

UNIVERSITY OF SOUTHAMPTON
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OPTIMUM ELASTIC DESIGN OF PRESTRESSED CONCRETE
STRUCTURES WITH PARTICULAR REFERENCE TO
STATICAL INDETERMINACY

By

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ABSTRACT

FACULTY OF ENGINEERING AND APPLIED SCIENCE

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OPTIMUM ELASTIC DESIGN OF PRESTRESSED CONCRETE STRUCTURES

WITH PARTICULAR REFERENCE TO STATICAL INDETERMINACY

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The thesis illustrates a general study for prestressed concrete structures. The main object is to develop an optimum solution for statically indeterminate prestressed concrete structures and provide an improved approach for the design engineer. A quadratic programming method is used to develop this approach.

A simple set of expressions is developed for the elastic design of both statically determinate and indeterminate prestressed concrete structures.

A computer program for the design of indeterminate prestressed concrete structures has been developed based on this analysis.

A brief experimental study was carried out on both the elastic and ultimate stages of statically indeterminate prestressed concrete frames and arches.

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INTRODUCTION

Since prestressed concrete was first used as a new technique in structural engineering, research workers in this field have been seeking an optimum design for such structures.

The optimum design is the design which for all cases of loading of the structure leads to the minimum section sizes giving the most economical form. This design should lead to stresses in the structure near or equal to the maximum allowable stresses, and should also be simple.

Morice²⁷ (1956) has proposed direct design methods for both statically determinate and indeterminate structures. He also established optimum designs for prestressed concrete sections and statically indeterminate prestressed concrete structures.

For statically indeterminate structures Guyon¹⁴ (1953) established the idea of the concordant profile and certain theorems concerning such profiles have been enounced by Morice and Lewis²⁷ (1956). This idea of a concordant profile leads conveniently to the solution of statically indeterminate prestressed concrete structures.

Lin T.Y.²³ (1963) introduced the load balancing method for design and analysis of prestressed concrete structures as an attempt to balance a portion of the load on the structure so that the flexural members would not be subjected to bending stresses under a

given loading condition. However, this method is usually only successful for structures subjected to one case of loading.

The aim of this thesis is to continue these researches into finding an optimum solution for prestressed concrete statically indeterminate structures and to provide an improved approach for the engineering design.

Chapter 1 gives a general background for the section design and an illustration of the load balancing method. It also discusses the optimum design of the statically determinate structures for both full and partial prestressed sections.

Chapter 2 presents a general background and description for the solution of statically indeterminate prestressed concrete structures.

Chapter 3 includes an optimum solution for statically indeterminate prestressed concrete structures.

Chapter 4 includes a theoretical description of the experimental work.

Chapter 5 discusses the experimental work for the statically indeterminate prestressed concrete frames and arches.

Chapter 6 includes a description of a prestressed concrete computer program with solved examples.

Chapter 7 presents the conclusions and proposals for further work.

CHAPTER (1)

1. PRESTRESSED SECTION DESIGN

1.1 GENERAL DESIGN PRINCIPLES

The basic idea of prestressed concrete is to use the elastic tensile force of stressed steel tendons (with a reasonable factor of safety) to produce compressive stresses in the concrete to minimize, as much as possible, the tensile concrete stresses caused by the applied loads.

In general, the centre of thrust of the prestressing force at some arbitrary section of a beam will be eccentric to the centroid of the section (Fig. 1.1). This will induce two superimposed stress systems:

Firstly, a uniformly distributed compressive stress $= H/A$ due to the direct action of the prestressing force over the section area A .

Secondly, the stress distribution produced by the moment M , where $M = H.e$ and e is the eccentricity of the force from the centroid.

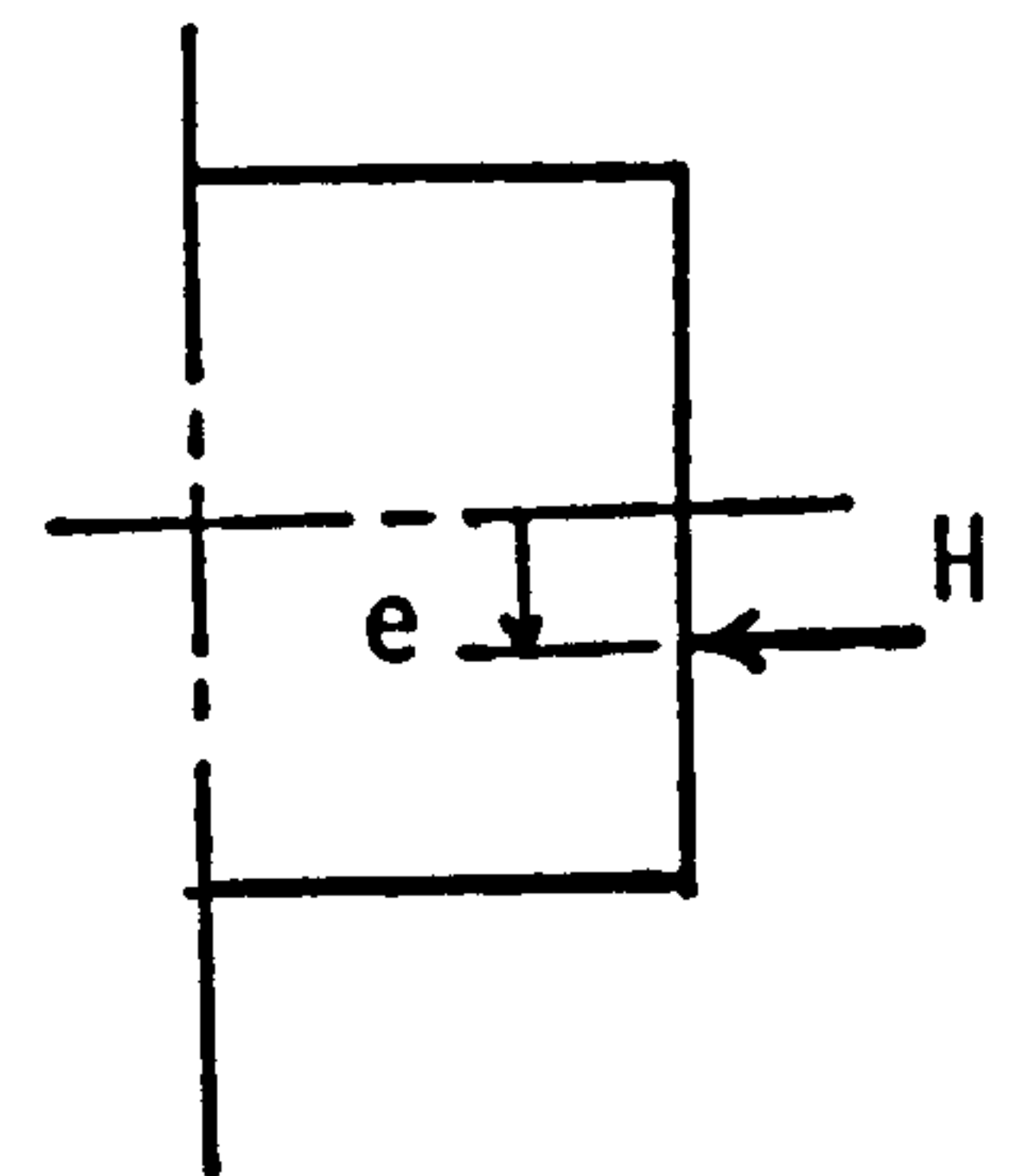


Fig. 1.1

The resultant effect is shown by the summation of the two diagrams Fig. 1.2c.

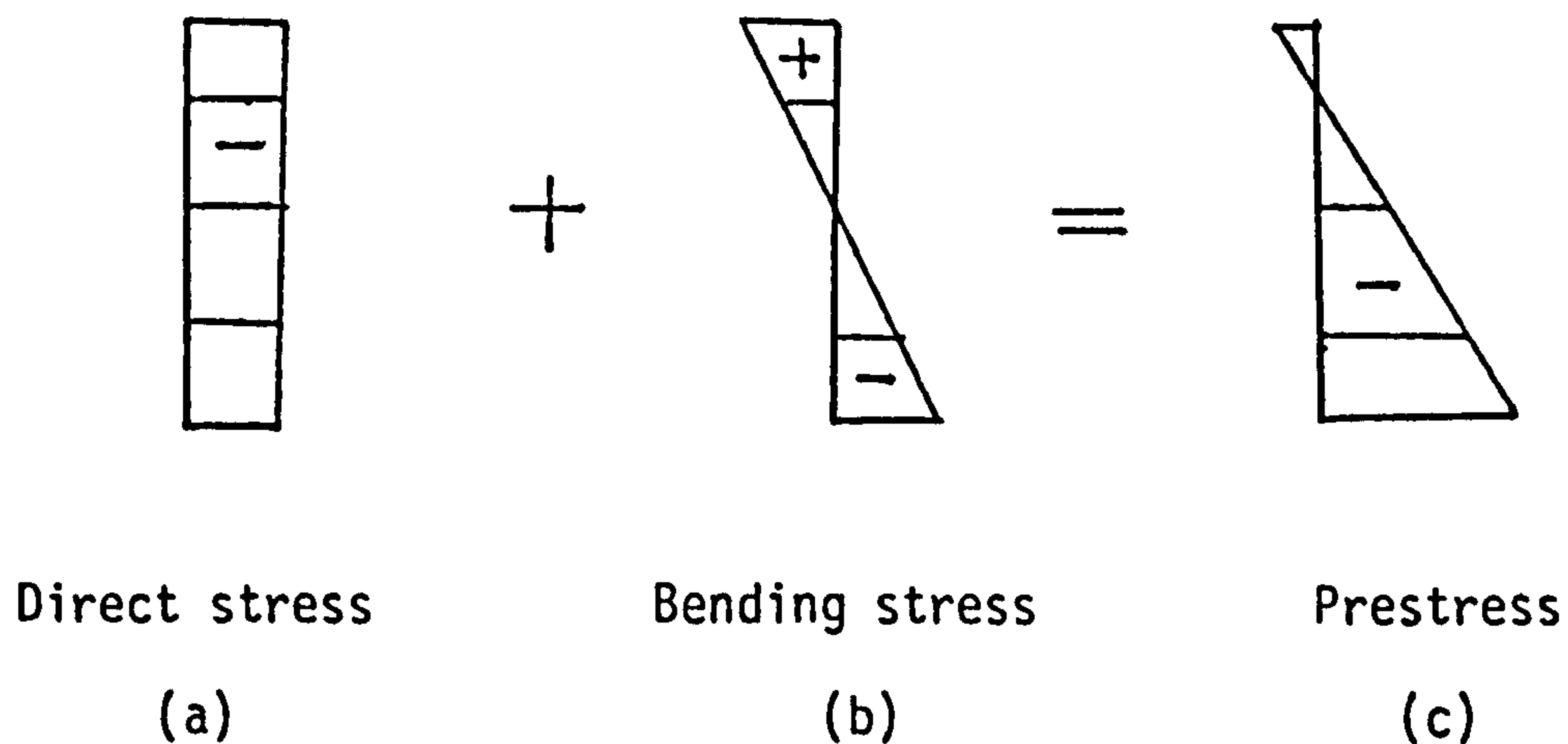


Fig. 1.2

To describe the effect of this resultant on a prestressed beam, we use the 'load balancing' method as an illustration. In this method, the prestressing stresses balance the stresses due to the applied loads so that the beam will not be subjected to bending stresses under a given loading condition. Thus, a structure carrying transverse loads is subjected only to axial stresses.

For example, in Fig. 1.3, a uniformly distributed load on a simple beam can be balanced by the transverse component from the tendon so that the beam is subjected only to a net axial force, which is given by:

$$H = F \cos \theta$$

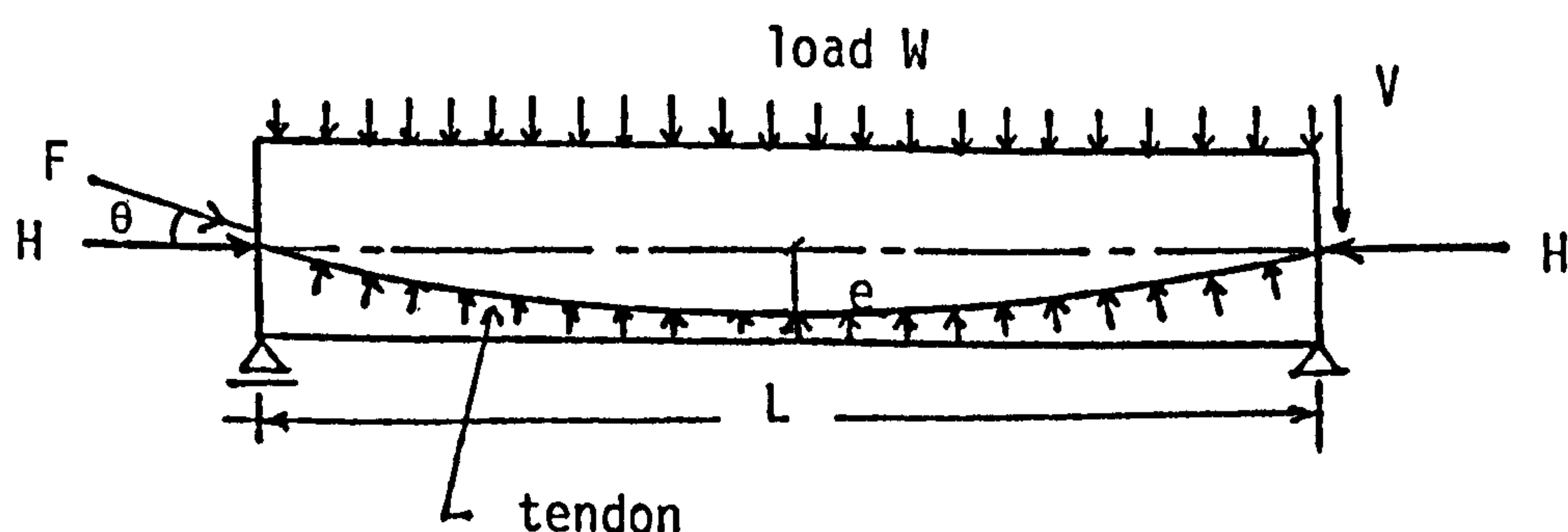


Fig. 1.3

Hence the stresses f in the concrete at any section are given by:

$$f = \frac{H}{A}$$

where A is the area of the concrete at the section.

1.2 THE LOSSES

Reduction in the initial value of the prestressing force occurs after the prestressing due to various effects caused by shrinkage, creep, elongation of the tendons, slip of the anchorage, ... etc.

The loss of forces depends on the time. The final stresses that are required in the structure in working conditions should be calculated for the reduced prestressed force H . Thus, at the time of prestressing the state of the structure will differ from the working design conditions due to an increased value H_i of H .

It is, therefore, important to investigate the stress conditions of this initial state so that it may be ensured that the short life initial stresses do not exceed the working stresses by more than a safe margin.

Let the initial prestressing force have as value $H_i = kH$ where $k > 1$.

If α is the factor representing the losses, then the final force becomes

$$H = (1 - \alpha)H_i = \frac{1}{k} H_i$$

For example if $\alpha = 20\%$

$$\text{then } k = \frac{1}{1 - \alpha} = \frac{1}{0.80} = 1.25$$

Hence $H_i = 1.25 H$.

The stresses at any section due to H_i can, therefore, be evaluated.

1.3 LOADING CONDITIONS

The loading conditions of the structure have a great effect on the design procedure. The design of a prestressed section depends mainly upon the range of the moment variations rather than their absolute value. The moment conditions for any section of a structure are defined as follows:

M_D is the dead load moment (permanent moment) which remains constant and exists from the time of prestressing through the life of the structure.

M_1 is the maximum live load hogging moment defined to be positive.

M_2 is the maximum live load sagging moment defined to be negative.

The direct force in the members may be considered as:-

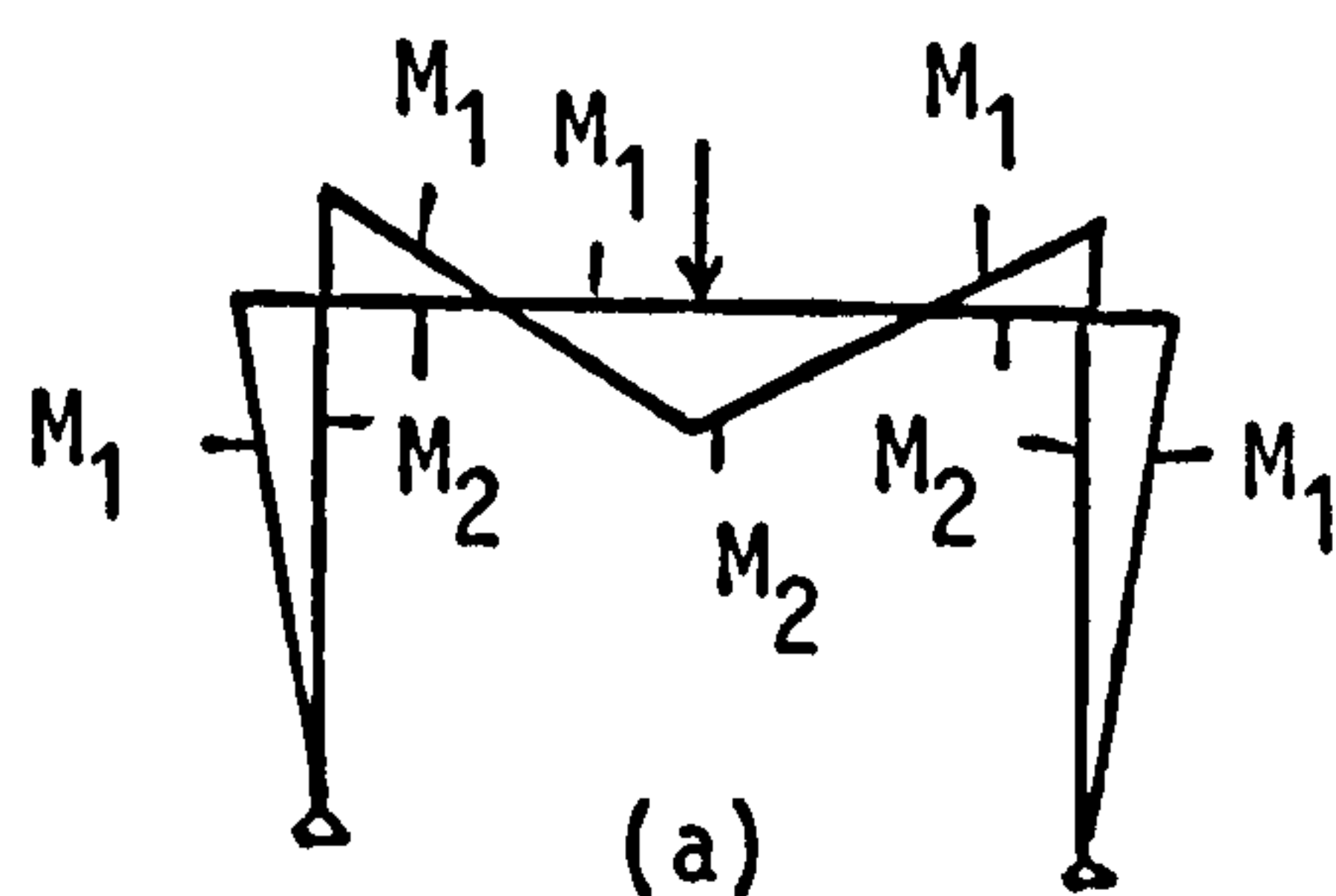
H_p due to prestressing force

H_D due to the dead load

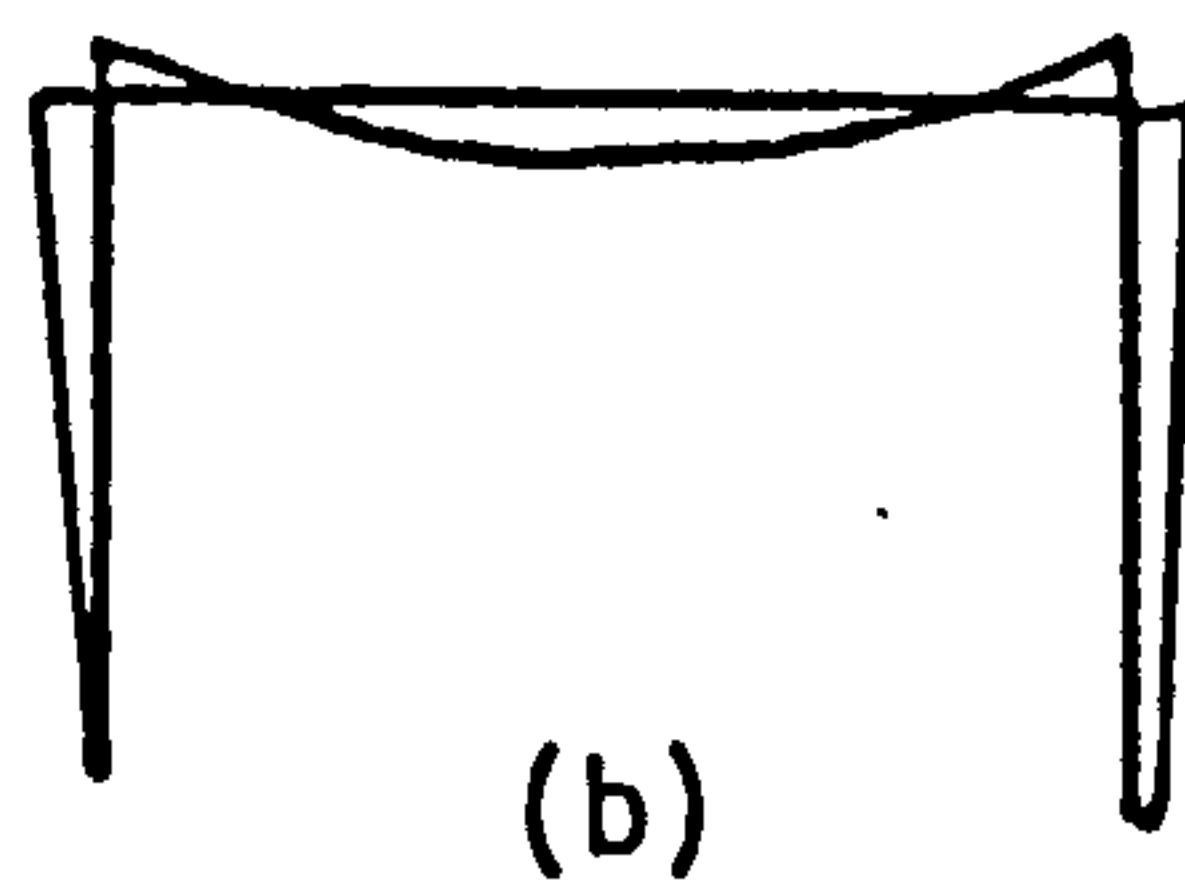
H_1 for maximum moment condition M_1 , and

H_2 for minimum moment condition M_2 .

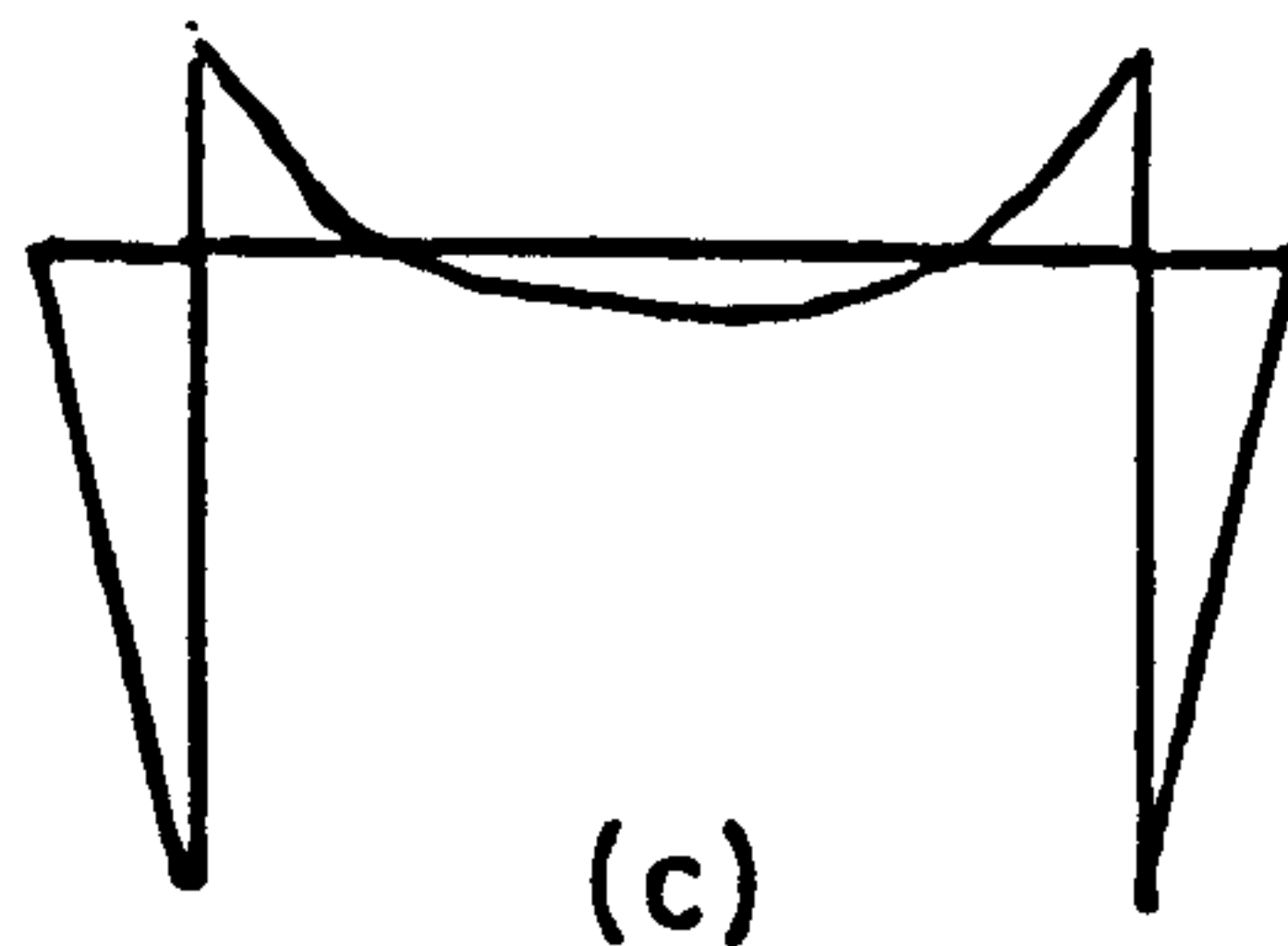
The moment conditions of a two-pinned portal frame (shown in Fig. 1.4) may be taken as an example.



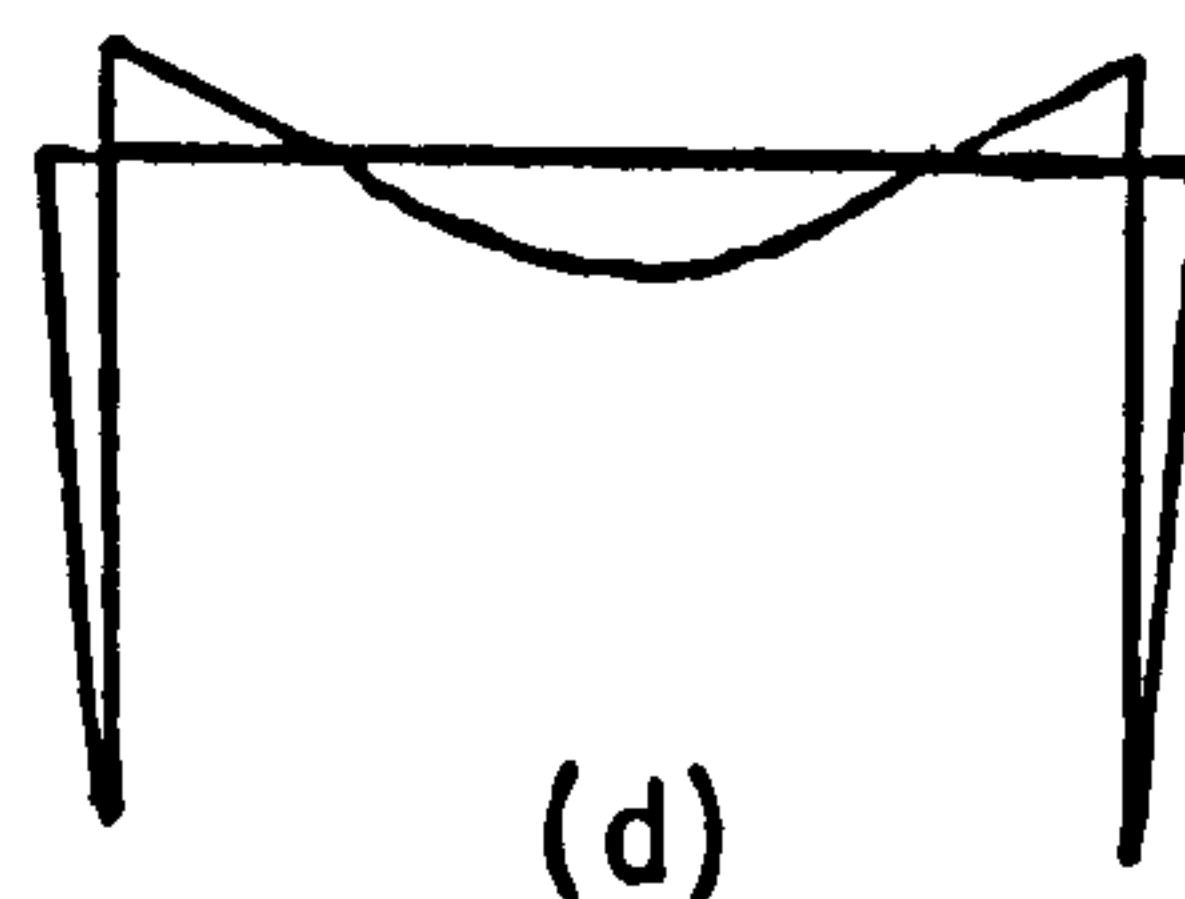
live load moment envelope



dead load moment M_D



$M_1 + M_D$



$M_2 + M_D$

Fig. (1.4) Moment diagrams for a two-pinned portal frame

1.4 PRESTRESSED SECTION DESIGN IN BENDING

(a) Partial Prestressed Section

Consider a section of a prestressed member (Fig. 1.5a) with depth d and its extreme fibres at distances $-y_1$ and y_2 from the centroidal axis.

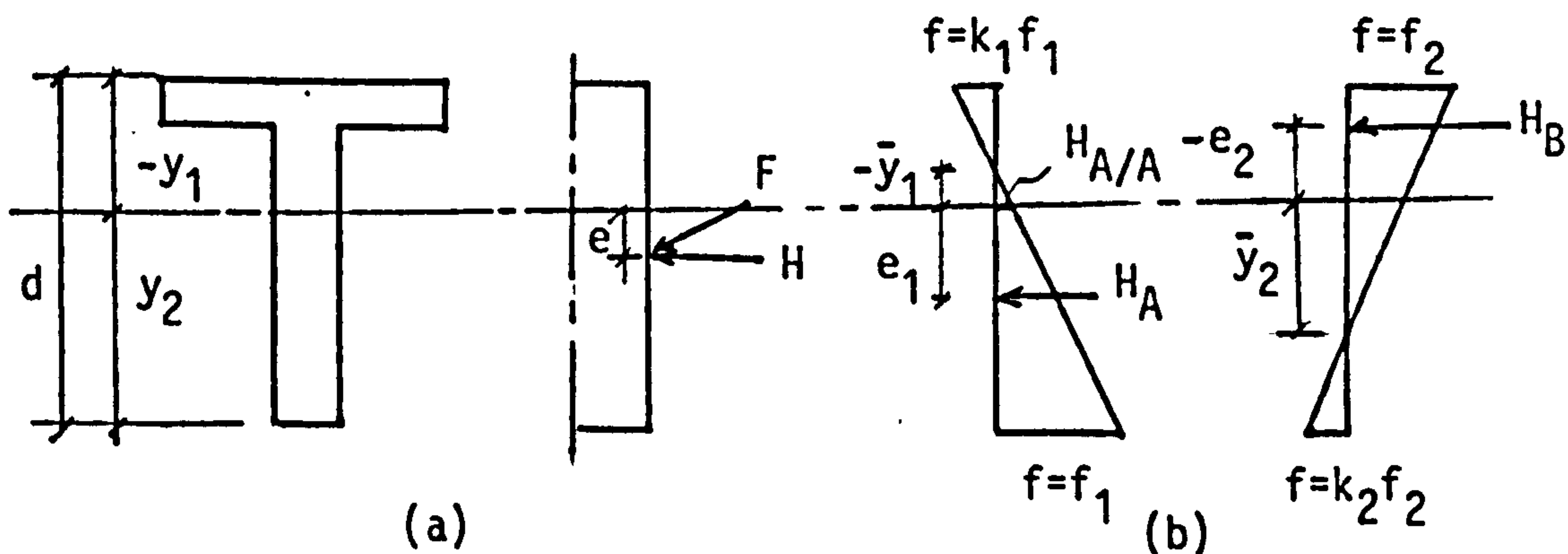


Fig. 1.5

If the section is subjected to a compressive force F whose horizontal component is H , which acts at distance e from the centroid, the total compression H in a member may be written for the loading $M_1 + M_D$ as follows:-

$$H = H_p + H_D + H_1 = H_A \text{ (say)} \quad 1.1$$

and for the loading $M_2 + M_D$:-

$$H = H_p + H_D + H_2 = H_B \text{ (say)} \quad 1.2$$

Then the longitudinal stress f at any point of the section, distance y from the centroid, is given by:-

$$f = H \left[\frac{1}{A} + \frac{y \cdot e}{I} \right] \quad 1.3$$

Where A is the area and I is the second moment of area of the beam section. The compressive stresses are taken as positive.

For a partial prestressed section, consider the values of the eccentricity e which will make $f = 0$ at distances $y = -\bar{y}_1$ and $y = \bar{y}_2$ from the centroid (Fig. 1.5b). These can be determined from equation 1.3 as follows:

$$\text{at } y = -\bar{y}_1 \quad e = \frac{I}{A\bar{y}_1} = e_1 \quad 1.4$$

$$\text{at } y = \bar{y}_2 \quad e = \frac{-I}{A\bar{y}_2} = -e_2 \quad 1.5$$

From Fig. 1.5b, the geometry of the diagram gives:

$$\bar{y}_1 = y_1 - \alpha d \quad 1.6$$

$$\bar{y}_2 = y_2 - \beta d \quad 1.7$$

$$\text{where } \alpha = \frac{k_1}{1+k_1} \rightarrow \beta = \frac{k_2}{1+k_2}$$

and k_1, k_2 are factors representing the allowable tensile stresses as a proportion of the allowable compressive stresses f_1 and f_2 . If the centre of pressure of the force H acts within the limits e_1 and $-e_2$, the stresses in the section will be within the allowable stress ranges.

NOTE: For concrete the tensile stresses would be small compared with the compressive values so that ($0 \leq k_1, k_2 \leq .25$)

(b) Fully prestressed section

For a fully prestressed section, the value of the eccentricity e which will make $f = 0$ (i.e. eliminate tensile stresses altogether), in the extreme fibres, where $y = -y_1$ and $y = -y_2$, can be directly determined by requiring $k_1 = 0$ and $k_2 = 0$.

Hence equations 1.4 and 1.5 become:

$$\text{at } y = -y_1, \quad e = \frac{I}{Ay_1} = e_1 \quad 1.8$$

$$\text{at } y = y_2, \quad e = -\frac{I}{Ay_2} = -e_2 \quad 1.9$$

If the centre of pressure of the force H acts within the limits e_1 and $-e_2$, the stresses in the beam section will be wholly compressive.

1.5 LIMITING ZONE FOR TENDON PROFILE

It is known that the effect of a moment M applied to a prestressed concrete section is equivalent to a displacement of the centre of pressure of the prestressed force by an amount M/H .

As shown in Fig. 1.6, using the loading states discussed above (section 1.3), it is possible to combine the actual prestressing tendon eccentricity e with the displacements of the centre of thrust due to extreme loading moments $M_D + M_1$ and $M_D + M_2$, so that the combined eccentricities are equal to the eccentricities e_1 and $-e_2$ evaluated in section 1.4 above, for both partial and fully prestressed sections.

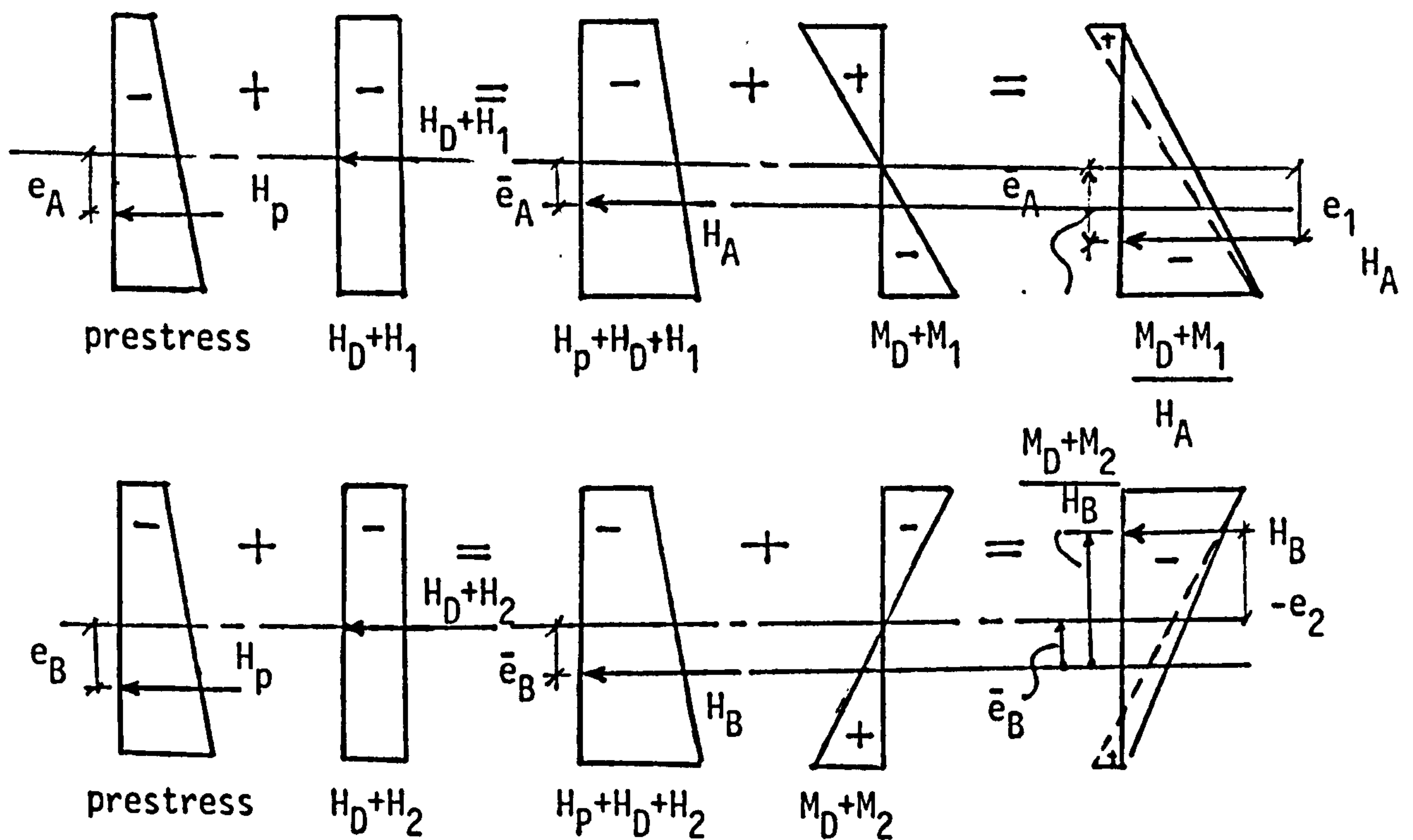


Fig. 1.6

For a partial prestressed section, therefore:

$$e_1 = \bar{e}_A + \frac{M_D + M_1}{H_A} \quad 1.10$$

$$-e_2 = \bar{e}_B + \frac{M_D + M_2}{H_B} \quad 1.11$$

Given that $e_1 = \frac{I}{A\bar{y}_1}$ $-e_2 = -\frac{I}{A\bar{y}_2}$ and

$$\bar{e}_A = e_A \left[\frac{H_p}{H_A} \right], \quad \bar{e}_B = e_B \left[\frac{H_p}{H_B} \right]$$

expressions 1.10 and 1.11 provide the two limits of the tendon eccentricity e . The lower limiting value of e which we will denote e_A is derived from 1.10 and the upper limiting value of e , denoted e_B , comes from 1.11.

Hence:

$$\text{lower limit } e_A = \frac{I}{A\bar{y}_1} \left[\frac{H_A}{H_p} \right] - \frac{M_D + M_1}{H_p} \quad 1.12$$

$$\text{upper limit } e_B = -\frac{I}{A\bar{y}_2} \left[\frac{H_B}{H_p} \right] - \frac{M_D + M_2}{H_p} \quad 1.13$$

For fully prestressed section, we obtain expressions of a similar form:

$$\text{lower limit } e_A = \frac{I}{Ay_1} \left[\frac{H_A}{H_p} \right] - \frac{M_D + M_1}{H_p} \quad 1.14$$

$$\text{upper limit } e_B = - \frac{I}{Ay_2} \left[\frac{H_B}{H_p} \right] - \frac{M_D + M_2}{H_p} \quad 1.15$$

Evidently the eccentricity e of an actual prestressing tendon may lie anywhere within the region e_A to e_B to ensure that the stresses in the section remain within the chosen stress range under any loading (for the given loading range). It is also clear that the values of e_A and e_B depend upon the applied loads and will vary along the length of the beam

(Note: For numerical calculations, we consider the absolute values of:

$$\frac{I}{Ay_1}, \frac{I}{Ay_2}, H_A, H_B, H_p,$$

and the signed values of M_D, M_1, M_2

according to the convention that hogging moments are positive and sagging moments negative.)

1.6. DESIGN IN BENDING

The design steps may be outlined as follows:-

- a) As discussed in Section (1.3) the distribution of bending moment and direct force due to known loading on statically determinate beam, frame or arch can be evaluated. Thus the live load moment range ($M_L = M_1 - M_2$) can be determined at all sections of the structure before the absolute section size is chosen.

b) The minimum size of beam, frame or arch section will be obtained when the width of the limiting zone is reduced to zero on at least one section along the length of the structure, i.e.

$$e_A - e_B = 0$$

For a partial prestressed section.

From equations 1.12 and 1.13, the width of the limiting zone reduces to zero when:

$$M_L = H_A e_1 + H_B e_2$$

$$M_L = \frac{I}{A} \left[\frac{H_A}{\bar{y}_1} + \frac{H_B}{\bar{y}_2} \right] \quad 1.16$$

Defining the maximum working stress to be f_c (assuming that this will be the stress f_1 , i.e. $f_1 = f_c$ and by taking the case illustrated in Fig. 1.5.

From the geometry of this diagram, we obtain:

$$H_A = \frac{\bar{y}_1}{d(1-\alpha)} A f_c \quad 1.17$$

Solving equations 1.1, 1.2, 1.16 and 1.17 gives:

$$M_L = \frac{I}{A[y_2 - \beta d]} \left[A f_c \frac{(1 - \alpha - \beta)}{(1-\alpha)} - H_1 + H_2 \right] \quad 1.18$$

The absolute section sizes can therefore be chosen on the basis of the most critical value of the compressive stress. This is given by equation 1.18.

- c) After choosing the absolute section size, the bending moment distribution, M_D , and the direct forces, H_D , can be determined.
- d) The prestressing force H_p which may have to vary along the length of the structure, if the structure cross section varies longitudinally, can be determined from the expression:

$$H_p = \frac{Af_c}{d} (y_1 - y_2 k_1) - H_D - H_1 \quad 1.19$$

- e) Statically Determinate Structures subject to Bending and Direct Axial force

At the points where the zone width is zero the tendon eccentricity e will be uniquely defined and is given by e_A or e_B . Taking the expression for M_L in equation 1.16 and substituting into equation 1.13 giving

$$e = - \left[\frac{M_1}{H_p} \left[\frac{(y_1 - \alpha d) H_B}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] + \frac{M_2}{H_p} \left[\frac{(y_2 - \beta d) H_A}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] + \frac{M_D}{H_p} \right] \quad 1.20$$

In the case when f_2 reaches the maximum working stress, i.e. $f_2 = f_c$, the design expressions may be rewritten.

$$M_L = \frac{I}{A(y_1 - \alpha d)} \left[\frac{Afc (1 - \alpha - \beta)}{(1-\beta)} + H_1 - H_2 \right] \quad 1.21$$

$$H_p = \frac{Afc}{d} (y_2 - y_1 k_2) - H_D - H_2 \quad 1.22$$

$$e = - \left[\frac{M_1}{H_p} \left[\frac{(y_1 - \alpha d) H_B}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] + \frac{M_2}{H_p} \left[\frac{(y_2 - \beta d) H_A}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] + \frac{M_D}{H_p} \right] \quad 1.23$$

Fully prestressed sections

The expressions for fully prestressed sections (no tensile stress) can be obtained by substituting $k_1 = k_2 = 0$ in the previous expressions.

In the case when f_1 reaches the maximum working stress, i.e. $f_1 = fc$, and $f_2 \leq fc$

$$M_L = \frac{I}{Ay_2} \left[Afc - H_1 + H_2 \right] \quad 1.24$$

$$H_p = \frac{y_1}{d} Afc - H_D - H_1 \quad 1.25$$

and

$$e = - \left[\frac{M_1}{H_p} \left[\frac{y_1 H_B}{y_1 H_B + y_2 H_A} \right] + \frac{M_2}{H_p} \left[\frac{y_2 H_A}{y_1 H_B + y_2 H_A} \right] + \frac{M_D}{H_p} \right] \quad 1.26$$

In the case when f_2 reaches the maximum working stresses f_c , i.e. $f_2 = f_c$, and $f_1 \leq f_c$:

$$M_L = \frac{I}{Ay_1} \left[Afc + H_1 - H_2 \right] \quad 1.27$$

$$H_p = \frac{y_2}{d} \times Afc - H_D - H_2 \quad 1.28$$

and

$$e = - \left[\frac{M_1}{H_p} \left[\frac{y_1 H_B}{y_1 H_B + y_2 H_A} \right] + \frac{M_2}{H_p} \left[\frac{y_2 H_A}{y_1 H_B + y_2 H_A} \right] + \frac{M_D}{H_p} \right] \quad 1.29$$

In using the above expressions, we must define which of f_1 or f_2 reaches the maximum working stresses according to the geometry of the section and values of H_1 and H_2 . This is done for the following cases

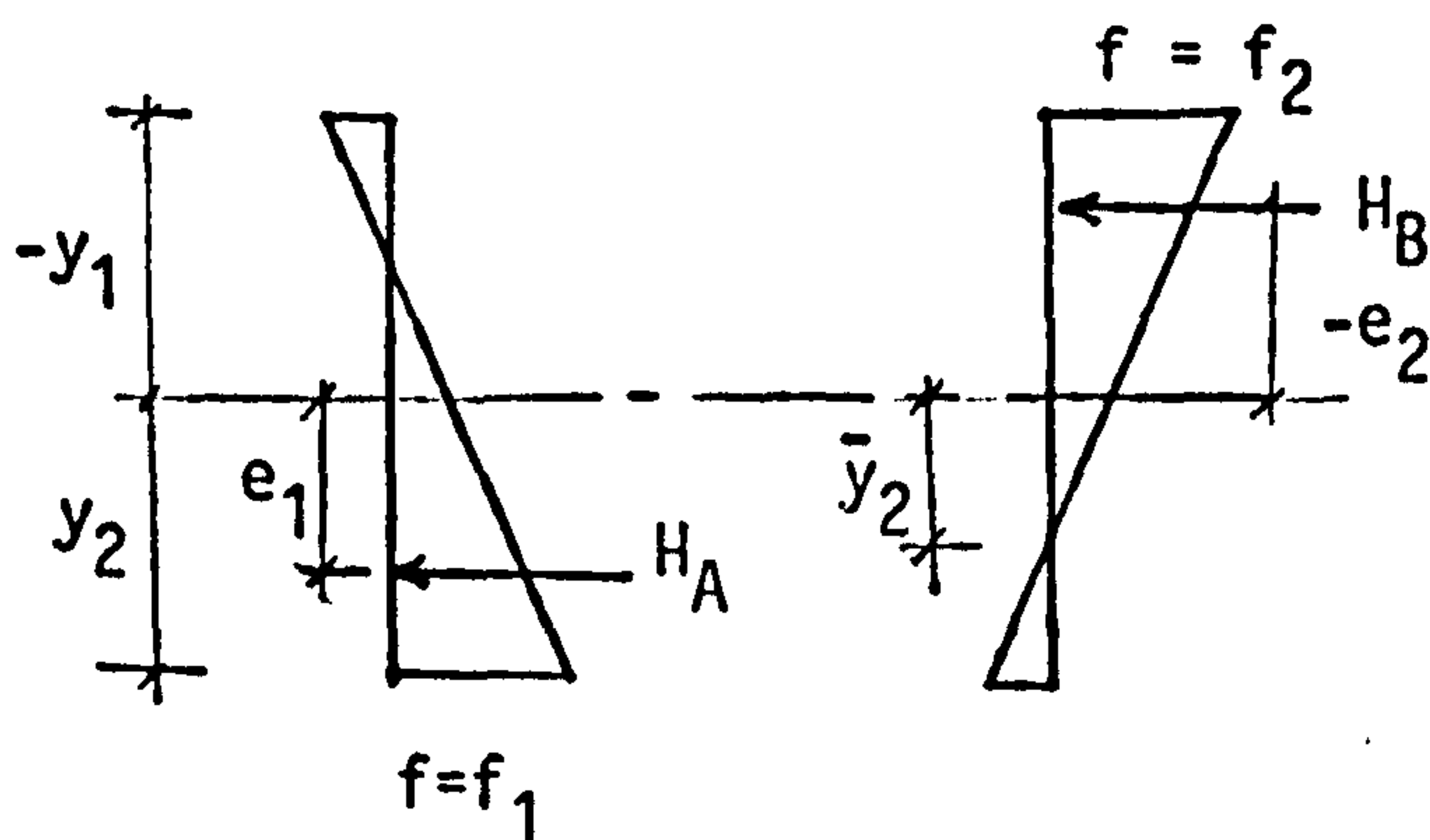


Fig. 1.7

- 1) If $y_1 = y_2$ and $H_2 > H_1$
 or $y_1 > y_2$ and $H_2 > H_1$
 or $y_1 > y_2$ and $H_2 = H_1$

Then expressions of $f_2 = f_c$ are to be considered

- 2) If $y_1 = y_2$ and $H_1 > H_2$
 or $y_2 > y_1$ and $H_1 > H_2$
 or $y_2 > y_1$ and $H_1 = H_2$

Then expressions of $f_1 = f_c$ are to be considered.

- 3) If $y_1 = y_2$ and $H_1 = H_2$

expressions of $f_1 = f_c$ or $f_2 = f_c$ are to be considered

- 4) If $y_1 > y_2$ and $H_1 > H_2$
 or $y_2 > y_1$ and $H_2 > H_1$

In this case, we substitute given data in the expression 1.18 (for partial prestressed sections) or expression 1.24 (for fully prestressed sections) to get the dimensions of the section and we compare them with the dimension given by expression 1.21 or 1.27 and hence find the design equations which give the maximum dimensions of the frame or arch sections.

Statistically Determinate Structures subject to Bending only

The design expressions can be obtained by substituting $H_1 = H_2 = H_D = 0$ in the previous expressions.

Partially Prestressed Section

In the case of an unsymmetrical section, in which $y_2 > y_1$, the design expressions are:

$$f_c = \left[\frac{M_L}{I} \left(\frac{y_2 - y_1 k_2}{1 - k_1 k_2} \right) \right]_{\max} \quad 1.30$$

$$H_p = \frac{A f_c}{d} (y_1 - y_2 k_1) \quad 1.31$$

$$e = - \left[\frac{M_D}{H_p} + \frac{1}{(1 - \alpha - \beta)} \left[\left(\frac{y_1}{d} - \alpha \right) \frac{M_1}{H_p} + \left(\frac{y_2}{d} - \beta \right) \frac{M_2}{H_p} \right] \right] \quad 1.32$$

In the case of a symmetrical section, where

$y_1 = y_2 = \frac{d}{2}$ and $k_1 = k_2 = k$, the design expressions may be rewritten as

$$f_c = \left[\frac{d \cdot M_L}{2I(1+k)} \right]_{\max} \quad 1.33$$

$$H_p = \frac{1}{2} A f_c (1 - k) \quad 1.34$$

$$e = - \left[\frac{M_D}{H_p} + \frac{M_1}{2H_p} + \frac{M_2}{2H_p} \right] \quad 1.35$$

In the case of an unsymmetrical section in which $y_1 > y_2$ the design expressions are:

$$f_c = \left[\frac{M_L}{I} \left(\frac{y_1 - y_2 k_1}{1 - k_1 k_2} \right) \right]_{\max} \quad 1.36$$

$$H_p = \frac{Afc}{d} (y_2 - y_1 k_2) \quad 1.37$$

$$e = - \left[\frac{M_D}{H_p} + \frac{1}{(1 - \alpha - \beta)} \left\{ \left(\frac{y_1}{d} - \alpha \right) \frac{M_1}{H_p} + \left(\frac{y_2}{d} - \beta \right) \frac{M_2}{H_p} \right\} \right] \quad 1.38$$

Fully Prestressed Section

The expressions for fully prestressed sections (no tensile stress) can be obtained by substituting $k_1 = k_2 = 0$ in the previous expressions.

In the case of an unsymmetrical section in which $y_2 > y_1$, the design expressions are:

$$z = \frac{I}{y_2} = \frac{M_L \max}{f_c} \quad 1.39$$

$$H_p = \frac{y_1}{d} Afc \quad 1.40$$

$$e = - \left[\frac{M_D}{H_p} + \frac{y_1}{d} \frac{M_1}{H_p} + \frac{y_2}{d} \frac{M_2}{H_p} \right] \quad 1.41$$

In the case of a symmetrical section where $y_1 = y_2 = \frac{d}{2}$, the design expressions may be rewritten as:

$$Z = \frac{2I}{d} = \frac{M_L}{f_c} \max \quad 1.42$$

$$H_p = \frac{1}{2} A f_c \quad 1.43$$

$$e = - \left[\frac{M_D}{H_p} + \frac{M_1}{2H_p} + \frac{M_2}{2H_p} \right] \quad 1.44$$

In the case of an unsymmetrical section in which $y_1 > y_2$, the design expressions are:

$$Z = \frac{I}{y_1} = \frac{M_L \max}{f_c} \quad 1.45$$

$$H_p = \frac{\bar{y}_2}{d} A f_c \quad 1.46$$

$$e = - \left[\frac{M_D}{H_p} + \frac{y_1}{d} \frac{M_1}{H_p} + \frac{y_2}{d} \frac{M_1}{H_p} \right] \quad 1.47$$

The previous expressions provide the optimal design of a statically determinate structure in bending.

1.7 DEAD LOAD LIMITS

The previous results have shown that the dead load moment has no influence upon the prestressed section size but it does affect the tendon position.

In general the tendon cannot be allowed to have an eccentricity which will take it outside the sections of the structures and in fact some cover must be provided. The dead load moment is limited to this extent. The value of the eccentricity e must not be less than $-a_1 y_1$ and not greater than $a_2 y_2$, where a_1 and a_2 are constants (<1) Fig. 1.8 ensuring that the tendon has adequate cover. Two limits to M_D can be obtained by substituting these value for e into expressions 1.20 these give.

For a partial prestressed section subject to bending and direct force.

$$M_D \geq -a_2 y_2 H_p - \left[\frac{(y_1 - \alpha d) H_B}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] M_1$$

$$- \left[\frac{(y_2 - \beta d) H_A}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] M_2$$

$$M_D \leq a_1 y_1 H_p - \left[\frac{(y_1 - \alpha d) H_B}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] M_1$$

$$- \left[\frac{(y_2 - \beta d) H_A}{(y_1 - \alpha d) H_B + (y_2 - \beta d) H_A} \right] M_2$$

1.48

and for fully prestressed section subject to bending and direct forces.

$$M_D \geq -a_2 y_2 H_p - \left[\frac{y_1 H_B}{y_1 H_B + y_2 H_A} \right] M_1 - \left[\frac{y_2 H_A}{y_1 H_B + y_2 H_A} \right] M_2$$

$$\begin{aligned}
M_D \leq a_1 y_1 H_B - \left(\frac{y_1 H_B}{y_1 H_B + y_2 H_A} \right) M_1 \\
- \left(\frac{y_2 H_B}{y_1 H_B + y_2 H_A} \right) M_2
\end{aligned} \quad 1.49$$

For a partial prestressed section subject to bending only.

$$\begin{aligned}
M_D \geq -a_2 y_2 H_p - \left(\frac{y_1 - \alpha d}{d (1 - \alpha - \beta)} \right) M_1 - \left(\frac{y_2 - \beta d}{d (1 - \alpha - \beta)} \right) M_2 \\
M_D \leq a_1 y_1 H_p - \left(\frac{y_1 - \alpha d}{d (1 - \alpha - \beta)} \right) M_1 - \left(\frac{y_2 - \beta d}{d (1 - \alpha - \beta)} \right) M_2
\end{aligned} \quad 1.50$$

and for fully prestressed section

$$\begin{aligned}
M_D \geq -a_2 y_2 H_p - \frac{y_1}{d} M_1 - \frac{y_2}{d} M_2 \\
M_D \leq a_1 y_1 H_p - \frac{y_1}{d} M_1 - \frac{y_2}{d} M_2
\end{aligned} \quad 1.51$$

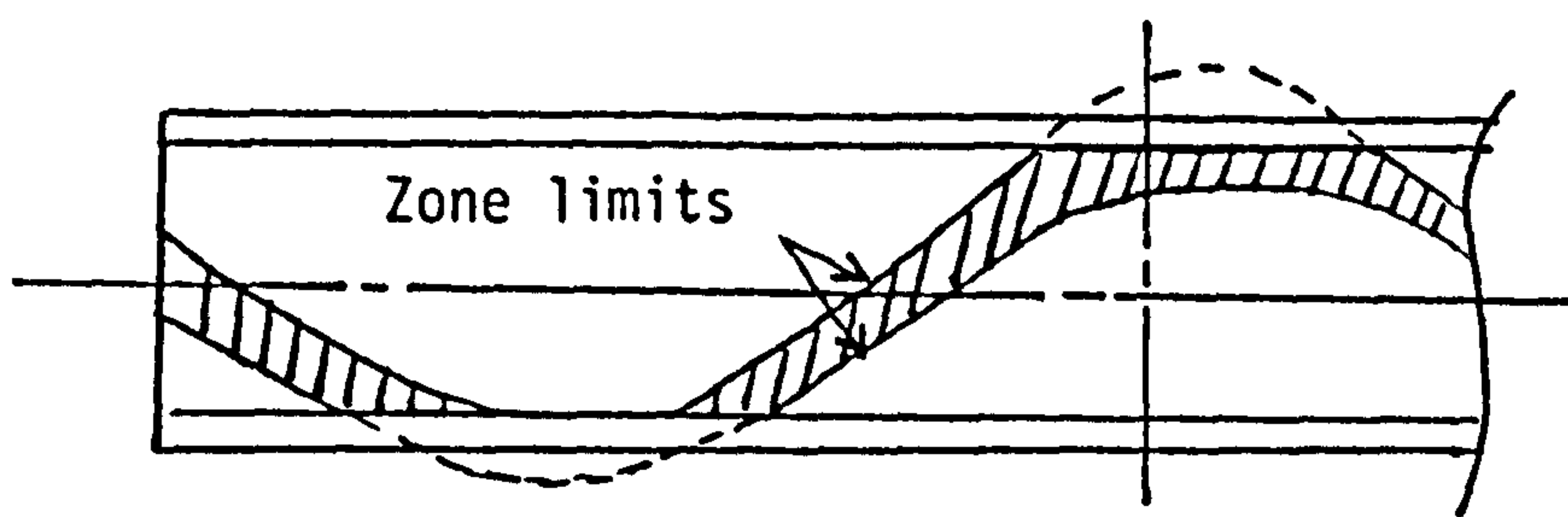
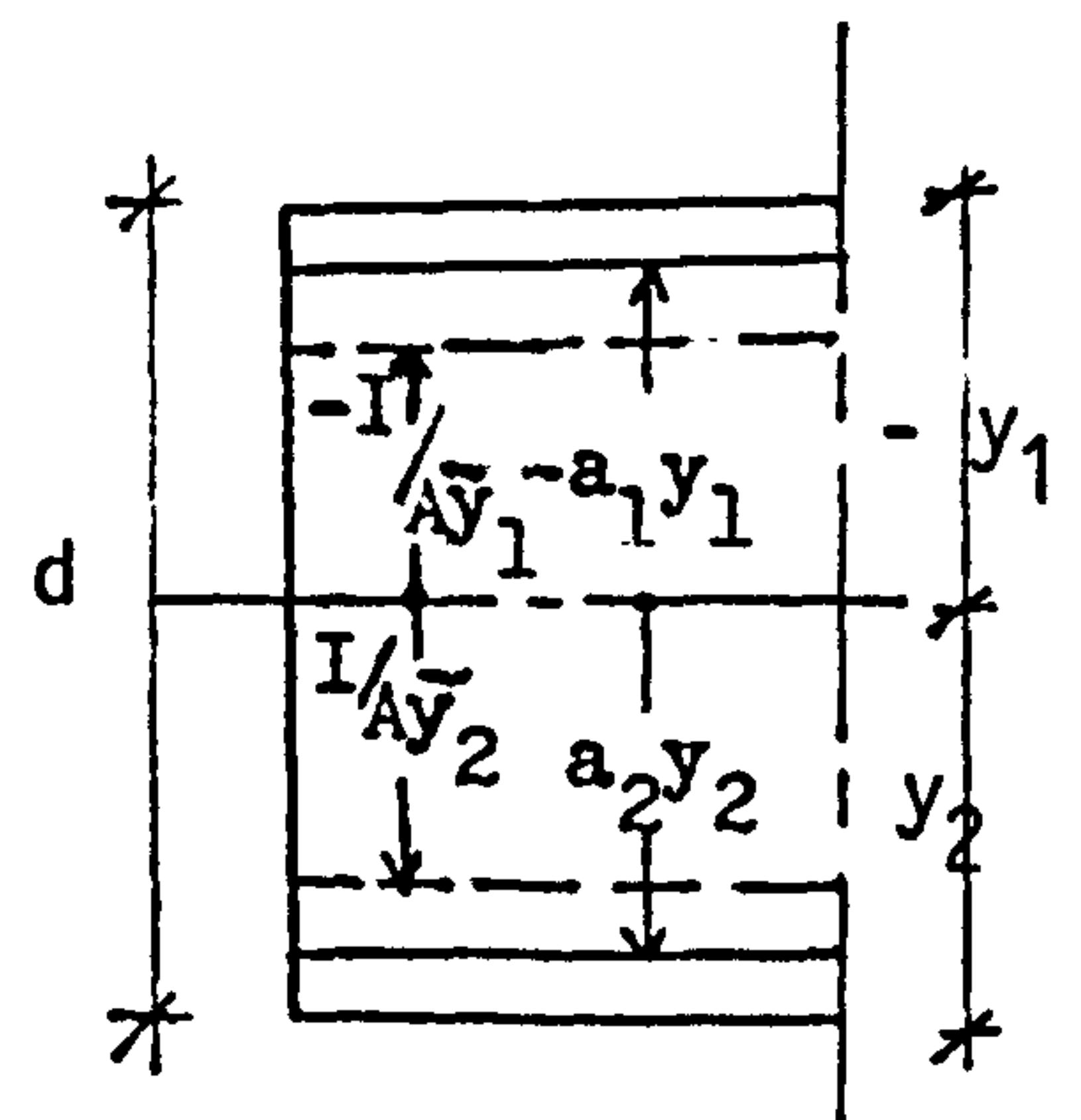


Fig. 1.8

Tendon Zone limits for a prestressed element with a dead load moment greater than the critical proportion (M_L)



For full prestressed section

$$\bar{y}_1 = y_1 \text{ and } \bar{y}_2 = y_2$$

1.8 Numerical Examples

The procedure of the design may be demonstrated by Numerical Examples:

Example 1

Consider a simple beam with cantliver, with the following data:

Simple span 9 m, cantliver span 3 m,

Rolling load 100 kN, working compressive beam

stress 1.4 kN/cm^2 and the beam density 25 kN/m^3

(no tensile stresses are allowed)

a) Live load moment envelope

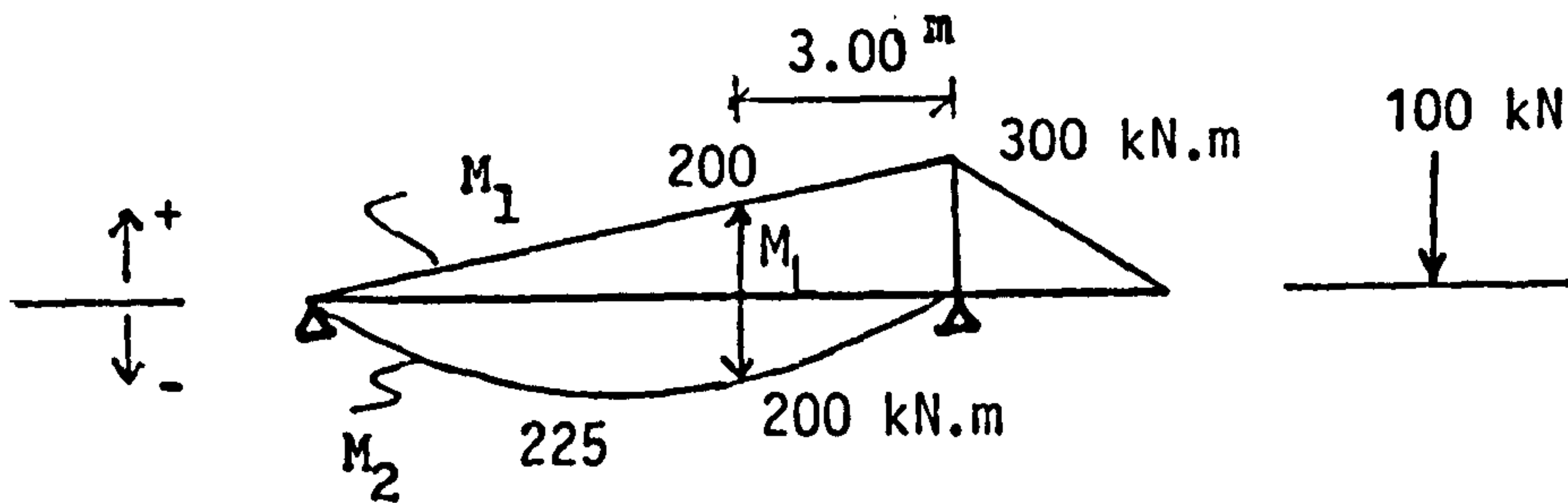


Fig. 1.9 Live load moment envelope

$$M_L (\text{max}) = M_1 - M_2 \text{ at } 3.0 \text{ m from the cantilever support}$$

$$= 200 - (-200) = 400 \text{ kN.m}$$

b) Section modules

$$Z = \frac{M_L}{f_c} = \frac{40000}{1.4} = 28571 \text{ cm}^3$$

taking a rectangular section and taking $b = 35 \text{ cm}$

$$\text{The value of the depth } d = \sqrt{\frac{6Z}{b}} = 70 \text{ cm}$$

c) Dead load moment

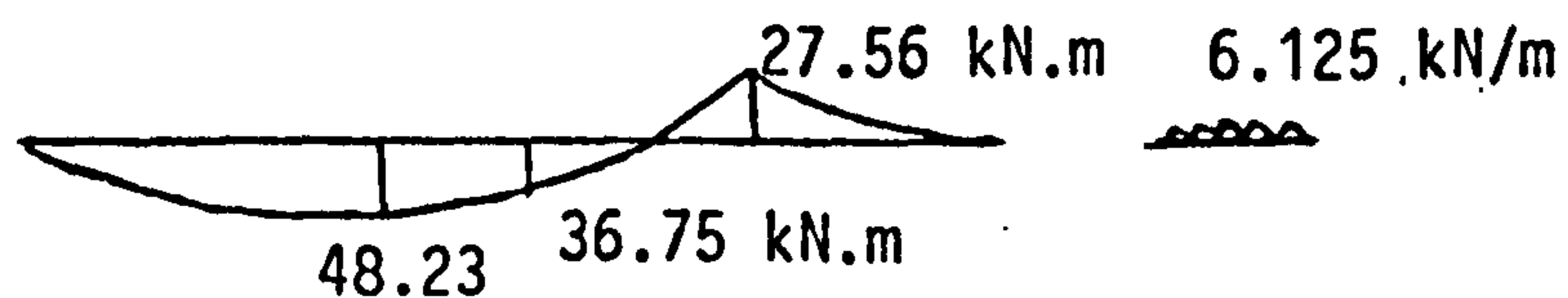


Fig. 1.10

d) The prestressing force:

$$H_p = \frac{1}{2} A_c f_c = \frac{1}{2} \times 35 \times 70 \times 1.4 = 1715 \text{ kN}$$

e) The tendon eccentricity:

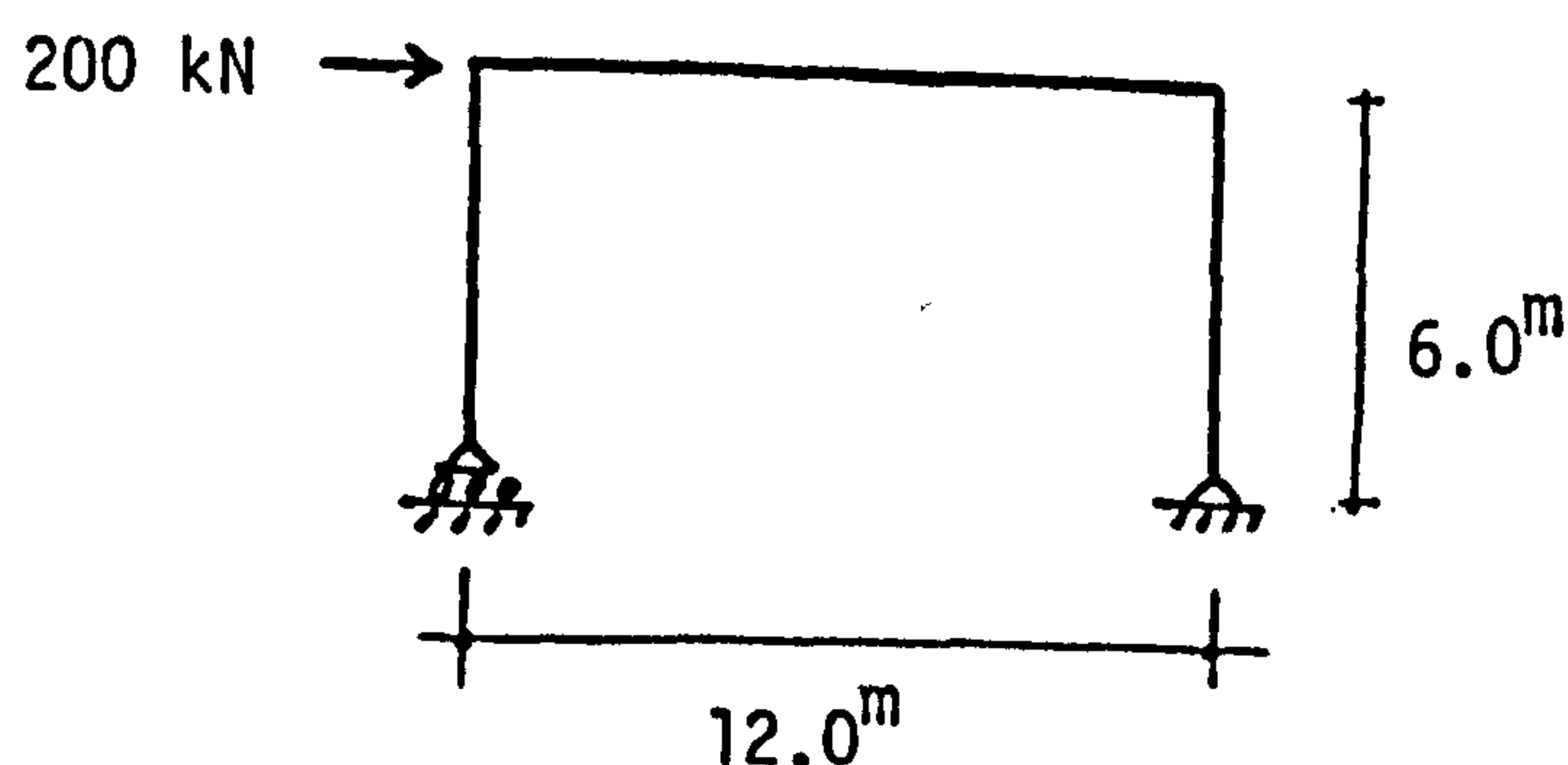
The tendon eccentricity is given by expression 1.44 and at the point of maximum moment variation is equal to:

$$e = - \left[\frac{M_D}{H_p} + \frac{M_1}{2H_p} + \frac{M_2}{2H_p} \right]$$
$$= - \left[- \frac{3675}{1715} + \frac{20000}{2 \times 1715} - \frac{20000}{2 \times 1715} \right] = 2.14 \text{ cm}$$

Example 2

Consider a statically determinate frame with a span of 12 m and a height of 6 m, which is designed to carry a horizontal load of 200 kN on the beam level or no load. The stress limits are 1.4 kN/cm^2 in compression and zero tension.

The cross section is rectangular and the concrete density 25 kN/m^3 it is required to design the beam of the frame as prestressed element.



a) Live load moment envelope

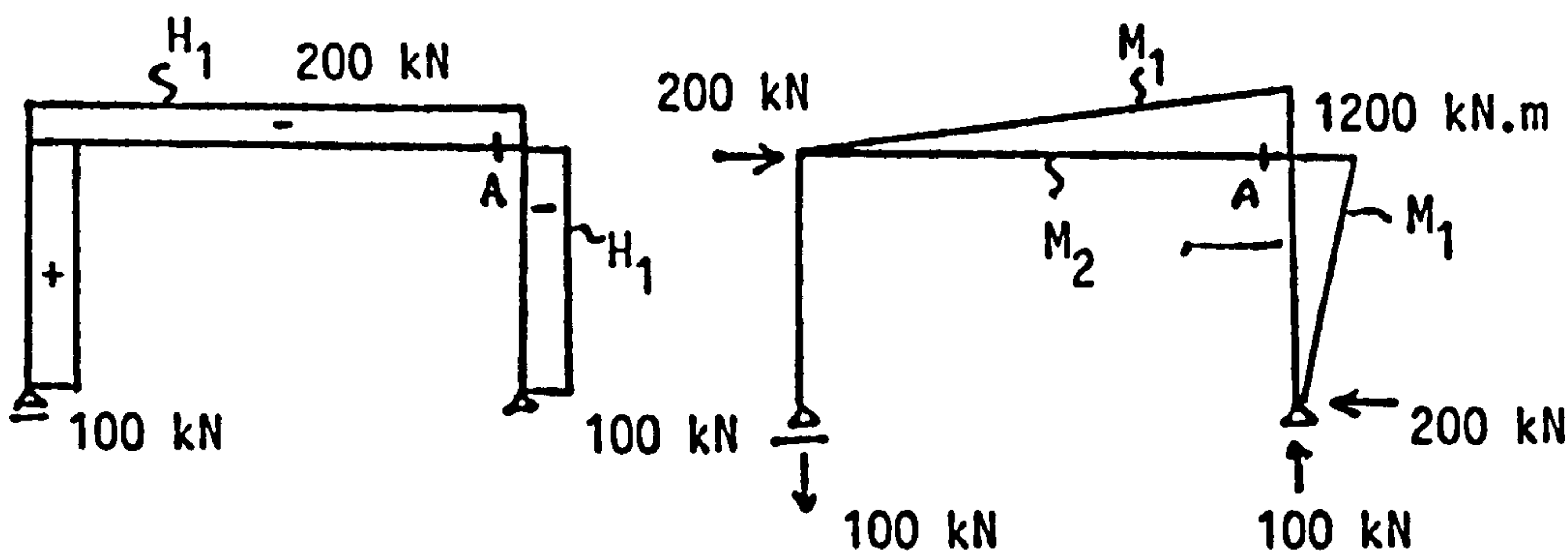


Fig. 1.11

b) Section Modules

<u>Beam</u>	$M_1 = 1200 \text{ kNm}$	$M_2 = 0$
	$H_1 = 200 \text{ kN}$	$H_2 = 0$

$$H_1 > H_2$$

from equation 1.24

$$M_L = \frac{1}{6} f_c b d^2 + \frac{1}{6} (H_2 - H_1) d$$

$$120000 = \frac{1}{6} \times 1.4 \times 40 \times d^2 + \frac{1}{6} (0 - 200) d$$

$$d = 115.2 \text{ cm}$$

115.2cm
40cm

c) Dead load moment

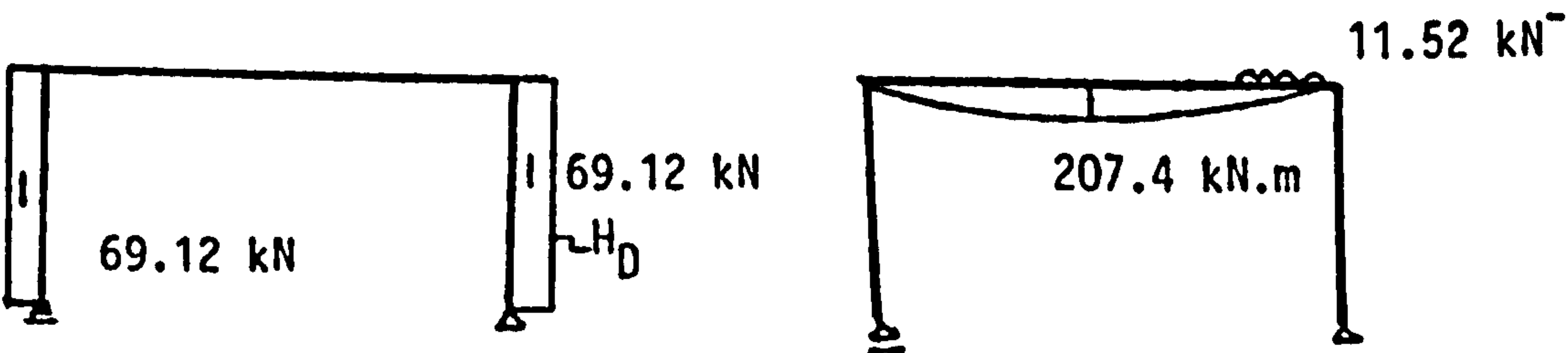


Fig. 1.12

d) The prestressing force:

$$H_1 > H_2$$

$$\begin{aligned} H_p &= \frac{1}{2} f_c b d - H_D - H_1 \\ &= \frac{1}{2} \times 1.4 \times 40 \times 115.2 - 0 - 200 \\ &= 3025.6 \text{ kN} \end{aligned}$$

e) The tendon eccentricity:

The tendon eccentricity is given by expression 1.26

$$\begin{aligned} e &= - \left[\frac{M_D}{H_p} + \left(\frac{H_B}{H_B + H_A} \right) \frac{M_1}{H_p} + \left(\frac{H_A}{H_B + H_A} \right) \frac{M_2}{H_p} \right] \\ &= - \left[0 + \left(\frac{3025.6}{(3025.6 + 3025.6 + 200)} \right) \frac{120000}{3025.6} + 0 \right] = -19.19 \text{ cm} \end{aligned}$$

CHAPTER (2)

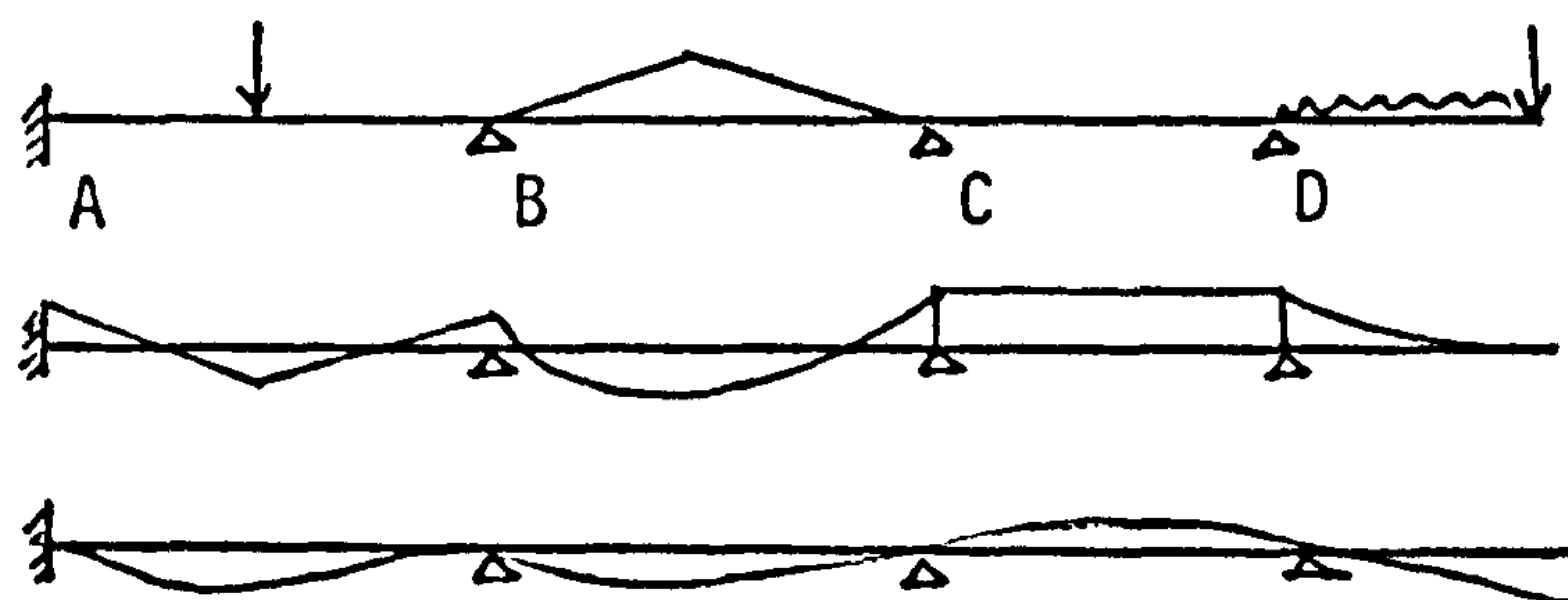
REVIEW OF THE DESIGN OF STATISTICALLY INDETERMINATE PRESTRESSED CONCRETE STRUCTURES

2.1 THE CONCEPT OF THE CONCORDANT PROFILE

One of the most important ideas in the design of prestressed concrete statically indeterminate structures is that of a concordant profile. Guyon¹⁴ defined the concordant profile as a tendon shape which may be stressed, in an otherwise unloaded structure, without causing support reactions (or the concordant profile is characterised by the fact that prestress itself does not produce any statically indeterminate reactions). Certain theorems concerning such profiles have been enunciated by Morice and Lewis²⁷. One of these theorems states "any tendon profile corresponding to the bending moment diagram due to any possible loading on a rigidly supported statically indeterminate beam is a concordant tendon profile, and a tendon laid to that profile will produce no support reactions".

To describe this theory, suppose that some form of statically indeterminate beam is loaded and that the application of the load induces no movement of the supports A, B, C and D (Fig. 2.1a), i.e. there is no vertical deflection at any support.

Normal methods of analysis enable the bending moment diagram and the deflection diagrams to be determined from the known values of the loads and the properties of the beam.



a) Loading, bending moment and deflection diagrams



b) Prestress

Fig. 2.1

From the previous statements it is evident that this bending moment could be produced in the unloaded beam, in addition to a uniform compression, by placing a prestressed tendon with an eccentricity given by $e=M/H$, Fig. 2.1b provided there is no restraint to the longitudinal compressive strain at the points of support. Since the bending moments came about with the beam undeflected at the supports, the introduction of the prestressing tendon to the bending moment profile would produce no tendency to deflect at A, B, C and D and thus would introduce no support reactions in the otherwise unloaded beam. Such a tendon profile is called a "Concordant" profile. This concordant profile leads to the solution of a statically indeterminate prestressed structures, but finding such a profile lying within the limiting zone can present considerable theoretical difficulty.

2.2 PRESTRESSED PORTAL FRAMES AND ARCHES

In structures such as portal frames and arches, where the overall longitudinal contraction due to prestressing sets up support reactions and where loading affects the value of the prestressing force, the usual simplifications of prestressed beam design are not strictly applicable and a more complex analysis may have to be used.

Total concordance

When the members of a structure are prestressed they contract in resisting the average longitudinal compression. In the normal design of beams this effect is unimportant and need not be considered but in the case of frames and arches it is not always possible to do so since a tendency to shorten will be restricted by support reactions, which will introduce additional bending stresses into the structure, and tend to reduce the prestressing force.

Thus, to ensure concordance, the tendon profile must differ from the "bending concordant profile" by an amount which exactly compensates for the shortening effects.

2.3 DIRECT DESIGN METHOD

This method solves two problems, the determination of the distribution of moment due to live and dead loading and the determination of the pressure line due to a tendon profile with an arbitrary varying eccentricity e , which may be assumed through the length of the structure without referring to the limiting zone.

Consider an example a two-pinned portal frame of an arbitrary varying section Fig. 2.2. This is once statically indeterminate structure and it may be made statically determinate by changing the hinged support at B to roller support.

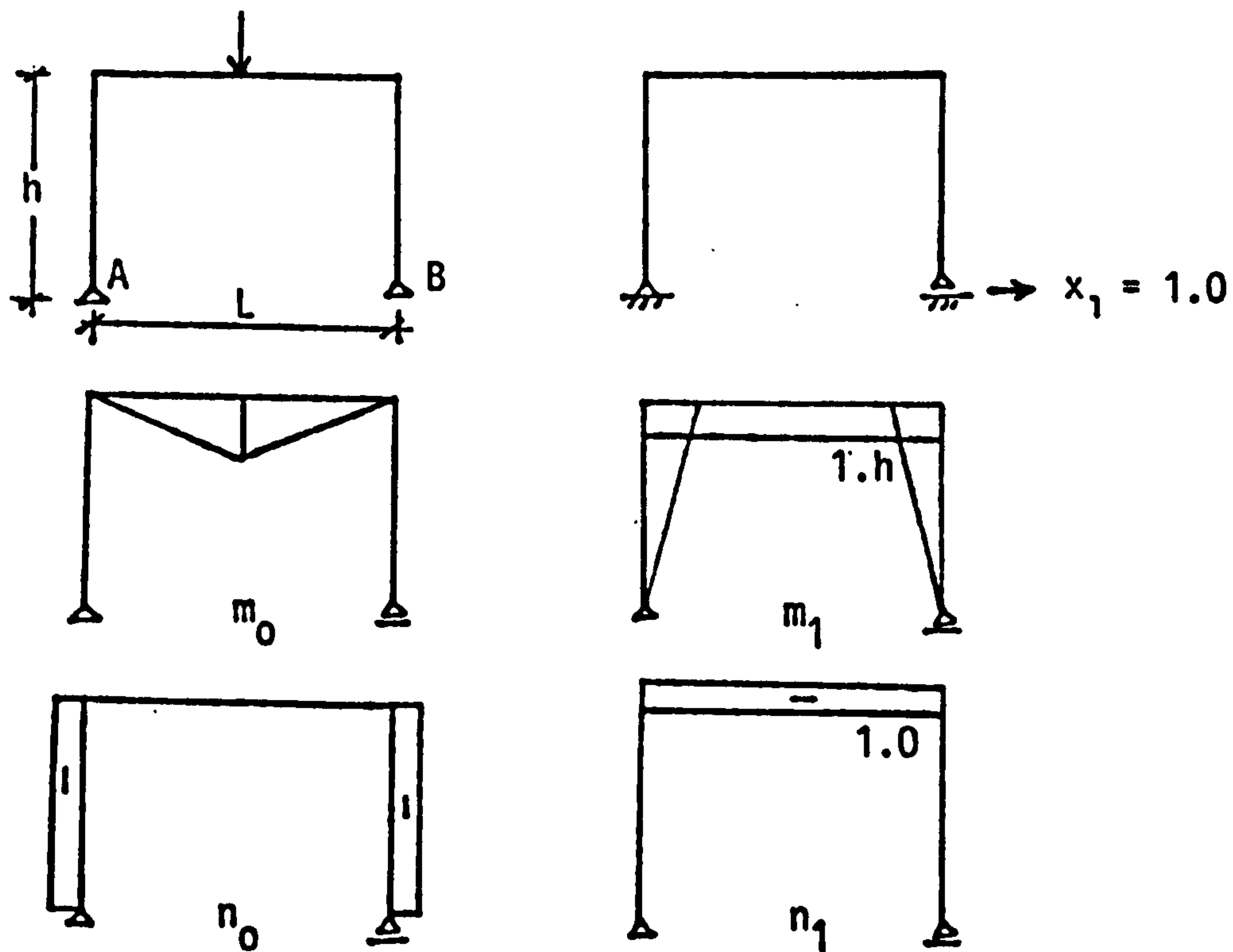


Fig. 2.2

Suppose $x_1=1$ acts alone on the structure Then the separation at the feet B may be shown to be

$$f_{11} = \int_s \frac{m_1^2}{EI} ds + \int_s \frac{n_1^2}{EA} ds \quad \dots \quad 2.1$$

where m_1 and n_1 are the distribution of bending moment and direct forces due to x_1 and the integration extends over the whole structure.

Consider a system of loading applied to the structure made statically determinate. This causes a distribution of moment m_0 and direct forces n_0 and the separation at the feet B is

$$u_1 = \int_s \frac{m_0 m_1}{EI} ds + \int_s \frac{n_0 n_1}{EA} ds \quad \dots \quad 2.2$$

Now to re-establish indeterminacy in the structure the separation at the feet (at hinge B) must be made zero

$$u_1 + f_{11} x_1 = 0$$

$$x_1 = - u_1 / f_{11} \quad \dots \quad 2.3$$

The final distribution of bending moment M and the direct force N through the loaded statically indeterminate structure will be given by

$$M = m_0 + x_1 m_1 \quad \dots \quad 2.4$$

$$N = n_0 + x_1 n_1 \quad \dots \quad 2.5$$

For checking the solution it may be seen that

$$\int_s \frac{M m_1}{EI} ds + \int_s \frac{N n_1}{EA} ds = 0 \quad \dots \quad 2.6$$

Consider now the loading produced by a prestressing tendon at an arbitrary varying eccentricity e , Fig 2.3

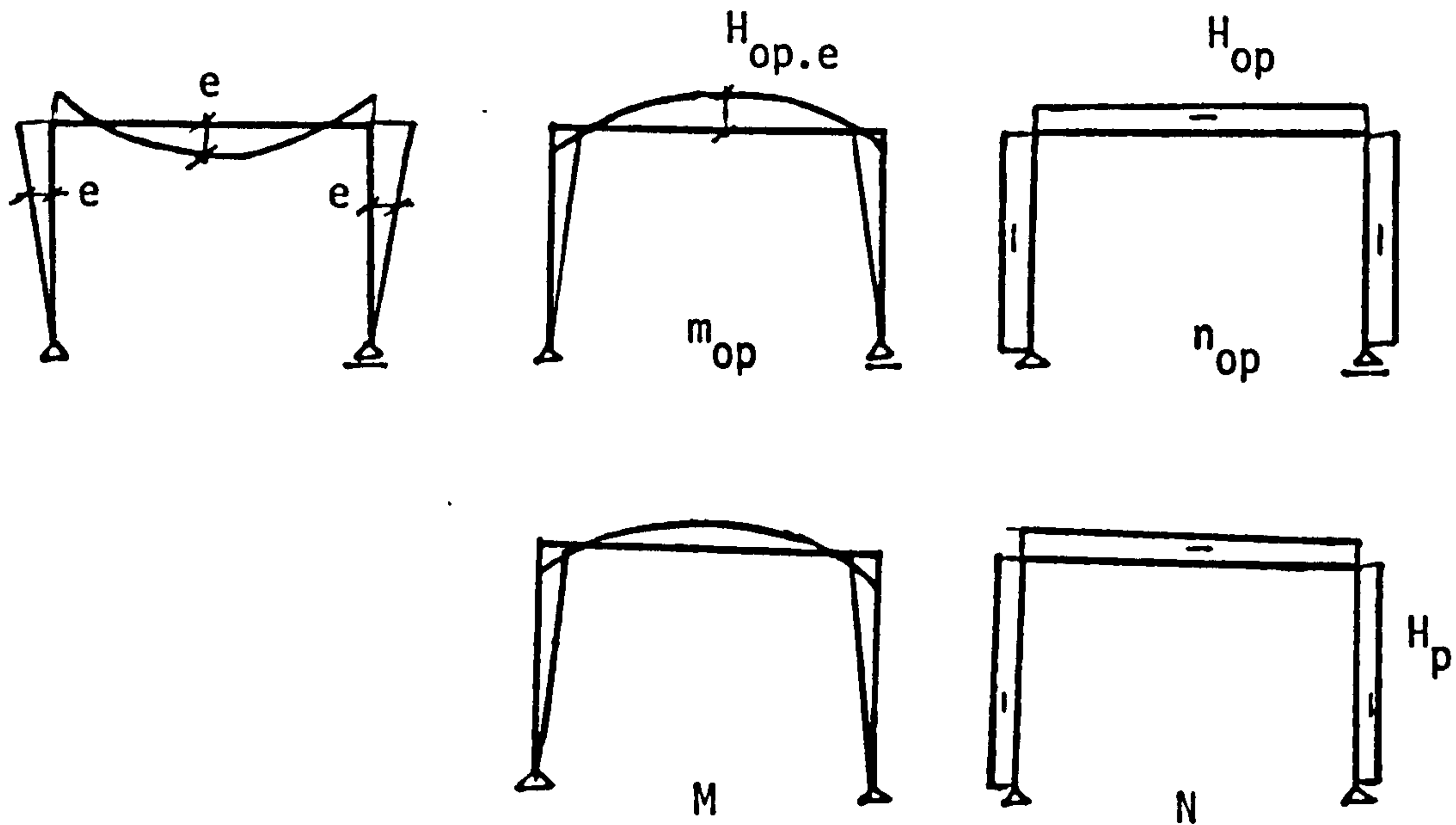


Fig. 2.3

The bending moment m_{op} due to the tendon in the statically determinate structure is

$$m_{op} = H_{op} \cdot e \quad \dots \quad 2.7$$

and the corresponding normal force distribution is n_{op} .

The separation at the feet B may be therefore calculated using 2.7 and the corresponding arbitrary constants computed from 2.3. The resultant distribution of moment M is then found using 2.4 and 2.5 which will become

$$M = m_{op} + x_1 m_1 \quad \dots \quad 2.8$$

$$N = n_{op} + x_1 n_1 \quad \dots \quad 2.9$$

If e is regarded as a tendon profile, it is seen from expression 2.6 that the separation at feet B is zero, that is e introduces no support reactions and it is therefore a concordant profile. This concordant profile may, however, not be in the limiting zone.

2.4 APPROXIMATE SOLUTION

In order to solve the prestressing problem for a statically indeterminate structure so that the stress conditions are satisfied, the centre of pressure of the prestressing force is required to lie within the limiting zone, i.e. a concordant profile must be found which lies within the given limiting zone.

