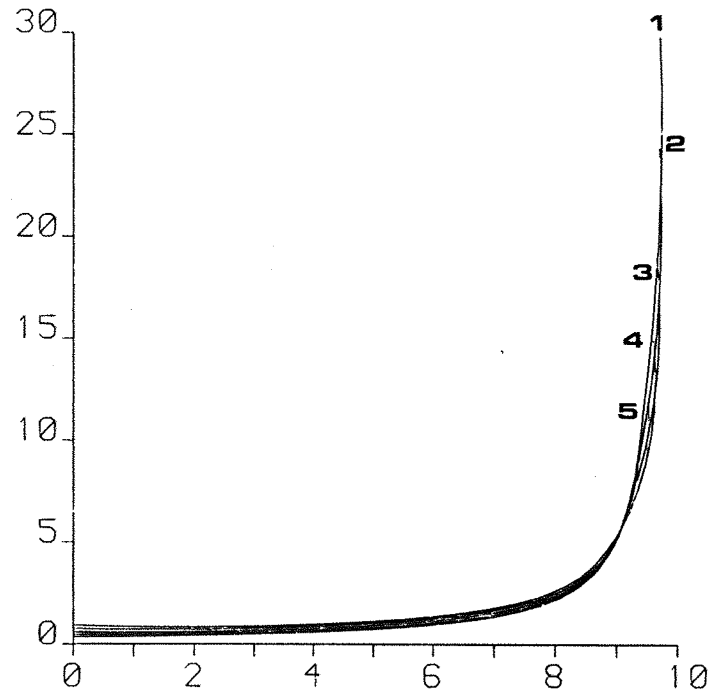

Fig.2.4b.Failure rate function

Figures 2.4a and 2.4b show the Ramberg density function and failure rate function for the values of L_3 and L_4 as shown in Table 2.3.

Table 2.3. Values of L_3 and L_4 for Fig.2.4a and Fig.2.4b.

Curve No.	1	2	3	4	5
L_3	1.05	1.25	1.50	1.75	1.95
L_4	1.95	1.75	1.50	1.25	1.05

Figures 2.5a and 2.5b show the Ramberg density function and failure rate function for the values of L_3 and L_4 as shown in Table 2.4.

Table 2.4. Values of L_3 and L_4 for Fig.2.5a and Fig.2.5b.

Curve No.	1	2	3	4	5
L_3	1.70	2.10	2.50	2.90	3.30
L_4	3.30	2.90	2.50	2.10	1.70

In region 3 the density function has a positive skewness when $L_3 > L_4$ and has a negative skewness when $L_3 < L_4$ with a very long tail to the right and to the left, respectively. Also, the mode of distribution shifted very fast to the right as L_3 decreases and L_4 increases. Figures 2.6a and 2.6b show the Ramberg density function and the failure rate function for the values of L_3 and L_4 as shown in Table 2.5.

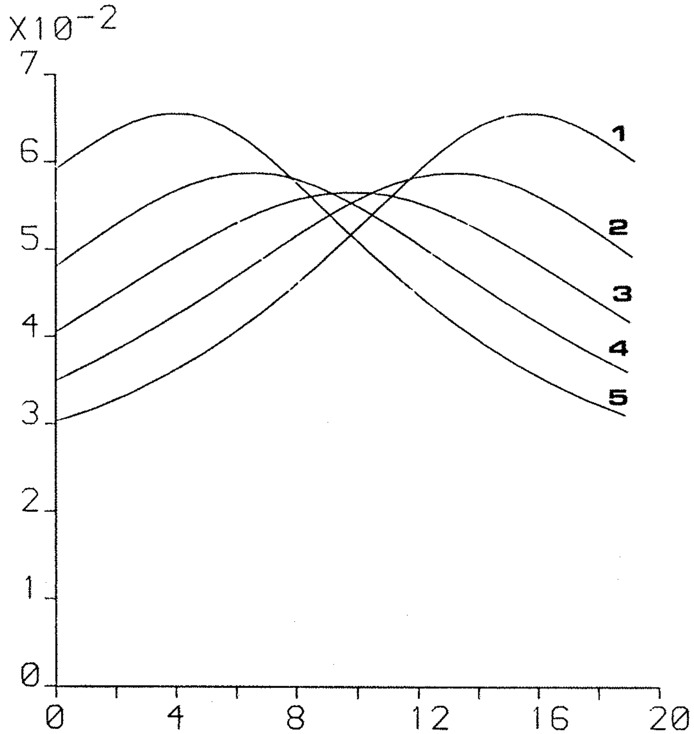

FIG.2.5a.Density function

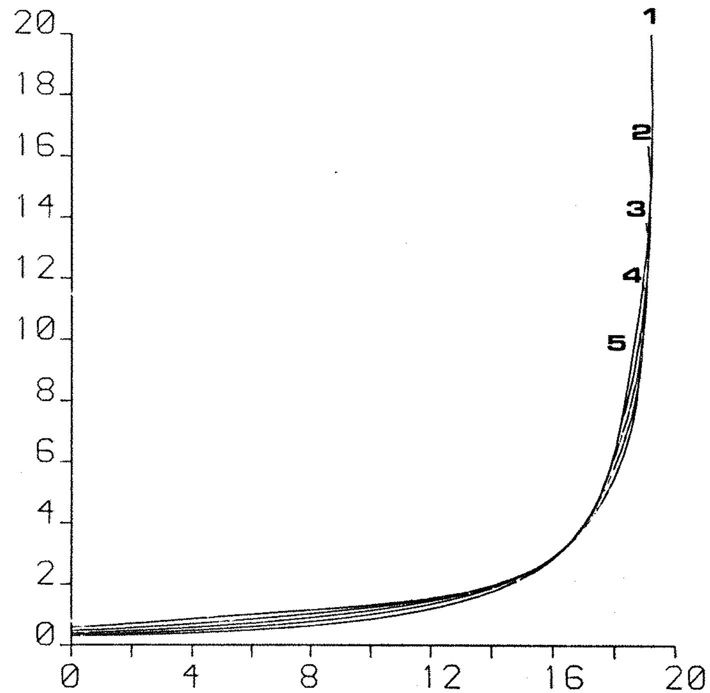

FIG.2.5b.Failure rate function

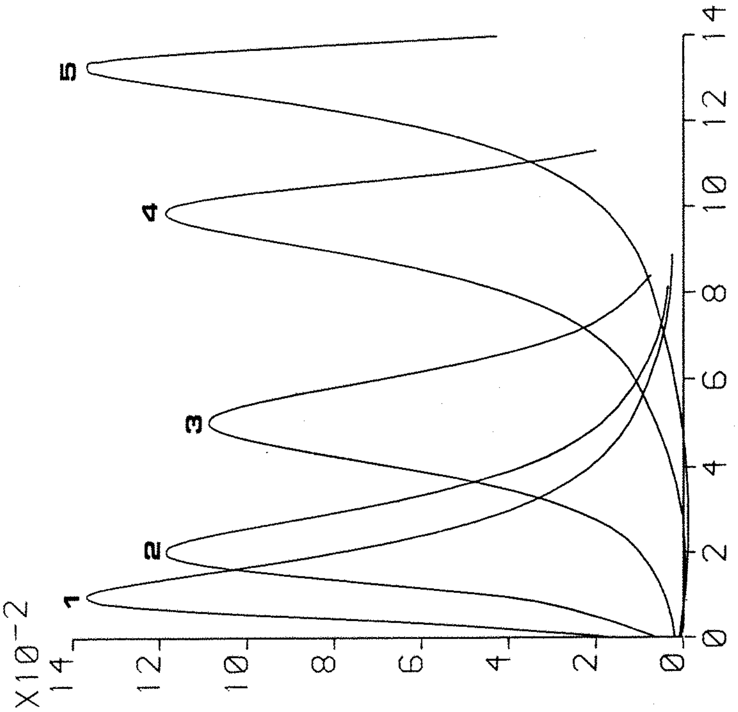

Fig.2.6a.Density function

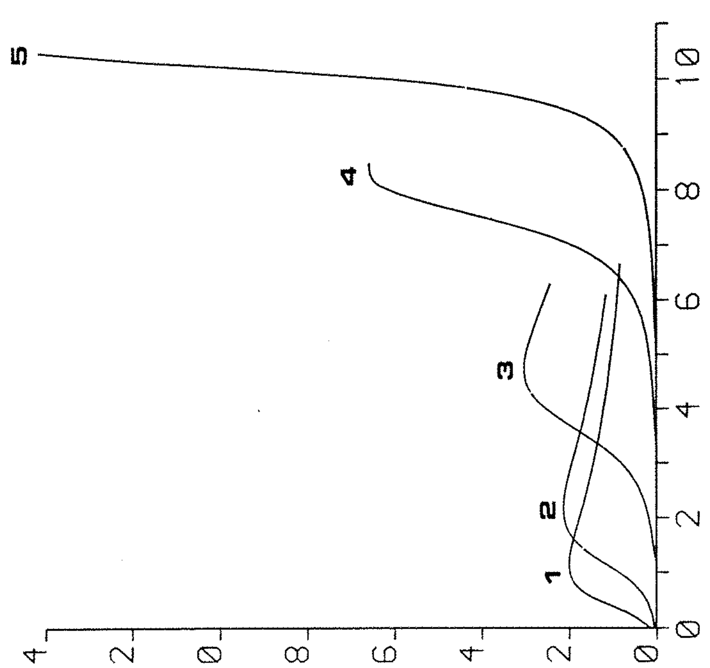

Fig.2.6b.Failure rate function

Table 2.5. Values of L_3 and L_4 for Fig.2.6a and Fig.2.6b.

Curve No.	1	2	3	4	5
$-L_3$	0.05	0.10	0.20	0.30	0.35
$-L_4$	0.35	0.30	0.20	0.10	0.05

In region 2 the distribution has a negative skewness with a very long tail to the left, while in region 4 the distribution has a positive skewness with a very long tail to the right. Figures 2.7a and 2.7b show the Ramberg density function and failure rate function in region 2 for the values of L_3 and L_4 as shown in Table 2.6.

Table 2.6. Values of L_3 and L_4 for Fig.2.7a and Fig.2.7b.

Curve No.	1	2	3	4	5
$-L_3$	1.30	1.40	1.50	1.60	1.70
L_4	1.70	1.60	1.50	1.40	1.30

Figures 2.8a and 2.8b show the Ramberg density function and failure rate function in region 4 for the values of L_3 and L_4 as shown in Table 2.7.

Table 2.7. Values of L_3 and L_4 for Fig.2.8a and 2.8b.

Curve No.	1	2	3	4	5
L_3	1.30	1.40	1.50	1.60	1.70
$-L_4$	1.70	1.60	1.50	1.40	1.30

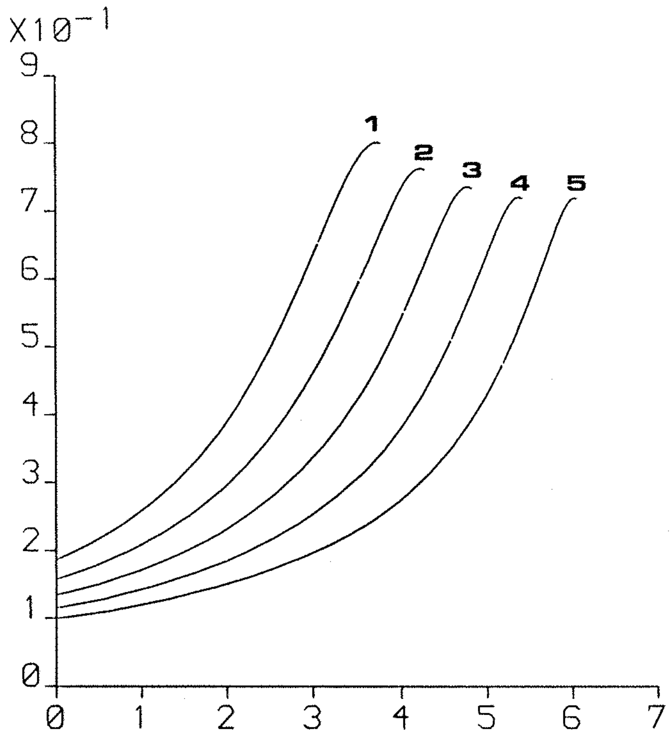

Fig.2.7a.Density Function

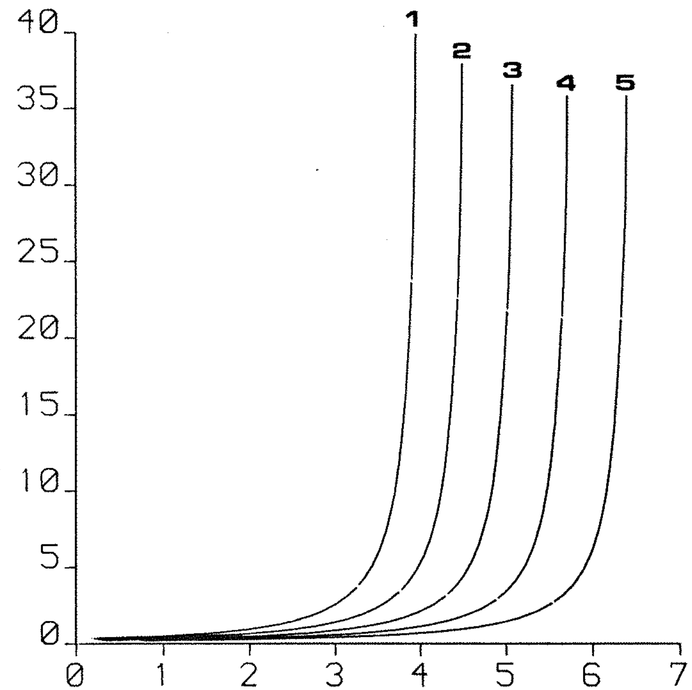

Fig.2.7b.Failure rate Function

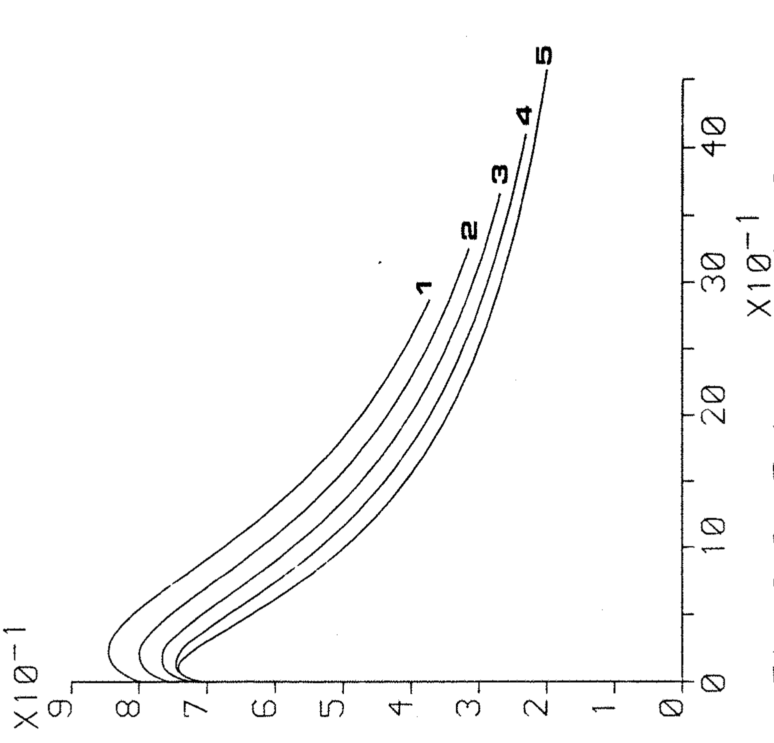

Fig.2.8b.Failure rate Function

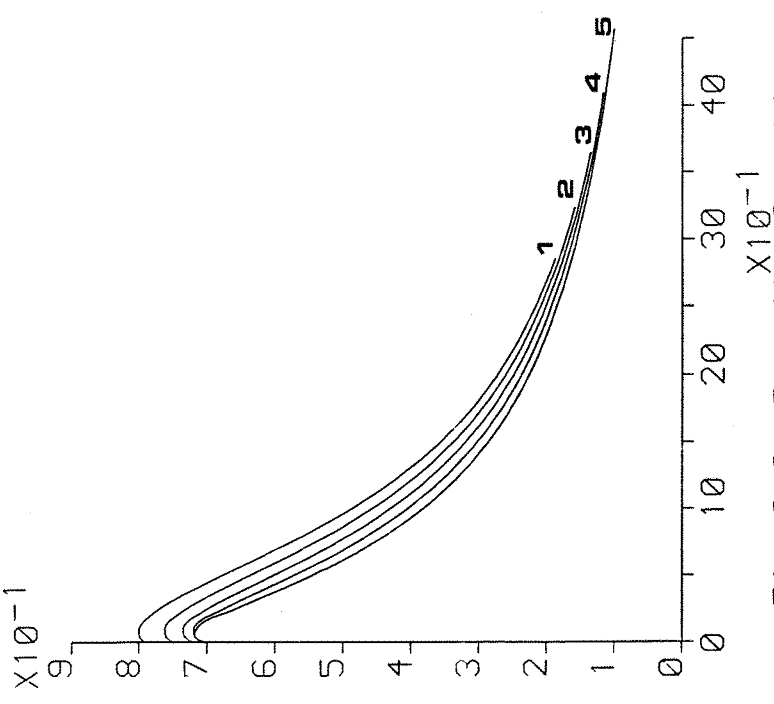

Fig.2.8a.Density Function

As we can see from the graphs that Ramberg distribution can be used for the life time distribution with a wide variety of failure rate models including a unimodal failure rate model.

Chapter 3.

THE DESIGN OF GAUGES.

3.1 General Comment On The Design.

Gauges can be used for any measurable quantity that can be arranged in an order. The use of a gauge leads to a classification of a set of items. It means that the observations are frequency counts in each class rather than exact measurements. The observed frequencies have to be used in the analysis. The physical form of gauges will depend on the nature of the observations. For instance, if our interest is in the weight of the items then a device like a balance can be a gauge. Similarly if our interest is in the life time of the material then probably an instrument that can check whether or not the life time of an item is greater than a particular value can be a gauge.

Now, consider an observation, using gauge, on the life time distribution in order to test the hypothesis about a parameter of distribution, say, for example the characteristic life of an item. Since the observation is concerned with a life time , we can place the instrument on the certain value of time T as a gauge wich will classify the items in the experiment into classes. The classification of the items would be based on whether or not an item passed the gauge. The number of classes will depend on the number of gauges being used in the experiment. For example if we use one gauge then we should have two disjoint regions where the items will be classified. If we use two gauges then we whould have three disjoint regions where the items will be classified, etc.

In the following discussion we will study of the setting of the gauges on the optimal position in the sense of maximizing the power of test. The optimal position of gauges would be as a fixed time being the value of gauges. Therefore, the design of gauges is concerned with the setting of gauges in the optimal position if possible, otherwise a good position would the result of the study. The study covers the use of one gauge and two gauges, respectively.

3.2 <u>One Gauge</u>.

Suppose n independent observations, using gauge, are made on the Weibull distribution in order to test the hypothesis about characteristic life θ. Suppose further, that one gauge has been chosen to be used in the observation. This means it would be similar to the life time test truncated at a preassigned time T. Therefore the observations would be classified into two classes A_1 and A_2. The two classes would be defined as follows:

A_1 is a set of the nonsurvival items prior to time T

A_2 is a set of the survival items beyond time T

Let n_i; $i=1,2$, be the number of items falling into A_i, and suppose we would like to to test the hypothesis:

$$H_0: \theta = \theta_0 \qquad \text{agianst}$$

$$H_1: \theta = \theta_1 \ , \ \theta_1 \le \theta_0$$

Consider $R_1=n_1$ as a test statistic. Clearly, if H_0 is false then we would like $R_1 > C$; C is an integer and commonly called acceptance number. Under both null and alternate hypothesis R_1 has a binomial distribution, hence the hypothesis test could be made equivalent to a test on the parameter in a binomial distribution.

Suppose, under null hypothesis H_0, p_i; $i=1,2$ denote the probability of an item falling into A_i, then H_0 implies the following binomial distribution:

$$P(R_1=r) \equiv P(r) = \binom{n}{r} p_1{}^r p_2{}^{n-r}$$

When P(rejection of $H_0 | \theta_0) = \alpha$; i.e Type I error, then

$$P(R_1 > C) = \alpha$$

or

$$\sum_{i=c+1}^{n} P(r_i) = \alpha$$

Now, under alternate hypothesis H_1, suppose q_i; i=1,2 denote the probability of an item falling into A_i, then H_1 implies the following binomial distribution:

$$P(r) = \binom{n}{r} q_1^r q_2^{n-r}$$

When P(acceptance of $H_0|\theta_1)=\beta$; i.e Type II error, then

$$P(R_1 \leq C) = \beta$$

or

$$\sum_{i=0}^{c} P(r_i) = \beta$$

The power of test is given by $Pw=1-\beta$.

It is likely that we could not expect a completely general solution of the value of gauge in terms of time t. However, when the probability term is used to express the position of a gauge, then we should be able to choose the single value of gauge such that the power of test is still, orclose enough to the maximum. Therefore we would express the position of a gauge in the probability terms rather than in the fixed time.

AS an illustration of this argument, consider the case of n=50 with hypothesis as follows:

$H_0:\theta_0=$ 1.000 hours against,

$H_1:\theta_1=$ {1.000,(-50),450} hours, respectively.

By using a normal approximation to the binomial distribution, the optimal position of one gauge has been carried out for a given Type I error α=.050 with the shape parameter β=1,(.1),1.5,2,3 as shown in Table 3.1a. As we can see, the positions of gauge are slightly different. However, we should be able to choose a single value as a common value of a gauge. Table 3.1b shows the power of test when the gauge is set at time t such that the probability of an item not surviving prior to time t is equal to 0.65.

It can be seen that the difference of the power of test in Table 3.1a and Table 3.1b is less than or equal to 0.002. Therefore, in this case, it is quite reasonable to take the probability p=0.65 as the value for one gauge.

It is interesting to note that for the practical purposes, when the power of test is greater or equal to 0.999 then may be we can reduce either the lengthtime of observation or the sample size. Alternatively, we can probably reduce both of them at the same time. Table 3.2 shows the power of test with β={ 1, 1.5, 2, 3} as an illustration for this argument. The final decision would depend on other consideration such as the cost of the observation.

Table 3.1a. The optimal position of one gauge.