One possible solution which may be used in arranging concordant profile is as follows:-

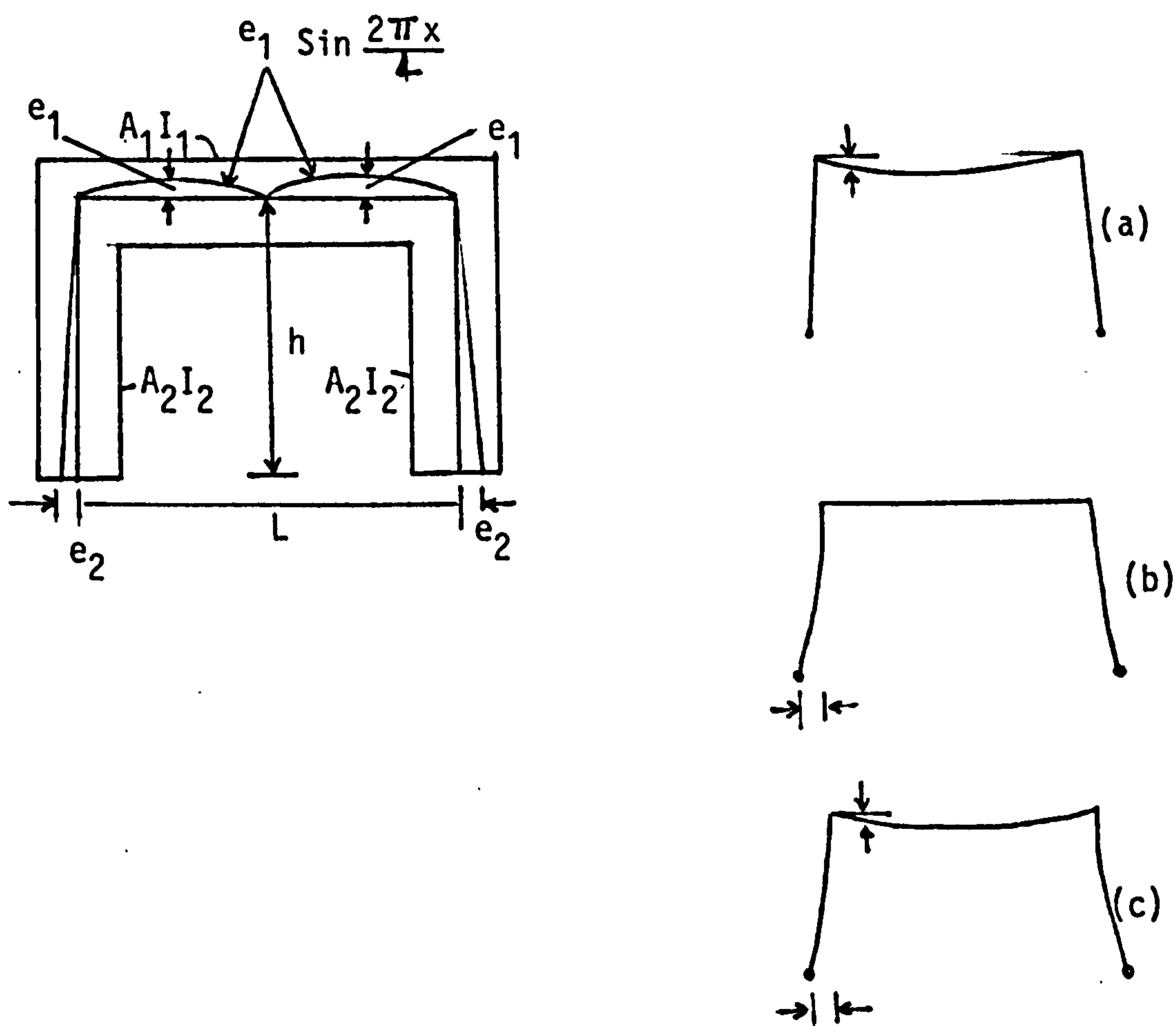
The frame may be in its final state of support at the time of prestressing and the tendon profile must be concordant to this system of support. In the case of frames the concordant profile has the added difficulty that the shortening of the beam due to prestressing must not introduce support reactions.

The amount of this difference is given by the bending moment diagram, which will produce a separation at the feet equal to the beam shortening.

In the case of a two-pinned portal, rotation is allowed and the required separation can be obtained either by bending of the beam or of the legs. It is to be noted that the change to the

bending concordant profile required to produce the feet separation must not take the final profile outside the limiting zone.

This means that there can be no displacement of the profile at the critical point or points and accounts for the form of the chosen separation profiles (Fig. 2.4).



Separation profiles for two-pinned portals

Fig. 2.4

The required eccentricities are as follows:-

a) by beam bending only (Fig. 2.4a)

$$e_1 = - \frac{\pi I_1}{2hA_1} \quad \dots \quad 2.10$$

b) by leg bending only (Fig. 2.4b)

$$e_2 = -3 \frac{H_1 I_2 L}{H_2 A_1 h^2} \quad \dots \quad 2.11$$

c) by combined beam and leg bending (Fig. 2.4c) either

$$e_1 = - \frac{\pi I_1}{2hA_1} \left(1 + \frac{1}{3} \frac{A_1}{I_2} \frac{H_2}{H_1} \frac{h^2}{L} e_2 \right) \quad \dots \quad 2.12$$

or

$$e_2 = - \frac{3H_1 I_2 L}{H_2 A_1 h^2} \left(1 + \frac{2hA_1}{\pi I_1} e_1 \right) \quad \dots \quad 2.13$$

in which the value of e_1 or e_2 may be chosen and the corresponding value of e_2 or e_1 calculated.

In the case of a frame with fixed feet the required separation must be accompanied by a state of no rotation at the feet. This means that there must be some leg bending whether beam bending takes place or not. These effect are covered by eccentricities as follows:-

a) leg bending only (Fig. 2.5a)

in this case $e_3 = -e_2$

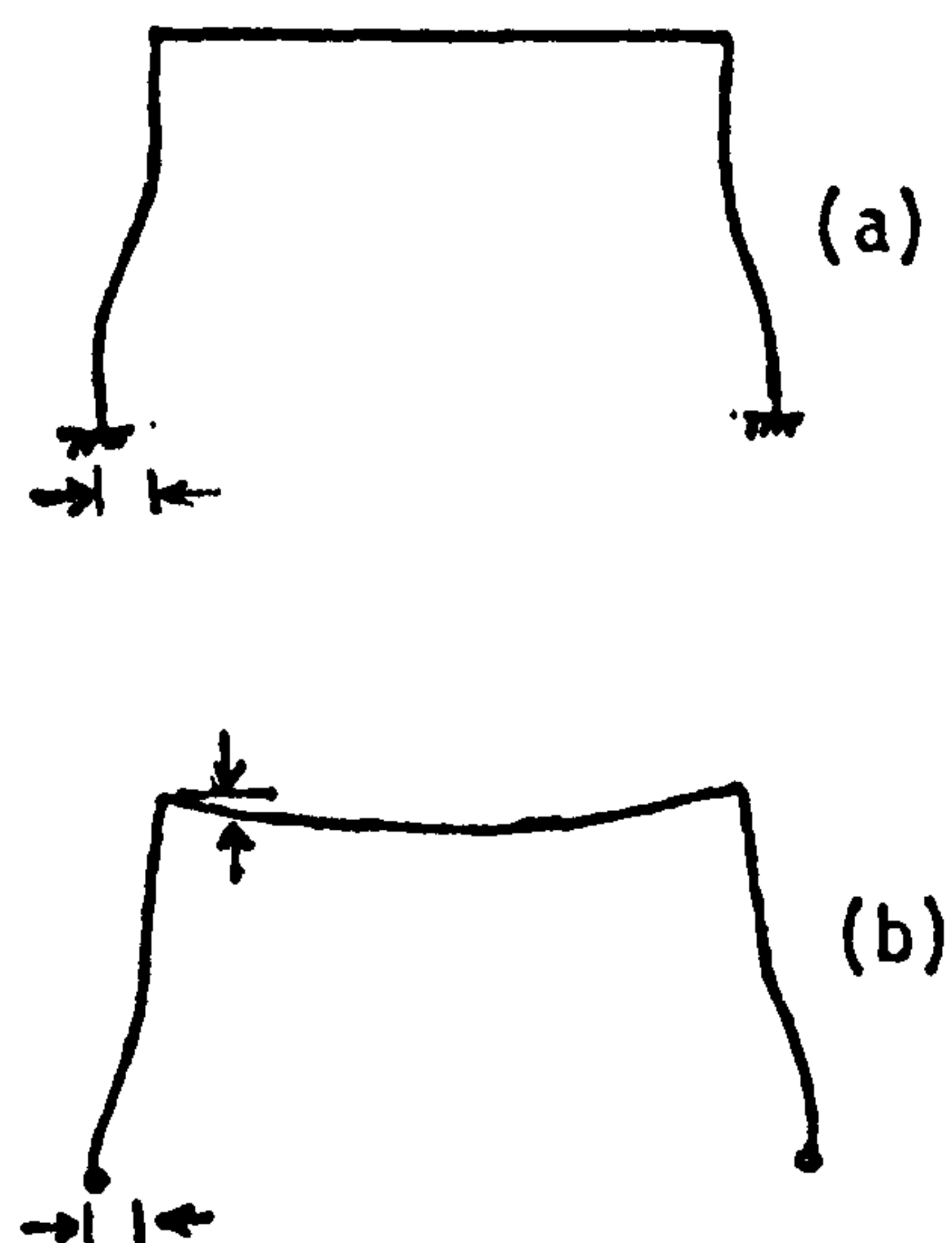
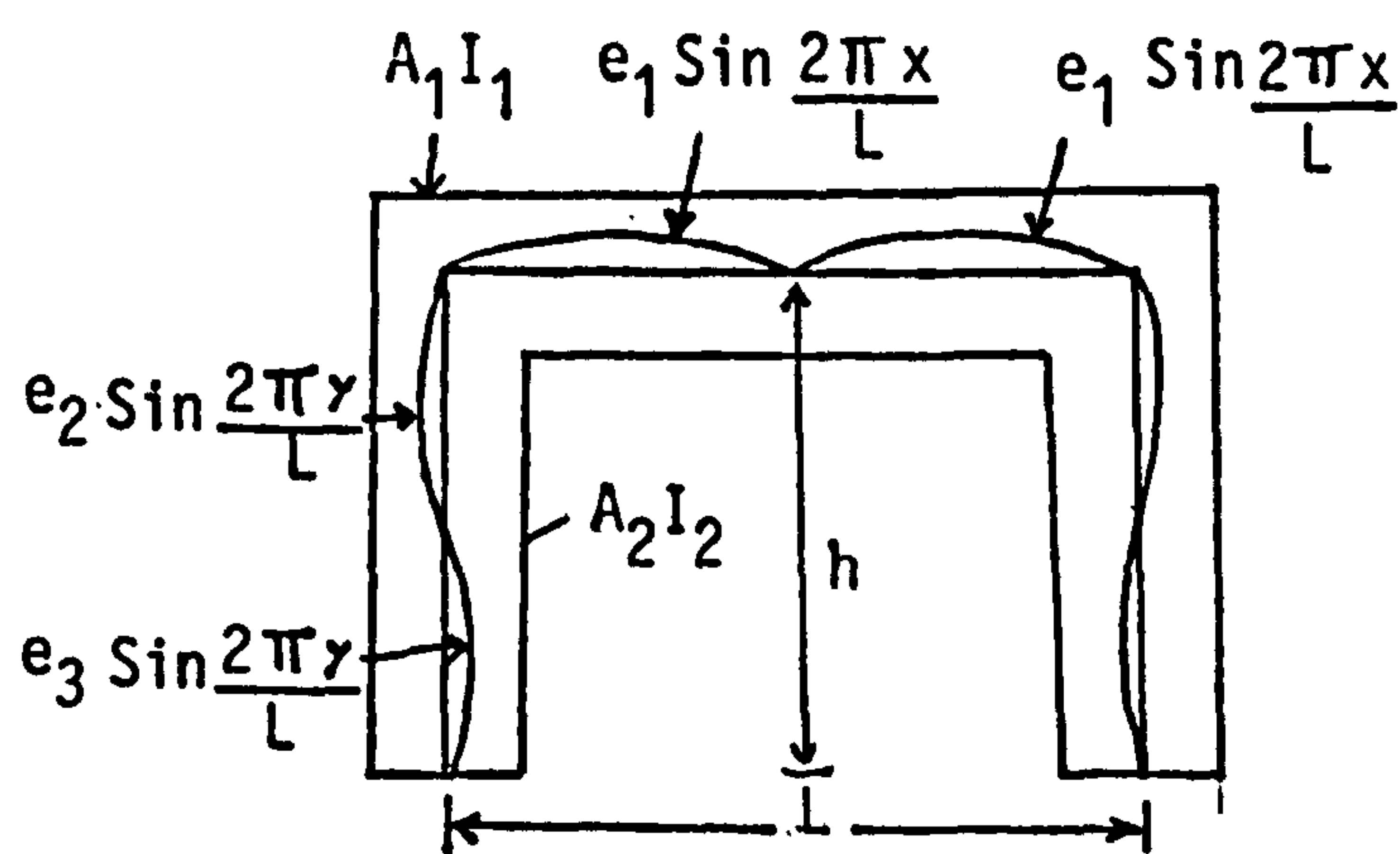
$$e_2 = -\pi \frac{H_1}{H_2} \frac{I_2}{A_1} \frac{L}{h^2} \dots 2.14$$

b) combined leg and beam bending (Fig. 2.5b)

$$e_2 + 3e_3 = 2\pi \frac{H_1}{H_2} \frac{I_2}{A_1} \frac{L}{h^2}$$

and

$$e_1 = -\frac{h}{L} \frac{I_1}{I_2} \frac{H_2}{H_1} (e_2 + e_3) \dots 2.15$$



Separation profiles for fixed portals
Fig. 2.5

Here we may chose two eccentricities; the third will then follow from the above relationships. The required true concordant profile is obtained by adding the "bending concordant profile" to the separation profile.

2.5 Stationary Unit Eccentricity Procedure

The main idea of this procedure depends on the basis of the flexibility method and can be outlined as follows:.

- (a) We will establish some "stations" along the length of the structure which shall denote 1, 2, 3 ... etc. Fig 2.6

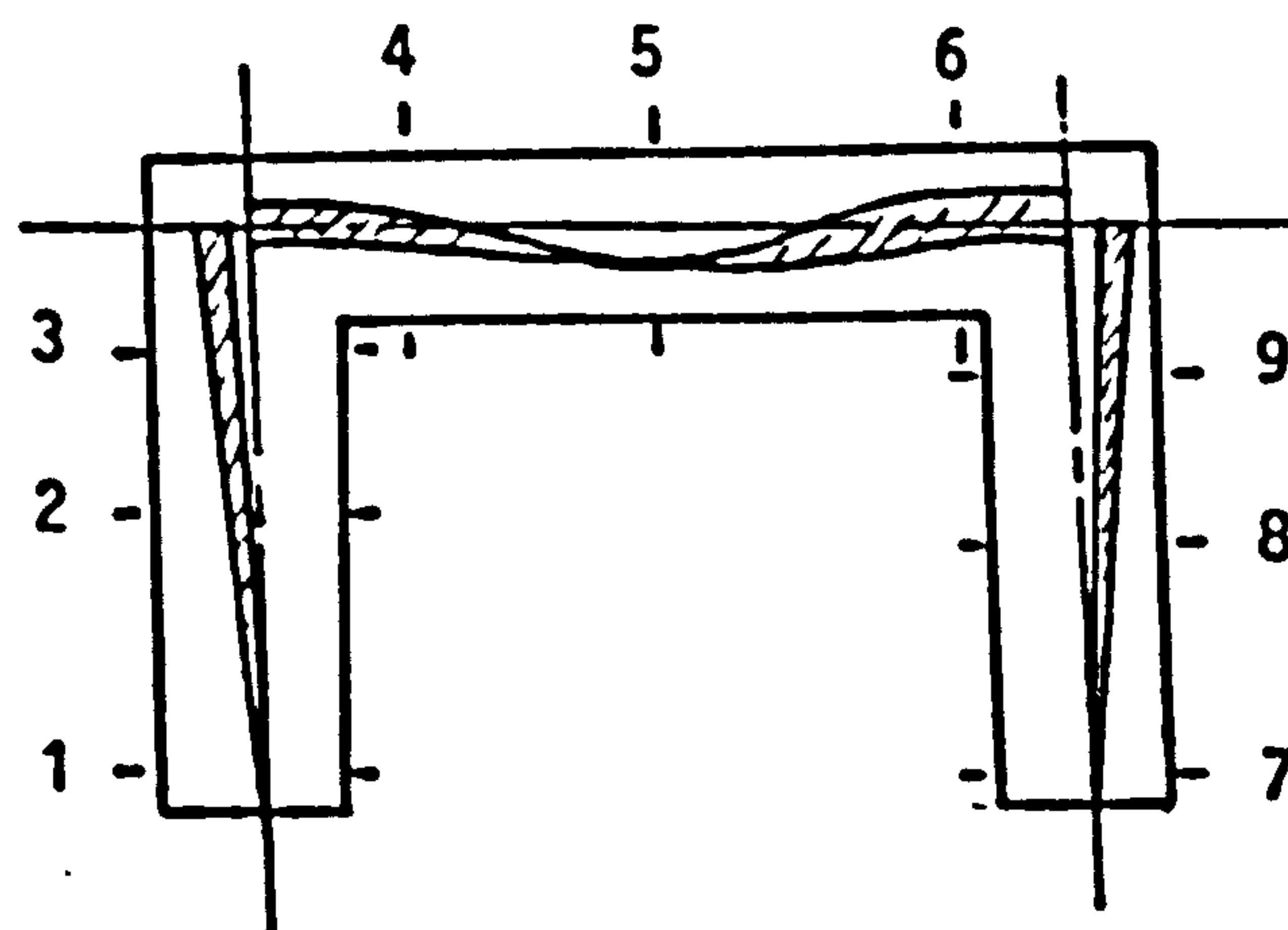


Fig. 2.6

- (b) We shall define the "Limiting Zone" at each station by the ordinates $e_{A_1}, e_{A_2}, e_{A_3} \dots e_{B_1}, e_{B_2}, e_{B_3}, \dots$
- (c) It is known that an arbitrary chosen prestressing tendon profile of eccentricity e can cause a "lack of concordancy" U_j , and this can be computed from the expression:

$$U_j = \int_s \frac{Mm_j}{EI} ds + \int_s \frac{Nn_j}{EA} ds \quad \dots \quad 2.16$$

where $j = 1, 2, \dots, n$ and n is the degree of indeterminacy of the structure.

Substituting by $M = H_p \cdot e$, $N = H_p$ in the above equation gives

$$U_j = U_{p_j} + k_j \quad j = 1, 2, \dots \quad 2.17$$

$$\text{where } U_{p_j} = \int_s \frac{H_p \cdot e \cdot m_j}{EI} ds \quad \text{and} \quad k_j = \int_s \frac{H_p \cdot n_j}{EA} ds$$

where m_j and n_j are respectively the bending moment and the normal force distribution due to a unit indeterminate stress resultant $x_1, x_2, \dots, x_j$ at the release points, and e is the distribution at tendon eccentricity along the structure Fig. 2.7.

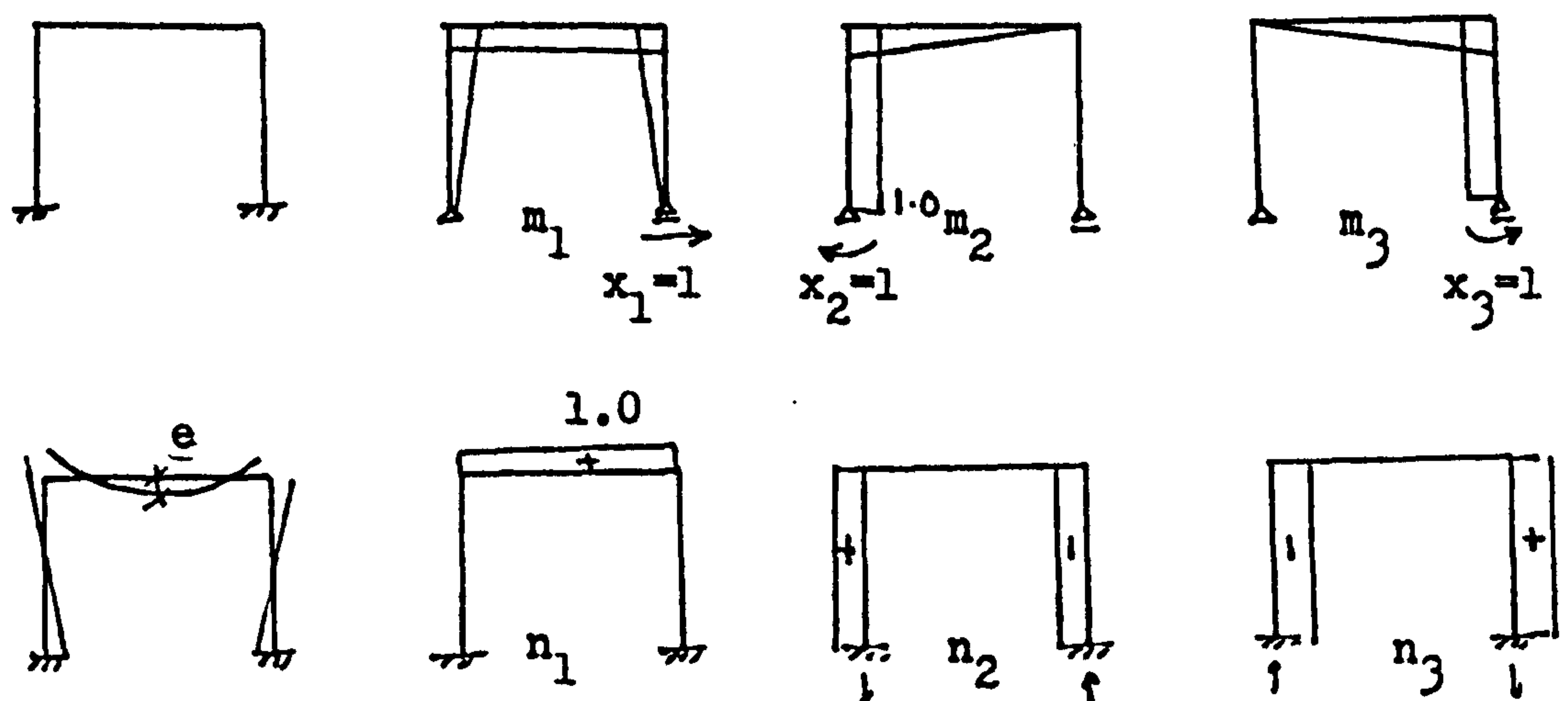


Fig. 2.7

The term k_j can be easily computed by integrating the prestressing force distribution diagram with the n_j diagrams. For the term (U_{p_j}) we can find a set of values for the contributions for a series of tendon profiles with a unit eccentricity, in turn of each "station" along the structure

and elsewhere lying on the centroidal axis in Fig. 2.8.

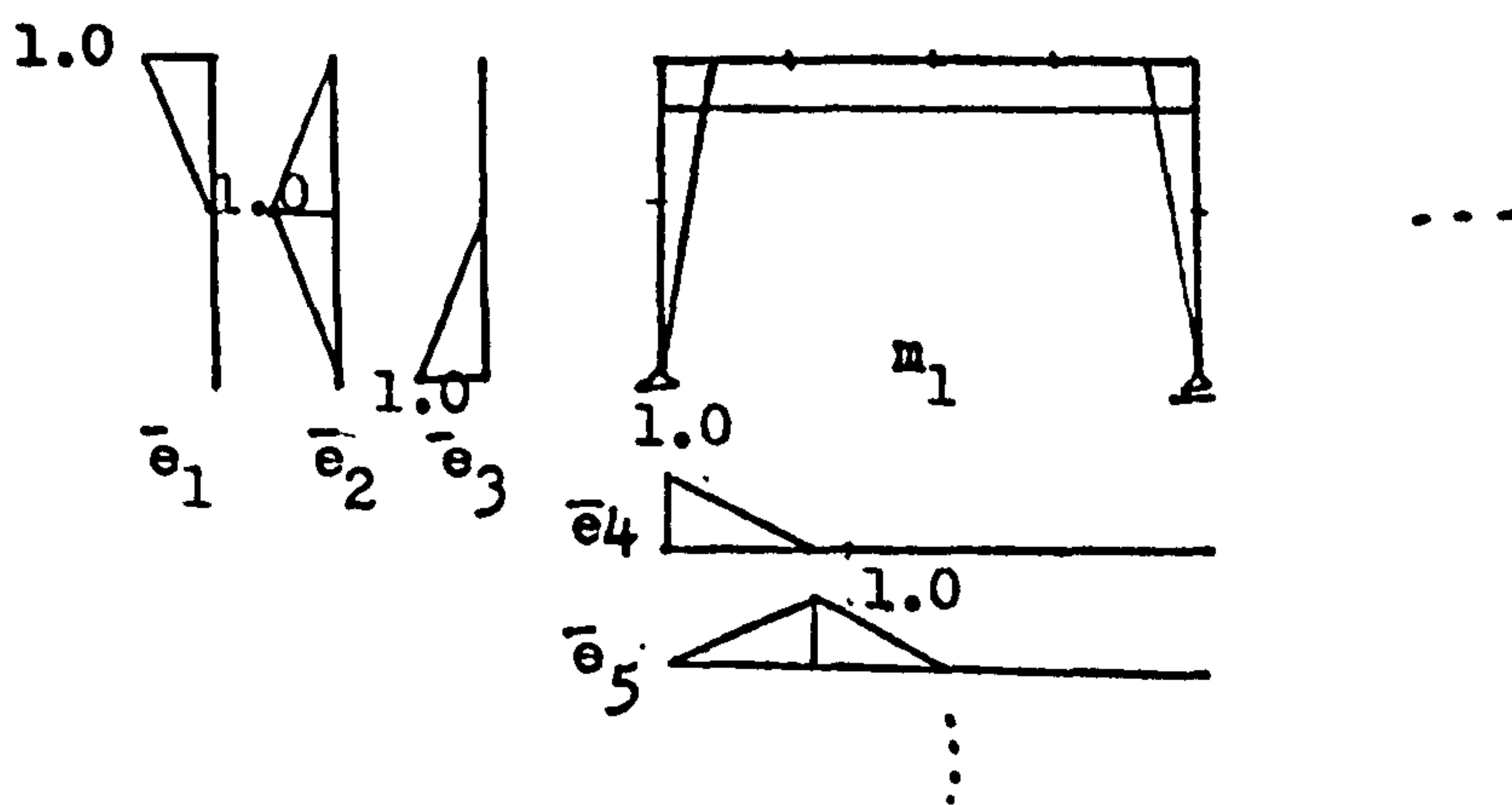


Fig. 2.8

by integrating these unit eccentricity profiles which we will call $\bar{e}_1, \bar{e}_2, \bar{e}_3 \dots$ etc. with the m_j diagrams. Let us call these values $U_{pj1}, U_{pj2}, U_{pj3} \dots$ etc.

$$\text{i.e. } U_{pj1} = \int_s \frac{H \bar{e}_1 m_j}{EI} ds \quad \dots \quad 2.18$$

- (d) Thus for an actual cable profile with eccentricities at stations 1, 2, 3... etc. having values $e_1, e_2, e_3 \dots$ etc. the term U_{pj} will be given by summing the product of the actual eccentricities and the unit effects at each station

$$\text{i.e. } U_{pj} = \sum_{i=1}^m U_{pj i} e_i \quad \dots \quad 2.19$$

where $i = 1, 2, 3 \dots m$ and m is the number of stations for the whole system.

- (e) In order to achieve concordancy we must try to choose

the values of $e_1, e_2, e_3 \dots$ etc. to make the expression 2.17 equal to zero for all

$$U_j = \sum_{i=1}^m U_{p_{ji}} e_i + k_j = 0 \text{ (concordant) } \dots \quad 2.20$$

At the same time, the values of $e_1, e_2 \dots$ etc will be constrained to lie within the limiting zone by requiring that

$$\begin{aligned} e_{B1} &\leq e_1 \leq e_{A2} \\ e_{B2} &\leq e_2 \leq e_{A2} \\ &\vdots \\ &\vdots \\ &\vdots \end{aligned} \quad \dots \quad 2.21$$

If we can satisfy these conditions we shall have found a concordant profile lying within the limiting zone. A solution derived by the quadratic programming procedure, and the concordant profile, was developed using the unit eccentricity procedure and was applied to different problems with the use of a computer program. However there is an error which exceeds 10% at some stations. The reason is that the factors derived from the integration of the unit eccentricity with the m_j diagrams do not give a reasonable factor for the cable eccentricity distributions, so we must try more accurate factors to minimize this error such as "Simpson's Factors".

CHAPTER (3)

DESIGN OF STATICALLY INDETERMINATE PRESTRESSED CONCRETE STRUCTURE

3.1 DESIGN STEPS

The design steps may be outlined as follows:

- a) For a given geometry of statically indeterminate structure in which the relative values of the stiffness distribution are defined, it is possible to determine the stress resultant distribution due to a given loading, without a knowledge of the absolute stiffness values. For example, in the case of a statically indeterminate beam, frame or arch, we can evaluate the distribution of the bending moment due to a known loading as long as we know the distribution of the cross-sectional bending stiffness along the length of the structure. Thus in the case of prestressed structure, provided we define the relative stiffness distribution (a uniform section is one particular case), we can find the live load moment range ($M_L = M_1 - M_2$) at all sections of the structure before the absolute section size is chosen.
- (b) The absolute section sizes can therefore be chosen on the basis of reaching the maximum allowable compressive stresses given by equation 1.18 or 1.21 (chapter 1), then the distribution

of the bending moment M_D and the direct forces H_D due to the dead loads can be determined.

- (c) The prestressing force H_p , which may have to vary along the length of the structure if the cross section varies, can be determined directly from expression 1.19 or 1.22 (chapter 1).
- (d) The limiting zone then can be derived using expressions 1.12, 1.13 or 1.14, 1.15, then the concordant profile can be produced (section 3.2).

3.2 THE CONCORDANT PROFILE

3.2.1 The procedure which will be used to determine the concordant profile within the "Limiting Zone" directly depends on the basis of the flexibility method discussed in chapter 2, section 2.3. Simpson's rule is used in the integration.

The procedure is outlined as follows:-

- a) We will choose to establish some "stations" along the length of the structure which we shall label 1, 2, 3, ... etc.

fig. 3.1

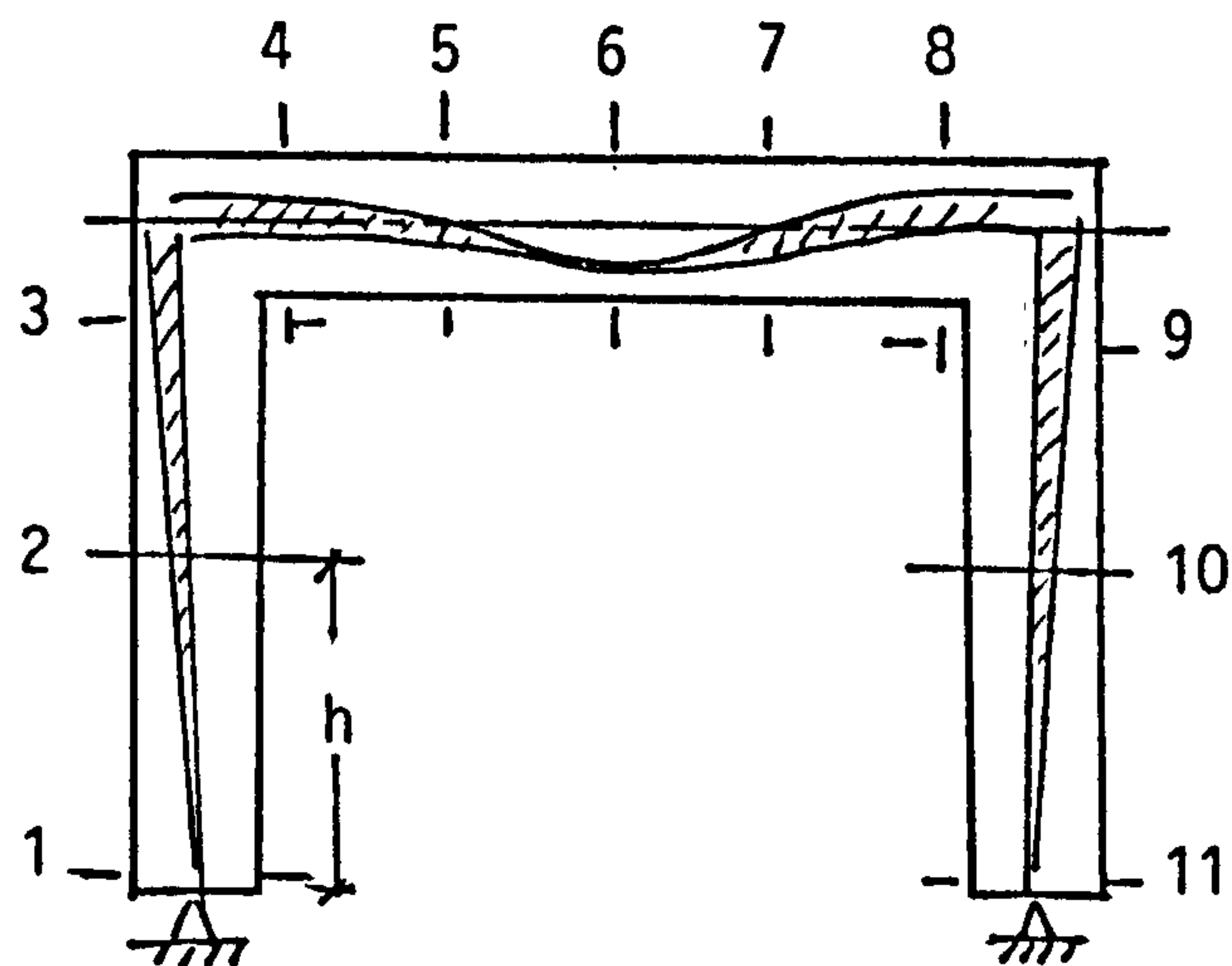


Fig. 3.1

- b) Then at each station we can define the "Limiting Zone" by the ordinates e_{A1} , e_{A2} , e_{A3} , ... e_{B1} , e_{B2} , e_{B3}
- c) It is known that an arbitrary chosen prestressing tendon profile of eccentricity e can cause a "Lack of concordancy" U , and can be computed from the expression

$$U = \int_s \frac{Mm_1}{EI} ds + \int_s \frac{Nn_1}{EA} ds$$

Substituting by $M=H_p \cdot e$, $N=H_p$ in the above equation gives

$$U = U_p + K \quad \dots 3.1$$

$$\text{where } U_p = \int_s \frac{H_p \cdot e m_1}{EI} ds \text{ and } K = \int_s \frac{H_p n_1}{EA} ds$$

where m_1 and n_1 are the bending moment and the normal force distribution respectively, due to a unit indeterminate stress resultant x_1 at the release point and e is the distribution at tendon eccentricity along the structure as shown in fig 3.2.

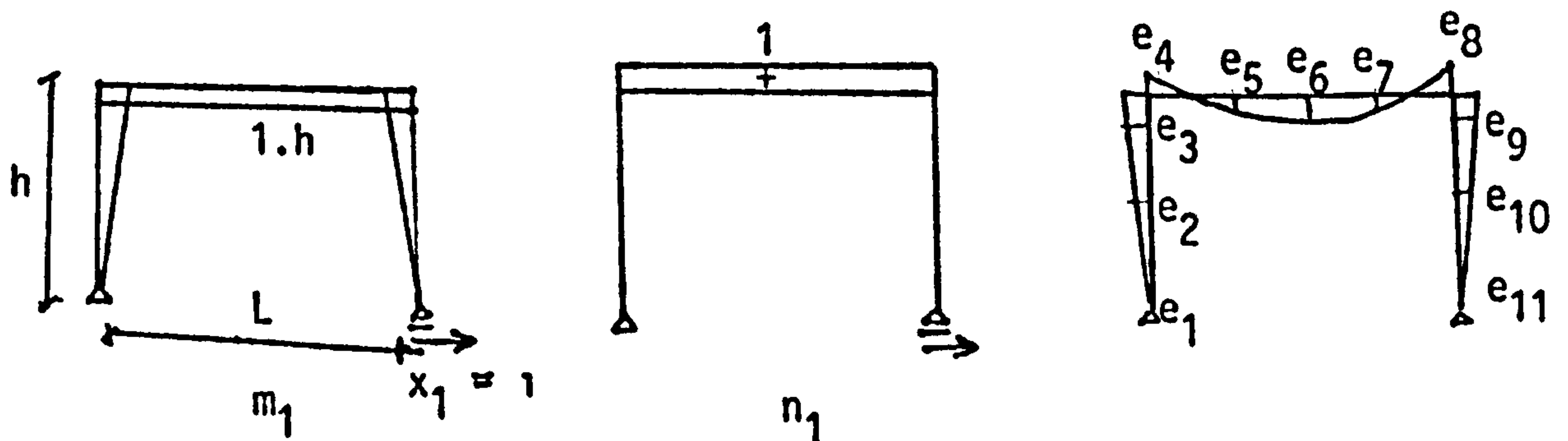


Fig. 3.2

The term (K) can be easily computed by integrating the prestressing force distribution diagram with the n_1 diagram. For the term (U_p) we can assume a profile with an eccentricity e_i (where $i=1,2,3,\dots,m$ and m is the number of stations for the whole structure), for each "station" in turn, along the structure and elsewhere lying on the centroidal axis Fig. 3.2.

d) In order to achieve concordancy we must obviously try to choose the values of $e_1, e_2, e_3, \dots$ etc. to make the expression (3.1) equal to zero.

$$\text{i.e. } U_p + k = 0 \quad \dots \quad 3.2$$

But at the same time, the values of $e_1, e_2, e_3 \dots$ etc. will be constrained to lie within the limiting zone by requiring that

$$e_{B1} \leq e_1 \leq e_{A1} \quad \dots \quad 3.3$$

$$e_{B2} \leq e_2 \leq e_{A2}$$

If we satisfy these conditions we shall have found a concordant profile lying within the "Limiting Zone".

e) In the case of a structure with more than one degree of indeterminacy, the "concordancy" equation (3.1) becomes a set of linear simultaneous equations each of which is equal to zero.

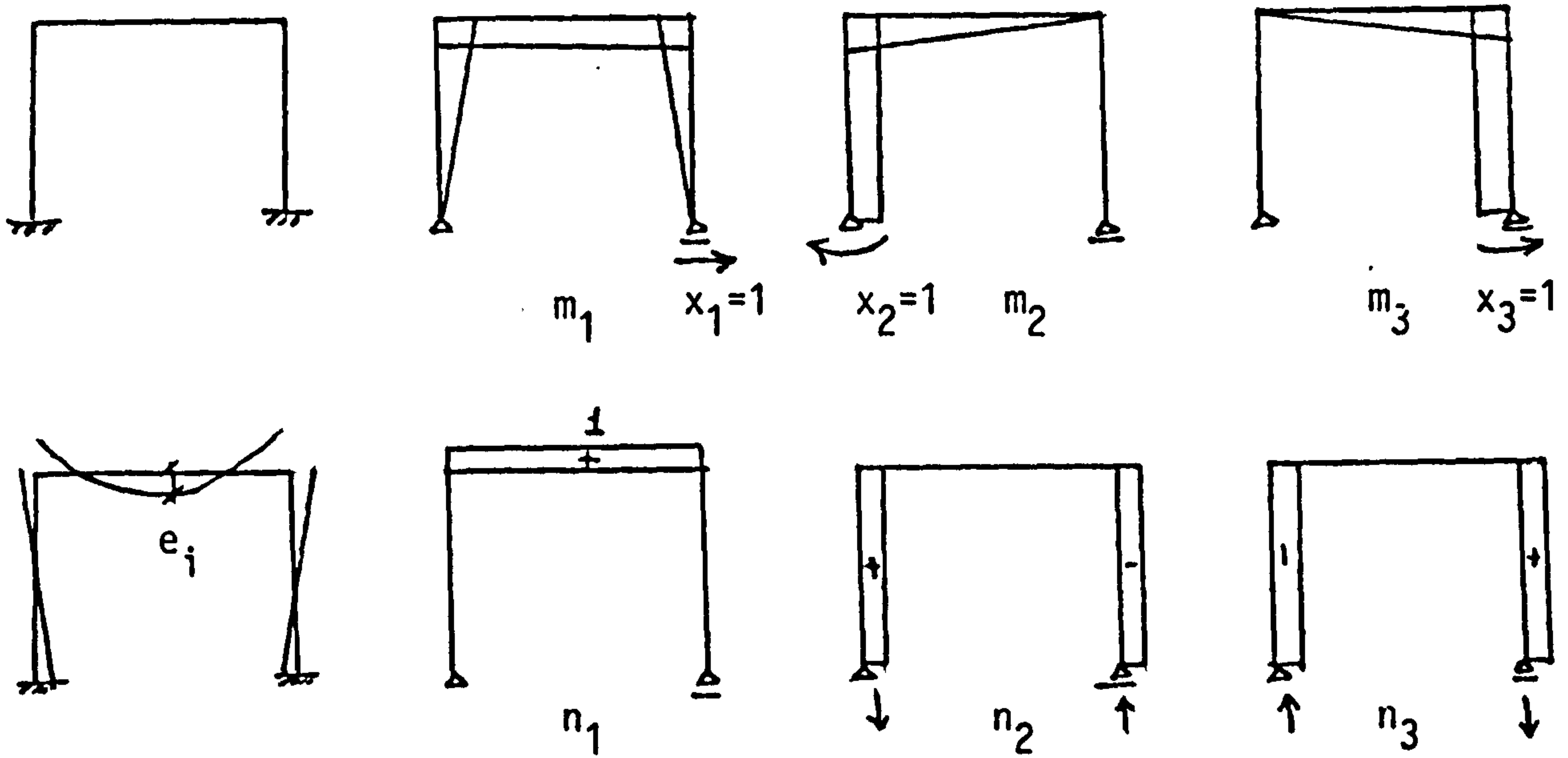


Fig. 3.3

Fig. 3.3 shows a frame with several degrees of indeterminacy and the bending moment, and the normal forces diagrams due to a unit indeterminate stress resultants x_1 , x_2 , x_3 . Clearly lack of "concordancy" must be removed at each of the release points.

f) Now to achieve concordancy we shall have as many objective functions (of type 3.1) as there are degrees of indeterminacy of the structure.

$$U_j = U_{p_j} + K_j$$

Where $j = 1, 2, 3, \dots, n$ and n is the number of degrees of indeterminacy of

$$U_j = \int_s \frac{H_p e_{mj}}{EI} ds + \int_s \frac{H_p n_j}{EA} ds = 0 \quad \dots \quad 3.4$$

we may notice that the constraints of (3.3) exist as before.

3.2.2 The mathematical solutions

a) Referring to equation (3.3) and taking

$$0 \leq x_i \leq 1 \quad \dots \quad 3.5$$

The eccentricity of the concordant profile may be expressed as follows

$$e_i = (e_{Ai} - e_{Bi}) x_i + e_{Bi} \quad \dots \quad 3.6$$

Substituting (3.6) into (3.4) and by using Simpson's rule in the integration gives

$$\sum_{i=1}^m H_{pi} \cdot ((e_{Ai} - e_{Bi}) x_i + e_{Bi}) \frac{1}{EI_i} \cdot \frac{h_i}{3} \cdot \gamma_i m_{ji} + \sum_{i=1}^m H_{pi} \cdot \frac{1}{EA_i} \cdot \frac{h_i}{3} \cdot \gamma_i \cdot n_{ji} = 0 \quad \dots \quad 3.7$$

Where the symbol i refers to the current section over the range $i = 1$ to m , which may count some sections twice if the interval width changes and m is the total number of sections

for the structure which should be an odd number of sections, h_i is the interval width which may vary from strip pair to strip pair, γ_i is the appropriate Simpson coefficient 1, 4 or 2 and $j = 1, 2 \dots n$ (note in using this numerical integration the functions H_{pi} , e_{Ai} , e_{Bi} , EI_i , $m_{ji} \dots$ etc. must be continuous over each pair of strips).

In order to develop a concordant profile within the "limiting zone", it is necessary to minimize the function:-

$$F = \sum_{i=1}^m (X_i - X_i^*)^2$$

with the constraints of equations (3.5) and (3.7), where X_i^* can take any values between zero and one at each station. This function may provide a solution according to the chosen values of X_i^* .

We may ignore the constant terms in this function and solve the problem by maximizing

$$F = - \sum_{i=1}^m X_i^2 + 2 \sum_{i=1}^m X_i^* X_i \quad \dots 3.8$$

The simplest procedure in this problem is to use the equality constraints of equation (3.7) to eliminate the corresponding values in equation (3.8), then equation (3.8) can be solved directly by two different methods:-

a) Quadratic programming (Wolfe's algorithm)²⁹

The equation (3.8) may be rewritten in matrix form as follows:-

Maximize

$$F = C X - \frac{1}{2} X^T D X \quad \dots \quad 3.9$$

Subject to

$$\begin{bmatrix} I \\ -I \end{bmatrix} X - \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 0 \\ 0 \\ \vdots \end{bmatrix} \leq 0$$

in which $X = (X_1, X_2, \dots, X_m)^T$

$$D = \begin{bmatrix} d_{11} & \dots & d_{1m} \\ \vdots & & \vdots \\ d_{m1} & \dots & d_{mm} \end{bmatrix}$$

and $C = (c_1, c_2, \dots, c_m)$

(note: D, C matrices are the constants produced from substituting equation (3.7) into equation (3.8) and I is the unit matrix.

c) Let $S = (S_1, S_2, \dots, S_m)^T$ be the Slack variables,

$Z = (Z_1, Z_2, \dots, Z_m)^T$

be the Artificial variables, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$

and $V = (V_1, V_2, \dots, V_m)^T$

be the Lagrangian multipliers corresponding to the two sets constraints $(X - I \leq 0)$ and $(-x \leq 0)$ respectively.

By the application of Kuhn-Tucker²⁹ conditions, the necessary conditions may be combined as

$$\left| \begin{array}{c|c|c|c|c|c} D & I & -I & I & 0 \\ \hline I & 0 & 0 & 0 & I \end{array} \right| \begin{array}{c} x \\ \lambda \\ v \\ z \\ s \end{array} = \begin{array}{c} C^T \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \dots 3.10$$

where $\lambda, v, z, s \geq 0$

and the objective function to be maximized is

$$W = - \sum_{i=1}^m z_i \dots 3.11$$

If we add the upper half of expression (3.10) and equation (3.11) together and multiplying them by (-1) we obtain a modified objective function for maximization as follows:

$$\sum_{i=1}^m (a_i x_i - \lambda_i + v_i) = W_n \dots 3.12$$

in which $a_i = - \sum_{j=1}^m D_{ji} \quad i = 1, 2, \dots, m$

$$W_n = - \sum_{j=1}^m C_j$$

and the constraints will be expression (3.10).

d) The quadratic programming problem has now been reduced to the linear programming problem given by (3.12) with the nonlinear conditions $x_i v_i = 0$ and $\lambda_i s_i = 0$ for all i

The latter problem is solved by means of the standard two phases of the simplex method as follows.

Tableau for two phases of simplex method

Consider the case $m = 2$, the objective function for maximization will be

$$W = -Z_1 - Z_2$$

subject to

$$\left[\begin{array}{cc|cc|cc|cc|c|c} D_{11} & D_{12} & 1 & 0 & -1 & 0 & 1 & 0 & 0 & x_1 \\ D_{21} & D_{22} & 0 & 1 & 0 & -1 & 0 & 1 & 0 & x_2 \\ \hline 1 & 0 & & & & & 1 & 0 & \lambda_1 \\ 0 & 1 & & & & & 0 & 1 & \lambda_2 \\ & & & & & & & & v_1 \\ & & & & & & & & v_2 \\ & & & & & & & & z_1 \\ & & & & & & & & z_2 \\ & & & & & & & & s_1 \\ & & & & & & & & s_2 \end{array} \right] = \left[\begin{array}{c} C_1 \\ C_2 \\ 1 \\ 1 \end{array} \right]$$

TABLE (3-1)

SIMPLEX TABLEAU FOR START CYCLES

Cycle	Basic Variables	Admissible Variables						Artificial Variables		Slack Variables		Values	Ratios
		x_1	x_2	x_1	x_2	v_1	v_2	z_1	z_2	s_1	s_2		
	z_1	D_{11}	D_{12}	1	0	-1	0	1	0	0	b	C_1	C_1/D_{11}
	z_2	D_{21}	D_{22}	0	1	0	-1	0	1	0	b	C_2	C_2/D_{21}
	s_1	1	0	0	0	0	0	0	0	1	0	1	$1/1 \leftarrow$ min(+ve)
	s_2	0	1	0	0	0	0	0	0	0	1	1	
I	-W form	0	0	0	0	0	0	1	1	0	0	0	
	W_n form	$-D_{11}-D_{21}$	$-D_{12}-D_{22}$	-1	-1	1	1	0	0	0	0	$-C_1-C_2$	
	Change	$\uparrow$ max -ve											
	z_1	0	D12	1	0	-1	0	1	0	- D_{11}	0	$-D_{11}+C_1$	$\leftarrow$
	z_2	0	D_{22}	0	1	0	-1	0	1	- D_{21}	0	$-D_{21}+C_2$	
	s_1	1	0	0	0	0	0	0	0	1	0	1	
	s_2	0	1	0	0	0	0	0	0	0	1	1	
II	W_n form	0	$-D_{12}-D_{22}$	-1	-1	1	1	0	0	$D_{11}+D_{21}$	0	$(D_{11}+D_{21}-C_1-C_2)$	
III			$\uparrow$										Continue as above

The final cycle gives the x_i values and, by substituting them back into equation (3.6), the concordant profile is now determined.

b) Differentiation

The results of the quadratic programming solution can be achieved by differentiating the function (3.8) with respect to X_i .

$$\text{i.e. } \frac{\partial F}{\partial x_i} = 0 \quad \text{for all } i \dots 3.9$$

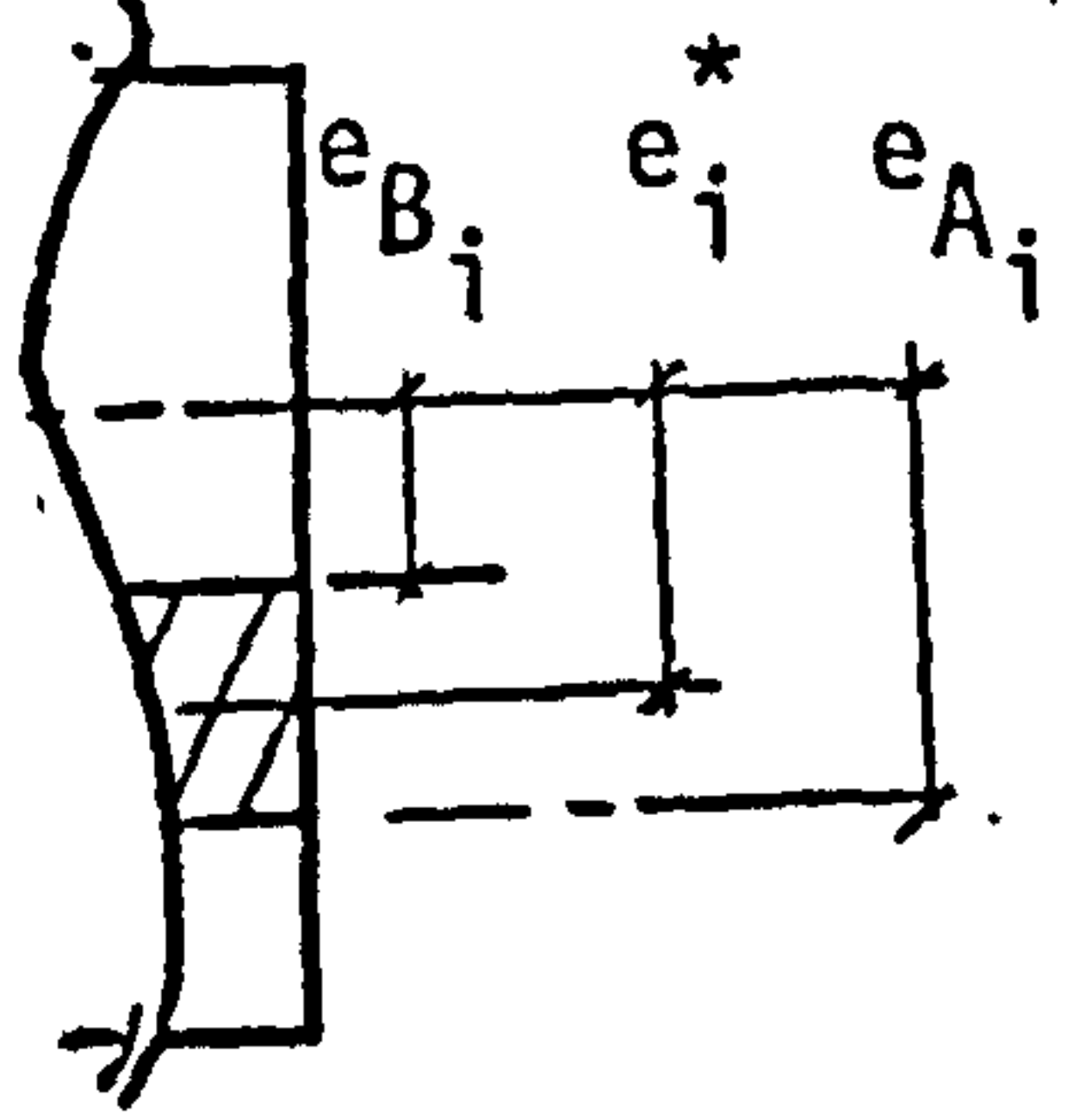
From equation (3.9), the values of X_i can be determined.

In an attempt to reach an optimum profile, it is necessary to achieve the minimum deviation around the line which produces the best stress distribution at each station under the loading conditions.

The equation of this line is:-

$$e_i^* = \frac{1}{H_p} \left[\frac{I}{Ad} (H_2 - H_1) - \frac{y_2}{d} (M_D + M_1) - \frac{y_1}{d} (M_D + M_2) \right]$$

in this case $X_i^* = \frac{e_i^* - e_{B_i}}{e_{A_i} - e_{B_i}}$



Another reasonable tendon profile may be obtained by minimizing the function (3.8) around the centroidal axis of the limiting zone.

In this case, $X_i^* = 0.5$ for all i

3.2.3 Direct Solution

For once (symetrically or unsymetrically) and twice (symetrically) statically indeterminate structures, the concordant profile can be determined directly as follows.

Referring to equation (3.3) and assuming

$$0 \leq x \leq 1 \quad \dots \quad 3.5a$$

The eccentricity of the concordant profile may be expressed in the general form as follows

$$e_i = (e_{Ai} - e_{Bi}) x + e_{Bi} \quad \dots \quad 3.6a$$

Substituting equation (3.6a) into equation (3.1) and by using Simpson's rule in the integration leads to

$$\sum_{i=1}^m H_{pi} ((e_{Ai} - e_{Bi}) x + e_{Bi}) \cdot \frac{1}{EI_i} \cdot \frac{h_i}{3} \cdot \gamma_i \cdot m_i +$$

$$\sum_{i=1}^m H_{pi} \cdot \frac{1}{EA_j} \cdot \frac{h_i}{3} \cdot \gamma_i \cdot n_i = 0 \quad \dots \quad 3.7a$$

By solving the above equation the value of x can be found. If the magnitude of x lies between zero and one ($0 \leq x \leq 1$) this means that a concordant profile exists within the limiting zone and the eccentricity of the profile can be found by substituting the value of x in equation 3.6a. Otherwise we use the quadratic programming or differentiation solutions.

The concordant profile of beams

In case of statically indeterminate beams, the terms (k) in equation (3.1) and (k_j) in equation (3.4) which represents the axial force effect should be replaced by zero and by following the same procedures as for frames and arches, the concordant profile of beams within the limiting zone can be found.

3.3 DESIGN STEPS

The design steps can be illustrated in a numerical example as follows:

Consider a frame with a variable rectangular cross section as shown in Fig. 3.4, subjected to live load of intensity 0.8 t/m^2 . The dead load is the external D.L of intensity 2.5 t/m^2 (the weight of the slab) and the self weight of the structure. The allowable compressive stresses = 15 N/mm^2 and no tensile stresses are allowed.

The concrete density = 25 kN/m^3

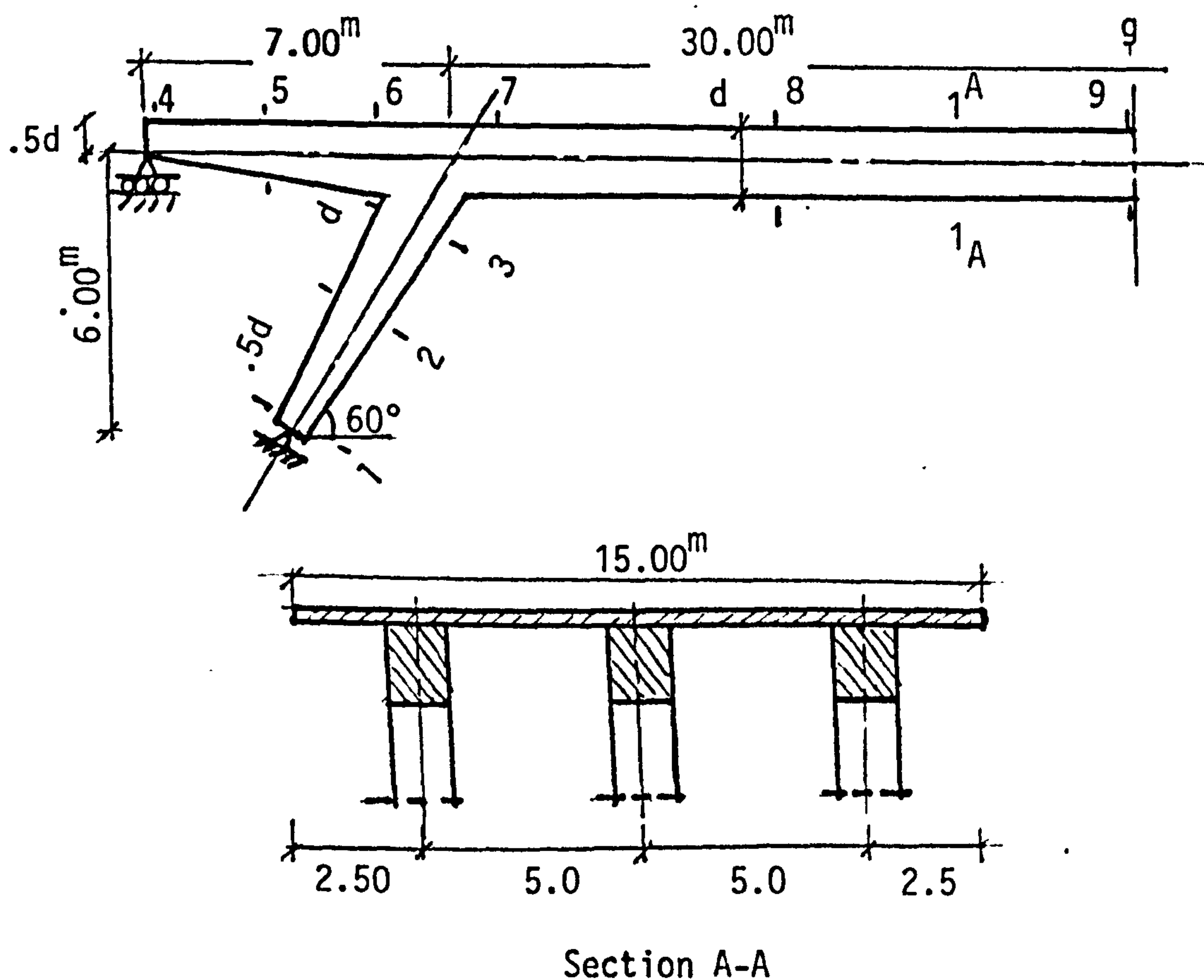
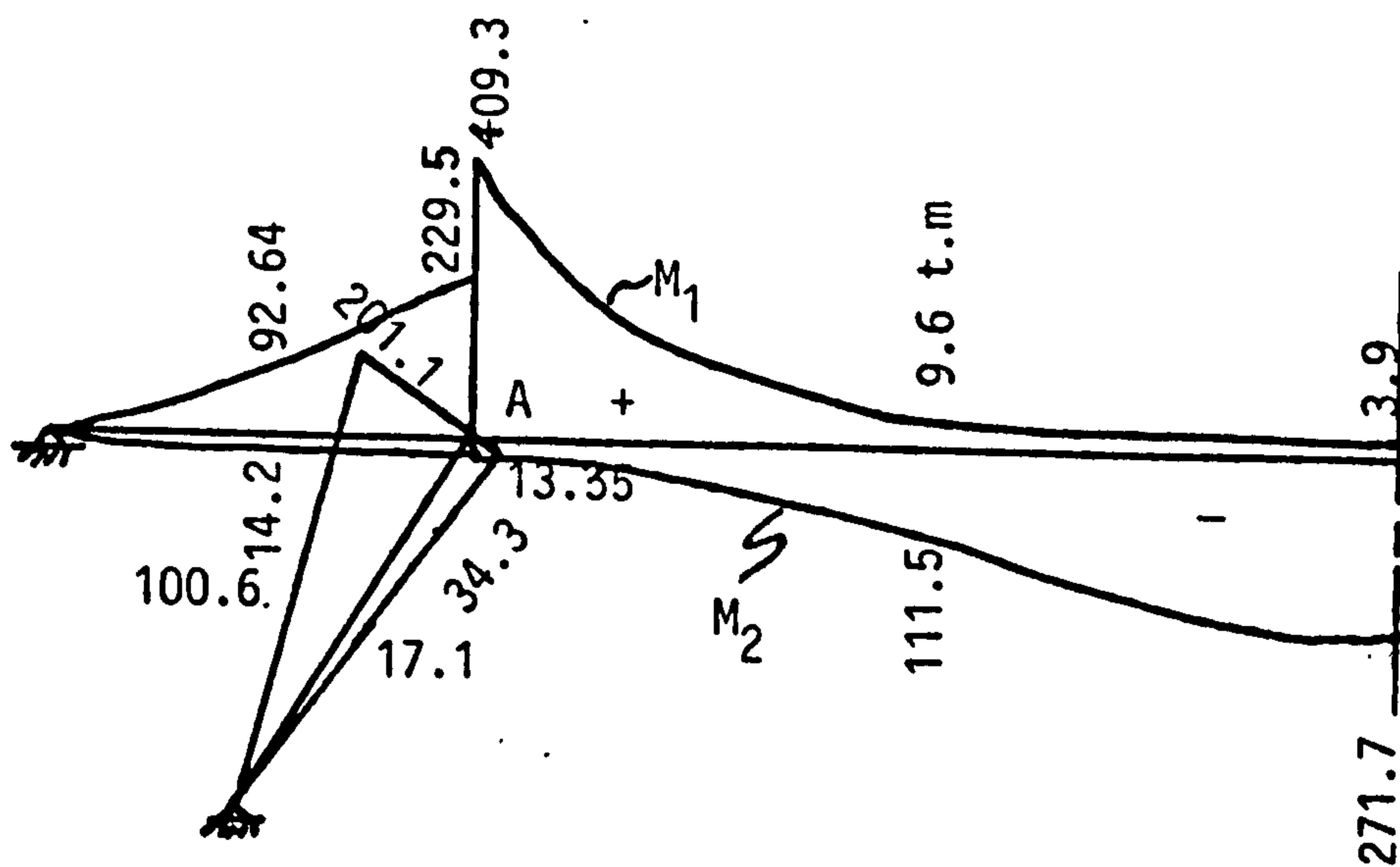
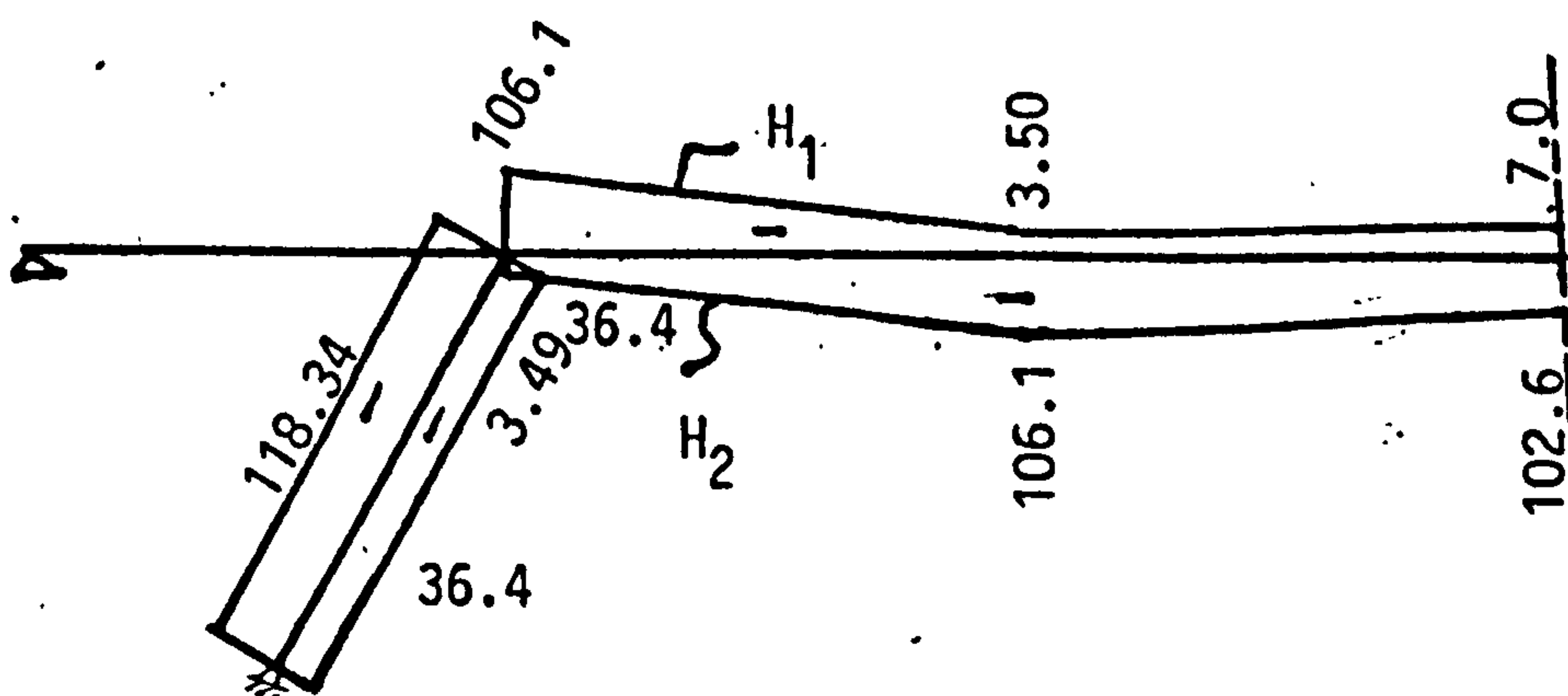


Fig.3.4

a) The live load + Ext. D.L. B.M. and N.F. envelopes



B.M. Envelope



N.F. Envelope

Fig. 3.5

b) Section modulus
(Sec. A)

$$H_1 > H_2$$

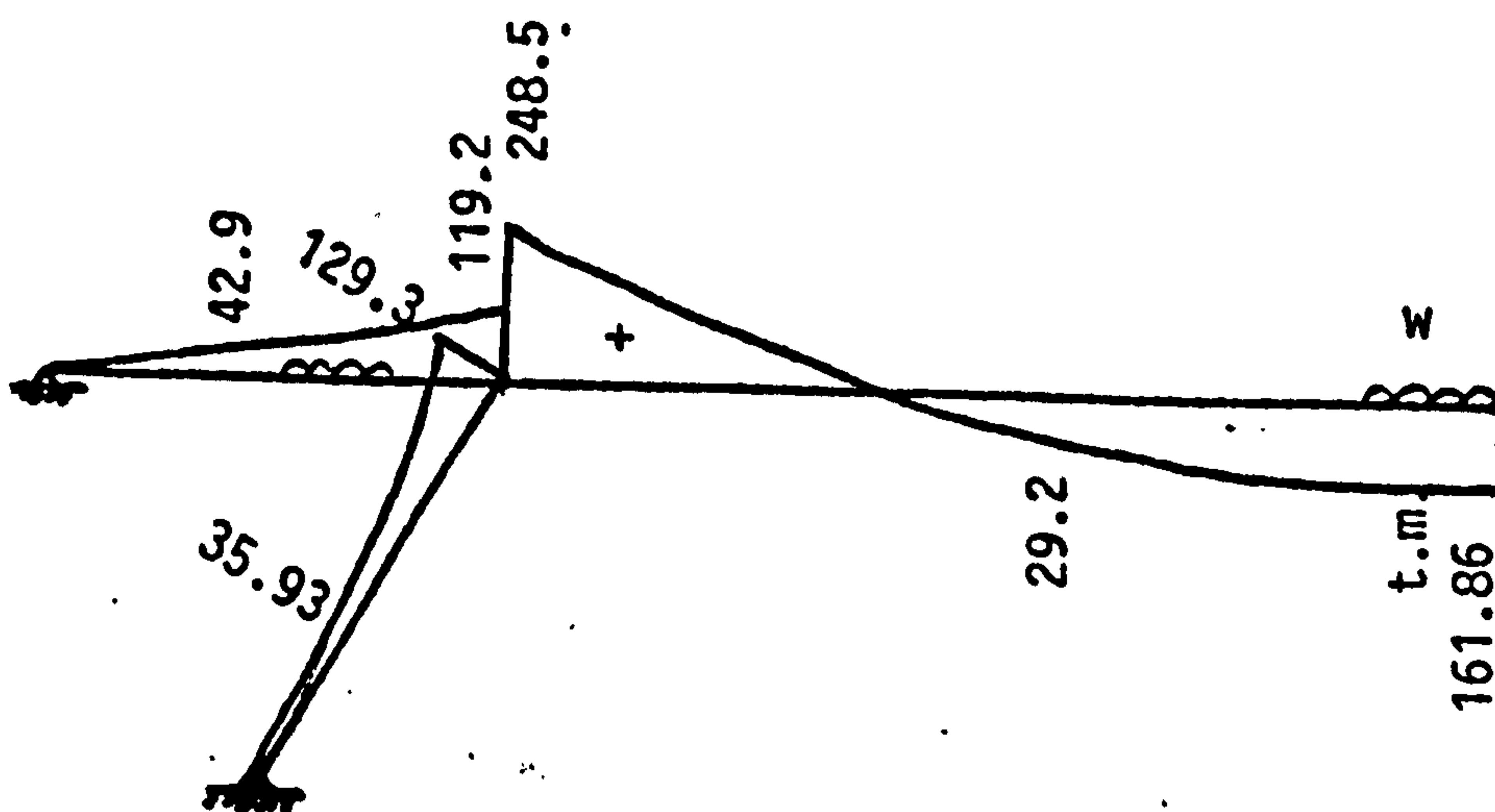
$$M_L = 1/6 f_c b d^2 + 1/6 (H_2 - H_1) d \quad (\text{eg}^n 1.24)$$

$$409.3 + 13.35 = 1/6 \times 1500 \times 1.2d^2 + 1/6 (3.49 - 106.1)d$$

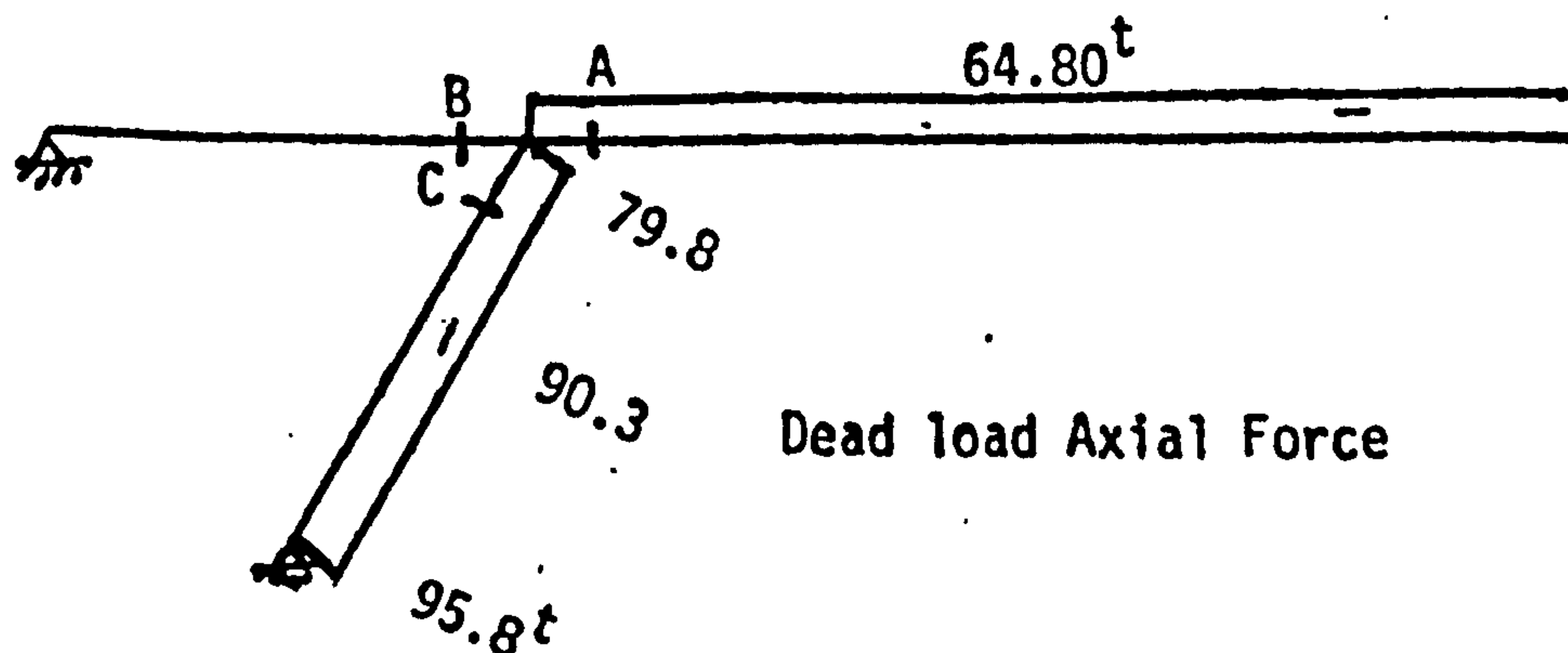
$$d = 1.216 \text{ m}$$

c) Dead load moment

$$w = 1.2 \times 1.216 \times 2.5 = 3.647 \text{ t/m}$$



Dead load moment



Dead load Axial Force

Fig. 3.6

d) The prestressing force
(Sec. A)

$$H_1 > H_2$$

$$\begin{aligned} H_p &= \frac{1}{2} f_c b d - H_D - H_1 \quad (\text{eq}^n \ 1.25) \\ &= \frac{1}{2} \times 1500 \times 1.2 \times 1.216 - 64.80 - 106.1 = 923.5 \text{ ton} \end{aligned}$$

(Sec. B)

$$f_c = \frac{6M_L}{bd^2}$$

$$H_p = \frac{1}{2} f_c b d = \frac{3M_L}{d} = \frac{3(229.5 + .44)}{1.216} = 567.3 \text{ ton}$$

(Sec. C)

$$H_1 > H_2$$

$$M_L = 1/6 f_c b d^2 + 1/6 (H_2 - H_1) d \quad (\text{eq}^n \ 1.24)$$

$$\begin{aligned} f_c &= [6M_L - (H_2 - H_1) d] / b d^2 \\ &= [6 (201.1 + 34.3) - (36.4 - 118.3) 1.216] / 1.2 \times 1.216^2 \\ &= 852.12 \text{ t/m}^2 \end{aligned}$$

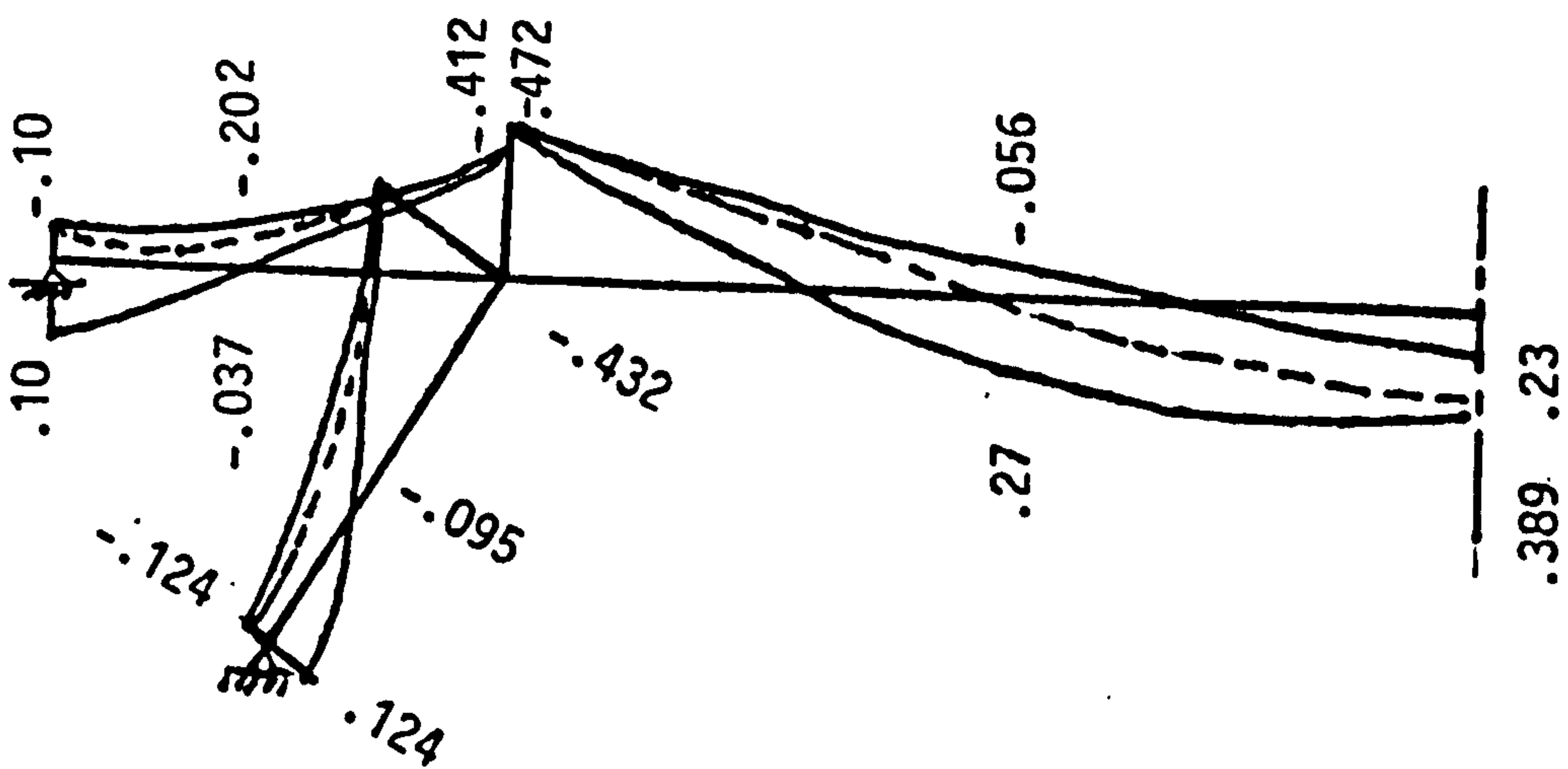
$$\begin{aligned} H_p &= \frac{1}{2} f_c b d - H_D - H_1 \quad (\text{eq}^n \ 1.25) \\ &= \frac{1}{2} \times 852.1 \times 1.2 \times 1.216 - 79.8 - 118.34 = 423.57 \text{ ton} \end{aligned}$$

e) The limiting zone

Referring to expressions 1.14 and 1.15 we obtain the shown zone

$$e_A = \frac{d}{6} \left(\frac{H_A}{H_p} \right) - \frac{M_D + M_1}{H_p}$$

$$e_B = -\frac{d}{6} \left(\frac{H_B}{H_p} \right) - \frac{M_D + M_2}{H_p}$$



----- The concordant profile

Fig. 3.7 The Limiting Zone

f) The concordant profile

Referring to expression (3.7) we obtain the following equations

$$7851.24 X_2 - 16957.8 X_5 - 71276 X_8 - 8687.13 X_9 + 52782.7 = 0$$

$$27229.8 X_2 + 241614 X_8 + 29447.9 X_9 - 123417 = 0$$

Solving the above equation gives

$$X_2 = -8.873 X_8 - 1.081 X_9 + 4.532$$

$$X_5 = -8.311 X_8 - 1.013 X_9 + 5.211$$

substituting the above into equation (3.8) and by considering

$$X_1, X_3, X_4, X_6, X_7$$

$$F = -(-8.873 X_8 - 1.081 X_9 + 4.532)^2$$

$$-(-8.311 X_8 - 1.013 X_9 + 5.211)^2$$

$$- X_8^2 - X_9^2$$

$$+ (-8.873 X_8 - 1.081 X_9 + 4.532)$$

$$+ (-8.311 X_8 - 1.013 X_9 + 5.211)$$

$$+ X_8 + X_9$$

$$\text{or } F = CX - \frac{1}{2} X^T D X$$

$$= (150.87, 19.266) \begin{bmatrix} X_8 \\ X_9 \end{bmatrix} - \frac{1}{2} (X_8 \ X_9) \begin{bmatrix} 297.62 & 36.03 \\ 36.03 & 6.391 \end{bmatrix} \begin{bmatrix} X_8 \\ X_9 \end{bmatrix}$$

substituting the above into equation (3.12) the objective function will be:

Maximize

$$-333.651 X_8 - 42.422 X_9 - \lambda_8 - \lambda_9 + v_8 + v_9 = -170.137$$

subject to

297.62	36.03	1	0	-1	0	1	0	0	x_8	150.87
36.03	6.39	0	1	0	-1	0	1		x_9	
-----		-----		-----		-----				
1	0						1	0	λ_8	1
0	1	0		0		0	0	1	λ_9	
									v_8	1
									v_9	
									z_8	
									z_9	
									s_8	
									s_9	

TABLE 3.2
SIMPLEX TABLEAU FOR THE EXAMPLE

Cycle	Basic Variables	Admissible Variables							Artificial Variables			Slack Variables	Solution
		x_8	x_9	λ_8	λ_9	v_8	v_9	z_8	z_9	s_8	s_9		
	z_8	297.621	36.030	1	0	-1	0	1	0	0	0	150.87	
	z_9	36.030	6.391	0	1	0	-1	0	1	0	0	19.266	
	s_8	1	0	0	0	0	0	0	0	1	0	1	
	s_9	0	1	0	0	0	0	0	0	0	1	1	
	w form	0	0	0	0	0	0	0	1	1	0	0	
	w_n form	-333.651	-42.422	-1	-1	1	1	0	0	0	0	-170.137	
		x_8	x_9	λ_8	λ_9	v_8	v_9	z_8	z_9	s_8	s_9		
	x_8	1	0.121	0.0034	0	-.0034	0	0.0034	0	0	0	0.5069	
I	x_9	0	2.030	-0.1125	1	0.1125	-1	-.1125	1	0	0	1.0017	
	s_8	0	-.121	-.0034	0	.0034	0	-.0034	0	1	0	0.4931	
	s_9	0	1	0	0	0	0	0	0	0	1	1	
	w_n form	0	-2.030	0.134	-1	-.134	1	1.134	0	0	0	-1.0017	

/cont...

Cycle	Basic Variables	Admissible Variables						Artificial Variables			Slack Variables	Solution
		x_8	x_9	λ_8	λ_9	v_8	v_9	z_8	z_9	s_8	s_9	
II	x_8	1	0	0.01	0.059	-0.01	0.059	0.01	-.059	0	0	0.4472
	x_9	0	1	-0.055	0.493	.055	-.493	-.055	.493	0	0	0.4936
	s_8	0	0	-.01	0.059	0.01	-.059	-0.01	0.059	1	0	0.5528
	s_9	0	0	0.055	-.493	-.055	.493	0.055	-.493	0	1	0.5064
	w_n form	0	0	0	0	0	0	1	1	0	0	≈ 0

from cycle II of simplex tableau we get

$$\begin{aligned}
 x_8 &= x_{10} = .4472 \\
 x_9 &= .4936 \\
 x_2 &= x_{16} = 0.0308 \\
 x_5 &= x_{13} = 0.9945 \\
 x_1 &= x_{17} = 0 \\
 x_3 &= x_{15} = 0 \\
 x_4 &= x_{14} = 0 \\
 x_6 &= x_{12} = 0 \\
 x_7 &= x_{11} = 0
 \end{aligned}$$

substituting in equation (3.6) the concordant profile will be

$$\begin{aligned}
 e_1 = e_{17} &= (.124 + .124) \cdot 0 - .124 = -.124 \text{ m} \\
 e_2 = e_{16} &= (-.095 + .242) \cdot 0.0308 - .242 = -.237 \\
 e_3 = e_{15} &= (-.482 + .432) \cdot 0 - .432 = -.432 \\
 e_4 = e_{14} &= (.101 + .101) \cdot 0 - .101 = -.101 \\
 e_5 = e_{13} &= (-.037 + .202) \cdot 0.9945 - .202 = -.038 \\
 e_6 = e_{12} &= (-.412 + .412) \cdot 0 - .412 = -.412 \\
 e_7 = e_{11} &= (-.472 + .472) \cdot 0 - .472 = -.472 \\
 e_8 = e_{10} &= (.271 + .055) \cdot .4472 - .055 = 0.091 \\
 e_9 &= (.339 - .230) \cdot .4936 + .23 = 0.284
 \end{aligned}$$

check lack of concordancy

$$\begin{aligned}
 \int_s \frac{H_p \cdot e \cdot m_1}{EI} ds + \int_s \frac{H_p \cdot n_1}{EA} ds &= 7.67 \times 10^{-7} \\
 \int_s \frac{H_p \cdot e \cdot m_2}{EI} ds + \int_s \frac{H_p \cdot n_2}{EA} ds &= 2.04 \times 10^{-6} \\
 \int_s \frac{H_p \cdot e \cdot m_3}{EI} ds + \int_s \frac{H_p \cdot n_3}{EA} ds &= 7.67 \times 10^{-7}
 \end{aligned}$$

PART II
THEORETICAL SOLUTION AND EXPERIMENTAL WORK

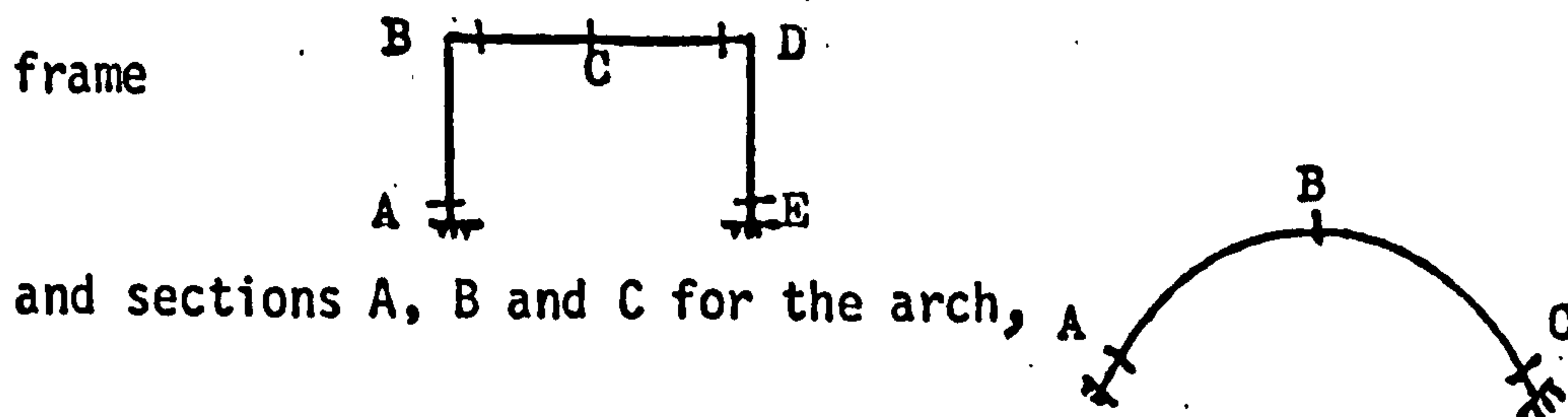
CHAPTER (4)

THE THEORETICAL DESCRIPTION OF THE EXPERIMENTAL WORK

4.1 DESIGN STEPS FOR THE TEST FRAME AND ARCH

4.1.1 In this part we shall find a concordant profile within the limiting zone by using the previously discussed quadratic programming solution and the computer programme.

After obtaining the concordant profile we are going to calculate the stresses at sections A, B, C, D and E for the



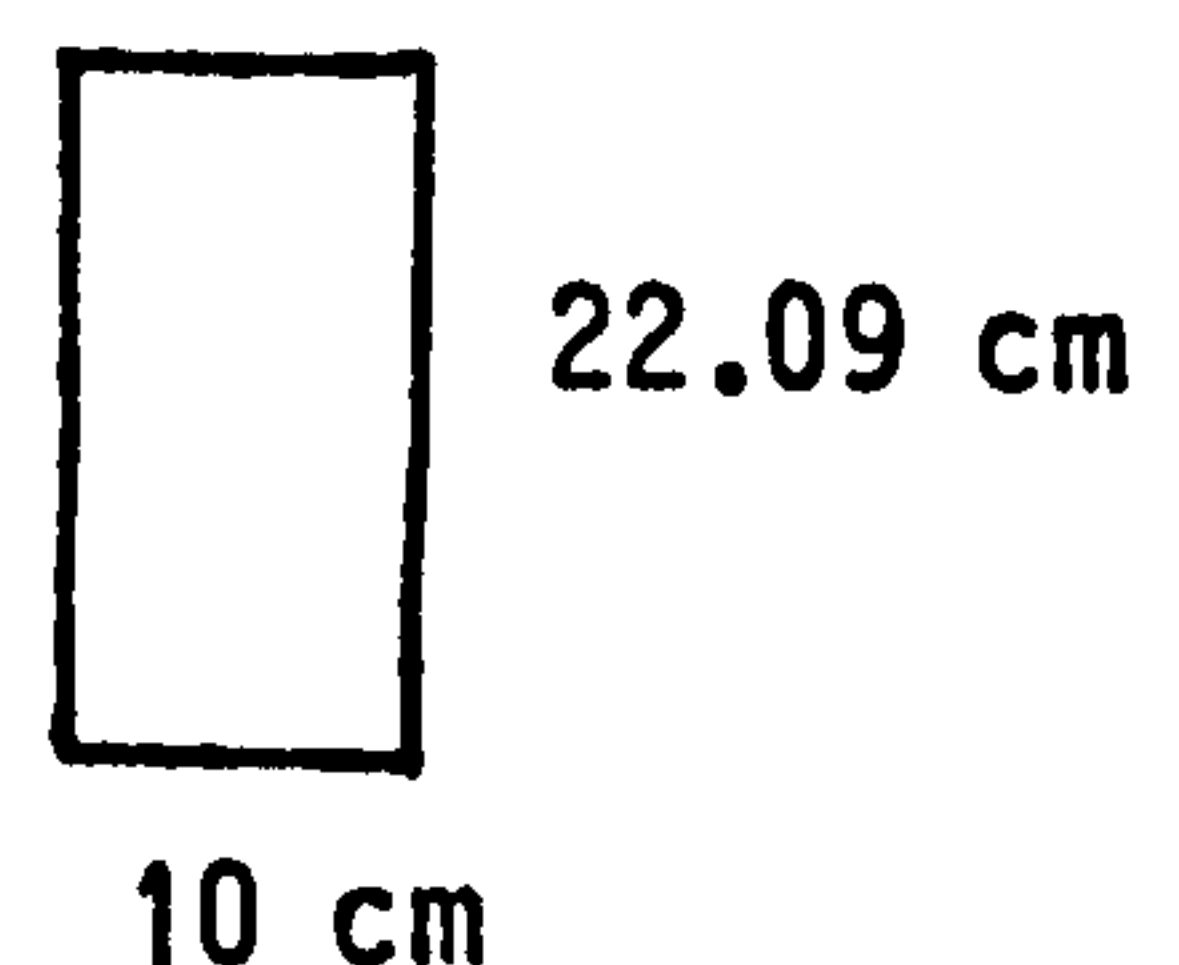
The deflections at section C for the frame and section B for the arch. Then we shall compare their values with the values obtained from the experiments.

4.1.2 Cross Section and Reinforcement of the Frame

The tested frame has a uniform rectangular cross-section.