β	1		1.1		1.2		1.3		1.4		1.5		2		3	
θ_1	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.
1.000	-	.050	-	.050	-	.050	-	.050	-	.050	-	.050	-	.050	-	.050
950	.65	.083	.65	.087	.65	.091	.65	.096	.65	.100	.65	.105	.65	.131	.66	.197
900	.65	.134	.65	.147	.65	.160	.65	.173	.65	.188	.65	.203	.65	.292	.66	.514
850	.65	.210	.65	.236	.65	.263	.65	.293	.65	.324	.65	.356	.66	.535	.66	.858
800	.65	.315	.65	.360	.65	.407	.65	.456	.65	.506	.66	.557	.66	.791	.66	.990
750	.65	.450	.65	.515	.66	.581	.65	.645	.67	.707	.66	.764	.67	.952	.66	*
700	.65	.606	.66	.685	.66	.757	.66	.821	.67	.875	.67	.917	.67	.997	-	*
650	.67	.763	.67	.838	.67	.897	.67	.940	.68	.969	.67	.985	-	*	-	*
600	.68	.890	.68	.942	.68	.974	.68	.990	.68	.997	-	*	-	*	-	*
550	.68	.966	.68	.988	.68	.997	.68	*	-	*	-	*	-	*	-	*
500	.68	.995	.68	.999	-	*	-	*	-	*	-	*	-	*	-	*
450	.68	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*

note: *= equal or nearly to 1

Table 3.1b. The power of test when the gauge is set on p=.65

β	1	1.1	1.2	1.3	1.4	1.5	2	3
θ_1								
1.000	.050	.050	.050	.050	.050	.050	.050	.050
950	.083	.087	.091	.096	.100	.105	.131	.197
900	.134	.147	.160	.173	.188	.203	.292	.514
850	.210	.236	.263	.293	.324	.356	.534	.857
800	.315	.360	.407	.456	.506	.556	.790	.990
750	.450	.515	.580	.645	.706	.763	.950	*
700	.606	.684	.756	.820	.873	.916	.996	*
650	.762	.836	.896	.938	.967	.984	*	*
600	.888	.940	.972	.989	.996	.999	*	*
550	.965	.987	.997	.999	*	*	*	*
500	.994	.999	*	*	*	*	*	*
450	*	*	*	*	*	*	*	*

note: *= equal or nearly to 1

Table.3.2.The power of test with n=50, p=.26;
n=40, p=.35; n= 30, 25, p=.65.

	n	50	40	30	25
	p	.26	.35	.65	.65
β	θ_1				
1	450	.963	.968	.979	.949
	400	.991	.993	.998	.990
	350	.999	.999	*	*
1.5	600	.948	.954	.967	.927
	550	.988	.991	.997	.986
	500	.999	.999	*	.999
2	650	.982	.985	.993	.977
	600	.998	.999	*	*
	550	*	*	*	*
3	750	.982	.986	.993	.977
	700	.999	*	*	*
	650	*	*	*	*

note *= equal or nearly to 1

3.3 Two Gauges.

As before, suppose n independent observations, using gauge, are made on the Weibull distribution in order to test the hypothesis about characteristic life θ. Now, instead of using one gauge, suppose two gauges have been chosen to be used in the observation. As a result of this, the observations would be classified into three disjoint classes A_1, A_2 and A_3. Let T_1 and T_2 denote the positions of the first and the second gauge, respectively, the three classes would be defined as :

A_1 is a set of the nonsurvival items prior to T_1

A_2 is a set of the nonsurvival items between T_1 and T_2

A_3 is a set of the suvival items beyond T_2

Suppose n_i nd p_i, i=1,2,3; denote the number of the items in each group and the probability of an item falling into ith class, respectively. Consider the statistic $R_2=n_1-n_3$ as the test statistic. There are 2n+1 possible values of R_2, i.e $-n,-(n-1),........,0,...........,(n-1),n$.

Consider the trinomial:

$$P(n_1,n_3)=\frac{n!}{n_1!n_2!n_3!}\, p_1^{n_1}p_2^{n_2}p_3^{n_3}$$

where $n_2=n-(n_1+n_3)$.

The probability distribution of R_2, Taj Hijri (1979), is given by:

$$P(R_2=r) = \begin{cases} \sum_{n_3=-r}^{[(n-r)/2]} P(n_3+r, n_3 & \text{for } r<0 \\ \\ \sum_{n_3=0}^{[(n-r)/2]} P(n_3+r, n_3 & \text{for } r\geq 0 \end{cases}$$

The exact distribution of R_2 may be generated by summing the trinomials probabilities.

As an illustration consider the case of n=5. The possible values of R_2 are -5,-4,......,0,.......4,5. Suppose the two gauges are setting such that $p_1=p_3=.20$. The probability of $R_2\leq-3$ is given by:

$$P(R_2\leq-3)=P(R_2=-5)+P(R_2=-4)+P(R_2=-3)$$

Using the obove formulae we find,

$$P(R_2\leq-3)=0.03552.$$

Taj Hijri (1979) has been investigated that a normal approximation can be used for R_2 where the symmetrical position of gauge, in the sense $p_1=p_3$, is desirable. In the study of the optimal position of two gauges, we will refer to this result by setting the gauges on the symmetrical position and assuming R_2 has a normal distribution. The first four moments of R_2 are given by,

$$E(R_2) \equiv \mu = n(p_1 - p_3)$$

$$\mu_2 = n\{(p_1 + p_3) - (p_1 - p_3)^2\}$$

$$\mu_3 = n(p_1 - p_3)\{2(p_1 - p_3)^2 - 3(p_1 + p_3) + 1\}$$

$$\mu_4 = 3\mu_2^2 - 6n(p_1 - p_3)^4 + n\{3(p_1 + p_3) - 1\}\{4(p_1 - p_3)^2 - (p_1 + p_3)\}.$$

For two gauges the length time of observation being conducted would be determined by the position of the second gauge, i.e T_2. However like the one gauge, it would not be a practical proposition to use the fixed time to express the position of gauges, in the sense that it would be difficult to obtain a single value of a fixed time as a value of gauges. Therefore, we use probability terms for the position of two gauges, i.e $p_1 = p_3 = p$.

As an illustration, consider the case of testing the hypothesis about θ as in the one gauge example. Table 3.3a shows the optimal position of gauges for a given Type I error $\alpha = .050$. From this table we can see that this is quite reasonable if we take $p = 0.30$ as a value for two gauges.

Table 3.3a. The optimal position of two gauges.

β	1		1.1		1.2		1.3		1.4		1.5		2		3	
θ_1	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.	p	Pw.
1.000	-	.050	-	.050	-	.050	-	.050	-	.050	-	.050	-	.050	-	.050
950	.30	.086	.30	.091	.30	.095	.30	.100	.30	.105	.30	.110	.30	.140	.30	.213
900	.30	.143	.30	.157	.30	.171	.30	.187	.30	.203	.30	.220	.30	.315	.30	.557
850	.30	.228	.30	.256	.30	.287	.30	.320	.30	.354	.30	.389	.30	.578	.30	.886
800	.30	.344	.30	.393	.30	.444	.30	.496	.30	.549	.30	.601	.30	.827	.30	.993
750	.30	.490	.30	.558	.30	.625	.30	.689	.30	.749	.30	.802	.30	.963	.30	*
700	.30	.651	.30	.728	.30	.796	.30	.854	.30	.900	.31	.936	.30	.997	-	*
650	.30	.801	.30	.868	.30	.919	.30	.954	.30	.976	.31	.989	.30	*	-	*
600	.30	.913	.30	.955	.31	.979	.31	.992	.31	.997	.31	*	-	*	-	*
550	.30	.974	.30	.991	.31	.997	.31	*	.31	*	-	*	-	*	-	*
500	.30	.996	.30	*	.31	*	-	*	-	*	-	*	-	*	-	*
450	.30	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*

note: *= equal or nearly to 1.

It should be noted that it is possible to reduce the length time of observation with a relatively small effect on the power of test. For instance if we set the gauges at the time T_1 and T_2 such that $p_1=p_3=p=0.35$, then the resulting power of test will be decreased only less than or equal to 0.006. Table 3.3b shows the power of test when the gauges are set such that p=0.35.

Table 3.3b. The power of test when gauges are set such that p=0.35.

β / θ_1	1	1.1	1.2	1.3	1.4	1.5	2	3
1.000	.050	.050	.050	.050	.050	.050	.050	.050
950	.086	.090	.095	.100	.104	.110	.138	.211
900	.142	.156	.170	.185	.201	.218	.315	.551
850	.226	.254	.284	.316	.349	.385	.573	.884
800	.341	.390	.439	.490	.543	.595	.823	.993
750	.487	.554	.619	.684	.744	.798	.962	*
700	.647	.725	.792	.851	.898	.934	.997	*
650	.798	.867	.917	.953	.976	.989	*	*
600	.912	.955	.979	.992	.997	*	*	*
550	.974	.991	.997	*	*	*	*	*
500	.996	*	*	*	*	*	*	*
450	*	*	*	*	*	*	*	*

note *= equal or nearly to 1.

As in the one gauge case, when the power of test is greater or equal to .999 then we can either reduce the lengthtime of observation or the sample size. Alternatively we can probably reduce both of them at the same time. Table 3.4 shows the power of test with β= 1, 1.5, 2 and 3 as an illustration of this argument.

Table 3.4. The power of test with n=50,p=.45; n=40,35,p=40; n=25,p=.30,.40.

	n	50	40	30	25	25
	p	.45	.40	.40	.40	.30
β	θ_1					
1	450	.999	.997	.984	.963	.964
	400	*	*	.998	.993	.993
	350	*	*	*	*	.999
1.5	600	.998	.995	.975	.947	.949
	550	*	*	.997	.991	.990
	500	*	*	*	.999	.999
2	650	*	.999	.995	.984	.984
	600	*	*	*	.999	.999
	550	*	*	*	*	*
3	750	*	.999	.995	.984	.984
	700	*	*	*	*	*

note: *= equal or nearly to 1.

Chapter 4.

APPLICATION IN ACCEPTANCE SAMPLING PLANS.

Gauges are most likely to be used when the quality of the items refer to the measurement on a continuous scale. This means the application of gauges in the acceptance sampling plans is concerned with the average quality and not with the fraction defective of the items. Therefore the operating characteristic (OC) function will give the probability of acceptance as a function of the average quality of an item. In the life testing case, since we are interested on the characteristic life or the mean life of the items, the OC function will give the probability ofacceptance as a function of this parameter.

Since the OC function is complement of the power function, the optimal position of a gauge in the acceptance sampling plans is the same as in the testing hypothesis. The optimal position in this case has as criteria the minimisation of type II error β or consumer's risk. However, the use of a gauge in the acceptance sampling plans is also concerned with a setting of a gauge in order to meet the required conditions of the plans. For instance we can specify the sample size n, the type I error α or producer's risk and the position of gauge (T) to find the critical point (C). Or to take another example, we can specify n, α and β to find T and C, etc.

Since C is cost independent, the plans are likely to specify α and β in order to find n and T. There may be a number of plans:i.e pairs of n and T which satisfy the required conditions. Therefore we shall impose an additional condition which will lead to a unique plan. The additional condition can be the minimum sample size n or the minimum of the cost function of observation. If a plan criteria is cost then from these several possibilities we can choose a pair of n and T such that the total cost of observation is at a minimum. We will discuss this problem later, in Chapter 5.

We consider a single sampling plan and a double sampling plan respectively.

4.1 Single Sampling Plan.

In a single sampling plan we will study the use of one gauge and two gauges.

a.One gauge.

Suppose a batch of items is presented for inspection. A single sampling plan, using one gauge, consists of a random sample of n items from the batch for inpection. The decision on the batch will depend on the result. If we decide to either accept or reject the batch then the batch would be accepted if the number of defectives d found prior to time T in the n items were less than or equal to the acceptance number c; T is the position of the gauge.

We have seen that for a given α then the gauge can be set as in the testing hypothesis to meet the minimum value of β. However in the practical use of acceptance sampling plans α and β are normally specified. For this plan we may have a number of pairs of n and T that satisfy the given conditions. Since one gauge is used we can consider that d has a binomial distribution hence we can use a binomial table to find the solutions that satisfy the required conditions.

For practical purposes, however, we can probably use either the Poisson or the normal approximation to the Binomial. Extensive examples of this case have been published in a number of text books. Guenther (1977) for example, pointed out the conditions in which the Poisson distribution can be use. In our case we will consider of the use of a normal approximation to calculate the plans; i.e the pair of n and T that satisfy a given condition. The related acceptance number of the plan can be calculated afterwards by putting mean μ=np and standard deviation $\sigma=\sqrt{np(1-p)}$. As an illustration of how good a normal approximation can be in use, consider the case of a single sampling plan consists of n=50 drawn from an exponential distribution with one gauge set at several positions. Suppose we wish the probability of acceptance at the mean life time θ=1000 hours to be about 95%, hence α=0.05. Table 4.1 shows the OC-curves which are calculated by using a binomial and a normal approximation. As we can see, as T increases the difference becomes smaller. Therefore, it is quite reasonable to use a normal approximation in this case.

Table 4.1 OC-curve for a single sampling plan using binomial and normal approximation

	Gauge position T hours					
	85.56		119.91		238.26	
θ	Bin.	Nor.	Bin.	Nor.	Bin.	Nor.
1,000	0.951	0.960	0.949	0.957	0.950	0.955
950	0.937	0.946	0.934	0.941	0.929	0.933
900	0.920	0.928	0.913	0.919	0.900	0.902
850	0.899	0.904	0.886	0.889	0.859	0.860
800	0.870	0.872	0.850	0.850	0.804	0.802
750	0.834	0.832	0.804	0.800	0.731	0.726
700	0.788	0.781	0.745	0.736	0.639	0.631
650	0.729	0.717	0.671	0.658	0.527	0.519
600	0.656	0.639	0.579	0.565	0.401	0.395
550	0.566	0.548	0.473	0.458	0.272	0.269
500	0.462	0.444	0.355	0.344	0.157	0.157
450	0.345	0.333	0.236	0.232	0.071	0.073
400	0.227	0.223	0.131	0.133	0.023	0.024
350	0.123	0.126	0.055	0.059	0.004	0.005
300	0.048	0.054	0.015	0.018	0.000	0.000

We will now investigate the determination of single sampling plans for situation in which $\theta_0, \theta_1, \alpha$ and β are specified. As an illustration consider the case of hypothesis :

$$H_0 : \theta_0 = 1{,}000 \text{ hours, against}$$

$$H_1 : \theta_1 = \{700, (-50), 350\} \quad , \quad \text{respectively}$$

For a given α and β and assuming life time T has an exponential distribution the plans that satisfy the required conditions have been calculated using a normal approximation as shown in Table 4.1a, b and c. Table 4.1a shows some of the plans that satisfy α=0.05 and β=0.05. Table 4.1b shows some of the plans that satisfy α=0.05 and β=0.10. Table 4.1c shows some of the plans that satisfy α=0.10 and β=0.10. As we can see from these Tables the sample size n decreases when the position of a gauge T increaseas. The minimum sample size n will occur when the gauge is set around optimal position.

Having determined the plans that satisfy the required conditions we can calculate the related acceptance number C for each plan. For example, if we take the case of alternate hypothesis $H_1:\theta_1=500$ with α=0.05 and β=0.05 then for the plan with n=38 and T=693.15 the acceptance number is about 24.

Table 4.1a Some of the plans that satisfy α=0.05 and β=0.05 for a single sampling plan, using one gauge.

Gauge position		θ_1							
p	T(hrs.)	700	650	600	550	500	450	400	350
0.50	693.15	158	106	74	52	38	27	20	15
0.51	713.35	156	105	73	52	37	27	20	15
0.52	733.97	154	103	72	51	37	27	20	15
0.53	755.02	152	102	71	50	36	27	20	14
0.54	776.53	150	100	70	50	36	26	19	14
0.55	789.51	148	99	69	49	36	26	19	14
0.56	820.98	146	98	68	49	35	26	29	14
0.57	843.97	144	97	68	48	35	26	19	14
0.58	867.50	143	96	67	48	35	26	19	14
0.59	891.60	141	95	66	48	35	26	19	14
0.60	916.29	140	94	66	47	34	25	19	14
0.61	941.61	139	93	65	47	34	25	19	14
0.62	967.58	137	93	65	47	34	25	19	14
0.63	994.25	136	92	65	46	34	25	19	14
0.64	1021.65	135	91	64	46	34	25	19	14
0.65	1049.82	134	91	64	46	34	25	19	14
0.66	1078.81	133	90	64	46	34	25	19	14
0.67	1108.66	133	90	63	46	34	25	19	15
0.68	1139.43	132	90	63	46	34	25	19	15
0.69	1171.18	132	89	63	46	34	26	19	15
0.70	1203.97	131	89	63	46	34	26	20	15

Table 4.1b Some of the plans that satisfy α=0.05 and β=0.10 for a single sampling plan, using one gauge.

Gauge position		θ_1							
p	T(hrs.)	700	650	600	550	500	450	400	350
0.50	693.15	126	85	59	42	30	22	16	12
0.51	713.35	124	83	58	41	30	22	16	12
0.52	733.97	123	82	57	41	30	22	16	12
0.53	755.02	121	81	57	41	30	22	16	12
0.54	776.53	120	80	56	40	29	22	16	12
0.55	798.51	118	80	56	40	29	22	16	12
0.56	820.98	117	79	55	39	29	21	16	12
0.57	843.97	115	78	55	39	29	21	16	12
0.58	867.50	114	77	54	39	29	21	16	12
0.59	891.60	113	77	54	39	28	21	16	12
0.60	916.29	112	76	53	38	28	21	16	12
0.61	941.61	111	75	53	38	28	21	16	12
0.62	967.58	110	75	53	38	28	21	16	12
0.63	994.25	110	74	52	38	28	21	16	12
0.64	1021.65	109	74	52	38	28	21	16	12
0.65	1049.82	108	74	52	38	28	21	16	12
0.66	1078.81	108	73	52	38	28	21	16	13
0.67	1108.66	107	73	52	38	28	21	16	13
0.68	1139.43	107	73	52	38	28	21	17	13
0.69	1171.18	106	73	52	38	28	22	17	13
0.70	1203.97	106	73	52	38	29	22	17	13

Table 4.1c Some of the plans that satisfy $\alpha=0.10$ and $\beta=0.10$ for a single sampling plan, using one gauge.

Gauge position		θ_1							
p	T(hrs.)	700	650	600	550	500	450	400	350
0.50	693.15	97	64	45	32	23	18	12	9
0.51	713.35	95	64	44	31	23	17	12	9
0.52	733.97	94	63	44	31	22	16	12	9
0.53	755.02	92	62	43	31	22	16	12	9
0.54	776.53	91	61	43	30	22	16	12	9
0.55	798.51	90	60	42	30	22	16	12	9
0.56	820.98	89	60	42	30	22	16	12	9
0.57	843.97	88	59	41	29	21	16	12	9
0.58	867.50	87	59	41	29	21	16	12	9
0.59	891.60	86	58	40	29	21	16	12	9
0.60	916.29	85	57	40	29	21	16	12	9
0.61	941.61	84	57	40	29	21	15	12	9
0.62	967.58	84	57	40	28	21	15	12	9
0.63	994.25	83	56	39	28	21	15	12	9
0.64	1021.65	82	56	39	28	21	15	12	9
0.65	1049.82	82	55	39	28	21	15	12	9
0.66	1078.81	81	55	39	28	21	15	12	9
0.67	1108.66	81	55	39	28	21	15	12	9
0.68	1139.43	80	55	39	28	21	15	12	9
0.69	1171.18	80	55	38	28	21	16	12	9
0.70	1203.97	80	54	38	28	21	16	12	9

b. Two gauges.

Suppose we choose to use two gauges in observation for a single sampling plans consisting of a sample of n items. Our decision on the batch will depend on the results of $R_2=n_1-n_3$. If the decision is to either accept or reject the batch then the batch would be accepted if R_2 were less than or equal to the acceptance number h.

Since we use a normal approximation to R_2 then under null hypothesis H_0 the acceptance number should be:

$$h= z_\alpha \sqrt{2np}.$$

where z_α is the appropriate normal deviate. This figure can be calculated for practical use, after we have decided on the most suitable plan. Since the lengthtime of the observation being conducted is determined by the position of the second gauge (T_2) the plan should be the pair of n and T_2.

As before, we investigate the determination of single sampling plans for situation in which $\theta_0, \theta_1, \alpha$ and β are specified. As an illustration consider the hypothetical case in the one gauge example. For a given α and β and assuming life time T has an exponential distribution the plans that satisfy the required conditions have been calculated as shown in Table 4.2a, b and c. Table 4.2a shows some of the plans that satisfy $\alpha=0.05$ and $\beta=0.05$. Table 4.2b shows some of the plans that satisfy

α=0.05 and β=0.10. Table 4.2c shows some of the plans that satisfy α=0.10 and β=0.10.

Table 4.2a Some of the plans that satisfy α=0.05 and β=0.05 for a single sampling plan, using two gauges.

Gauge Position		θ_1							
p	T_2(hrs.)	700	650	600	550	500	450	400	350
0.25	1386.29	128	87	61	44	33	25	19	14
0.26	1347.07	127	86	61	44	32	24	18	14
0.27	1309.33	126	86	60	43	32	24	18	14
0.28	1272.97	126	85	60	43	32	24	18	13
0.29	1237.87	126	85	60	43	31	23	17	13
0.30	1203.97	126	85	59	43	31	23	17	13
0.31	1171.18	126	85	59	43	31	23	17	13
0.32	1139.43	127	85	59	42	31	23	17	13
0.33	1108.66	127	85	59	42	31	23	17	12
0.34	1078.81	128	86	60	43	31	23	17	12
0.35	1049.82	128	86	60	43	31	23	17	12
0.36	1021.65	129	87	60	43	31	23	17	12
0.37	994.25	130	87	61	43	31	23	17	12
0.38	967.58	132	88	61	43	31	23	17	12
0.39	941.61	133	89	62	44	32	23	17	12
0.40	916.29	134	90	62	44	32	23	17	12

Table 4.2b Some of the plans that satisfy α=0.05 and β=0.10 for a single sampling plan, using two gauges.

Gauge Position		θ_1							
p	T_2(hrs.)	700	650	600	550	500	450	400	350
0.25	1386.29	102	70	49	36	27	20	15	12
0.26	1347.07	102	69	49	35	26	20	15	11
0.27	1309.33	101	69	48	35	26	19	15	11
0.28	1272.97	101	68	48	35	26	19	14	11
0.29	1237.87	101	68	48	35	25	19	14	11
0.30	1203.97	101	68	48	34	25	19	14	11
0.31	1171.18	101	68	48	34	25	19	14	11
0.32	1139.43	101	68	48	34	25	19	14	10
0.33	1108.66	101	68	48	34	25	19	14	10
0.34	1078.81	102	69	48	34	25	18	14	10
0.35	1049.82	102	69	48	34	25	18	14	10
0.36	1021.65	103	69	48	35	25	19	14	10
0.37	994.25	104	70	49	35	25	19	14	10
0.38	967.58	105	70	49	35	25	19	14	10
0.39	941.61	106	71	49	35	26	19	14	10
0.40	916.26	107	72	50	36	26	19	14	10

Table 4.2c Some of the plans that satisfy α=0.10 and β=0.10 for a single sampling plan, using two gauges.