The cross-section area $A_c = 220.9\text{cm}^2$

The section modulus $Z_c = 813.28\text{cm}^3$



The modulus of elasticity of the concrete $E_c = 33 \text{ kN/mm}^2$

The density of concrete $\gamma_c = 25 \text{ kN/m}^3$

The cross-section area of the wire on the span of the
frame $A_s = 122.72 \text{ mm}^2$

The design load per wire $F = 140.3 \text{ kN}$

The cross-section area of the wire on the leg of the
frame $A_s = 67.93 \text{ mm}^2$

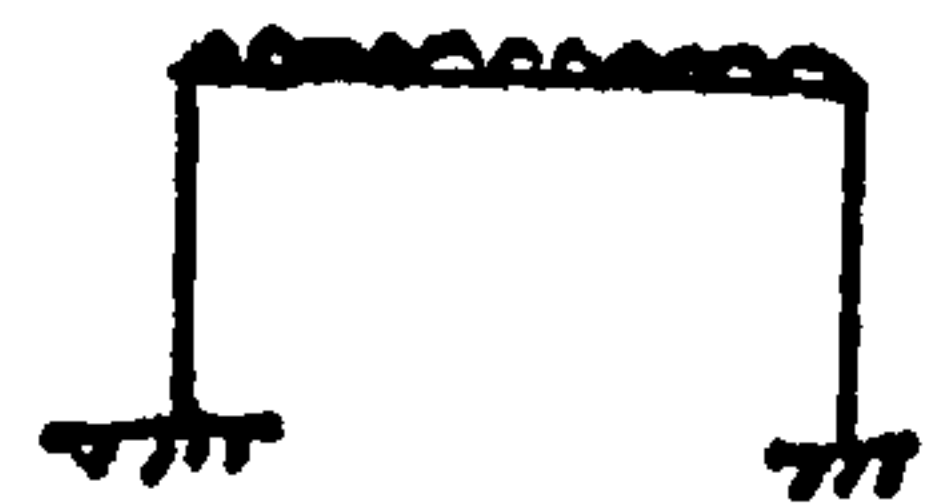
The design load per wire $F = 79.5 \text{ kN}$

The modulus of elasticity of the steel $E_s = 2.2 \times 10^5 \text{ N/mm}^2$

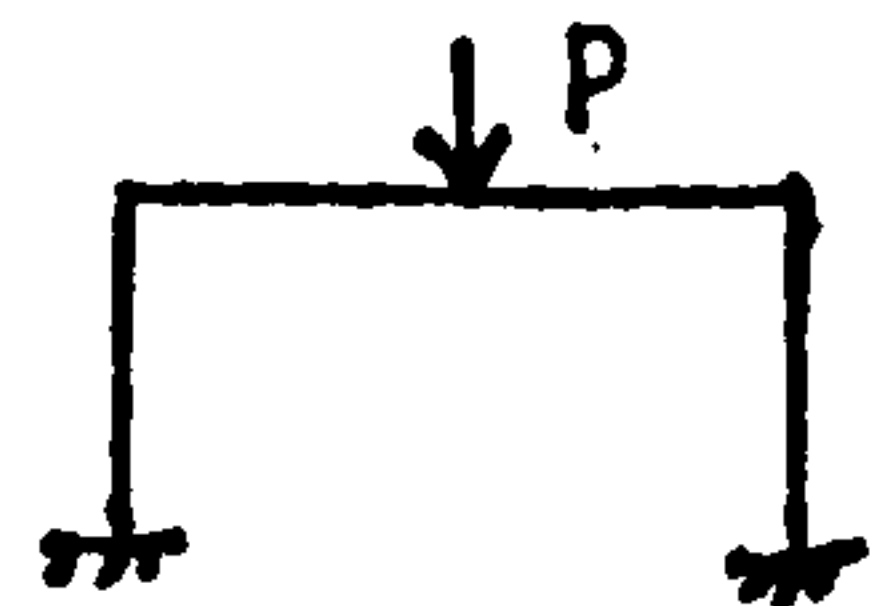
The number of wires per frame are three, i.e. one for the
span ($\phi 12.5 \text{ mm}$) and one for each leg ($\phi 9.3 \text{ mm}$)

4.1.3 The Load Condition

The dead load:- The weight of the frame only giving a
uniformly distributed load $w = .552 \text{ kN/m}$

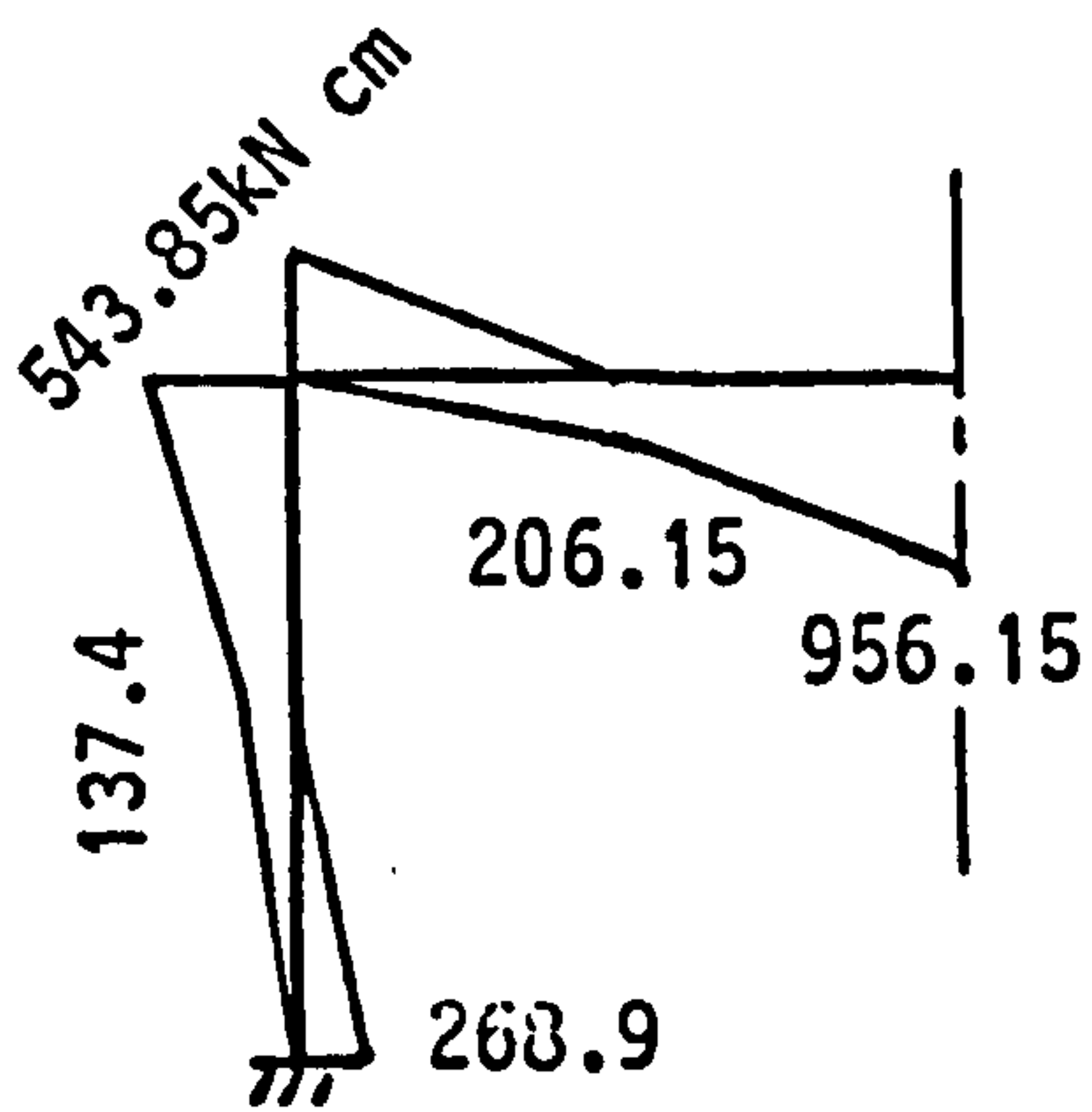


The live load:- The live load is a concentrated load applied
at the mid point of the span and varying between $p = 0$ and
 $p = 30 \text{ kN}$

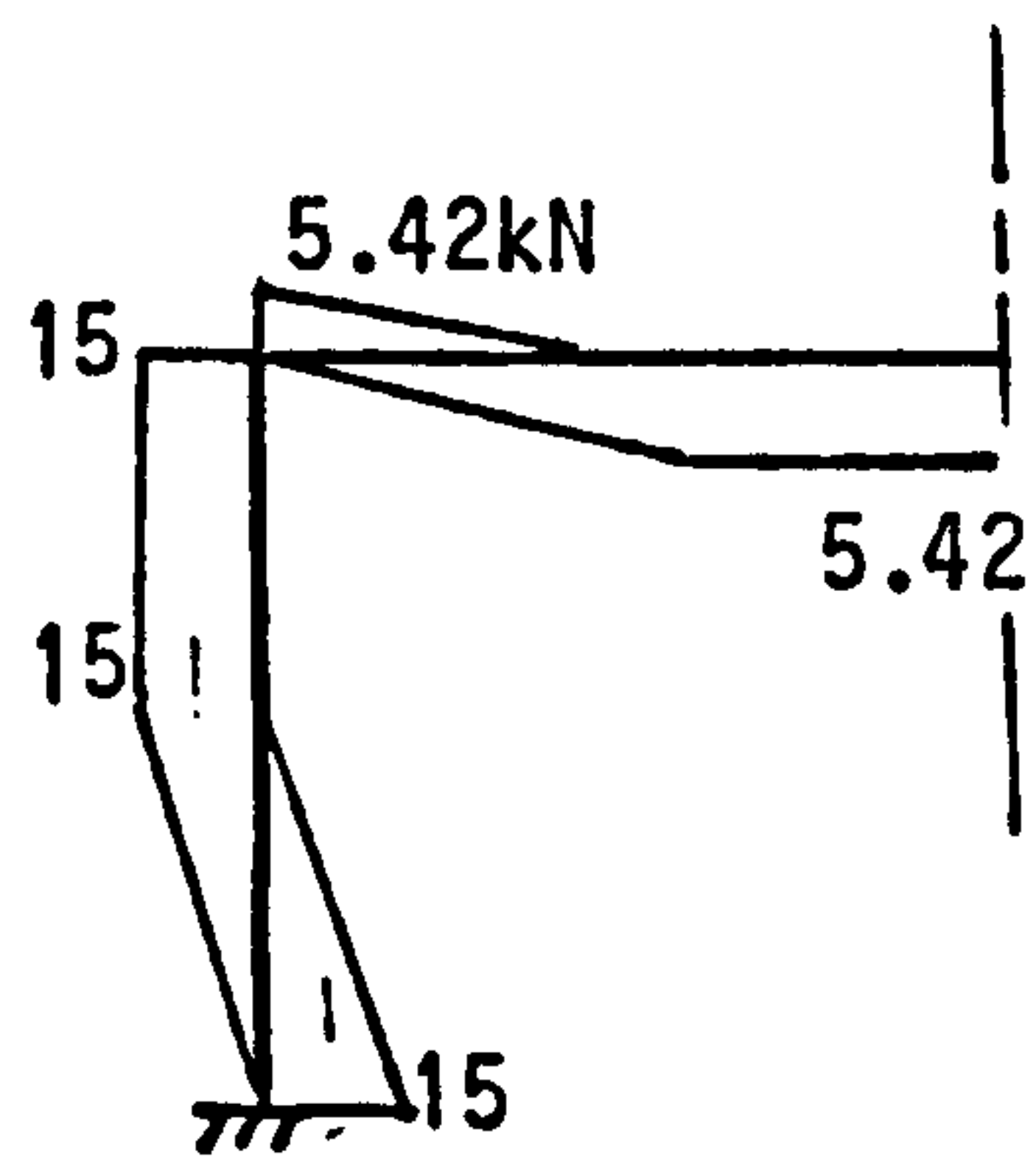


4.1.4 The Frame Profiles

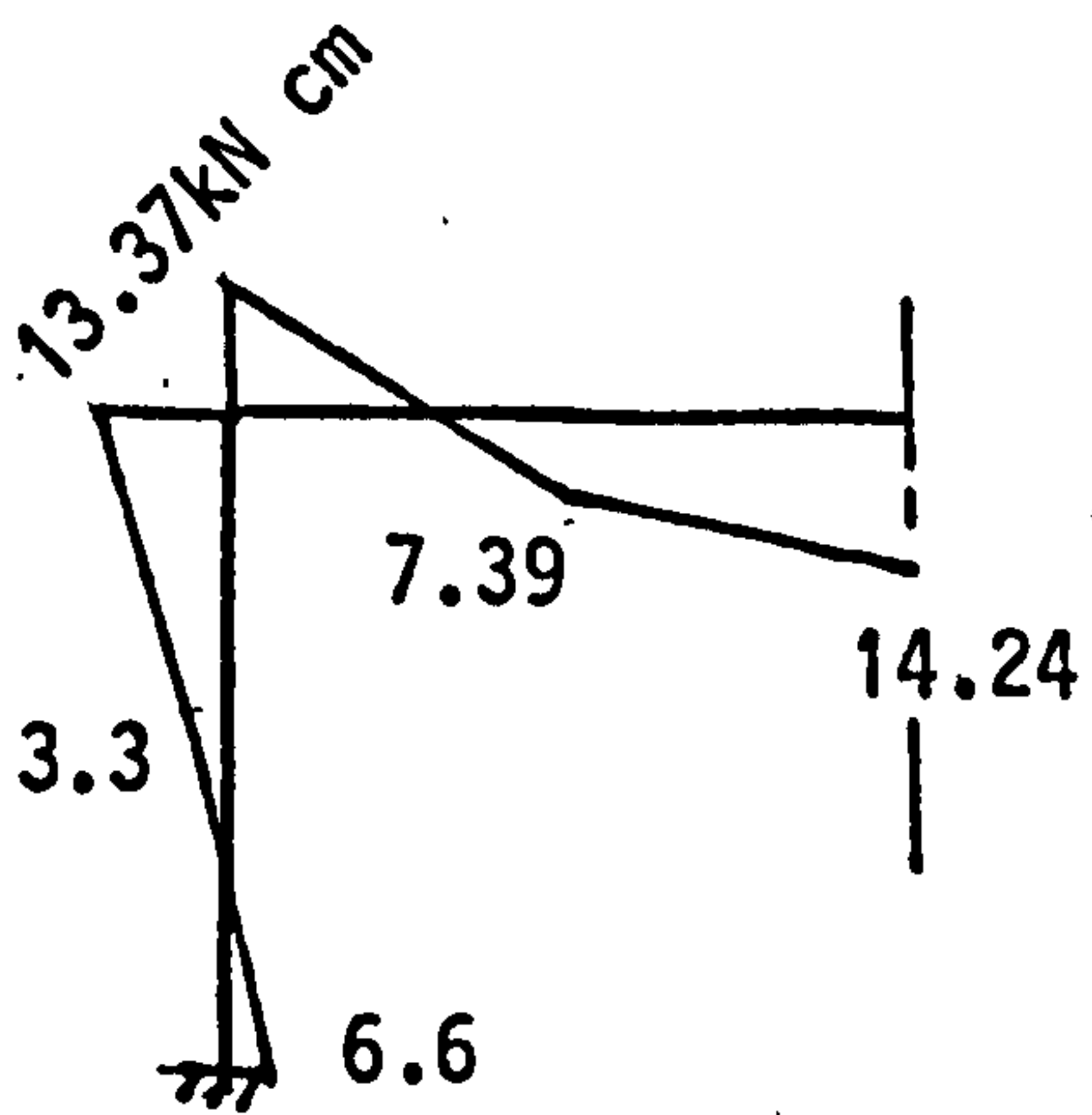
From the computer program we find the following profiles
for the frame, and the output data of the program shown in
Appendix B.



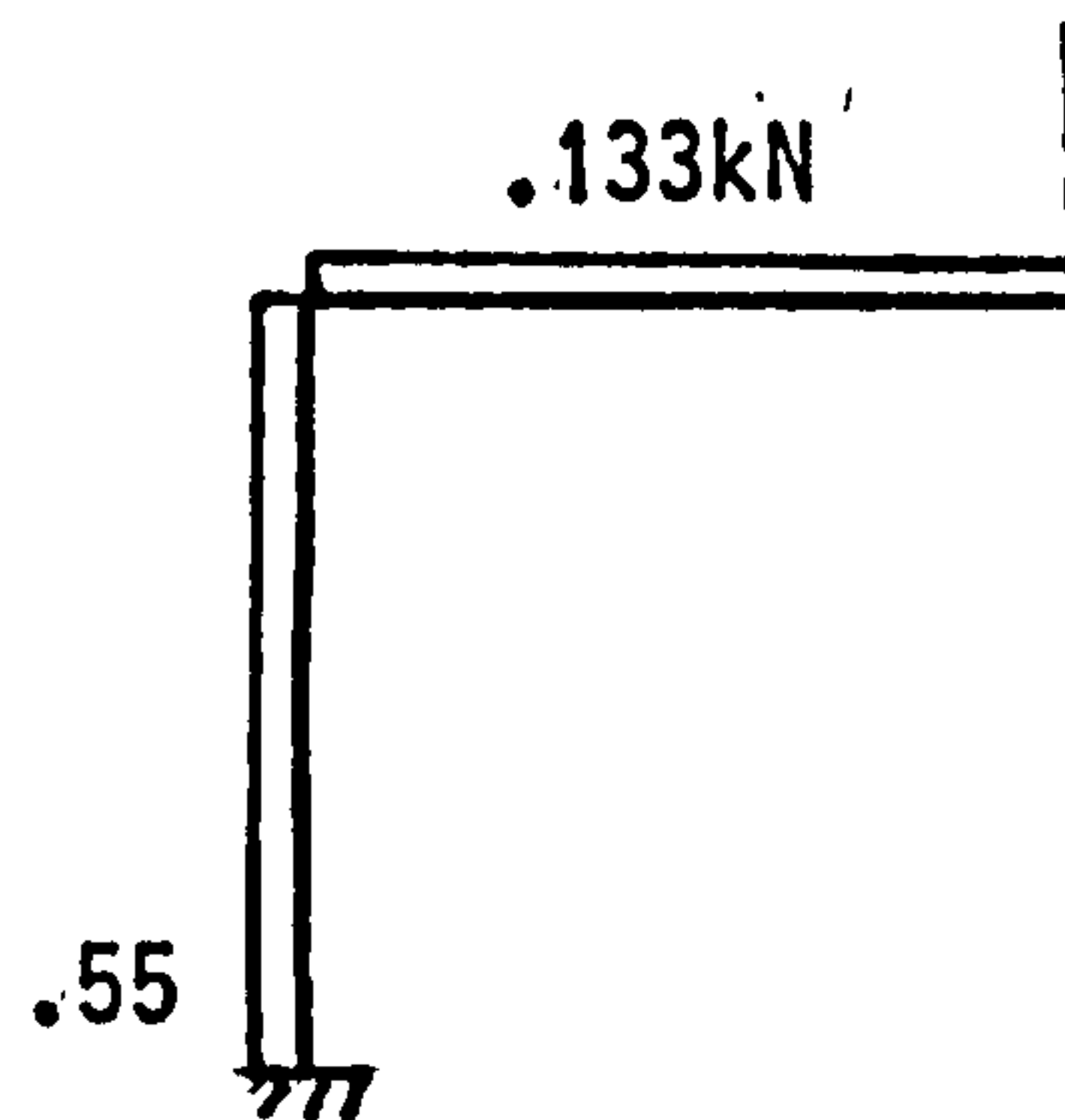
a) M_1, M_2 envelope



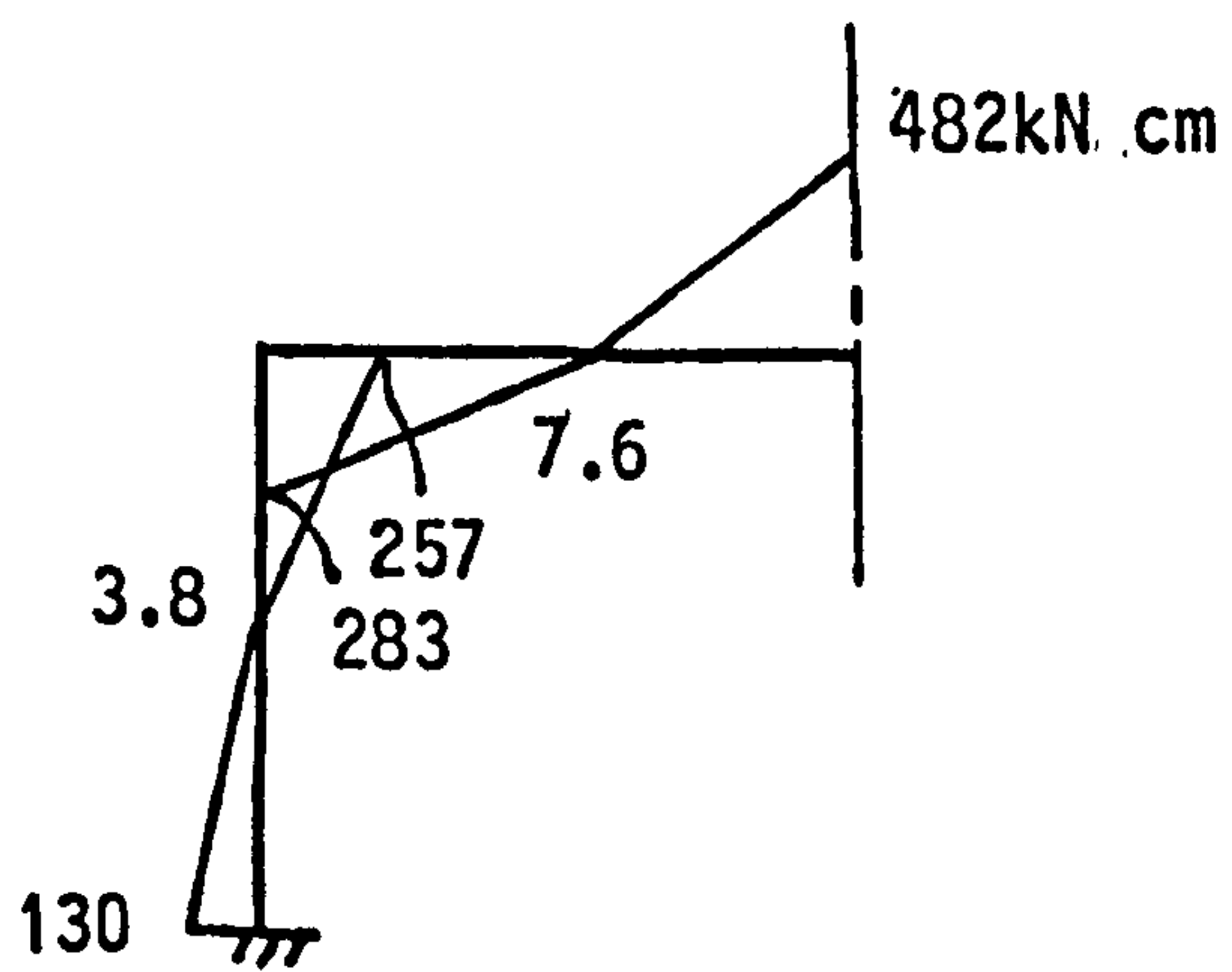
b) H_1, H_2 envelope



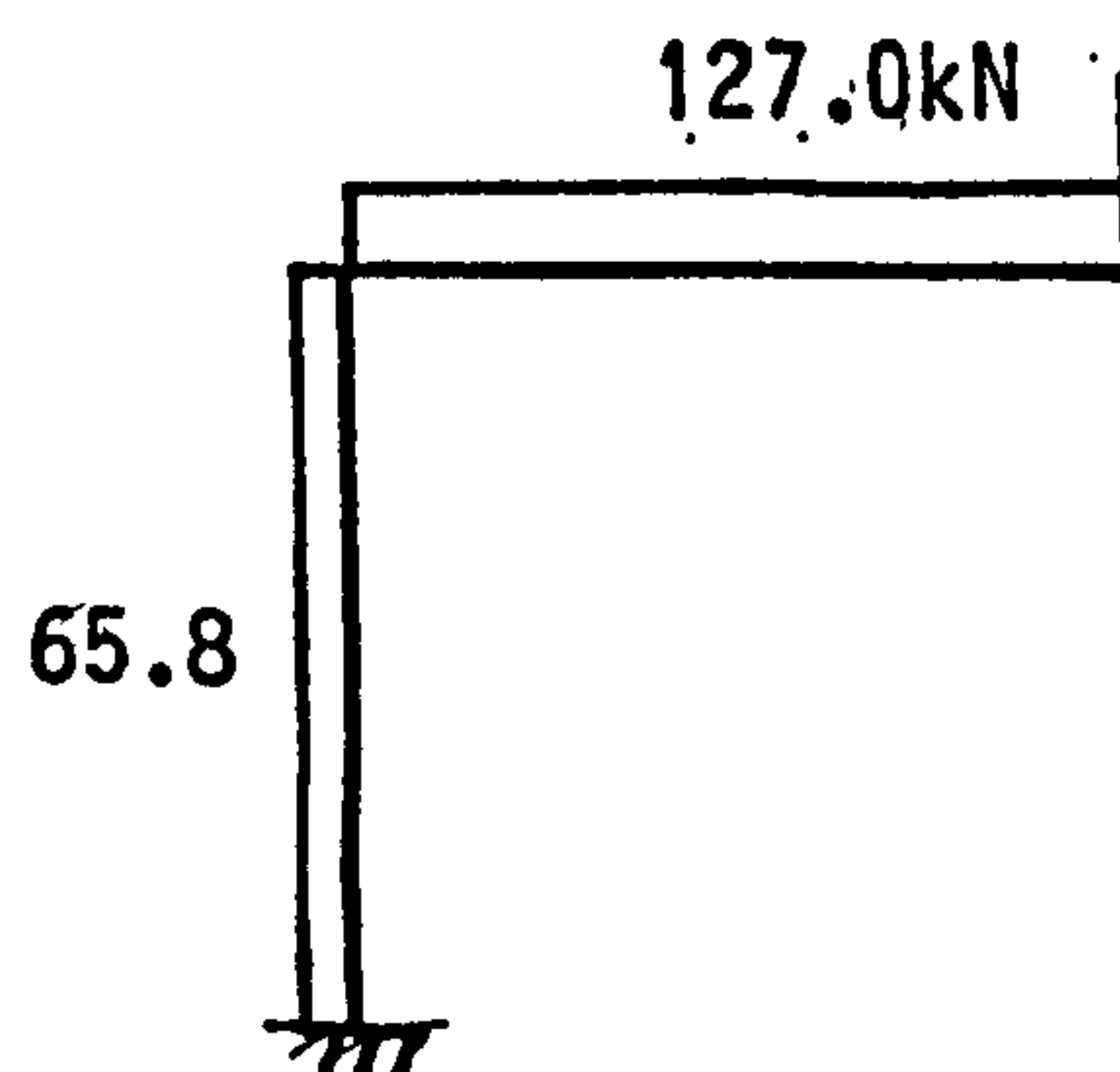
c) the dead load moment



d) the dead load axial force



e) the prestressing moment



f) the prestressing force

Fig. 4.1

The upper limit	-3.791	-2.161	3.798 cm
The concordant	-2.23	-.06	3.798 cm
The lower limit	-.544	3.744	3.798 cm

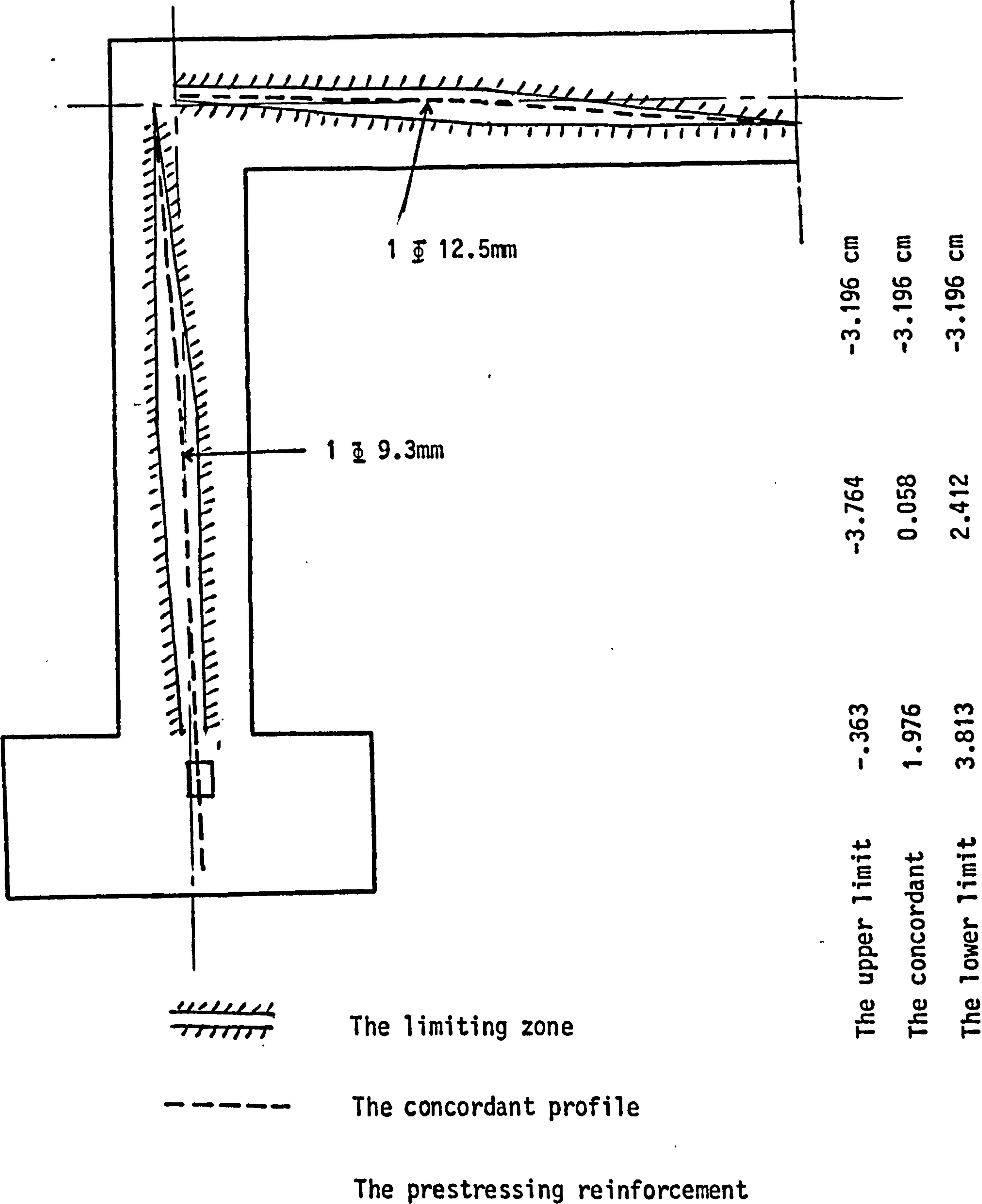
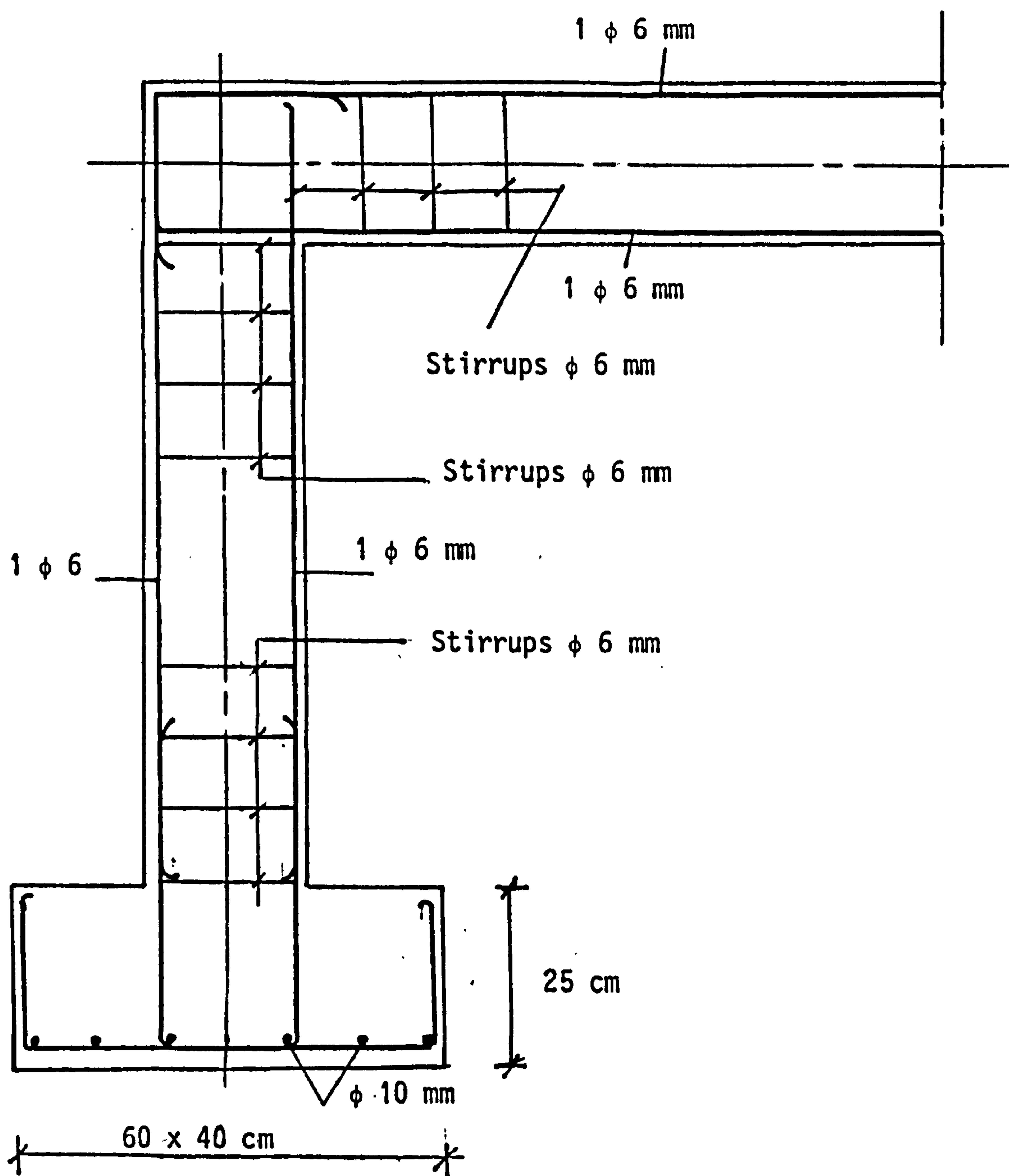


Fig. 4.2



The Ordinary Reinforcement

Fig. 4.3

4.1.5 Cross Section and Reinforcement of the Arch

The tested arch has a uniform rectangular cross section.

The cross section area $A_c = 142.7\text{cm}^2$

The section modulus $Z_c = 339.388\text{cm}^3$

The modulus of elasticity of the concrete $E_c = 33\text{kN/mm}^2$

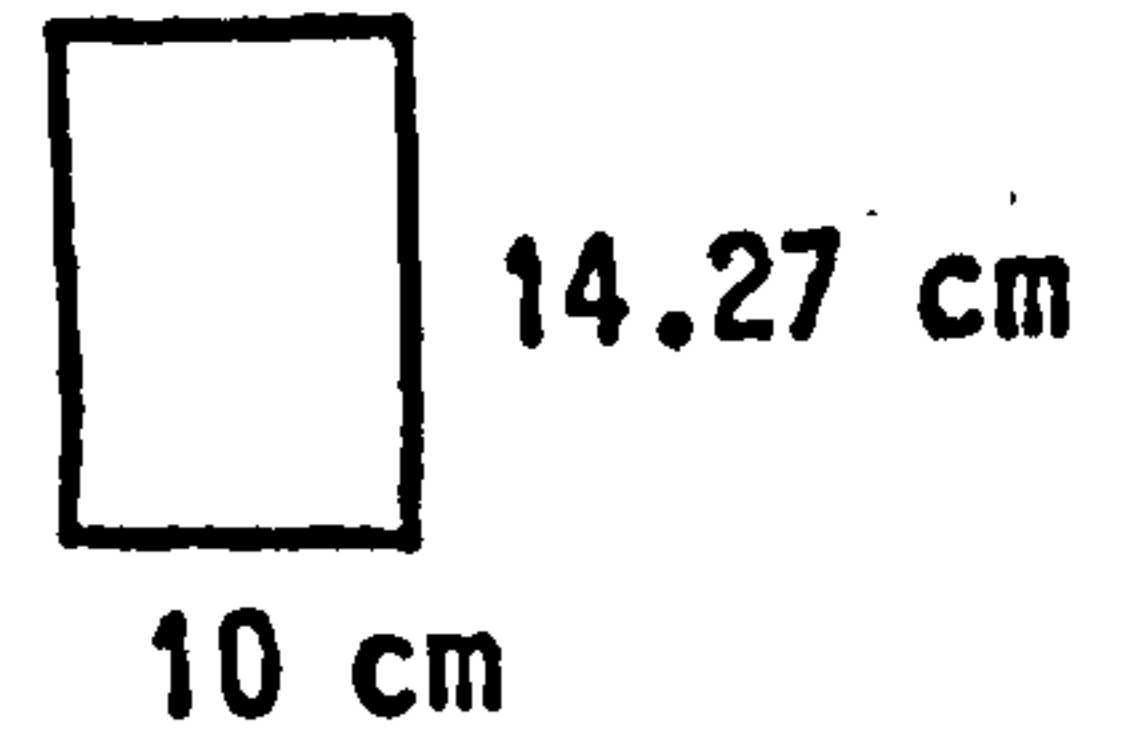
The density of concrete $\gamma_c = 25\text{kN/m}^3$

The cross section area of the wire $A_s = 49.017\text{mm}^2$

The design load per wire 58.7kN

The modulus of elasticity of the steel $E_s = 2.2 \times 10^5\text{N/mm}^2$

The number of wires per arch is one



4.1.6 The Load Condition

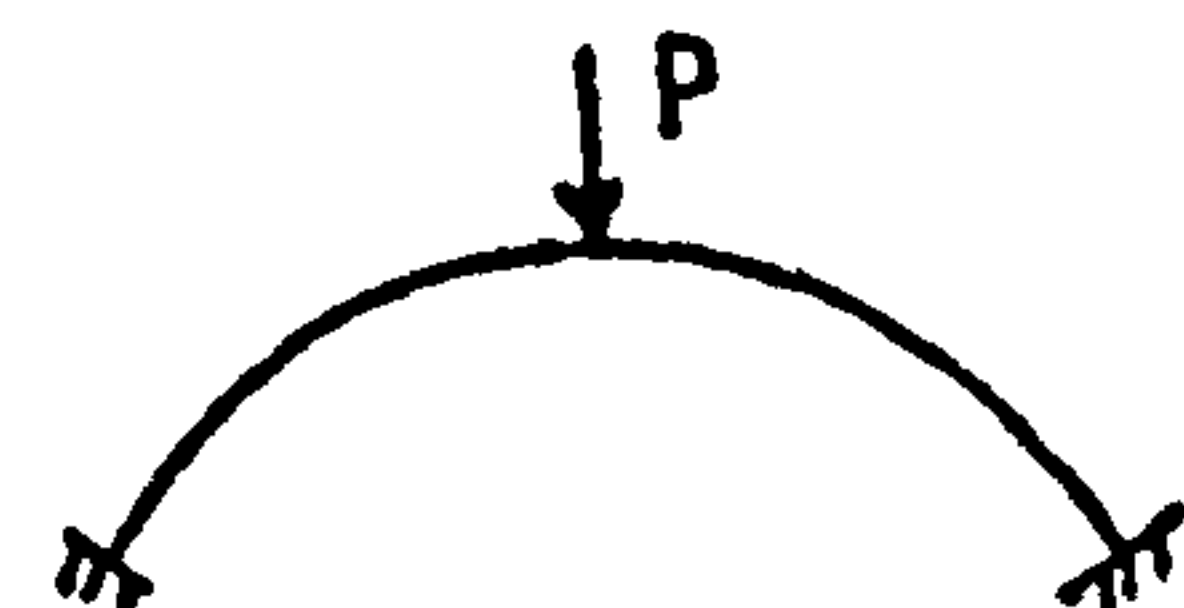
The dead load:- The weight of the arch only giving a uniformly distributed load

$$W = .357\text{kN/m}$$



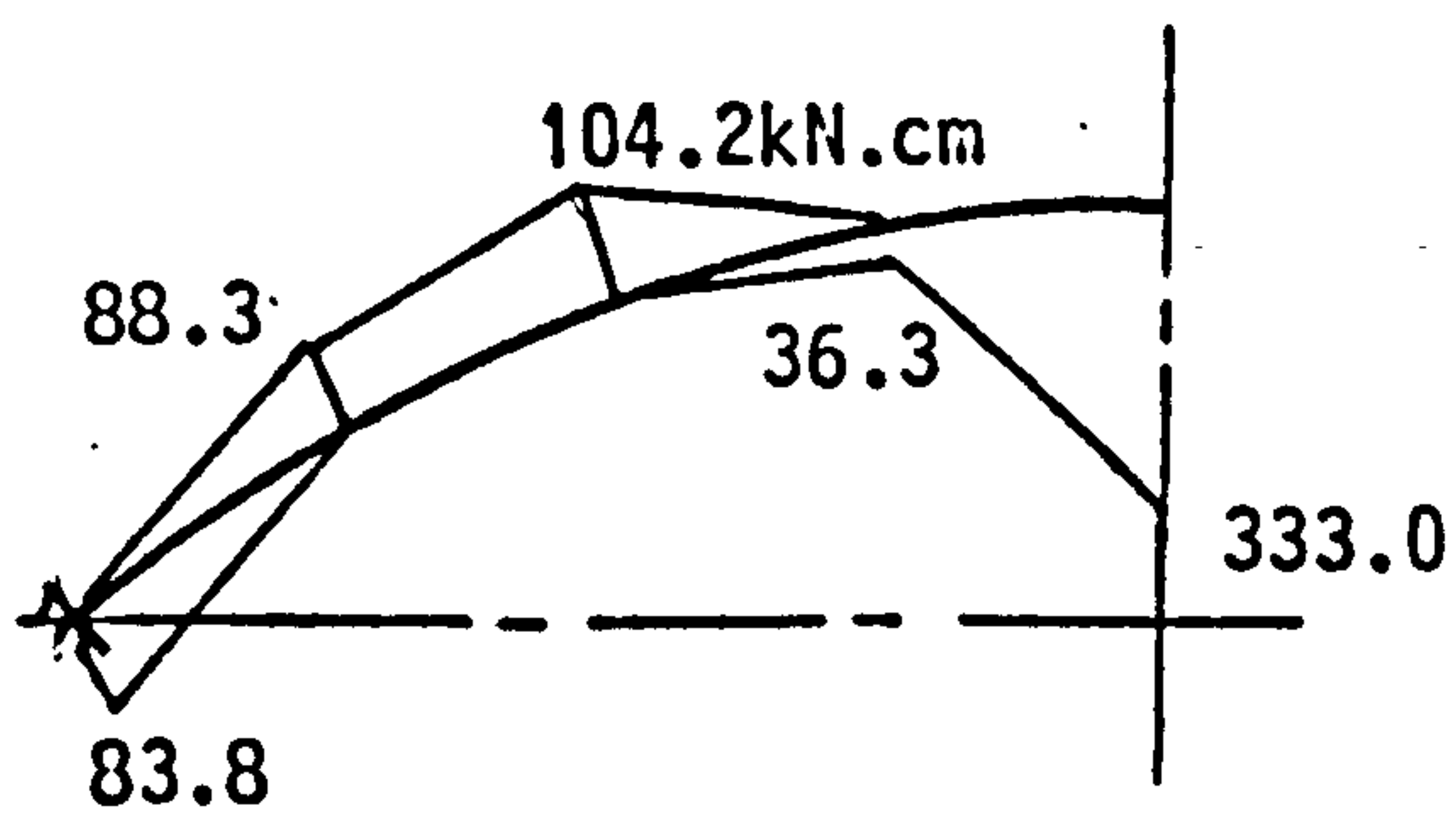
The live load:- The live load is a concentrated load applied at the mid point of the span and varying between

$$P = 0 \text{ and } P = 30\text{kN}$$

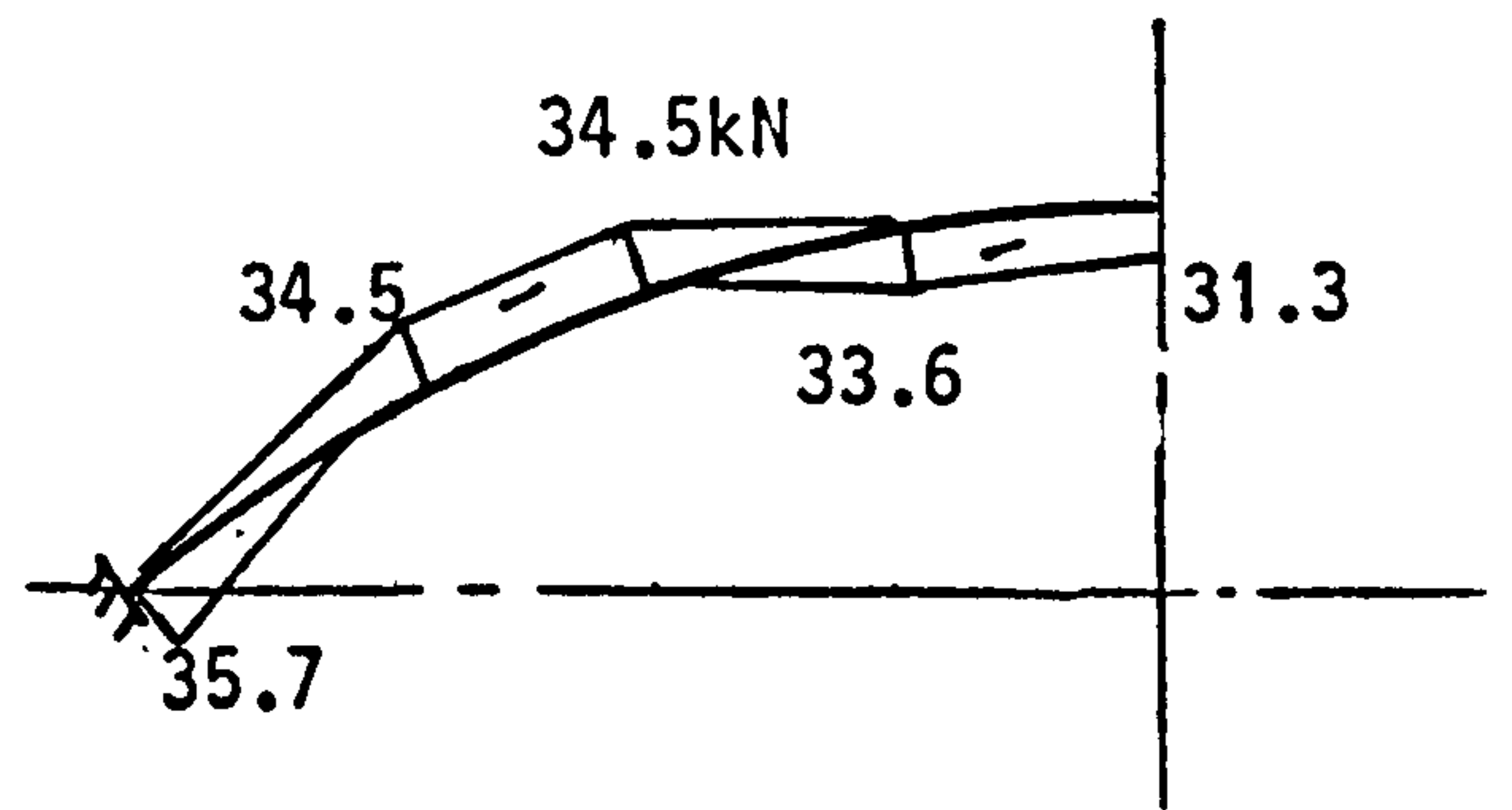


4.1.7 The Arch Profiles

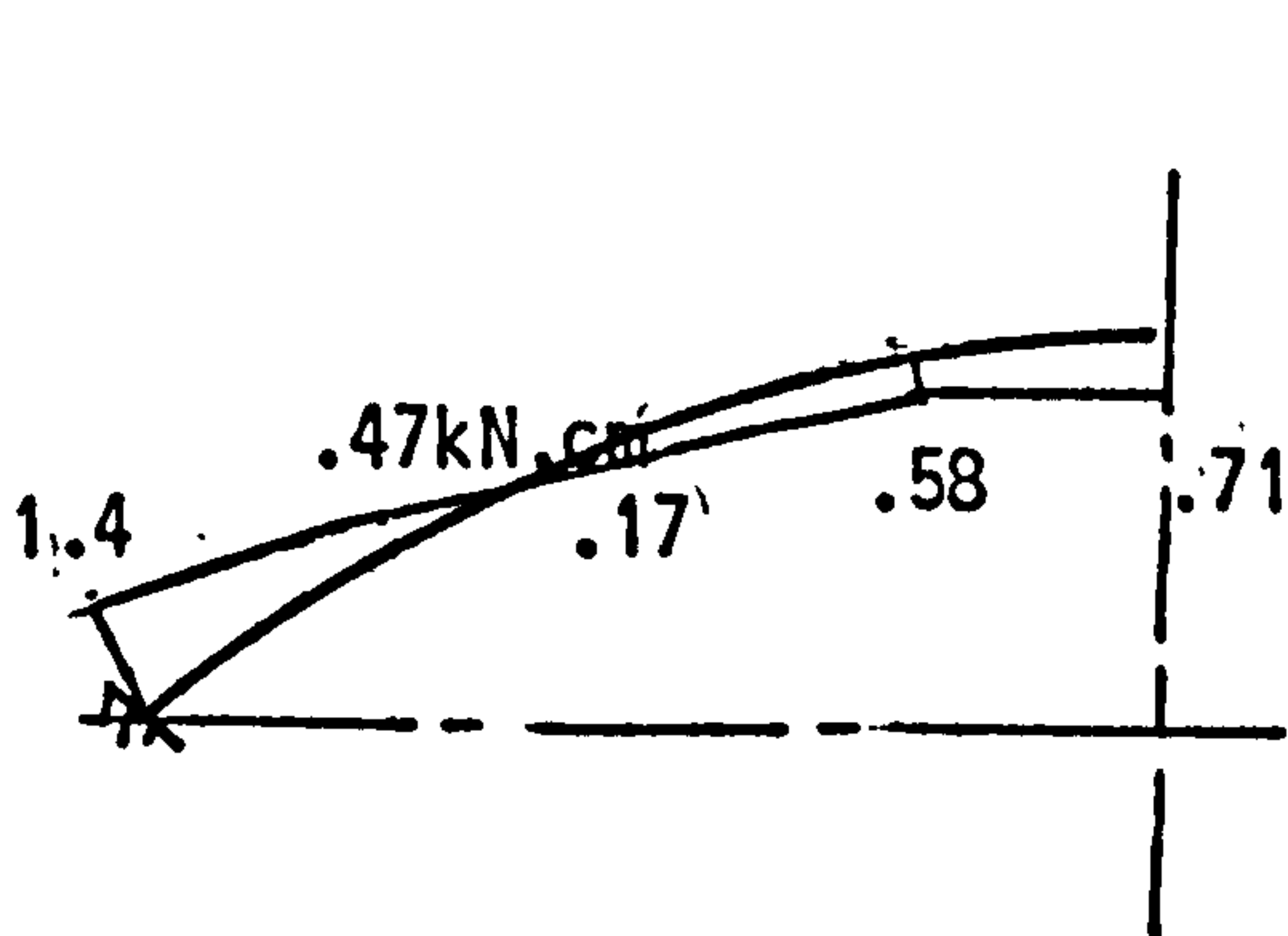
From the computer programme we find the following profiles for the arch, and the output data of the programme shown in Appendix C.



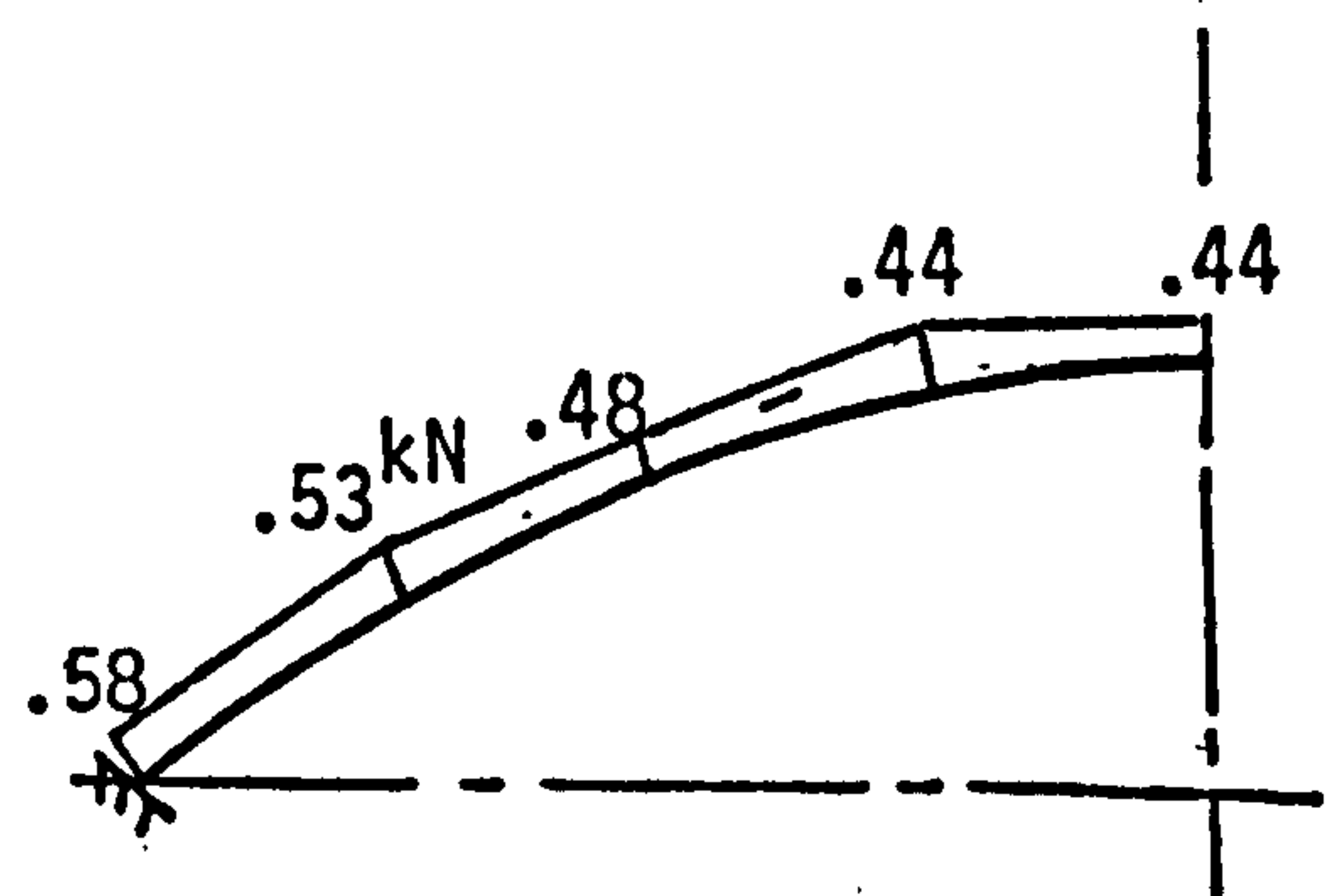
a) M_1, M_2 envelope



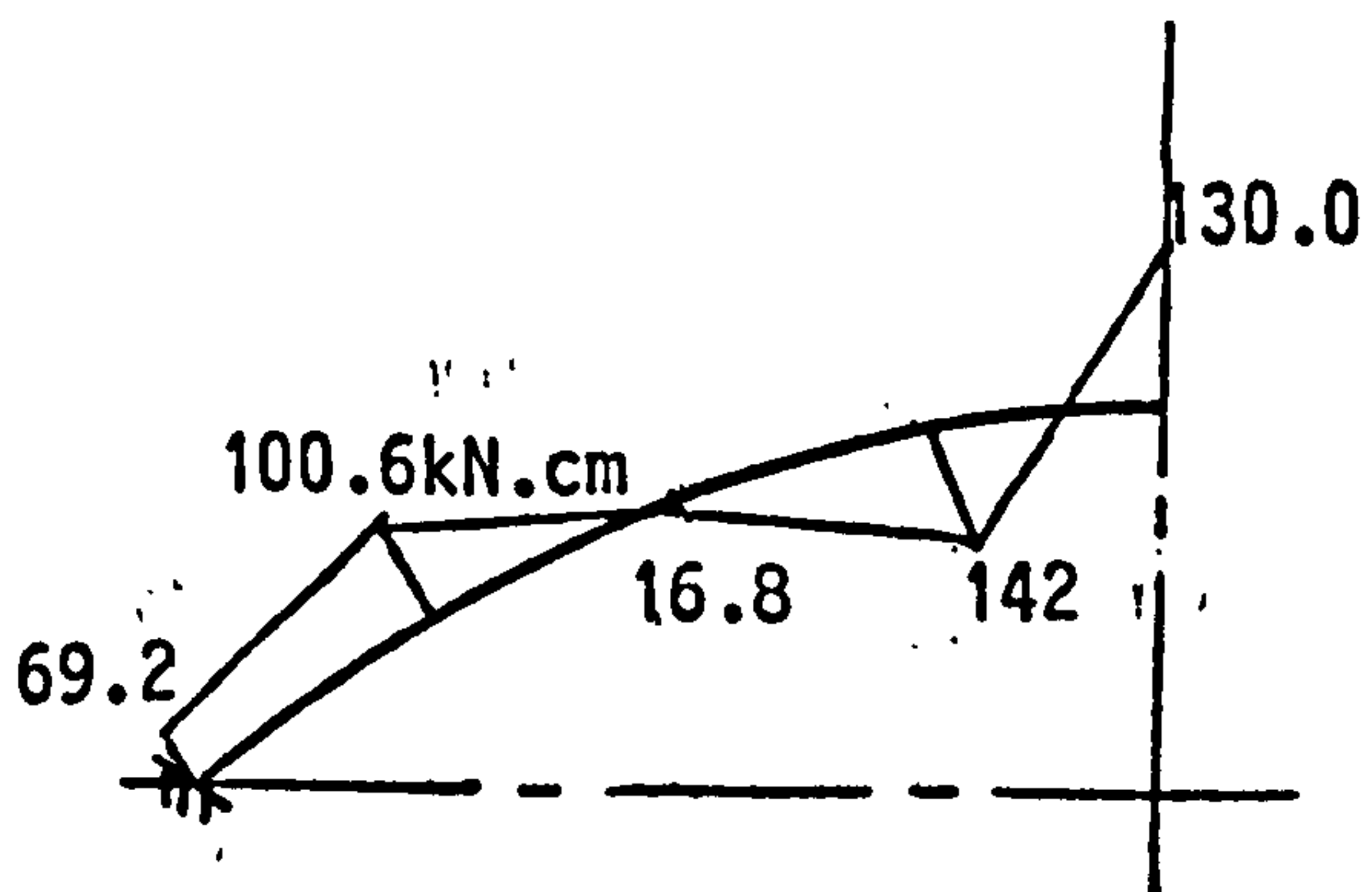
b) H_1, H_2 envelope



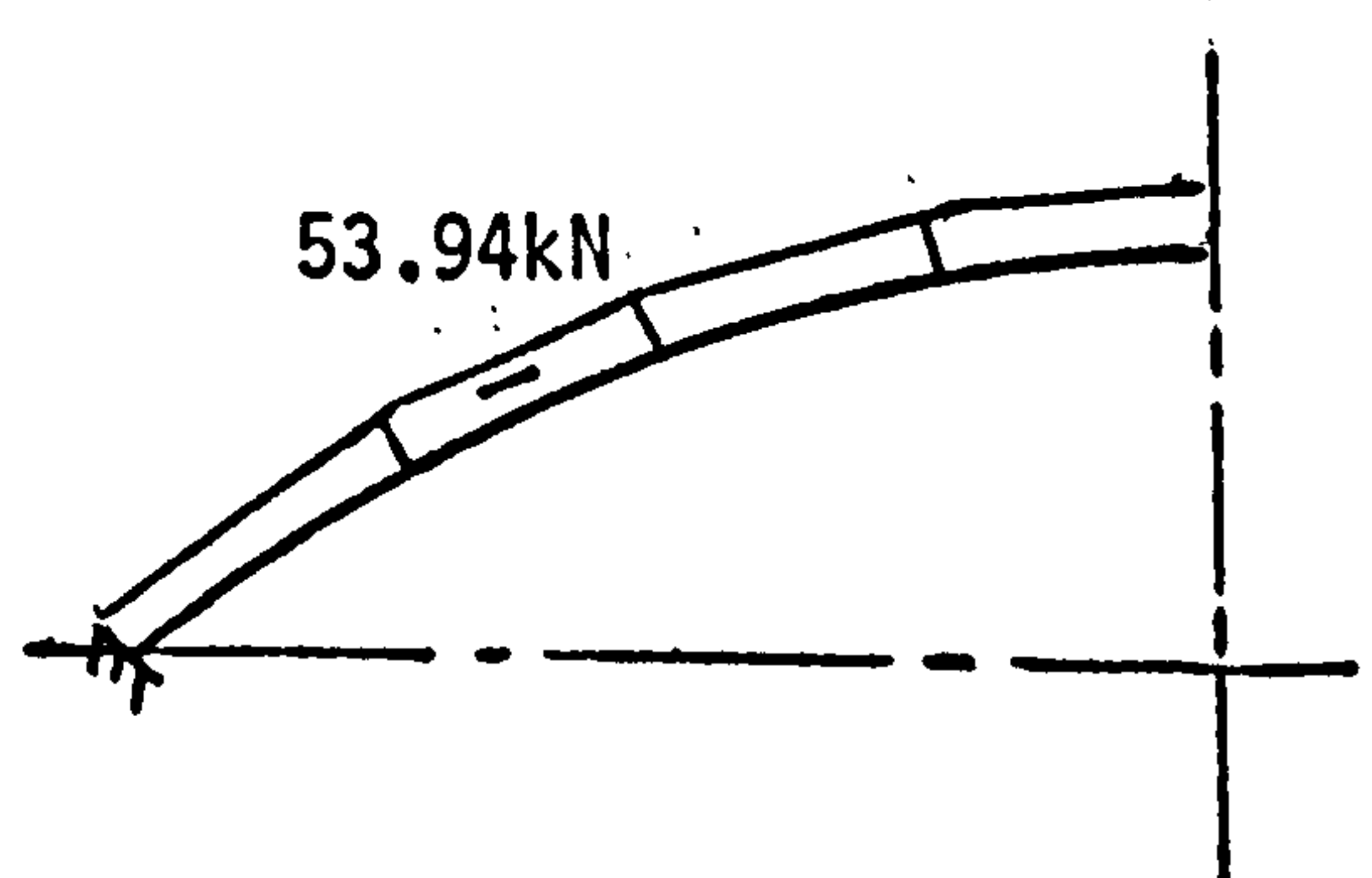
c) dead load moment



d) dead load axial force

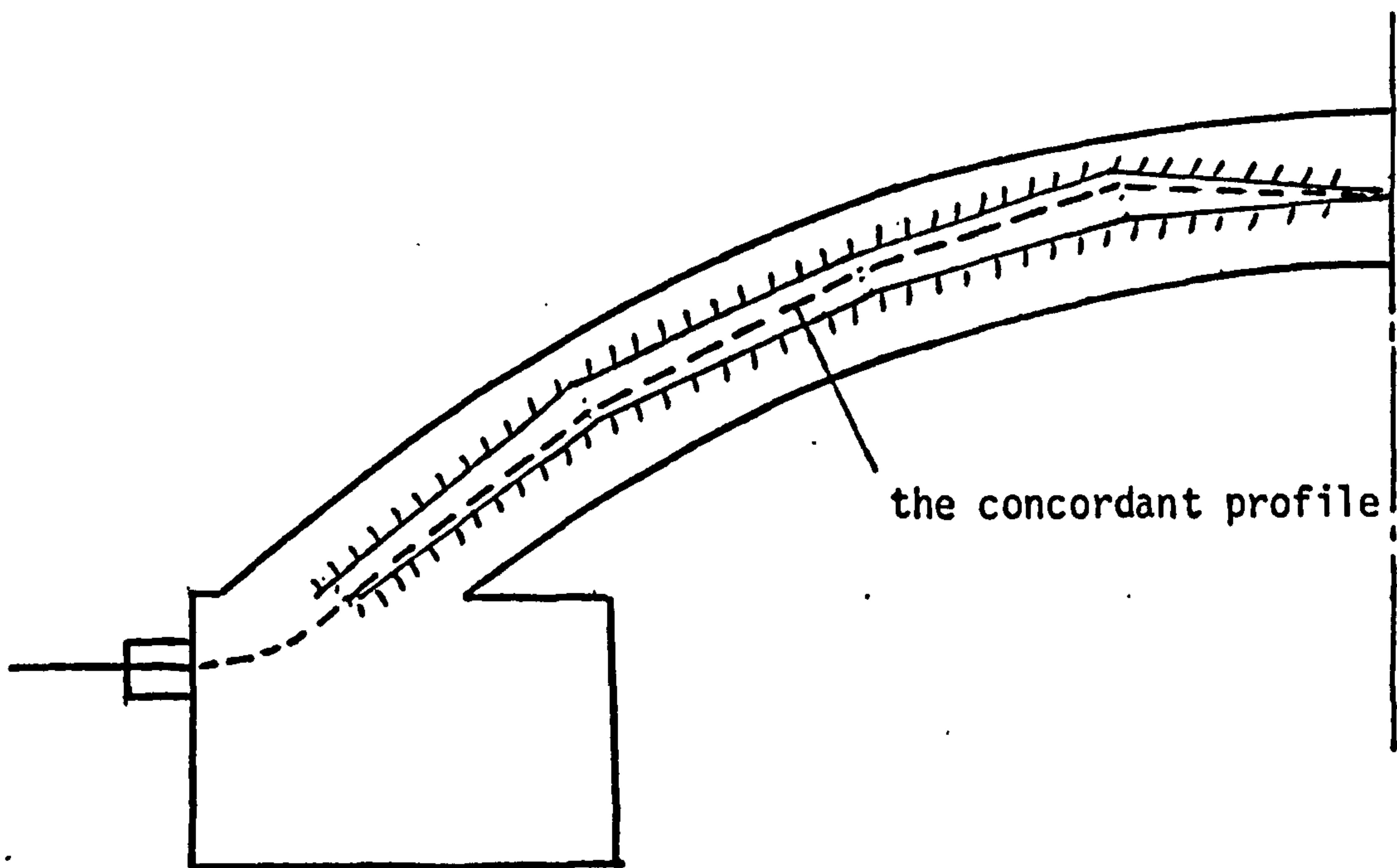


e) the prestressing moment

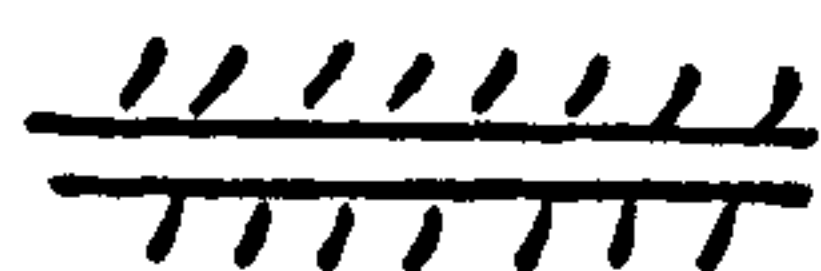


f) the prestressing force

Fig. 4.4



The upper limit	-2.367	-2.411	-2.397	-3.198	2.421 cm
The concordant	1.283	1.865	.312	-2.632	2.411 cm
The lower limit	2.378	2.279	1.996	2.409	2.411 cm



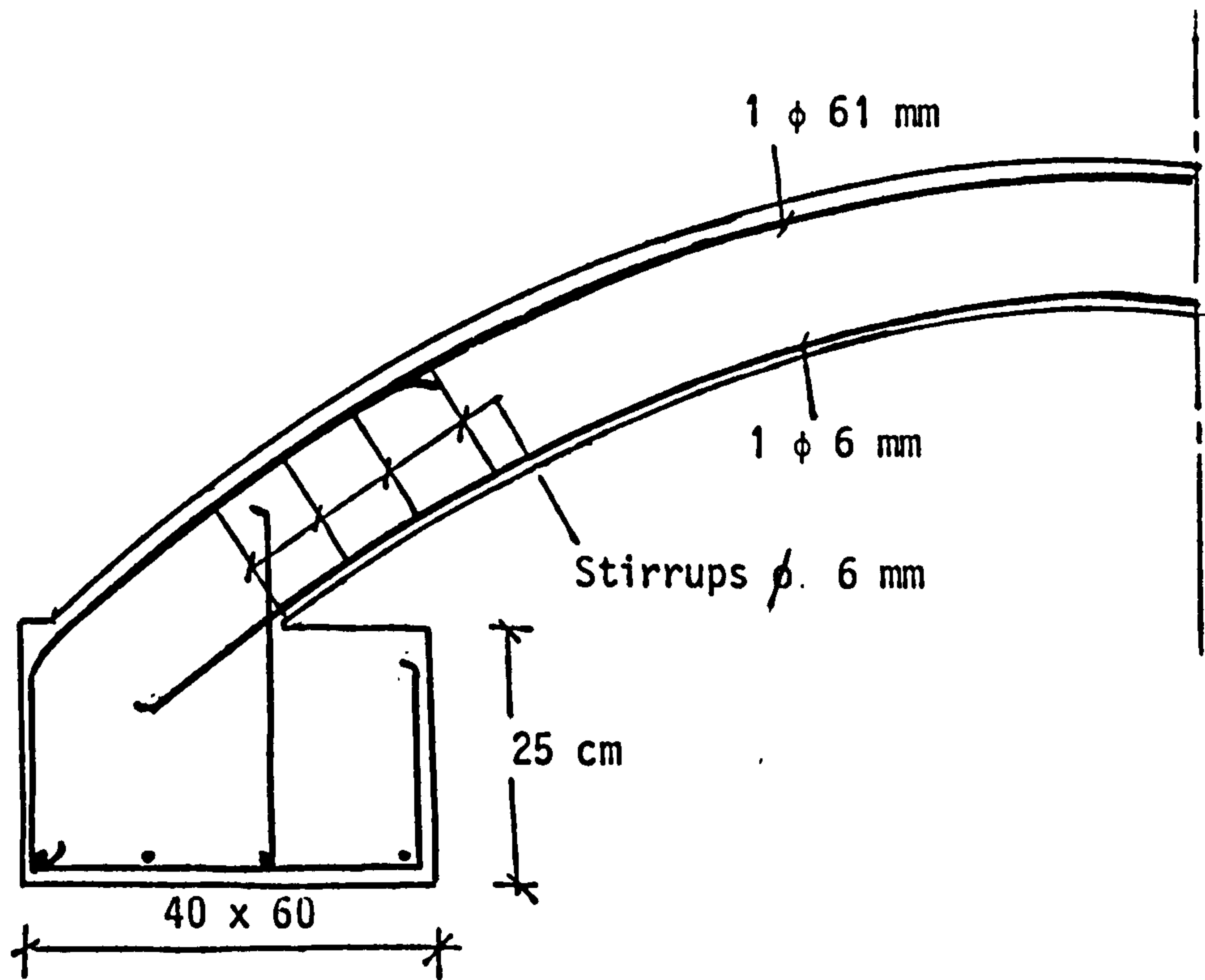
The limiting zone



The concordant profile

The prestressing reinforcement

Fig. 4.5

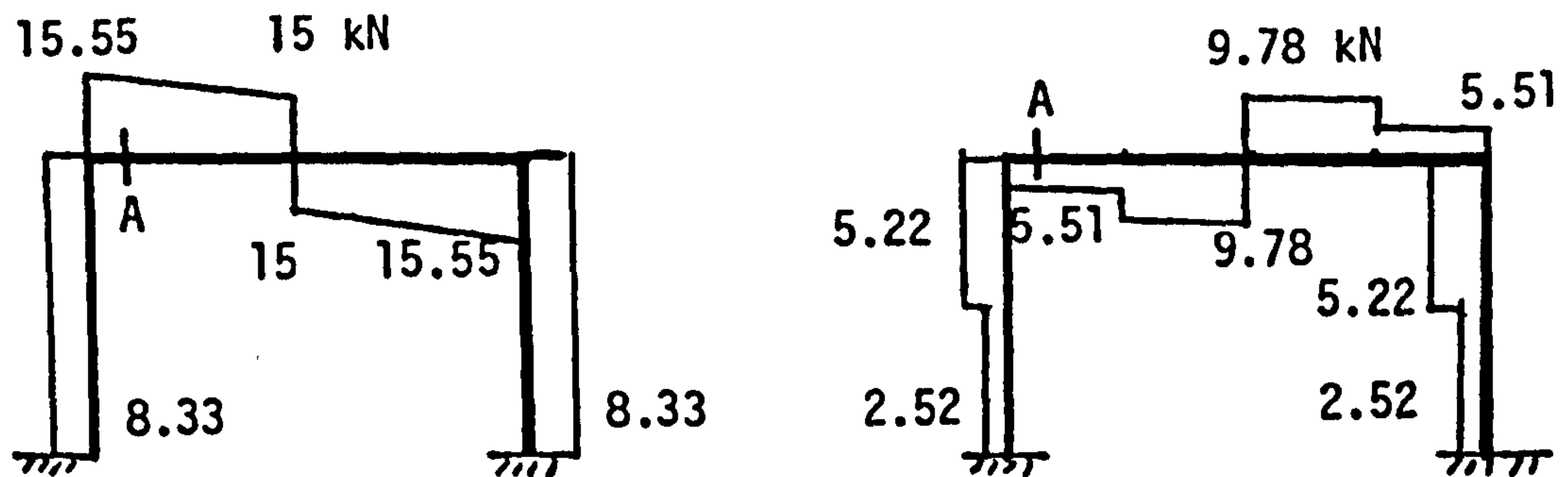


The ordinary reinforcement

Fig. 4.6

4.2 CHECK OF SHEAR

4.2.1 Shear Stress for the Frame



a) S.F.D. due (D.L. + L.L.)

b) S.F.D. due (prestressing force)

Fig. 4.7

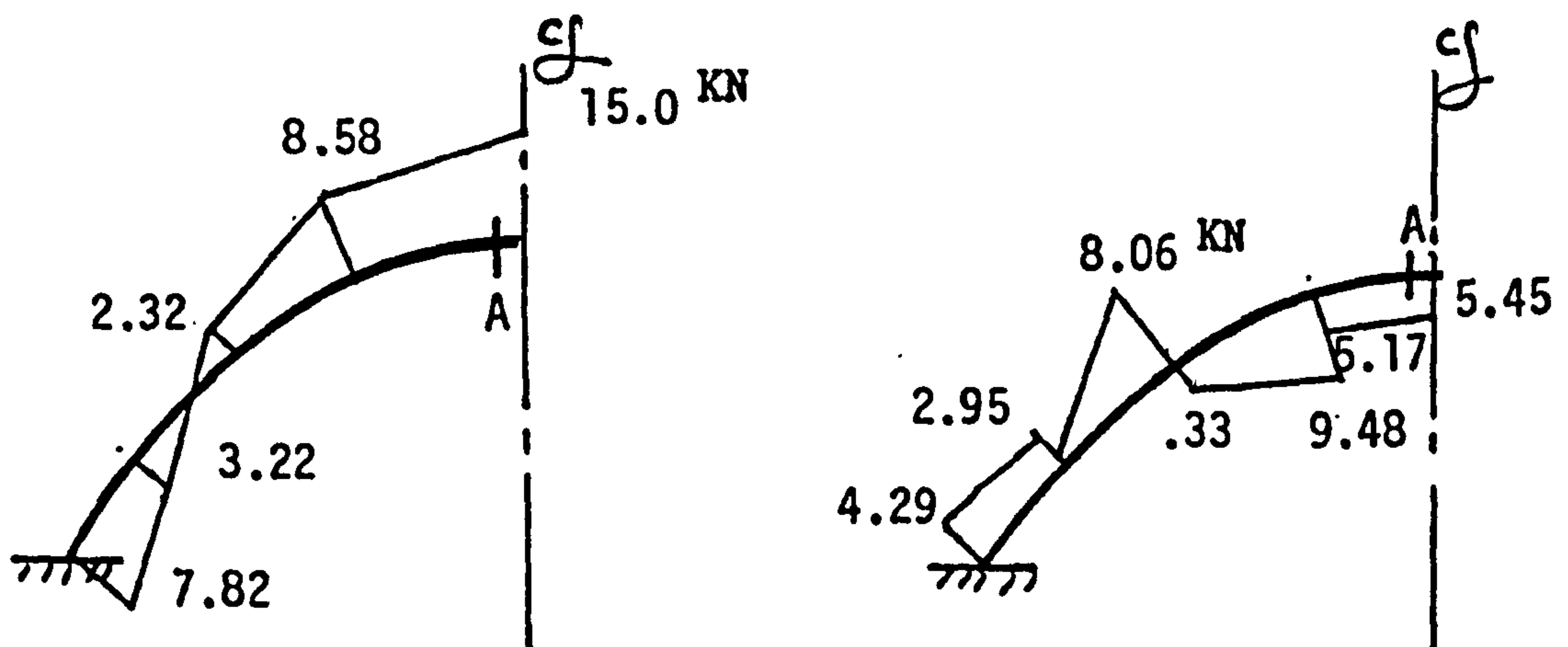
From Fig. 4.7 a and b (Sec.A).

The maximum shear force = $15.55 - 5.51 = 10.04 \text{ kN}$

The maximum shear stress = $\frac{3}{2} \frac{Q}{A} = \frac{3 \times 10.04 \times 10^3}{2 \times 220.9 \times 100}$

$$= 0.682 \text{ N/mm}^2$$

4.2.2 Shear Stress for the Arch



a) S.F.D. due (D.L. + L.L.)

b) S.F.D. due (prestressing force)

Fig. 4.8

The maximum shear force = $15.0 - 5.45 = 9.55 \text{ kN}$

$$\begin{aligned} \text{The maximum shear stress} &= \frac{3 Q}{2 A} = \frac{3 \times 9.55 \times 10^3}{2 \times 142.7 \times 100} \\ &= 1.004 \text{ N/mm}^2 \end{aligned}$$

4.3 ULTIMATE MOMENT OF RESISTANCE

4.3.1 Ultimate Moment of Resistance of the Frame

The reinforcement ratio at the critical section of the frame $\alpha = 0.56\%$.

In order to drive the expression for the ultimate moment when untensioned mild or high-tensile steel is included in the section the strain conditions at the critical section must be taken into consideration. Fig. 4.9 a and b show the stress and strain conditions respectively of the critical section resisting its ultimate moment.

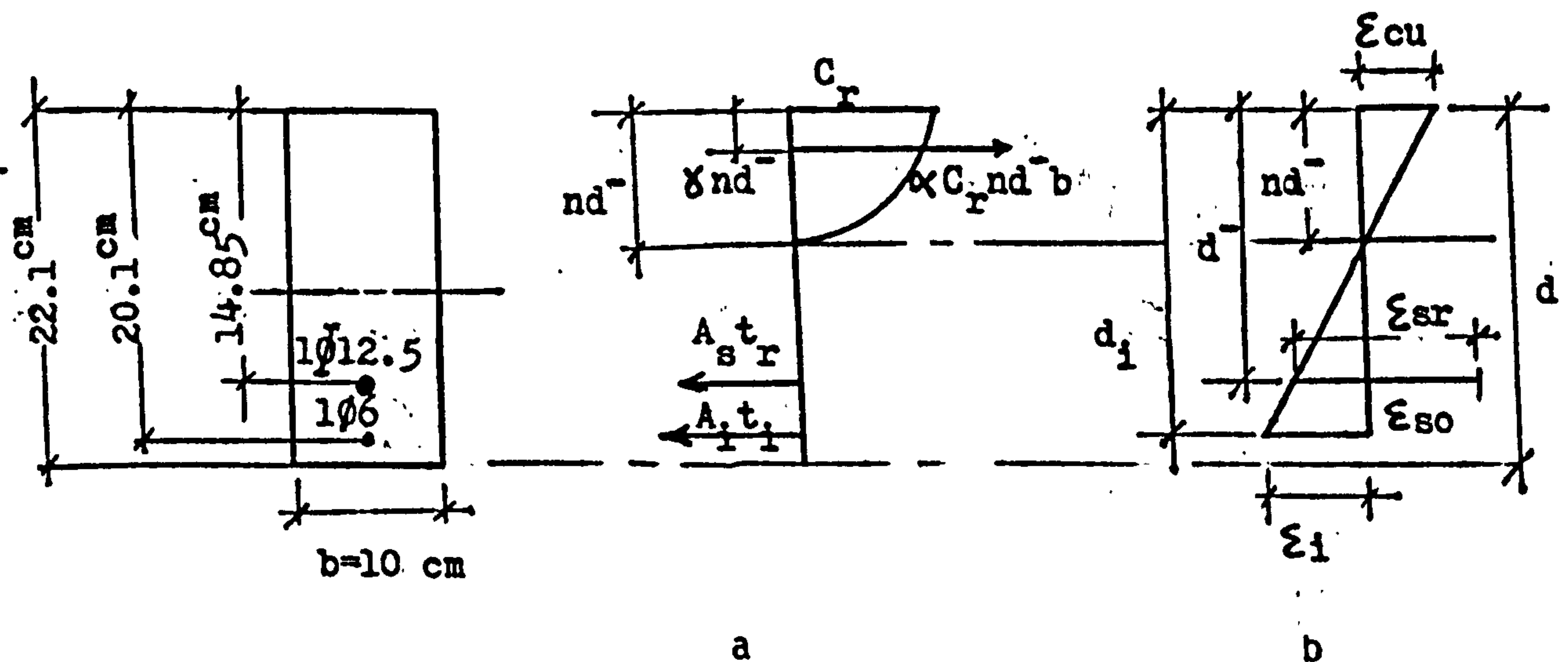


Fig. 4.9

Let A_s be the area of the tensioned steel at a depth d and t_r its tensile stress.

A_i is the area of the untensioned steel at a depth d_i and t_i its tensile stress.

In the strain diagram Fig.4.9b the ultimate strain in the concrete is ϵ_{cu} , the initial strain in the tensioned steel is ϵ_{so} , while its total strain is ϵ_{sr} and the strain in the untensioned steel is ϵ_i .

From a consideration of longitudinal forces we get

$$\alpha c_r n \bar{d} b = A_s t_r + A_i t_i \quad \dots\dots\dots (4.a)$$

$$n = \frac{A_s t_r + A_i t_i}{\alpha c_r b \bar{d}} \quad \dots\dots\dots (4.b)$$

From Fig. 4.9 the geometry of the diagram gives the strain in the tensioned steel.

$$\epsilon_{sr} = \epsilon_{so} + \epsilon_{cu} \left(\frac{1}{n} - 1 \right) \quad \dots\dots\dots (4.c)$$

and that in the untensioned steel is

$$\epsilon_i = \epsilon_{cu} \left(\frac{d_i}{n\bar{d}} - 1 \right) \quad \dots\dots\dots (4.d)$$

From a knowledge of the strains in the steel it is possible to determine the steel stresses from the stress/strain diagrams of the steel. Now the ultimate moment will be

$$Mult = A_s t_r (1 - \gamma n) \bar{d} + A_i t_i (d_i - \gamma n \bar{d}) \quad \dots\dots\dots (4.e)$$

It is to be noticed that the steel stress values are required in order to determine the depth of the neutral axis.

The process starts by guessing a depth of the neutral axis $n\bar{d}$ and a knowledge of the ultimate concrete strain ϵ_{cu} and then by

trial and error we get the final position of the neutral axis. For the calculation the empirical values of α and γ are $\alpha = 0.5$ and $\gamma = 0.4$. Fig.4.10 shows the ultimate concrete strain ϵ_{cu} with the average cube crushing strength, c_r .

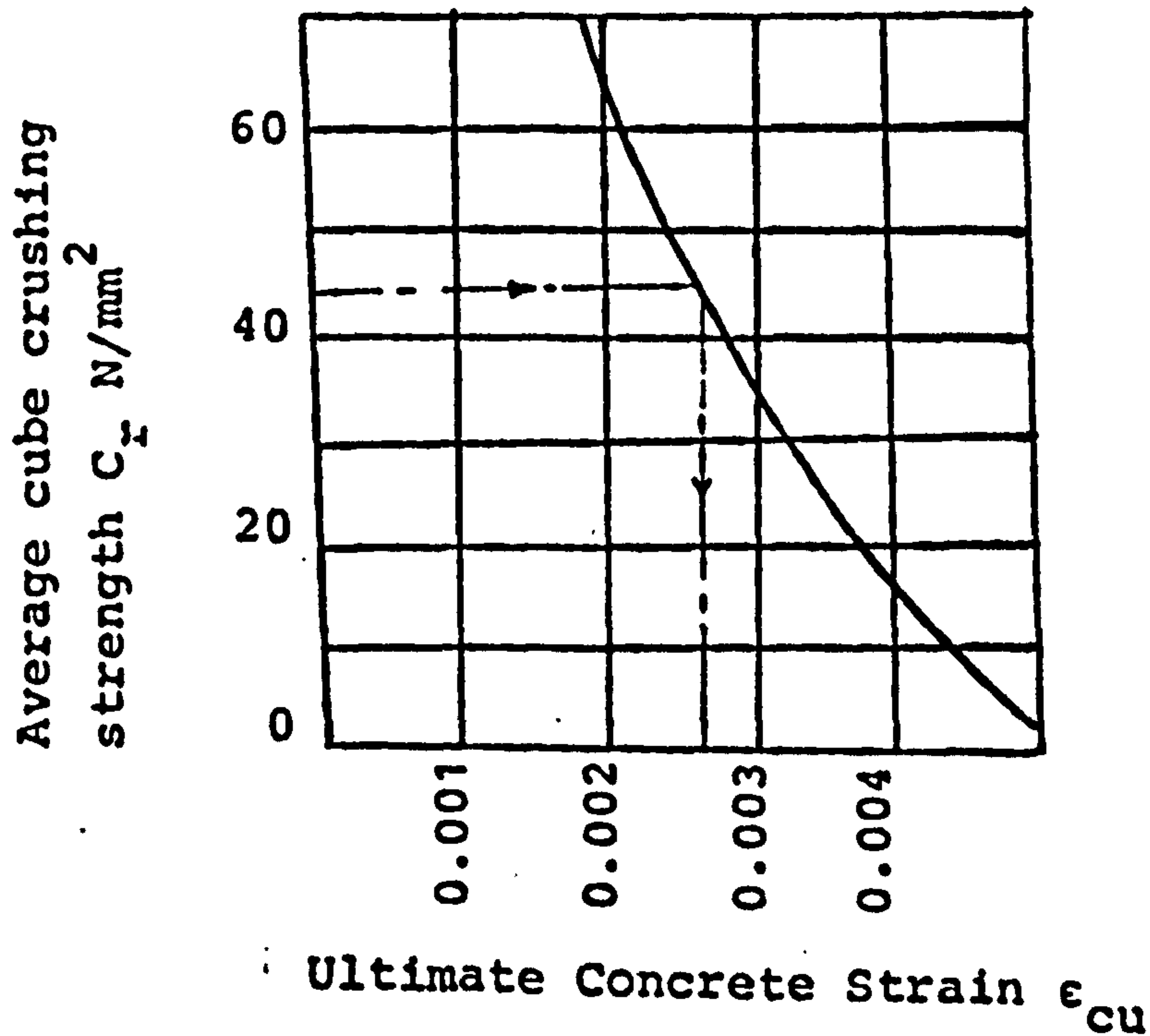


Fig. 4.10

The critical section properties are:

$b = 10 \text{ cm}$	$d = 22.1 \text{ cm}$	
$A_s = 122.72 \text{ mm}^2$	$d' = 14.85 \text{ cm}$	$E_s = 2.2 \times 10^5 \text{ N/mm}^2$
$A_i = 28.27 \text{ mm}^2$	$d_i = 20.1 \text{ cm}$	$E_i = 2 \times 10^5 \text{ N/mm}^2$
$c_r = 43.42 \text{ N/mm}^2$	$\epsilon_{cu} = 0.00267 \text{ (from Fig. 4.10)}$	

The stress in the steel can be determined from Figs. 5.3 and 5.4 provided in Chapter 5.

The initial stress in the tension wire

$$t_r = \frac{\text{The prestressing force}}{\text{The area of the wire}} = \frac{127 \times 10^3}{122.72} = 1034.8 \text{ N/mm}^2$$

the initial strain in the tensioned wire

$$\epsilon_{so} = \frac{t_r}{E_s} = \frac{1034.8}{2.2 \times 10^5} = 0.0047$$

by assuming $n = 0.5$ in the expression 4.c we obtain:

$$\epsilon_{sr} = 0.0047 + 0.00267 \left(\frac{1}{.5} - 1 \right) = 0.00737$$

The steel stress at this strain is 1621.4 N/mm^2 .

The strain in the untensioned steel is given by equation 4.d.

$$\epsilon_i = 0.00267 \left(\frac{20.1}{0.5 \times 14.85} - 1 \right) = 0.00456$$

The steel stress at this strain is 371 N/mm^2 .

Using expression 4.b the depth of the neutral axis assuming $\alpha = 0.5$

$$n = \frac{122.72 \times 1621.4 + 28.27 \times 271}{0.5 \times 43.42 \times 100 \times 148.5} = 0.65$$

A second trial of 0.6 results in a value of 0.601 from expression 4.b. Thus, 0.6 is the required value in this case and the stress in the tensioned steel is 1425.6 N/mm^2 and that in

the untensioned steel is 370 N/mm^2 and the ultimate moment of resistance from expression 4.e, assuming $\gamma = .4$, is

$$M_{ult} = [122.72 \times 1425.6 (1 - 0.4 \times .6) 148.5 + 28.27 \times 370 (201 - 0.4 \times 0.6 \times 148.5)] 10^{-4}$$

$$= 2147.45 \text{ kN.cm}$$

From the symmetry, the frame will collapse by beam mechanism.

$$W \frac{L}{2} \theta = 4 M_p \theta$$

$$W = 8 \frac{M_p}{L}$$

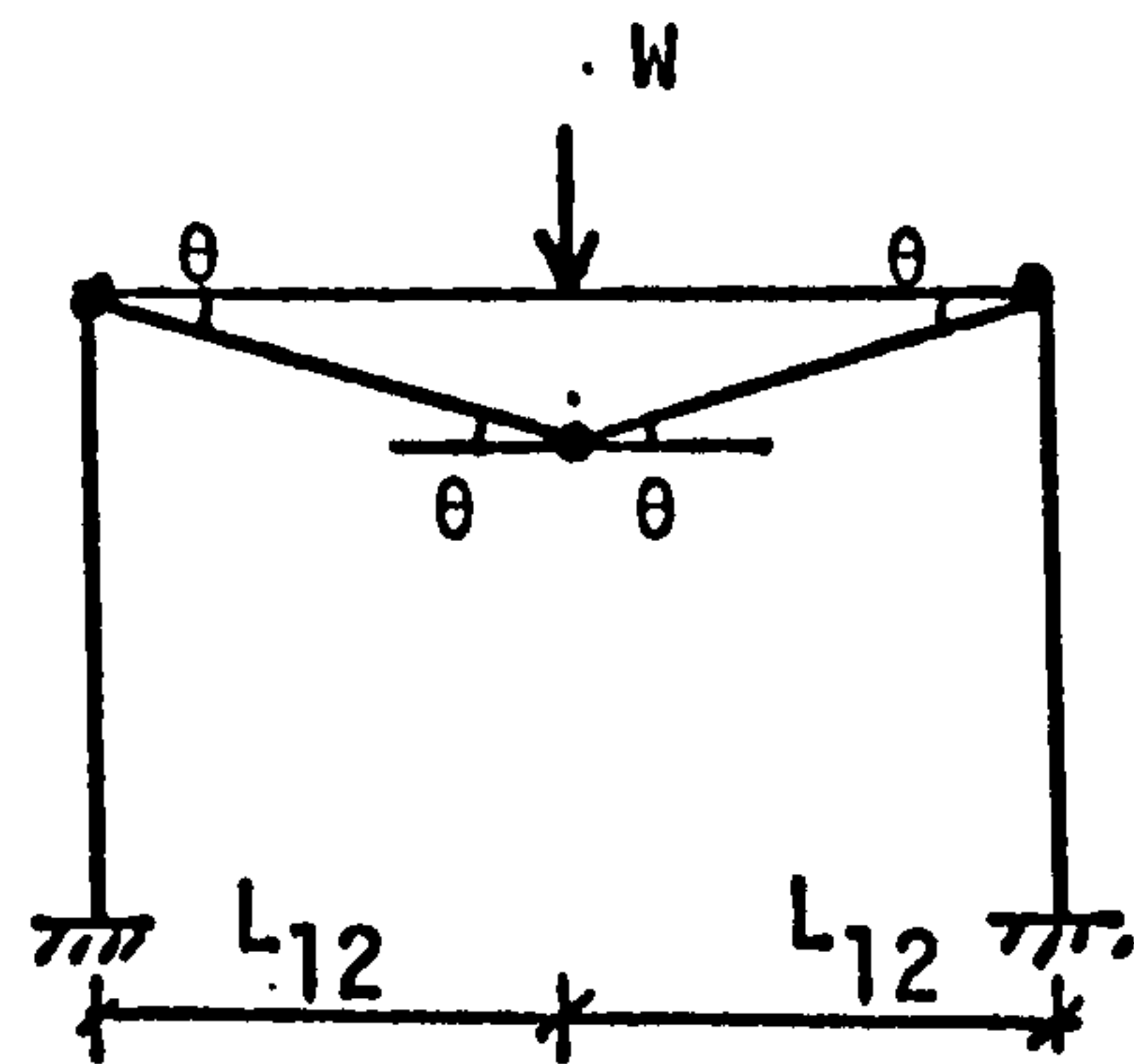


Fig. 4.11

$$= \frac{8 \times 2147.45}{200} = 85.9 \text{ kN}$$

4.3.2 Ultimate Moment of Resistance of the Arch

The critical section properties are

$$b = 10 \text{ cm} \quad d = 14.27 \text{ cm}$$

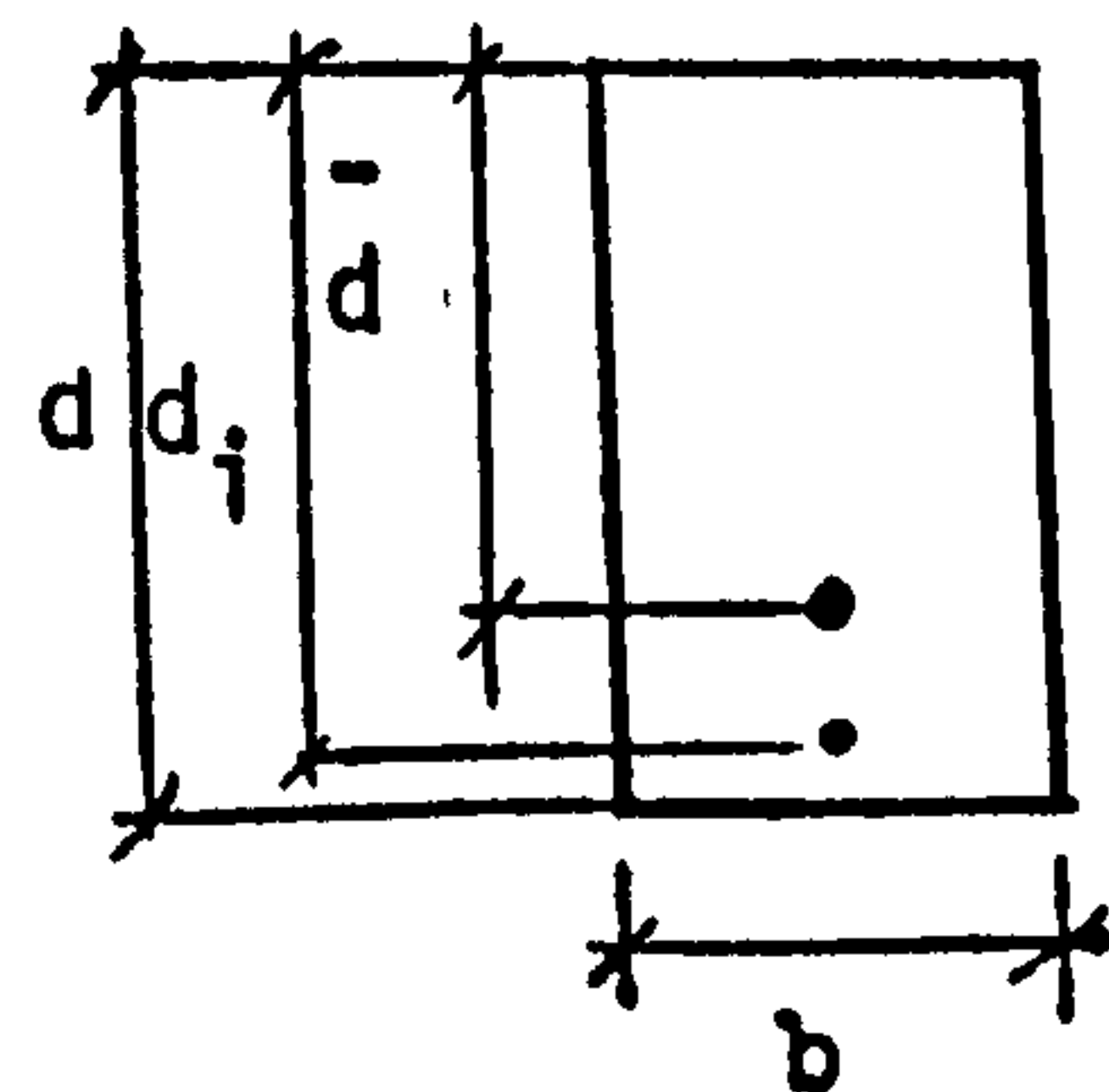
$$A_s = 67.93 \text{ mm}^2 \quad d = 9.55 \text{ cm}$$

$$E_s = 2.2 \times 10^5 \text{ N/mm}^2$$

$$A_i = 28.27 \text{ mm}^2 \quad d_i = 12.27 \text{ cm}$$

$$E_i = 2 \times 10^5 \text{ N/mm}^2$$

$$C_r = 43.42 \text{ N/mm}^2 \quad \epsilon_{cu} = 0.00267$$



The initial stress in the tensioned wire:

$$t_r = \frac{53.94 \times 10^3}{67.93} = 794.05 \text{ N/mm}^2$$

The initial strain in the tensioned wire

$$\epsilon_{so} = \frac{t_r}{E_s} = \frac{794.05}{2.2 \times 10^5} = 0.0036$$

By assuming $n = 0.5$ in the expression 4.c we obtain

$$\epsilon_{sr} = 0.0036 + 0.00267 \left(\frac{1}{.5} - 1 \right) = 0.00627$$

The steel stress at this strain is 1379.4 N/mm^2 .