Gauge position		θ_1							
p	T_2(hrs.)	700	650	600	550	500	450	400	350
0.25	1386.29	78	53	37	27	20	15	11	9
0.26	1347.07	77	53	37	27	20	15	11	8
0.27	1309.33	77	52	37	26	19	15	11	8
0.28	1272.97	77	52	36	26	19	14	11	8
0.29	1237.87	77	52	36	26	19	14	11	8
0.30	1203.97	77	52	36	26	19	14	10	8
0.31	1171.18	77	52	36	26	19	14	10	8
0.32	1139.43	77	52	36	26	19	14	10	8
0.33	1108.66	77	52	36	26	19	14	10	8
0.34	1078.81	78	52	36	26	19	14	10	8
0.35	1049.82	78	53	37	26	19	14	10	8
0.36	1021.65	79	53	37	26	19	14	10	8
0.37	994.25	79	53	37	26	19	14	10	8
0.38	967.58	80	54	37	26	19	14	10	8
0.39	941.61	81	54	38	28	19	14	10	8
0.40	916.29	82	55	38	28	19	14	10	8

4.2 Double sampling plan.

Suppose for the same required conditions we wish to replace a single sampling plan by a double sampling plan. This means we shall require both plans to possess the same or approximately the same OC curve. As we have observed in the previous section, there may be a number of plans whose OC curve satisfy the given conditions. In order to find a unique plan we have to impose an additional condition.

For a situation in which $\theta_0, \theta_1, \alpha$ and β are specified we have calculated the plans that satisfy the required conditions, using a normal approximation, in a single sampling plans. In a double sampling case however, we cannot use the same technique to find the plans that satisfy the required conditions, since a double sampling plan requires 5 parameters for its full specification. The five parameters are two sample sizes n_1 and n_2 and three decision numbers C_1, C_2 and C_3; where C_1 and C_2 are acceptance number and rejection number in the first sampling with sample size n_1 and C_3 is acceptance number for the combination of the first sample of n_1 and the second sample of n_2.

To overcome this problem we can use the information given by the solution of the single sampling plan. Guenther (1977) recomended the procedure of using the information from a single sampling solution for the double sampling case. The summary of the procedure as follows:

1. List the single sample solutions and non solutions
2. Select any C_2 for which solution exist
3. Select any C_1 such that $0 \le C_1 < C_2$. In a number of plans used in practical situations we have $C_1 \le 0.5C_2$
4. For chosen C_1, C_2 determine bounds on n_1 such the OC at $\theta_1 \le \beta$
5. By trial for the chosen C_1, C_2, n_1 find n_2 such that the two conditions on OC curve are satisfied.

Repeat step 5 for another n_1, C_1 and C_2 and terminate the trial by an additional condition.

We will apply this same idea to our case, though probably not using excactly the same procedure as Guenther. We might determine the solution for a double sampling plan by reference to the single sampling plan chosen earlier; i.e a pair of n and T, from a number of plans that satisfy the required conditions in a single sampling plan. With the same position (T) of a gauge we can determine n_1

proportional to n. Also choose C_1,C_2 and C_3 such that $C_1<C_2\leq C\leq C_3$; C is the acceptance number in a single sampling plan. Using the fact that $n\leq n_1+n_2$, we can find n_2 by trial for chosen C_1,C_2,C_3 and n_1. Since no claim is made on the determinetion of all these figures then we can choose by trial.

Let us suppose that the parameters of a double sampling plan are determine as follows:

1.Select n_1 to be about 0.8n

2.select $C_3 \geq C+1$

3.Select C_2 to be about C-2

4.Select $C_1\leq 0.5C$

for the case in which θ_0=1,000 hours, θ_1=500 hours, α=0.05 and β=0.05. Suppose the plan with n=160 and T=105.36 (p=0.10) has been chosen from a number of plans that satisfy the given conditons in a single sampling plan. For this plan the acceptance number C=22. Table 4.3 shows the OC at θ_0 and θ_1 for several combinations of C_1,C_2 and C_3 with n_1=0.8n. As a reference we put the OC of a single sampling plan in the first row. As we can see from the Table, we can have a number of plans that satisfy the required conditions. In order to find a unique double sampling plan we can use n_2 as a criteria on parameter, since we have specified T and n_1. In our example we can see that plans No.3 and 5 are most suitable choices for a double sampling plans.

Intuitively the results in Table 4.3 suggest that the total sample $n_t=n_1+n_2$ will depend on C_3, and C_2 will depend on the chosen value of n_1 proportional to n. This is probably much clearer if we describe our results using a random walk diagram as shown in Fig.4.1; see Hamaker(1955), by putting the total sample as abscissa and the number of defectives observed as ordinate. Hamaker (1955) pointed out that apart from random deviations a random walk created by the inspection of items taken from a homogeneous lot will move in a long straight line through the origin. Hence if we draw a stright line from the origin to the divided point C_3 in the third screen it is preferable that this line should pass somewhere through the centre of the open area between C_1 and C_2, otherwise the judgements based on the first and the total sample are not balanced.

Table 4.3 OC of a double sampling plan at θ_0=1,000 hours θ_1=500 hours for a given C_1,C_2,C_3 with n_1=0.8n

single sampling plan					0.951	0.052
Plan.No.	C_1	C_2	C_3	n_2	OC(θ_0)	OC(θ_1)
1	5	21	23	40	0.950	0.051
2	8	20	23	37	0.951	0.050
3	8	21	23	40	0.950	0.044
4	11	20	23	37	0.951	0.050
5	11	21	23	40	0.950	0.045
6	13	21	23	40	0.950	0.044
7	15	20	23	38	0.950	0.051
8	15	21	23	40	0.952	0.049
9	15	21	24	49	0.950	0.042
10	15	21	26	67	0.950	0.033
11	18	21	23	80	0.950	0.092

Since the plans for double sampling are made with reference to the solutions in a single sampling plan the sample size n and the acceptance number C in the single sampling plans would be used as a reference for drawing the diagram. Hence if we draw a stright line from the origin to the dividing point C in the second screen then for any $n_1 \leq n$ we can have pairs of C_1 and C_2. Since a straight line OC is fixed then the total sample n_t will increase as well as C_3 on the third screen increased.

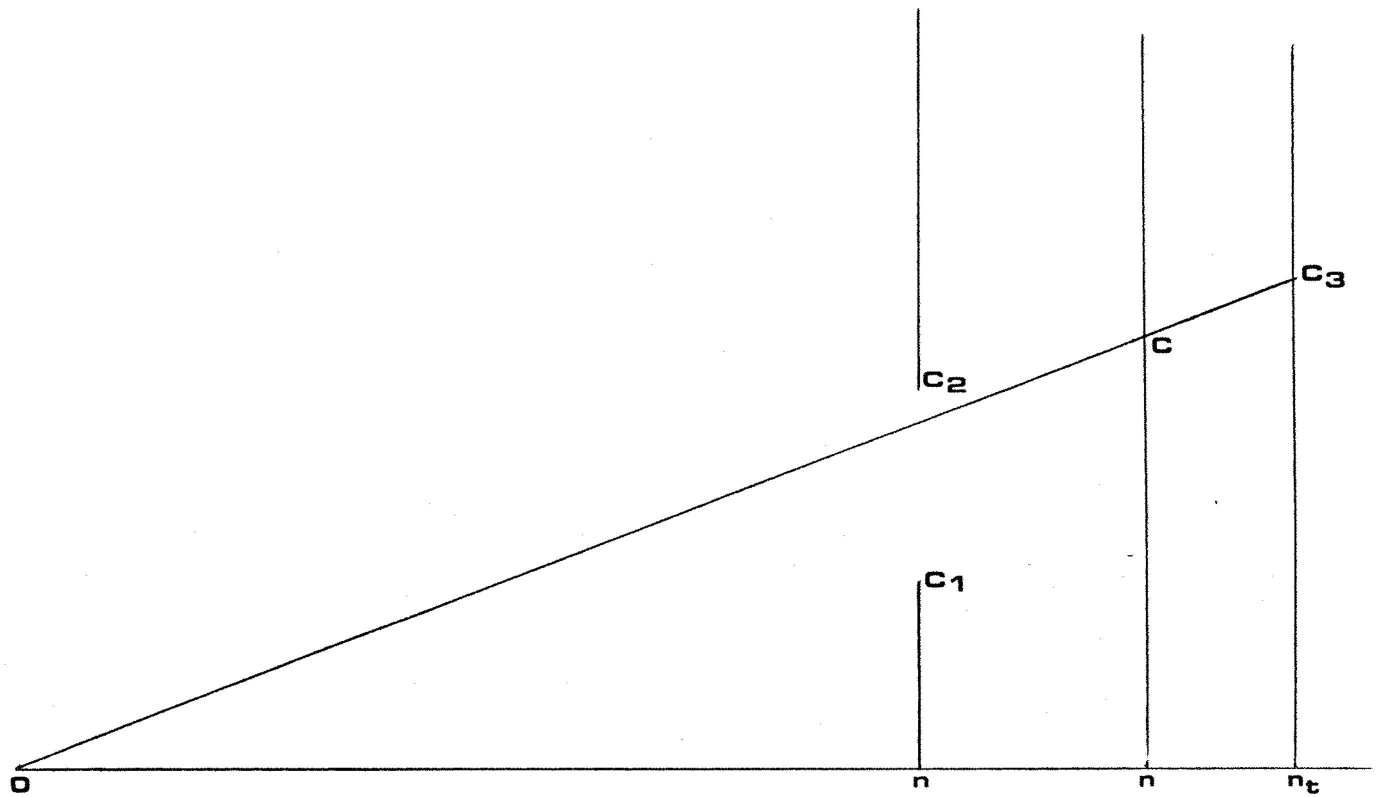

Fig.4.1 Random walk diagram for a double sampling plan

Chapter 5.

EFFICIENCY AND ROBUSTNESS OF TEST.

Clearly the advantages of gauging over exact measurement are its speed and ease of operation. Gauging can lead to an automatic quality control system that could replace nonproductive inspection work. Furthermore, the simplicity of the statistical results of gauging make it attractive.

In this chapter we will consider the other measures of the merits of the test such as its relative efficiency and robustness of test. Also we will consider the costs that could be involved in observations using gauging.

5.1 Efficiency.

A comparison of gauges with exact measurement is obviously of interest. The efficiency of test is calculated for a given α and β with reference to the number of observations required for a gauge based test and a test based on exact measurement. We will compare the test which is based on one gauge (R_1) and two gauges (R_2) with the U test, Bain(1978), which is based on exact measurement. According to Bain, for a random sample of size n from the Weibull distribution $W(\beta,\theta)$, the distribution of U is approximated to normal distribution as sample size n is increased. The statistic U is given by:

$$U=\beta \ln(\theta_0/\theta)\sqrt{n}$$

θ_0 = observed characteristic life time

Using a normal approxmation to U, R_1 and R_2, we calculate the number of observations required for R_i; $i=1,2$ and U tests, for a given α and β. As an illustration consider the following hypothetical case:

$H0:\theta_0=1.000$ hours against,

$H_1:\theta_1=\{800,(-50),500)$, respectively.

For a given α and β the number of observations required has been calculated as shown in the Table 5.1a, b and c with shape parameter $\beta=1$, 1.5 and 2. n_U and n_R denote the number of obsevations for U and R_i test, respectively. From these Tables we can see that the efficiency of gauging relative to exact measurement is over 70%. We can also see that the R test when based on two gauges, is of relatively high efficiency than when based on the one gauge.

Table 5.1a. The number of observations required for α=0.05 and β=0.05.

β	1			1.5			2		
		n_R			n_R			n_R	
θ_1	n_U	o.g	t.g	n_U	o.g	t.g	n_U	o.g	t.g
800	267	351	341	119	153	147	67	84	80
750	161	209	201	71	91	86	40	50	46
700	105	134	128	47	58	54	26	32	29
650	72	91	86	32	39	36	18	22	19
600	51	64	60	23	28	25	13	15	13
550	37	46	43	17	20	18	9	11	9
500	28	34	31	12	15	13	7	9	7

Table 5.1b. The number of observations reqiured for α=0.05 and β=0.10.

β	1			1.5			2		
		n_R			n_R			n_R	
θ_1	n_U	o.g	t.g	n_U	o.g	t.g	n_U	o.g	t.g
800	212	280	271	94	123	117	53	69	64
750	127	168	160	57	73	67	32	41	37
700	83	108	102	37	47	44	21	27	24
650	57	74	69	25	32	29	14	18	16
600	40	52	48	18	23	20	10	13	11
550	30	38	34	13	17	14	7	10	8
500	22	28	25	10	13	10	5	8	6

Table 5.1c. The number of observations required for α=0.10 and β=0.10.

β	1			1.5			2		
		n_R			n_R			n_R	
θ_1	n_U	o.g	t.g	n_U	o.g	t.g	n_U	o.g	t.g
800	163	214	208	72	93	89	41	51	49
750	98	127	123	44	55	52	25	30	29
700	64	82	78	28	35	33	16	19	18
650	44	55	53	19	24	22	11	13	12
600	31	39	37	14	17	15	8	9	8
550	23	28	26	10	12	11	6	7	6
500	17	21	19	8	9	8	4	5	4

5.2 Robustness.

The use of a gauge in testing the hypothesis $H_0:\theta=\theta_0$ we assumed that the distribution of the parent population is known. However, if the actual distribution of the parent population is different from the assumption, then for the same critical region CR the actual type I errors will differ from the assumed type I errors. A test that is less sensitive to the departures from assumptions made about the known distribution is said to be more robust.

Suppose, that in order to test the hypothesis $H_0:\theta=1.000$ hours, a sample of size n has been drawn from the population with an exponential distribution. For sample size n=25,(5),75 and assuming $\alpha=0.050$ the CR's are calculated. With the same CR, α's are calculated when the true parent population is a Weibull distribution. Table 5.2a shows the effect on type I error α of assuming T has an exponential distribution when T has a Weibull distribution, using one gauge set at time T such that P=p. P is the probability of an item nonsurvive prior to time T. Table 5.2b shows the effect on α of assuming T has an exponential distribution when T has a Weibull distribution, using two gauges set at time T_1 and T_2 such that $P_1=P_3=p$.

As we can see from Table 5.2a and 5.2b an R test which is based on the two gauges is less sensitive than that based on one gauge to the departure from assumption about a known distribution as exponential. Therefore, we can say that the test using two gauges is more robust then the test using one gauge.

Table 5.2a Effect on α of assuming T has an exponential distribution when T has a Weibull distribution, one gauge set at time T.

n	25	30	35	40	45	50	55	60	65	70	75
CR	6	7	8	9	10	11	12	13	14	15	16
p	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.14	0.14	0.14	0.14
β											
exp.	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
1.1	0.0157	0.0146	0.0136	0.0128	0.0120	0.0113	0.0107	0.0101	0.0095	0.0091	0.0086
1.2	0.0037	0.0031	0.0026	0.0923	0.0019	0.0017	0.0015	0.0013	0.0011	0.0010	0.0009
1.3	0.0006	0.0005	0.0003	0.0003	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001

Table 5.2b Effect on α of assuming T has an exponential distribution when T has a Weibull distribution, two gauges set at T_1 and T_2.

n	25	30	35	40	45	50	55	60	65	70	75
CR	3	4	5	6	7	8	9	10	11	12	12
p	0.09	0.12	0.16	0.20	0.23	0.27	0.30	0.34	0.38	0.41	0.38
β											
exp.	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
1.1	0.0328	0.0353	0.0373	0.0390	0.0405	0.0418	0.0430	0.0441	0.0451	0.0460	0.0453
1.2	0.0195	0.0233	0.0265	0.0294	0.0320	0.0343	0.0365	0.0385	0.0404	0.0421	0.0408
1.3	0.0103	0.0142	0.0178	0.0212	0.0245	0.0276	0.0305	0.0332	0.0359	0.0383	0.0365
1.4	0.0047	0.0078	0.0112	0.0147	0.0182	0.0216	0.0250	0.0284	0.0316	0.0347	0.0324
1.5	0.0018	0.0038	0.0065	0.0096	0.0130	0.0165	0.0202	0.0239	0.0276	0.0312	0.0285
1.6	0.0005	0.0016	0.0034	0.0059	0.0089	0.0122	0.0159	0.0198	0.0238	0.0279	0.0248
1.7	0.0001	0.0006	0.0016	0.0034	0.0058	0.0089	0.0123	0.0162	0.0204	0.0248	0.0215

5.3 Cost function.

Suppose the total cost associated with an observation is represented by an equation:

$$C= a_1n + a_2nt$$

The constant a_1 denotes the cost per item in the sample. This could be the cost of the sample unit, the part of the cost of test equipment which depends on the number of units tested, etc. The constant a_2 denotes the cost per unit item in a unit time of conducted observation. It could represent the cost of carrying out the observations, the cost incurred from waiting for the result, etc.

We have seen in Chapter 4 that for a given α and β in a sampling plans we can have several pairs of n and T that satisfy the required conditions. A minimum sample size n occured when the gauge is set on the optimal position as in Chapter 3. However, we cannot say whether or not this plan is suitable one. In such a situation, perhaps a plan reflecting costs other than those associated with the sample size may be of more interest. In other words we can chose a plan that can minimize the total cost of the observations. For example, suppose a single sampling plan consists of a sample of size n drawn from an exponential distribution. Let us suppose that we wish the probability of acceptance of a batch of an average quality θ_0=1.000

hours to be about 95%. At the same time, we wish the probability of acceptance of a batch of an average quality θ_1=700 hours to be about 5%. Hence, the plan specifies α=0.05 and β=0.05. Let us also suppose a_1=£1.50 and a_2=£0.50. Table 5.3a shows the total cost of the observations using one gauge for each pair of n and T that satisfy the above specification. We can see from this Table that the minimum cost of £38,578.49 is achieved when n=1021 and T=72.57 hours. Compare this with the much greater total cost of £70,538.94 when the gauge is set on the optimal position;i.e T=1049.82 and n=134.

Table 5.3a Total cost of each plan using one gauge (single sampling plan).

n	T(hours)	cost(£)
1,426	51.29	38,708.77
1,190	61.88	38,603.60
1,021	**72.57**	**38,578.49**
894	83.38	38,611.86
795	94.31	38,680.73
717	105.36	38,847.06
652	116.53	38,966.78
599	127.83	39,183.59
553	139.26	39,334.89
514	150.82	39,531.74

When the two gauges are used in a sinlge sampling plan, the minimum cost of the observation is achieved when both gauges are set at the same position;i.e p_1=p_3=0.50. This would appear to suggest that we should use one gauge rather than two gauges, but since one gauge is very sensitive to the assumption about the distribution of the parent population, this is probably not a practical proposition. On the other hand, when the two gauges are set on time T1 and T2 such that $p_1=p_3\leq 0.36$, then the lengthtime of the observation being conducted is quite long. This of cause is something we are trying to avoid as far as possible. Therefore, for any practical purpose it might be a good compromise to set the two gauges at time T_1 and T_2 such that $0.36<p_1=p_3<0.50$. As an illustration of this, consider the case in the one gauge example. Table 5.3b shows the total cost of the observations using two gauge for each pair of n and T that satisfy given specifications. The minimum total cost is equal to £54,995.85 achieved when the two gauges are set in the same position, i.e $p_1=p_3=0.50$ ($T_1=T_2=693.15$ hours) which is equivalent to using one gauge such that the probability of an item failing prior to time T (position of the gauge) is p=0.50.

Table 5.3b Total cost of each plan using two gauges (single sampling plan).

n	T(hours)	cost(£)
126	1,139.43	71,973.09
127	1,108.66	70,590.41
128	1,049.82	67,380.48
129	1,021.65	66,089.93
130	994.25	64,821.25
132	967.58	64,058.28
133	941.61	62,816.56
134	916.29	61,592.43
136	891.60	60,832.80
138	867.50	60,064.50
140	843.97	59,287.90
142	820.98	58,502.58
144	798.51	57,708.72
147	776.53	57,295.46
149	755.02	56,472.49
152	733.97	56,009.72
155	713.35	55,517.12
158	**693.15**	**54,995.85**

Chapter 6.

SUGGESTION FOR FURTHER WORK.

In this thesis, we have considered the use of one and two gauges in life testing. We have considered the use of gauges to test hypotheses about the mean life time of a Weibull distribution. Since the exponential distribution is a special case of the Weibull distribution it will be covered in the study. As Hirji and Shahani (1978) pointed out, it is shown that the test is of higher efficiency and greater robustness when based on two gauges than when based on one gauge. Therefore it would be reasonable to expect that the performance of a test would further improve if we use more gauges.

We have considered the using gauges in both a single sampling plan and a double sampling plan. By reference to the solution in a single sampling plan and using a random walk diagram we might be able to predict the required parameters for a double sampling plan. It is shown that for a given set of conditions the minimum sample size reqiured is achieved when the gauges are set around the optimal position. It is also shown that although the use of two gauges substantially improved the performance of the test, in some cases it will probably be more economical to use one gauge, particularly when the cost per unit time is quite large. Since the required sample size for two gauges is much less than for one gauge, it would be interesting to investigate the use of a gauge when the quality of items is not based on the time term.

Perhaps gauges can be used to test hypotheses about the other parameters of parent population. Shahani (1979) has used gauges for testing hypotheses about correlation coefficient in a bivariate normal case. But as Hirji(1979) recorded, apart from Stevens(1948) work on estimation of the variance, no work has been yet done on testing hypotheses about variance using gauges. In a life testing context it would be interesting to investigate the use of gauges on testing hypotheses about the other parameters of life time such as shape and scale parameters. Since our considerations have been limited only to certain distribution; i.e Weibull distribution, it would be interesting to investigate more generally the design and use of gauges in other life time distributions.

It would be interesting to investigate the use of gauges in several dimensions random variable.

Appendix 1.

Gaussian Quadrature.

The idea behind Gaussian Quadrature is to find an integration formula:

$$I(f)=\int_a^b w(x)f(x)dx$$

by,

$$I_n(f)=\sum_{j=1}^{n} w_j(x)f(x_j).$$

The weights w_j and nodes x_j are restricted to be real, and nodes must belong to the interval of integration. The weight function should ,be nonnegative and satisfy the hypotheses:

1. $\int_a^b |x|^n w(x)dx$, is integrable and finite for all $n \geq 0$
2. Suppose that $\int_a^b w(x)g(x)dx=0$ for some nonnegative continuous function then the function $g(x)\equiv 0$ on (a,b).

The weights w_j and nodes x_j are determine such that the error

$$E_n(f) = I(f) - I_n(f) = 0$$

This will be achieved for as high a degree polynomial $f(x)$ as possible.

As an illustration, consider the special case

$$\int_{-1}^{1} f(x)dx = \sum_{j=1}^{n} w_j f(x) \qquad (1)$$

The weights w_j and nodes x_j are to be determined to make the error $E_n(f)$ equal to zero. To derive equations for the nodes and weights, we first note that

$$E_n(a_0+a_1x+.....+a_mx^m) = a_0E_n(1)+a_1E(x)+.......+a_mE_n(x^m).$$

Thus $E_n(f)=0$ for every polynomial of degree $\leq m$ if and only if

$$E_n(x^i)=0 \qquad i=0,1,.....,m.$$

Now, suppose $n=1$, then we have two parameters w_1 and x_1. Since there are two parameters we consider requiring

$$E_1(1)=0 \text{ and } E_1(x)=0$$

This gives

$$\int_{-1}^{1} 1dx - w_1=0 \quad \text{and} \quad \int_{-1}^{1} xdx - w_1x_1=0$$

This implies $w_1=2$ and $x_1=0$. Thus the formula (1) becomes

$$\int_{-1}^{1} f(x)dx = 2f(0)$$

which is similar to midpoint rule.