The strain in the untensioned steel is given by equation 4.d.

$$\epsilon_i = 0.00267 \left(\frac{12.27}{0.5 \times 9.55} - 1 \right) = 0.0042$$

The steel stress at this strain is 370.5 N/mm^2 .

Using expression 4.b the depth of the neutral axis assuming $\alpha = 0.5$

$$= \frac{67.93 \times 1379.4 + 28.27 \times 370.5}{0.5 \times 43.42 \times 100 \times 95.5} = 0.502$$

From expression 4.e the ultimate moment of resistance assuming $\gamma = 0.4$ is

$$\begin{aligned} M_{ult} &= [67.93 \times 1379.4 (1 - 0.4 \times 0.5) 95.5 + 28.27 \times 370.5 \\ &\quad (122.7 - 0.4 \times 0.5 \times 95.5)] 10^{-4} \\ &= 824.4 \text{ kN.cm} \end{aligned}$$

The final collapse of the arch occurred when the arch had five hinges as shown in Fig. 4.12.

4.4.1 Splitting Force for the Frame

$$P_t = 127 \text{ kN} \quad 2d = 10 \text{ cm} \quad 2\hat{d} = 5.0 \text{ cm}$$

$$T_{sp} = \frac{127}{4} \left(1 - \frac{2.5}{5}\right) = 15.875 \text{ kN}$$

$$A_s = \frac{T_{sp}}{f_s} = \frac{15.875}{14.0} = 1.134 \text{ cm}^2$$

4.4.2 Splitting Force for the Arch

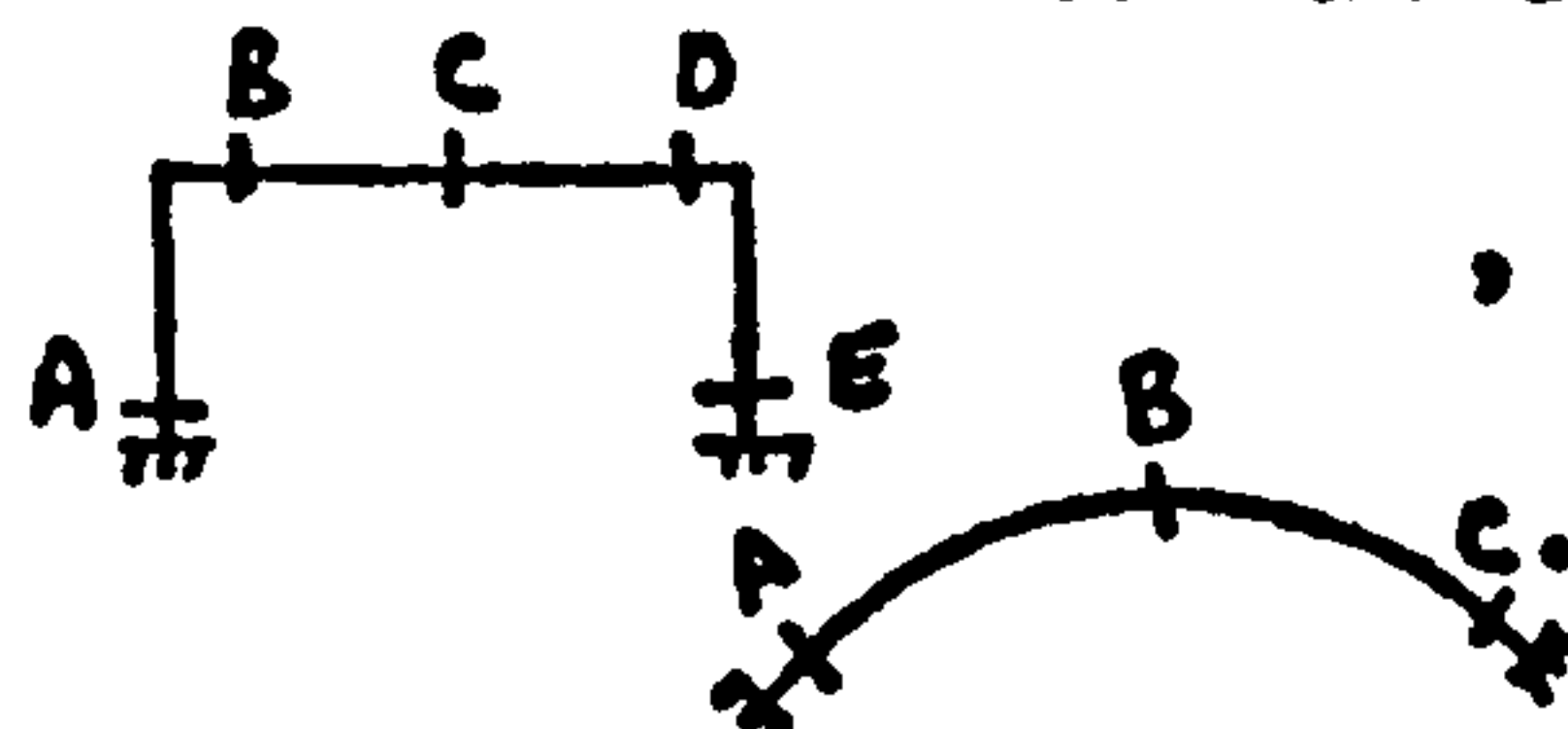
$$P_t = 53.94 \text{ kN} \quad 2d = 10 \text{ cm} \quad 2\hat{d} = 5.0 \text{ cm}$$

$$T_{sp} = \frac{53.94}{4} \left(1 - \frac{2.5}{5}\right) = 6.743 \text{ kN}$$

$$A_s = \frac{T_{sp}}{f_s} = \frac{6.743}{14.0} = 0.482 \text{ cm}^2$$

4.5 STRESSES AND DEFLECTIONS

The stress distribution at sections A, B, C, D and E for the frame, at sections A, B and C for the arch. Also the deflection at section C for the frame and B for the arch were calculated using the computer programme.



Note: the unit force method was used for calculating the deflections.

CHAPTER (5)

THE EXPERIMENTAL WORK

5.1 DESCRIPTION OF THE EXPERIMENTAL WORK

The experimental work consists of four parts: firstly the preparation and testing of the materials, secondly the casting of the frames and the arches, thirdly the prestressing operation and fourthly the testing of the frames and arches which had a concordant prestressing wire.

The tests were made on two frames having the same size, shape and materials. The two arches used were also of the same size, shape and materials.

The frames and the arches have fixed supports with the footings which are attached to the floor by steel beams and a screw system. A point load is applied at the mid point of the span of each element as shown in plate No. 6 for the frame and plate No. 16 for the arch.

The prestressing wires are placed inside the concrete within the limiting zone and fixed in a concordant position as shown in plate No. 1 for the frame and plate No. 14 for the arch.

For the prestressing operation a jack system was used. The tensile force in the wires was measured by the pressure gauge indicator of the pump which was connected to the jack as shown in plate No. 16

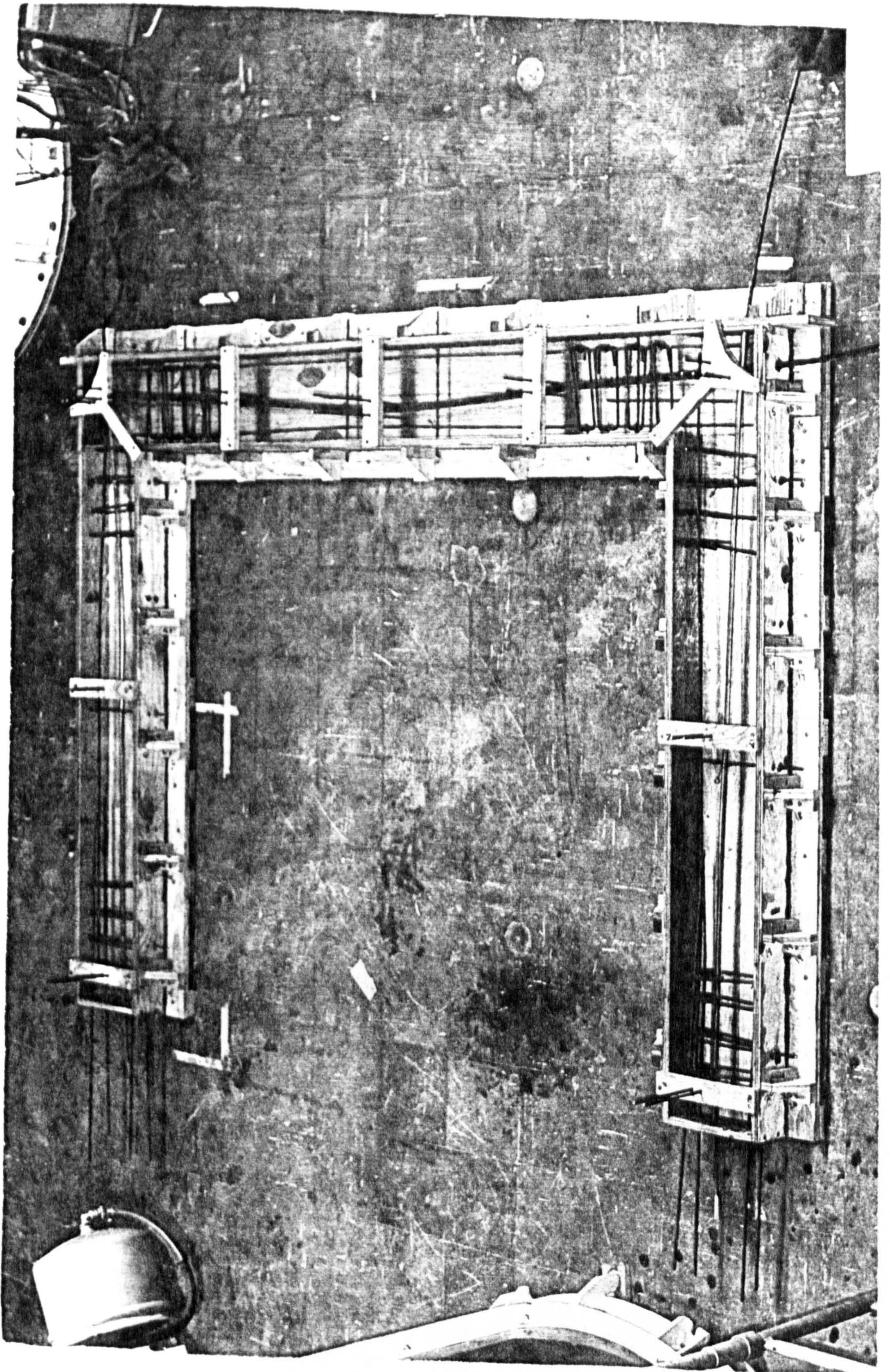


Plate No. 1 The frame mould

5.2 THE MATERIALS

5.2.1 The Concrete Mix

a) Sieve analysis was done for the three mixes, the maximum aggregate size being 20 mm. Figs. 5.1 (a) and (b) show the average sieve analysis for two sieve operations for both fine and coarse aggregates.

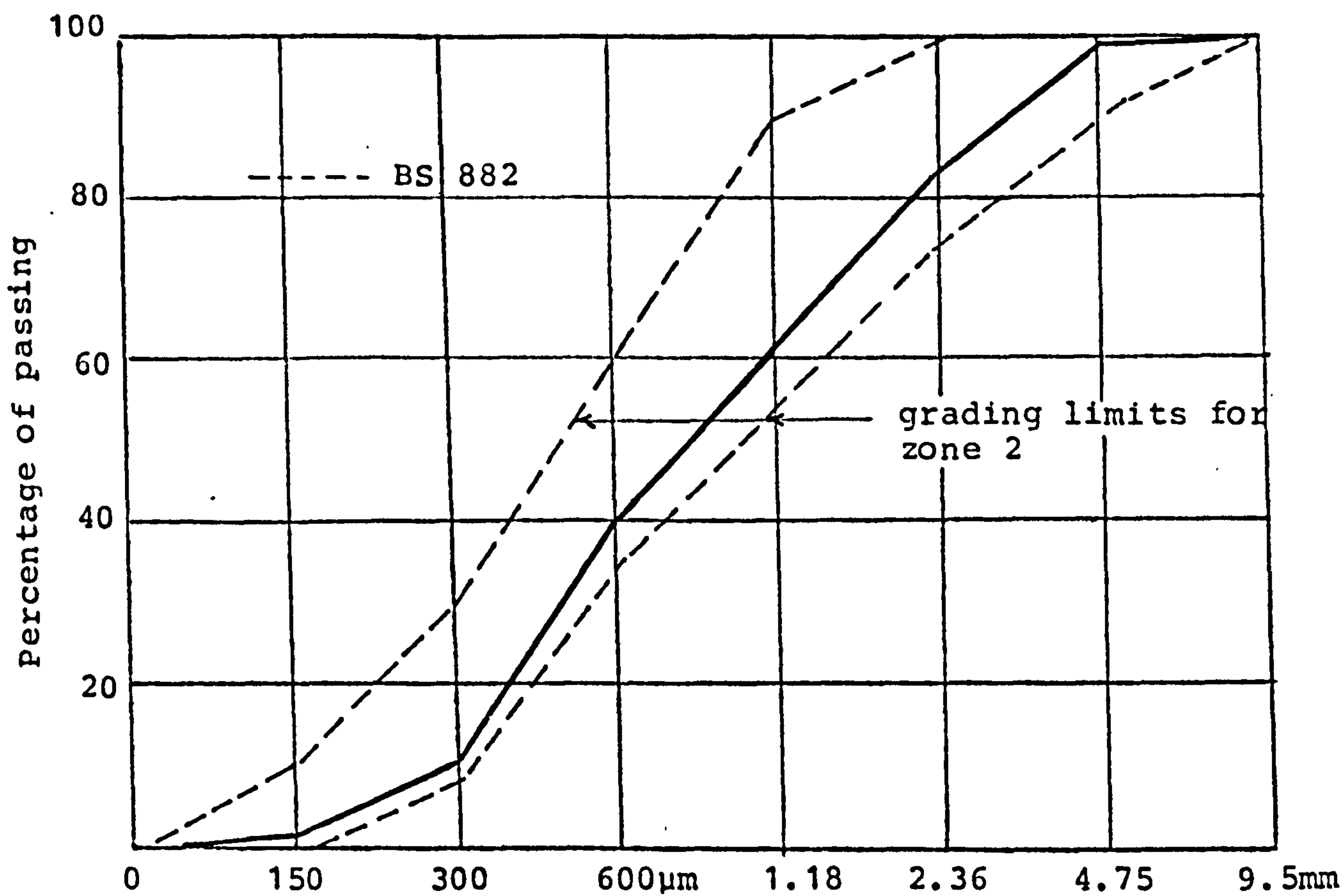
b) Three trial mixes described as shown in Table 5.1 were made and nine cubes 10 x 10 x 10 cm were cast from each mix.

TABLE 5.1

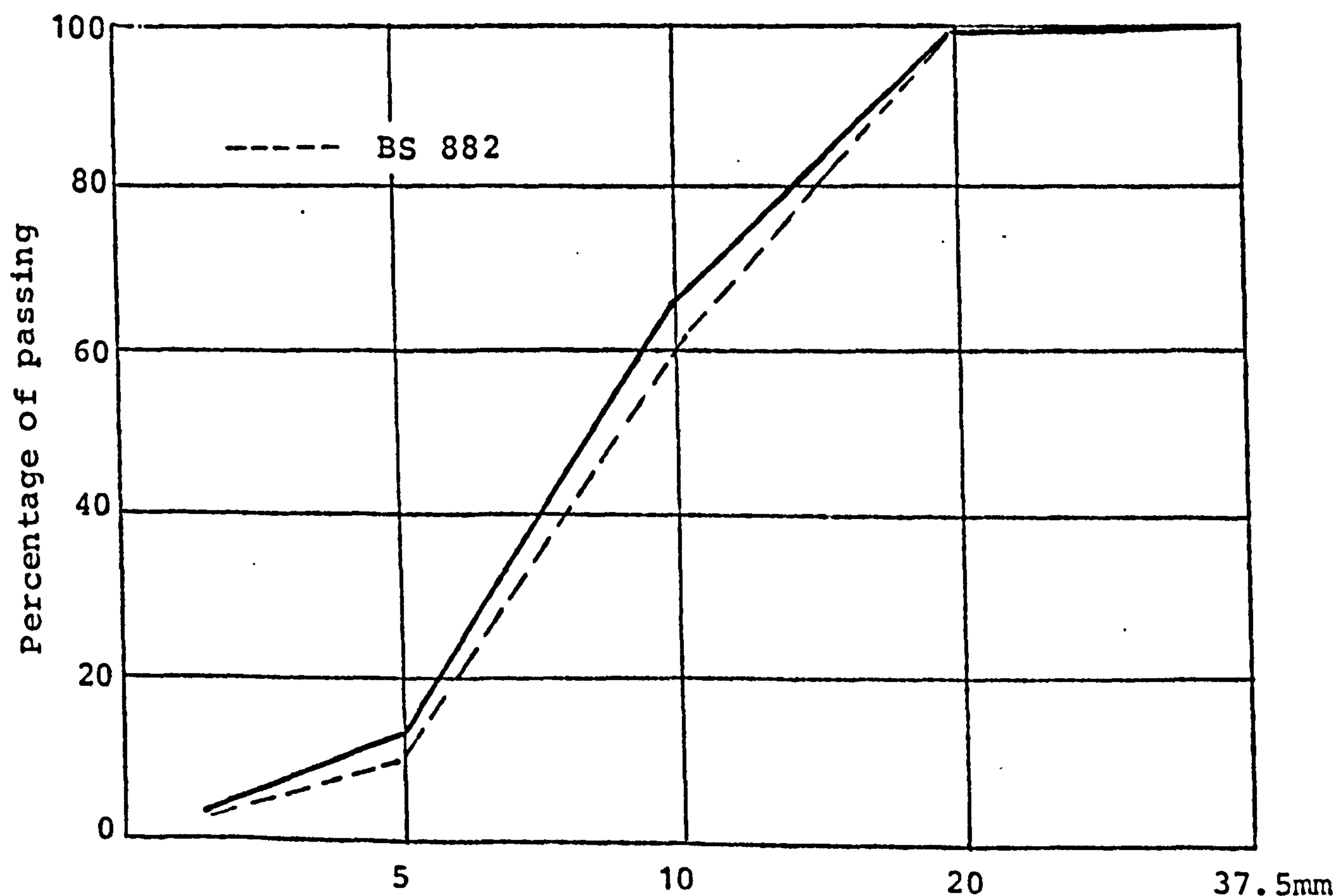
Mix No.	Cement	Water	Fine Agg.	Coarse Agg.
I	1	0.505	1.87	3.77
II	1	0.53	1.64	2.94
III	1	0.53	1.63	2.89

c) Compression tests were done on the cubes after 7, 14 and 28 days. Tables 5.2a, b and c show the strength of the concrete for the above mixes.

d) Slump tests were done on the three mixes in order to measure the degree of workability of the concrete. The average slump was 20 mm for mix III (the specified value of CP110 is between 0-25 mm).



(a) Sieve's aperture (Fine aggregate)



(b) Sieve's aperture (Coarse aggregate)

Fig. 5.1

TABLE (5.2)

(a)

Mix No.	Cube No.	Age Days	Strength N/mm ²	Average N/mm ²
I	1	7	43.7	43.6
	2		43.0	
	3		43.3	
	4	14	47.0	47.17
	5		50.5	
	6		44.0	
	7	28	51.0	51.5
	8		53.0	
	9		51.5	

(b)

II	1	7	45.7	46.57
	2		47.9	
	3		46.1	
	4	14	53.0	53.0
	5		53.0	
	6		50.0	
	7	28	59.5	59.17
	8		59.5	
	9		58.5	

(c)

III	1	7	33.5	34.5
	2		35.0	
	3		35.0	
	4	14	40.5	39.33
	5		40.5	
	6		37.5	
	7	28	44.5	43.33
	8		42.5	
	9		43.0	

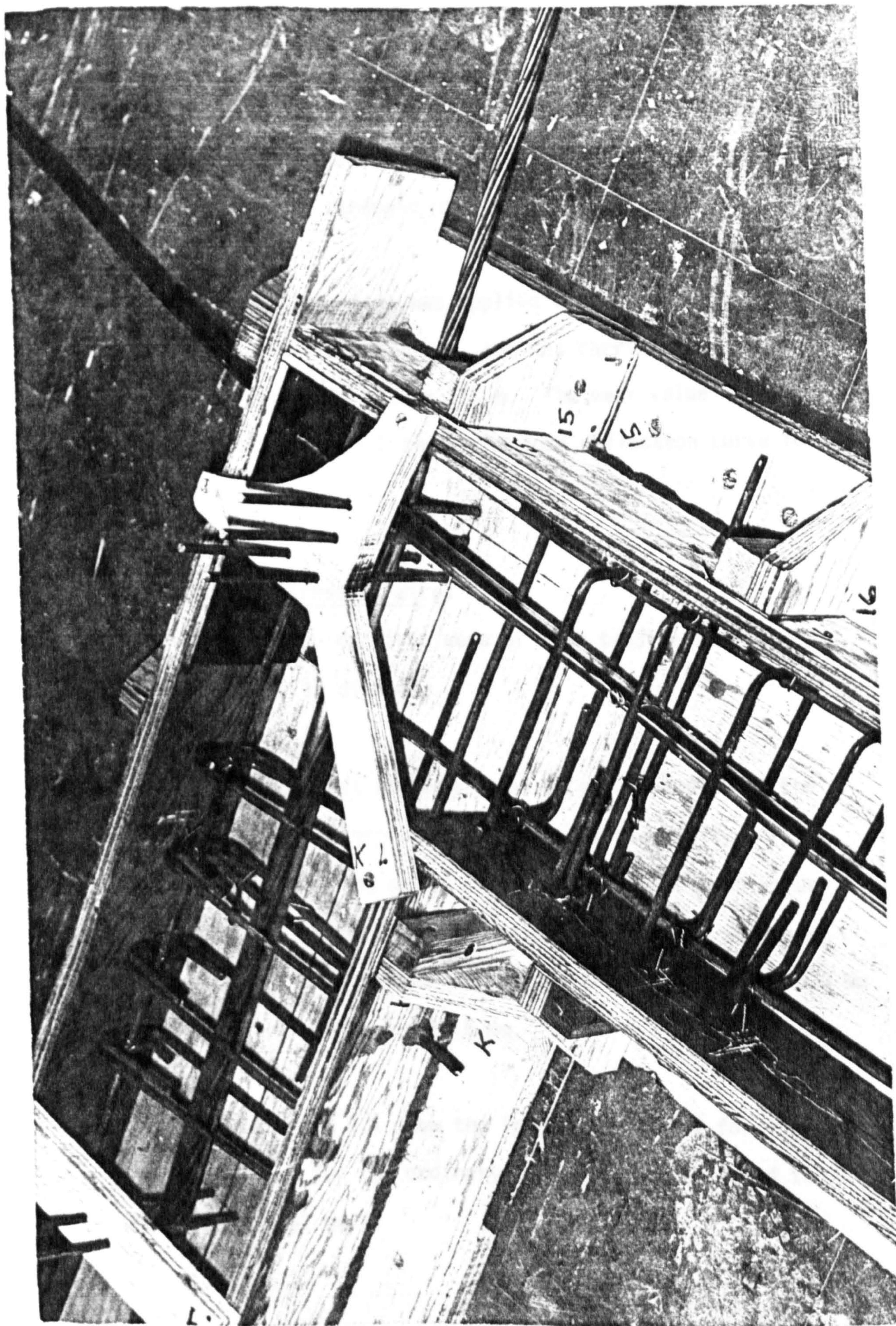


Plate No. 2 Details of reinforcement at the frame corner

e) The Vebe consistometer test was carried out and the Vebe degree was found to be 7 (CP 110 specifies a value between 7-20). Mix III was chosen to be the design mix because it gives a reasonable strength with good workability.

f) The bending test was applied to find the modulus of elasticity for the concrete, and was carried out after 28 days on four beams 10 x 10 x 50 cm. The mean value for E_c was 33 kN/mm^2 . Fig. 5.2 shows the load deflection curve for the concrete.

5.2.2 The prestressing wires

The prestressing wires were of high tensile steel 7.9, 9.3 and 12.5 mm in diameter.

The tension test had been done on three samples of the wire in order to determine the modulus of elasticity for the prestressing steel and chose a design load. The mean value was $E_s = 2.2 \times 10^5 \text{ N/mm}^2$. The design tensile load was taken to be 58.7 kN for diameter 7.9 mm, 79.5 kN for diameter 9.3 mm and 140.3 kN for diameter 12.5 mm.

Figs. 5.3 and 5.4 show the load strain curve for one sample wire and for the ordinary mild steel sample used in the experimental work.

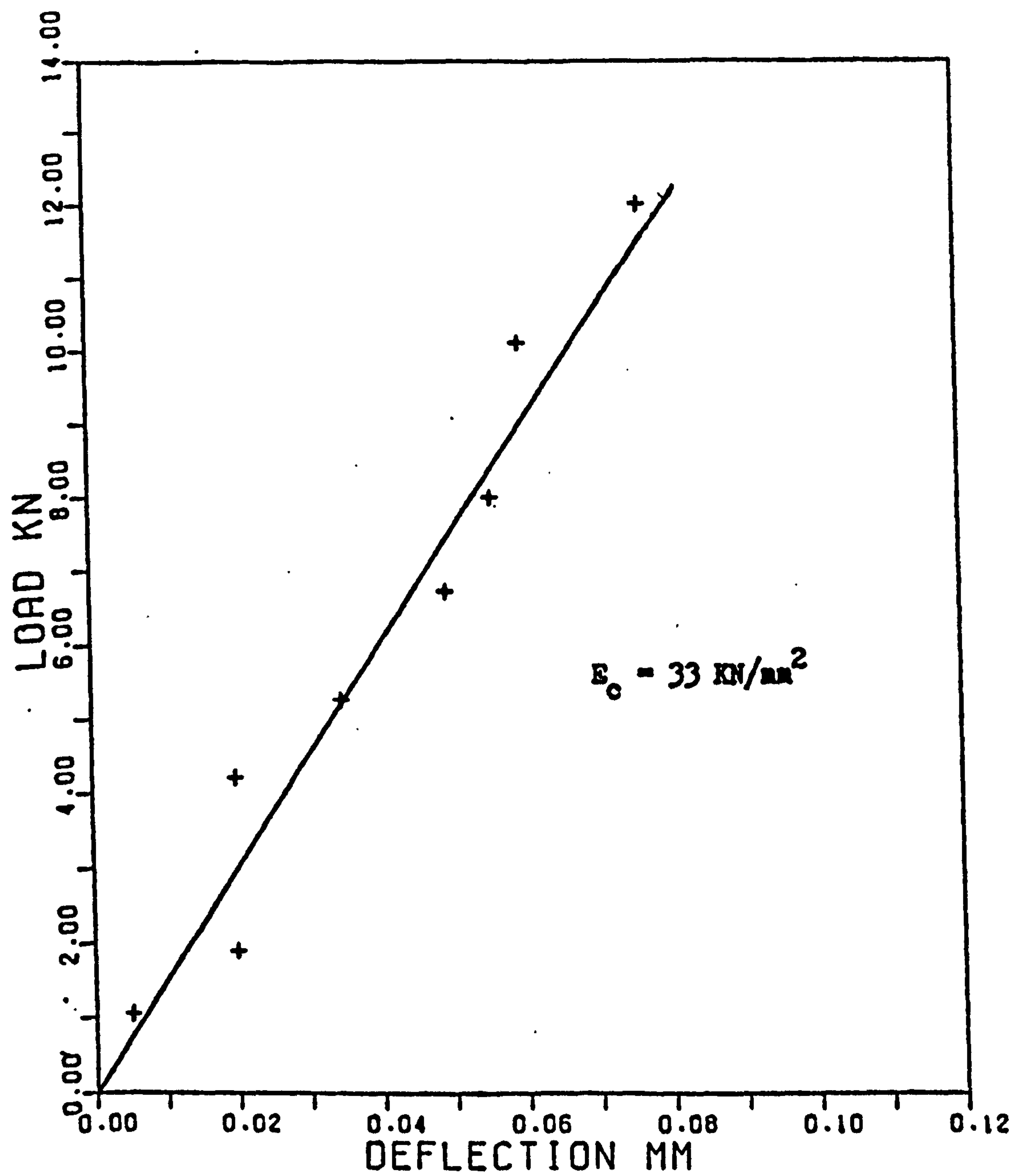


FIG.5.2 MODULUS OF ELASTICITY OF THE CONCRETE

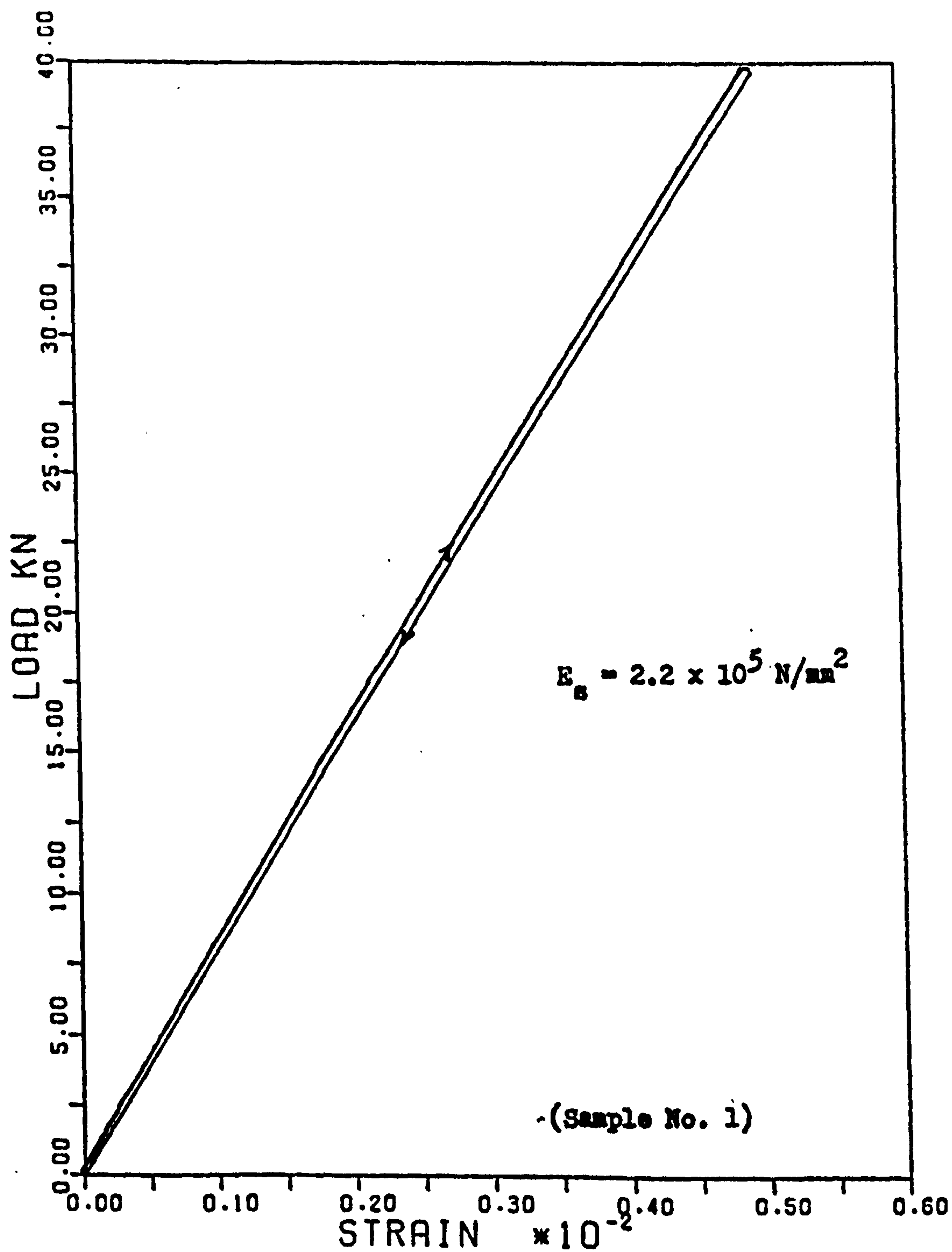


FIG.5.3 MODULUS OF ELASTICITY OF THE
PRESTRESSING CABLE

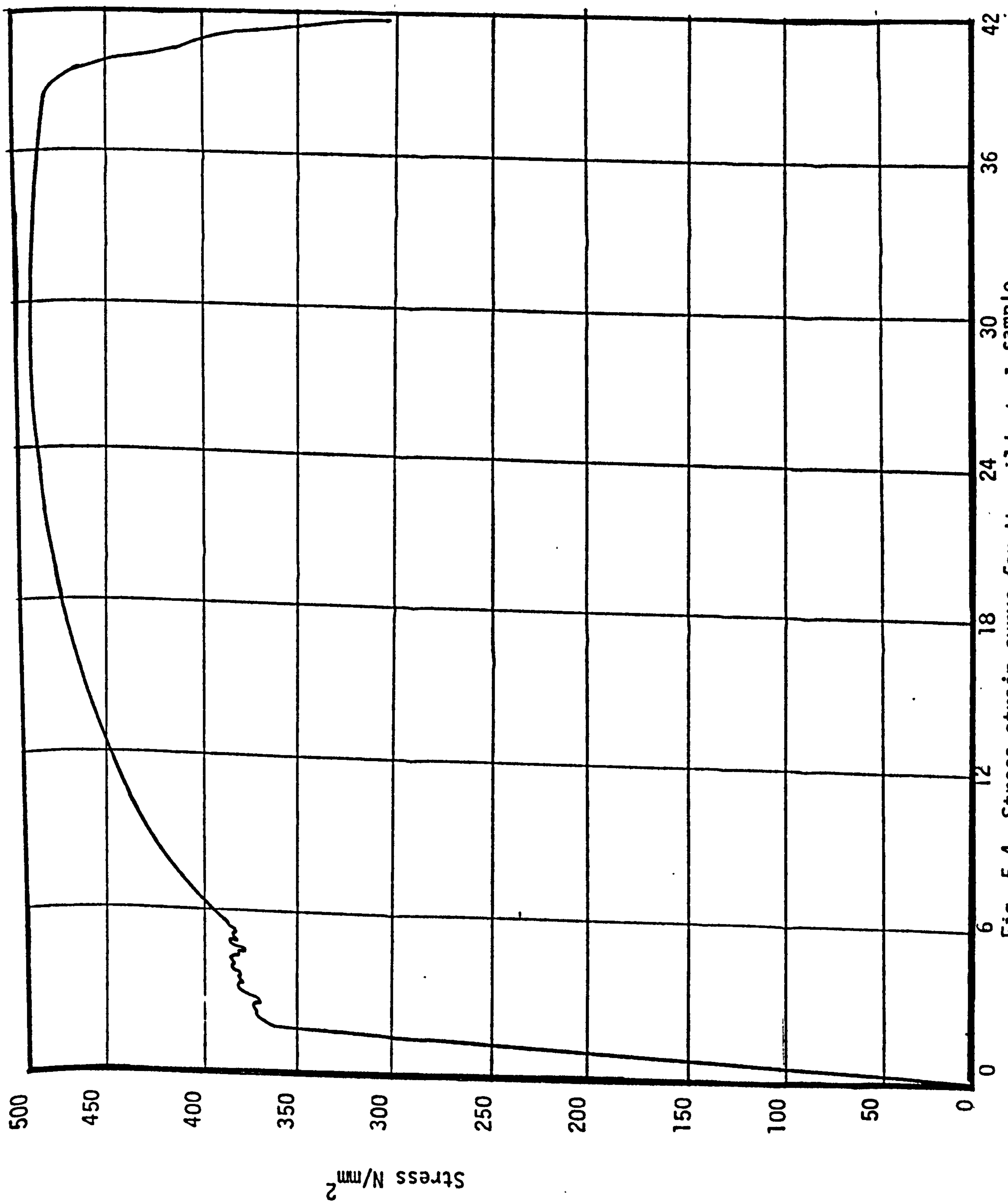


Fig. 5.4 Stress-strain curve for the mild steel sample

5.3 CASTING THE FRAMES AND THE ARCHES

The frames and the arches were cast using Mix III. Nine cubes (10 x 10 x 10 cm) were also made from the mix. Table (5.3) shows the averages of the results of the compression tests.

TABLE (5.3)

Days	Average strength N/mm ²
7	34.60
14	40.67
28	43.42

The process started by fixing the wires on a wooden mould with pieces of steel rod as shown in plates No. 1 and 14. Next the concrete was cast. 24 hours after casting the mould was removed. The concrete was cured by spraying it with curing liquid, covering it with plastic sheeting and leaving it at a temperature of about 20⁰ C for 28 days. The casting processes are illustrated by plates No. 3, 4 and 15.

5.4 THE PRESTRESSING

5.4.1 The prestressing system

The prestressing wires came through plastic tubes as shown in plates No. 1 and 14 and were passed through a steel plate at each end.

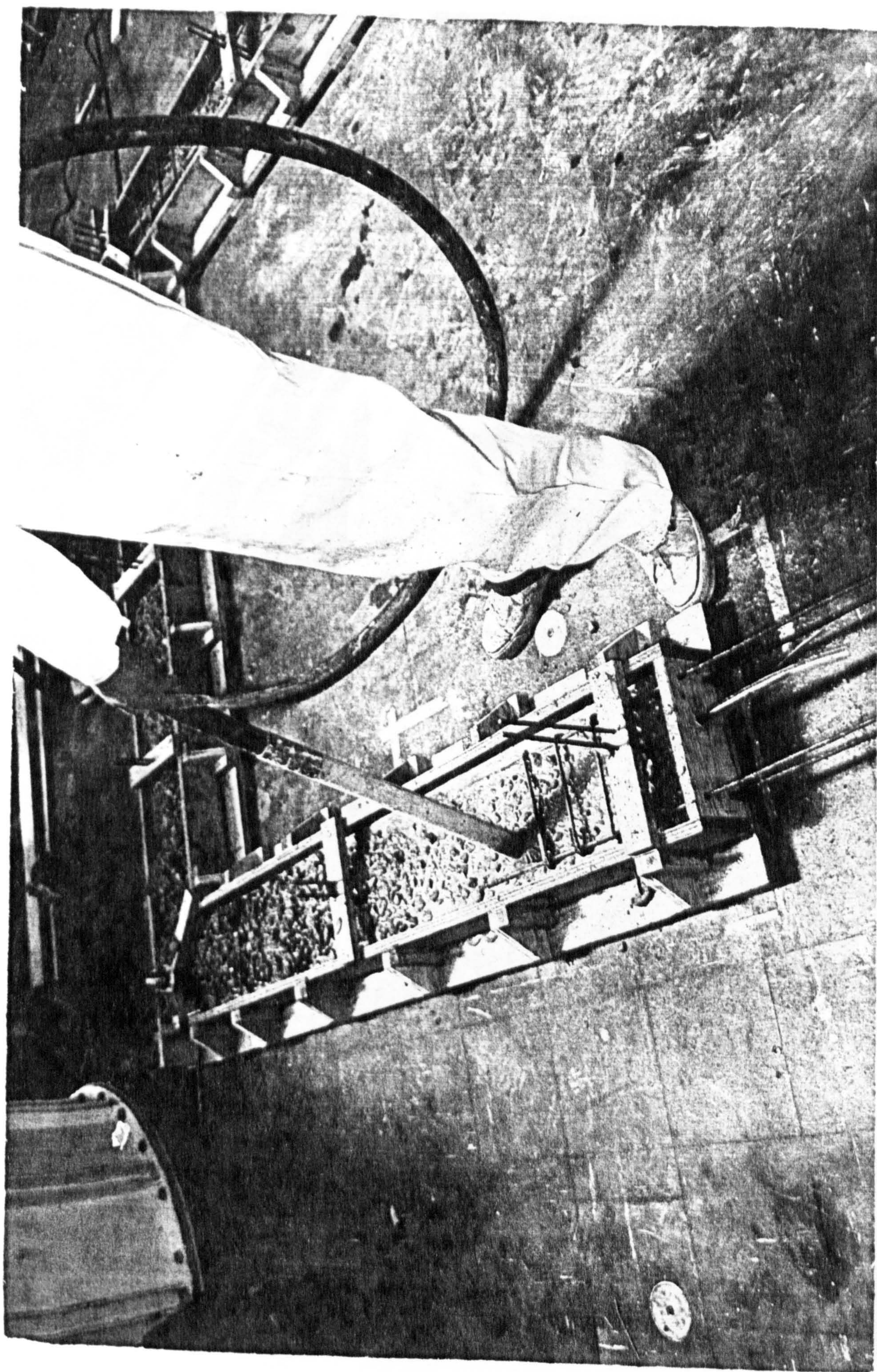


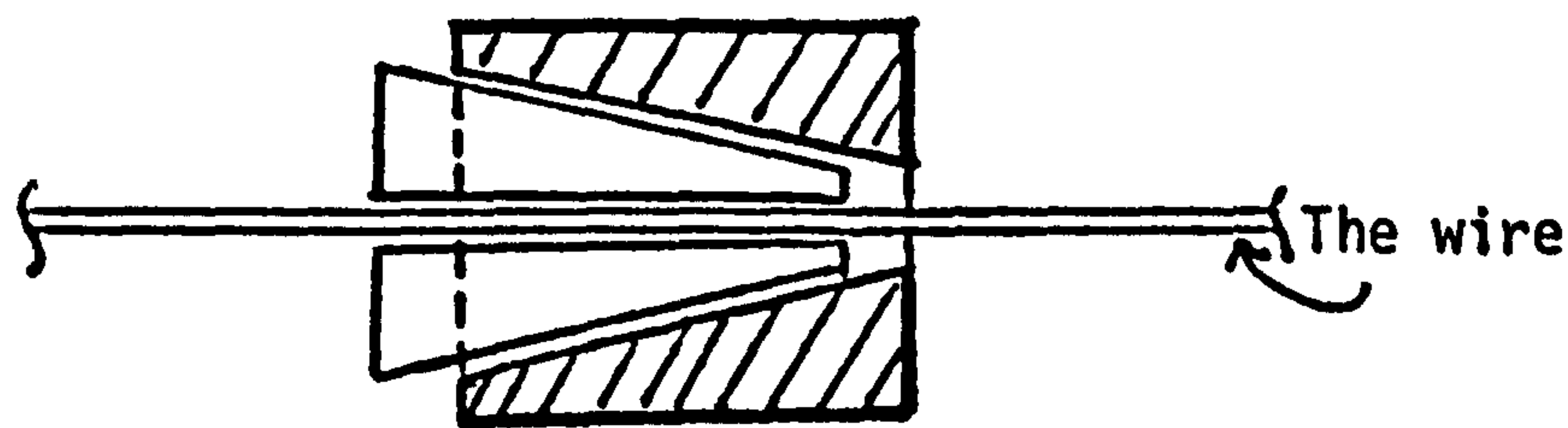
Plate No. 3 The concrete vibrator



Plate No. 4 Curing the concrete

The live and dead ends consist of an anchorage unit as shown in plate No. 7.

The anchorage unit (Fig. 5.5) is a cylinder which has a cone-shaped hollow and another cone divided into two pieces which can be fitted together inside the hollow. The unit has a hole through which the wire can be placed.



An anchorage unit

Fig. 5.5

The end plates which are attached to the sides of the frames and arches were used to distribute the load on the faces of the frames and arches.

5.4.2 The prestressing process

The jack was calibrated in order to study its behaviour as shown in Fig. 5.6

The prestressing operation is described as follows:-

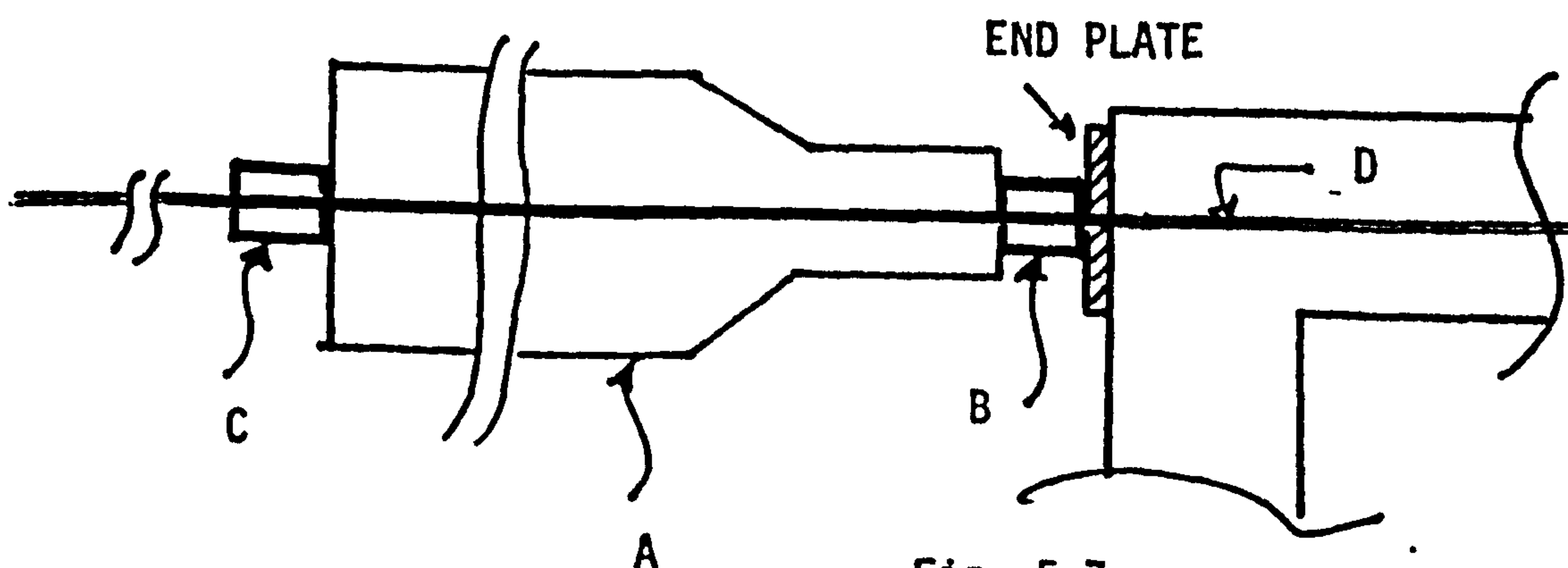


Fig. 5.7

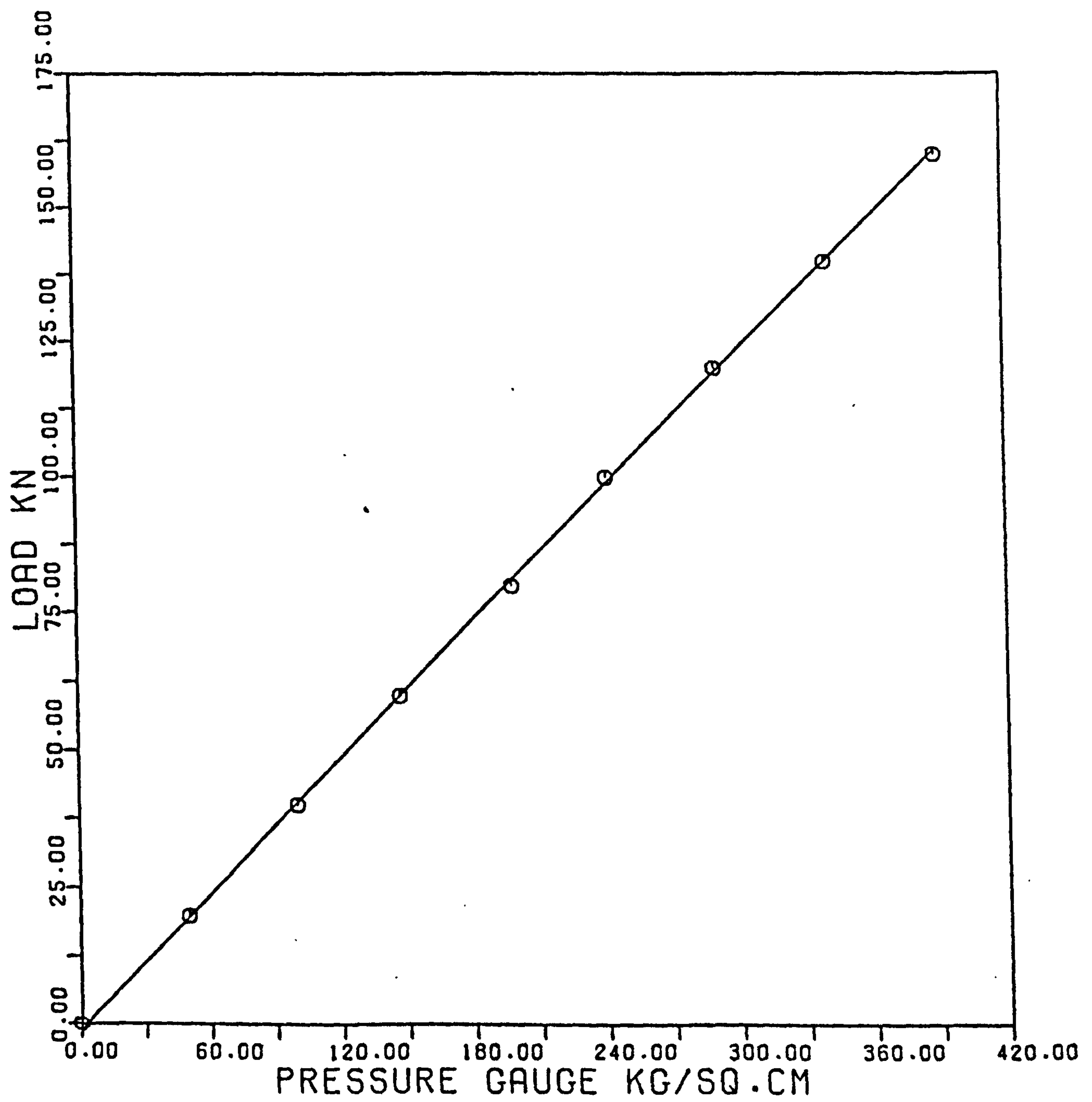


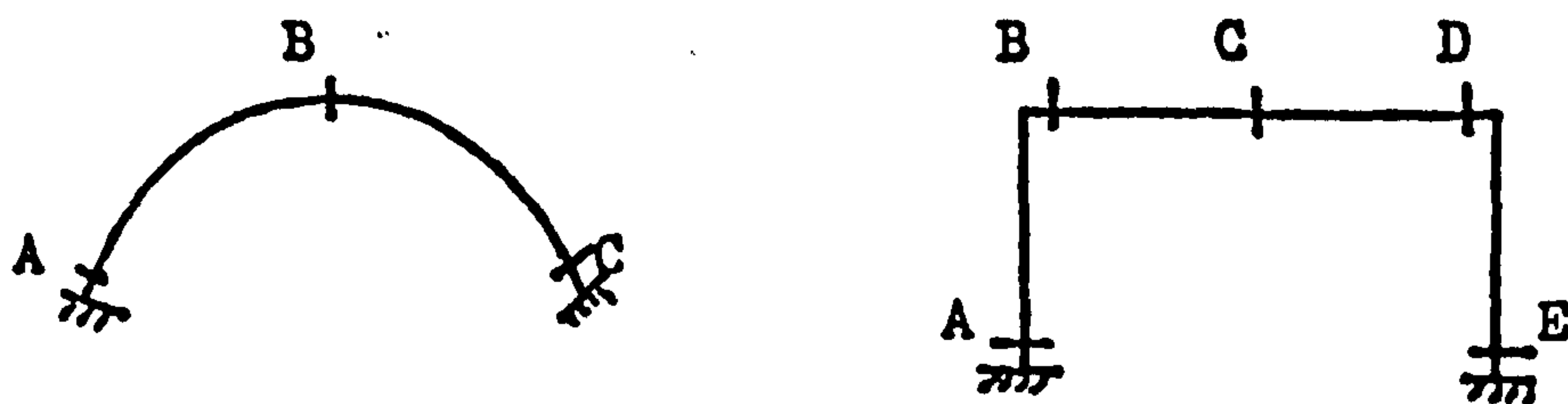
FIG.5.6 CALIBRATION OF THE JACK

- I) The jack A was reacted against the anchorage B and by using the jack's pump the pressure was transmitted into the anchorage C, which produces the tensile force in the wire D. The anchorage B was used to retain this force and the jack's indicator gave the value of the force.
- II) The anchorage C was removed.
- III) The jack was removed.

5.5 TESTING THE FRAMES AND ARCHES

In order to study the behaviour of the frames and arches in both elastic and ultimate stages, the following tests were carried out:-

- I) In the elastic stage:- The strains were measured in sections A, B and C for the arches and in sections A, B, C, D and E for the frames.



The deflections were measured at the section at the mid point of the span under the applied load. The results are shown in Figs. 5.8, 5.9, 5.10, 5.11, 5.12, 5.13, 5.14, 5.17, 5.18, 5.19, 5.20 and 5.21.

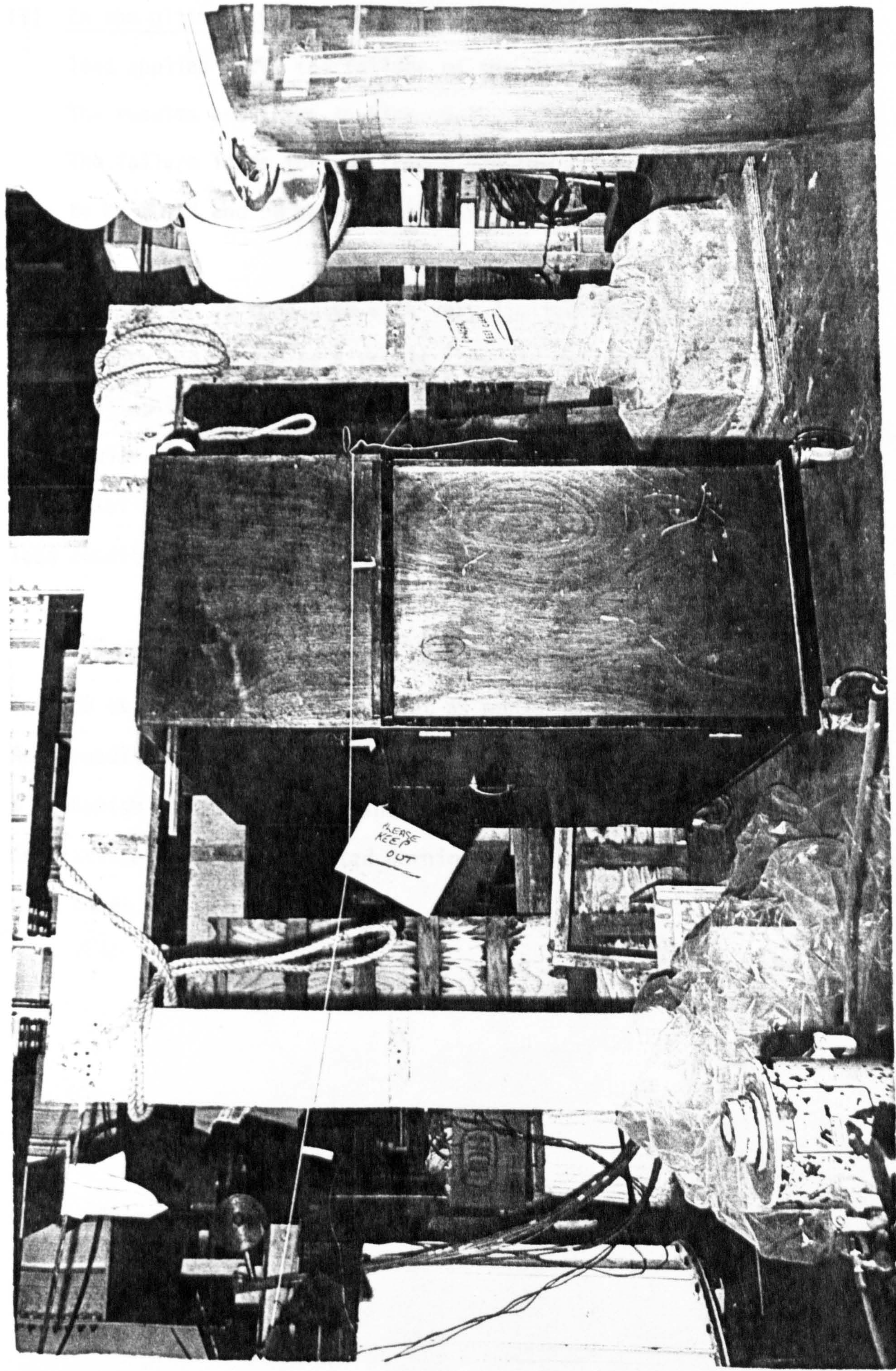


Plate No. 5 Casting the footings of the frame

II) In the ultimate stage:- Deflections were measured with the load applied until the failure of the frames and the arches. The results are shown in Figs. 5.15, 5.16, 5.22 and 5.23. The failure loads for the frames and the arches were found to be 80 kN and 58 kN respectively.

The tests had to be conducted with very limited resources in both finance and time. As a result they did not yield as full and detailed information as would have been desirable. Nevertheless, the results showed that the structures, designed according to the theory set out in this thesis, behaved satisfactorily under working load conditions.

In the case of the arches, however, the design did not include as much shear reinforcement as would have been desirable. As a result the form of failure was by shear. This is, of course, a mechanism of failure which should be avoided, as far as possible, in practice due to the limited warning of failure which occurs in such cases.

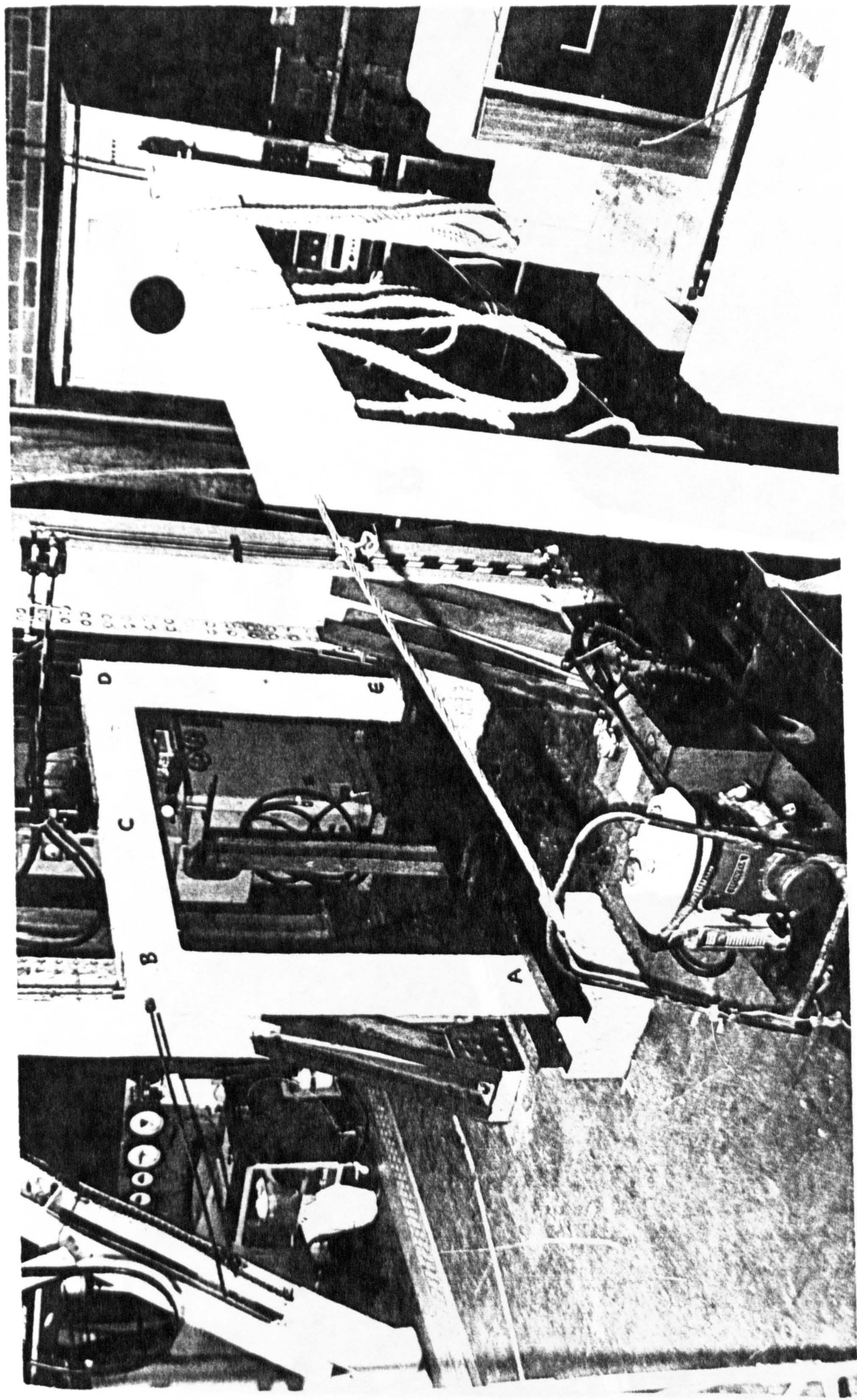


Plate No. 6 The frame under the applied load of the testing machine

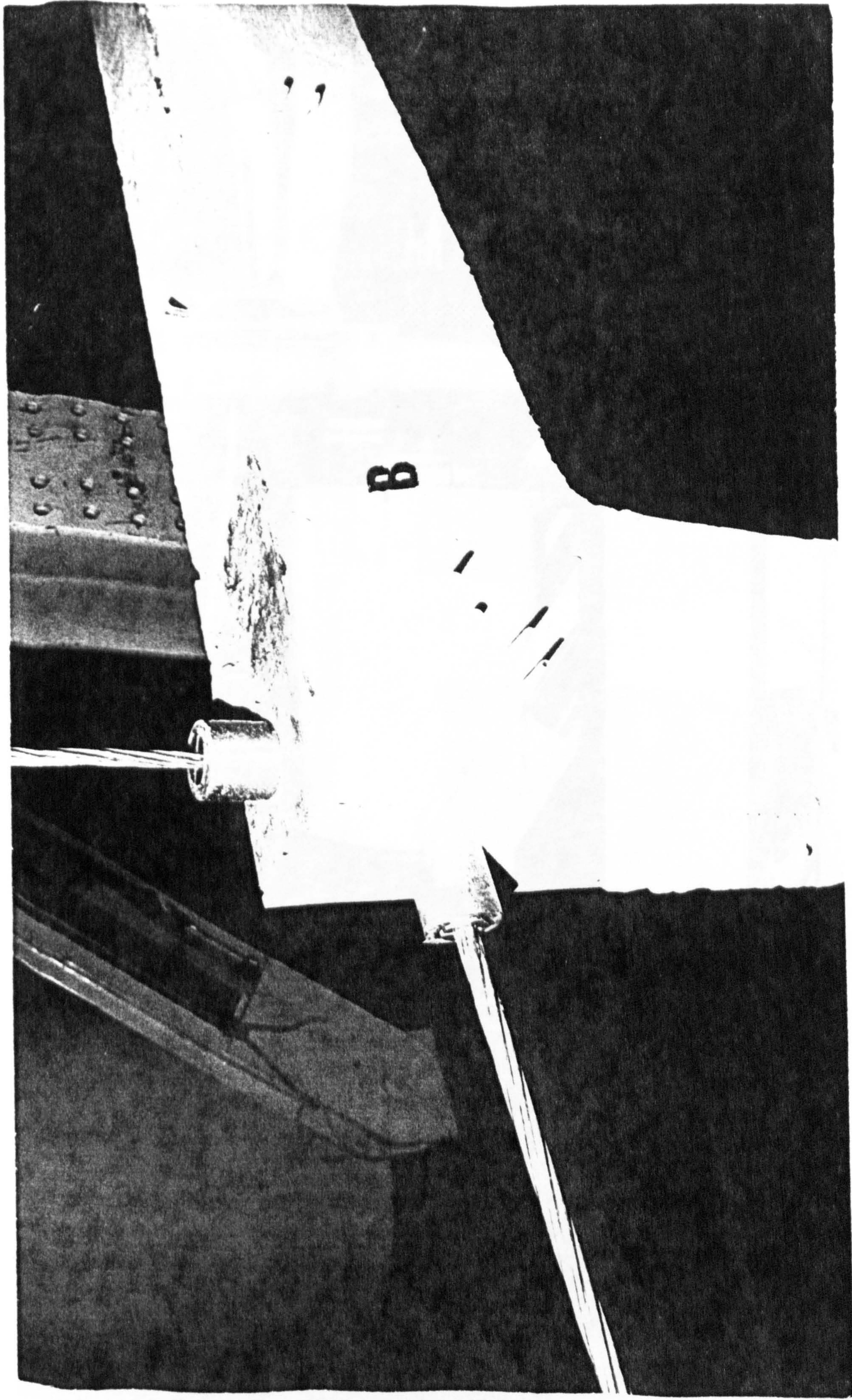


Plate No. 7 Details of anchorage

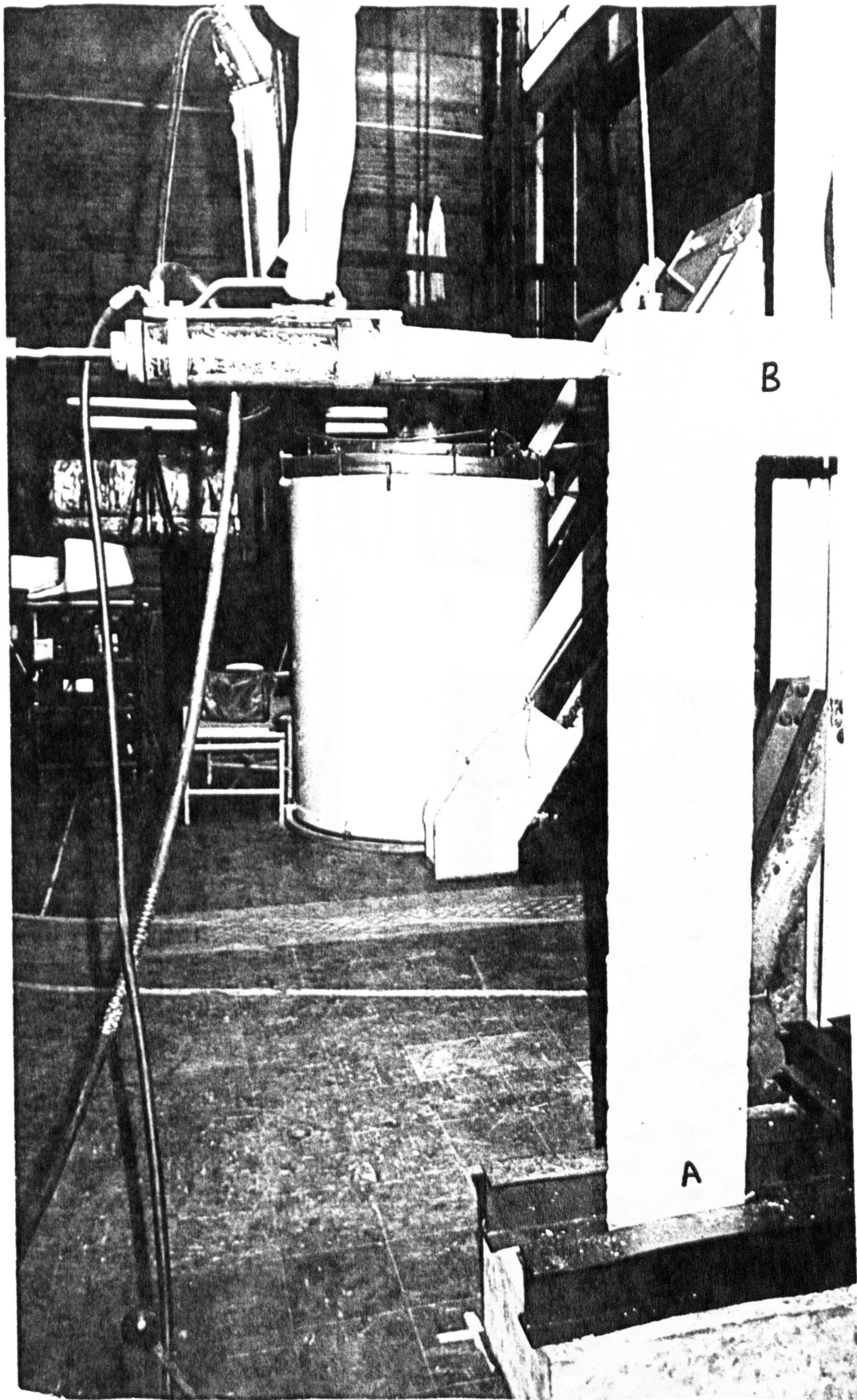


Plate No. 8 Prestressing the horizontal wire

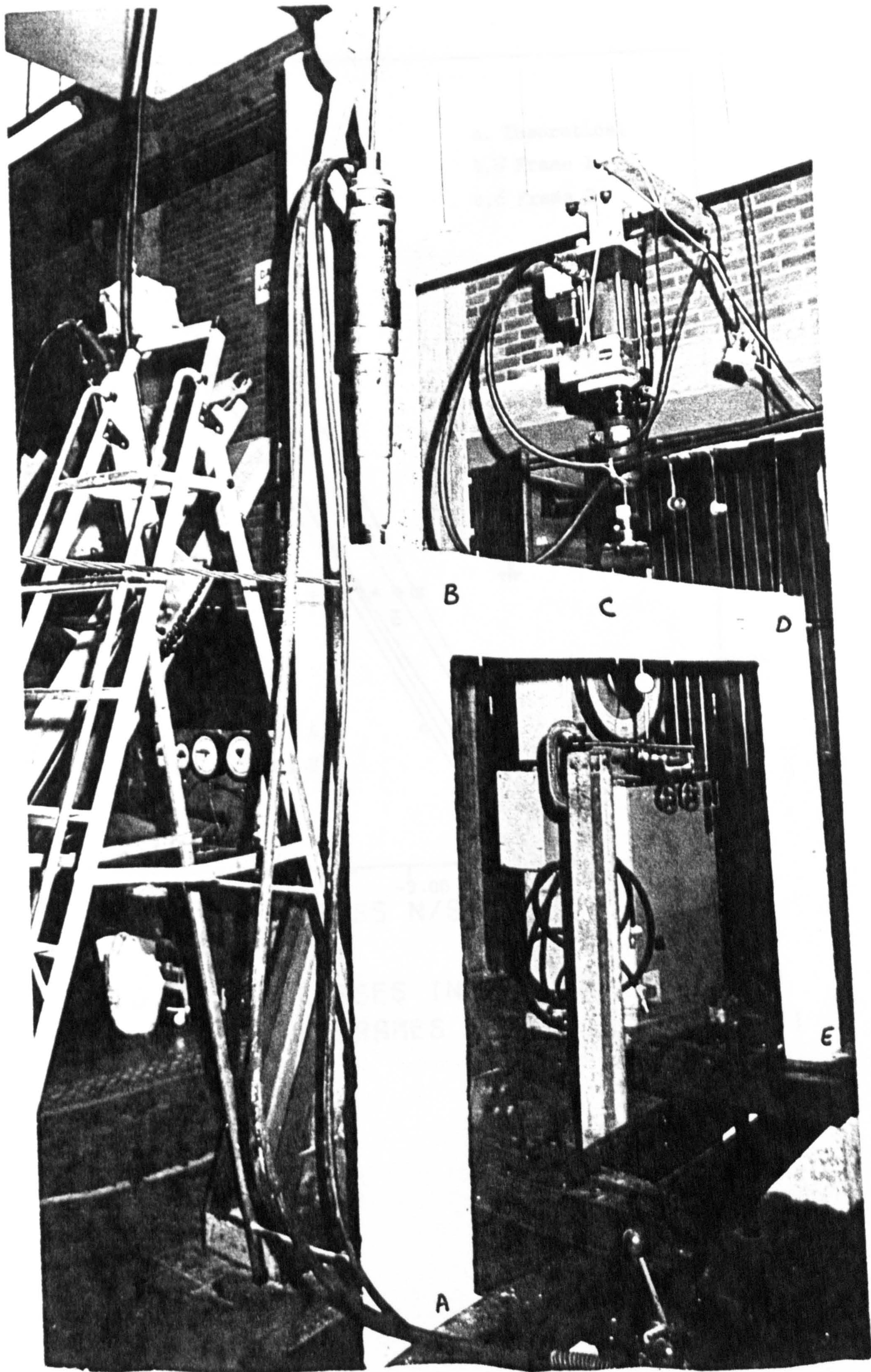


Plate No. 9 Prestressing the vertical wire

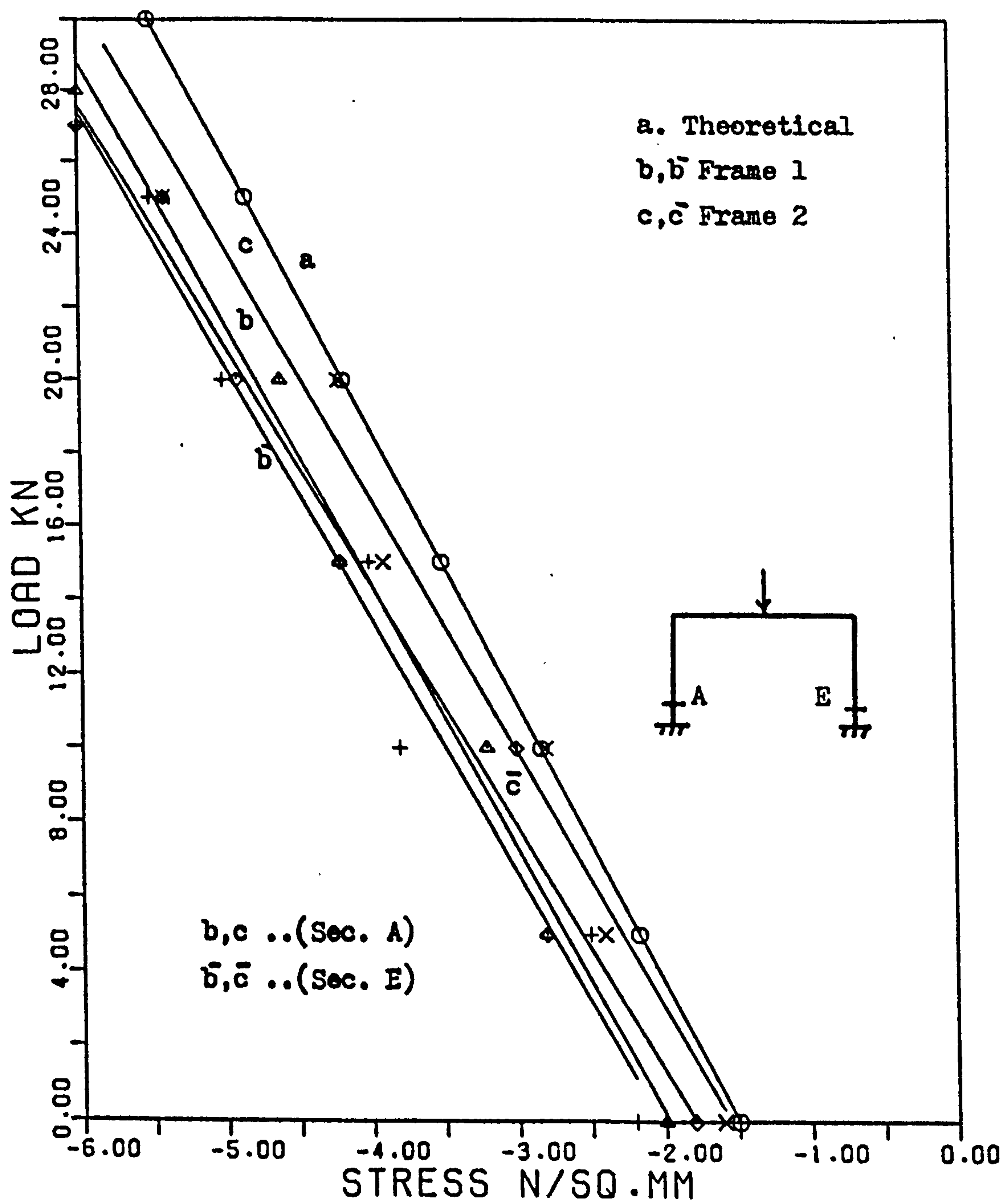


FIG.5.8 STRESSES IN THE TOP FIBRE AT THE FRAMES SUPPORTS (SEC.A&E)

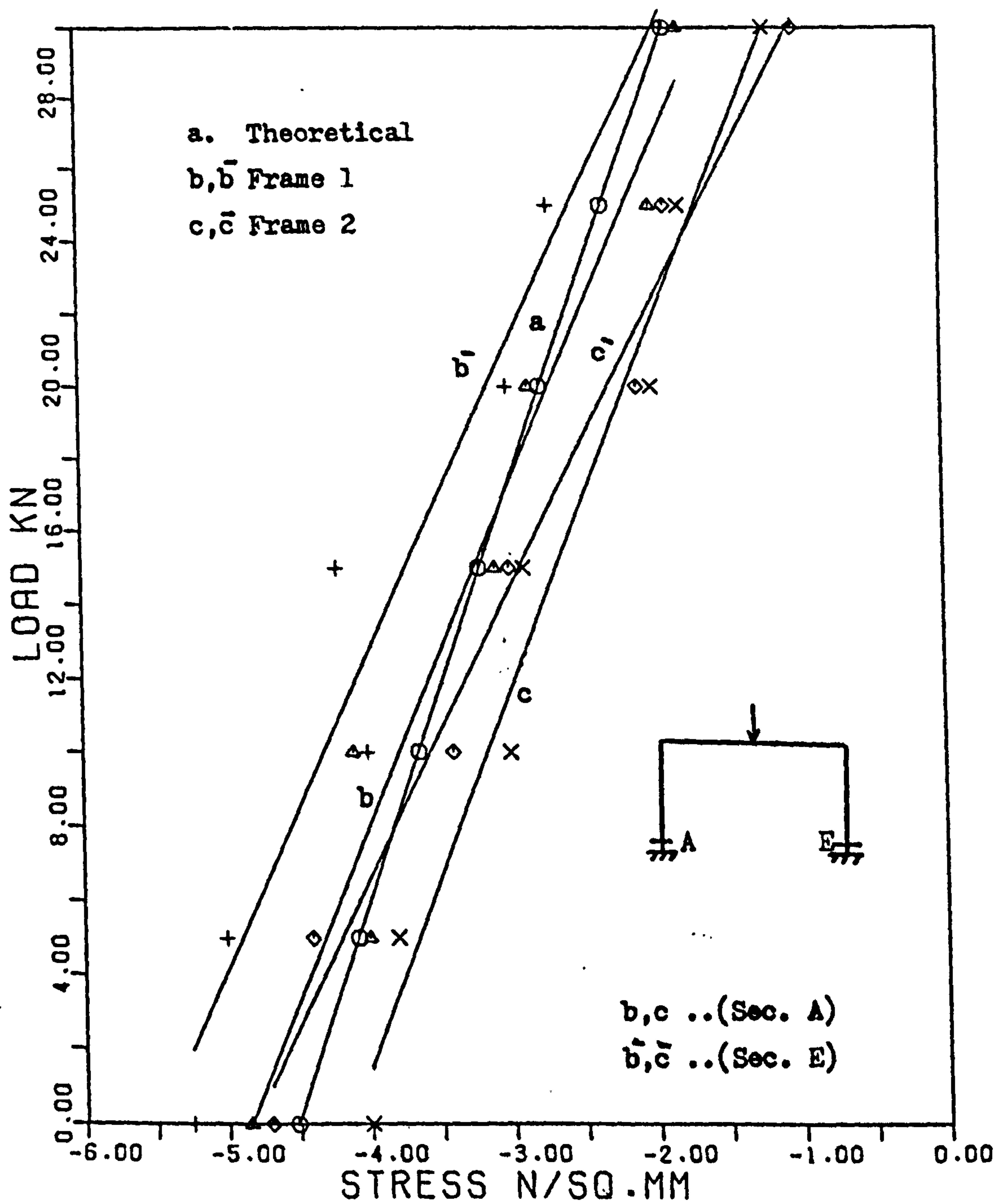


FIG.5.9 STRESSES IN THE BOTTOM FIBRE AT THE FRAMES SUPPORTS (SEC. A&E)

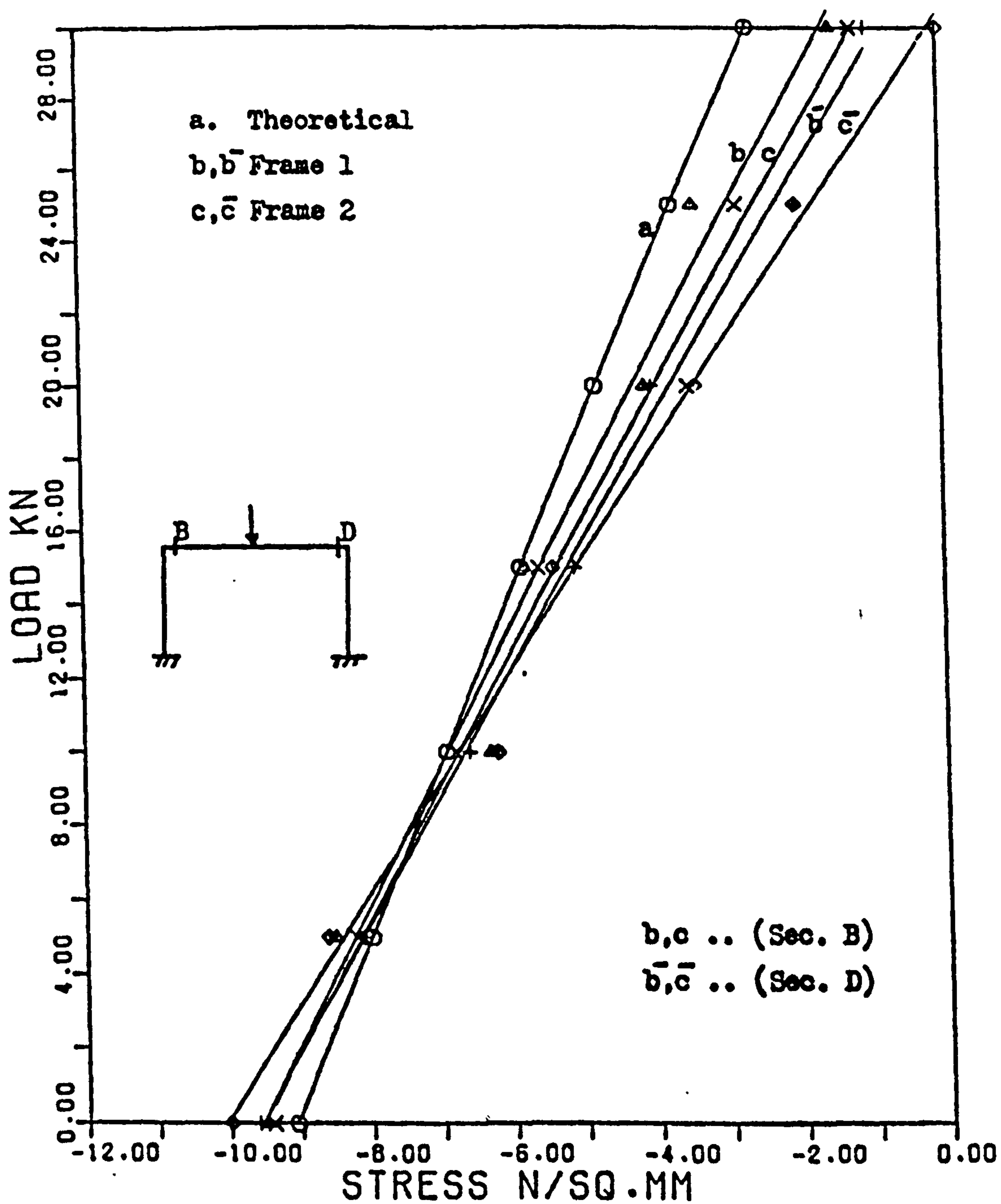


FIG.5.10 STRESSES IN THE TOP FIBRE AT
THE JOINTS OF THE FRAMES (SEC.B&D)

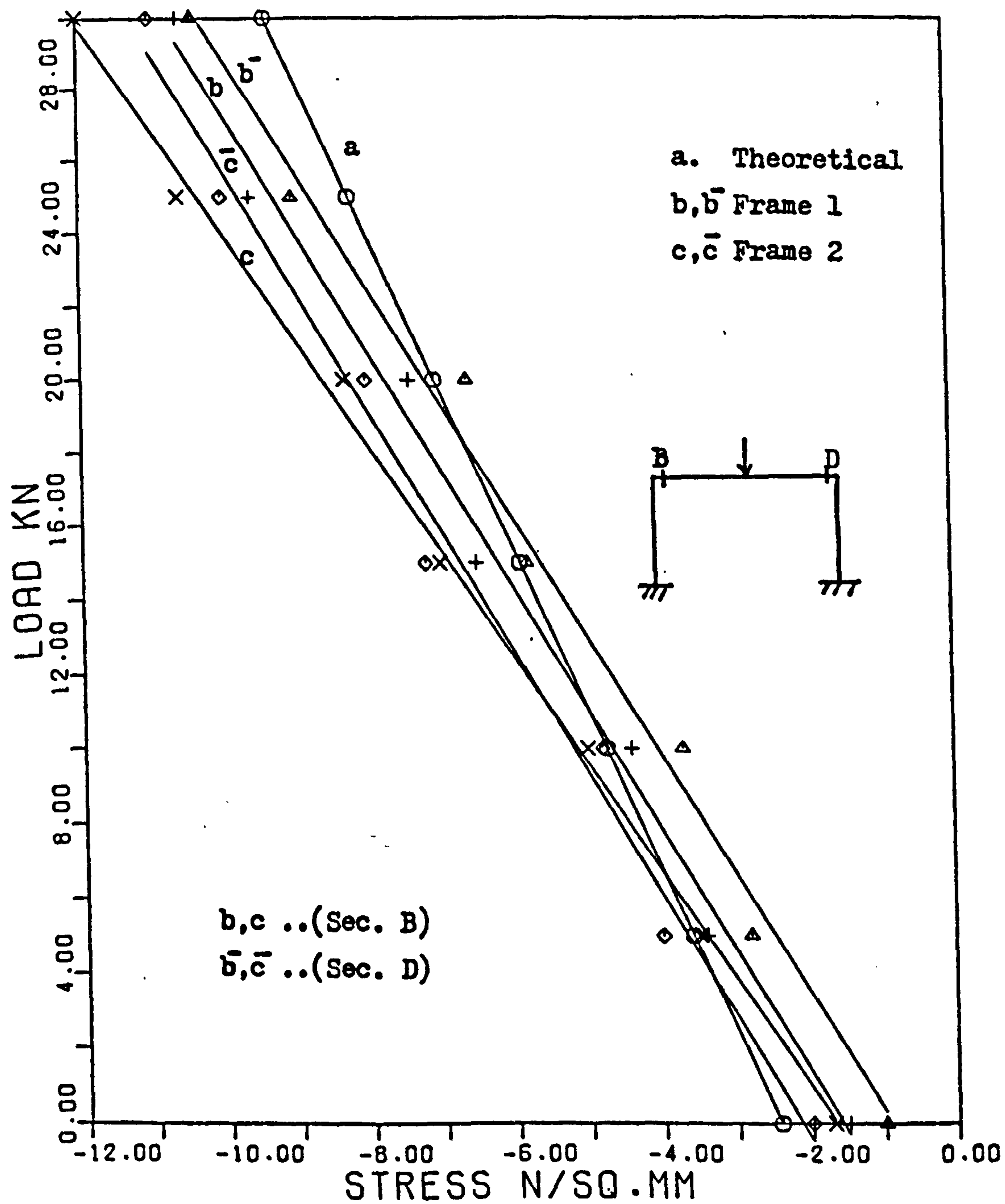


FIG.5.11 STRESSES IN THE BOTTOM FIBRE AT THE JOINTS OF THE FRAMES (SEC.B&D)

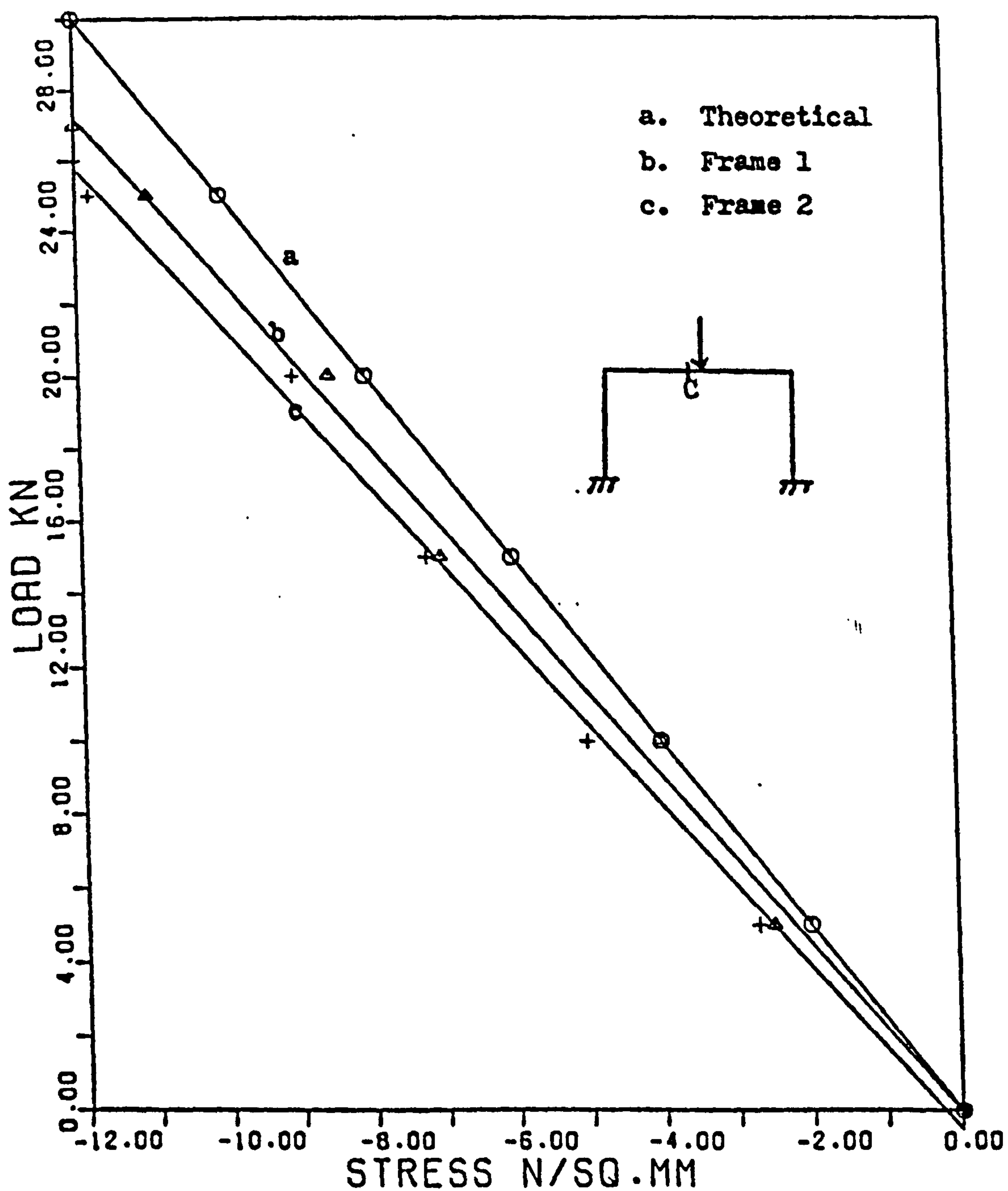


FIG.5.12 STRESSES IN THE TOP FIBRE UNDER THE APPLIED LOAD FOR THE FRAMES (SEC. C)

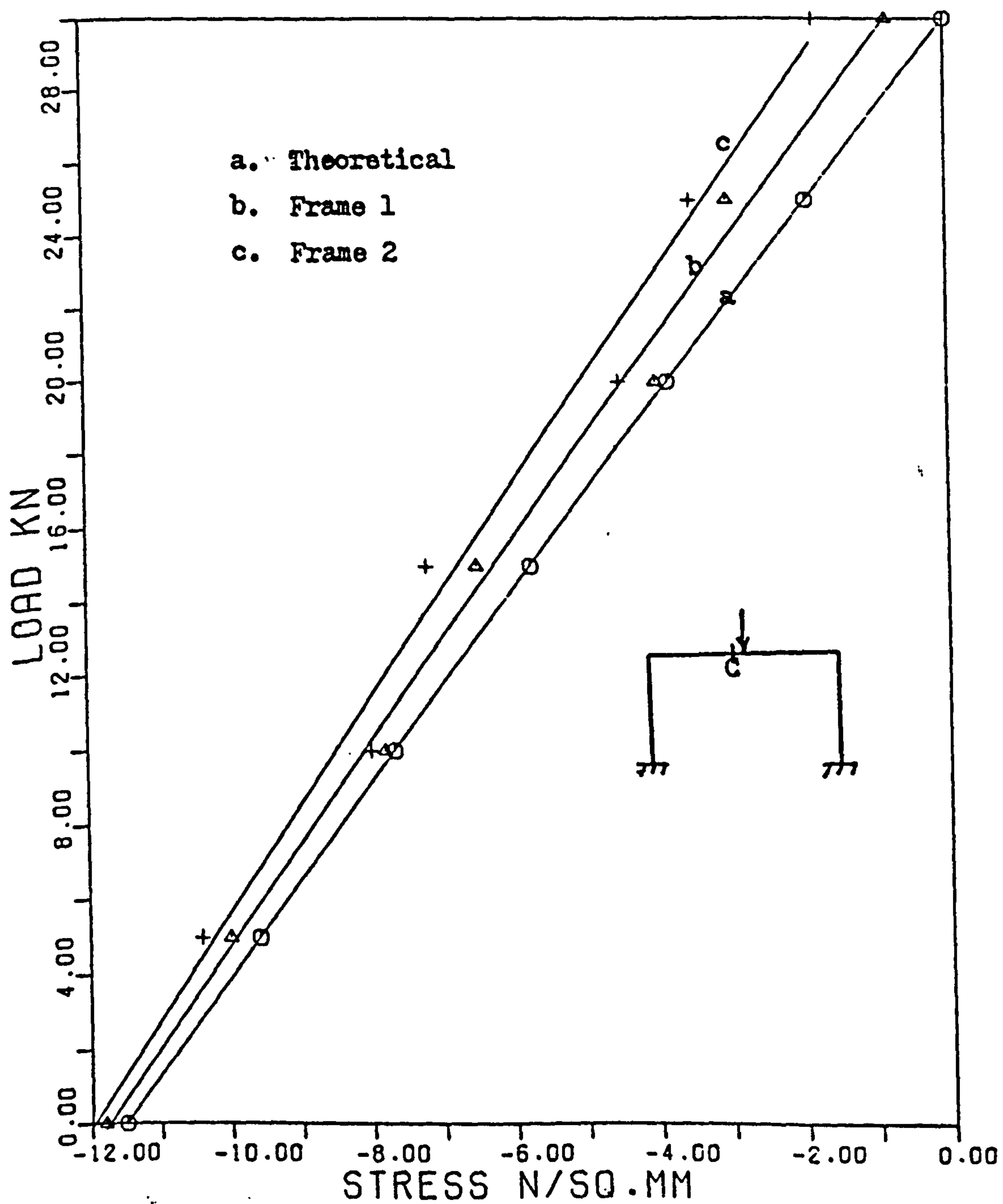


FIG.5.13 STRESSES IN THE BOTTOM FIBRE UNDER THE APPLIED LOAD FOR THE FRAMES (SEC. C)

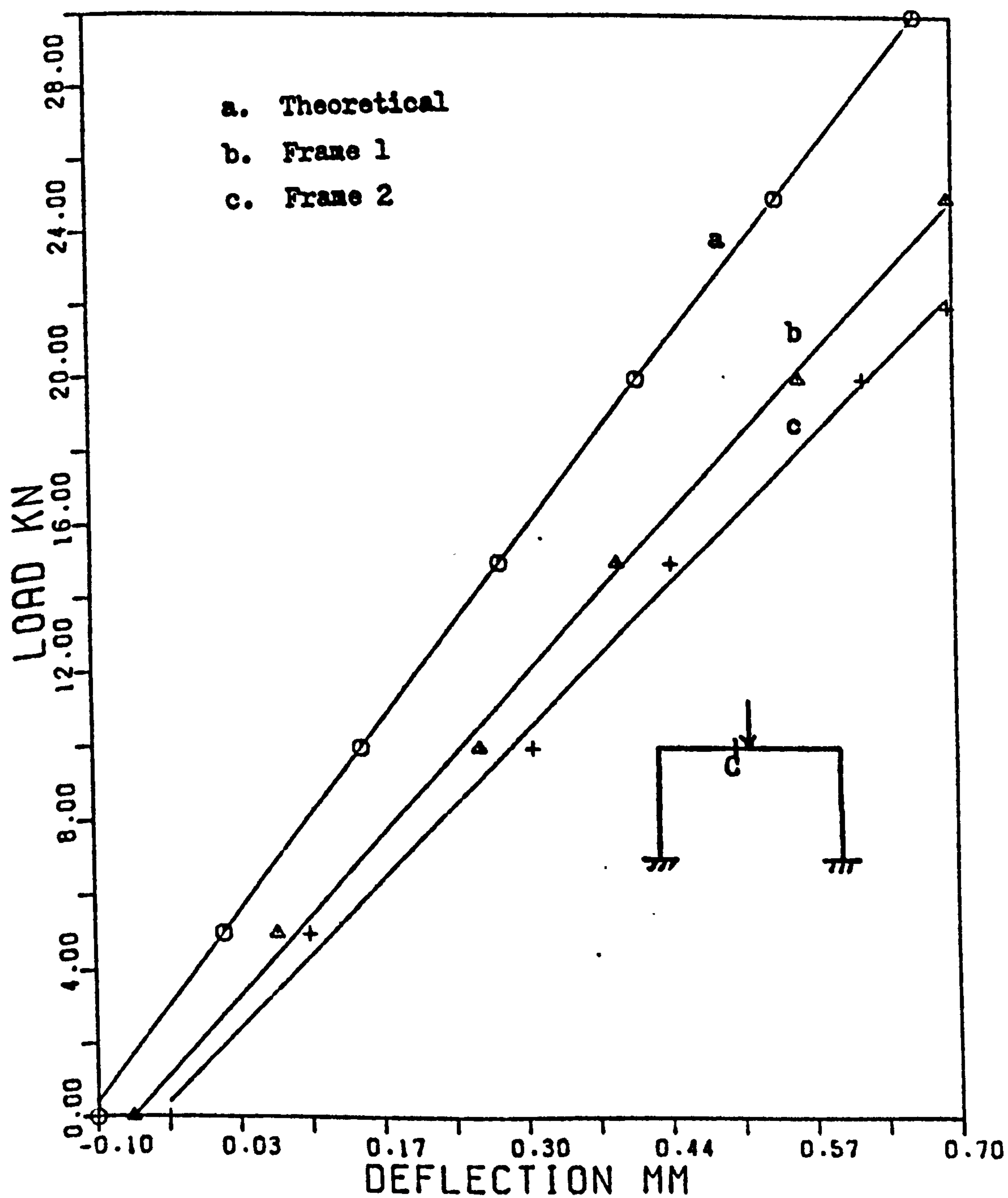


FIG.5.14 DEFLECTIONS AT MID SPAN POINT UNDER THE APPLIED LOAD FOR THE FRAMES (SEC. C)

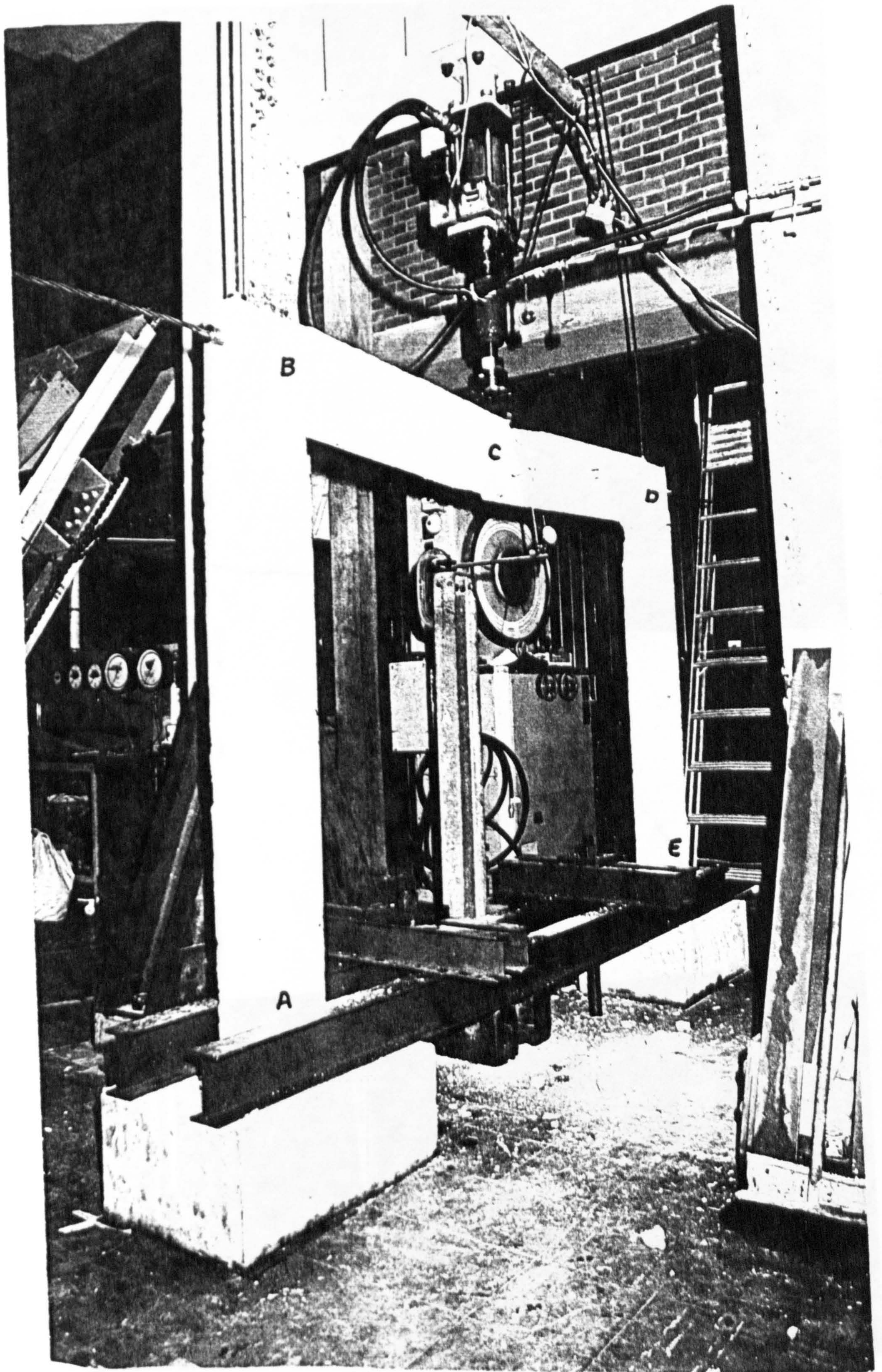


Plate No. 10 The frame after failure

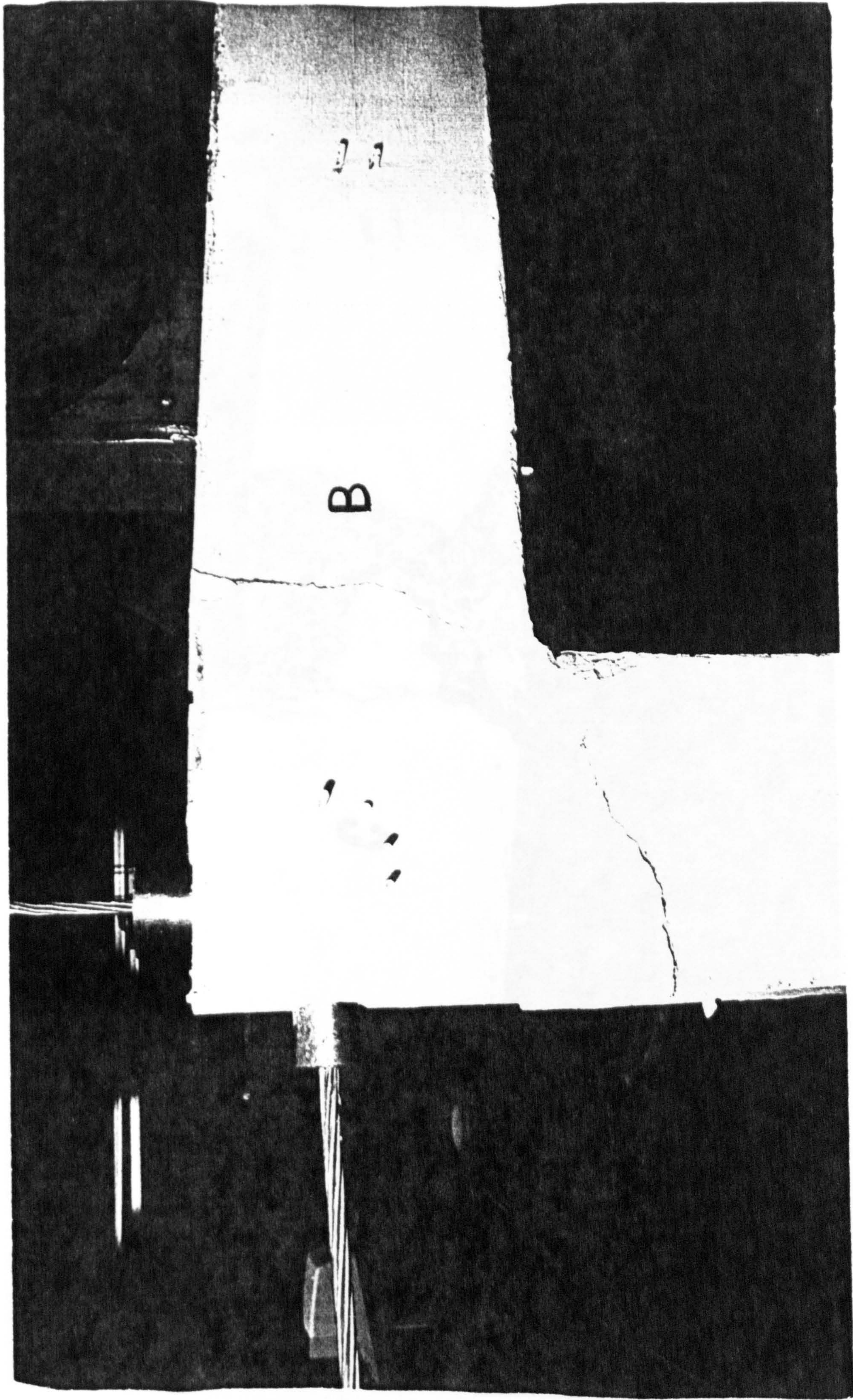


Plate No. 11 The left joint of the frame after failure

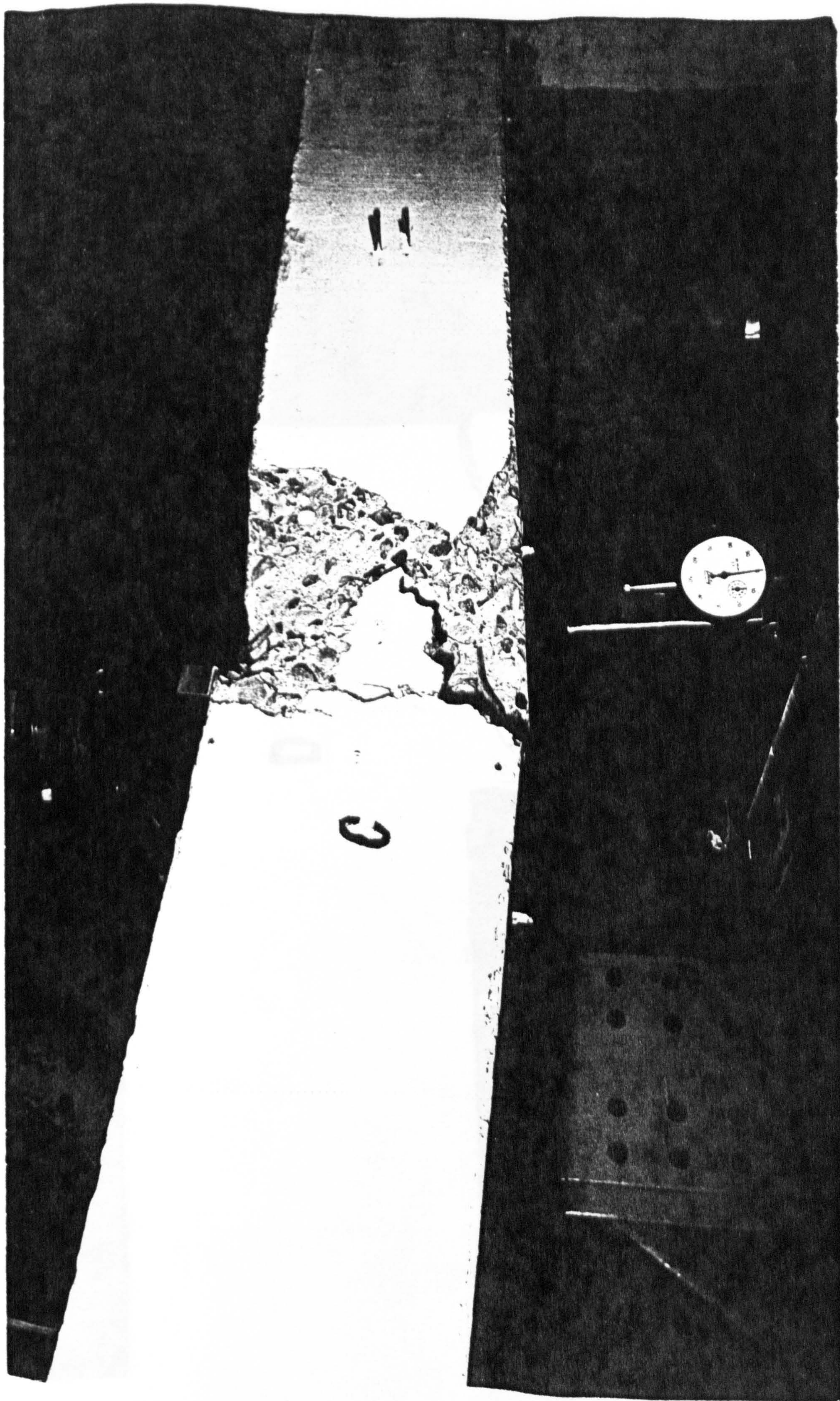


Plate No. 12 The section under applied after failure

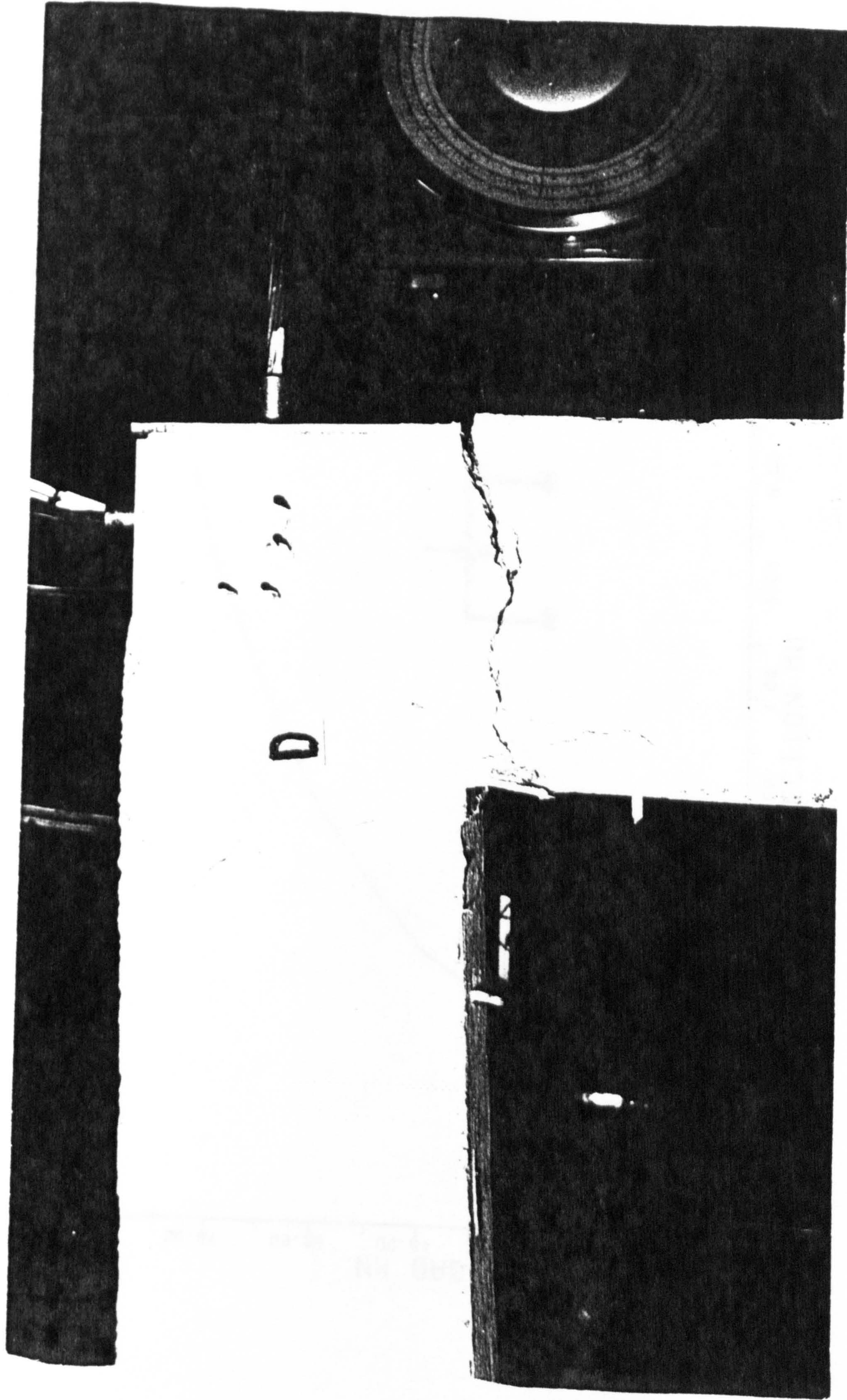


Plate No. 13 The right joint of the frame after failure

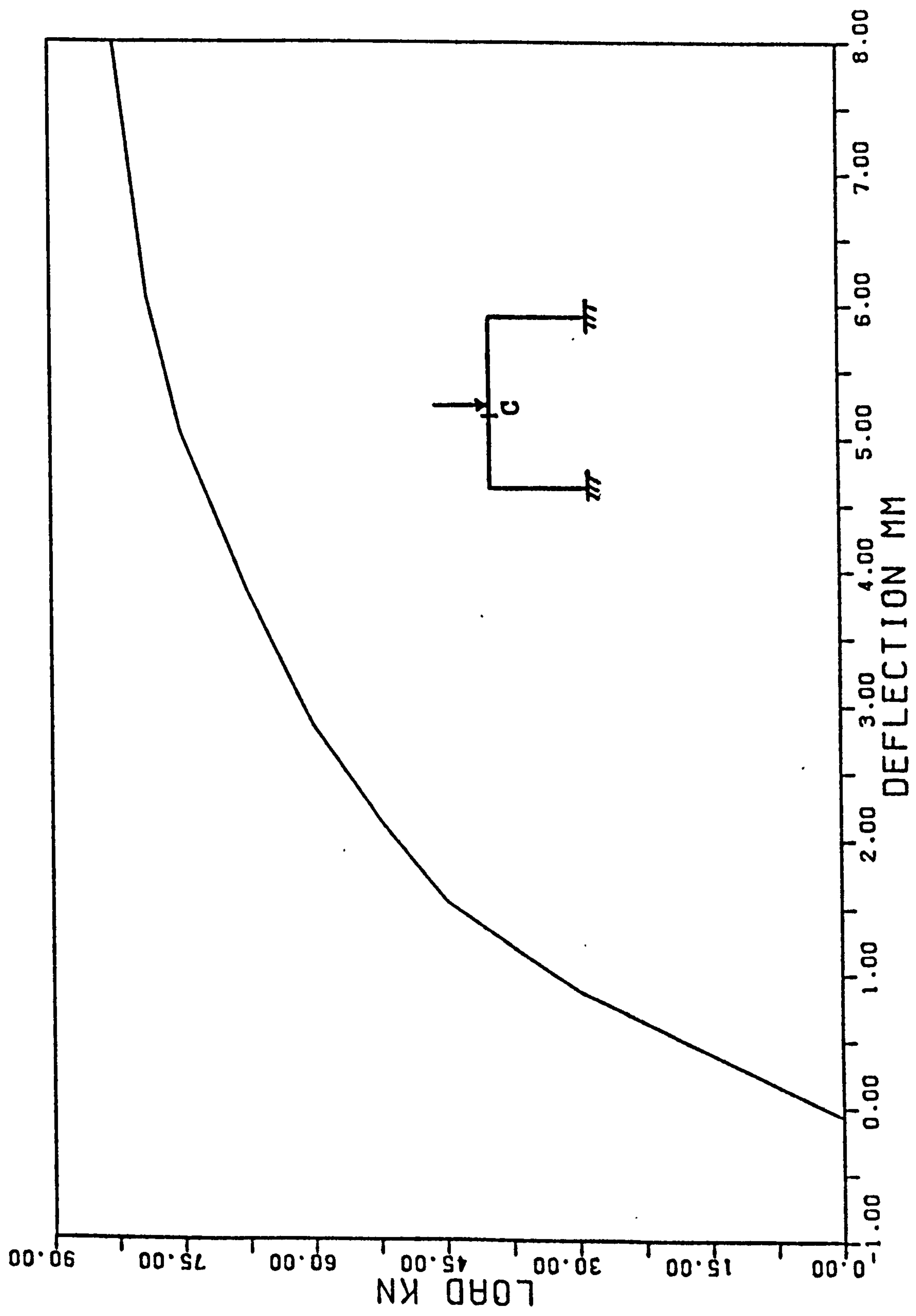


FIG.5.15 LOAD DEFLECTION CURVE FOR FRAME 1 (SEC. C)

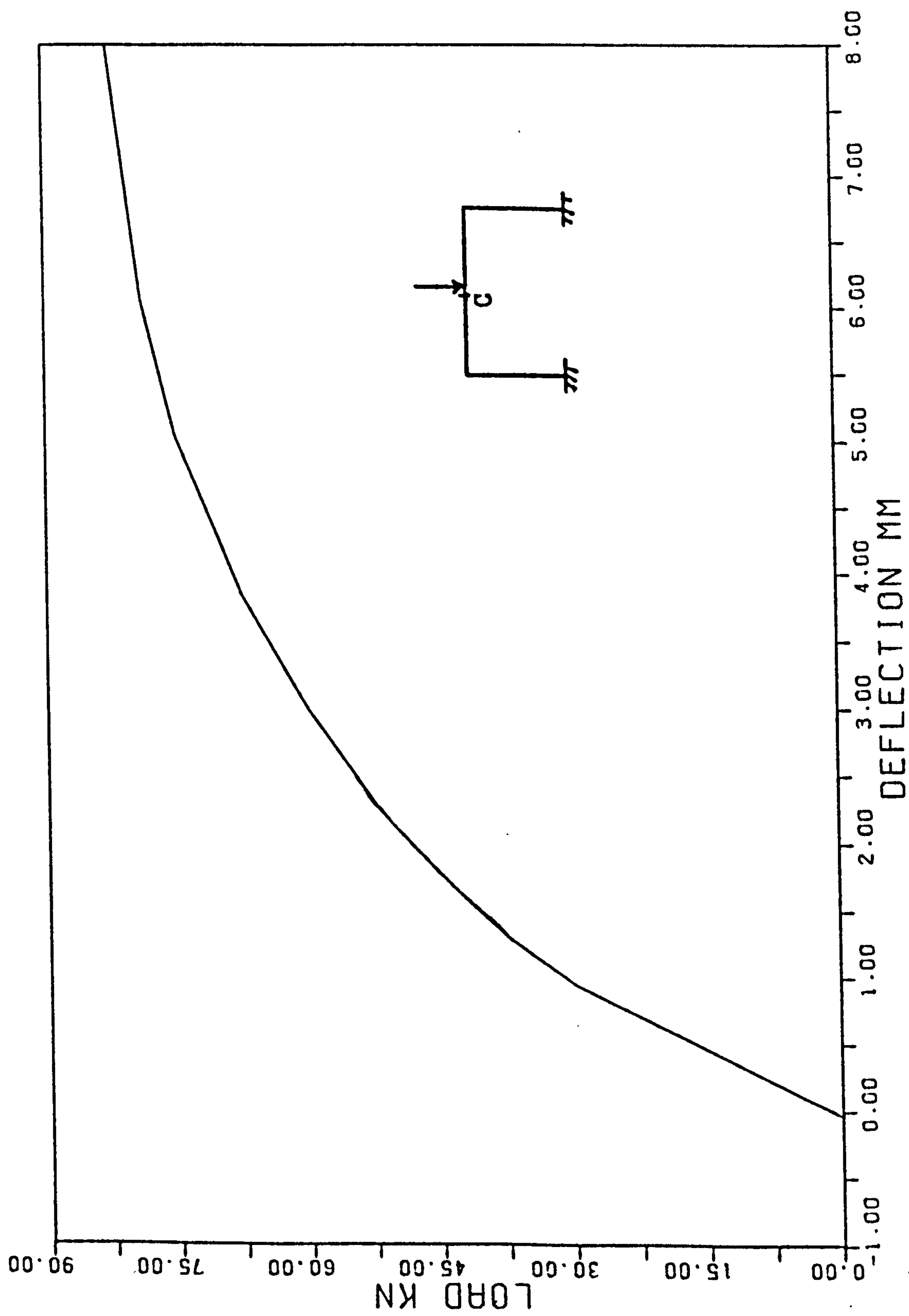


FIG.5.16 LOAD DEFLECTION CURVE FOR FRAME 2 (SEC. C)

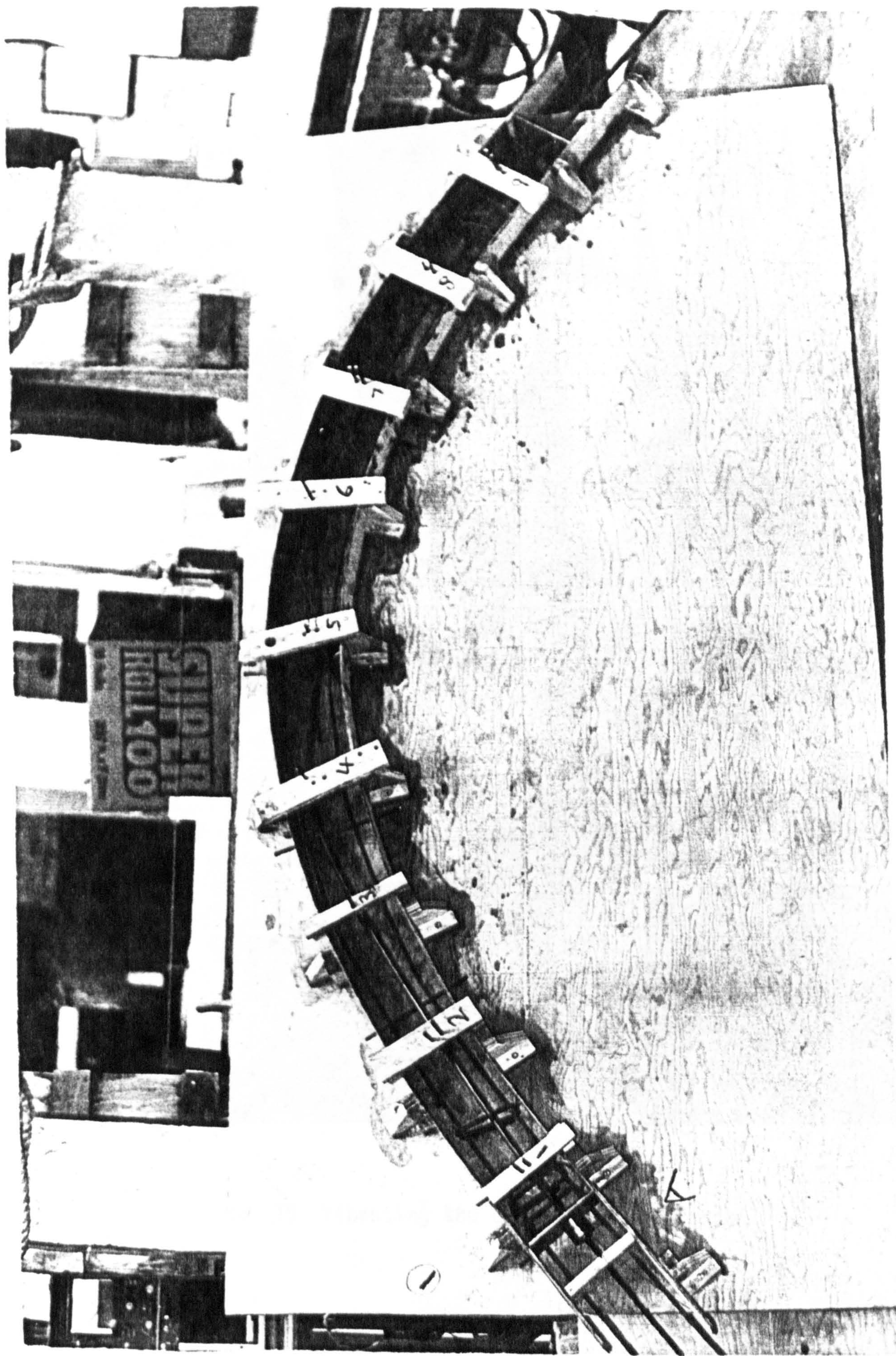


Plate No. 14 The arch mould

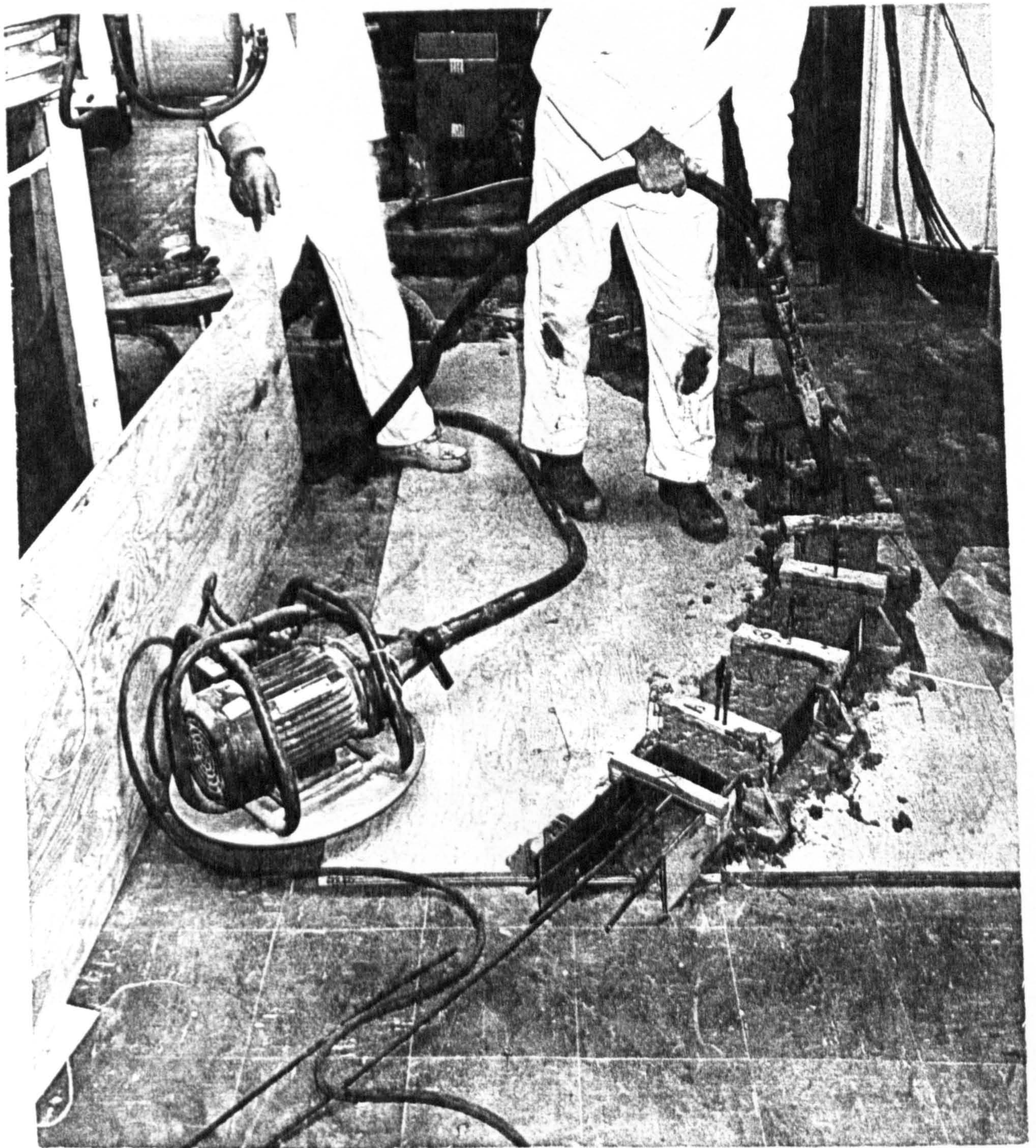


Plate No. 15 Vibrating the concrete of the arch

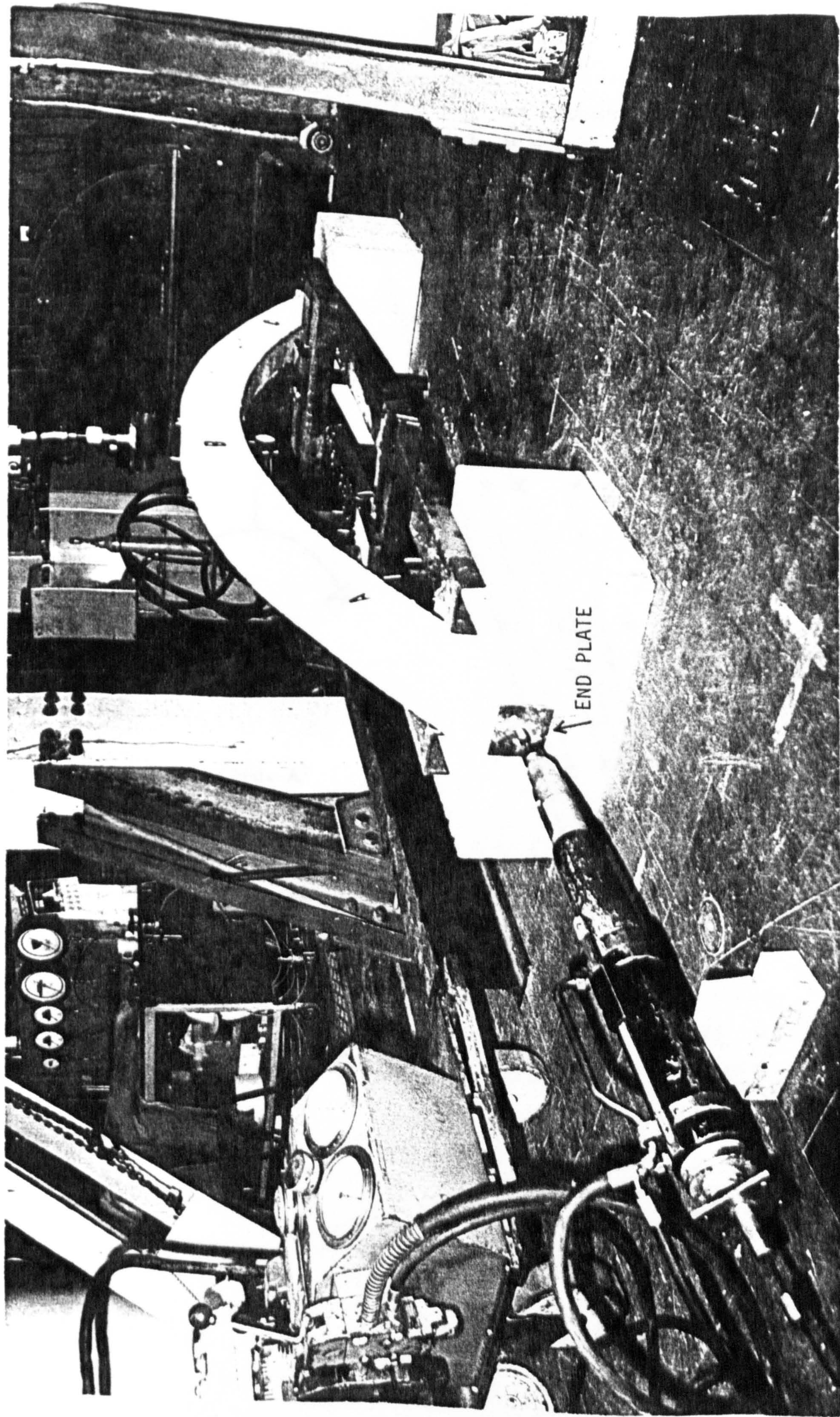


Plate No. 16 The arch under the applied load - prestressing the wire

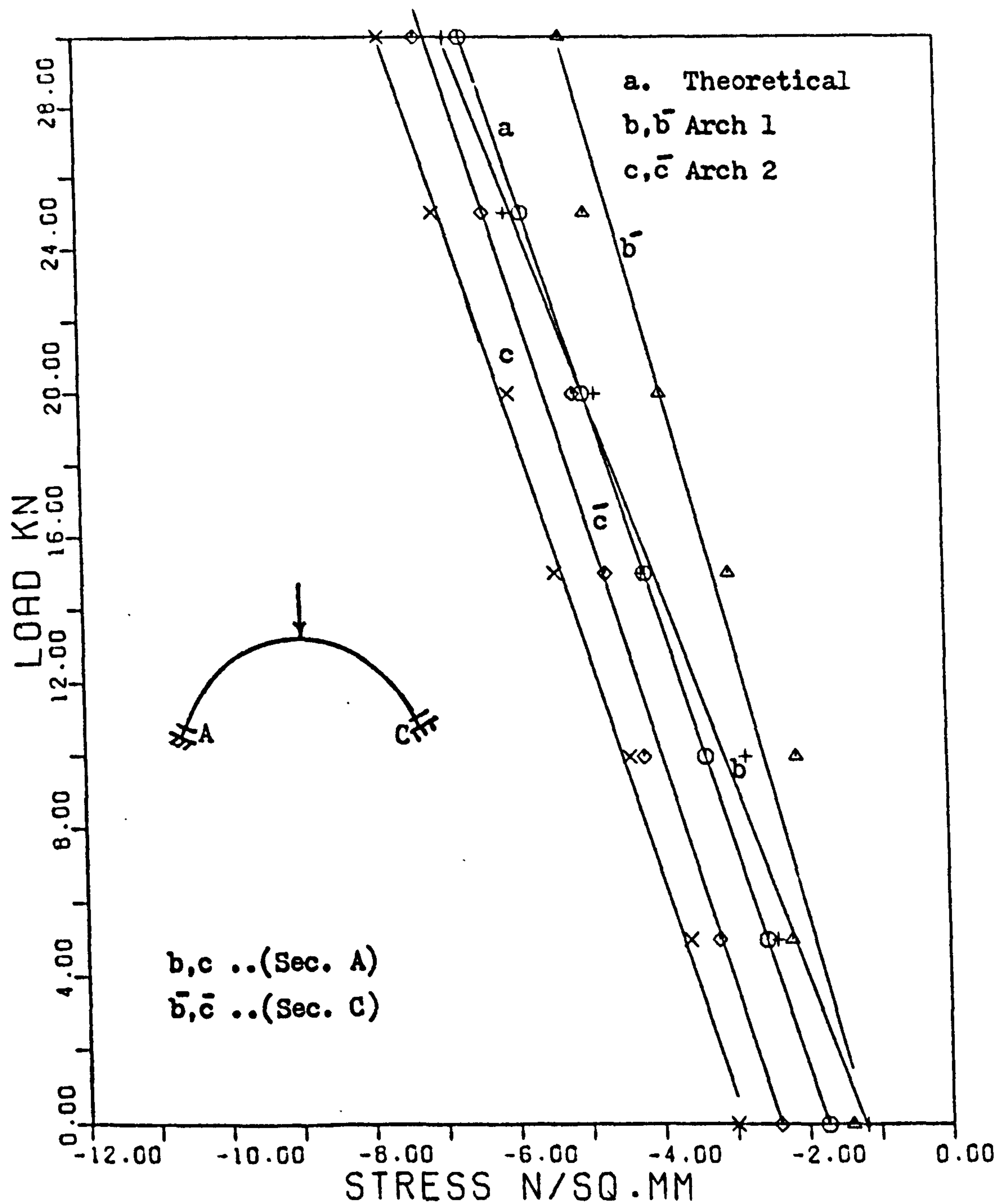


FIG.5.17 STRESSES IN THE TOP FIBRE AT THE SUPPORTS OF THE ARCHES (SEC. A&C)

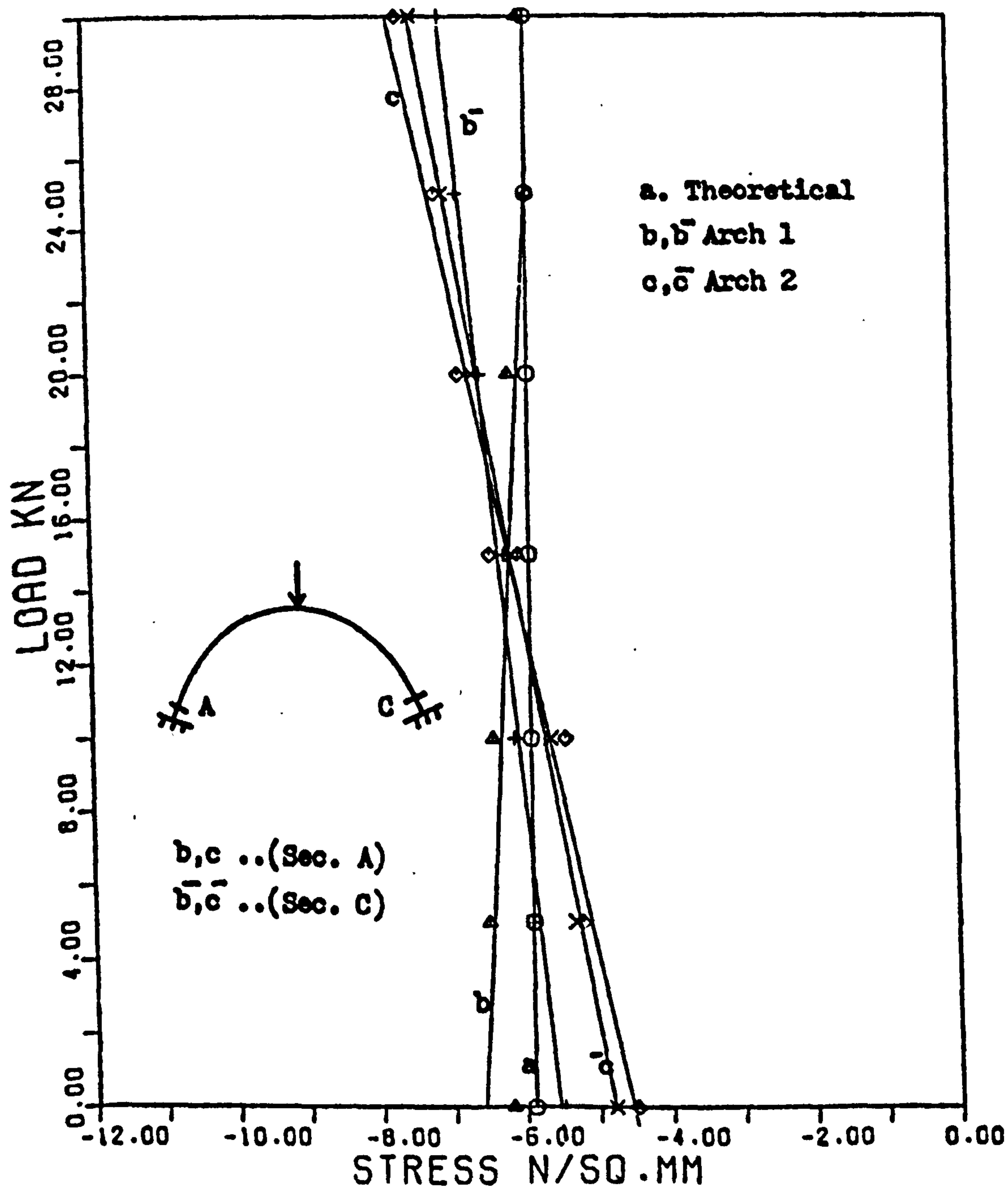


FIG.5.18 STRESSES IN THE BOTTOM FIBRE AT THE SUPPORTS OF THE ARCHES (SEC. A&C)

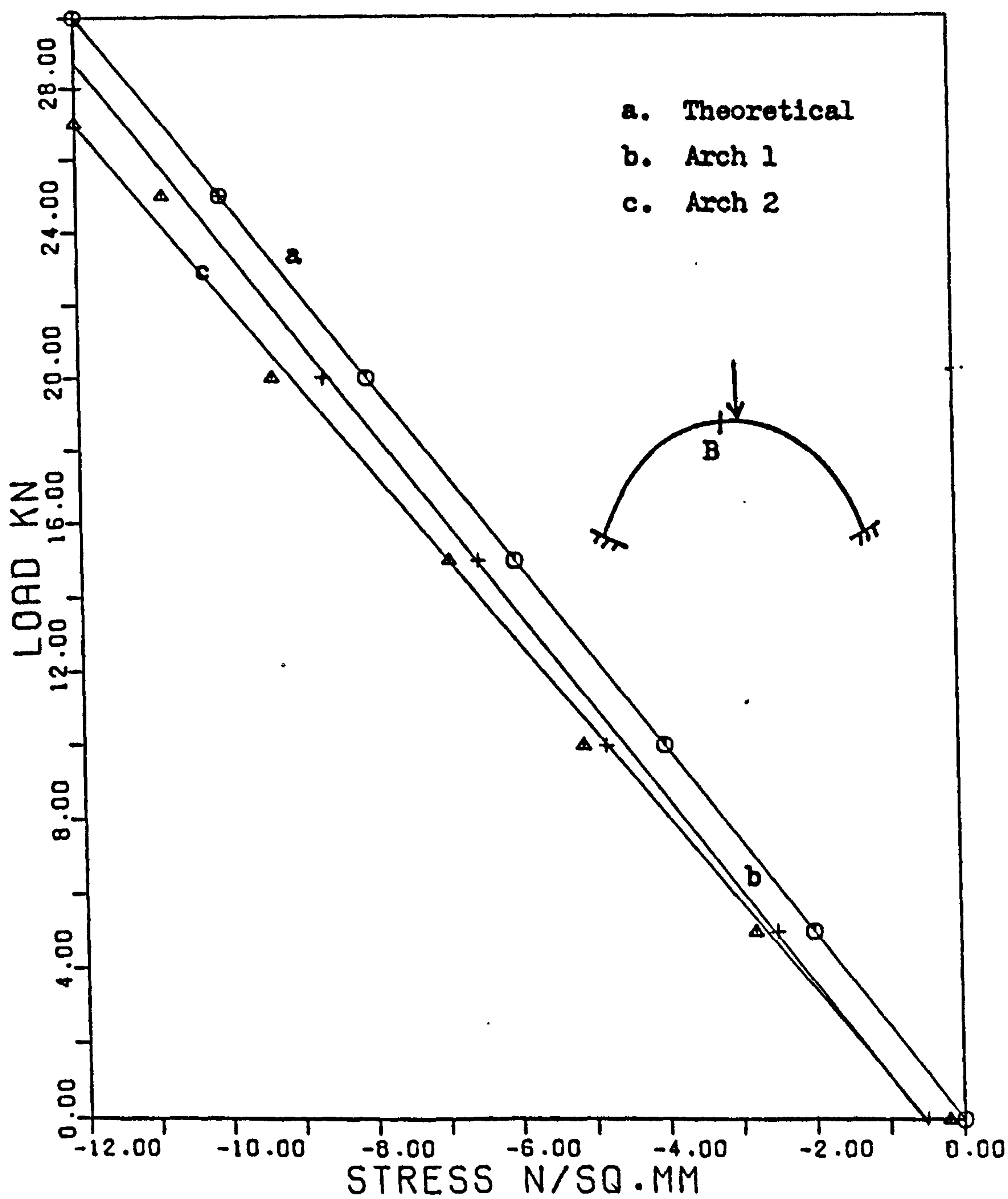


FIG.5.19 STRESSES IN THE TOP FIBRE UNDER THE APPLIED LOAD FOR THE ARCHES (SEC. B)

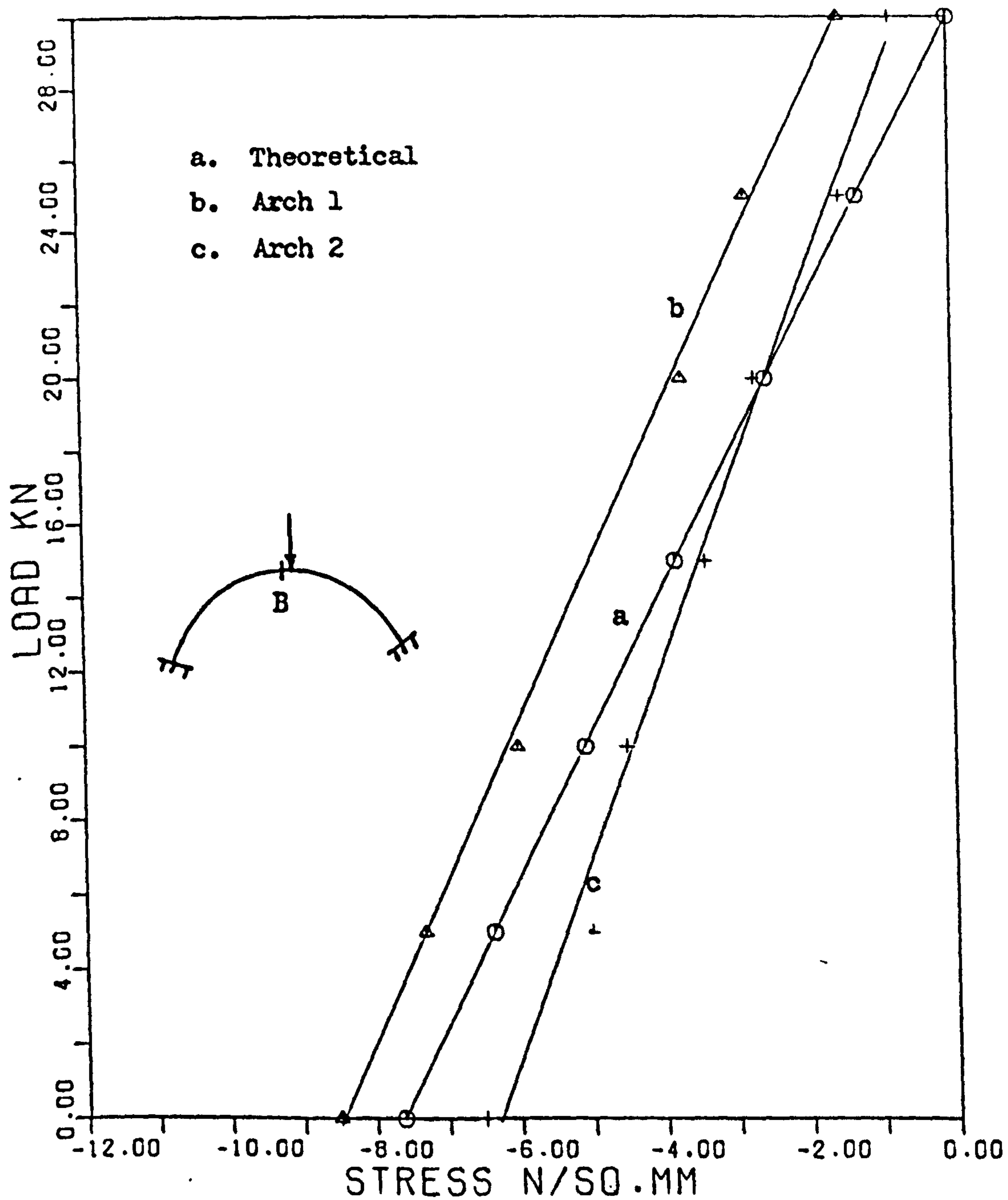


FIG.5.20 STRESSES IN THE BOTTOM FIBRE UNDER THE APPLIED LOAD FOR THE ARCHES (SEC. B)

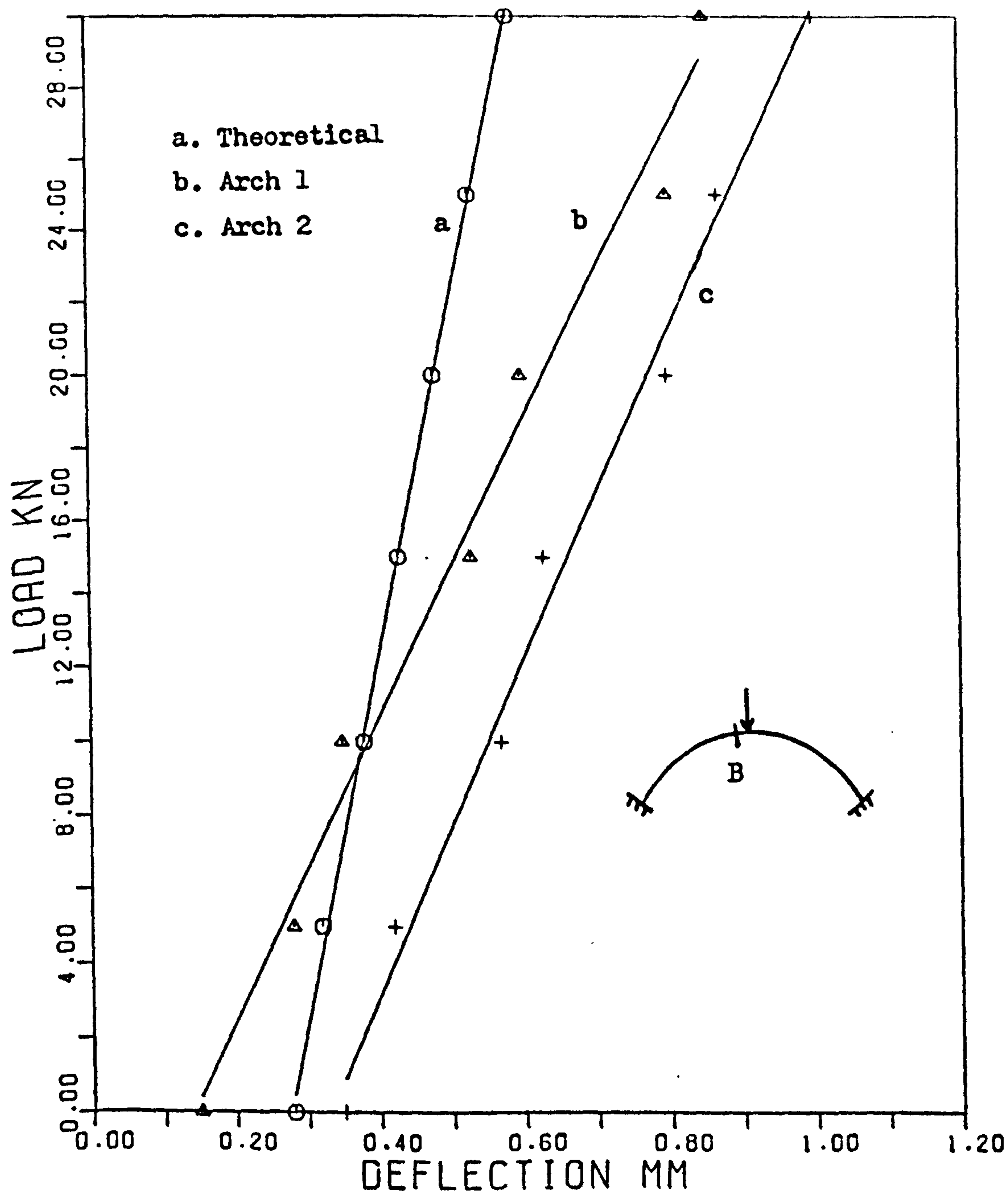


FIG.5.21 DEFLECTIONS AT MID SPAN POINT UNDER THE APPLIED LOAD FOR THE ARCHES (SEC. B)

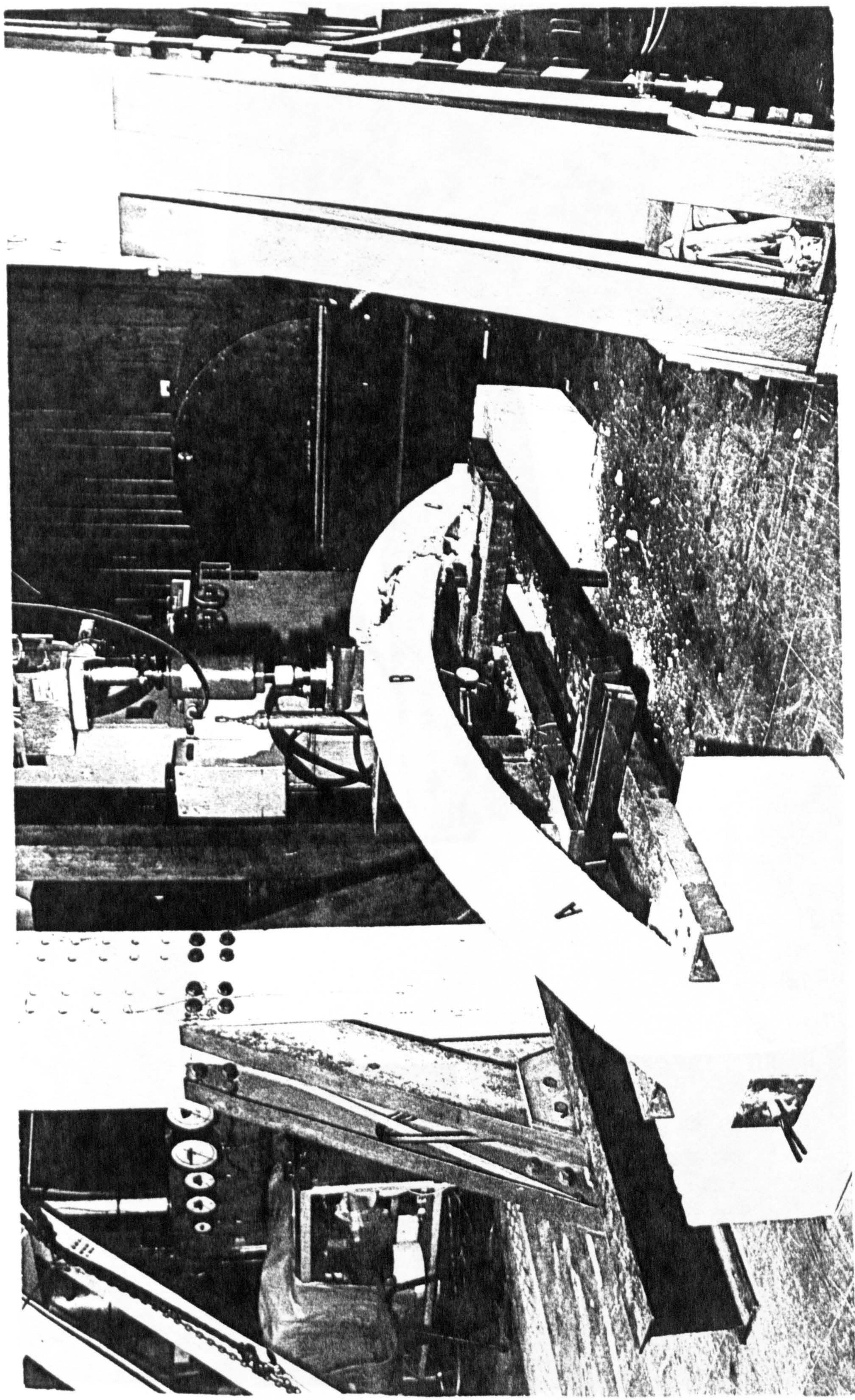


Plate No. 17 The arch after failure

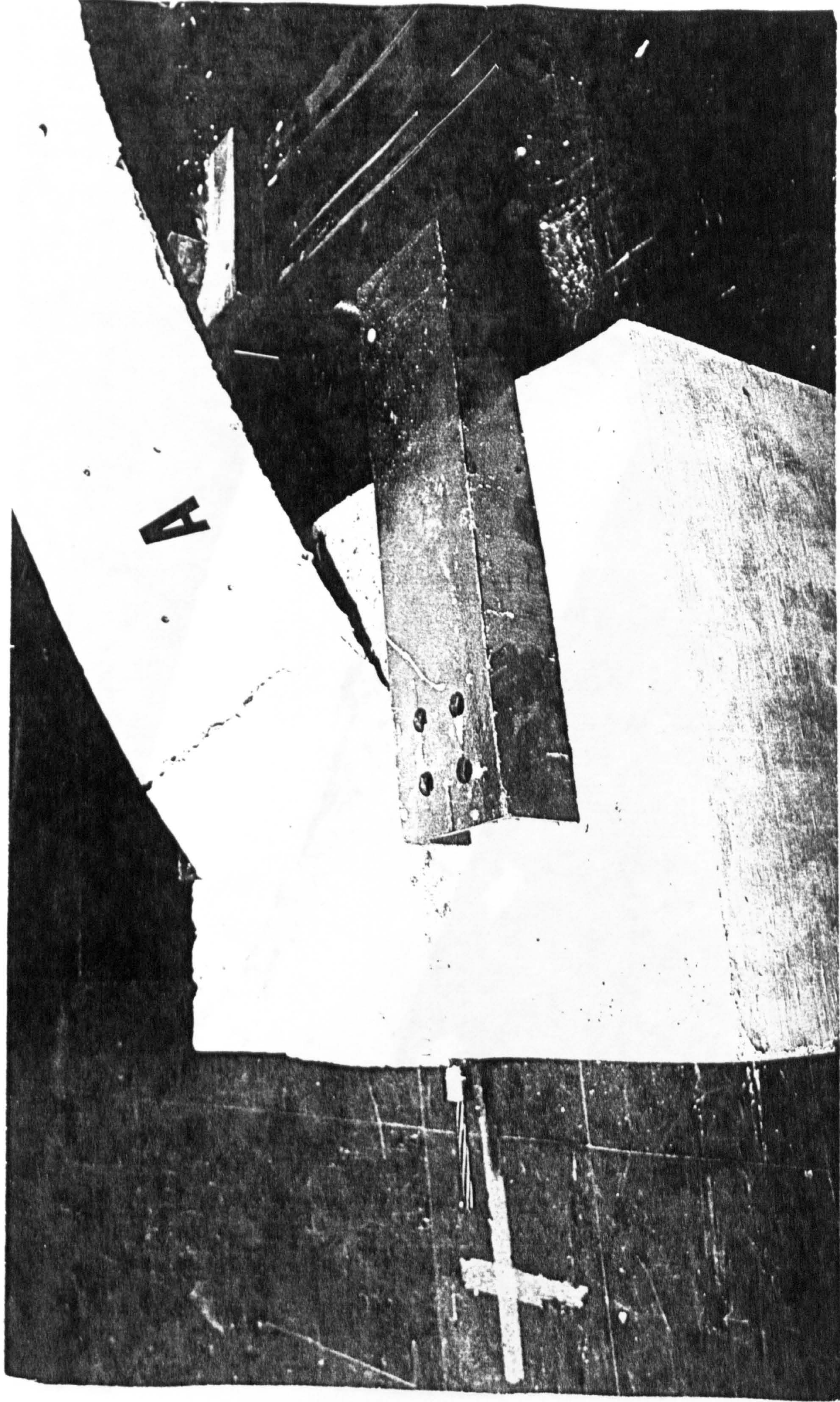


Plate No. 18 The section at the left support after failure

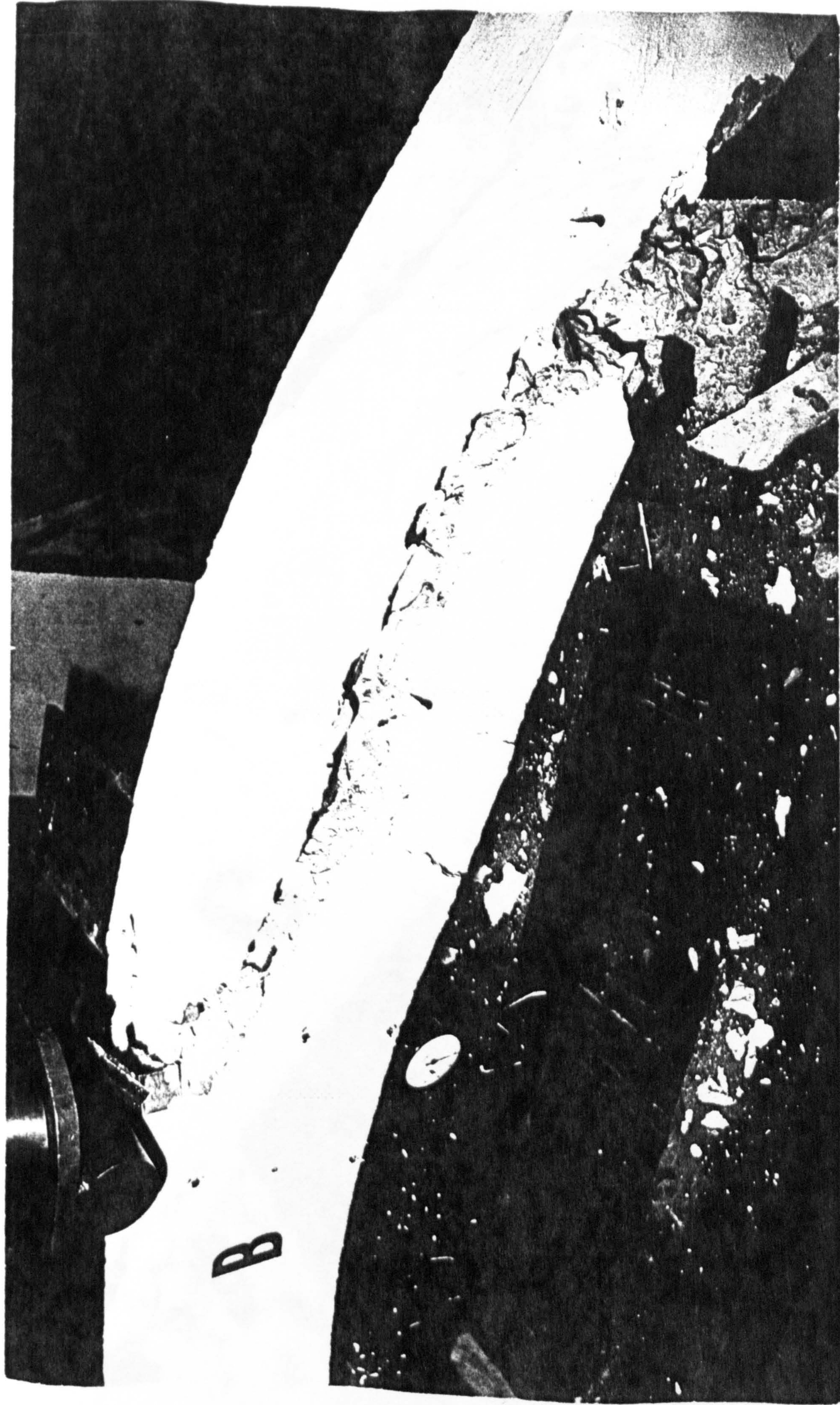


Plate No. 19 The section under applied load after failure

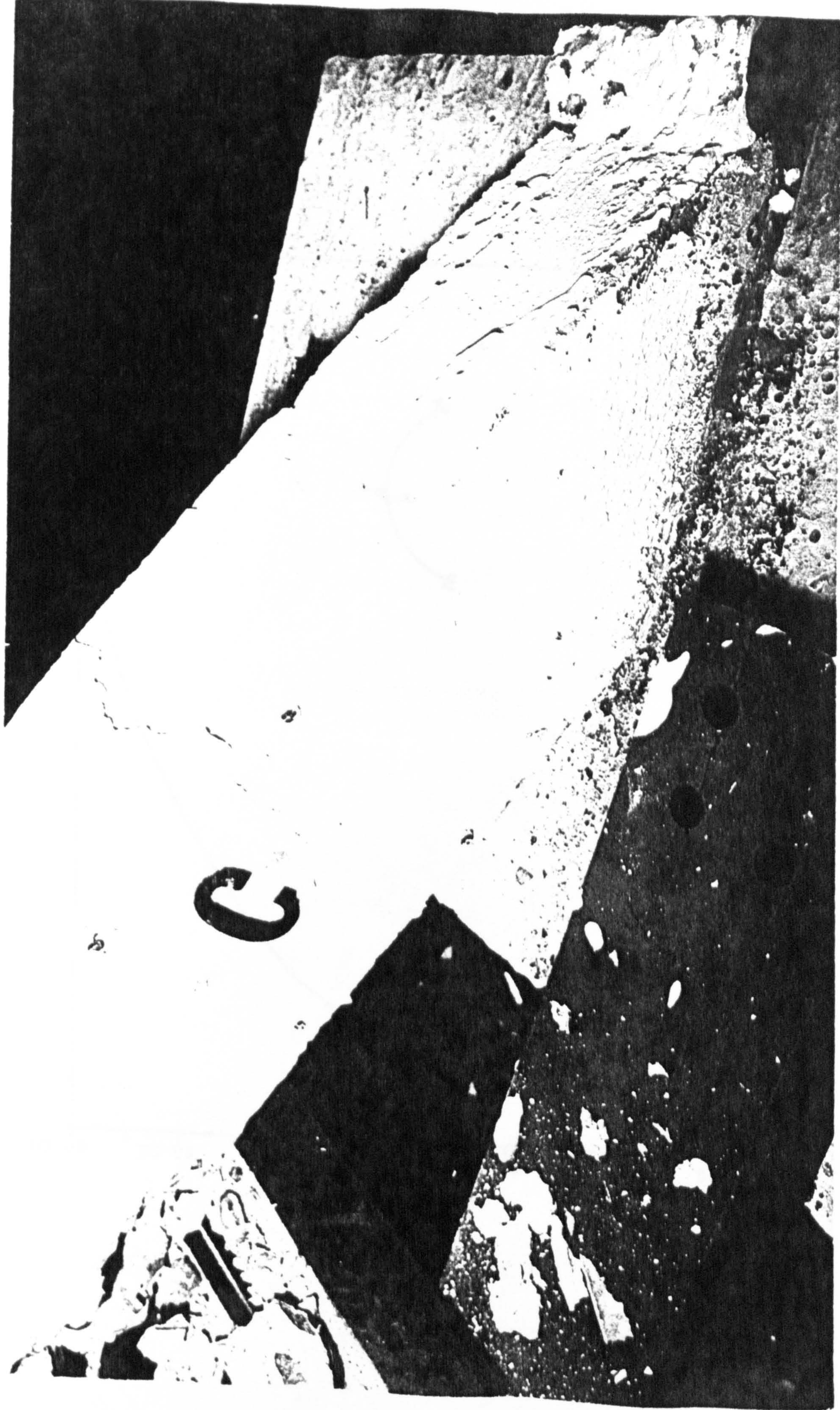


Plate No. 20 The section at the right support after failure

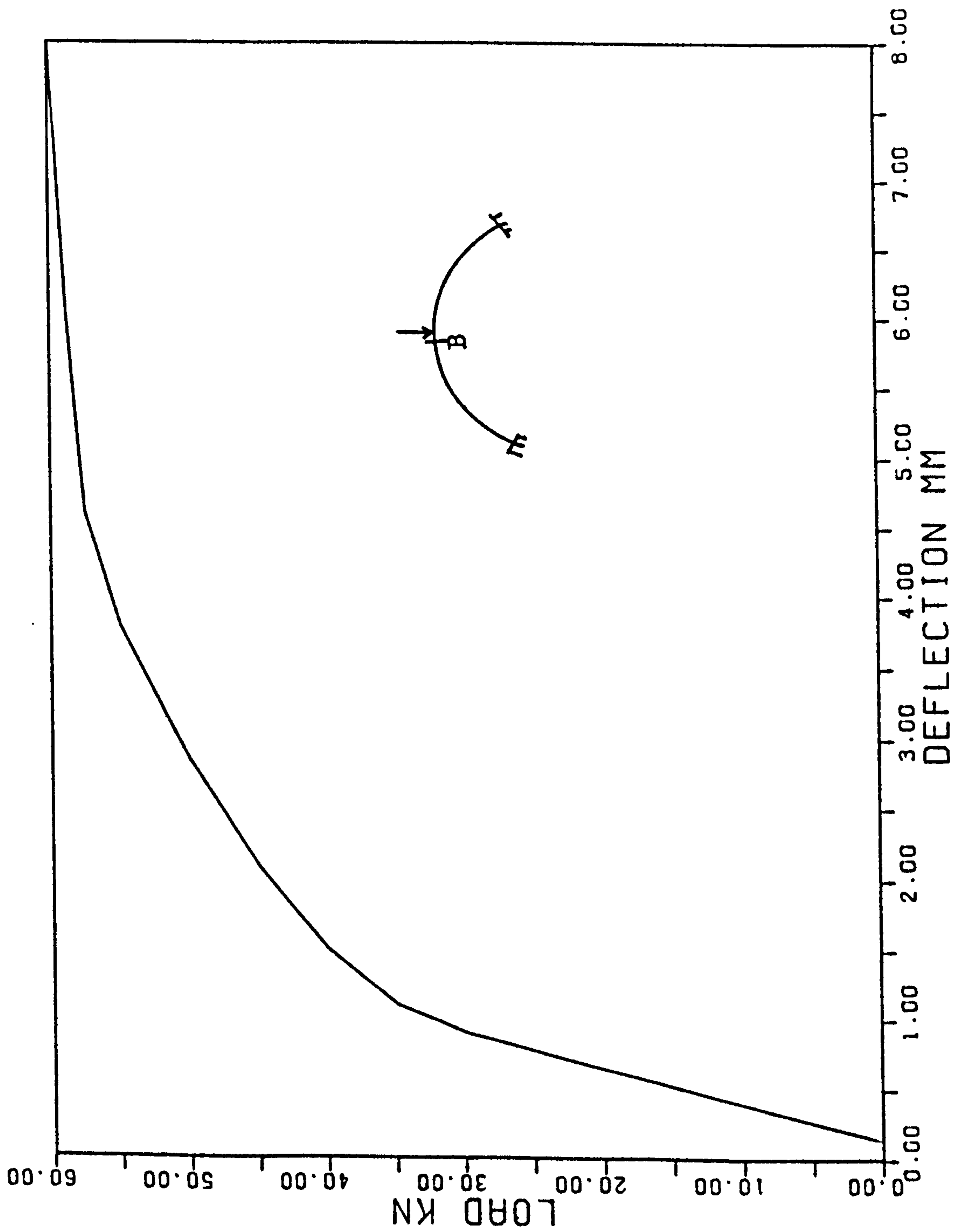


FIG.5.22 LOAD DEFLECTION CURVE FOR ARCH 1 (SEC. B)

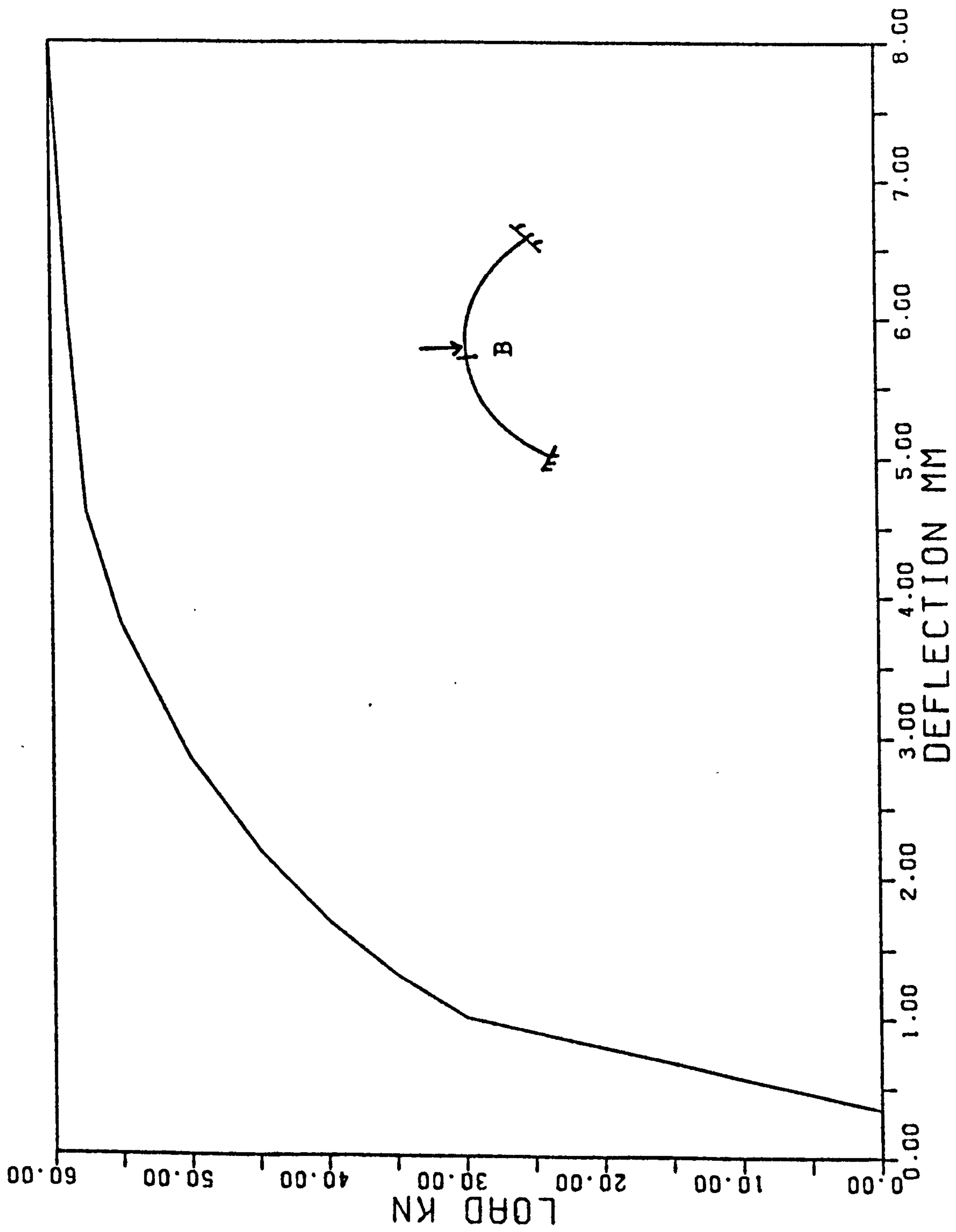


FIG.5.23 LOAD DEFLECTION CURVE FOR ARCH 2 (SEC. B)

PART III
CHAPTER (6)
COMPUTER PROGRAM

6.1 INTRODUCTION

The program consists of two parts; the first part computes the live load moment envelope and the second part computes the design steps discussed in Chapter (1) and Chapter (3).

The first part depends on the flexibility method the structure may be constant or Variable Inertia, and the program is appropriate for all types of loading. Also it computes the absolute resultant values. The second part gives the design of the prestressed concrete structures using the concordant profile idea, by the direct and quadratic programming solutions. The prestressing force can be variable or constant for the whole structure or constant per each span.

The program solves the second part using Gaus's elimination method, Quadratic programming procedure, Simplex method and is valid for beams, frames and arches.

The program contains Input data corrections. Minimum capacity is used in this program. Finally the program is iterated until the design output data coincides with the resultant stresses.

6.2 Program Description

The first part of the program can be outlined as follows:-

- (a) The final moment and direct forces for any cases of loading in which the structure is subjected may be expressed in matrix form:

$$[M] = [m_0] + [m] [x],$$

$$[N] = [n_0] + [n] [x] \quad \dots \quad 6.1$$

$$\text{where } [x] = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad [m] = [m_1, m_2, \dots, m_n]$$

$$\text{and } [n] = [n_1, n_2, \dots, n_n]$$

where $[x]$ is the indeterminate forces which can be established from compatability equations

$$[x] = - [F]^{-1} [u] \quad \dots \quad 6.2$$

whilst the principal of virtual forces leads to the form of the elements of the matrices as follows

$$\text{Element of } [F] \quad f_{ij} = \int_s \frac{m_i m_j}{EI} ds + \int_s \frac{n_i n_j}{EA} ds$$

$$\text{Element of } [u] \quad u_i = \int_s \frac{m_i m_0}{EI} ds + \int_s \frac{n_i n_0}{EA} ds$$

where $[F]$ is the flexibility matrix and it can be written as

$$[F] = \int_s [m]^T \left[\frac{1}{EI} \right] [m] ds + \int_s [n]^T \left[\frac{1}{EA} \right] [n] ds \dots 6.3$$

$$= \int_s \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix} \left[\frac{1}{EI} \right] [m_1 \ m_2 \ \dots \ m_n] ds + \int_s \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix} \left[\frac{1}{EA} \right] [n_1 \ n_2 \ \dots \ n_n] ds$$

and the same way the matrix of u values is

$$[u] = \int_s [m]^T \left[\frac{1}{EI} \right] [m_0] ds + \int_s [n]^T \left[\frac{1}{EA} \right] [n_0] ds \dots 6.4$$

$$= \int_s \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix} \left[\frac{1}{EI} \right] [m_0] ds + \int_s \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix} \left[\frac{1}{EA} \right] [n_0] ds$$

Thus, if we substitute (6.3) and 6.4) into (6.2) and again substitute the result into (6.1) we can obtain an expression of the form.

$$[M] = [m_0] - [m] \left(\int_s [m]^T \left[\frac{1}{EI} \right] [m] ds + \int_s [n]^T \left[\frac{1}{EA} \right] [n] ds \right)^{-1}$$

$$\left(\int_s [m]^T \left[\frac{1}{EI} \right] [m_0] ds + \int_s [n]^T \left[\frac{1}{EA} \right] [n_0] ds \right)$$

and

$$[N] = [n_0] - [n] \left(\int_s [m]^T \left[\frac{1}{EI} \right] [m] ds + \int_s [n]^T \left[\frac{1}{EA} \right] [n] ds \right)^{-1} \left(\int_s [m]^T \left[\frac{1}{EI} \right] [m_0] ds + \int_s [n]^T \left[\frac{1}{EA} \right] [n_0] ds \right) \dots 6.5$$

(b) For numerical integration each of these integral functions can be determined using Simpson's rule. So we shall have an odd number of stations per each span and hence the integrations 6.3 and 6.4 may be replaced by a summation of the form

$$\int_s m_i \frac{1}{EI} m_j ds + \int_s n_i \frac{1}{EA} n_j ds = \sum_{\alpha=1}^m m_{i(\alpha)} \cdot \frac{1}{EI(\alpha)} \cdot \frac{h(\alpha)}{3} \cdot \gamma(\alpha) m_{j(\alpha)} + \sum_{\alpha=1}^m n_{i(\alpha)} \frac{1}{EA(\alpha)} \cdot \frac{h(\alpha)}{3} \cdot \gamma(\alpha) n_{j(\alpha)} \dots 6.6$$

when the symbol α refers to the current section over the range $\alpha = 1$ to m which may count some sections twice if the interval width change, $h(\alpha)$ is the interval width and $\gamma(\alpha)$ is the appropriate Simpson co-efficient 1, 2 or 4.

For example for a twice times statically indeterminate structure the flexibility matrix will have matrices products of the form

$$[F] = \begin{bmatrix} m_1(1) & m_1(2) & \dots & m_1(m) & n_1(1) & n_1(2) & \dots & n_1(m) \\ m_2(1) & m_2(2) & \dots & m_2(m) & n_2(1) & n_2(2) & \dots & n_2(m) \end{bmatrix} \begin{bmatrix} EI(1) \\ EI(2) \\ \vdots \\ EI(m) \\ EA(1) \\ EA(2) \\ \vdots \\ EA(m) \end{bmatrix} \quad -1$$

$$\begin{bmatrix} h/3(1) \\ h/3(2) \\ \vdots \\ h/3(m) \\ h/3(1) \\ h/3(2) \\ \vdots \\ h/3(m) \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ \vdots \\ 1 \\ 1 \\ 4 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} m_1(1) & m_2(1) \\ m_1(2) & m_2(2) \\ \vdots & \vdots \\ m_1(m) & m_2(m) \\ n_1(1) & n_2(1) \\ n_1(2) & n_2(2) \\ \vdots & \vdots \\ n_1(m) & n_2(m) \end{bmatrix} \quad \dots \quad 6.7$$

(c) The "Analytical check", (the lack of indeterminacy), which comes from considering the solution [M] and [N] as a new "applied load" will be

$$[T] = \int_s [m] \left[\frac{1}{EI} \right] [M] ds + \int_s [n] \left[\frac{1}{EA} \right] [N] ds \quad \dots \quad 6.8$$

$$\begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ T_n \end{bmatrix} = \int_s \begin{bmatrix} m_1 \\ m_2 \\ \cdot \\ \cdot \\ m_n \end{bmatrix} \left[\frac{1}{EI} \right] [M] ds + \int_s \begin{bmatrix} n_1 \\ n_2 \\ \cdot \\ \cdot \\ n_n \end{bmatrix} \left[\frac{1}{EA} \right] [N] ds$$

The matrix $[T]$ should be equal to zero. For practical purposes the final stress resultants (using the final design Inertia) usually vary from the initial stress resultants (using relative Inertia) by an amount which is equal to

$$\Delta = [m] [F]^{-1} [T] \quad \dots \quad 6.9$$

The program calculates this error Δ and repeats continuously using the currently computed design Inertia until the error becomes infinitesimally small. This happens when the final design Inertia approximately is equal to the previously computed one.

The second part of the program computer the design steps, discussed in chapter (1) and chapter (3) can be outlined as follows:-

- (a) The absolute section sizes can be chosen on the basis of reaching the maximum allowable compressive stresses given by the following equations:

$$\text{IF } [H_1] > [H_2]$$

$$[M_L] = (1/6)fc \quad b \quad [D^2] + (1/6)([H_2] - [H_1]) [D]$$

$$\text{IF } [H_2] > [H_1]$$

$$[M_L] = (1/6)fc \quad b \quad [D^2] + (1/6)([H_1] - [H_2]) [D] \quad \dots \quad 6.10$$

$$\text{where } [H_1] = \begin{bmatrix} H_{1(1)} \\ H_{1(2)} \\ \cdot \\ \cdot \\ H_{1(m)} \end{bmatrix}, \quad [H_2] = \begin{bmatrix} H_{2(1)} \\ H_{2(2)} \\ \cdot \\ \cdot \\ H_{2(m)} \end{bmatrix}$$

$$[M_L] = \begin{bmatrix} M_{L(1)} \\ M_{L(2)} \\ \cdot \\ \cdot \\ M_{L(m)} \end{bmatrix} \quad \text{and} \quad [D] = \begin{bmatrix} D_1(1) \\ D_1(2) \\ \cdot \\ \cdot \\ D_1(m) \end{bmatrix}$$

(b) The program computes the dead load moment $[M_D]$ and the direct forces $[H_D]$ using the first part after computing the section sizes.

(c) The prestressing force $[H_p]$ may be either:

- i) variable along the length of the structure
- ii) constant for the whole structure
- iii) constant per each span

and can be determined from the equations

$$\text{IF } [H_1] > [H_2]$$

$$[H_p] = \frac{1}{2} f_c b [D] - [H_D] - [H_1]$$

$$\text{IF } [H_2] > [H_1]$$

$$[H_p] = \frac{1}{2} f_c b [D] - [H_D] - H_2 \quad \dots \quad 6.11$$

where

$$[H_p] = \begin{bmatrix} H_{p(1)} \\ H_{p(2)} \\ \cdot \\ \cdot \\ H_{p(m)} \end{bmatrix} \quad \text{and} \quad [H_D] = \begin{bmatrix} H_{D[1]} \\ H_{D[2]} \\ \cdot \\ \cdot \\ H_{D(m)} \end{bmatrix}$$

The program computes the prestressing force per each section and calculates the maximum force if it is required to be constant along the length of the structure; or the maximum force per each span if the prestressing force required to be constant per each span.

(d) The program computes the limiting zone from the expressions:-

$$\text{the lower limit: } [e_A] = \frac{[D]}{6} \left(\frac{[H_A]}{[H_p]} \right) - \frac{[M_D] + [M_1]}{[H_p]}$$

$$\text{the upper limit: } [e_B] = - \frac{[D]}{6} \left(\frac{[H_B]}{[H_p]} \right) - \frac{[M_D] + [M_2]}{[H_p]} \quad \dots \quad 6.12$$

$$\text{where } [e_A] = \begin{bmatrix} e_{A(1)} \\ e_{A(2)} \\ \cdot \\ \cdot \\ e_{A(m)} \end{bmatrix}, \quad [e_B] = \begin{bmatrix} e_{B(1)} \\ e_{B(2)} \\ \cdot \\ \cdot \\ e_{B(m)} \end{bmatrix}$$

$$, \quad [H_A] = \begin{bmatrix} H_{A(1)} \\ H_{A(2)} \\ \cdot \\ \cdot \\ H_{A(m)} \end{bmatrix} = [H_1] + [H_D] + [H_p]$$

$$, \quad [H_B] = \begin{bmatrix} H_{B(1)} \\ H_{B(2)} \\ \cdot \\ \cdot \\ H_{B(m)} \end{bmatrix} = [H_2] + [H_D] + [H_p]$$

$$\text{and } [M_D] = \begin{bmatrix} M_{D(1)} \\ M_{D(2)} \\ \cdot \\ \cdot \\ M_{D(m)} \end{bmatrix}$$

(e) The program computes the concordant profile by the Direct solution given from the equation

$$[e]_i = [e_B]_i + ([e_A]_i - [e_B]_i) * X \text{ where}$$

$$X = \frac{\sum_{i=1}^m [H_p]_i [e_B]_i [ET]_i \frac{1}{3} [h]_i [\lambda]_i [m]_i + \sum_{i=1}^m [H_p]_i [EA]_i \frac{1}{3} [h]_i [\lambda]_i [n]_i}{\sum_{i=1}^m [H_p]_i ([e_A]_i - [e_B]_i) [ET]_i \frac{1}{3} [h]_i [\lambda]_i [m]_i + \sum_{i=1}^m [H_p]_i [EA]_i \frac{1}{3} [h]_i [\lambda]_i [n]_i} \quad \dots \quad 6.13$$

$$\text{where } [e]_i = \begin{bmatrix} e_{(1)} \\ e_{(2)} \\ \vdots \\ e_{(m)} \end{bmatrix}, \quad [e_A]_i = \begin{bmatrix} e_{A(1)} \\ e_{A(2)} \\ \vdots \\ e_{A(m)} \end{bmatrix}, \quad [e_B]_i = \begin{bmatrix} e_{B(1)} \\ e_{B(2)} \\ \vdots \\ e_{B(m)} \end{bmatrix}$$

(f) The program also computes the condordant profile using the quadratic programming solution as follows:

(i) By using Gausiane elimination the equation

Maximize

$$F = c x - \frac{1}{2} x^T d x \quad \dots \quad 6.14$$

can be derived from the equations

$$\sum_{i=1}^m [H_p]_i ([e_A]_i - [e_B]_i) [x]_i + [e_B]_i [ET]_i \frac{1}{3} [h]_i [\lambda]_i [m]_i + \sum_{i=1}^m [H_p]_i [EA]_i \frac{1}{3} [h]_i [\lambda]_i [n]_i = 0$$

$$\text{and maximizing } F = - \sum_{i=1}^m [x^2]_i + \sum_{i=1}^m [x]_i$$

subject to $1 \leq [x] \leq 0$

where $[C] = [C_1, C_2, \dots, C_m]$

$$\text{and } [D] = \begin{bmatrix} d_{11} & d_{1m} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ d_{m1} & d_{mn} \end{bmatrix}$$

- (i) With the use of quadratic programming the necessary conditions may be combined as

$$\left| \begin{array}{c|c|c|c|c} -D & -I & -I & -I & 0 \\ \hline I & 0 & 0 & 0 & I \end{array} \right| \begin{array}{c} x \\ \lambda \\ v \\ z \\ s \end{array} = \begin{array}{c} C^T \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \quad \dots 6.15$$

The program computed the matrix $[D]$ and $[C]$ and rearranges the equations to be solved by the simplex method.