Similarly, when n=2 then we will have four parameters w_1, w_2, x_1, x_2 and thus we put four constraints on these parameters:

$$E2(x^i)=\int_{-1}^{1} x^i dx - (w_1x_1{}^i+w_2x_2{}^i)=0 \qquad i=0,1,2,3.$$

or

$$w_1+w_2=2$$
$$w_1x_1+w_2x_2=0$$
$$w_1x_1{}^2+w_2x_2{}^2=2/3$$
$$w_1x_1{}^3+w_2x_2{}^3=0$$

These nonlinear equations have the unique solution

$$w_1=w_2=1 \quad \text{and} \quad x_2=-x_1=\sqrt{3}/3$$

and the formula (1) becomes

$$\int_{-1}^{1} f(x)dx = f(-\sqrt{3}/3) + f(\sqrt{3}/3)$$

For a general n thereare 2n parameters w_j and x_j, hence the equations to be solved are

$$E_n(x^i)=0 \qquad i=0,1,\dots\dots\dots,2n-1$$

or

$$\sum_{j=1}^{n} w_j x_j^{\,i} = \begin{cases} 0 & i=1,3,\dots\dots,2n-1 \\ 2/(i+1) & i=0,2,\dots\dots,2n-2 \end{cases}$$

Table I shows the values of weights w_j and nodes x_j for n=2,(1),10 for Gaussian integration. The details of the weights and nodes values can be seen in some references such as Abramowitz and Stegun (1976).

In our case we take n=7 to calculate the integral P(X) for a normal distribution. The list of the computer program used to calculate the OC curve using normal approximation is given as an example.

To obtain some intuition the degree of precision of this method, the integral P(X) of a normal standard have been calculated for x=0,(0.1),3. The results are compare with the integral P(X) in Pearson & Hartley as shown in the Table II.

Table I. Nodes and weight factors for Gaussian integration.

Nodes ($\pm x_j$)	n	Weight factors (w_j)
0.57735 02691 89626	2	1.00000 00000 00000
0.00000 00000 00000 0.77459 66692 41483	3	0.88888 88888 88889 0.55555 55555 55556
0.33998 10435 84856 0.86113 63115 94053	4	0.65214 51548 62546 0.34785 48451 37454
0.00000 00000 00000 0.53846 93101 05683 0.90617 98459 38664	5	0.56888 88888 88889 0.47862 86704 99366 0.23692 68850 56189
0.23861 91860 83197 0.66120 93864 66265 0.93246 95142 03152	6	0.46791 39345 72691 0.36076 15730 48139 0.17132 44923 79170
0.00000 00000 00000 0.40584 51513 77397 0.74153 11855 99393 0.94910 79123 42759	7	0.41795 91836 73469 0.38183 00505 05119 0.27970 53914 89277 0.12948 49661 68870
0.18343 46424 95650 0.52553 24099 16329 0.79666 64774 13627 0.96028 98564 97536	8	0.36268 37833 78362 0.31370 66458 77887 0.22238 10344 53374 0.10122 85362 90376
0.00000 00000 00000 0.32425 34234 03809 0.61337 14327 00590 0.83603 11073 26636 0.96816 02395 07626	9	0.33023 93550 01260 0.31234 70770 40003 0.26061 06964 02935 0.18064 81606 94857 0.08127 43883 61574
0.14887 43389 81631 0.43339 53941 29247 0.67940 95682 99024 0.86506 33666 88985 0.97390 65285 17172	10	0.29552 42247 14753 0.26926 67193 09996 0.21908 63625 15982 0.14945 13491 50581 0.06667 13443 08688

Table II. The integral P(X) for a normal standard

X	P(X)	
	Pearson & Harteley	Gaussian quadrature
0.0	0.500 000 0	0.500 000 0
0.1	0.539 827 8	0.539 827 8
0.2	0.579 259 7	0.579 259 7
0.3	0.617 911 4	0.617 911 4
0.4	0.655 421 7	0.655 421 7
0.5	0.691 462 5	0.691 462 5
0.6	0.725 746 9	0.725 746 9
0.7	0.758 036 3	0.758 036 3
0.8	0.788 144 6	0.788 144 6
0.9	0.815 939 9	0.815 939 9
1.0	0.841 344 7	0.841 344 7
1.1	0.864 333 9	0.864 333 9
1.2	0.884 930 3	0.884 930 3
1.3	0.903 199 5	0.903 199 5
1.4	0.919 243 3	0.919 243 3
1.5	0.933 192 8	0.933 192 8
1.6	0.945 200 7	0.945 200 7
1.7	0.955 434 5	0.955 434 5
1.8	0.964 069 7	0.964 069 7
1.9	0.971 283 4	0.971 283 4
2.0	0.977 249 9	0.977 249 9
2.1	0.982 135 6	0.982 135 6
2.2	0.986 096 6	0.986 096 6
2.3	0.989 275 9	0.989 275 9
2.4	0.991 802 5	0.991 802 5
2.5	0.993 790 3	0.993 790 3
2.6	0.995 338 8	0.995 338 8
2.7	0.996 533 0	0.996 533 0
2.8	0.997 444 9	0.997 444 9
2.9	0.998 134 2	0.998 134 2
3.0	0.998 650 1	0.998 650 1

```
09  REM  ********************************************
10  REM  * PROG OC-CURVE FOR SINGLE SAMPLING PLAN. *
20  REM  * USING BINOMIAL AND NORMAL APPROXIMATION *
25  REM  ********************************************
30  DIM PO(7),WG(7)
40  INPUT"MEAN LIFE TIME H0";M0
50  INPUT"LOWER MEAN LIFE TIME H1";M1
60  INPUT"STEP";S
70  INPUT"SAMPLE SIZE";N
80  INPUT"CRITICAL POINT";C
90  INPUT"GAUGE POSITION";P
100 INPUT"SHAPE PARAMETER";B0
110 PRINT"WHICH ONE DO YOU LIKE"
120 PRINTTAB(5)"1.BINOMIAL"
130 PRINTTAB(5)"2.NORMAL"
140 INPUT TAB(10)"Type 1 or 2";A%
150 PRINT'
160 IF A%=1 THEN PROCbin
170 IF A%=2 THEN PROCnor
180 END
190 REM*******************d*i*n*a*r**********************
200 DEF PROCnor
210 P1=1-P
220      X0=LN(1/P1)
230      X1=LNX0/B0+LNM0
240      T=EXPX1
250 @%=&2020A
260 PRINT"T=";T
270 @%=10
280 R=C+.5
290 FOR K0=M0 TO M1 STEP -S
300    H=T/K0
310    Q1=EXP-H
320    Q=1-Q
330    M=SQR(N*Q*Q1)
340    Z=(R-N*Q)/M
350 IF Z<0 THEN PROCneg ELSE PROCpos
360 PRINTTAB(10)"CH.=";K0;
370 @%=&2030A
380 PRINTTAB(30)"OC1=";OC1
390 @%=10
400 NEXT K0
450 ENDPROC
460 REM*******************m*a*s*s*a*u*********************
470 DEF PROCbin
480 P1=1-P
490      X0=LN(1/P1)
500      X1=LNX0/B0+LNM0
510      T=EXPX1
520 @%=&2020A
530 PRINT"T=";T
540 @%=10
550 FOR K0=M0 TO M1 STEP -S
560    H=T/K0
570    Q1=EXP-H
```

```
580    Q=1-Q
590  P0=0
600  FOR I=0 TO C
610    P0=FNG(I)*Q^I*Q1^(N-I)+P0
620  NEXT I
630  PRINTTAB(10)"CH.=";K0;
640  @%=&2030A
650  PRINTTAB(30)"OC1=";OC1
670  @%=10
680  NEXT K
694  ENDPROC
700  REM********************r*a*f*l*l*l*a*********************
710  DEF PROCpos
720  I=7
730  B=Z
740  P0(1)=0.0:P0(2)=.230458315955135:P0(3)=.448492751036447
750  P0(4)=.642349339440340:P0(5)=.801578090733310
760  P0(6)=.917598399222978:P0(7)=.984183054718588
770  WG(1)=.232551553230874:WG(2)=.226283180262897
780  WG(3)=.207816047536889:WG(4)=.178145980761946
790  WG(5)=.138873510219787:WG(6)=.092121499837728
800  WG(7)=.040484004765316
810  SUM=0
820  FOR J=1 TO I
830   X=((B-A)*P0(J)+A+B)/2
840   Y=((A-B)*P0(J)+A+B)/2
850  SUM=SUM+WG(J)*FNF(X)
860  IF P0(J)<>0 THEN SUM=SUM+WG(J)*FNF(Y)
870  NEXT J
880  OC1=(B-A)*SUM/2+.5
890  ENDPROC
900  REM*****************e*i*s*e*n*h*a*r*t*********************
910  DEF FNG(I)
920  SUM=1
930  IF I=0 THEN GOTO 970
940  FOR L+1 TO I
950    SUM=(N-(L-1))*SUM/L
960  NEXT L
970  =SUM
980  REM********************************************************
990  DEF PROCneg
1000  DEF PROCpos
1010  I=7
1020  B=-Z
1030  P0(1)=0.0:P0(2)=.230458315955135:P0(3)=.448492751036447
1040  P0(4)=.642349339440340:P0(5)=.801578090733310
1050  P0(6)=.917598399222978:P0(7)=.984183054718588
1060  WG(1)=.232551553230874:WG(2)=.226283180262897
1070  WG(3)=.207816047536889:WG(4)=.178145980761946
1080  WG(5)=.138873510219787:WG(6)=.092121499837728
1090  WG(7)=.040484004765316
1100  SUM=0
1110  FOR J=1 TO I
1120   X=((B-A)*P0(J)+A+B)/2
1130   Y=((A-B)*P0(J)+A+B)/2
```

```
1140  SUM=SUM+WG(J)*FNF(X)
1150  IF PO(J)<>0 THEN SUM=SUM+WG(J)*FNF(Y)
1160  NEXT J
1170  OC1=.5-(B-A)*SUM/2
1180  ENDPROC
1190  REM*****************************************************
1200  DEF FNF(X)
1210  S1=1/(SQR(2*PI)
1220  T0=X^2/2
1230  =S1*EXP-T0
```

Appendix 2.

GINOGRAF.

GINOGRAF was developed at the Computer Aid Design Centre, Cambridge. It is a library of subroutine used in conjuction with the graphics package GINO-F.

GINO-F stands for graphical input/output-FORTRAN version. It is a graphics package that takes the form of a library of drawing and administrative subroutines. Most of the routines are written in standard ANSI FORTRAN making GINO-F virtually independent. GINO-F is also device independent a change to one line of a user program being all that is required to convert the program to produce output on a different device. The routines in GINO-F that produce this output are code generators, there being one for each device available on each line.

GINOGRAF has facilities for producing graphs, histogram, bar charts and pie charts by two different methods. The first method,is produced the graph by a simple single call routine which automatically performs all the scaling and annotation. The second method, is produced the graph built up from series of routines which allow the user to define each aspect of the graph and axis system independently. A set of defaults is available for items not explicity set by the user program.

GINOGRAF assumes the GINO-F defaults for all drawing. Thus graphs are drawn in unilimeters. The space co-ordinate system is the same as the picture co-ordinate system with the origin at the bottom left-hand corner of the device, The X axis horizontal and the Y axis vertical. The graphical axis system is with respect to this space co-ordinate system. GINOGRAF has a set of defaults for any aspect of the built-up axis system which have not been defined by the user, for example the position and scaling of a pictural axis. The axis system of the composite routines, which is provided automatically, is made up of these default.

The list of the computer program using GINOGRAF used to draw the graphs of the density function and the failure rate function for the Weibull and Ramberg distributions is given as an example.

In this program, several routines have been used, they are:

- axis definition
- axis drawing
- graphical drawing.