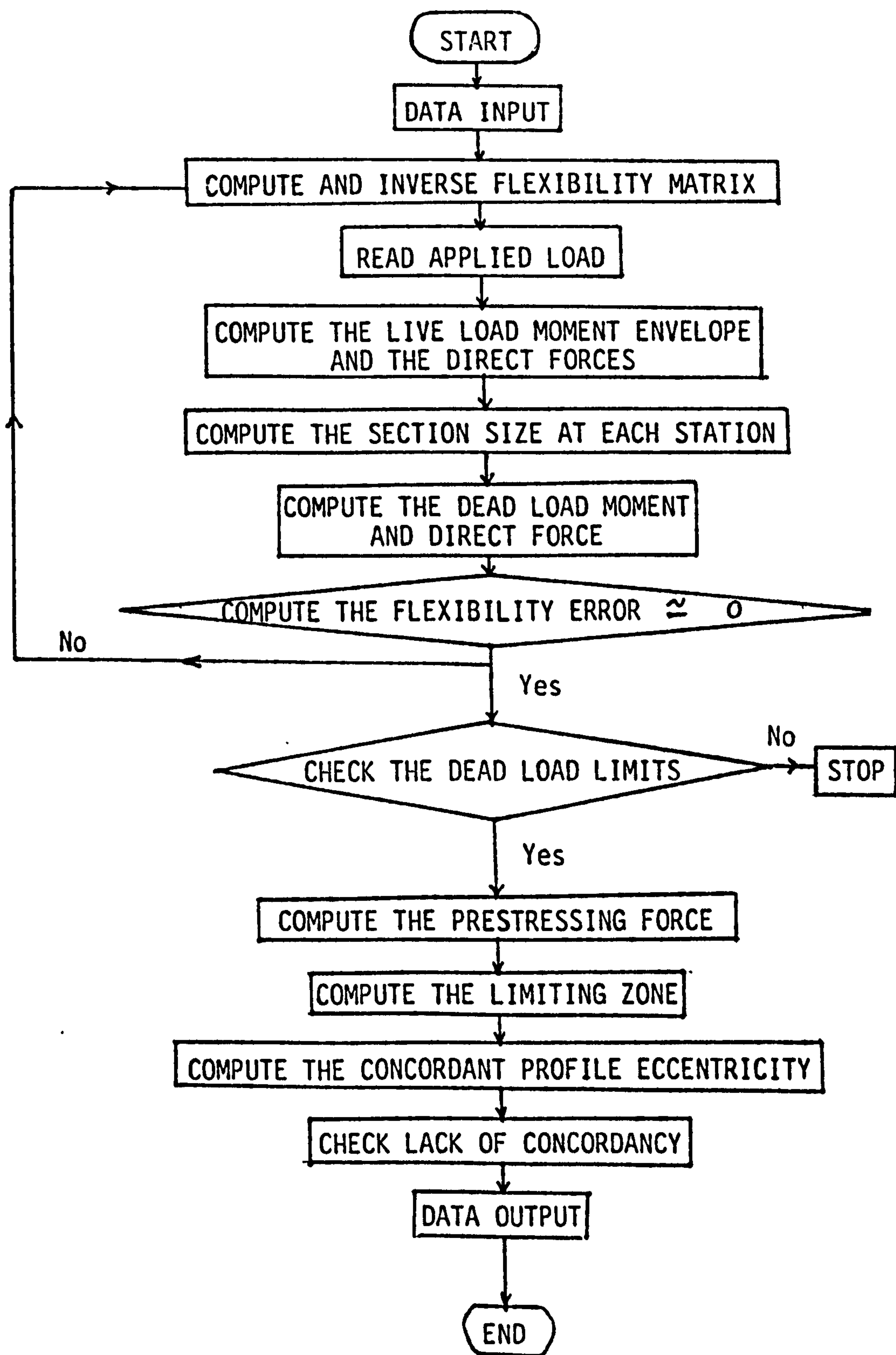
- (ii) The program follows the steps in the table (6.1) and finds the deviation around the centroidal axis of the limiting zone $[x]$; and the concordant profile eccentricity

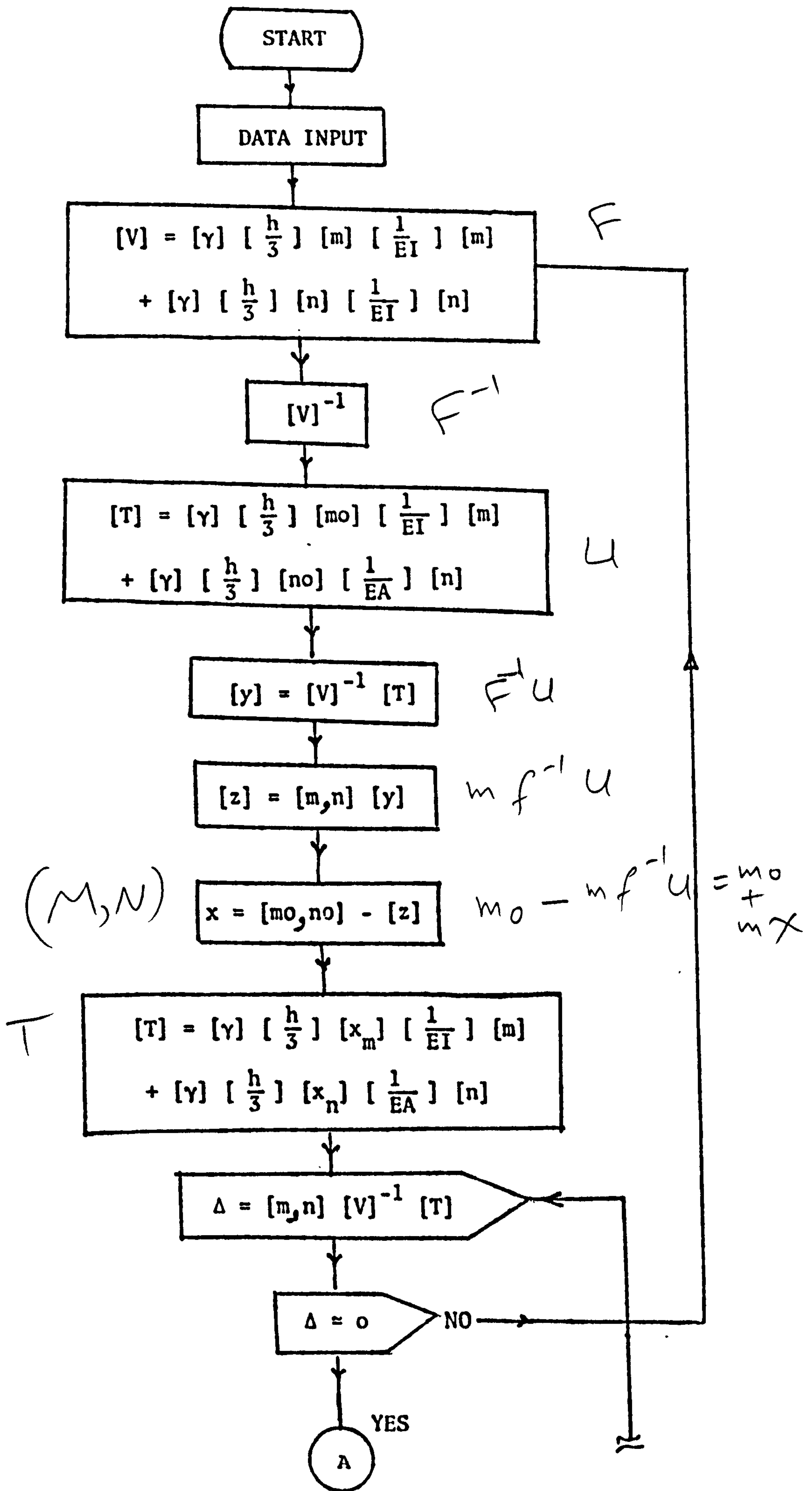
$$[e]_i = ([e_A]_i - [e_B]_i) [x]_i + [e_B]_i \quad \dots 6.16$$

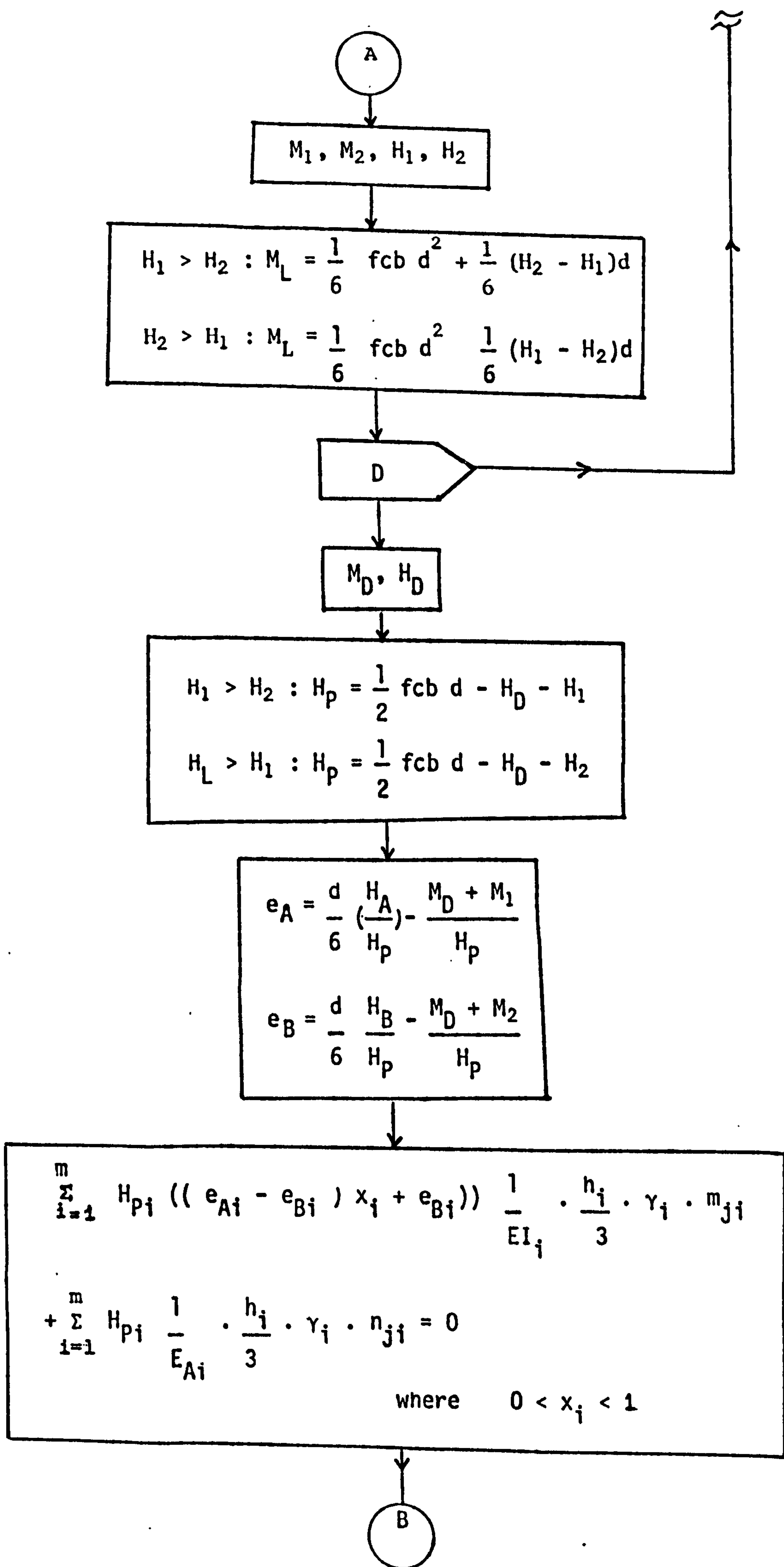
- (g) And finally the computer computes the lack of concordancy

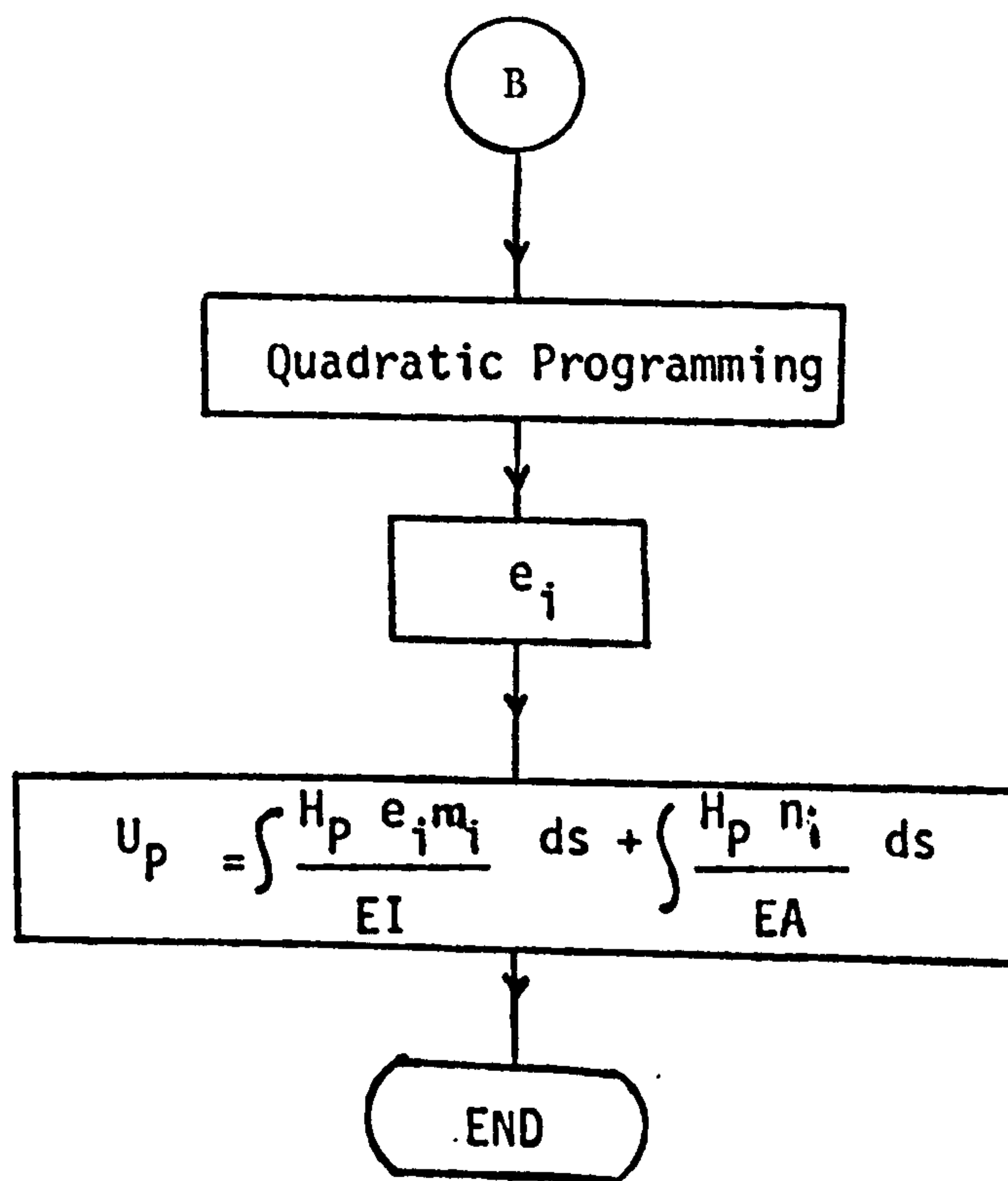
$$U_p = \int_s \frac{H_p e_i m_j}{EI} ds + \int_s \frac{H_p n_j}{EA} ds$$

6.3 Flowchart





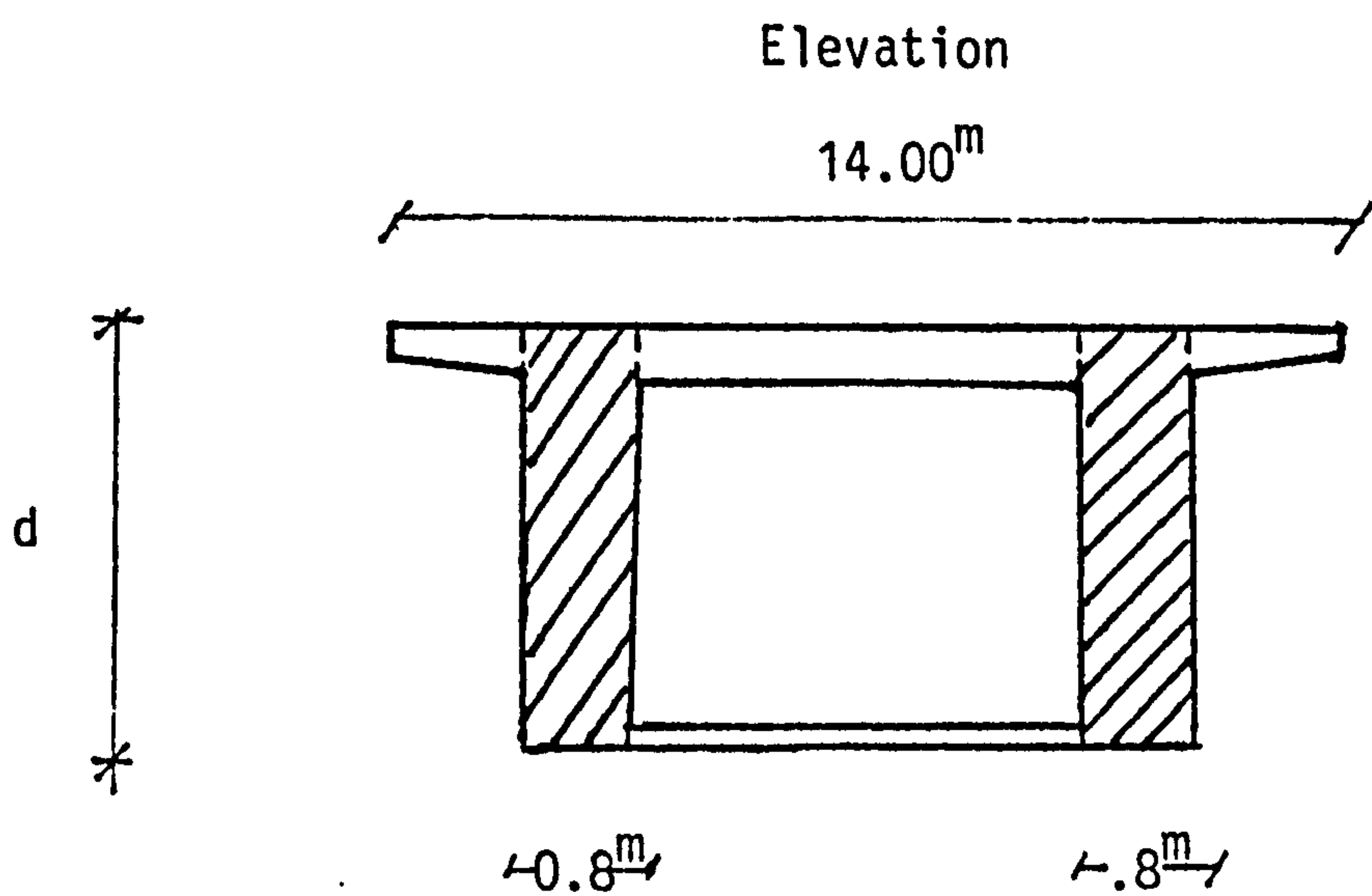
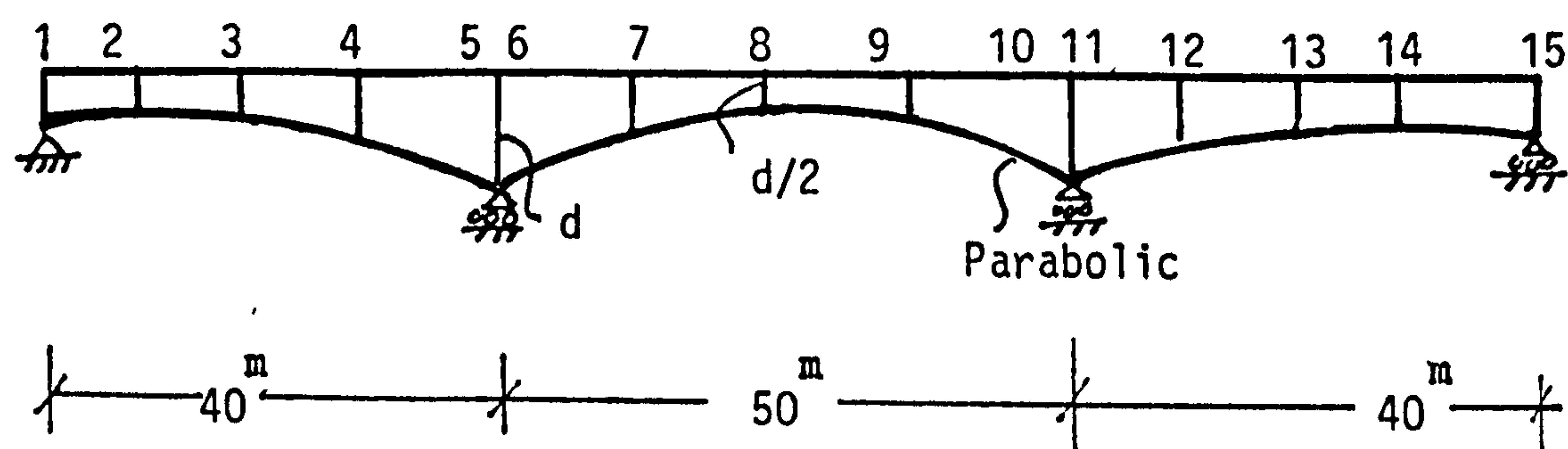




6.4 NUMERICAL EXAMPLES

EXAMPLE 1

As an example of the case where the prestressing force is variable per each section, consider a symmetrical continuous bridge (Fig.6.1). Let the length of the spans be 40,50 and 40 metres, and the width constant and taken as 0.8m. Each span is divided into 5 stations, the depth and the second moments of area for each station being given in Table (6.1) for half the symmetrical structure.



Section A - A

Fig. 6.1

TABLE (6.1)

VARIATION OF BEAM SECTION PROPERTIES

Section	1	2	3	4	5	6	7	8
Depth	.5d	.531d	.625d	.78d	d	d	.625d	.5d
I	.0104d ³	0.0125d ³	.0203d ³	.395d ³	0.1833d ³	0.0833d ³	0.0203d ³	0.0104d ³

It will be convenient to take $E = 1$. The loading on the structure was taken as a uniformly distributed load of $.7\text{t/m}^2$ and the dead load was taken as the weight of the slabs which gives a uniformly distributed load of 5.25 t/m . The o.w of the girder was taken as a function of the width and the depth of the girders. Fig. (6.2). The working stress limits were taken to be 15 N/mm^2 compression and 0 N/mm^2 tension.

Cases of loading

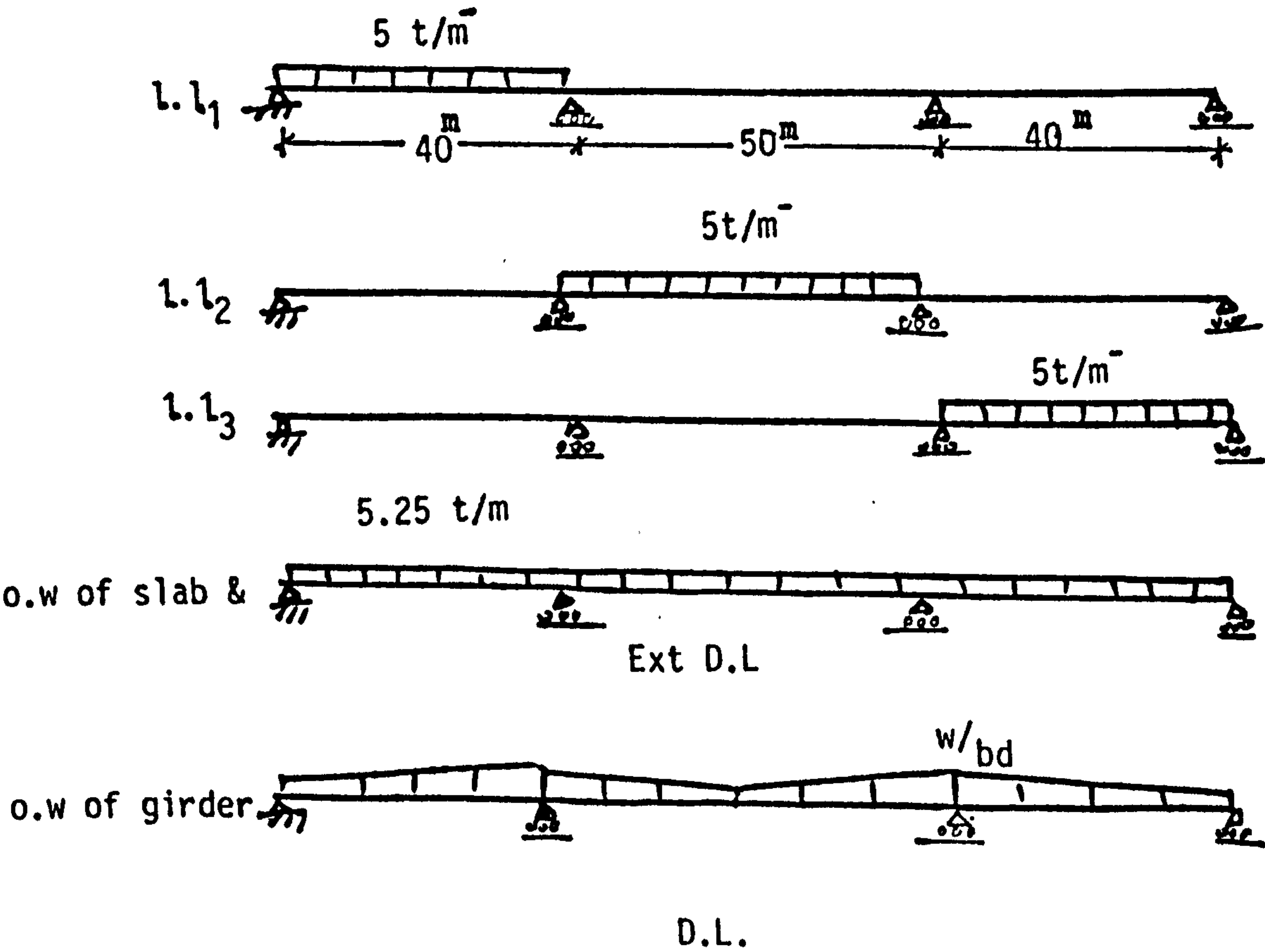
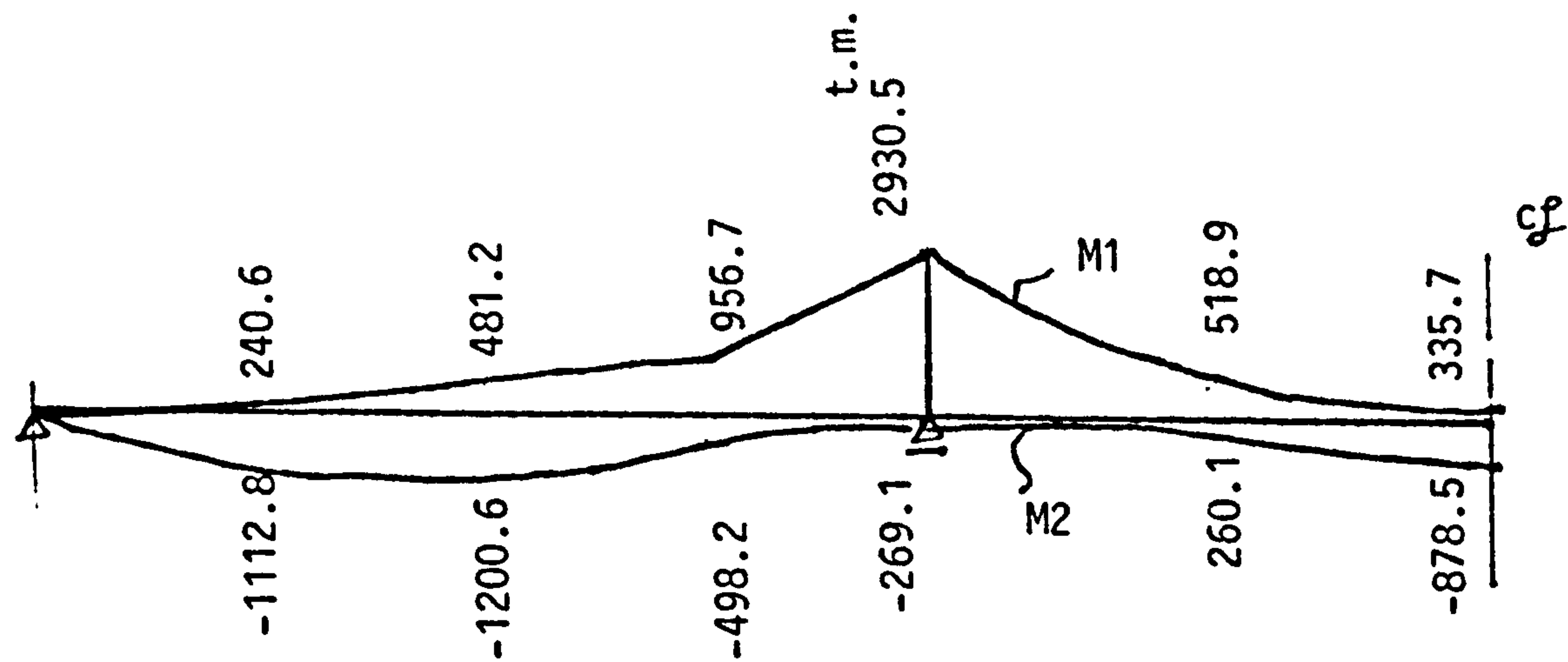
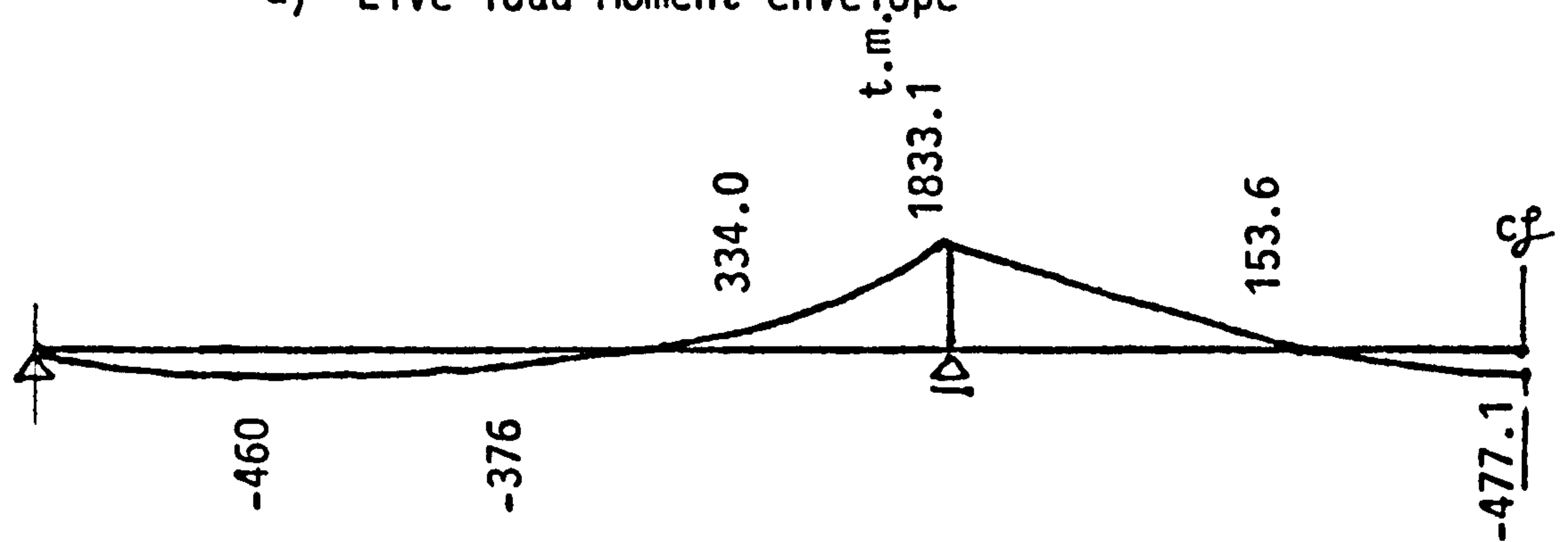


Fig. 6.2

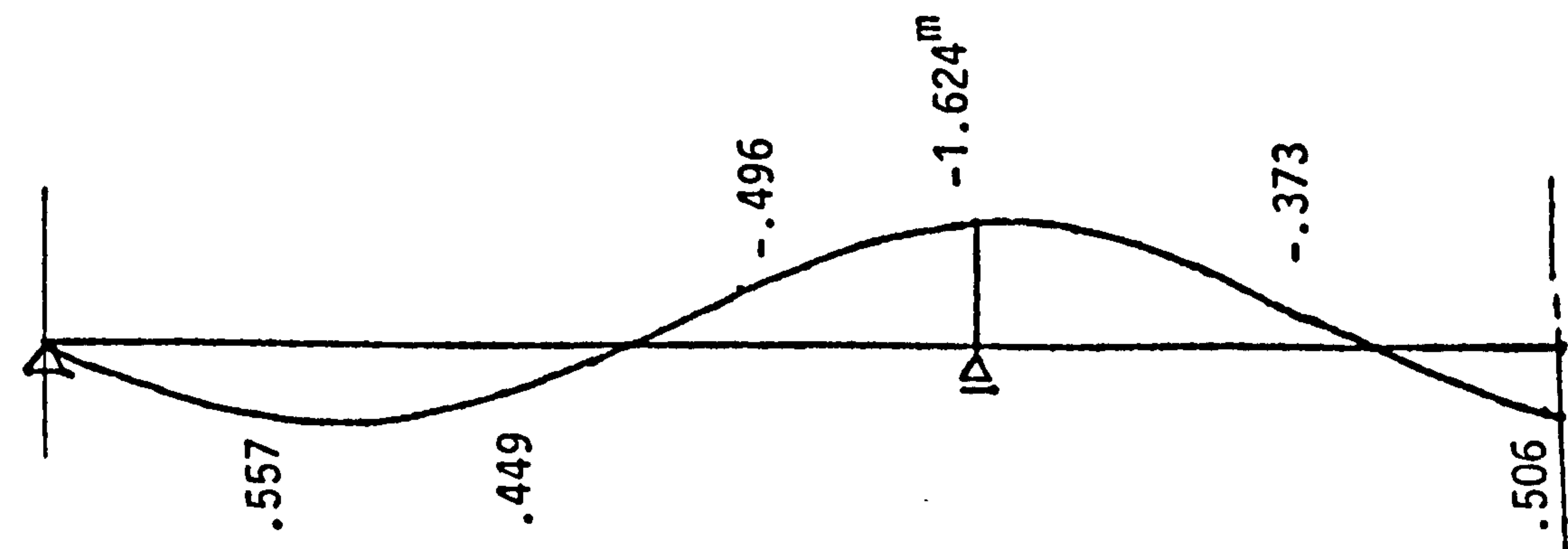
The results were given by the computer program and shown in Figs. 6.3, 6.4 and Table 6.2. The maximum required depth is 4.93m.



a) Live load Moment envelope



b) Dead load Moment



c) The Limiting zone

Fig. 6.3

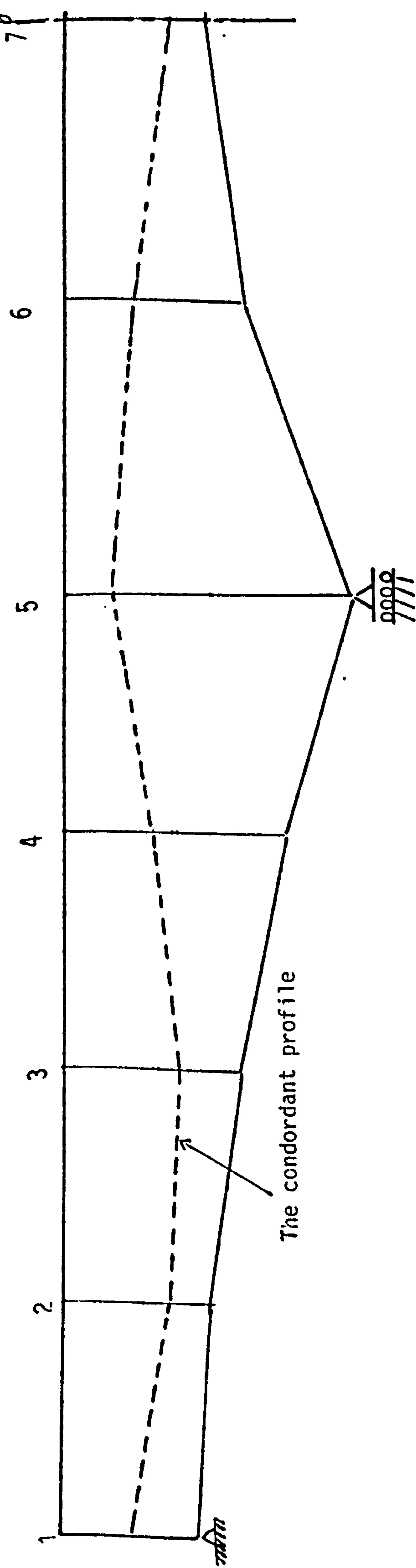
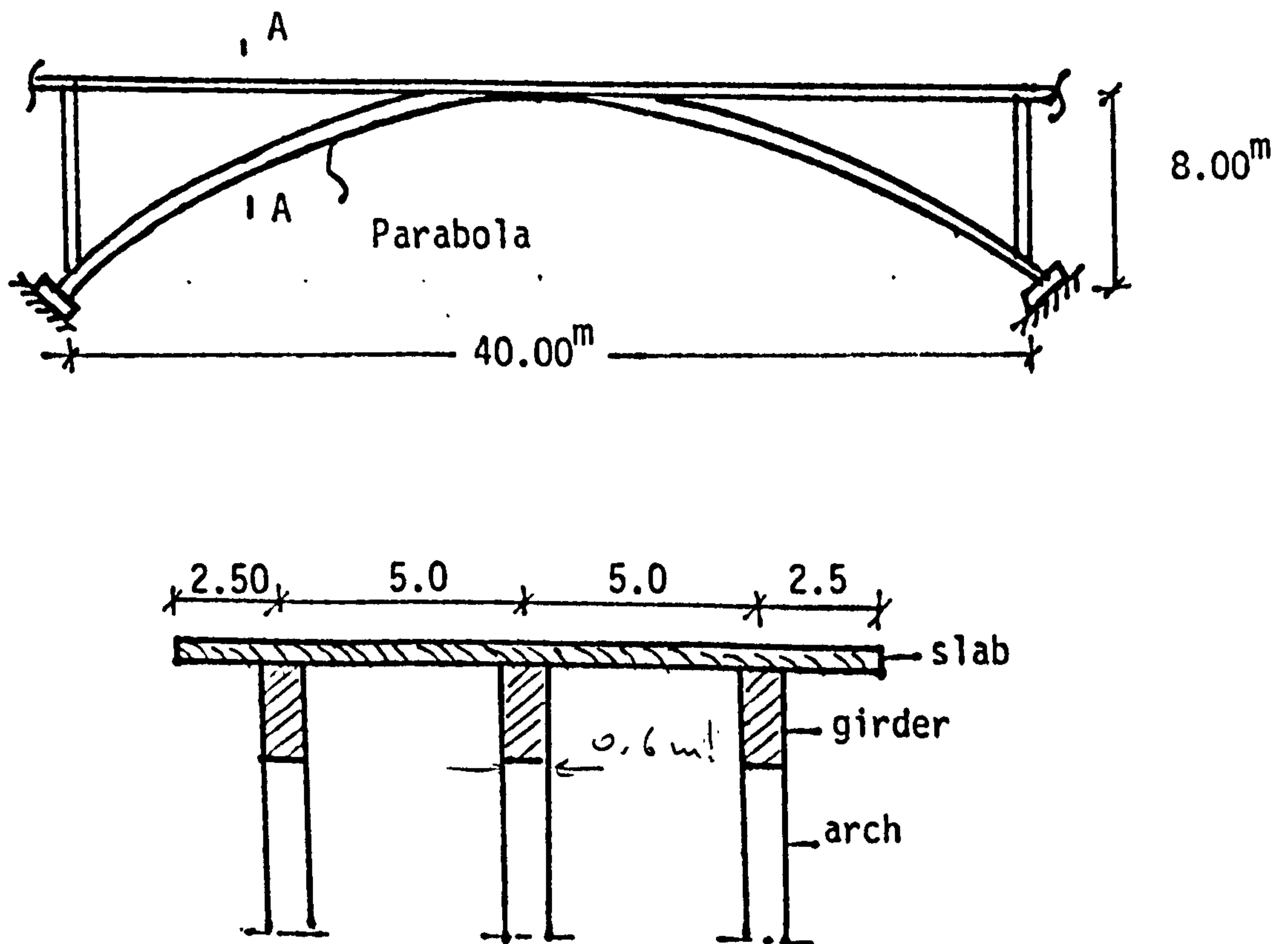


Fig. 6.4

Station N ^o	1	2	3	4	5	6	7
the depth (m)	2.464	2.617	3.08	3.843	4.928	3.08	2.464
the prestressing force (ton)	-	1551.6	1638.1	1135.5	1947.9	758.8	1478.4
the eccentricity of the cables measured from the centroid of the Section (m.)	-	-.577	.449	-.496	-1.624	-.373	0.506

EXAMPLE 2

As an example of Fixed Arch, consider an arch bridge of span 40^m and height 8^m carrying a continuous bridge of spans 20^m with a width taken as 0.6^m . The arch span is divided into 9 stations and each span of the girder divided into 5 stations. The depth is constant for each element of the structure, and the width is constant along the structure.



Section A - A

Fig. 6.5

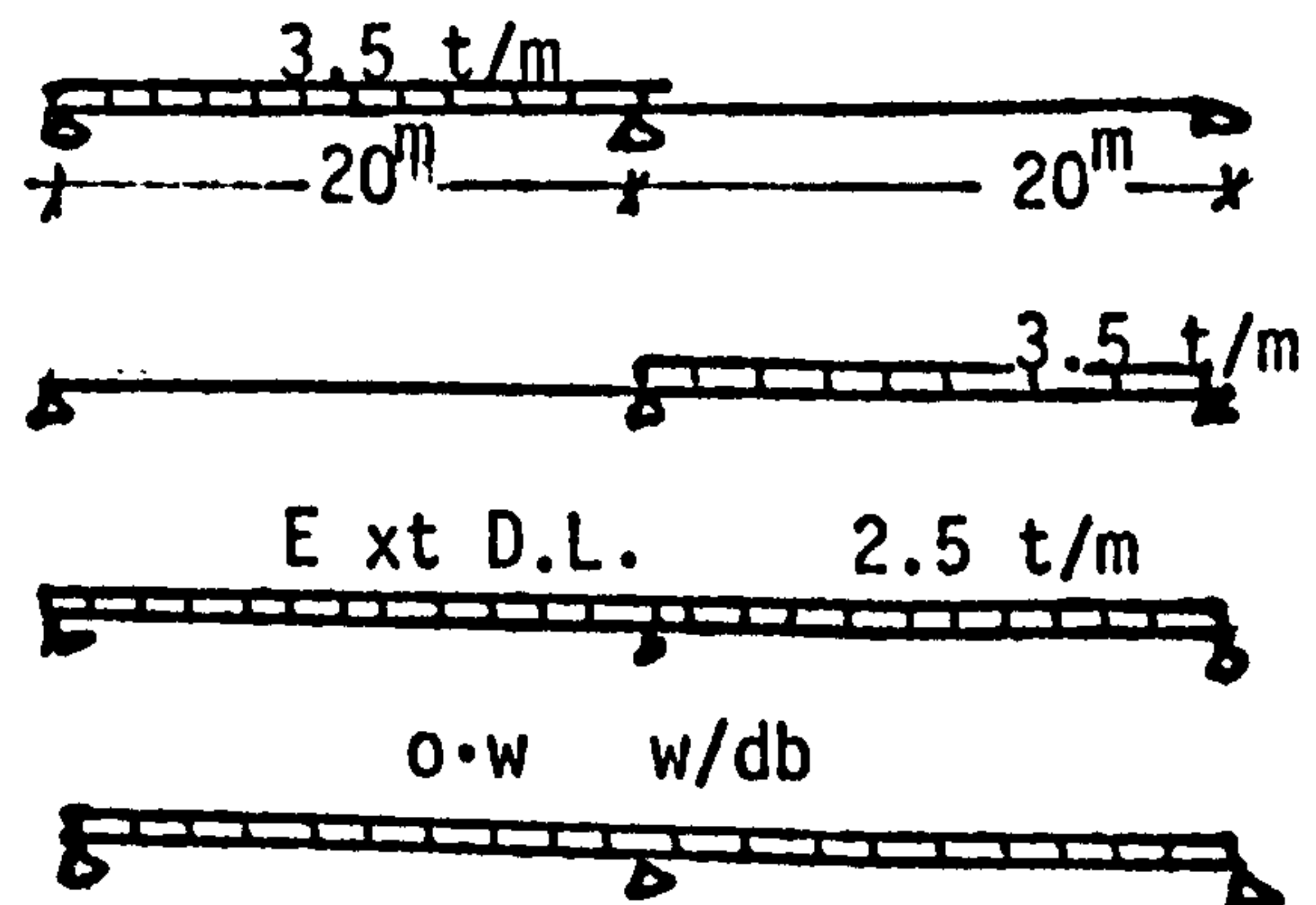
The loading on the structure was taken as a uniformly distributed load of $.7 \text{ t/m}^2$ and the dead load was taken as the weight of the slab which gives a uniformly distributed load of 2.5 t/m^2 on the girders. The o.w of the girders and the arch were taken as a function of the depth and the width. The working stress limits were taken to be 15 N/mm^2 compression and 0 N/mm^2 tension.

Case of loading

Girders

L.L₁

L.L₂



Arch

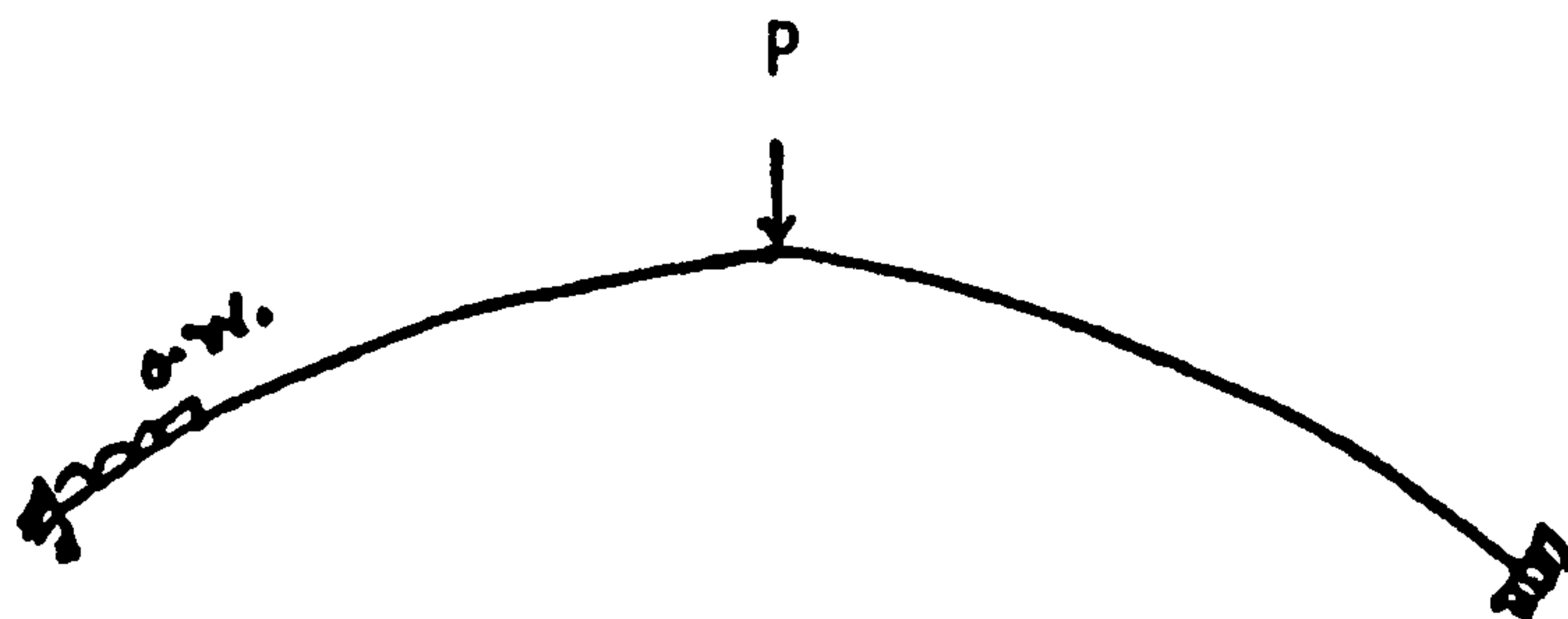


Fig. 6.6

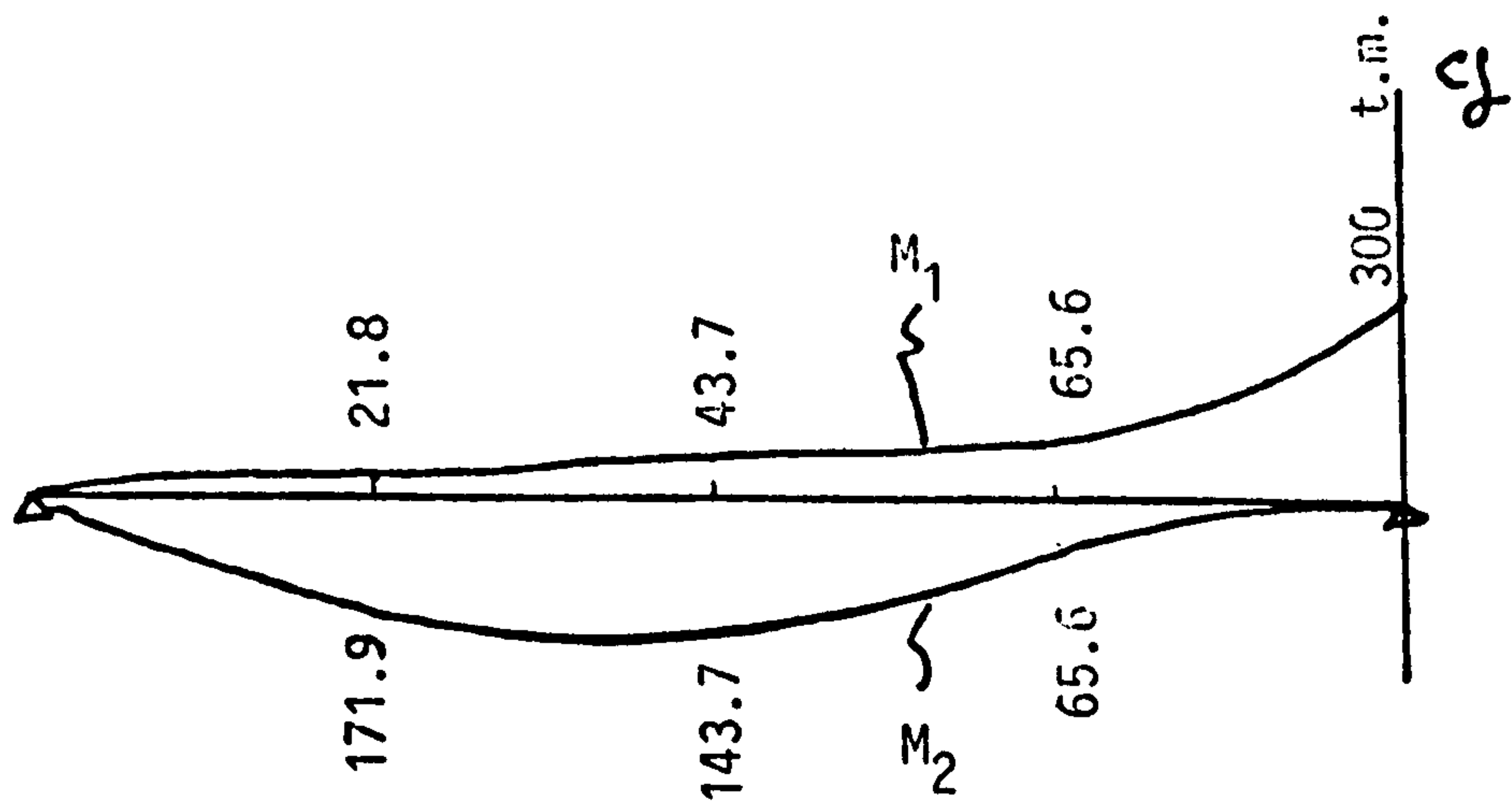
$$P_{L.L} = .7 \times 5 \times 20 = 70^t$$

$$P_{D.L} = .2 \times 2.5 \times 5 \times 20 + .6 \times 1.5 \times 2.5 \times 20 = 95^t$$

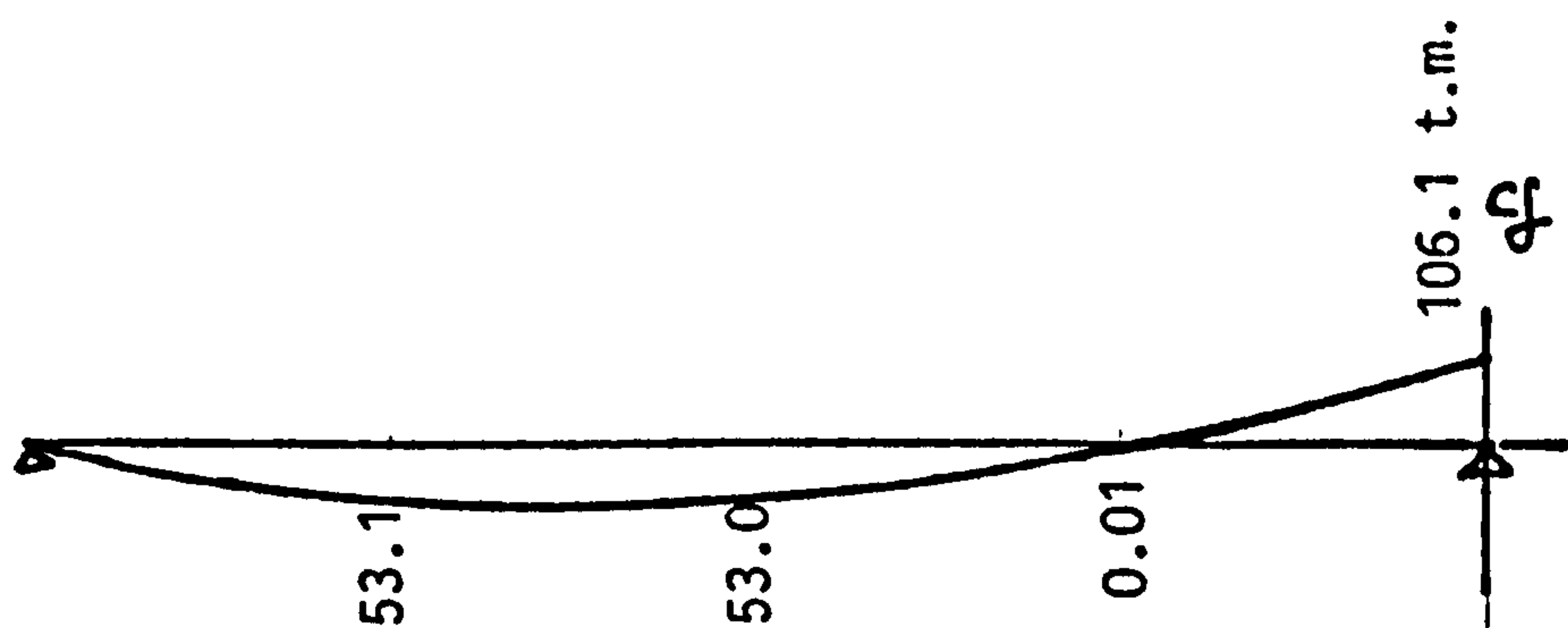
$$o.w = W/bd \text{ t/m}$$

The results were given by the computer program and shown in Figs. 6.7, 6.8, 6.9, 6.10 and 6.11.

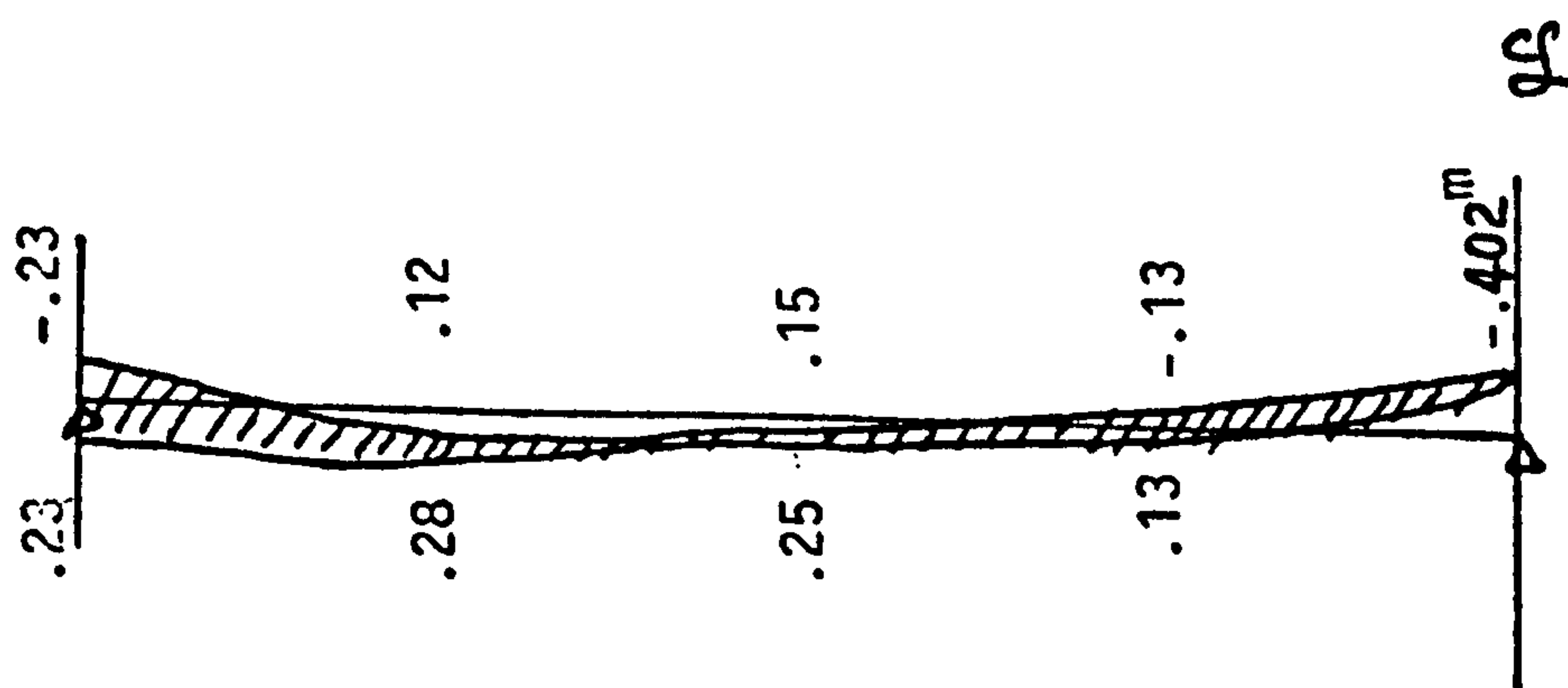
The maximum required depth for the girder is 1.41 m and for the arch is 1.59 m.



a) Live load Moment envelope (the girder)



b) Dead load Moment (the girder)



c) the limiting Zone (the girder)

Fig. 6.7

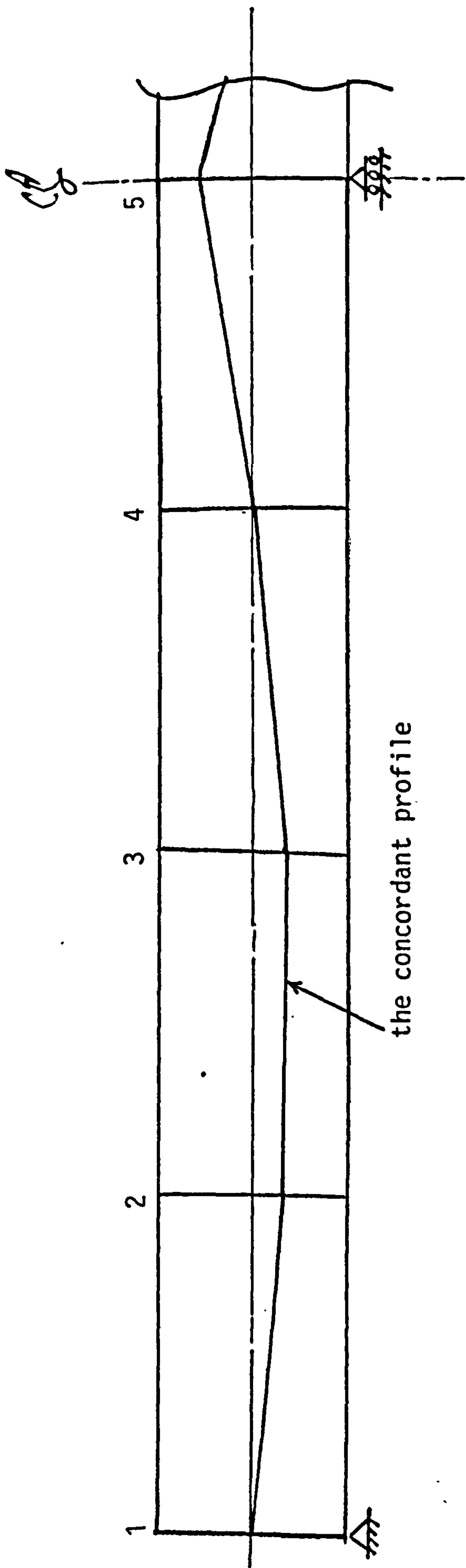


Fig. 6.8

Station No.	1	2	3	4	5
the depth (m)	1.414				
the prestressing force (ton)	636.5				
the eccentricity of the cables measured from the centroid of the section (m)	≈ 0	0.201	0.201	≈ 0	-0.402

Table 6.3

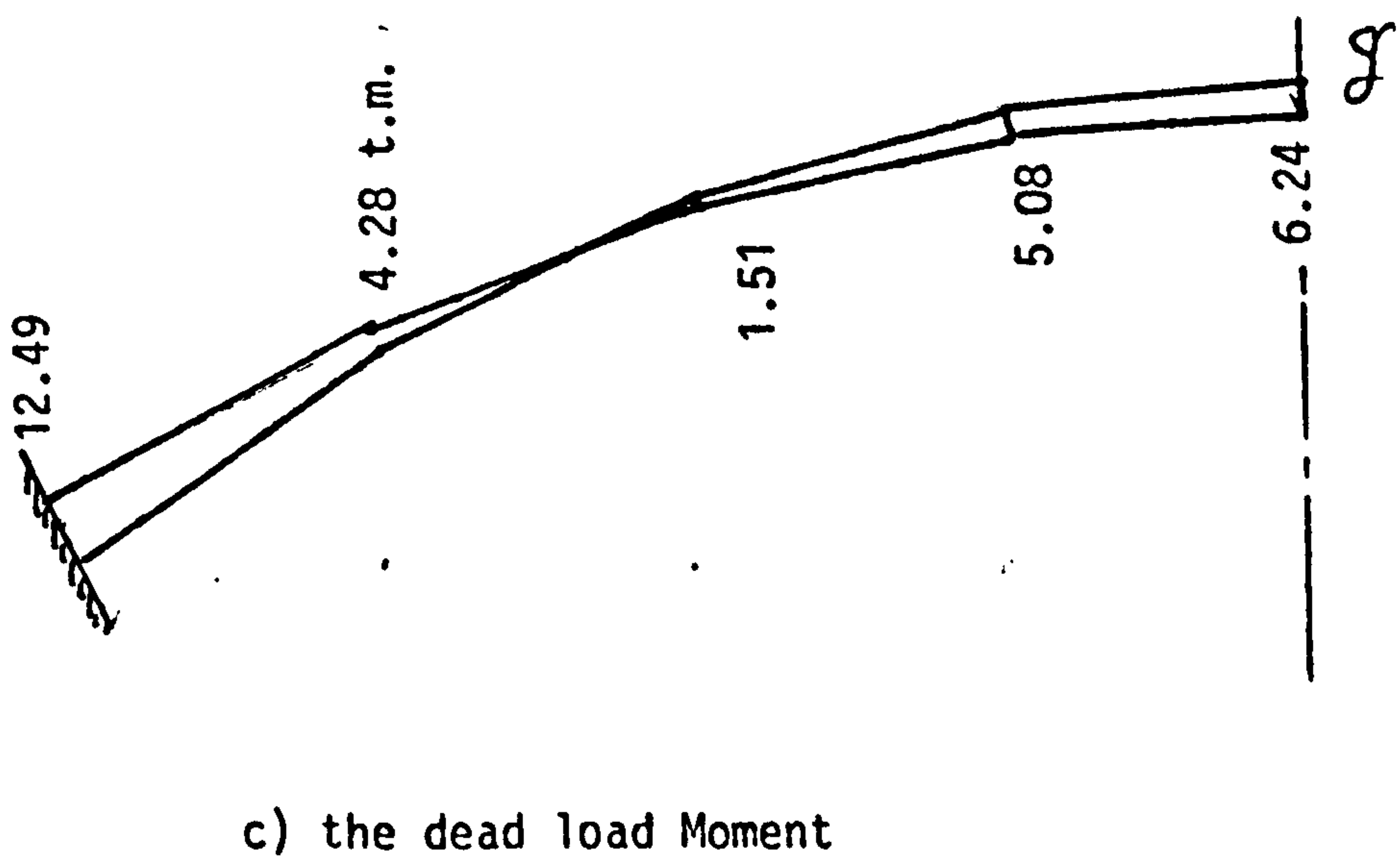
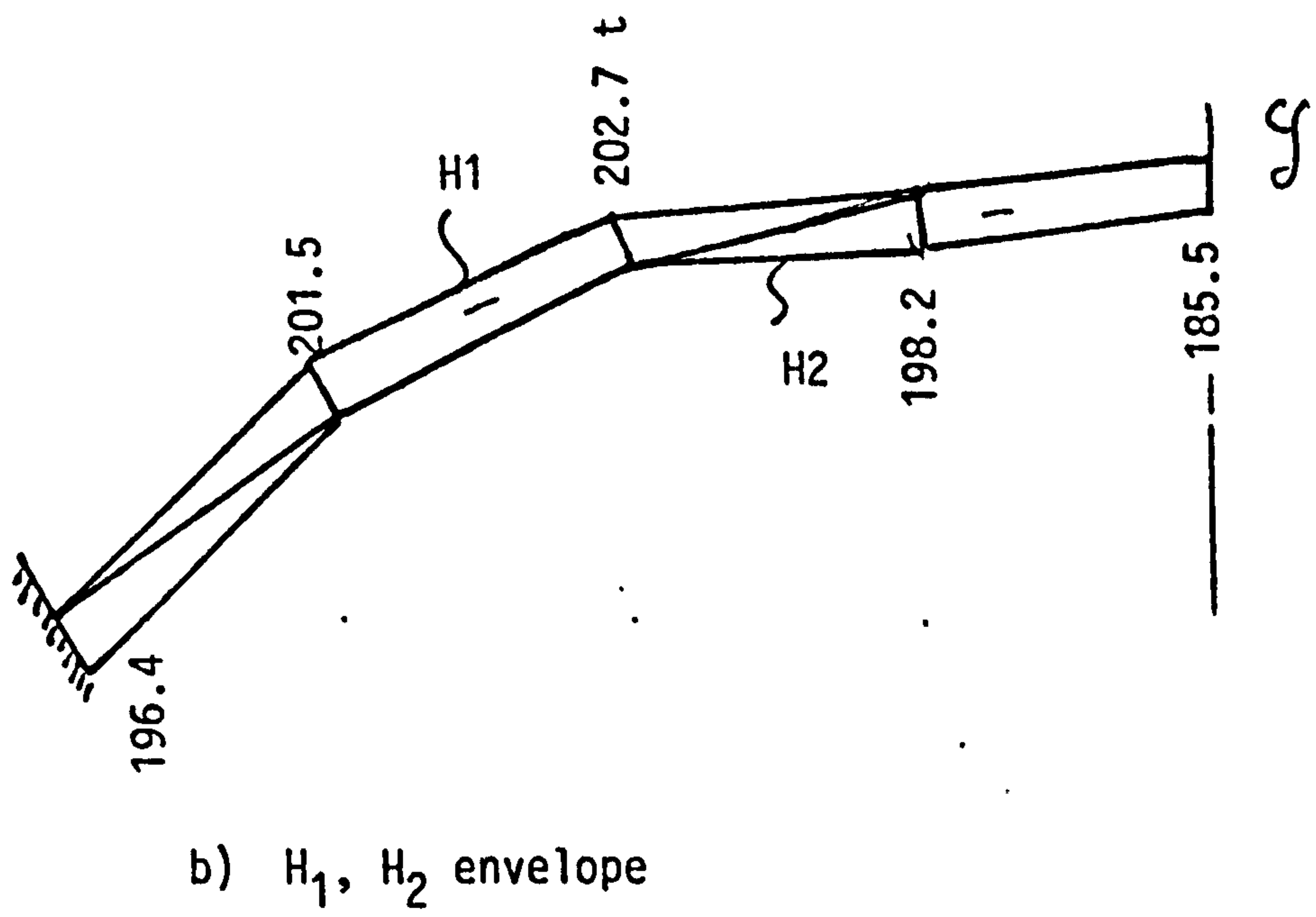
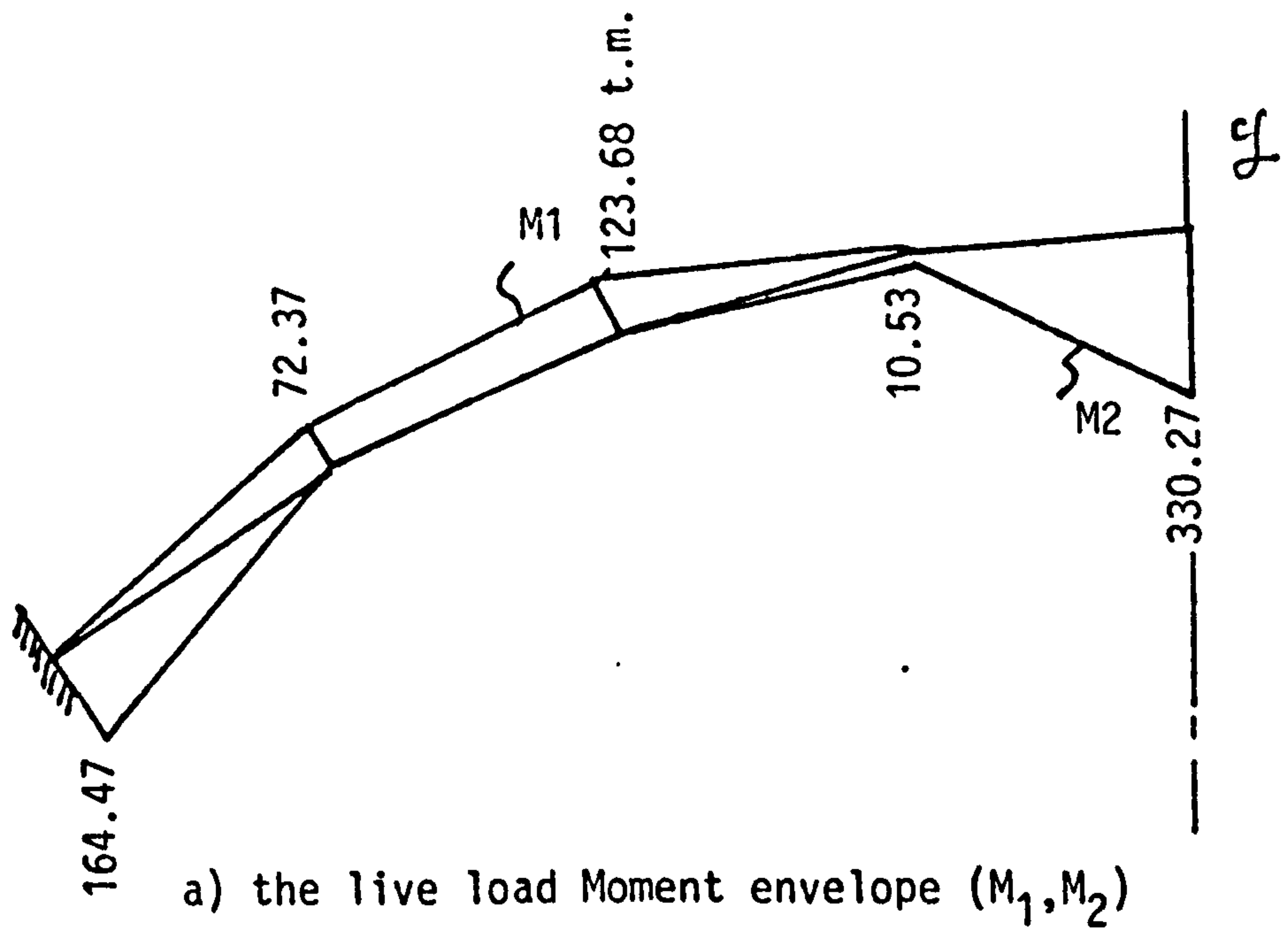
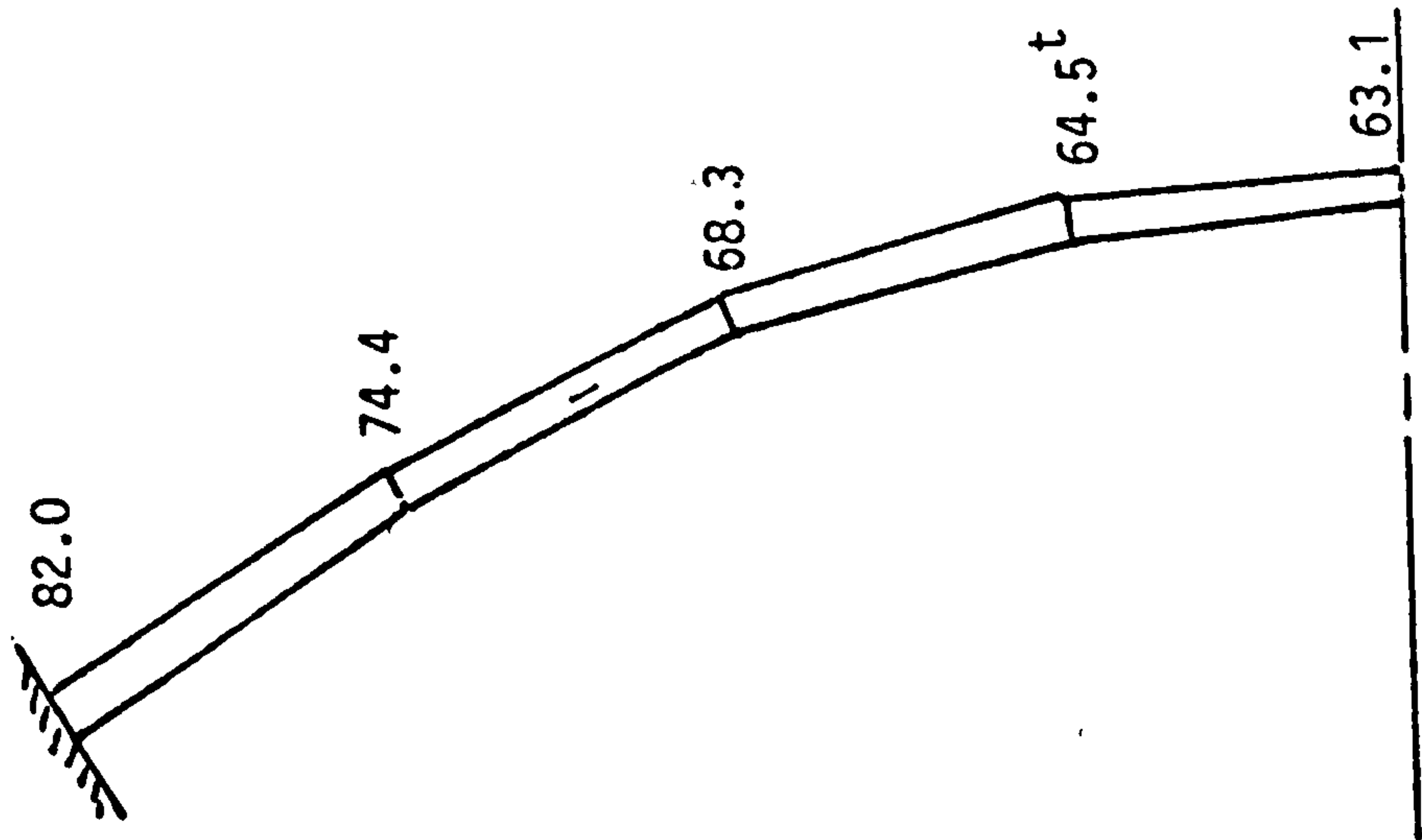
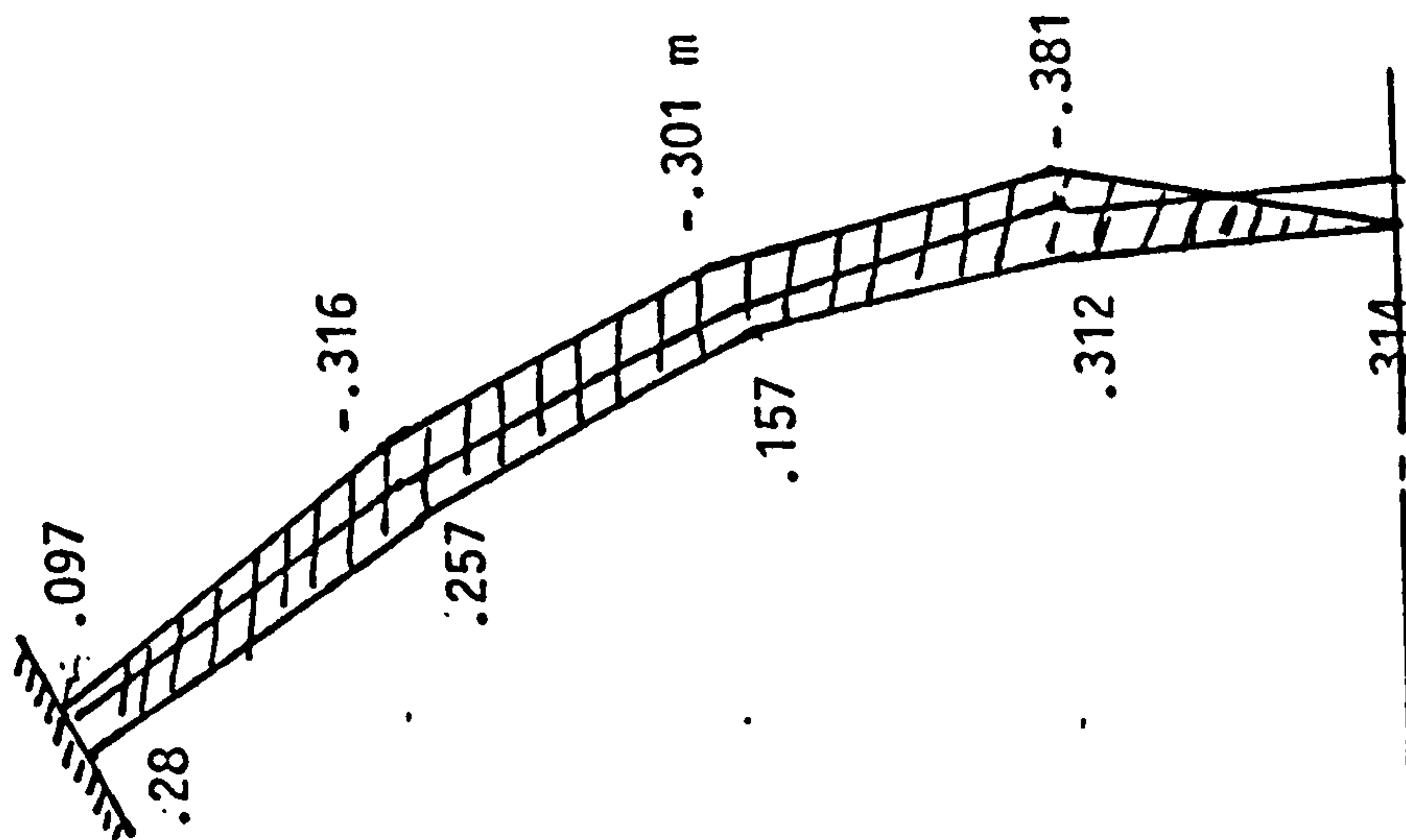


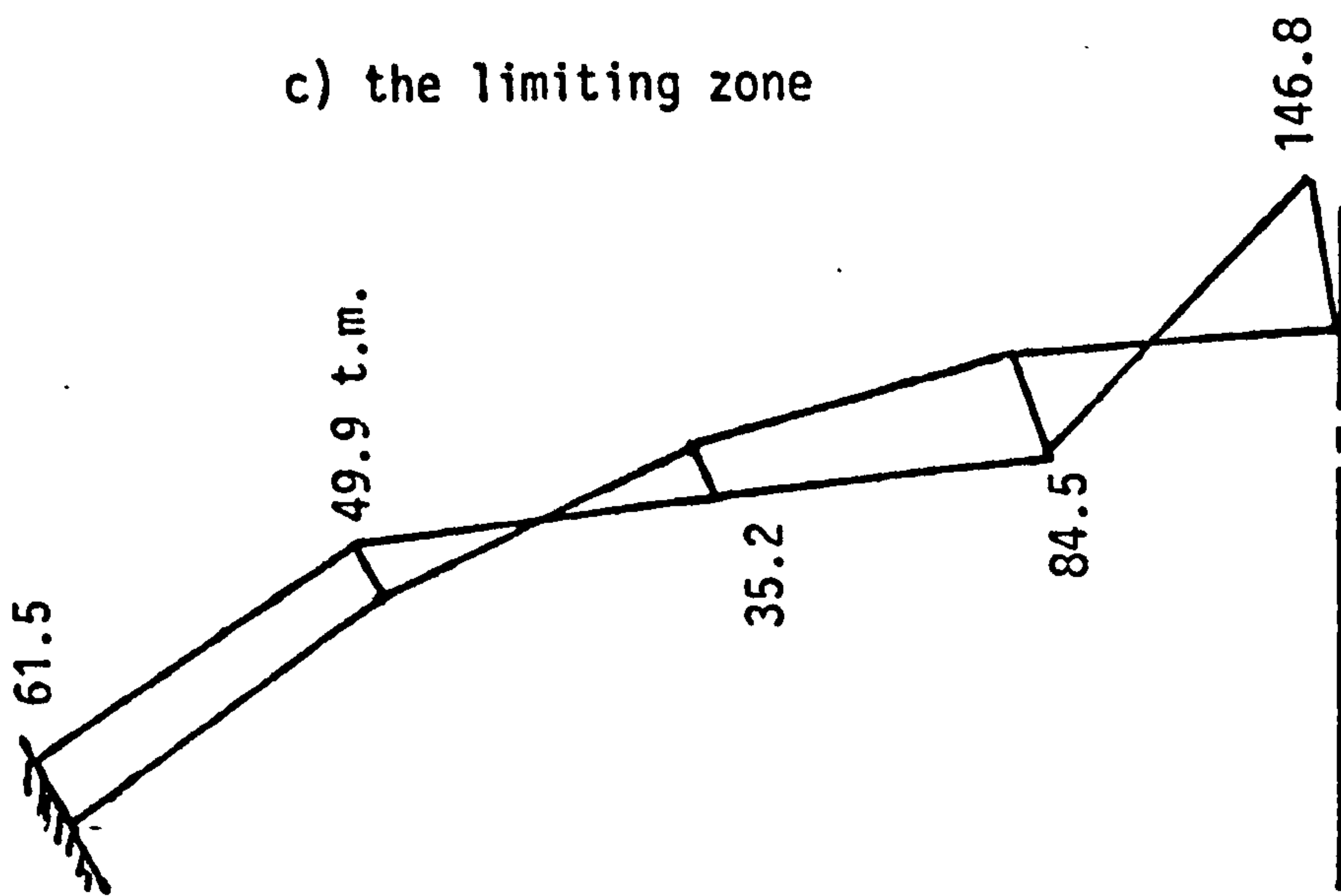
Fig. 6.9



d) the dead load axial force



c) the limiting zone



f) the prestressing moment

Fig. 6.10

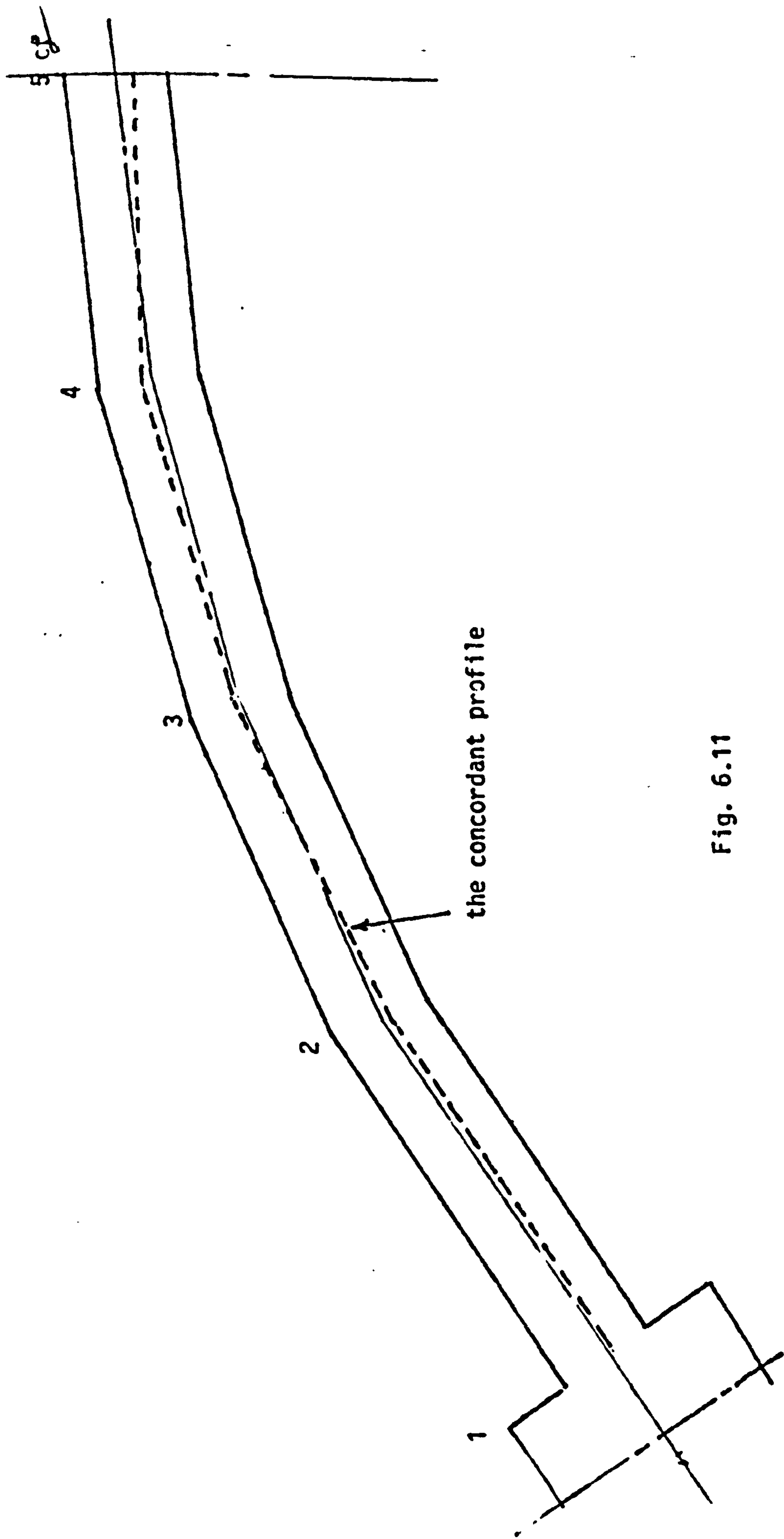
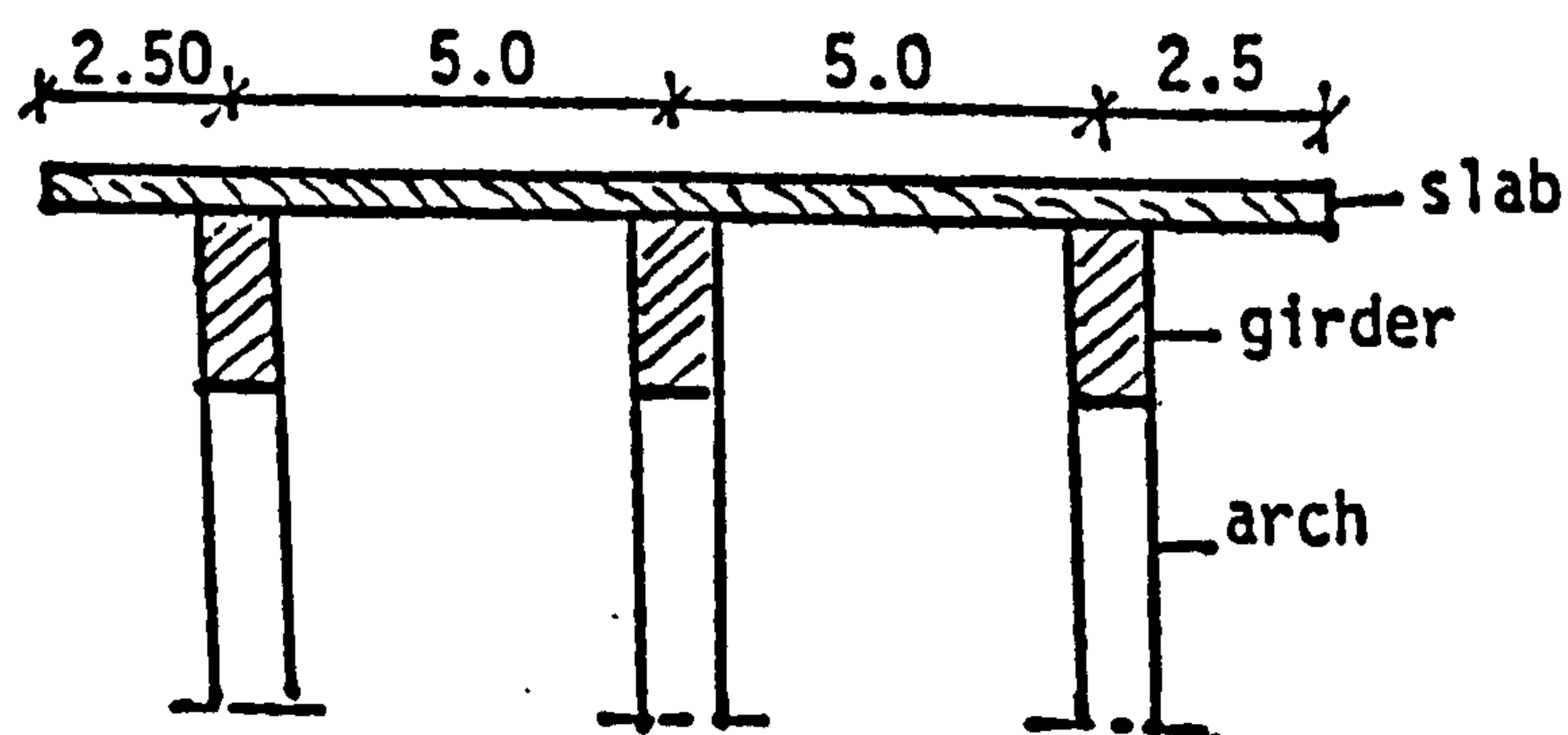
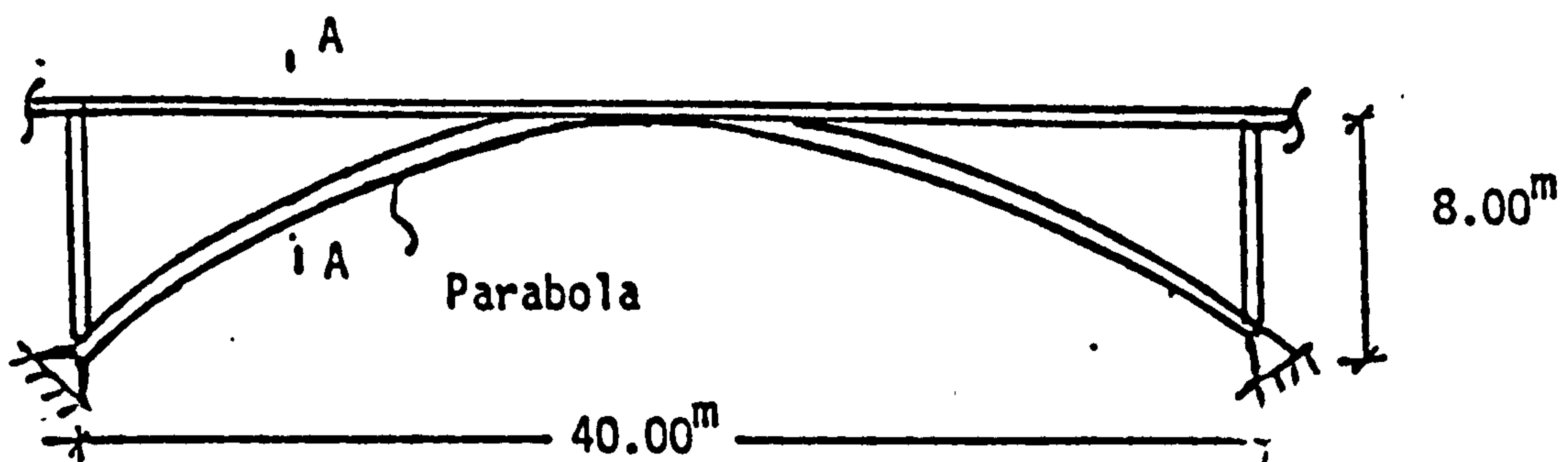


Fig. 6.11

Station N ^o	1	2	3	4	5
the depth (m)	1.59				
the prestressing force (ton)	467.0				
the eccentricity of the cables measured from the centroid of the Section (m.)	.131	.107	-.075	-.181	0.314

EXAMPLE 3

As an example of a hinged arch, consider the arch of Example 2 with the same dimensions and loading conditions but with hinged supports at the ends.



Section A - A

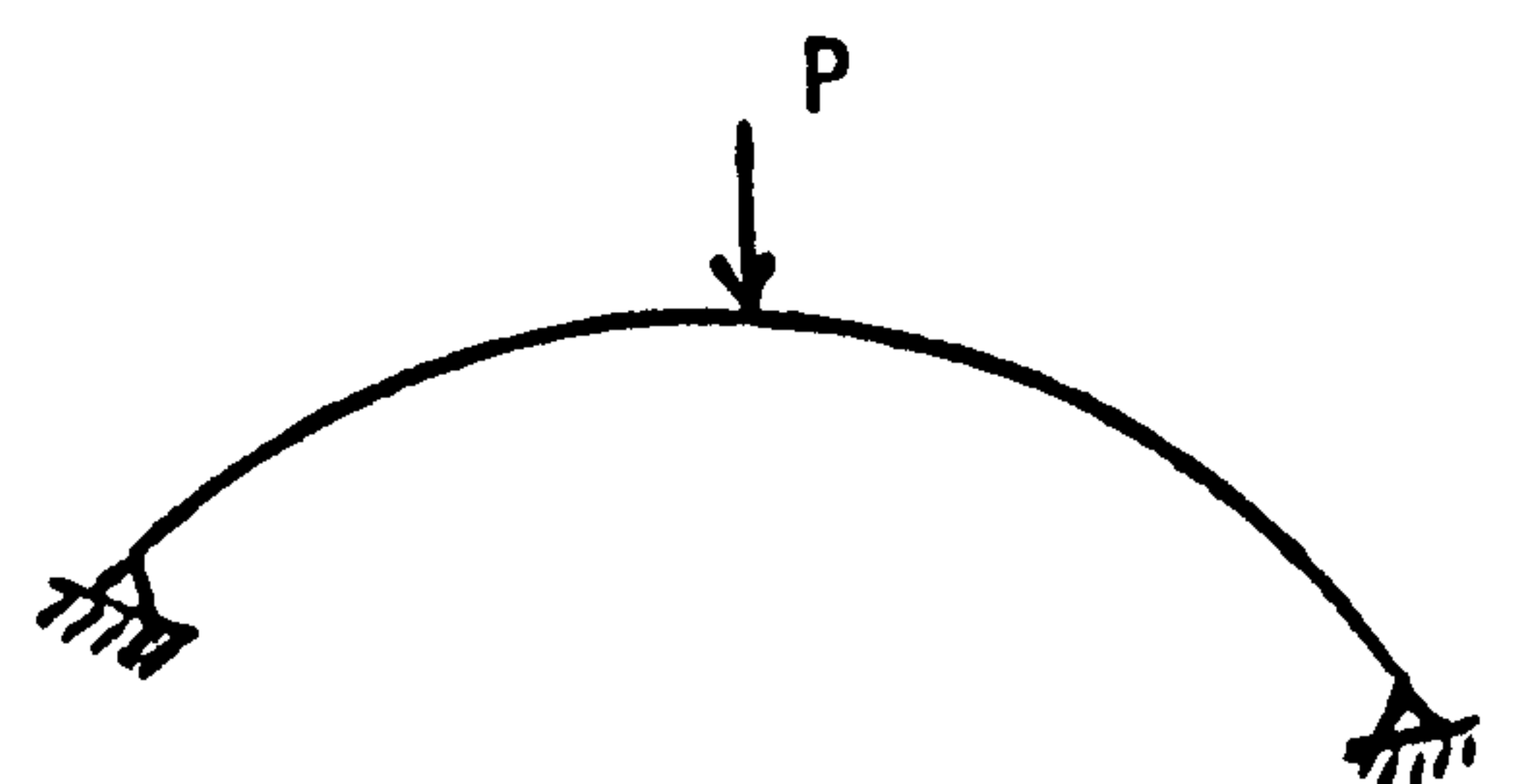
Fig. 6.12

Cases of loading

$$P_{L.L} = 70^t$$

$$P_{D.L} = 95^t$$

$$o.w. = W/bd \text{ t/m}$$



The results were given by the computer program and are shown in Figs. 6.13, 6.14 and 6.15.

The minimum required depth for the arch is 1.663 m and the width is = 0.6 m.

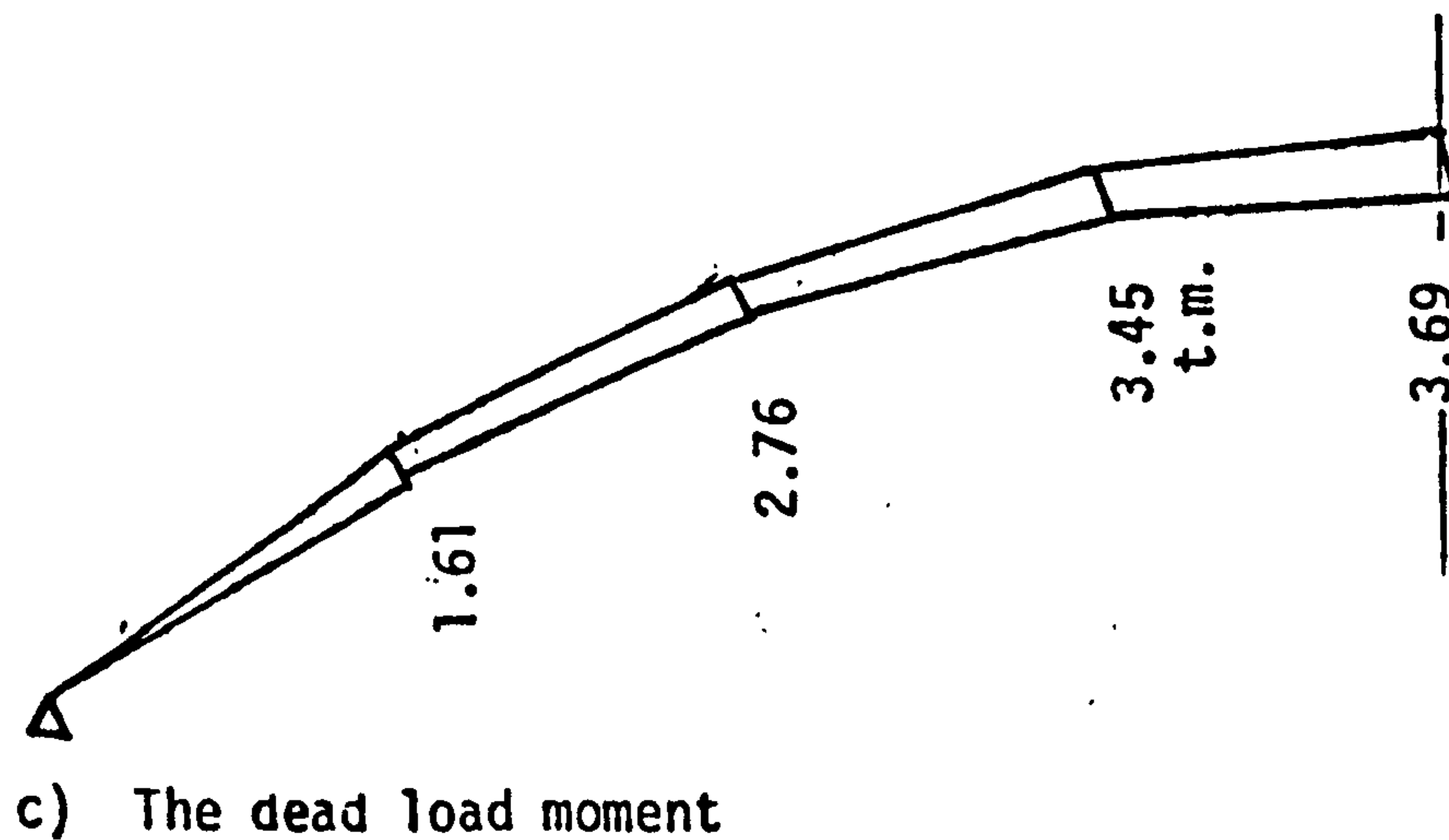
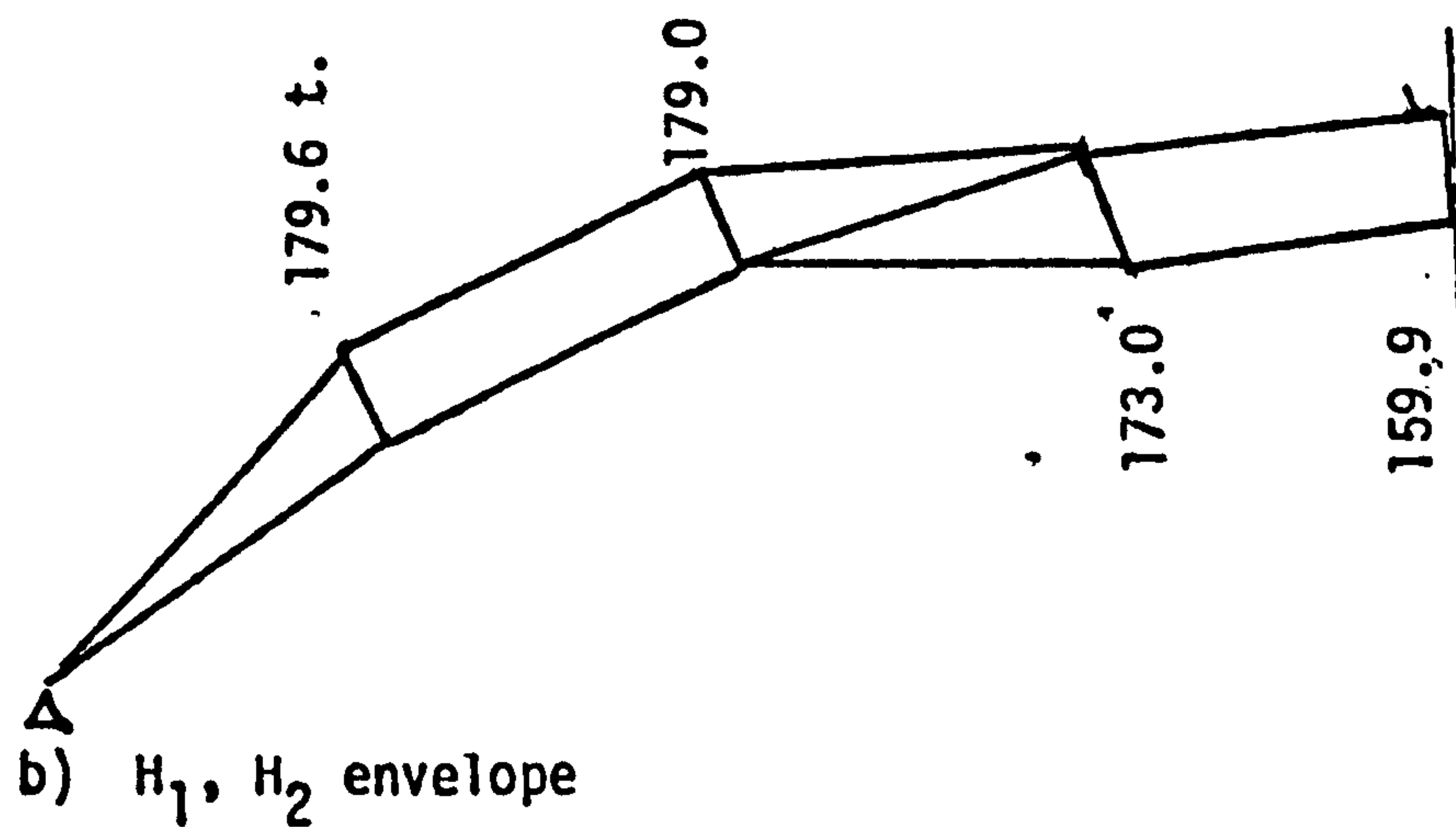
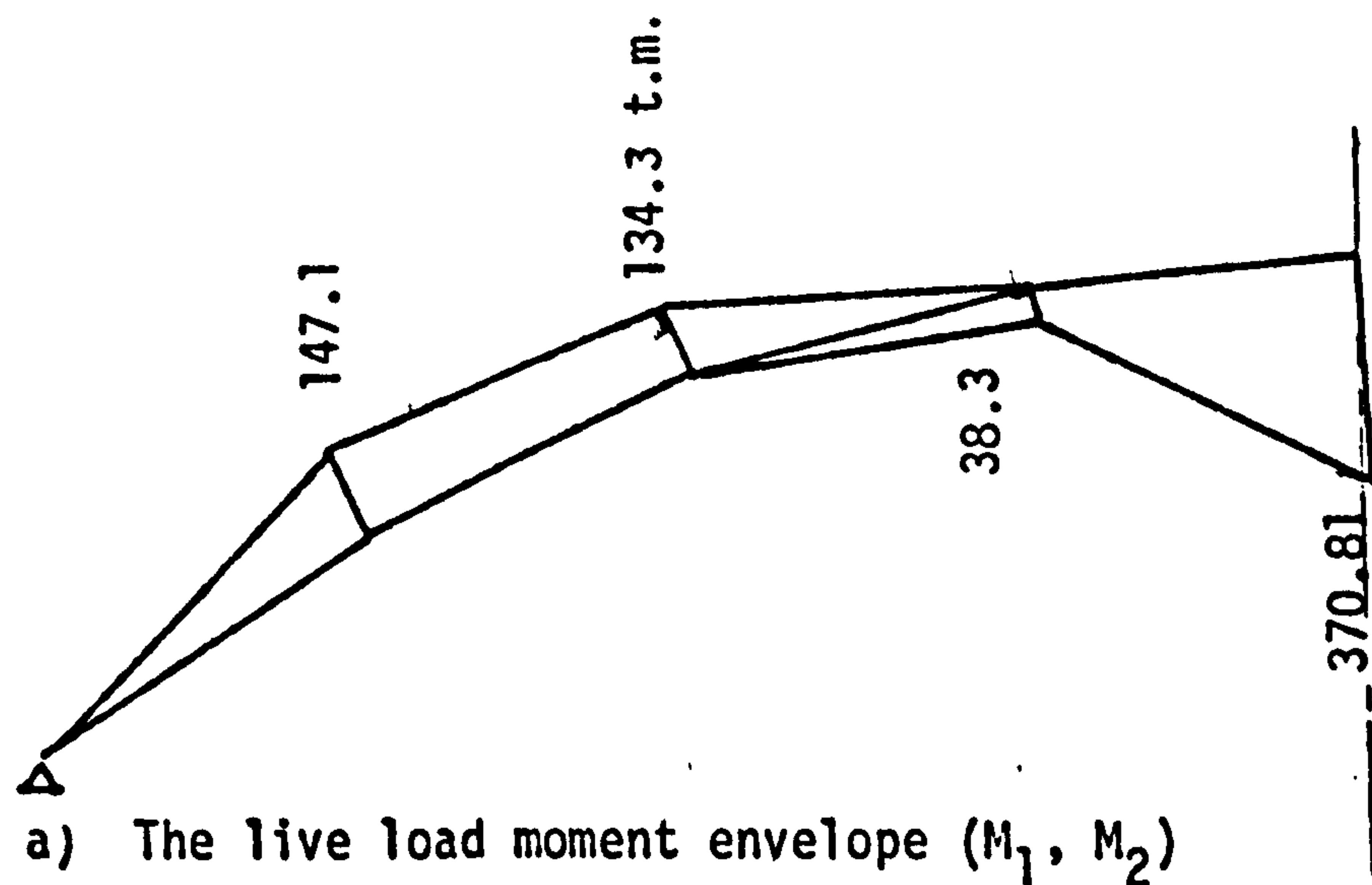
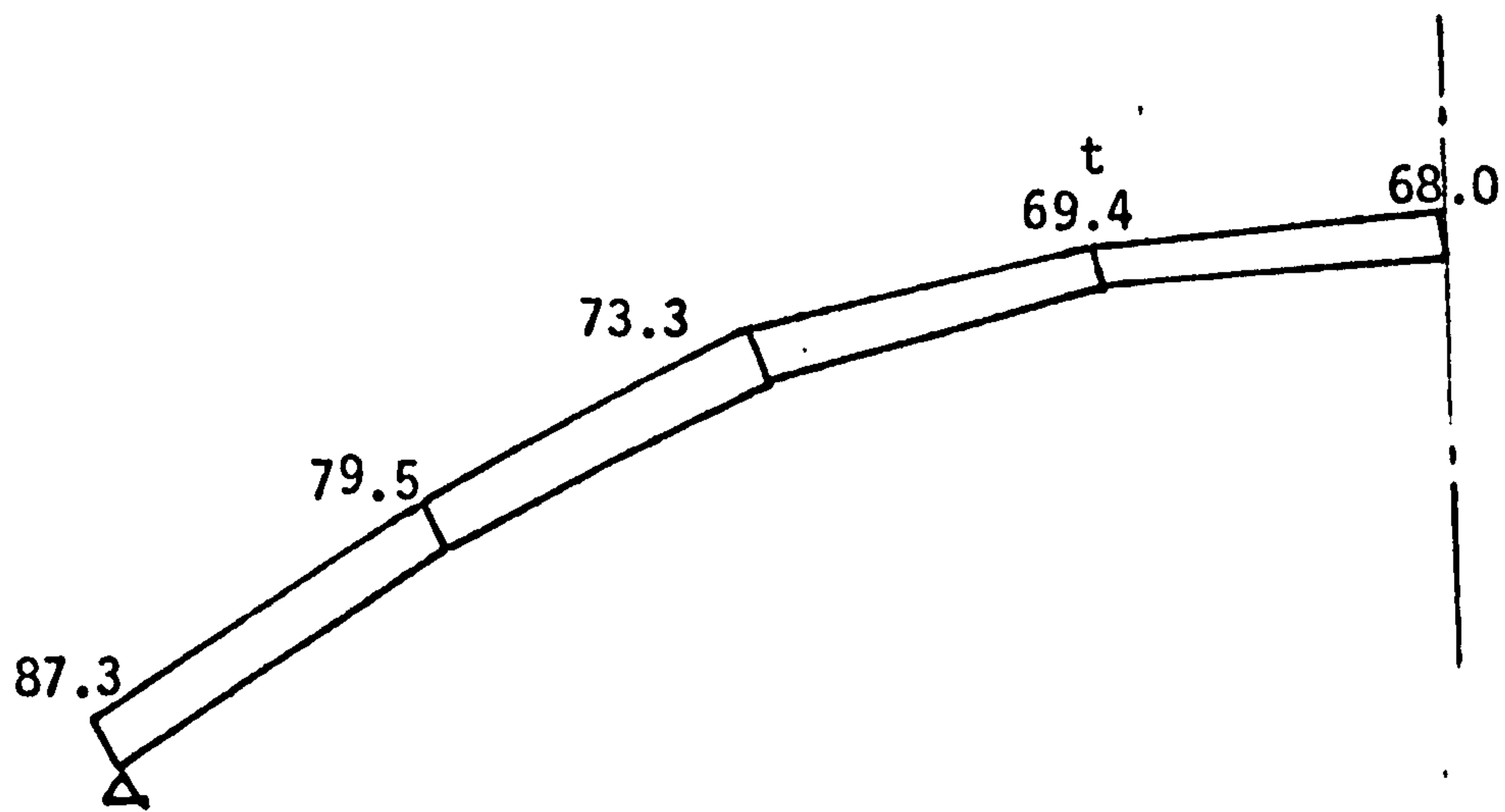
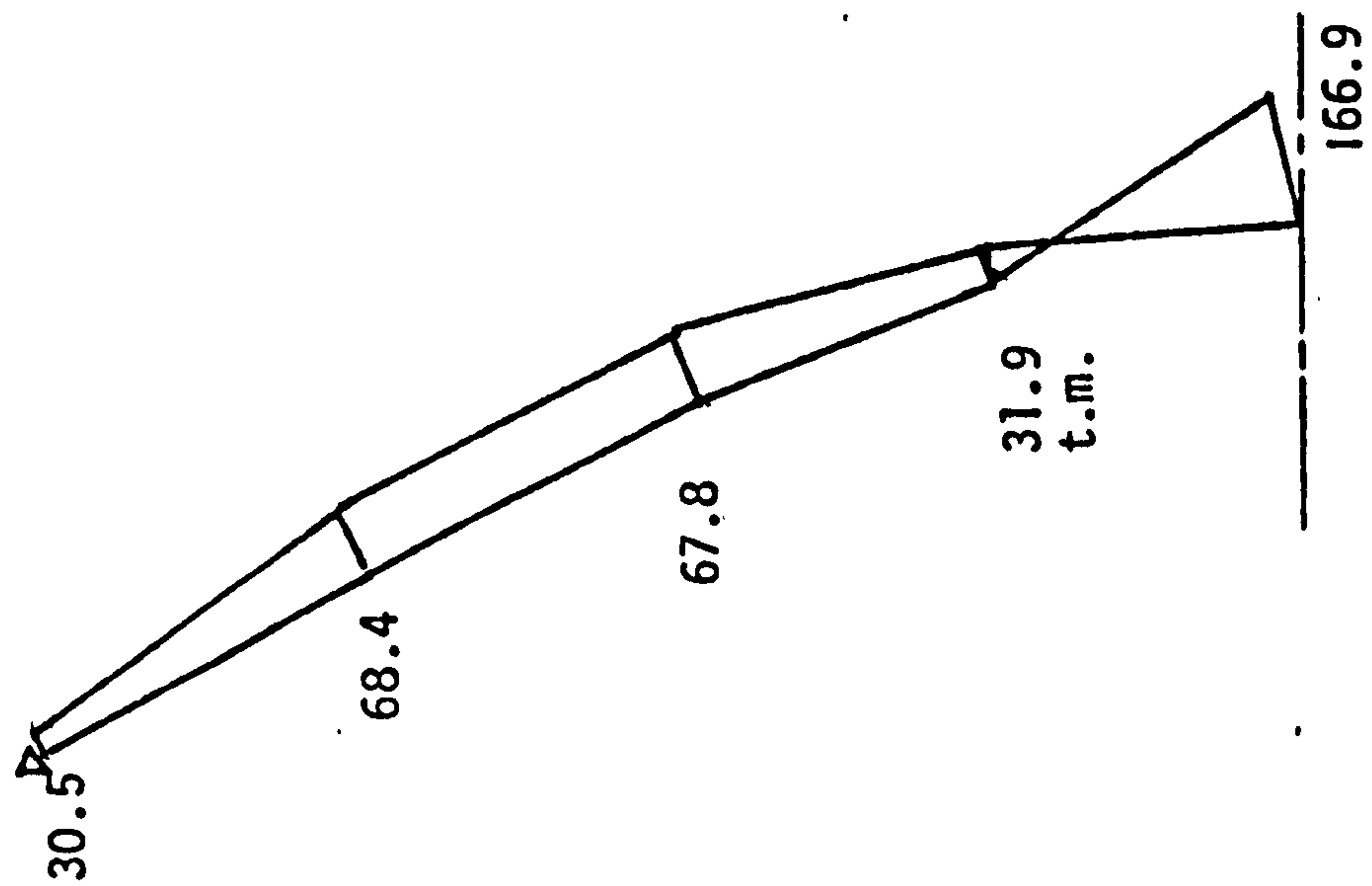


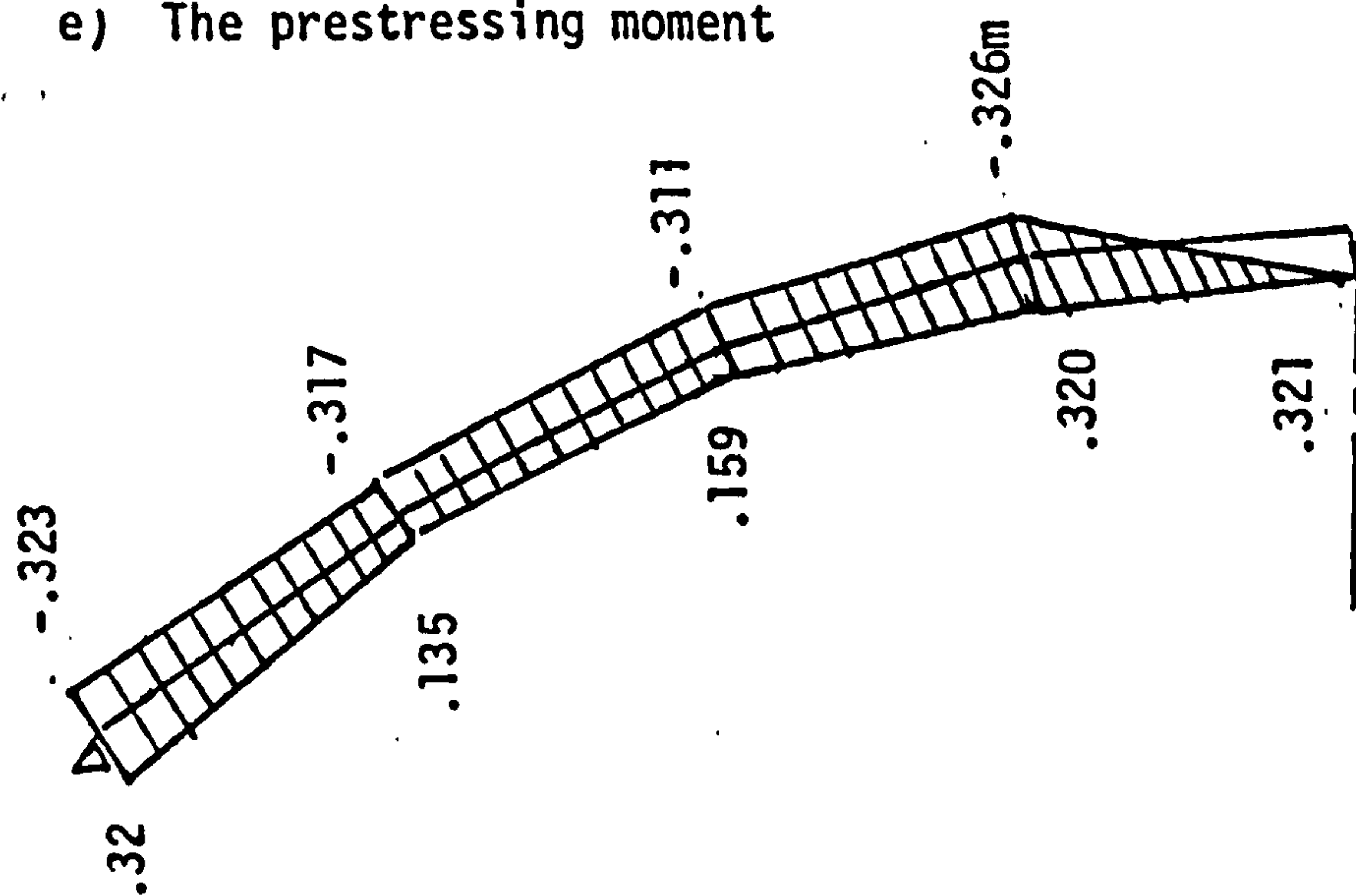
Fig. 6.13



d) The dead load axial force



e) The prestressing moment



f) The limiting zone

Fig. 6.14

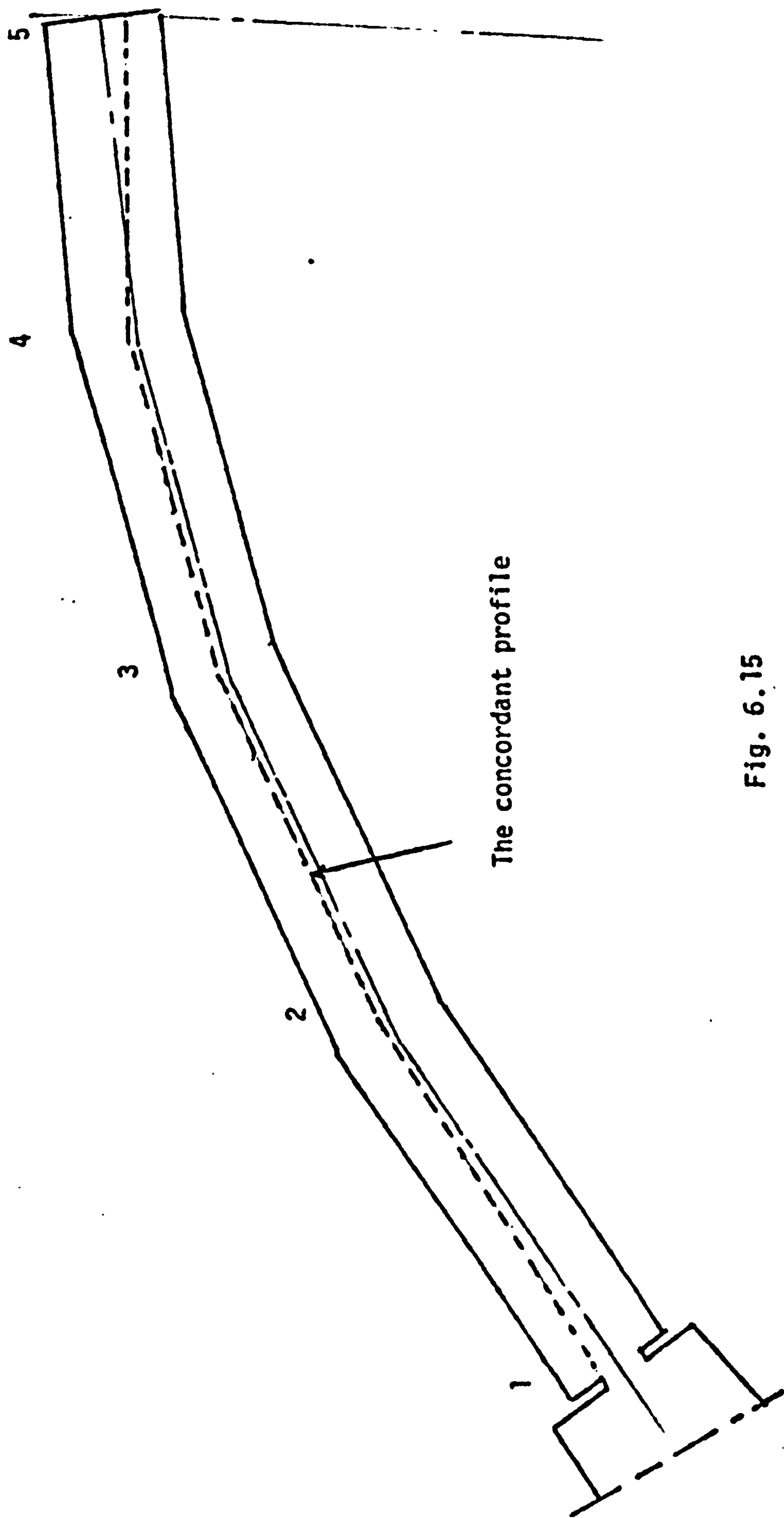


Fig. 6.15

Station No.	1	2	3	4	5
The depth (m)	1.664				
The prestressing force (ton)	520.74				
The eccentricity of the cables measured from the centroid of the station(m)	-.324	-.108	-.092	-.082	.321

Table 6.5

EXAMPLE 4

As an example of a fixed frame, consider the bridge frame as shown in Fig. 6.16 of span 12.0 m and height 6.0 m with double cantilevers of spans 3.0 m each. The loading conditions come from the consideration of uniformly distributed load of 0.7 t/m^2 on the structure and from the weight of the slabs and the secondary girders on the main frame, as shown in Fig. 6.17. The width of the structure is constant and is taken as 1.0 m and the depth is variable as shown in Fig. 6.16. The o.w. of the frame was taken as a function of the width and the depth of the frame. The working stress limits were taken to be 14 N/mm^2 compression and 0 N/mm^2 tension and the wind load was taken into consideration.

The frame column is divided into 3 stations per column and the span is divided into 5 stations. The results were given by the computer program and are shown in Figs. 6.18, 6.19.

The minimum required d is 1.791 m.

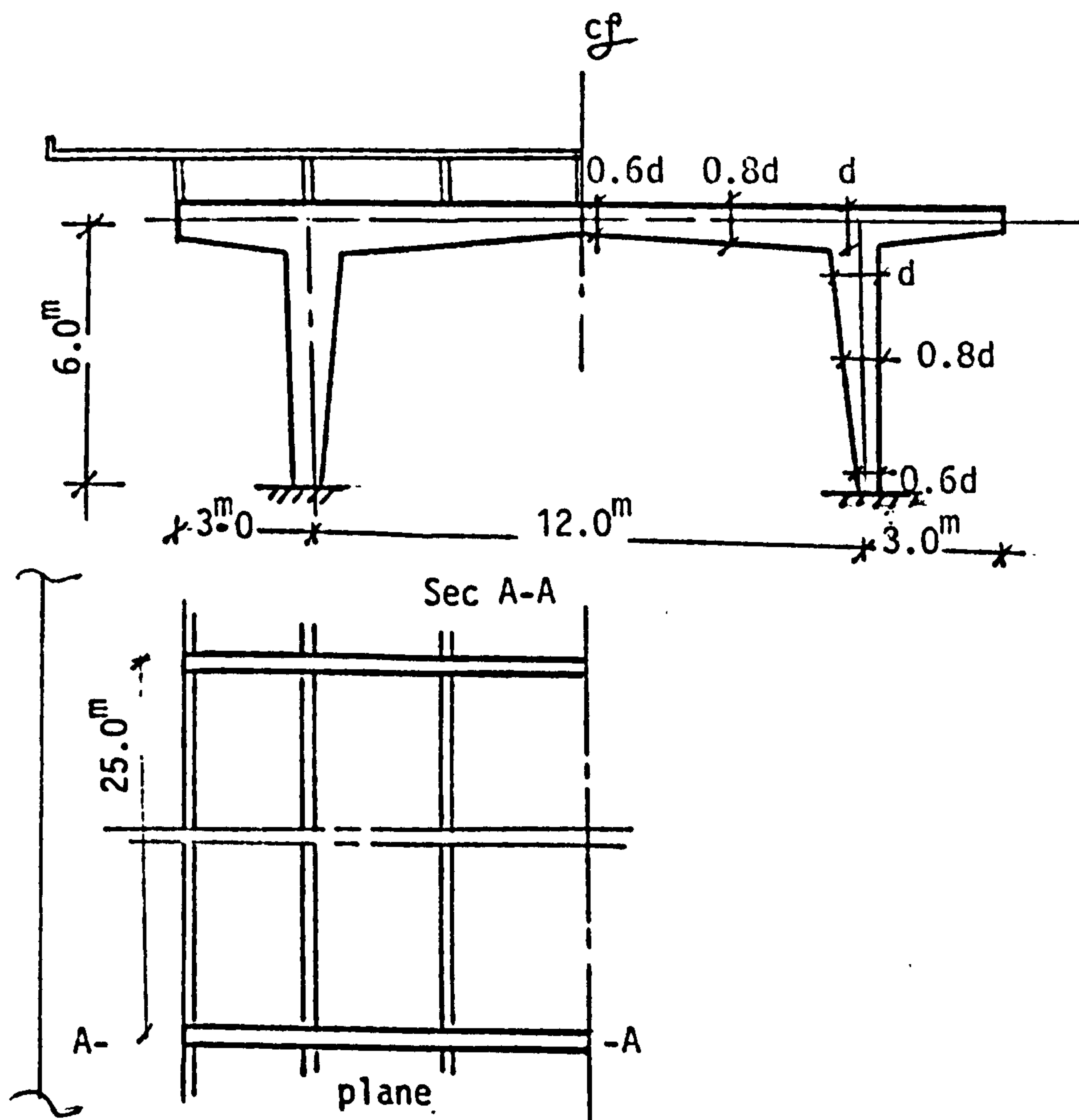
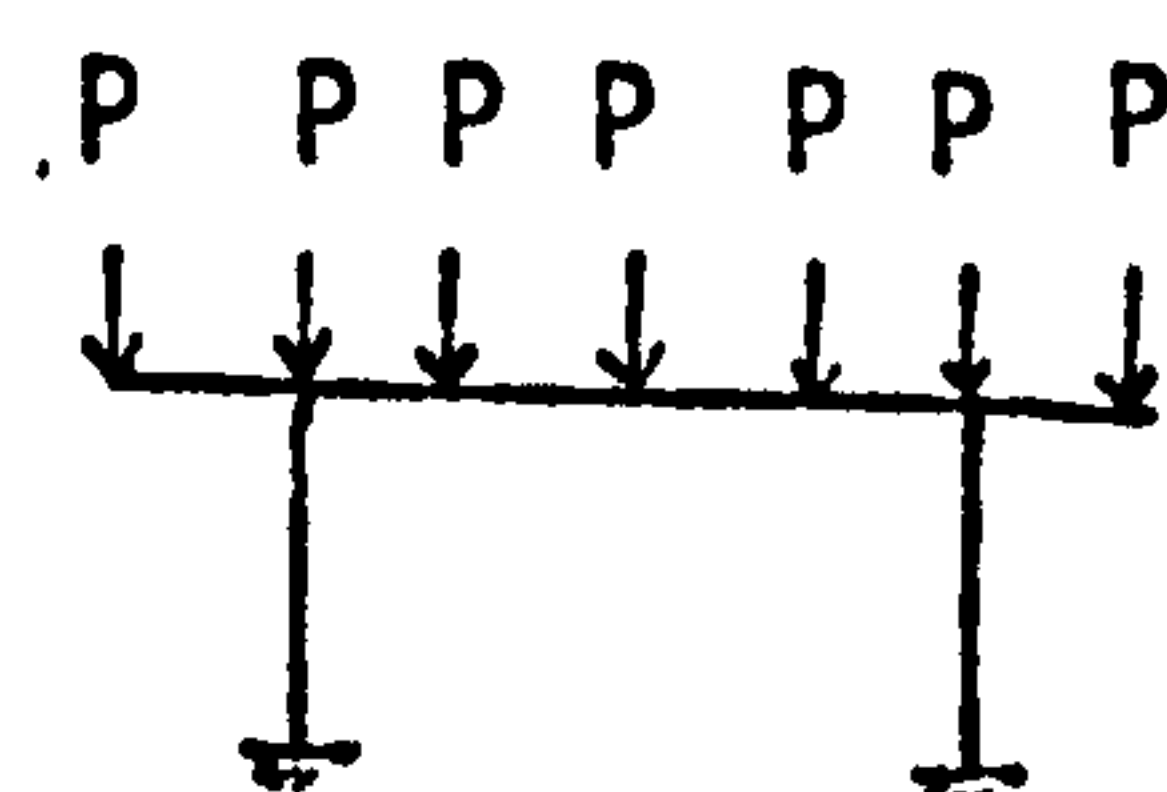
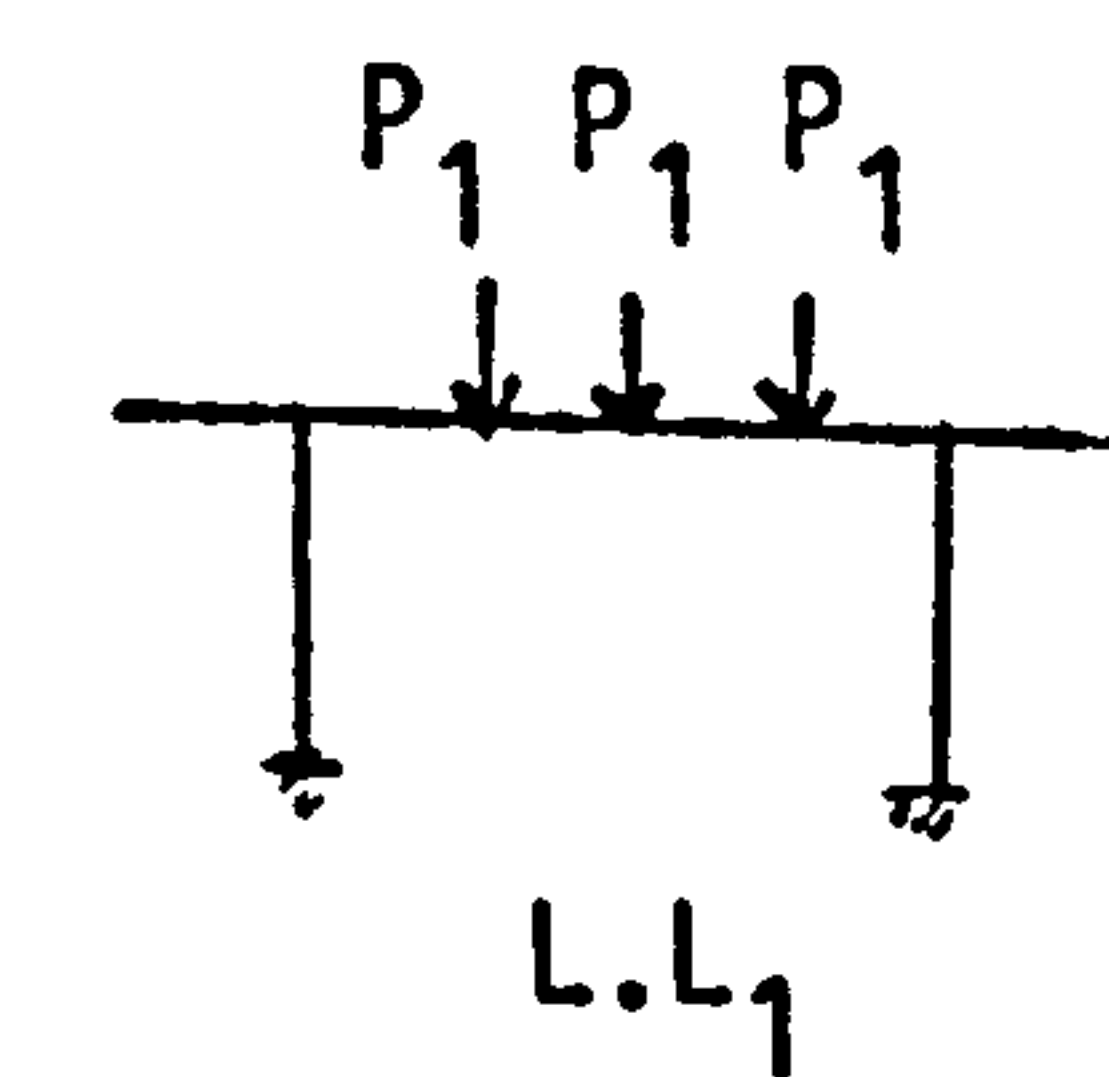
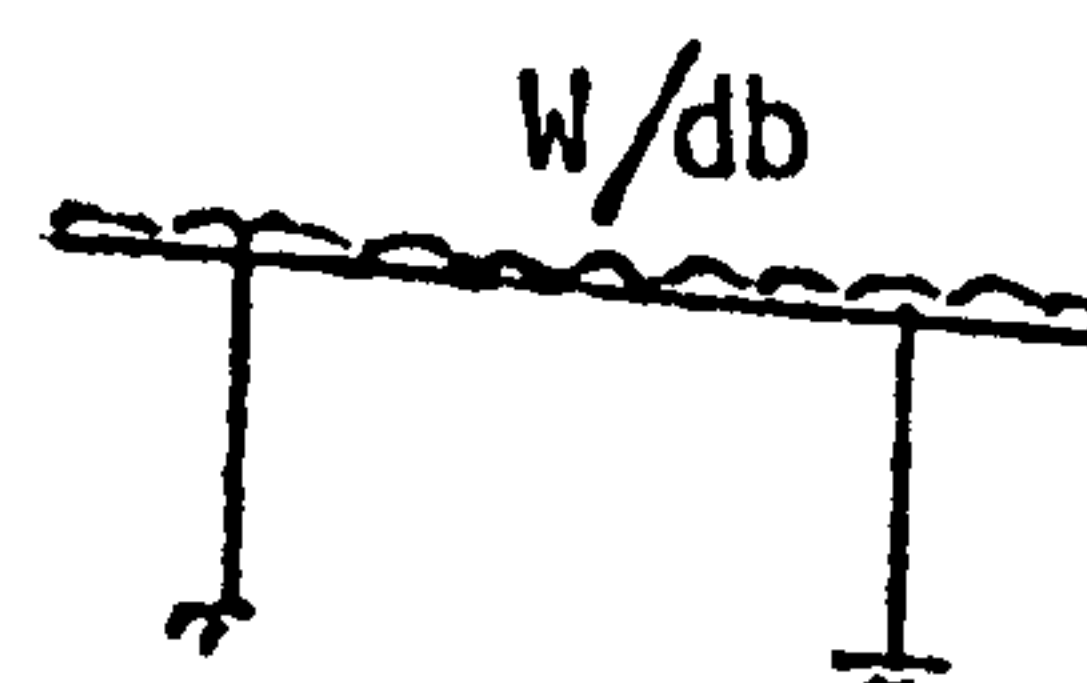
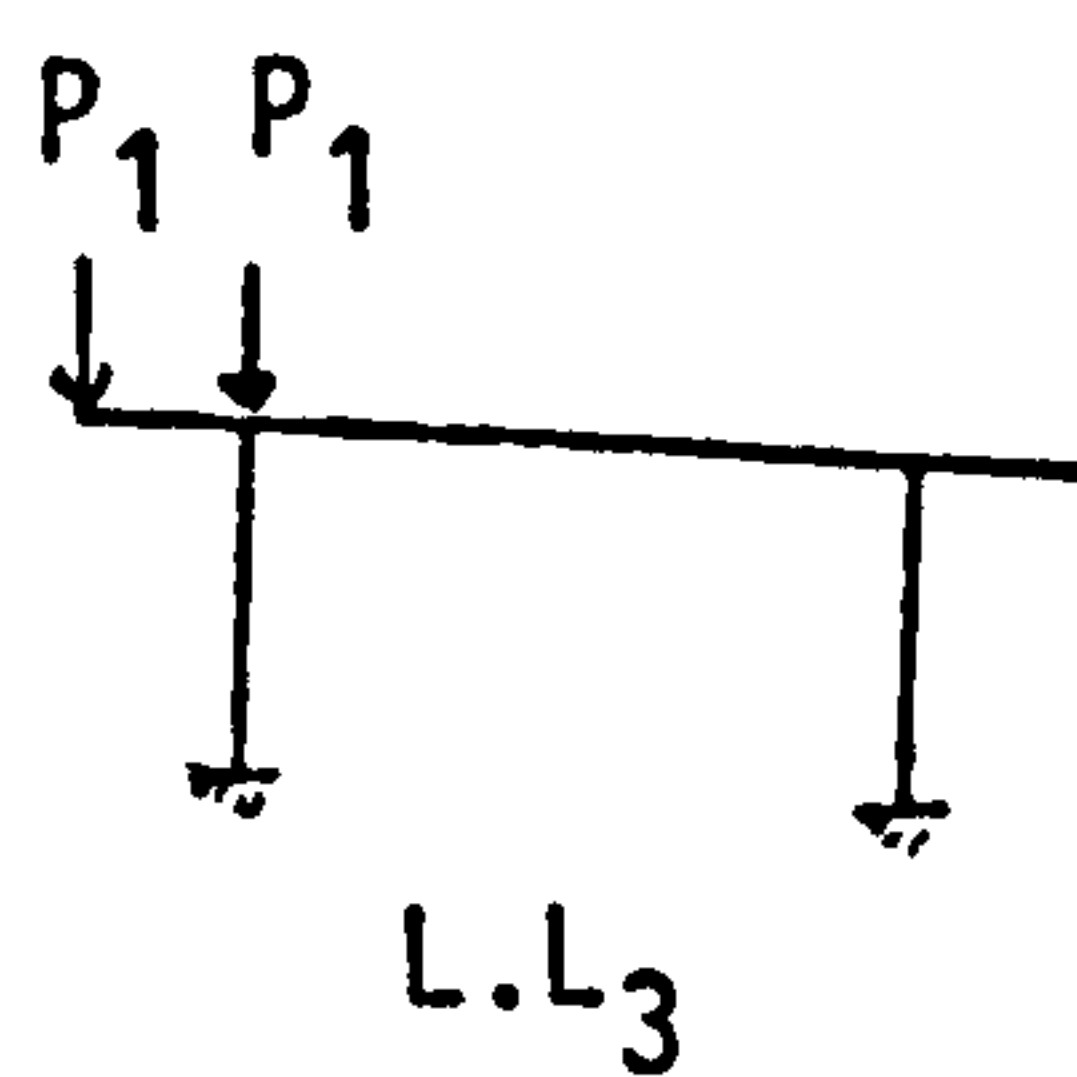
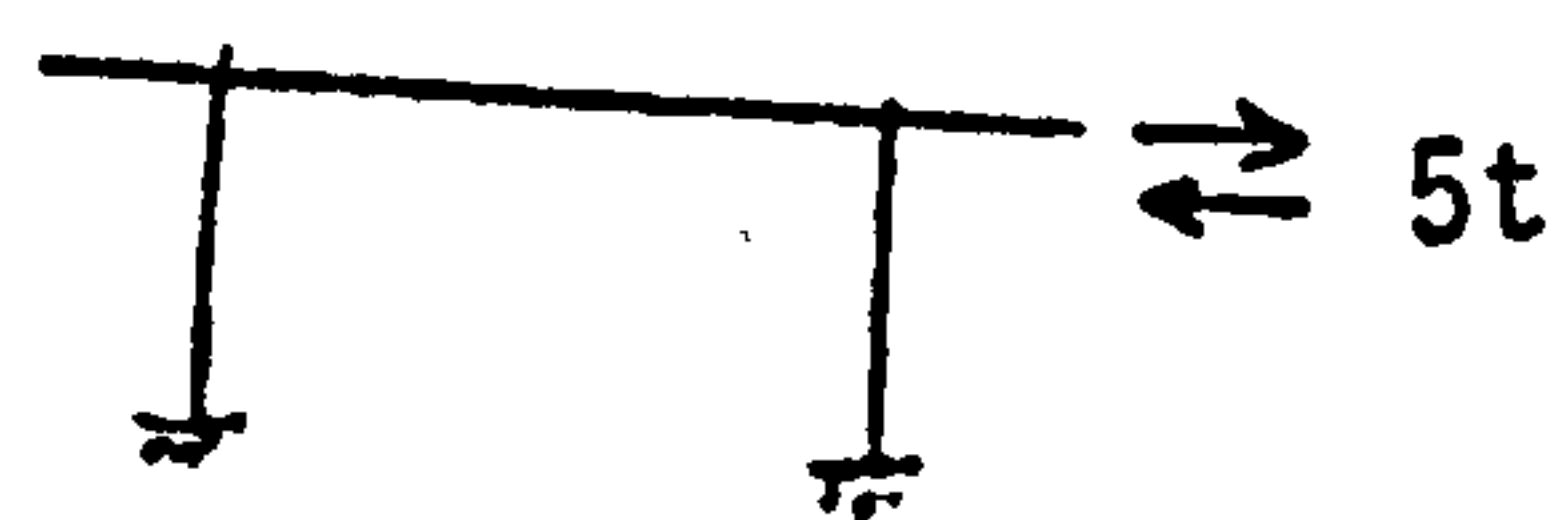
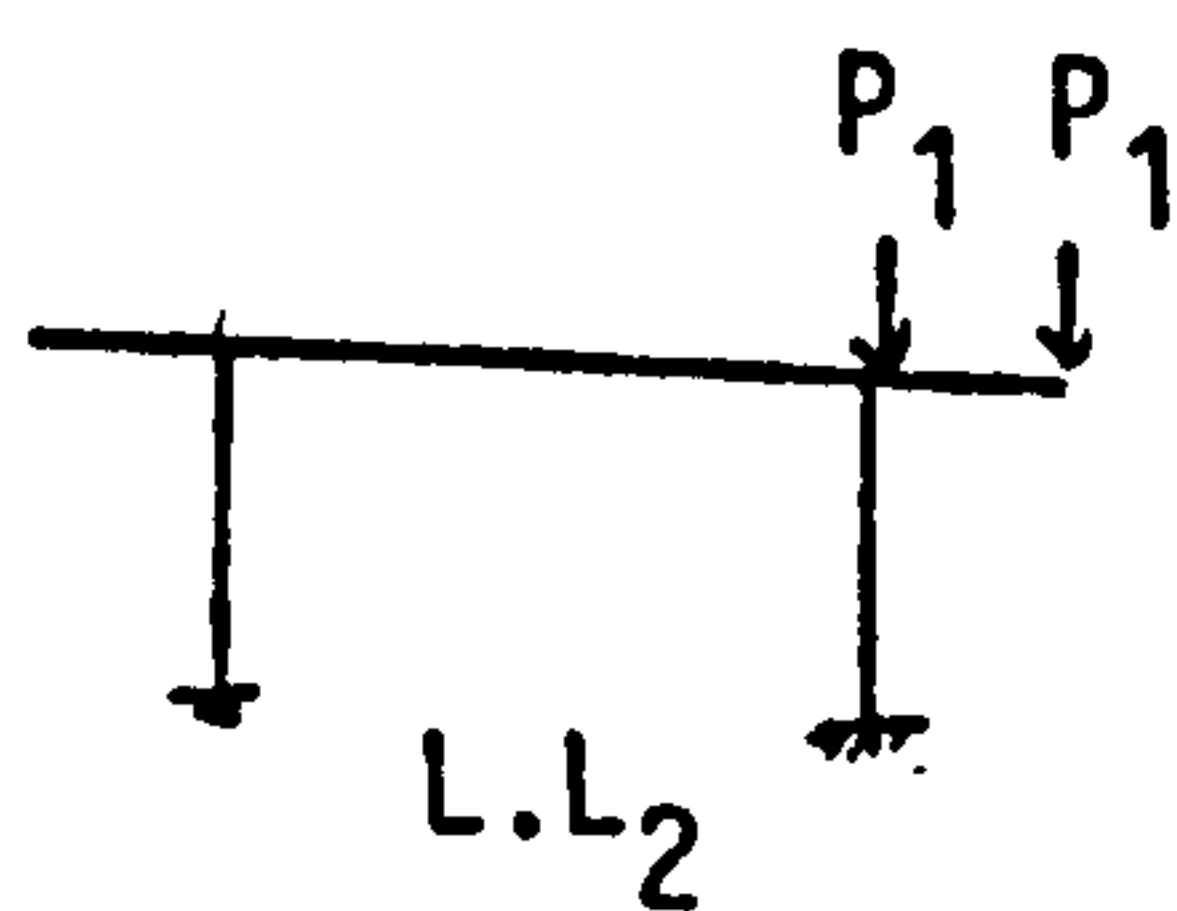


Fig. 6.16

Cases of loadings

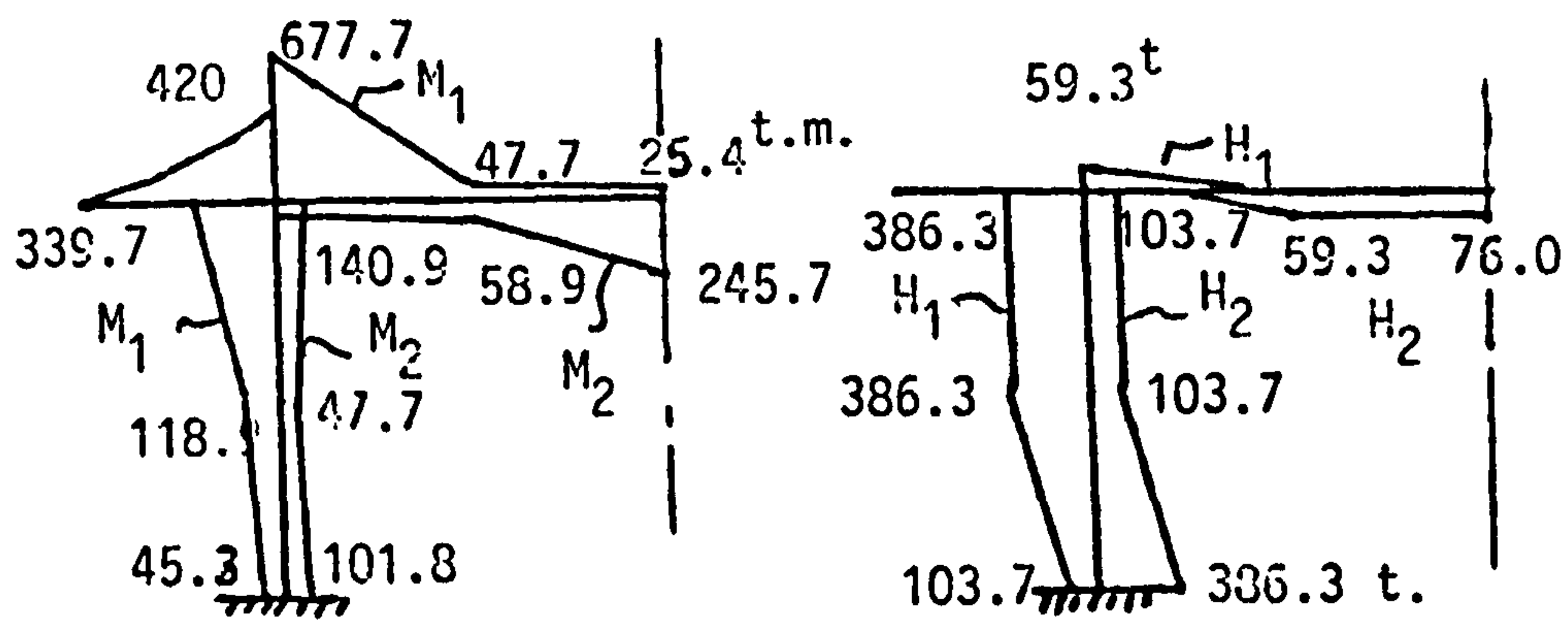


$$P = 85.5 \text{ t}$$



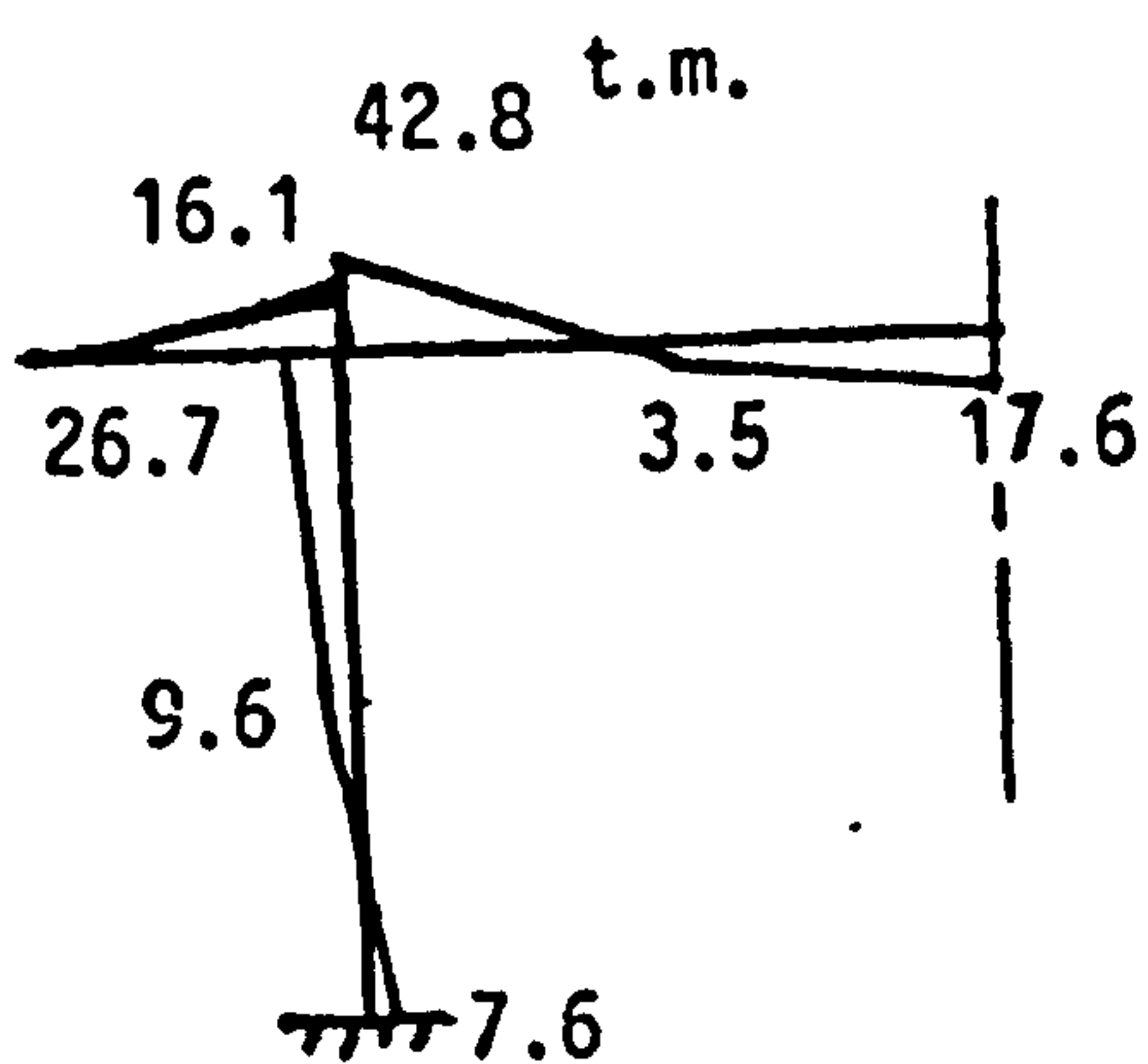
$$P_1 = 52.5 \text{ t}$$

Fig. 6.17

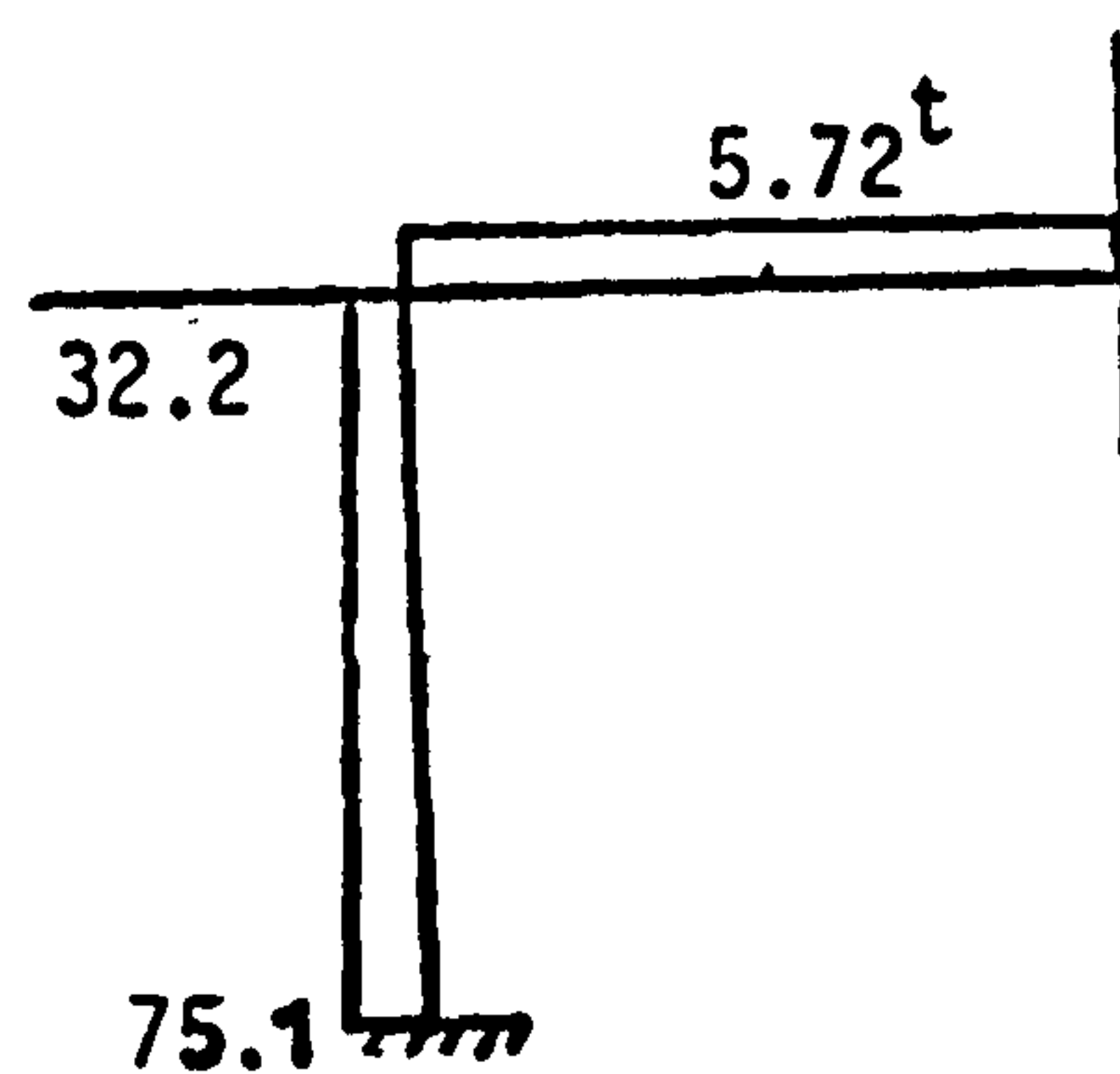


M_1, M_2 Envelope

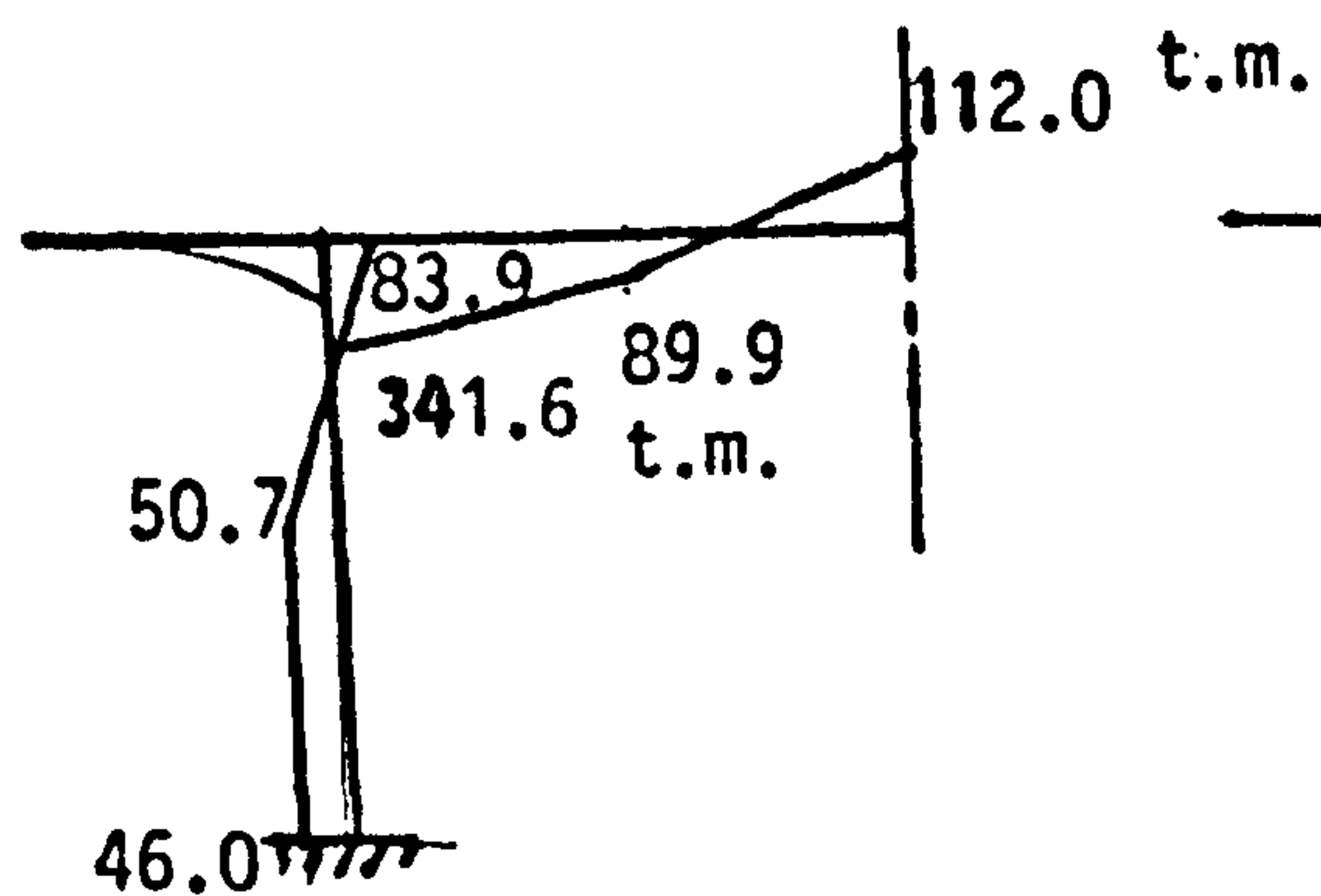
H_1, H_2 Envelope



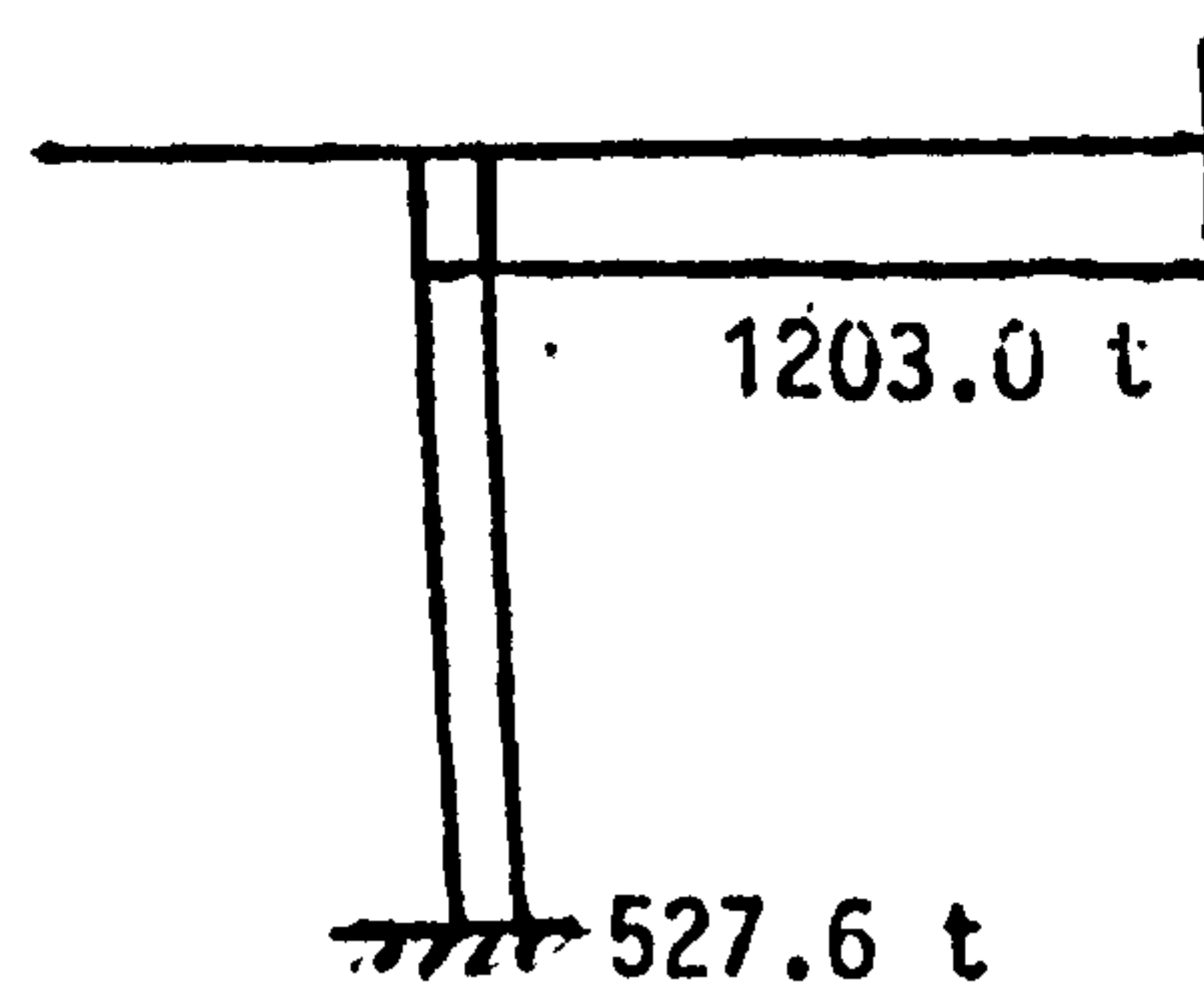
D.L. Moment



D.L. Axial force



the prestressing moment



the prestressing force

Fig. 6.18

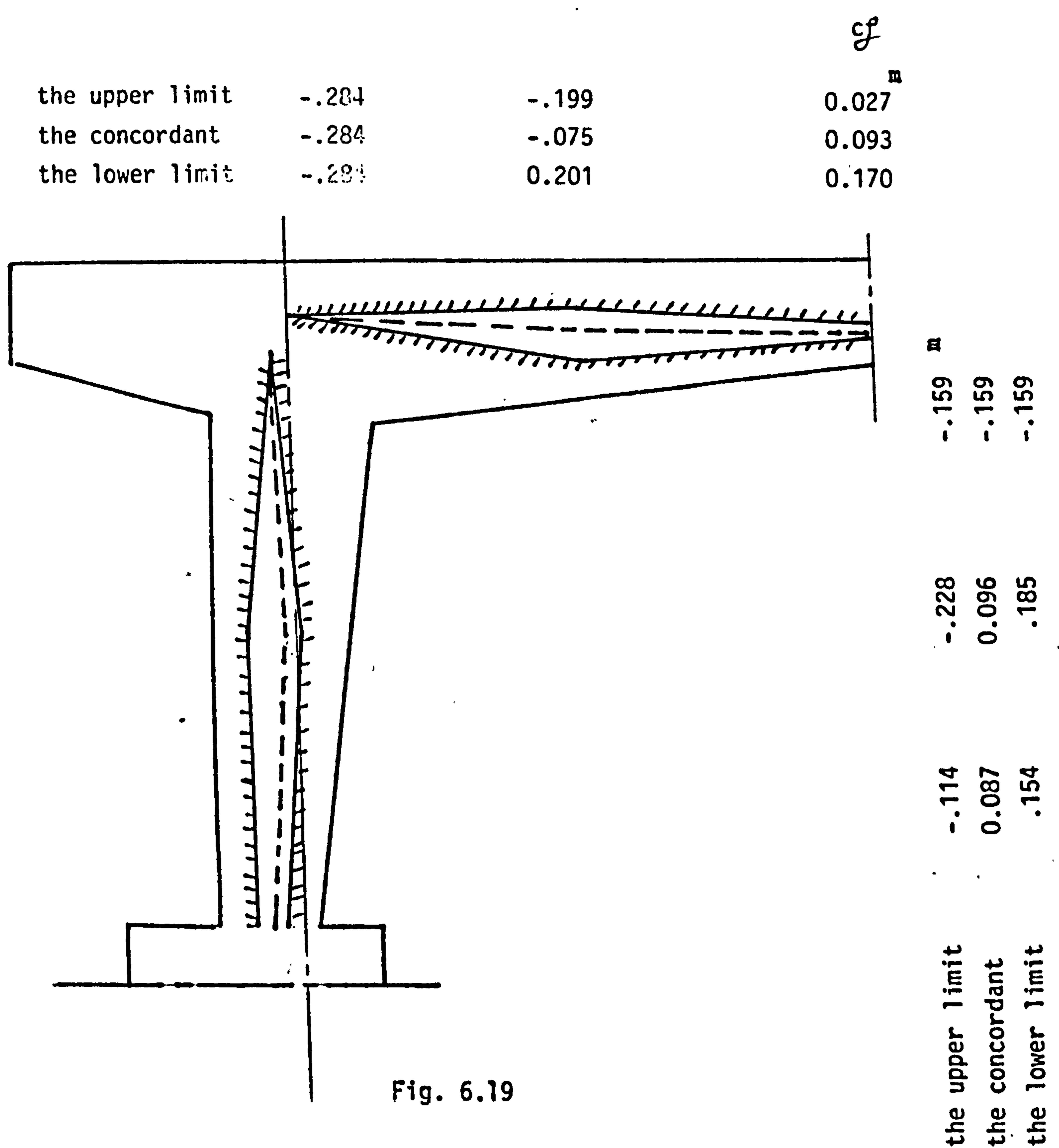
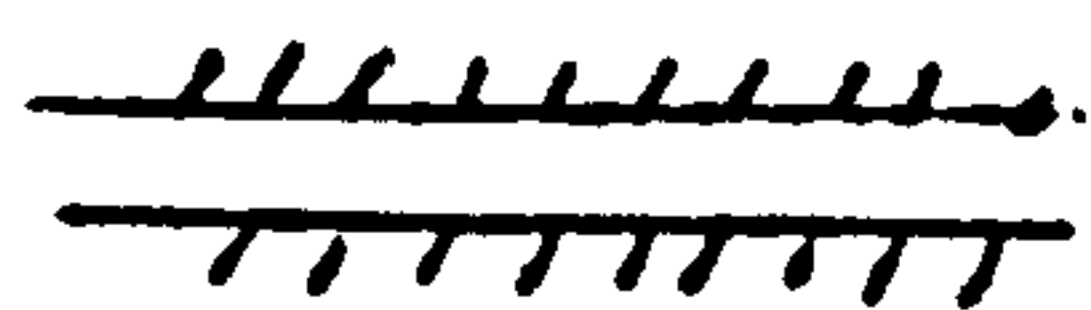


Fig. 6.19



the limiting zone



the concordant profile

Distances measured from the centroid of the section at each station

EXAMPLE 5

As an example of a hinged frame, consider the frame of example 4 with the same dimensions and loading conditions but with hinged supports at the ends.

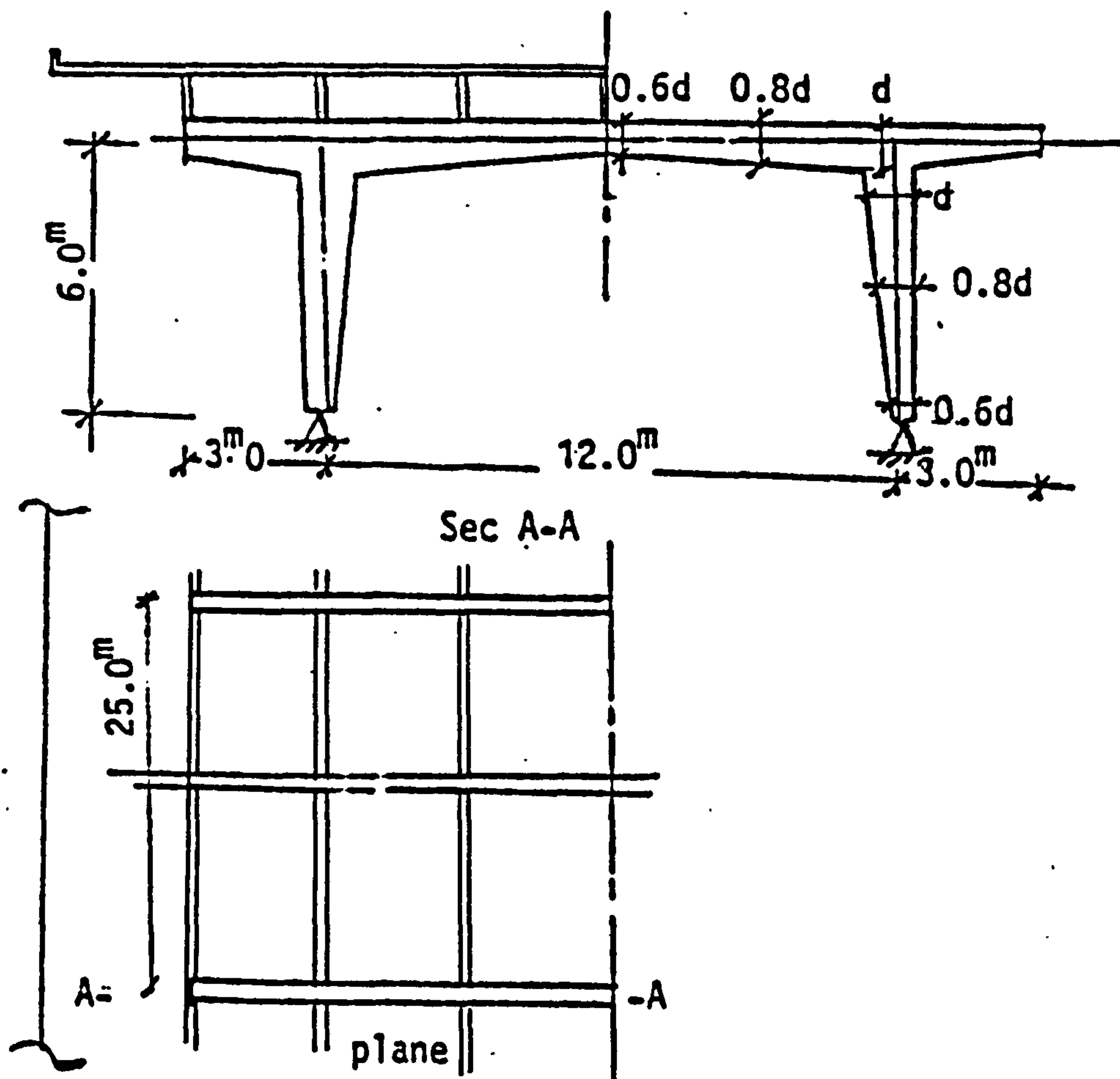
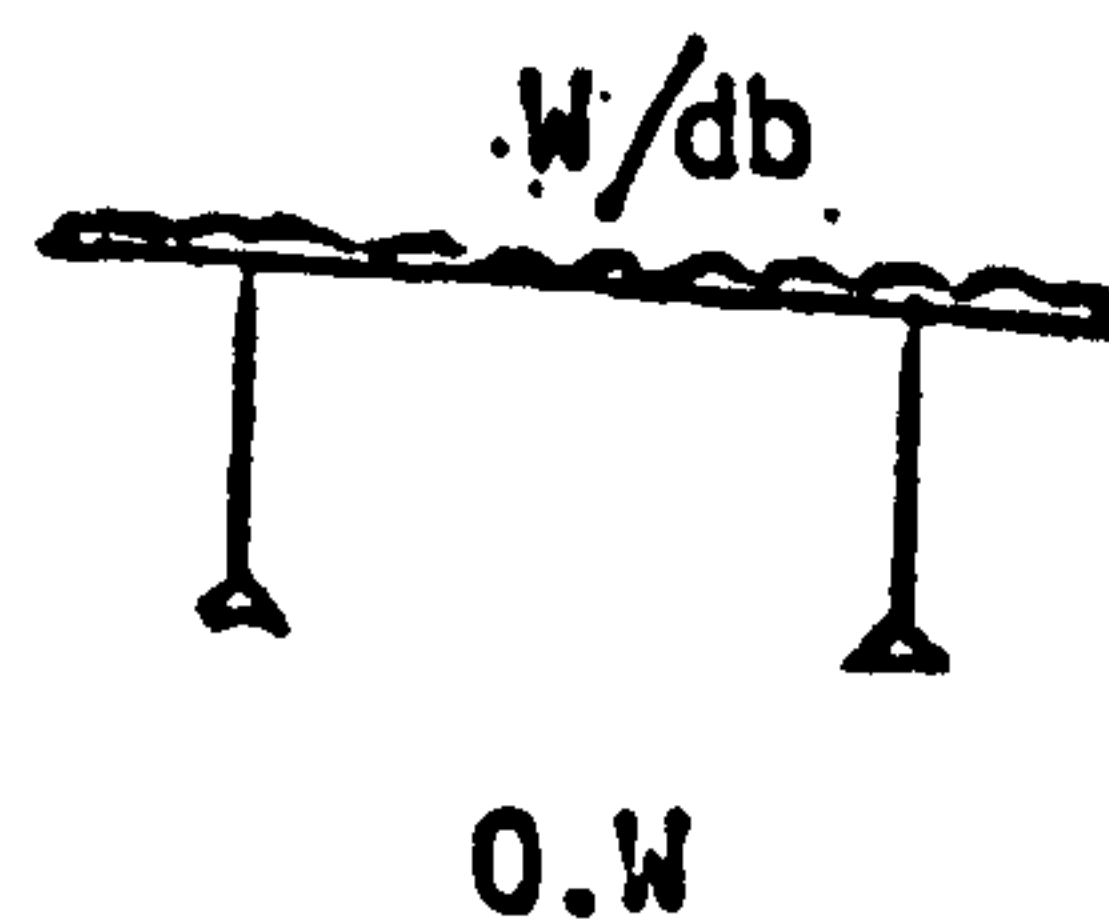
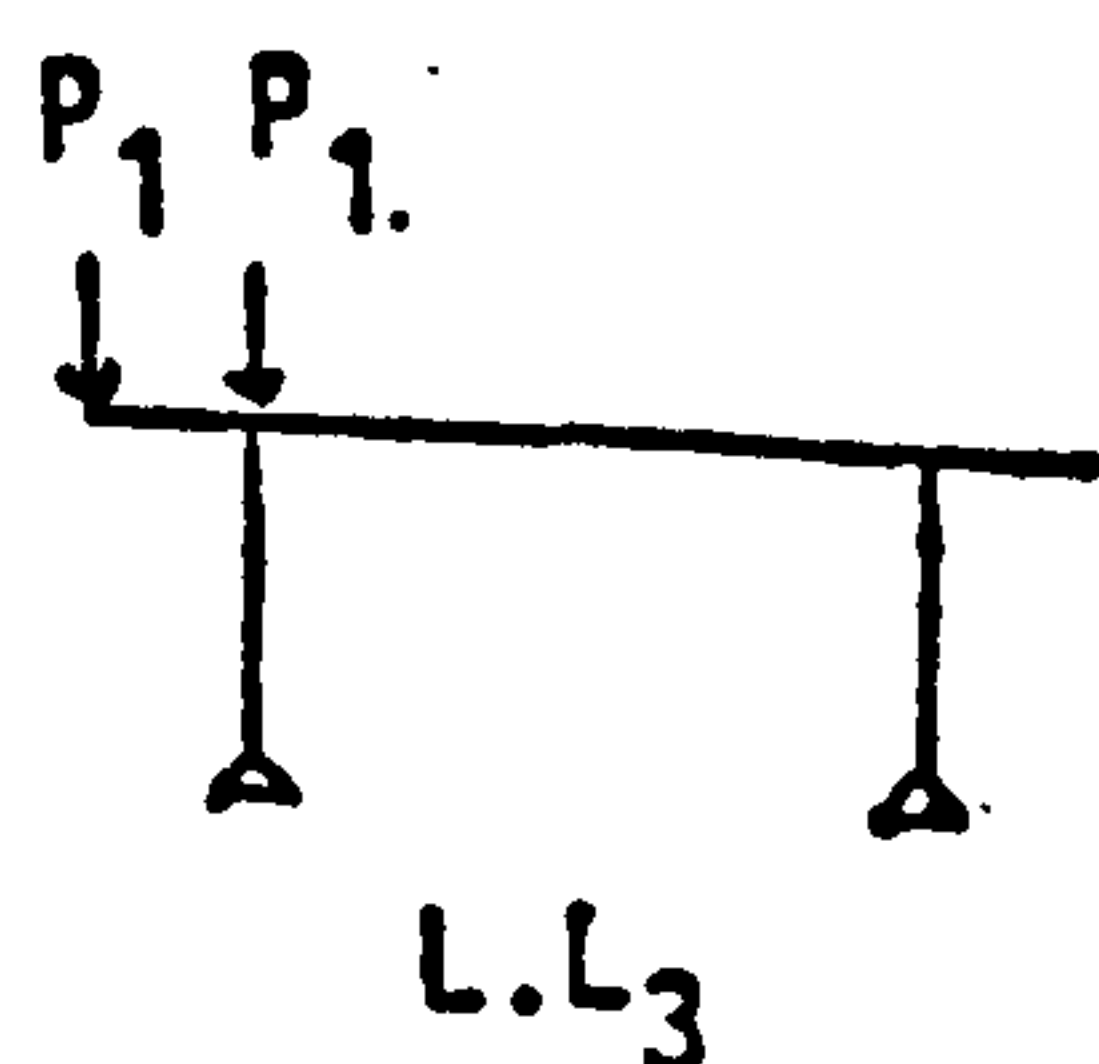
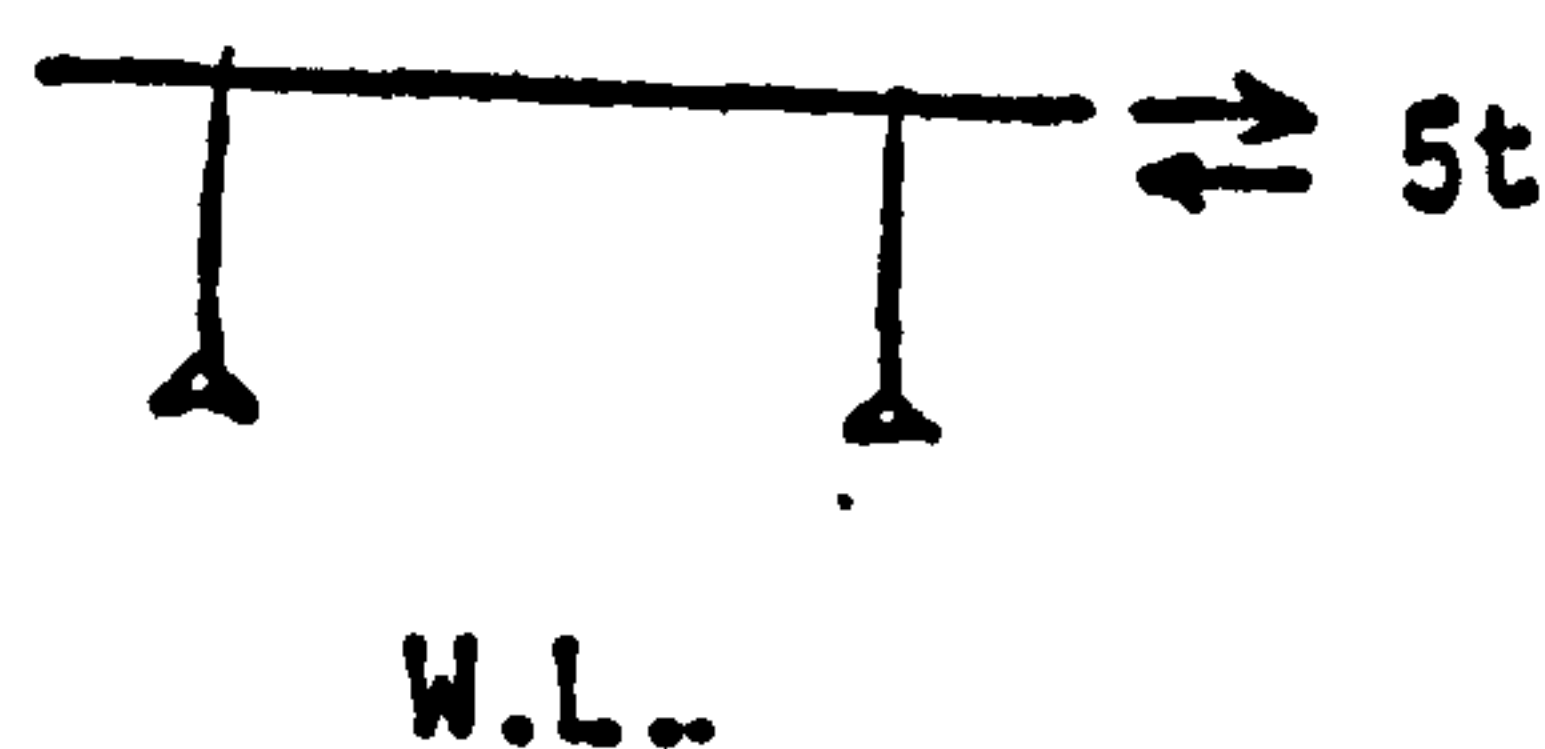
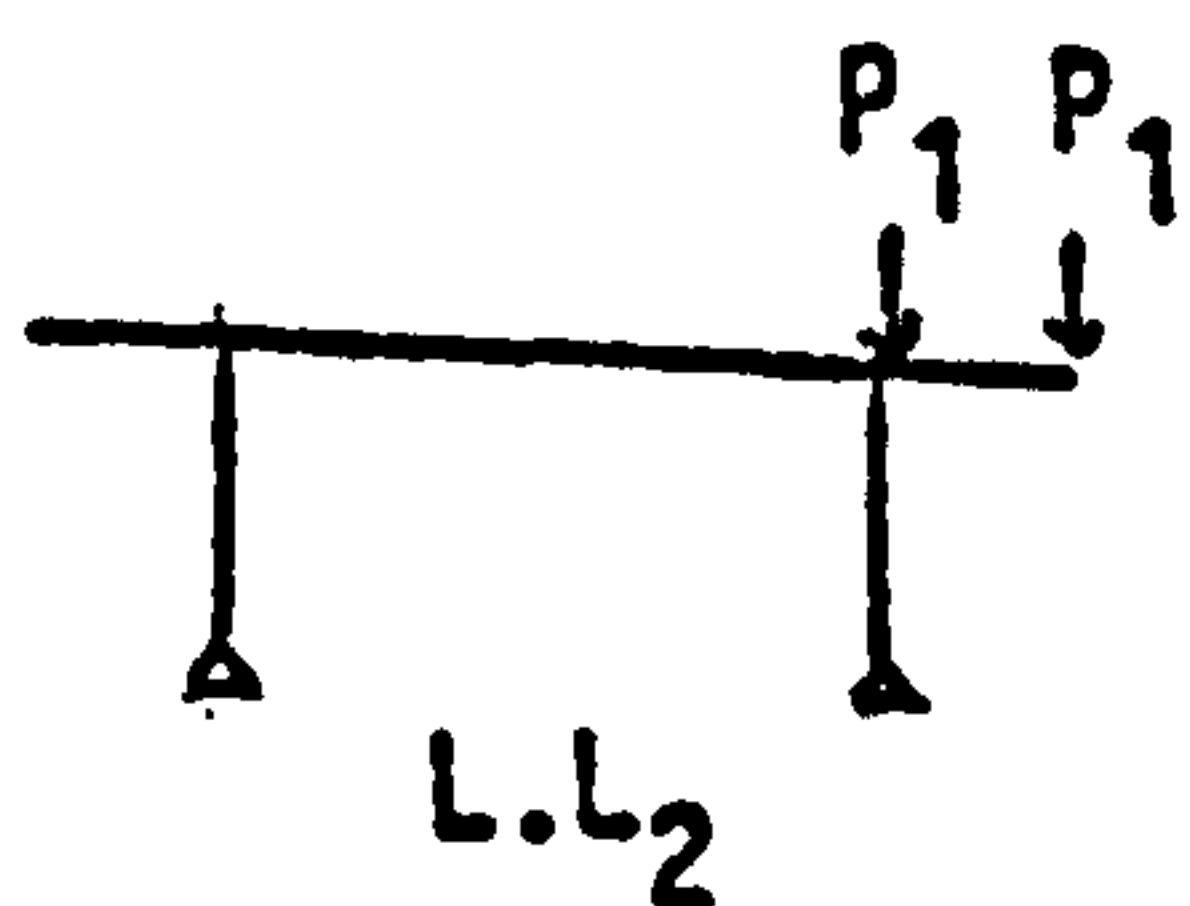
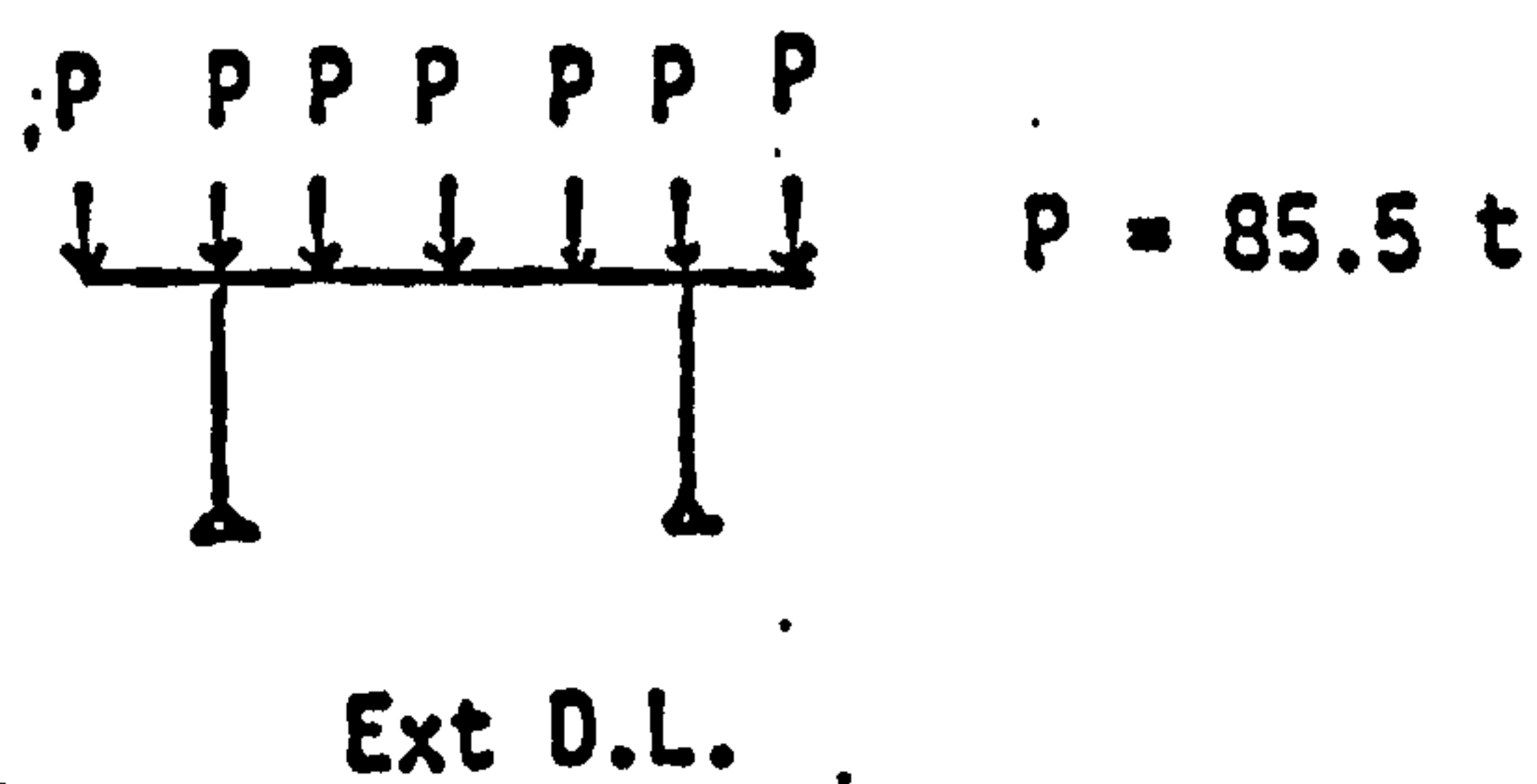
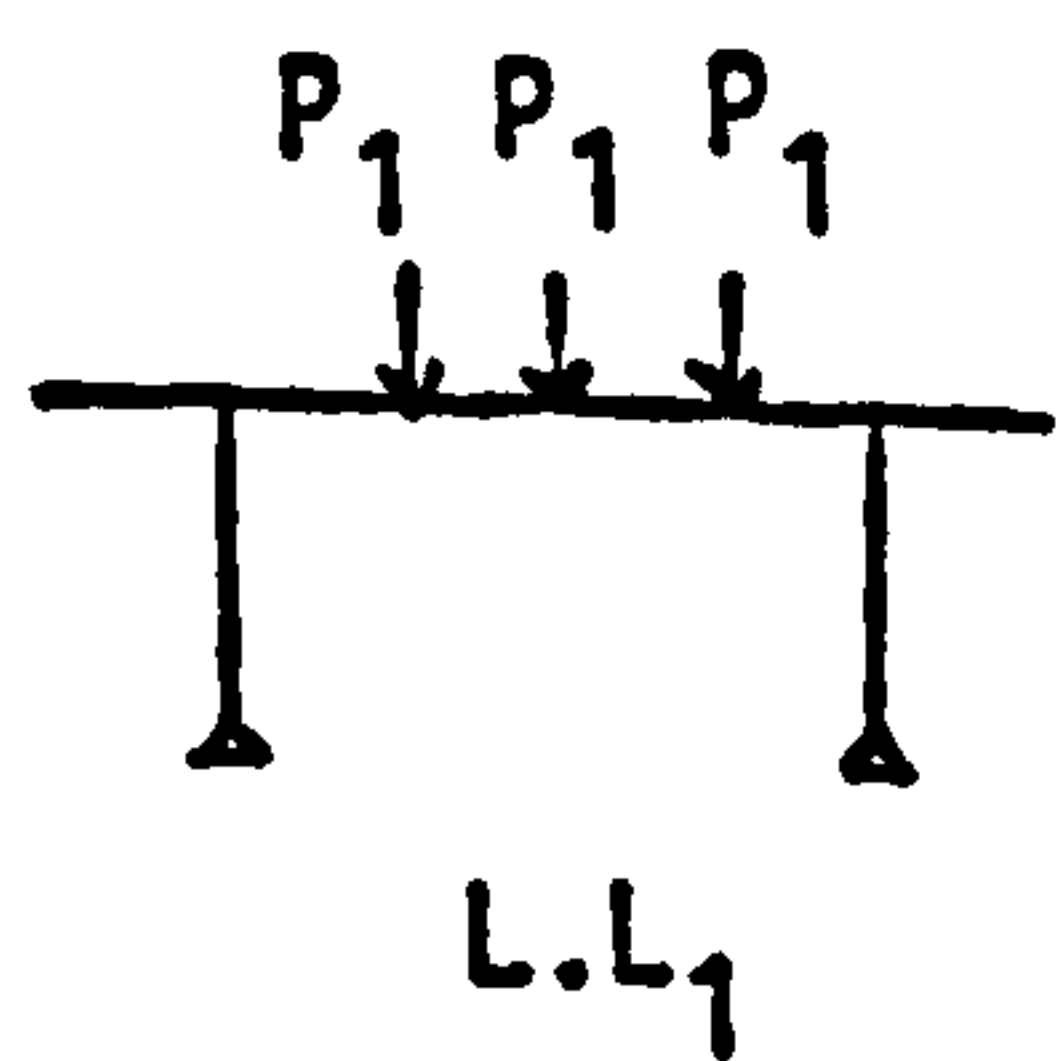


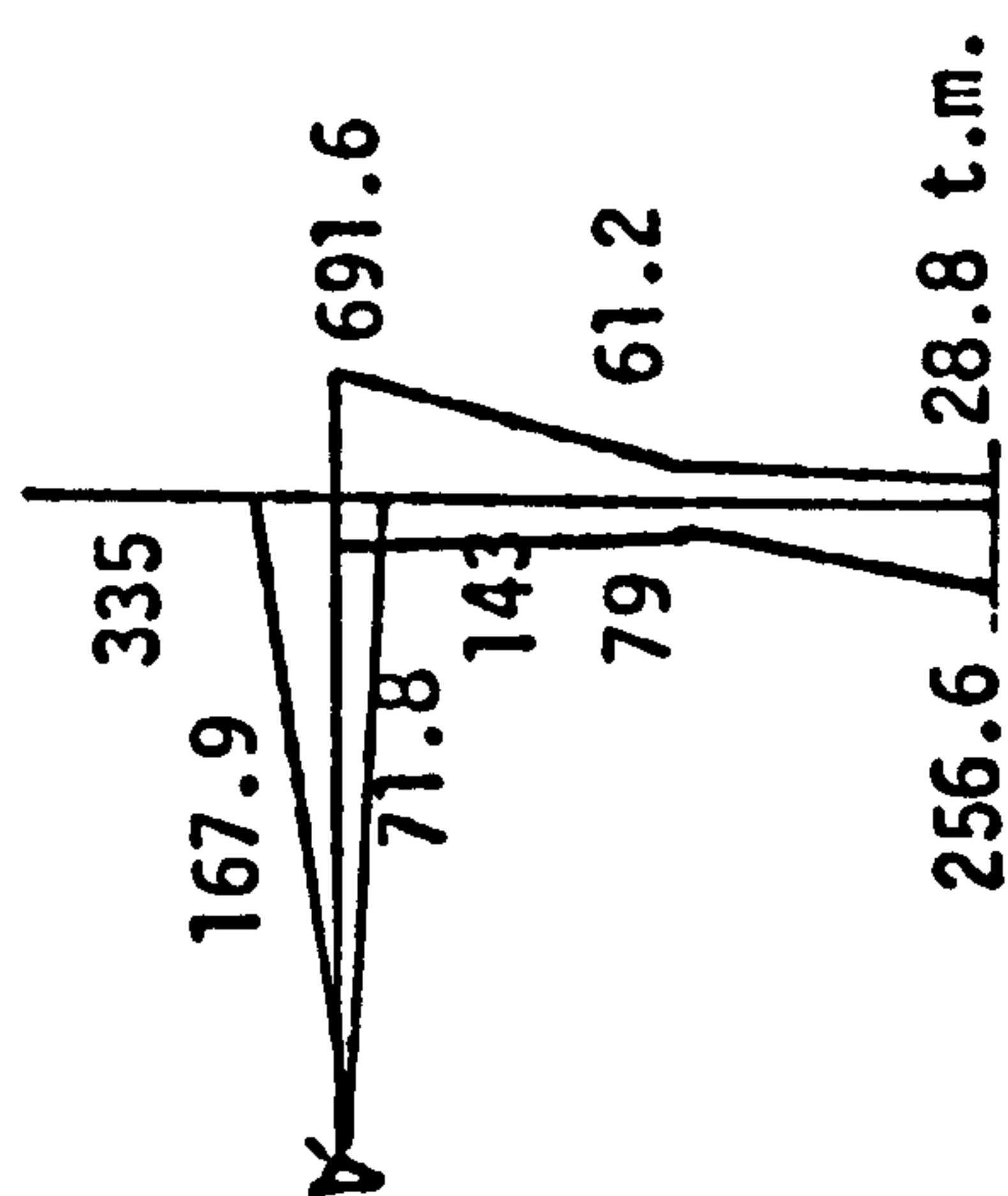
Fig. 6.20

Cases of loadings

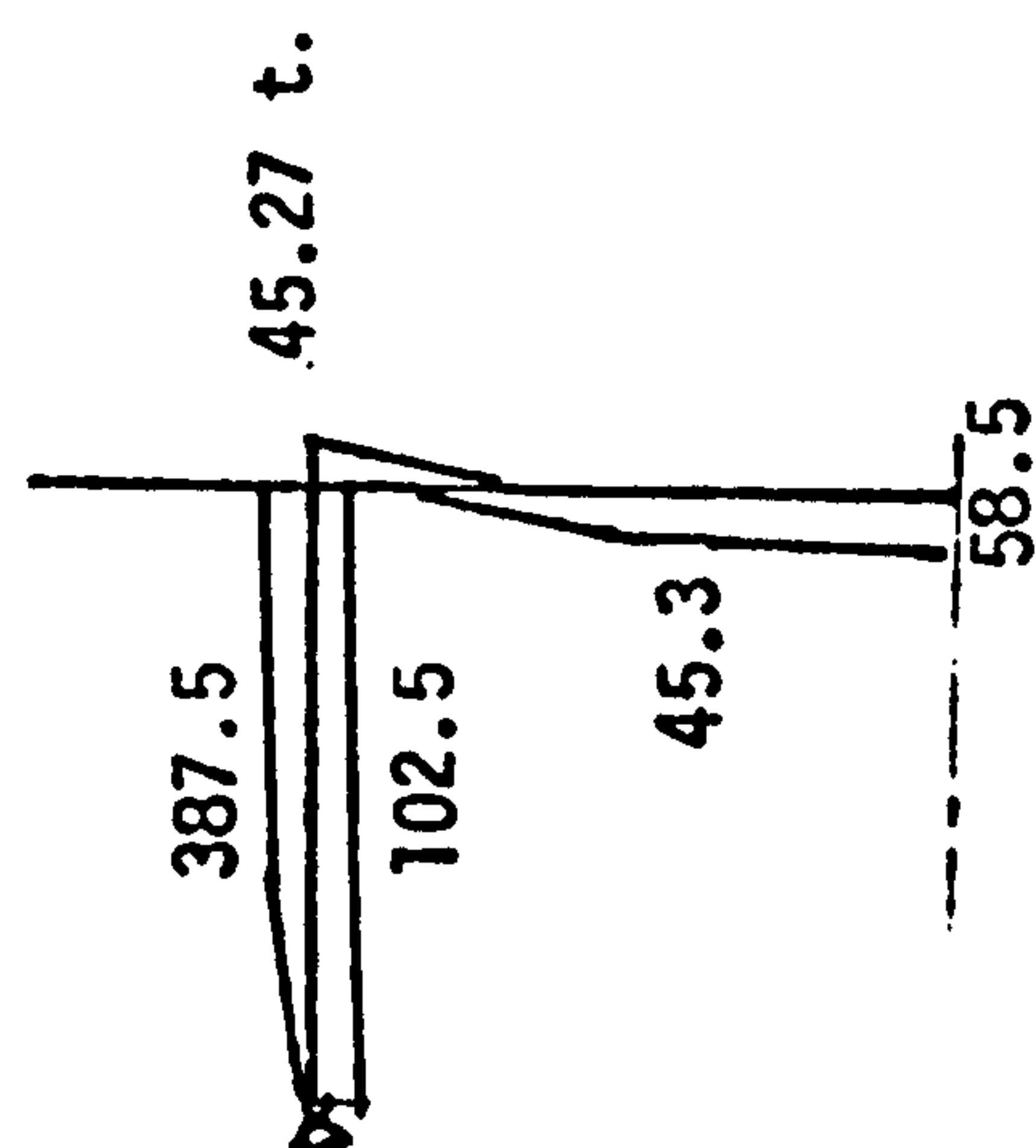


$$P_1 = 52.5 \text{ t}$$

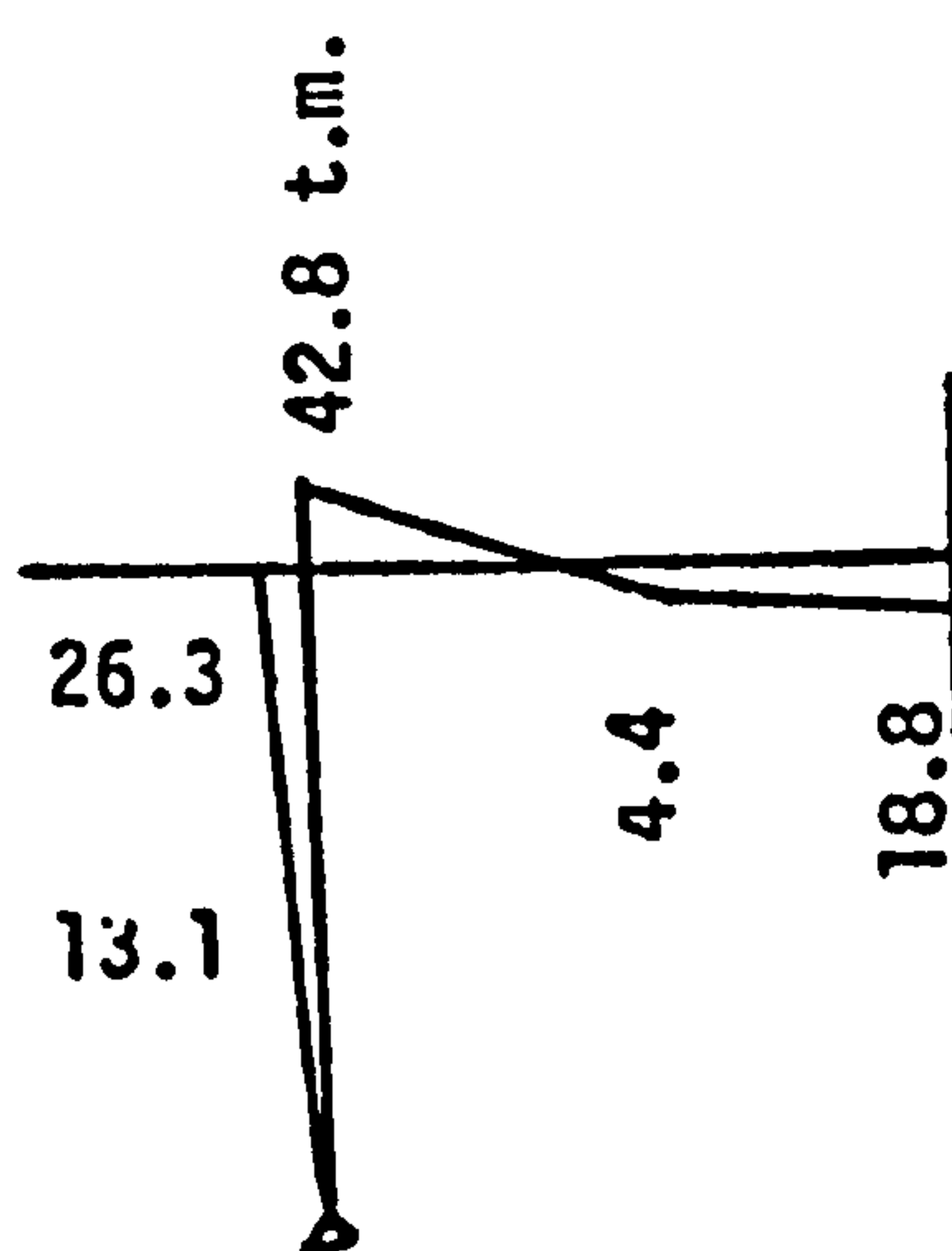
The results were given by the computer program and are shown in Figs. 6.21 and 6.22, the minimum required d is 1.826 m.