The axis definition consists of two routines which define the position and scaling of an axis. The position of the axis is defined by:

AXIPOS(IOR,XOR,YOR,AXLEN,IXORY)

The current and the length of axis are specified by IXORY and AXLEN. For the current X axis IXORY=1 and the current Y axis IXORY=2. The IOR indicates the starting point (XOR,YOR). If IOR=1, the axis starts at the point (XOR,YOR) and if IOR=0 then the axis is positioned such that the point (XOR,YOR) is at the natural origin as defined in the data.

The scale of axis is defined by:

AXISCA(ISCALE,NINTS,VBEG,VEND,IXORY).

This routine gives a choice of linear,log or histogram scales indicated by ISCALE. The axis including the step of interval (NINTS) and a range of data values specified by VBEG,VEND.

The axis drawing is to draw an axis with or without tick marks and scale values. It will depend on the values of ITICK and IVAL. The axis drawing is specified by:

AXIDRA(ITICK,IVAL,IXORY).

The graphical drawing represent the data in a graph form. The data may be represented in a number of ways, such as:

- points joints by stright lines.
- points joints by a smooth curve.
- symbol at the points.
- histogram.
- bar charts.

In our case, the data are representad by a smooth curve and specified by routine:

GRACUR(X,Y,NPTS).

GRACUR draws a smooth curve through a number of points (NPTS) in arrays X and Y.

Before any of GINOGRAF routines is called, the output device must be nominated. The following calls to GINO-F subroutines as device nomination:

-CALL SAVDRA for the plotter

-CALL APDS4 for the Imlac 3205 terminal

-CALL T4010 for the Tektronix 4010 terminal.

A call to subroutine DEVEND should be used to terminate graphical output in each case.

```
C  **********************************************************
C  *PROGRAM USING GINOGRAF TO DRAW WIEBULL AND RAMBERG PDF*
C  *HONEYWELL TERMINAL                                     *
C  **********************************************************
*#FRN=(ULIB)LIBRARY/GINOGRAF;LIBRARY/GINO-F#A"01";B"02";C"03";D"04";
*#E"05";F"06";A0"07";B0"09";C0"10";D0"11";E0"12";F0"13";DINAR"08"
      DIMENSION X1(100),X2(100),X3(100),X4(100),X5(100),X6(100),X7(100)
      DIMENSION X8(100),X9(100),X10(100),X11(100),X12(100),X13(100)
      DIMENSION Y1(100),Y2(100),Y3(100),Y4(100),Y5(100),Y6(100),Y7(100)
      NN=12
      DO 1 I=1,25
      READ(1,101)X,Y
  101 FORMAT(V)
      X1(I)=X
      Y1(I)=Y
      N1=I
    1 CONTINUE
      DO 2 I=1,30
      READ(2,101)X,Y
      X2(I)=X
      Y2(I)=Y
      N2=I
    2 CONTINUE
      DO 2 I=1,30
      READ(2,101)X,Y
      X2(I)=X
      Y2(I)=Y
      N2=I
    2 CONTINUE
      DO 3 I=1,30
      READ(3,101)X,Y
      X3(I)=X
      Y3(I)=Y
      N3=I
    3 CONTINUE
      DO 3 I=1,30
      READ(3,101)X,Y
      X3(I)=X
      Y3(I)=Y
      N3=I
    3 CONTINUE
      DO 4 I=1,30
      READ(4,101)X,Y
      X4(I)=X
      Y4(I)=Y
      N4=I
    4 CONTINUE
      DO 5 I=1,30
      READ(4,101)X,Y
      X5(I)=X
      Y5(I)=Y
      N5=I
    5 CONTINUE
```

```
      DO 6 I=1,30
      READ(6,101)X,Y
      X6(I)=X
      Y6(I)=Y
      N6=I
    6 CONTINUE
      DO 7 I=1,30
      READ(7,101)X,Y
      X7(I)=X
      Y7(I)=Y
      N7=I
    7 CONTINUE
      DO 8 I=1,30
      READ(9,101)X,Y
      X9(I)=X
      Y9(I)=Y
      N9=I
    8 CONTINUE
      DO 9 I=1,30
      READ(10,101)X,Y
      X10(I)=X
      Y10(I)=Y
      N10=I
    9 CONTINUE
      DO 10 I=1,30
      READ(11,101)X,Y
      X11(I)=X
      Y11(I)=Y
      N11=I
   10 CONTINUE
      DO 11 I=1,30
      READ(12,101)X,Y
      X12(I)=X
      Y12(I)=Y
      N12=I
   11 CONTINUE
      DO 12 I=1,30
      READ(13,101)X,Y
      X13(I)=X
      Y13(I)=Y
      N13=I
   12 CONTINUE
      CALL SAVDRA
      CALL CHASWI(1)
      CALL PICCLE
      CALL DEVPAP(1000.,280.,0)
      CALL AXIPOS(1,10.,60.,80.,1)
      CALL AXIPOS(1,10.,60.,90.,2)
      CALL AXISCA(1,.01,0.,1.6,2)
      CALL AXISCA(1,.01,0.,3.,1)
      CALL AXIDRA(1,1,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRACUR(X1,Y1,25)
      CALL GRACUR(X2,Y2,30)
      CALL GRACUR(X3,Y3,30)
```

```
      CALL GRACUR(X4,Y4,30)
      CALL GRACUR(X5,Y5,30)
      CALL GRACUR(X6,Y6,30)
      CALL AXIPOS(1,105.,60.,80.,1)
      CALL AXIPOS(1,105.,60.,90.,2)
      CALL AXISCA(1,.1,0.,2.,1)
      CALL AXISCA(1,.01,0.,5.,2)
      CALL AXIDRA(1,1,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRACUR(X7,Y7,20)
      CALL GRACUR(X9,Y9,20)
      CALL GRACUR(X10,Y10,20)
      CALL GRACUR(X11,Y11,20)
      CALL GRACUR(X12,Y12,20)
      CALL GRACUR(X13,Y13,20)
      CALL MOVTO2(15.0,45.)
      CALL CHAHOL('Fig.2.1a.Density Function*.')
      CALL MOVTO2(105.0,45.)
      CALL CHAHOL('Fig.2.1b.Failure Rate Function*.')
      CALL DEVEND
      STOP
      END
```

Appendix 3.

Computer Listing for Sampling Plan.

```
 10 REM ****************************************
 20 REM *PROG FOR PLAN OF SINGLE SAMPLING PLAN*
 30 REM ****************************************
 40 INPUT"SHAPE PARAMETER";B0
 50 INPUT"MEAN LIFE TIME M0";M0
 60 INPUT"ABSISCA FOR A GIVEN ALPHA";A
 70 INPUT"ABSISCA FOR A GIVEN BETA";B
 80 PRINT'
 90 PRINT"DO YOU WISH ANOTHER ALPHA AND BETA?"
100 IF GET$="Y" THEN 40
110 PRINTTAB(5)"SELECT ONE PLEASE:"
120 PRINTTAB(5)"1.ONE GAUGE"
130 PRINTTAB(5)"2.TWO GAUGES"
140 INPUT TAB(5),"TYPE 1 or 2";A%
150 PRINT'
160 IF A%=1 THEN PROCone
170 IF A%=2 THEN PROCtwo
180 REM******************************************
190 DEF PROCone
200 PRINTTAB(15)"sample size for a single sampling plan using one gauge"
210 FOR P=.5 TO .75 STEP .01
210 P1=1-P
220   X0=LN(1/P1)
230   X1=LNX0/B0+LNM0
240   T=EXPX1
250 @%=&2020A
260 PRINTTAB(0)"p=";P;TAB(10)"T=";T
270 @%=10
280 FOR M1=700 TO 350 STEP-50
290    H=(T/M1)^B0
300    Q1=EXP-H
310    Q=1-Q1
320    X=B*SQR(Q*Q1)-A*SQR(P*P1)
350    N=X^2/(P-Q)^2
360
370 @%=&2020A
380 PRINT N;
390 @%=10
400 NEXT M1
410 NEXT P
420 PRINTTAB(25)"Do you like another procedure"
430 IF GET$="Y" THEN 90
440 PRINT
450 END
460 REM***********************************************************
```

```
470 DEF PROCtwo
480 PRINTTAB(15)"sample size for single sampling plan using two gauges"
490 FOR P1=.25 TO .46 STEP .01
500 P3=1-P1
510   C0=LN(1/P3)
520   C1=LN(1/P1)
530   Y0=LNC0/B0+LNM0
540   Y1=LNC1/B0+LNM0
550   T1=EXPY0
560   T2=EXPY1
570 @%=&2020A
580 PRINTTAB(0)"p=";P1;TAB(10)"T1=";T1;TAB(25)"T2=";T2
590 @%=10
600 FOR M1=700 TO 350 STEP -50
610     G=(T1/M1)^B0
620     G0=(T2/M1)^B0
630     Q1=1-EXP-G
640     Q3=EXP-G0
650     D=(Q3+Q1)-(Q3-Q1)^2
660 N=(A*SQR(2*P1)-B*SQRD)^2/(Q3-Q1)^2
670 @%=&2020A
680 PRINT N;
690 @%=10
700 NEXT M1
710 NEXT P1
720 PRINTTAB(25)"Do you want another procedure"
730 IF GET$="Y" THEN GOTO 90
740 PRINT
750 END
```

```
 10 REM ********************************************************
 20 REM *PROG FOR PLAN OF DOUBLE SAMPLING PLAN USING ONE GAUGE*
 30 REM ********************************************************
 40 INPUY"SHAPE PARAMETER";B5
 50 INPUT"GAUGE POSITION";P5
 60 INPUT"SAMPLE SIZE (SINGLE)";N
 70 INPUT"RATIO FOR THE FIRST SAMPLE";R1
 80 INPUT"FIRST ACC. NUMBER";F
 90 INPUT"FIRST REJ. NUMBER";S
100 INPUT"SECOND ACC. NUMBER";C
110 INPUT"MEAN LIFE TIME H0";M0
120 INPUT"LOWER MEAN LIFE TIME H1";M1
130 INPUT"STEP";S5
140  N0=R1*N
150  N1=INT(N0)
160  N3=N-N1
170  D=F+1
180  G=S-1
190  P6=1-P5
200   X0=LN(1/P6)
210   X1=LNX0/B5+LNM0
240   T=EXPX1
250 PRINT"N1=";N1;
250 @%=&2020A
260 PRINTTAB(15)"T=";T
270 @%=10
280 FOR N2=N3 TO 2*N1
290 PRINT"N2=";N2
300 FOR R=M0 TO M1 STEP -S5
310  P=1-EXP(-T/R)
320 Pr=0
330 FOR J=D TO G
340   Q=C-J
350   SUM=1
360   A1=(1-P)^N2
370 P1=0
380 FOR K=1 TO Q
390   SUM=(N2-(K-1))/K*SUM
400   P1=SUM*P^K*(1-P)^(N2-K)+P1
410 NEXT K
420 B1=A1+P1
430 P2=FNF(J)*P^J*(1-P)^(N1-J)
440 B2=B1*P2
450 Pr=Pr+B2
460 NEXT J
470    A=(1-P)^N1
480    X=1
490    P0=0
500    IF F=0 THEN GOTO  550
510 FOR I=1 TO F
520    X=(N1-(I-1))/I*X
530    P0=X*P^I*(1-P)^(N1-1)+P0
540 NEXT I
```

```
550 B0=P0+A
560 OC=Pr+B0
570 @%=&2040A
580 PRINT OC;
590 @%=10
600 NEXT R
610 PRINT'
620 NEXT N2
630 PRINT
640 END
650 REM*****************************************************
660 DEF FNF(J)
670 M=1
680 IF J=0 THEN 720
690 FOR L=1 TO J
700  M=(N1-(L-1))/L*M
710 NEXT L
720 =M
```

References

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