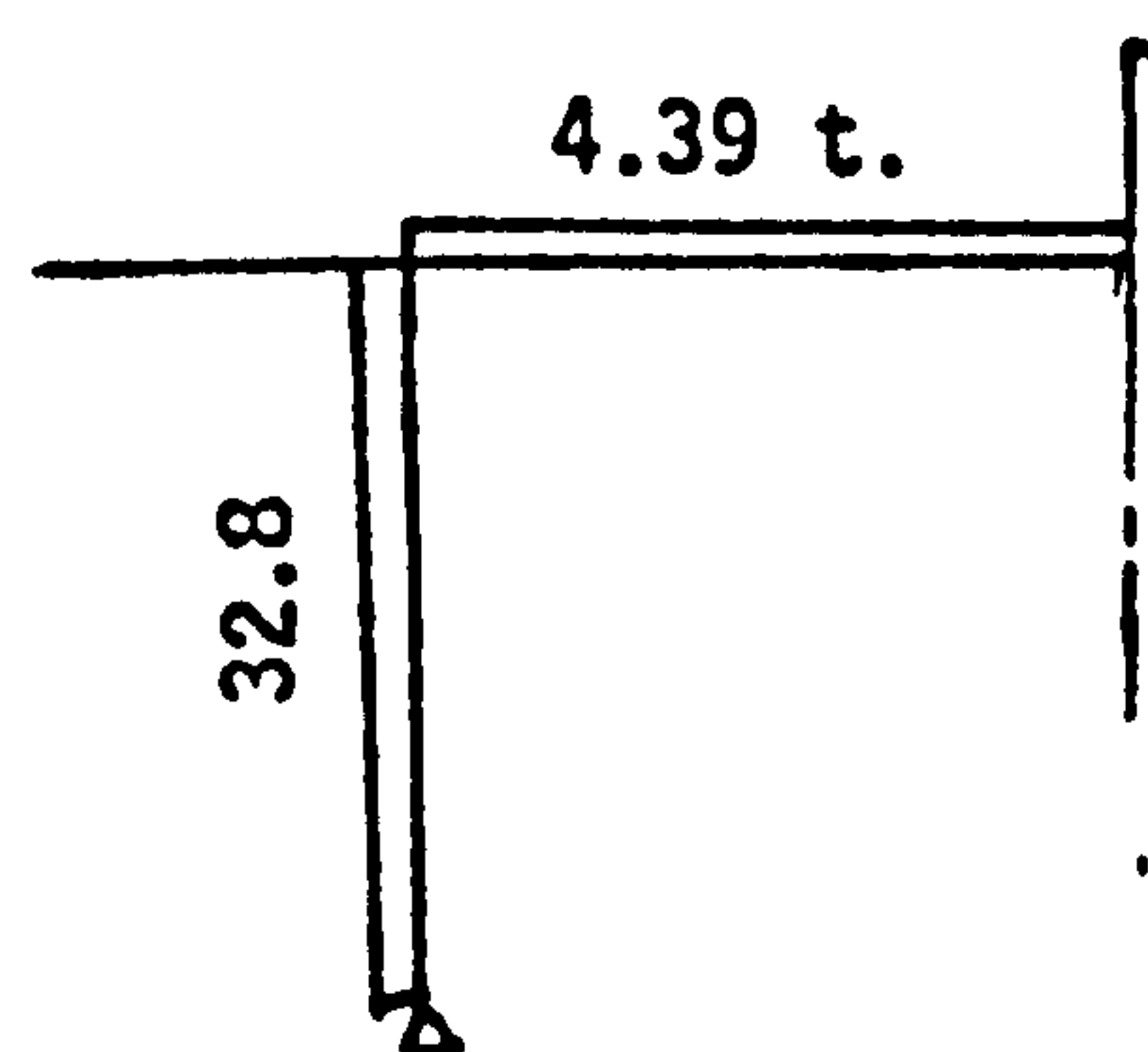
M_1, M_2 Envelope



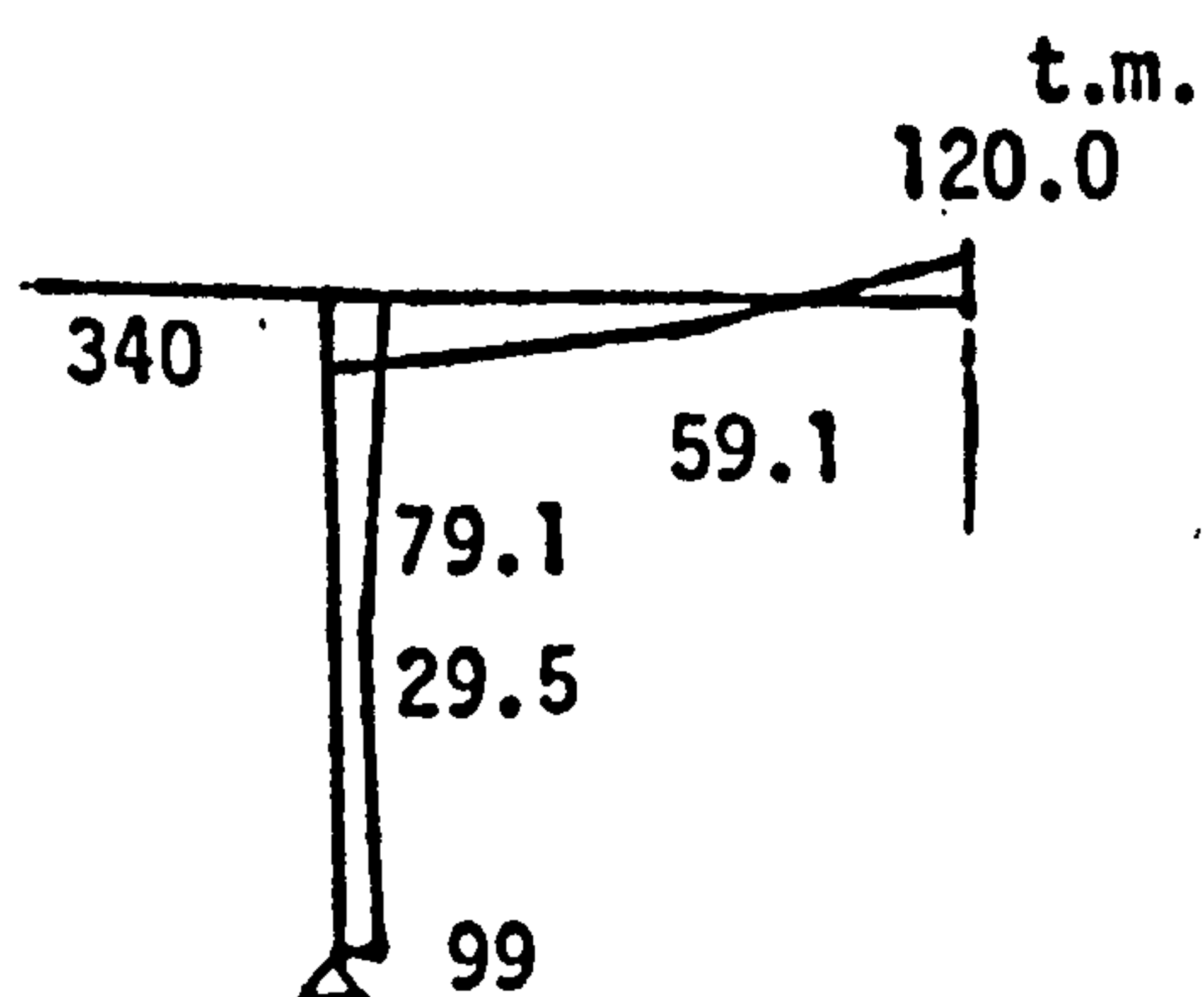
H_1, H_2 Envelope



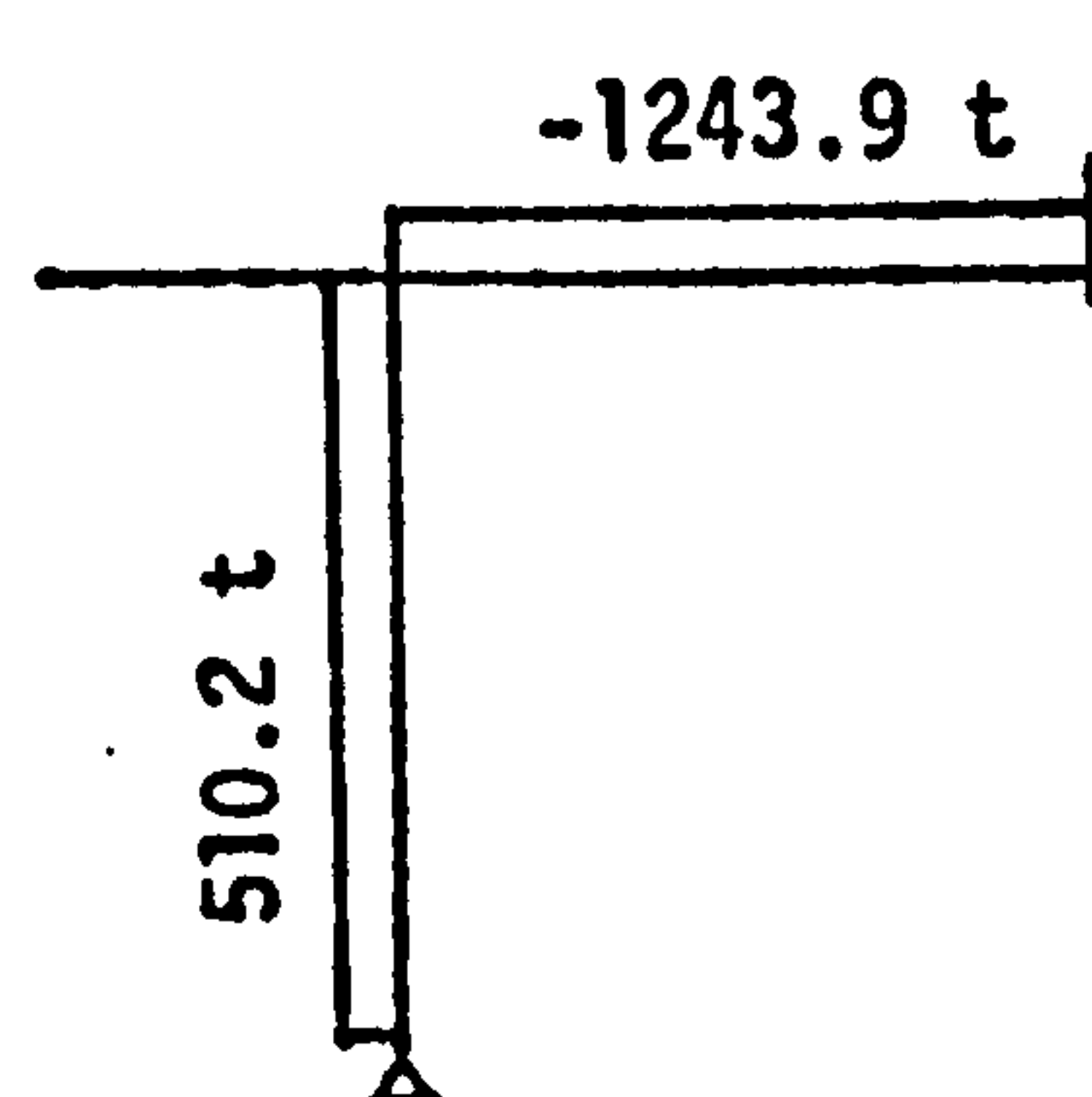
D.L. Moment



D.L. Axial force



The prestressing moment



The prestressing force

Fig. 6.21

The upper limit	-.274	-.186	0.029 ^m
The concordant	-.274	-.048	0.096
The lower limit	-.274	.197	.172

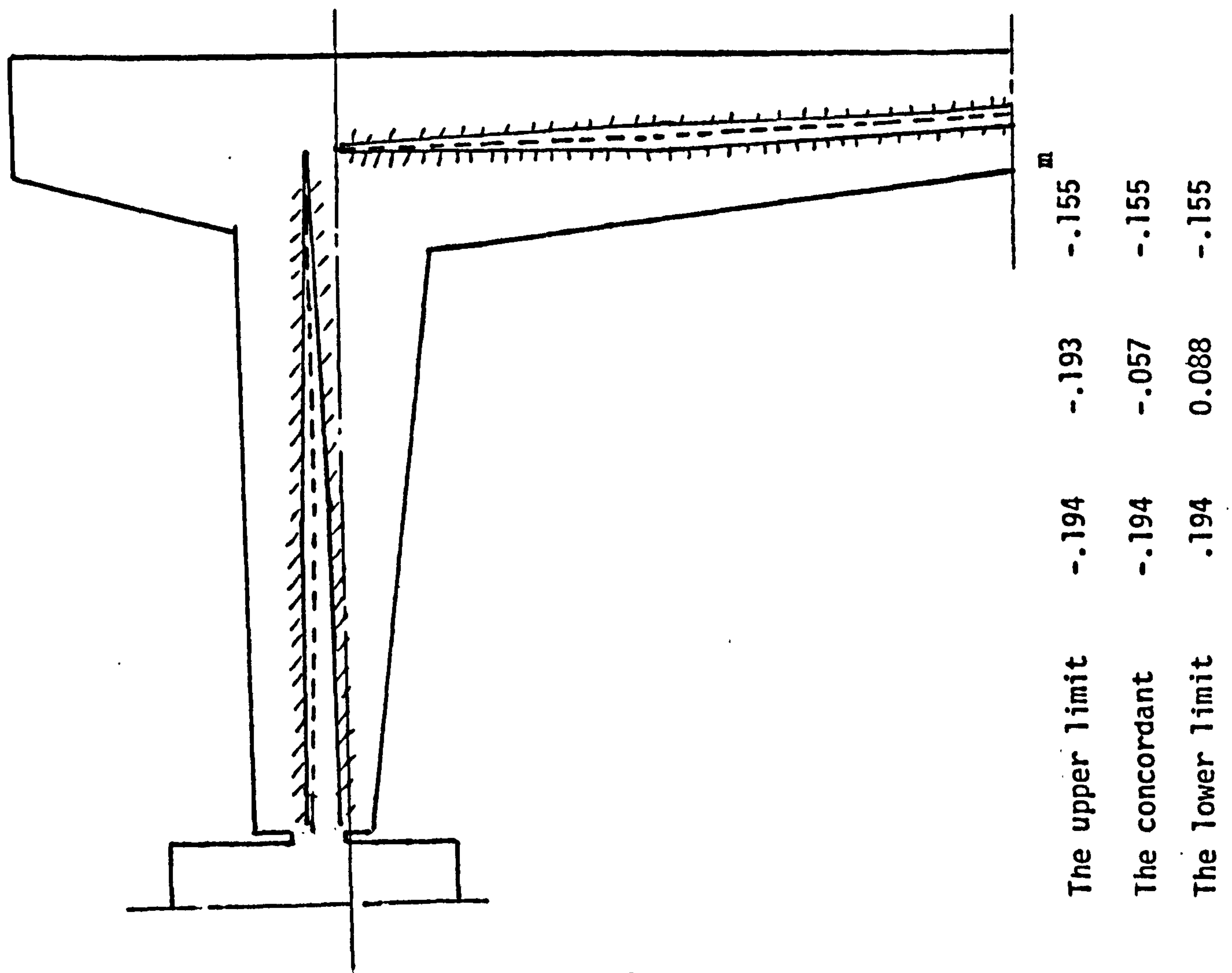
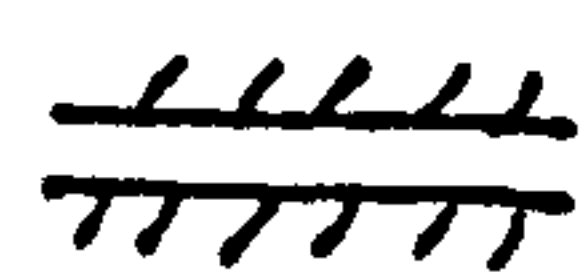


Fig. 6.22



The limiting zone



The concordant profile

Distances measured from the centroid of the section
at each station

CHAPTER (7)

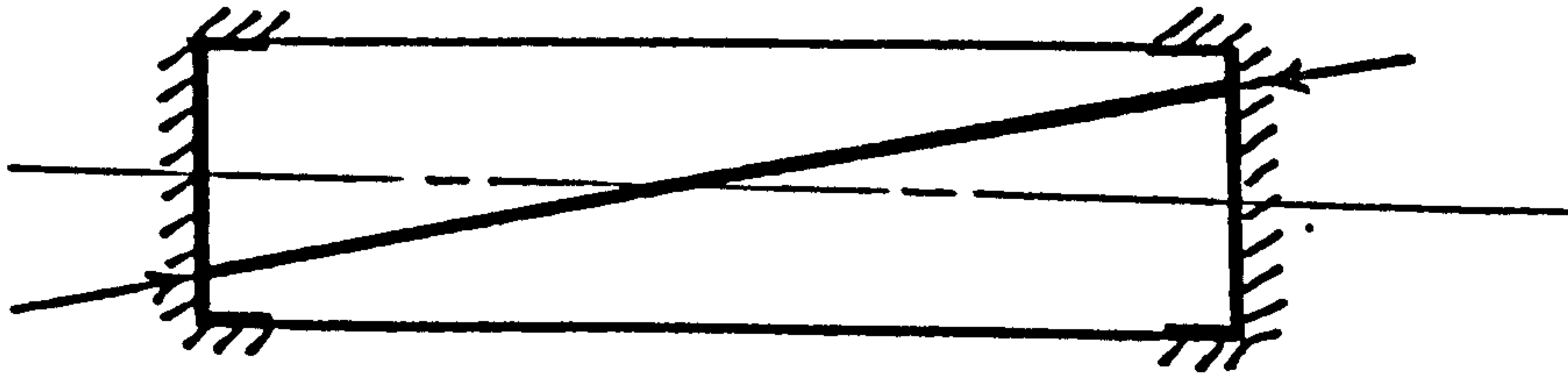
CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

This thesis presents a number of new ideas which, it is believed, will be of direct practical use to the designer of prestressed concrete structures.

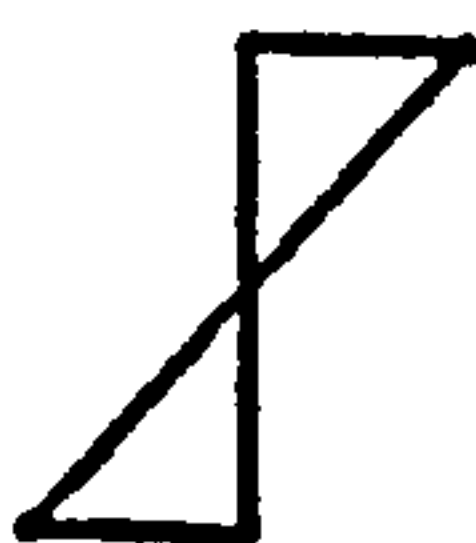
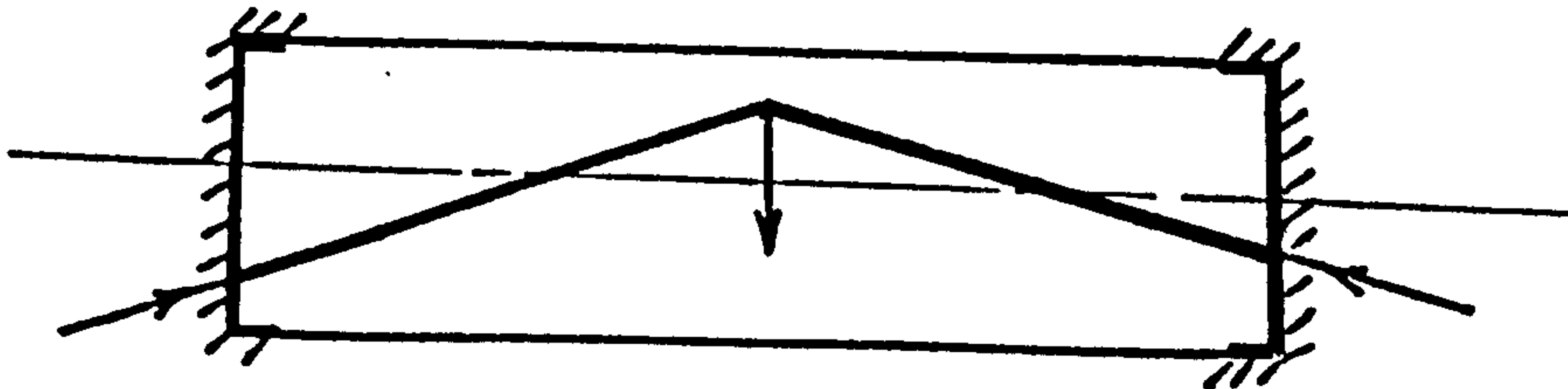
In the first place the usual analysis for prestressed concrete sections has been extended to cover the case of partial prestressing (i.e. allowing defined tensile stresses) and of the effects of axial forces due to the applied loading. This latter effect occurs in frames, arches, shells, tanks and silos structures. It may also acquire some special importance in the case of flat arches and low rise portal frames where the effective axial stress is high. However, the most significant contribution is believed to be that of enabling the designer to obtain a direct solution to the problem of finding concordant profiles within the limiting zones in the case of statically indeterminate structures. Previously, only laborious methods of successive approximation were available. The solution presented here is still dependant upon the flexibility method of structural analysis and the use of chosen stations around the structure. However, the solution process is based upon the optimization of the error functions by the procedure of quadratic programming subject to the constraints on the tendon eccentricities lying within the limiting zone. This technique is capable of being

automated on a computer and a program has been written. Numerical examples demonstrate the way the program can be used for the solution of practical structural problems.

It must be remembered, of course, that it is not theoretically possible to prestress satisfactorily all statically indeterminate structures. For example a fully clamped beam with a straight tendon would experience no stress due to tensioning the cable since all the loads would be transferred to the supports.



However, a change of direction of the tendon (or curvature) within the structure will lead to transverse forces which will introduce bending stresses.



Nevertheless, no direct stresses will result which can provide the prestressing component if the beam is rigidly supported.

This is an extreme example of an effect which can occur in statically indeterminate structures. Such difficulties will be clearly indicated in the application of the computer program, should they exist, since no solution will be forthcoming. As a consequence such errors in design will be avoided.

The main burden of this work was the theoretical study, but an attempt was made to carry out a few simple experiments to obtain some experience of actual structural behaviour. Unfortunately, there were problems with both time and finance in conducting the tests. It was therefore not possible to make as many or as accurate measurements as would have been desirable.

It is suggested that further experimental studies, both in the laboratory and on full sized structures, would be a useful extension to the general body of knowledge of the behaviour of prestressed concrete structures.

REFERENCES

1. Abbles, P.W. and Bardhan, B.K.: "Prestressed Concrete Designer's Hand Book" Concrete Publications, London 1981.
2. Abbles, P.W.: "Introduction to Prestressed Concrete" Concrete Publications, London Volume 1 1963.
3. Abbles, P.W.: "Introduction to Prestressed Concrete" Concrete Publications, London Volume 2 1965.
4. Allen, A.H. "An Introduction to Prestressed Concrete" Cement and Concrete Association, London 1978.
5. Au and Stolen: "Introduction to Systems Engineering, Deterministic Models" Addison-Wesley Publishing Co. 1969.
6. Brebbia, C.A. and Ferrante, A.J.: "Computational Methods for the Solution of Engineering Problems" Crane, Russak & Co. Inc, New York, 1978.
7. British Standard Institution: "Code of Practice for Structural Use of Concrete (p. 110).
8. Chi, Diberstein: "Theory of Prestressed Concrete" Englewood Cliffs, N.J. Prentice Press 1963.
9. Cowan, H.J.: "Theory of Prestressed Concrete Design" London, Macmillan, New York St. Martin 1956.
10. Crandall, S.H.: "Engineering Analysis" McGraw Hill New York, 1956.
11. Farina: "Elementary Basic with Applications" Prentice-Hall U.S.A. 1970.
12. Kong, F.K. and Evans, R.H.: "Reinforced and Prestressed Concrete" Van Nostrand and Reinhold Co. Ltd. 1983.
13. Graves Smith, T.R.: "Linear Analysis of Frameworks" Ellis Horwood Limited 1983.
14. Guyon, Y.: "A Theoretical Treatment of Continuity in Prestressed Concrete" Proceedings of Symposium on Prestressed Concrete Statically Indeterminate Structures. Cement and Concrete Association, London, 1953 pp 131-145.
15. Guyon, Y.: "Limit-State Design of Prestressed Concrete". London, Applied Science, Volume 1, 1972.
16. Guyon, Y.: "Limit-State Design of Prestressed Concrete". London, Applied Science Volume 2, 1974.

17. Guyon, Y.: "Prestressed Concrete" Volume II Statically Indeterminate Structures. London, New York 1960.
18. Jabakharzi, M.: "M.Sc. Thesis, Southampton Univ." 1977.
19. Kong, Evans: "Reinforced and Prestressed Concrete" London, Nelson 1975.
20. Loentivich: "Frames and Arches" Eng. Societies monographs 1959.
21. Leonhart, F.: "Prestressed Concrete Design and Construction" Berlin 1964.
22. Libby: "Modern Prestressed Concrete Design Principles and Construction Methods". San Diego, California, 1971.
23. Lin, T.Y.: "Load-Balancing Method for Design and Analysis of Prestressed Concrete Structures". Journal of American Concrete Institute, June 1963, pp. 719.
24. Morice, P.B.: "A Direct Design Method for Statically Indeterminate Prestressed Concrete Structures" Proceedings of the World Conference on Prestressed Concrete, July 1957.
25. Morice, P.B.: "The Analysis of Prestressed Concrete Structures and the Application of the Recent Research" Proceedings of the Institute of Civil Engineering, Vol. 6. March 1957. pp. 445-492.
26. Morice, P.B. and Cooley: "Prestressed Concrete Theory and Practice". Sir Isaac Pitman, Sons, Ltd. London 1958.
27. Morice, P.B. and Lewis, H.E.: "Prestressed Continuous Beams and Frames" Proceedings of the American Society of Civil Engineering. Vol. 82, September 1956. Structural Division Paper 1055 pp. 29.
28. Moy, S.S.J.: "Plastic Methods for Steel and Concrete Structures" London, Macmillan 1981.
29. Taha, H.A. : "Operation Research" Macmillan Publishing Co. Inc. New York, 1976.
30. "Undemi Siècle de Technique Française de la Précontrainte". Association Scientifique de la Précontrainte. Travaux: Organe, de la Technique Française de Travaux Publics et du Ciment Armé 6, Avenue Pierre, DEE Serbie - Paris.

APPENDIX A

Computer program for prestressed concrete structures

```

1  C                      MAIN PROGRAM
2  C
3  C
4  C      PRESTRESSED CONCRETE DESIGN AND FLEXIBILITY METHOD PROGRAM
5  C      COMPUTS THE DESIGN OF RECTANGULAR CROSS SECTION STATICALLY
6  C      INDETERMINATE STRUCTURES, AND THE CONCORDANT PROFILE FOR ONCE
7  C      (SYMETRICALLY, UNSYMETRICALLY) OR TWICE (SYMETRICALLY) INDETERMINATE
8  C      STRUCTURES BY THE DIRECT METHOD.
9  C
10     DIMENSION H(100,100), XN(100,100), V(100,100), T(100,100)
11     &, XM(100), Y(100,100), Z(100,100), T1(100,100), X(100,100)
12     &, XK(100), XK1(100), XK2(100), C(100), S1(100), NS2(100)
13     &, XM1(100), H1(100), XM2(100), H2(100), XK3(100), IV1(100)
14     INTEGER W, S, R
15     WRITE(6,8)
16     8 FORMAT(5X, 'THE STRUCTURE, INPUT 1 FOR BEAM, 0 FOR FRAME OR ARCH?')
17     READ(5,*) IST
18     WRITE(6,9) IST
19     9 FORMAT(5X, I3)
20     WRITE(6,11)
21     11 FORMAT(5X, 'NO. OF SPANS, INDETERMIANCY, NO. OF LOADING'
22     *' CONDITIONS ?')
23     READ(5,*) N1, N, W
24     WRITE(6,46) N1, N, W
25     46 FORMAT(5X, 3I10)
26     WRITE(6,13)
27     13 FORMAT(5X, 'THE LENTH OF EACH SPAN IN MATRIX S1(N1) ?')
28     CALL INPUT(S1, N1, 1)
29     WRITE(6,14)
30     14 FORMAT(5X, 'THE NO. OF STATIONS PER EACH SPAN IN MATRIX NS2(N1)?')
31     DO 15 I=1, N1
32     READ(5,*) NS2(I)
33     15 WRITE(6,49) NS2(I)
34     49 FORMAT(5X, I10)
35     S=0
36     DO 20 I=1, N1
37     S=S+NS2(I)
38     20 CONTINUE
39     R=2*S
40     WRITE(6,17)
41     17 FORMAT(5X, 'THE MOMENTS DUE TO UNIT LOADS AT THE RELASE POINTS')
42     WRITE(6,25)
43     25 FORMAT(5X, 'IN MATRIX H(S,N) BY ROWS ?')
44     CALL INPUT (T1, S, N)
45     DO 30 I=1, S
46     DO 30 J=1, N
47     H(I,J)=T1(I,J)
48     30 CONTINUE
49     IF(IST.EQ.1) GO TO 29
50     WRITE(6,21)
51     21 FORMAT(5X, 'THE NORMAL FORCES DUE TO UNIT LOADS AT THE RELASE')
52     WRITE(6,35)
53     35 FORMAT(5X, 'POINTS IN MATRIX H(S,N) BY ROWS ?')
54     CALL INPUT(T1, S, N)
55     DO 40 I=1, S
56     DO 40 J=1, N
57     H(I+S,J)=T1(I,J)
58     40 CONTINUE
59     GO TO 37

```

```

60      29 DO 33 I=1,S
61          DO 33 J=1,N
62      33 H(I+S,J)=0.
63      37 WRITE(6,23)
64      23 FORMAT(5X,'THE APPLIED LOAD MATRIX MO(S,W) BY ROWS?')
65          CALL INPUT(T1,S,W)
66          DO 50 I=1,S
67          DO 50 J=1,W
68          XN(I,J)=T1(I,J)
69      50 CONTINUE
70          IF(IST.EQ.1)GO TO 38
71          WRITE(6,26)
72      26 FORMAT(5X,'THE APPLIED LOAD MATRIX NO(S,W) BY ROWS ?')
73          CALL INPUT(T1,S,W)
74          DO 60 I=1,S
75          DO 60 J=1,W
76          XN(I+S,J)=T1(I,J)
77      60 CONTINUE
78          GO TO 39
79      38 DO 41 I=1,S
80          DO 41 J=1,W
81      41 XN(I+S,J)=0.
82      39 WRITE(6,32)
83      32 FORMAT(5X,'THE INERTIA, INPUT 1 FOR CONSTANT, 0 FOR VARIABLE')
84          READ(5,*)INERT
85          WRITE(6,64)INERT
86      64 FORMAT(5X,I3)
87          IF(INERT.EQ.1)GO TO 36
88          WRITE(6,27)
89      27 FORMAT(5X,'THE INERTIA RATIOS EI AT EACH STATIONS IN MATRIX'
90          * 'K(S) ?')
91          CALL INPUT(XK,S,1)
92          DO 70 I=1,S
93          XK1(I)=XK(I)
94      70 CONTINUE
95          IF(IST.EQ.1)GO TO 36
96          WRITE(6,28)
97      28 FORMAT(5X,'THE AREA RATIOS (EA) AT EACH STATION IN MATRIX K(S) ?')
98          CALL INPUT (XK,S,1)
99          DO 80 I=1,S
100         XK1(I+S)=XK(I)
101      80 CONTINUE
102          GO TO 62
103      36 DO 44 I=1,S
104          XK1(I)=1.
105      44 XK1(I+S)=1.
106      62 DO 61 I=1,S
107      61 XM(I)=XK1(I+S)
108          L=0
109          DO 90 I=1,N1
110          L=L+NS2(I)
111          IV1(I)=L
112      90 CONTINUE
113          DO 95 I=1,N1
114          JJ=(IV1(I)-NS2(I))+1
115          KK=IV1(I)
116          DO 950 J=JJ,KK
117      950 XK2(J)=S1(I)/((NS2(I)-1)*3)
118      95 CONTINUE
119          DO 100 I=1,S
120          XK2(I+S)=XK2(I)
121      100 CONTINUE

```

```

122      CALL SIMPSON(C,N1,NS2,IV1)
123      DO 110 I=1,S
124      XK3(I+S)=C(I)
125      XK3(I)=C(I)
126      110 CONTINUE
127      WRITE(6,600)
128      600 FORMAT(5X,'THE WIDTH,THE ALLOWABLE CONCRETE STRESS ?')
129      READ(5,*)B1,F1
130      WRITE(6,605)B1,F1
131      605 FORMAT(5X,3F15.5)
132      R1=1.
133      GO TO 610
134      750 DO 745 I=1,S
135      XK1(I)=XK1(I)*(R1**3)/(R2**3)
136      745 XK1(I+S)=XK1(I+S)*R1/R2
137      C      FORM THE FLIXIBILITY MATRIX
138      610 DO 115 I=1,N
139      DO 115 J=1,N
140      V(I,J)=0.
141      R=2*S
142      DO 115 L=1,R
143      V(I,J)=V(I,J)+(H(L,I)*H(L,J)*XK2(L)*XK3(L)/XK1(L))
144      115 CONTINUE
145      C      INVERSE OF THE FLIXIBLITY MATRIX
146      CALL INVER(V,N)
147      DO 130 I=1,N
148      DO 130 J=1,W
149      T(I,J)=0.
150      DO 130 L=1,R
151      T(I,J)=T(I,J)+H(L,I)*XN(L,J)*XK2(L)*XK3(L)/XK1(L)
152      130 CONTINUE
153      DO 135 I=1,N
154      DO 135 J=1,W
155      Y(I,J)=0.
156      DO 135 K=1,N
157      Y(I,J)=Y(I,J)+V(I,K)*T(K,J)
158      135 CONTINUE
159      DO 140 I=1,R
160      DO 140 J=1,W
161      Z(I,J)=0.
162      DO 140 K=1,N
163      Z(I,J)=Z(I,J)+H(I,K)*Y(K,J)
164      140 CONTINUE
165      DO 145 I=1,R
166      DO 145 J=1,W
167      X(I,J)=XN(I,J)-Z(I,J)
168      145 CONTINUE
169      DO 150 I=1,S
170      DO 150 J=1,W
171      T1(I,J)=X(I,J)
172      150 CONTINUE
173      IF(R1.EQ.1.)GO TO 177
174      IF(R2.LT.R1)GO TO 177
175      WRITE(6,5)
176      5 FORMAT(5X,'THE FINAL BENDING MOMENTS RESULTANTS')
177      CALL OUTPUT(T1,S,W)
178      DO 160 I=1,S
179      DO 160 J=1,W
180      T1(I,J)=X(I+S,J)
181      160 CONTINUE

```

```

182      WRITE(6,12)
183      12 FORMAT(5X,'THE CORRESPONDING AXIAL FORCES RESULTANTS')
184      CALL OUTPUT (T1,S,W)
185      177 CONTINUE
186      DO 180 I=1,S
187      XM1(I)=0.
188      H1(I)=0.
189      XM2(I)=0.
190      H2(I)=0.
191      DO 180 J=1,W-1
192      IF(X(I,J))170,170,165
193      165 XM1(I)=XM1(I)+X(I,J)
194      H1(I)=H1(I)+X(I+S,J)
195      GO TO 180
196      170 XM2(I)=XM2(I)+X(I,J)
197      H2(I)=H2(I)+X(I+S,J)
198      180 CONTINUE
199      IF(R1.EQ.1.)GO TO 755
200      IF(R2.LT.R1)GO TO 755
201      WRITE(6,16)
202      16 FORMAT(5X,'THE HOGGING MOMENT M1 VALUES')
203      CALL OUTPUT(XM1,S,1)
204      WRITE(6,18)
205      18 FORMAT(5X,'THE AXIAL FORCE H1 CORRESPONDING TO M1 VALUES')
206      CALL OUTPUT(H1,S,1)
207      WRITE(6,22)
208      22 FORMAT(5X,'THE SAGGING MOMENT M2 VALUES')
209      CALL OUTPUT (XM2,S,1)
210      WRITE(6,24)
211      24 FORMAT(5X,'THE AXIAL FORCE H2 CORRESPONDING TO M2 VALUES')
212      CALL OUTPUT(H2,S,1)
213      755 CALL PRESTRESS(H1,H2,X,IV1,NS2,N1,N,S,H,XK2,XK3,XM1,XM2
214      *,W,R1,XK1,B1,F1,R2,XM)
215      IF(R2.LE.R1)GO TO 750
216      STOP
217      END
218  C *****
219  C
220      SUBROUTINE INPUT(T1,M,N)
221  C
222      DIMENSION T1(100,100)
223      DO 200 I=1,M
224      READ (5,*)(T1(I,J),J=1,N)
225      200 WRITE(6,42)(T1(I,J),J=1,N)
226      42 FORMAT(5X,5F15.5)
227      RETURN
228      END
229  C *****
230  C
231      SUBROUTINE SIMPSON(C,N1,NS2,IV1)
232  C
233      DIMENSION C(100),A1(100,100),NS2(100),IV1(100)
234      IR=0
235      DO 492 I=1,N1
236      IF(NS2(I).GT.IR)GO TO 494
237      GO TO 492
238      494 IR=NS2(I)
239      492 CONTINUE
240      DO 500 I=1,IR
241      A1(I,((IR+1)-I))=1.
242      500 A1(I,((IR-1)+I))=1.

```

```

243      DO 505 I=1,(IR-1)/2.
244      N8=2*I
245      DO 510 J=N8,IR
246      A1(J,((N8+IR)-J))=4.
247      A1(J,((IR-N8)+J))=4.
248      A1((J+1),((N8+IR)-J))=2.
249      510 A1((J+1),((IR-N8)+J))=2.
250      505 CONTINUE
251      DO 515 L=1,IR
252      K=(2*IR)-L
253      N9=(IR+1)-L
254      DO 520 I=1,N1
255      IF(NS2(I).EQ.((2*N9)-1)) GO TO 525
256      GO TO 520
257      525 JJ=(IV1(I)-NS2(I))+1
258      KK=IV1(I)
259      KD=JJ-L
260      DO 530 J=L,K
261      J1=J+KD
262      530 C(J1)=A1(N9,J)
263      520 CONTINUE
264      515 CONTINUE
265      RETURN
266      END
267  C *****
268  C
269      SUBROUTINE INVER(A,N)
270  C
271      DIMENSION A(100,100)
272      N1=N-1
273      N2=2*N
274  C
275      DO 2 I=1,N
276      DO 1 J=1,N
277      J1=J+N
278      1 A(I,J1)=0.
279      J1=I+N
280      2 A(I,J1)=1.
281  C
282      DO 310 K=1,N1
283      C1=A(K,K)
284      IF(ABS(C1)-0.000001)3,3,7
285      3 WRITE(6,4)K
286      4 FORMAT(' *** SINGULARITY IN ROW',I5)
287      GO TO 400
288      7 K1=K+1
289      DO 6 J=K1,N2
290      6 A(K,J)=A(K,J)/C1
291      DO 310 I=K1,N
292      C1=A(I,K)
293      DO 310 J=K1,N2
294      310 A(I,J)=A(I,J)-C1*A(K,J)
295      NP1=N+1
296      IF(ABS(A(N,N))-0.000001)3,3,19
297      19 DO 320 J=NP1,N2
298      320 A(N,J)=A(N,J)/A(N,N)
299  C
300      DO 330 L=1,N1
301      K=N-L
302      K1=K+1

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303      DO 330 I=NP1,N2
304      DO 330 J=K1,N
305      330 A(K,I)=A(K,I)-A(K,J)*A(J,I)
306  C
307      DO 350 I=1,N
308      DO 350 J=1,N
309      J1=J+N
310      350 A(I,J)=A(I,J1)
311      400 RETURN
312      END
313  C *****
314  C
315      SUBROUTINE PRESTRESS(H1,H2,X,IV1,NS2,N1,N,IS,H,XK2,XK3,XM1
316      *,XM2,IW,R1,XK1,B1,F1,R2,XM)
317  C
318      DIMENSION XK1(100),B(100),H1(100),H2(100),D(100),X(100,100)
319      &,H4(100),C2(100),IV1(100),NS2(100),F(100),H(100,100),XK2(100)
320      &,XK3(100),XM1(100),XM2(100),XM(100)
321  C      THE DEPTH AT EACH STATION
322      IF(R2.EQ.R1)GO TO 631
323      DO 615 I=1,IS
324      IF(H1(I).GE.H2(I))GO TO 620
325      B(I)=(H1(I)-H2(I))/6.
326      GO TO 615
327      620 B(I)=(H2(I)-H1(I))/6.
328      615 CONTINUE
329      A2=F1*B1/6.
330      DO 625 I=1,IS
331      C2(I)=XM2(I)-XM1(I)
332      625 D(I)=(-B(I)+SQRT(B(I)**2-4*A2*C2(I)))/(2*A2*XM(I))
333      R2=R1
334      R1=D(1)
335      DO 630 I=2,IS
336      630 IF(D(I).GT.R1)R1=D(I)
337      IF(R2.LE.R1)GO TO 740
338      631 DO 640 I=1,IS
339      640 D(I)=R1*XM(I)
340  C      THE DEAD LOAD MOMENT AND NORMAL FORCE AT EACH STATION
341      DO 645 I=1,IS
342      XM(I)=(B1*R1)*X(I,IW)
343      645 H4(I)=(B1*R1)*X(I+IS,IW)
344      WRITE(6,650)
345      650 FORMAT(5X,'THE DEAD LOAD MOMENT AT EACH STATION')
346      CALL OUTPUT(XM,IS,1)
347      WRITE(6,655)
348      655 FORMAT(5X,'THE DEAD LOAD AXIAL FORCE AT EACH STATION')
349      CALL OUTPUT(H4,IS,1)
350      WRITE(6,601)
351      601 FORMAT(5X,'THE DEPTH AT EACH STATION')
352      CALL OUTPUT(D,IS,1)
353      DO 665 I=1,IS
354      665 F(I)=-((B(I)*D(I))+C2(I))*6./(B1*D(I)**2)
355      DO 670 I=1,IS
356      IF(H1(I).GE.H2(I))GO TO 675
357      F(I)=-((B1*F(I)*D(I)/2.)+H4(I)+H1(I))
358      GO TO 670
359      675 F(I)=-((B1*F(I)*D(I)/2.)+H4(I)+H2(I))
360      670 CONTINUE
361      WRITE(6,672)
362      672 FORMAT(5X,'THE PRESTRESS. FORCE INPUT 1 FOR CONST. 0 FOR VAR.')
363      READ(5,*)IPF
364      WRITE(6,674)IPF
365      674 FORMAT(5X,I2)

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366      IF(IPF.EQ.0)GO TO 692
367      DO 676 I=1,IS
368      IF(F(I).EQ.0.)GO TO 677
369      GO TO 676
370      677 F(I)=0.000001
371      676 CONTINUE
372      DO 680 I=1,N1
373      C2(I)=0.
374      J=IV1(I)-NS2(I)+1.
375      K=IV1(I)
376      DO 680 J1=J,K
377      IF(F(J1).LT.C2(I))GO TO 685
378      GO TO 680
379      685 C2(I)=F(J1)
380      680 CONTINUE
381      DO 690 I=1,N1
382      J=(IV1(I)-NS2(I))+1
383      K=IV1(I)
384      DO 690 J1=J,K
385      690 F(J1)=C2(I)
386      692 WRITE(6,700)
387      700 FORMAT(5X,'THE PRESTRESSING FORCE AT EACH STATION')
388      CALL OUTPUT(F,IS,1)
389      C      THE LIMITING ZONE AT EACH STATION
390      DO 705 I=1,IS
391      H1(I)=H4(I)+F(I)+H1(I)
392      H2(I)=H4(I)+F(I)+H2(I)
393      B(I)=(D(I)*H1(I)/(6.*F(I)))+((XM1(I)+XM(I))/F(I))
394      705 C2(I)=- (D(I)*H2(I)/(6.*F(I)))+((XM2(I)+XM(I))/F(I))
395      WRITE(6,710)
396      710 FORMAT(5X,'THE LOWER LIMIT OF THE LIMITING ZONE AT EACH STATION')
397      CALL OUTPUT(B,IS,1)
398      WRITE(6,715)
399      715 FORMAT(5X,'THE UPPER LIMIT OF THE LIMITING ZONE AT EACH STATION')
400      CALL OUTPUT(C2,IS,1)
401      C      THE CONCORDANT PROFILE
402      DO 720 I=1,IS
403      DO 720 J=1,N
404      720 X(I,J)=-12.*F(I)*H(I,J)*XK2(I)*XK3(I)/(D(I)**3)
405      IF(N.GT.1)GO TO 632
406      XL=0.
407      XL1=0.
408      XL2=0.
409      DO 725 I=1,IS
410      XL=XL+X(I,1)
411      XL1=XL1+X(I,1)*C2(I)
412      725 XL2=XL2+F(I)*H(I+IS,1)*XK2(I)*XK3(I)/D(I)
413      DO 730 I=1,IS
414      730 B(I)=C2(I)+((B(I)-C2(I))*(XL1+XL2))/(XL1-XL)
415      WRITE(6,735)
416      735 FORMAT(5X,'THE CONCORDANT PROFILE')
417      CALL OUTPUT(B,IS,1)
418      IF(R2.EQ.R1)GO TO 632
419      740 RETURN
420      632 STOP
421      END
422      C      *****
423      C
424      SUBROUTINE OUTPUT(T1,IS,IW)
425      C
426      DIMENSION T1(100,100)
427      DO 418 I=1,IS
428      418 WRITE(6,34)I,(T1(I,J),J=1,IW)
429      34 FORMAT(5X,I5,5F15.5)
430      RETURN
431      END
TERMINATED BY EOF

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10 REM PRESTRESSED CONCRETE DESIGN PROGRAM
11 PRINT"THE PROGRAM:TYPE F FOR FLIXABILITY, P FOR PRESTRESSED"
12 INPUT B$
13 PRINT"THE STRUC.:TYPE B FOR BEAMS,F FOR FRAME OR ARCH"
14 INPUT E$
19 PRINT"THE INERTIA:TYPE C FOR CONSTANT,V FOR VARIABLE"
20 INPUT D$
21 IF B$="F"GOTO 29
22 PRINT"THE MAX.PRESTRESSEING FORCE REQUIRED TO BE CONSTANT:TYPE C(FOR THE WHOLE STRUC.)"
23 PRINT",S (FOR EACH SPAN),V (FOR EACH SECTION)"
24 INPUT F$
25 IF E$="F"THEN 29 : IF F$="V"THEN 26 ELSE GOTO 29
26 PRINT"THE END SUPPORT:TYPE 1L(FOR LEFT HINGE(ROLLAR)),1R(FOR RIGHT HINGE(ROLLAR))"
27 PRINT",2(FOR TWO HIGED OR ROLLERS),N(FOR FIXED) ,NOTE:FOR BEAMS ONLY."
28 INPUT C$
29 PRINT"THE LOAD CONDITIONS:L.L1,L.L2,.....,EXT.(D.L.),O.W.(D.L.)"
30 PRINT"INPUT THE NO OF SPANS,INDETERMIANCY,NO OF LOAD CONDITIONS"
31 INPUT N1,N,W
32 DIM S1(N1),S2(N1),V1(N1)
33 PRINT"INPUT THE LENGTH OF EACH SPAN IN MATRIX S1(N1)"
34 MAT INPUT S1(N1)
35 PRINT"INPUT THE NO OF STATIONS PER EACH SPAN IN MATRIX S2(N1)"
36 MAT INPUT S2(N1)
37 S=0 : FOR I=1 TO N1 : S=S+S2(I) : NEXT I : R=2*S
38 DIM H(R,N),N(R,W),Z(R,W),X(S,W),T1(S,W),V(N,N),T(N,W),Y(N,W),F1(S,5)
39 DIM K1(R),K2(R),K3(R),K(S),H1(S),H2(S),H3(S),H4(S),M(S)
40 DIM M1(S),M2(S),D(S),P(S),B(S),C(S),F(S),C1(N1)
44 PRINT"INPUT THE MOMENTS DUE TO UNIT LOADS AT RELASE POINTS IN MATRIX H(S,N) BY ROWS"
45 MAT INPUT T1(S,N)
50 FOR I=1 TO S : FOR J=1 TO N : H(I,J)=T1(I,J) : NEXT J : NEXT I : GOTO 5000
66 PRINT"INPUT THE NORMAL FORCES DUE TO UNIT LOADS AT RELASE POINTS IN MAT. H(S,N)BY ROWS"
67 MAT INPUT T1(S,N)
68 FOR I=1 TO S : FOR J=1 TO N : H(I+S,J)=T1(I,J) : NEXT J : NEXT I : GOTO 6000
70 PRINT"INPUT APPLIED LOAD MATRIX MO(S,W) BY ROWS"
71 MAT INPUT T1(S,W)
72 FOR I=1 TO S : FOR J=1 TO W : N(I,J)=T1(I,J) : NEXT J : NEXT I : GOTO 7000
75 PRINT"INPUT APPLIED LOAD MATRIX NO(S,W) BY ROWS"
76 MAT INPUT T1(S,W)
77 FOR I=1 TO S : FOR J=1 TO W : N(I+S,J)=T1(I,J) : NEXT J : NEXT I : GOTO 8000
79 IF D$="C"THEN 107
80 PRINT"INPUT EI RATIOS AT EACH STATION"
81 MAT INPUT K(S)
83 R=2*S : FOR I=1 TO S : K1(I)=K(I) : NEXT I
88 PRINT"INPUT EA RATIOS AT EACH STATION"
90 MAT INPUT K(S)
95 FOR I=1 TO S : K1(I+S)=K(I) : NEXT I
96 GOTO 100
97 FOR I=1 TO S : K1(I+S)=1 : NEXT I
100 PRINT"INPUT THE DEPTH RATIO AT EACH STATION"
102 MAT INPUT K(S)
105 GOTO 130
107 FOR I=1 TO S : K1(I)=1/12 : K1(I+S)=1 : NEXT I
130 L=0 : FOR I=1 TO N1 : L=L+S2(I) : V1(I)=L : NEXT I
135 FOR I=1 TO N1 : FOR J=V1(I-1)+1 TO V1(I) : K2(J)=S1(I)/((S2(I)-1)*3) : NEXT J
140 NEXT I
145 FOR I=1 TO S : K2(I+S)=K2(I) : NEXT I
170 PRINT"INPUT SIMPSON`S COEFFICIENTS FOR EACH SPAN K3(S)"
180 MAT INPUT C(S)
185 R1=1 : FOR I=1 TO S : K3(I+S)=C(I) : K3(I)=C(I) : NEXT I

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186 IF B$="F"THEN 210
187 PRINT"INPUT THE WIDTH,THE ALLOW. CONCRETE STRESS,CONCRETE MODULAS"
188 INPUT B1,F1,E2
189 IF F$="V"THEN 210
195 PRINT"TYPE Q FOR QUADRATIC PROG. SOLUTION,D FOR DIRECT SOLUTION"
196 INPUT W$
210 REM FORM THE FLIXIPILITY MATRIX AND PUT IN V
214 LET R2=1 : GOTO 220
216 FOR K=1 TO 100 : PRINT"ITERATION NO.";K
218 FOR I=1 TO S : K1(I)=K1(I)*(R1^3)/(R2^3) : K1(I+S)=K1(I+S)*R1/R2 : NEXT I
220 FOR I=1 TO N : FOR J=1 TO N : V(I,J)=0 : R=2*S : FOR L=1 TO R
230 V(I,J)=V(I,J)+H(L,I)*H(L,J)*K2(L)*K3(L)/K1(L) : NEXT L
235 NEXT J : NEXT I
238 FOR I=1 TO N : FOR J=1 TO N : IF V(I,J)=0 THEN V(I,J)=1E-21 : NEXT J : NEXT I
240 REM FORM THE INVERSE OF THE FLEXIBILITY MATRIX IN V
250 MAT V=INV(V)
280 REM FORM THE INTEGRAL OF MO.M1/EI&NO.N1/EA AND PUT IN T
290 FOR I=1 TO N : FOR J=1 TO W : T(I,J)=0 : FOR L=1 TO R
300 T(I,J)=T(I,J)+H(L,I)*N(L,J)*K2(L)*K3(L)/K1(L) : NEXT L
305 NEXT J : NEXT I
310 REM FORM PRIDUCT OF FLEXIBILITY MATRIX INVERSE AND APPLIED LOAD IN Y
320 MAT Y=V*T
330 REM MULTIBLY THIS PRODUCT BY THE B.M.&N.F DUE TO UNIT LOADS AT RELASE POINTS
340 MAT Z=H*Y
350 REM STRESS RESULTANTS GIVEN BY XO-THIS PRODUCT
360 MAT X=N-Z
370 REM PRINT STRESS RESULTANTS AT EACH OF THIS STATIONS
371 FOR I=1 TO S : FOR J=1 TO W : T1(I,J)=X(I,J) : NEXT J : NEXT I
372 PRINT"THE FINAL BENDING MOMENTS RESULTANTS"
373 MAT PRINT T1
375 FOR I=1 TO S : FOR J=1 TO W : T1(I,J)=X(I+S,J) : NEXT J : NEXT I
380 PRINT"THE CORESPONDING AXIAL FORCES RESULTANTS"
382 MAT PRINT T1
384 IF Z$="N"THEN 404
385 IF B$="F"THEN E2=2000000 ELSE GOTO 386
386 FOR I=1 TO N : FOR J=1 TO W : T(I,J)=0 : FOR L=1 TO R
387 T(I,J)=T(I,J)+H(L,I)*X(L,J)*K2(L)*K3(L)/K1(L) : Y(I,J)=T(I,J)/E2 : NEXT L
388 NEXT I
389 PRINT"THE LACK OF INDETERMIANCY AT EACH RELASE POINT (USING RELATIVE INERTIA)"
390 MAT PRINT Y
392 MAT Y=V*T
393 MAT Z=H*Y
396 FOR I=1 TO S : FOR J=1 TO W : T1(I,J)=Z(I,J) : NEXT J : NEXT I
397 PRINT"THE B.M. ERROR (USING RELATIVE INERTIA)"
398 MAT PRINT T1
400 FOR I=1 TO S : FOR J=1 TO W : T1(I,J)=Z(I+S,J) : NEXT J : NEXT I
401 PRINT"THE N.F. ERROR (USING RELATIVE INERTIA)"
402 MAT PRINT T1
404 FOR I=1 TO S : M1(I)=0 : H1(I)=0 : FOR J=1 TO W-1
405 IF X(I,J)>0 THEN(M1(I)=M1(I)+X(I,J) : H1(I)=H1(I)+X(I+S,J))
406 NEXT J : NEXT I
407 FOR I=1 TO S : M2(I)=0 : H2(I)=0 : FOR J=1 TO W-1
410 IF X(I,J)<0 THEN(M2(I)=M2(I)+X(I,J) : H2(I)=H2(I)+X(I+S,J))
415 NEXT J : NEXT I
504 IF B$="F"GOTO 640
510 FOR I=1 TO S : IF H1(I)>=H2(I)THEN 520 ELSE GOTO 540
520 B(I)=(H2(I)-H1(I))/6
530 GOTO 550
540 B(I)=(H1(I)-H2(I))/6
550 NEXT I
552 IF D$="C"THEN 554
553 GOTO 560

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554 FOR I=1 TO S : K(I)=1 : NEXT I
560 A=(F1*B1)/6 : FOR I=1 TO S : C(I)=M2(I)-M1(I) : D(I)=(-B(I)+SQR(B(I)^2-4*A*C(I)))
570 NEXT I
590 LET R2=R1 : R3=R2 : R1=0 : FOR I=1 TO S : IF D(I)>R1 THEN R1=D(I)
600 NEXT I
602 LET Z$="N" : IF R3=1 GOTO 216 : IF(R2/R1)<1 THEN NEXT K
610 FOR I=1 TO S : D(I)=R1*K(I)
611 NEXT I
612 FOR I=1 TO N : FOR J=1 TO W : T(I,J)=0 : Y(I,J)=0 : FOR L=1 TO S
613     T(I,J)=T(I,J)+H(L,I)*X(L,J)*K2(L)*K3(L)*12/(D(L)^3)
614     Y(I,J)=Y(I,J)+H(L+S,I)*X(L+S,J)*K2(L)*K3(L)/D(L)
615 NEXT L : NEXT J : NEXT I
616 FOR I=1 TO N : FOR J=1 TO W : T(I,J)=Y(I,J)+T(I,J) : Y(I,J)=T(I,J)/(B1*E2) : NEXT J
619 PRINT"THE LACK OF INDETERMIANCY AT EACH RELASE POINT (ITERATIONN NO.";K")"
620 MAT PRINT Y
621 MAT Y=V*T
622 MAT Z=H*Y
623 FOR I=1 TO S : FOR J=1 TO W : T1(I,J)=Z(I,J) : NEXT J : NEXT I
624 PRINT"THE FINAL B.M. ERROR (ITERATION NO.";K")"
625 MAT PRINT T1
627 FOR I=1 TO S : FOR J=1 TO W : T1(I,J)=Z(I+S,J) : NEXT J : NEXT I
628 PRINT"THE FINAL N.F. ERROR (ITERATION NO.";K")"
629 MAT PRINT T1
640 PRINT"THE HOGGING MOMENT M1 VALUES"
641 MAT PRINT M1
648 PRINT"THE AXIAL FORCE H1 CORESPONDING TO M1 VALUES"
649 MAT PRINT H1
650 PRINT"THE SAGGING MOMENT M2 VALUES"
651 MAT PRINT M2
653 PRINT"THE AXIAL FORCE H2 CORESPONDING TO M2 VALUES"
654 MAT PRINT H2
655 IF B$="F" THEN 9100
656 FOR I=1 TO S : M(I)=B1*R1*X(I,W) : H4(I)=B1*R1*X(I+S,W)
657 NEXT I
660 PRINT"THE DEAD LOAD MOMENT AT EACH STATION"
670 MAT PRINT M
680 PRINT"THE AXIAL FORCE HD DUE TO THE DEAD LOAD AT EACH STATION"
690 MAT PRINT H4
692 PRINT"THE DEPTH AT EACH STATION"
694 MAT PRINT D
728 FOR I=1 TO S : F(I)=-((B(I)*D(I))+C(I))*6/(B1*D(I)^2)
730 NEXT I
732 PRINT"THE PRESTRESSING FORCE AT EACH STATION"
734 FOR I=1 TO S : IF H1(I)>=H2(I) THEN 736 ELSE GOTO 740
736 P(I)=-((B1*F(I)*D(I)/2)+H4(I)+H2(I))
738 GOTO 741
740 P(I)=-((B1*F(I)*D(I)/2)+H4(I)+H1(I))
741 NEXT I
742 FOR I=1 TO S : IF P(I)=0 THEN P(I)=-1E-17 : NEXT I
751 IF F$="C" THEN 753
752 GOTO 756
753 T=0 : FOR I=1 TO S : IF P(I)<T THEN T=P(I) : NEXT I
754 FOR I=1 TO S : P(I)=T : NEXT I
755 GOTO 765
756 IF F$="S" THEN 758
757 GOTO 765
758 FOR I=1 TO N1 : C1(I)=0 : FOR J=V1(I-1)+1 TO V1(I) : IF P(J)<C1(I) THEN C1(I)=P(J)
759 FOR I=1 TO N1 : FOR J=V1(I-1)+1 TO V1(I) : P(J)=C1(I) : NEXT J : NEXT I
765 MAT PRINT P
770 FOR I=1 TO S : H3(I)=H4(I)+P(I)+H1(I) : H4(I)=H4(I)+P(I)+H2(I)
780 NEXT I

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790 PRINT"THE LOWER LIMIT EA VALUES OF THE LIMITING ZONE AT EACH STATION"
800 FOR I=1 TO S : B(I)=(D(I)*H3(I)/(6*P(I)))+(M1(I)+M(I))/P(I)
810 NEXT I
812 IF F$="V"THEN 813 ELSE GOTO 820
813 IF C$="1L"THEN 814 ELSE GOTO 815
814 B(1)=0 : GOTO 820
815 IF C$="1R"THEN 816 ELSE GOTO 817
816 B(S)=0 : GOTO 820
817 IF C$="2"THEN B(1)=0 : B(S)=0
820 MAT PRINT B
830 FOR I=1 TO S : C(I)=-(D(I)*H4(I)/(6*P(I)))+(M2(I)+M(I))/P(I)
840 NEXT I
841 IF F$="V"THEN 842 ELSE GOTO 849
842 IF C$="1L"THEN 843 ELSE GOTO 844
843 C(1)=0 : GOTO 849
844 IF C$="1R"THEN 845 ELSE GOTO 846
845 C(S)=0 : GOTO 849
846 IF C$="2"THEN C(1)=0 : C(S)=0
849 PRINT"THE UPPER LIMIT EB VALUES OF THE LIMITING ZONE AT EACH STATION"
850 MAT PRINT C
852 IF F$="V"THEN 2994
855 FOR I=1 TO S : IF B(I)-C(I)<=1E-03 THEN 860 ELSE GOTO 874
860 B(I)=B(I) : NEXT I
865 GOTO 2994
874 FOR I=1 TO S : FOR J=1 TO N : F1(I,J)=-12*P(I)*H(I,J)*K2(I)*K3(I)/(D(I)^2) : NEXT J
877 IF W$="Q"THEN 900
878 REM DIRECT SOLUTION METHOD
882 IF N>1 GOTO 896
884 L=0 : L1=0 : L2=0 : FOR I=1 TO S : L=L+F1(I,1)*B(I) : L1=L1+F1(I,1)*C(I)
886 L2=L2+P(I)*H(I+S,1)*K2(I)*K3(I) : NEXT I
892 FOR I=1 TO S : B(I)=C(I)+(B(I)-C(I))*(L1+L2)/(L1-L) : NEXT I
893 S7=(L1+L2)/(L1-L) : IF S7<0 THEN 2910 : IF S7>1 THEN 2910 : PRINT S7
894 GOTO 2994
896 IF E$="F"THEN 900
898 IF N<=2 GOTO 884
900 R=2*S : DIM T1(S,N),T2(S+1,N),U(N,S+1),X1(S+1,S+1),X2(S+1,S+1),C2(S+1),S1(N),S2(N)
903 REM INTEG. OF (HP*N1/EA)ds
904 FOR J=1 TO N : S1(J)=0 : FOR I=1 TO S : S1(J)=S1(J)+P(I)*H(I+S,J)*K2(I)*K3(I)
905 FOR I=1 TO S : IF B(I)-C(I)<1E-04 THEN B(I)=C(I) : NEXT I
913 FOR J=1 TO N : S2(J)=0 : FOR I=1 TO S : S2(J)=S2(J)+F1(I,J)*C(I) : NEXT I : NEXT J
914 FOR I=1 TO S : FOR J=1 TO N : F1(I,J)=F1(I,J)*(B(I)-C(I)) : NEXT J : NEXT I
915 FOR I=1 TO N : S2(I)=S2(I)+S1(I) : NEXT I
916 REM GAUSE ELEMINATION METHOD
917 FOR I=1 TO S+1 : FOR J=1 TO N : T2(I,J)=0 : NEXT J : NEXT I
918 IF E$="B"THEN 919 ELSE GOTO 920
919 IF N<=2 THEN 920 ELSE GOTO 939
920 FOR I=1 TO S : IF B(I)-B(S+1-I)<9.9E-03 THEN 921 ELSE GOTO 938
921 GOTO 922 : NEXT I
922 FOR I=1 TO(S-1)/2 : FOR J=1 TO N : T2(I,J)=F1(I,J)+F1(S+1-I,J)
923 S=(S+1)/2 : IF E$="F"THEN 939
924 N1=N : N=1
938 FOR I=1 TO S : FOR J=1 TO N : T2(I,J)=F1(I,J) : NEXT J : NEXT I
939 MAT U=TRN(T2)
940 FOR J=1 TO N : U(J,S+1)=S2(J) : NEXT J
941 IF N<=2 THEN 947
942 FOR I=1 TO S+1 : K1(I)=U(2,I) : NEXT I
943 FOR I=1 TO S+1 : IF U(1,I)=U(2,I)THEN 944 ELSE GOTO 945
944 U(2,I)=U(3,I) : U(3,I)=K1(I) : NEXT I : GOTO 947
945 FOR I=1 TO S+1 : IF U(2,I)=U(3,I)THEN 946 ELSE GOTO 947
946 U(2,I)=U(1,I) : U(1,I)=K1(I) : NEXT I
947 A=U(1,1) : FOR I=1 TO 2 : IF U(1,I)=0 THEN A=U(1,I+1) : J1=I : NEXT I
948 FOR I=1 TO S+1 : U(1,I)=U(1,I)/A : NEXT I

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949 IF N<=1 THEN 1110
950 B=U(2,1) : FOR I=1 TO S+1 : U(2,I)=U(2,I)-(B*U(1,I)) : NEXT I
955 IF U(2,2)=0 THEN U(2,2)=1E-19
960 C=U(2,2) : FOR I=1 TO S+1 : U(2,I)=U(2,I)/C : NEXT I
961 IF N<=2 THEN 1005
962 FOR I=1 TO S : IF F1(I,3)=0 THEN 973 ELSE GOTO 963
963 A8=U(1,1)/U(3,1) : B8=U(1,2)/U(3,2) : FOR I=1 TO S : FOR J=1 TO S+1
964 IF A8-B8<1E-03 THEN(F1(I,3)=0 : U(3,J)=0) : NEXT J : NEXT I
965 D=U(3,1) : FOR I=1 TO S+1 : U(3,I)=U(3,I)-(D*U(1,I)) : NEXT I
970 E=U(3,2) : FOR I=1 TO S+1 : U(3,I)=U(3,I)-(E*U(2,I)) : NEXT I
973 FOR I=1 TO S : IF F1(I,3)<=0 THEN 974 ELSE GOTO 975
974 GOTO 985 : NEXT I
975 FOR I=1 TO S+1 : IF U(1,I)=U(3,I) THEN 976 ELSE GOTO 980
976 GOTO 985 : NEXT I
980 IF U(3,3)=0 THEN U(3,3)=1E-19 : F=U(3,3) : GOTO 990
985 U(3,3)=0 : F=1E-21
990 FOR I=1 TO S+1 : U(3,I)=U(3,I)/F : NEXT I
995 G=U(2,3) : FOR I=1 TO S+1 : U(2,I)=U(2,I)-(G*U(3,I)) : NEXT I
1000 H=U(1,3) : FOR I=1 TO S+1 : U(1,I)=U(1,I)-(H*U(3,I)) : NEXT I
1005 K=U(1,2) : FOR I=1 TO S+1 : U(1,I)=U(1,I)-(K*U(2,I)) : NEXT I
1110 REM TRANSFORMATION TO QUADRATIC PROGRAMMING
1115 FOR I=1 TO N : U(I,I)=0 : NEXT I
1118 IF N>1 THEN 1120 : FOR I=1 TO N : U(I,J1)=0 : NEXT I
1120 MAT T2=TRN(U)
1130 FOR I=1 TO S+1 : FOR J=1 TO S+1 : X2(I,J)=0 : FOR L=1 TO N
1135 X2(I,J)=X2(I,J)+T2(I,L)*U(L,J) : NEXT L : NEXT J : NEXT I
1140 FOR I=1 TO S : IF X2(I,I)=0 THEN H3(I)=0 ELSE H3(I)=1 : NEXT I
1145 FOR I=1 TO S : H4(I)=0 : FOR J=1 TO N : H4(I)=H4(I)+T2(I,J) : NEXT J : NEXT I
1150 FOR I=1 TO S : X2(I,I)=X2(I,I)+H3(I) : NEXT I
1155 FOR I=1 TO S : X2(I,S+1)=X2(I,S+1)+H4(I) : NEXT I
1160 FOR I=1 TO S : X2(I,S+1)=X2(I,S+1)-H3(I) : NEXT I
1165 FOR I=1 TO S+1 : FOR J=1 TO S+1 : X1(I,J)=X2(I,J)+X2(J,I) : NEXT J : NEXT I
1170 FOR I=1 TO S+1 : X1(I,S+1)=-X1(I,S+1) : NEXT I
1175 FOR J=1 TO S+1 : C2(J)=0 : FOR I=1 TO S : C2(J)=C2(J)+X1(I,J) : NEXT I : NEXT J
1180 FOR I=1 TO S+1 : X1(S+1,I)=-C2(I) : NEXT I
1185 FOR I=1 TO S : IF X1(I,S+1)<0 THEN X1(I,S+1)=-X1(I,S+1) : NEXT I
1190 IF X1(S+1,S+1)>0 THEN X1(S+1,S+1)=-X1(S+1,S+1)
2705 REM SIMPLEX METHOD PROGRAMMING
2750 R1=0 : FOR J=1 TO S : IF X1(S+1,J)>=R1 THEN 2754
2752 R1=X1(S+1,J) : C1=J
2754 NEXT J
2765 FOR I=1 TO S : IF X1(I,C1)=0 THEN X1(I,C1)=1E-20 : NEXT I
2770 FOR I=1 TO S : X1(I,S+1)=X1(I,S+1)/X1(I,C1) : NEXT I
2775 FOR I=1 TO S : IF X1(I,S+1)<=0 THEN X1(I,S+1)=90000000 : NEXT I
2780 A2=1E+15 : FOR I=1 TO S : IF X1(I,S+1)>=A2 THEN 2784
2782 A2=X1(I,S+1) : W1=I
2784 NEXT I
2786 FOR I=1 TO S : IF X1(I,S+1)>9000000 THEN X1(I,S+1)=0 : NEXT I
2790 Y1=X1(W1,C1) : FOR I=1 TO S : X1(I,S+1)=X1(I,S+1)*X1(I,C1) : NEXT I
2795 FOR J=1 TO S+1 : X1(W1,J)=X1(W1,J)/Y1 : NEXT J
2800 FOR I=1 TO S+1 : FOR J=1 TO S+1 : X2(I,J)=X1(I,J) : NEXT J : NEXT I
2805 FOR J=1 TO S+1 : C2(J)=X1(W1,J) : NEXT J
2810 FOR I=1 TO S+1 : FOR J=1 TO S+1 : X1(I,J)=X1(I,J)-X2(I,C1)*X2(W1,J) : NEXT J
2815 FOR J=1 TO S+1 : X1(W1,J)=C2(J) : NEXT J
2816 FOR I=1 TO S : FOR J=1 TO S+1 : IF X1(I,J)<1E-10 THEN X1(I,J)=0 : NEXT J
2820 FOR I=1 TO S : IF X1(S+1,I)<0 GOTO 2750 : NEXT I
2822 FOR I=1 TO S+1 : IF X1(I,S+1)<1E-05 THEN X1(I,S+1)=0 : NEXT I
2828 REM CONCORDANT PROFILE CALCULATION
2830 FOR J=1 TO N : S2(J)=0 : FOR I=1 TO S+1 : S2(J)=S2(J)+U(J,I)*X1(I,S+1) : NEXT I
2832 FOR I=1 TO N : S2(I)=S2(I)+U(I,S+1) : NEXT I
2835 FOR I=1 TO N : H3(I)=-S2(I) : NEXT I
2836 IF N<=2 THEN 2840 ELSE GOTO 2838
2838 FOR I=1 TO S+1 : IF U(N,I)=0 THEN H3(N)=X1(N,S+1) ELSE GOTO 2840 : NEXT I
2840 FOR I=N+1 TO S : H3(I)=X1(I,S+1) : NEXT I

```

```

2842 IF N<=2 THEN(H3(J1)=H3(N) : H3(N)=0)
2845 FOR I=1 TO S : B(I)=C(I)+(B(I)-C(I))*H3(I) : NEXT I
2850 R3=2*S : IF R3<R THEN 2852 ELSE GOTO 2900
2852 S=R/2 : FOR I=1 TO S : H3(S+1-I)=H3(I) : NEXT I
2855 FOR I=1 TO S : B(S+1-I)=B(I) : NEXT I
2900 FOR I=1 TO S : IF H3(I)<0 THEN 2910 : IF H3(I)>1 THEN 2910 : NEXT I : GOTO 2994
2910 PRINT"NO CONCORDANT PRIFILE EXIST" : END
2915 IF E$="F"THEN 2996
2920 IF N=N1 THEN N=N ELSE N=N1
2994 S=R/2 : DIM S1(N),S2(N),V2(N) : FOR I=1 TO S : M1(I)=-P(I)*B(I) : NEXT I
2997 PRINT"THE PRESTRESSING MOMENT AT EACH STATION"
2998 MAT PRINT M1
3000 FOR J=1 TO N : S1(J)=0 : S2(J)=0 : FOR I=1 TO S
3010     S1(J)=S1(J)+M1(I)*H(I,J)*K2(I)*K3(I)*12/(D(I)^3)
3020     S2(J)=S2(J)+(P(I)*H(I+S,J)*K2(I)*K3(I)/D(I))
3030 NEXT I : NEXT J
3050 FOR I=1 TO N : V2(I)=S1(I)+S2(I) : NEXT I
3051 FOR I=1 TO N : S1(I)=V2(I)/(B1*E2) : NEXT I
3062 PRINT"THE LACK OF CONCORDANCY AT EACH RELASE POINT"
3063 MAT PRINT S1
3064 MAT Y=V*V2
3065 MAT Z=H*Y
3067 FOR I=1 TO S : M(I)=Z(I,1) : C(I)=Z(I+S,1) : NEXT I
3068 PRINT"THE ERROR IN THE PRESTRESSING MOMENT"
3069 MAT PRINT M
3070 PRINT"THE ERROR IN THE PRESTRESSING FORCE"
3072 MAT PRINT C
4000 GOTO 9091
4900 REM SUBROUTINE INPUT DATA CORRECTION
5000 MAT PRINT T1
5010 PRINT"IF THE DATA COORRECT? TYPE Y IF NOT TYPE N"
5020 INPUT A$
5030 IF A$="Y"GOTO 66
5040 PRINT"INPUT THE ROW NO,COLUMN NO OF THE ERROR"
5050 INPUT A1,A2
5060 PRINT"INPUT THE CORRECT VALUE OF THE DATA"
5070 INPUT A3
5080 T1(A1,A2)=A3
5090 GOTO 5000
6000 MAT PRINT T1
6010 PRINT"IF THE DATA CORRECT? TYPE Y IF NOT TYPE N"
6020 INPUT A$
6030 IF A$="Y"GOTO 70
6040 PRINT"INPUT THE ROW NO,CLIU MN NO OF THE ERROR"
6050 INPUT A1,A2
6060 PRINT"INPUT THE CORRECT VALUE OF THE DATA"
6070 INPUT A3
6080 T1(A1,A2)=A3
6090 GOTO 6000
7000 MAT PRINT T1
7010 PRINT"IF THE DATA CORRECT? TYPE Y IF NOT TYPE N"
7020 INPUT A$
7030 IF A$="Y"GOTO 75
7040 PRINT"INPUT THE ROW NO,COLUMN NO OF THE ERROR"
7050 INPUT A1,A2
7060 PRINT"INPUT THE CORRECT VALUE OF THE DATA"
7070 INPUT A3
7080 T1(A1,A2)=A3
7090 GOTO 7000
8000 MAT PRINT T1
8010 PRINT"IF THE DATA CORRECT? TYPE Y IF NOT TYPE N"
8020 INPUT A$
8030 IF A$="Y"GOTO 79

```

```

8040 PRINT"INPUT THE ROW NO,COLUMN NO OF THE ERROR"
8050 INPUT A1,A2
8060 PRINT"INPUT THE CORRECT VALUE OF THE DATA"
8070 INPUT A3
8080 T1(A1,A2)=A3
8090 GOTO 8000
9091 PRINT"THE DESIGN OUT PUT DATA AT EACH STATION"
9092 FOR I=1 TO S : IF P(I)>-1E-03 THEN P(I)=0 : NEXT I
9094 PRINT"STATION NO.," THE DEPTH","THE P. FORCE","THE PROFILE"," THE MAX. STRESSES"
9095 FOR I=1 TO S : F1(I,1)=I : F1(I,2)=D(I) : F1(I,3)=P(I) : F1(I,4)=B(I)
9098 MAT PRINT F1
9099 PRINT"THE WIDTH=";B1 : END
9105 PRINT"STRESSES AND DEFLECTIONS AT THE CRITICAL SECTION OF THE STRUC. UNDER"
9110 PRINT"THE APPLIED LOADS"
9112 PRINT"LOAD.,"TOP STRESSES","BOT.STRESSES","DEFLECTION"
9115 FOR I=1 TO R : N(I,1)=N(I,1)/30 : K1(I)=X(I,1)/30 : NEXT I
9120 FOR L=0 TO 30 : FOR I=1 TO S
9125   X(I,1)=K1(I)*L+X(I,2)*B1*D(I)+M1(I) : X(I+S,1)=K1(I+S)*L+X(I+S,2)*B1*D(I)+P(I)
9130   FOR I=1 TO S : Y1=Y1+N(I,1)*X(I,1)*K2(I)*K3(I)*12/(E2*B1*D(I)^3)
9135   NEXT I : Y=Y1+Y2
9140   Z=(S+1)/2 : Z1=X((Z+S),1)/(B1*D(Z)) : Z2=6*X(Z,1)/(B1*D(Z)^2) : T=Z1+Z2 : B=Z1-Z2
9155   PRINT L,T,B,Y : NEXT L
9160 PRINT"DIMENTIONS IN KN.CM."
9170 END

```

APPENDIX B

Program output data for the test frame

:KUN

THE PROGRAM:TYPE F FOR FLIXABILITY, P FOR PRESTRESSED

? P

THE STRUC.:TYPE B FOR BEAMS,F FOR FRAME OR ARCH

? F

THE INERTIA:TYPE C FOR CONSTANT,V FOR VARIABLE

? C

THE MAX.PRESTRESSEING FORCE REDURIED TO BE CONSTANT:TYPE C(FOR THE WHOLE STRUC.)
,S (FOR EACH SPAN),V (FOR EACH SECTION)

? S

THE LOAD CONDITIONS:L.L1,L.L2,.....,D.W.(D.L.),EXT.(D.L.)

INPUT THE NO OF SPANS,INDETERMIANCY,NO OF LOAD CONDITIONS

? 3,3,3

INPUT THE LENGTH OF EACH SPAN IN MATRIX S1(N1)

? 150,200,150

INPUT THE NO OF STATIONS PER EACH SPAN IN MATRIX S2(N1)

? 3,5,3

INPUT THE MOMENTS DUE TO UNIT LOADS AT RELASE POINTS IN MATRIX H(S,N) BY ROWS

? -1,-150,0

?? -1,-75,0

?? -1,0,0

?? -1,0,0

?? -1,0,0

?? -1,0,0

?? -1,0,0

?? -1,0,0

?? -1,0,0

?? -1,-75,0

?? -1,-150,0

-1 -150 0

-1 -75 0

-1 0 0

-1 0 0

-1 0 0

-1 0 0

-1 0 0

-1 0 0

-1 0 0

-1 -75 0

-1 -150 0

IF THE DATA COORRECT?-TYPE Y IF NOT TYPE N

? Y

INPUT THE NORMAL FORCES DUE TO UNIT LOADS AT RELASE POINTS IN MAT. H(S,N)BY ROWS

? 0,0,0

?? 0,0,0

?? 0,0,0

?? 0,-1,0,

?? C,-1,0

?? 0,-1,0

?? 0,-1,0

?? 0,-1,0

??

0 0 0

0 0 0

0 0 0

0 -1 0

0 -1 0

0 -1 0

0 -1 0

0 -1 0

0 0 0

0 0 0

0 0 0

IF THE DATA CORRECT? TYPE Y IF NOT TYPE N

INPUT APPLIED LOAD MATRIX NO(S,W) BY ROWS

? 1500,.125,0
?? 1500,.125,0
?? 1500,.125,0
?? 1500,.125,0
?? 750,.031,0
?? 0,0,0
?? 750,.031,0
?? 1500,.125,0
?? 1500,.125,0
?? 1500,.125,0
?? 1500,.125,0

1500	0.125	0
1500	0.125	0
1500	0.125	0
1500	0.125	0
750	0.031	0
0	0	0
750	0.031	0
1500	0.125	0
1500	0.125	0
1500	0.125	0
1500	0.125	0

IF THE DATA CORRECT? TYPE Y IF NOT TYPE N

? Y

INPUT APPLIED LOAD MATRIX NO(S,W) BY ROWS

? -15,-.0025,0
?? -15,-.0025,0
?? -15,-.0025,0
?? 0,0,0
?? 0,0,0
?? 0,0,0
?? 0,0,0
?? 0,0,0
?? -15,-.0025,0
?? -15,-.0025,0
?? -15,-.0025,0

-15	-2.5E-03	0
-15	-2.5E-03	0
-15	-2.5E-03	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
-15	-2.5E-03	0
-15	-2.5E-03	0
-15	-2.5E-03	0

IF THE DATA CORRECT? TYPE Y IF NOT TYPE N

? Y

INPUT SIMPSON'S COEFFICIENTS FOR EACH SPAN K3(S)

? 1,4,1
?? 1,4,2,4,1
?? 1,4,1

INPUT THE WIDTH,THE ALLOW. CONCRETE STRESS,CONCRETE MODULAS

? 10,1.2,3400

TYPE Q FOR QUADRATIC PROG. SOLUTION,D FOR DIRECT SOLUTION

? Q

THE FINAL BENDING MOMENTS RESULTANTS

-272.72	-0.0303628	0
136.366	0.0151821	0
545.451	0.0607269	0
545.451	0.0607269	0
-204.549	-0.0332731	0

-954.549	-0.0642731	0
-204.549	-0.0332731	0
545.451	0.0607269	0
545.451	0.0607269	0
136.366	0.0151821	0
-272.72	-0.0303628	0

THE CORRESPONDING AXIAL FORCES RESULTANTS

-15	-2.5E-03	0
-15	-2.5E-03	0
-15	-2.5E-03	0
-5.45447	-6.07265E-04	0
-5.45447	-6.07265E-04	0
-5.45447	-6.07265E-04	0
-5.45447	-6.07265E-04	0
-5.45447	-6.07265E-04	0
-15	-2.5E-03	0
-15	-2.5E-03	0
-15	-2.5E-03	0

THE LACK OF INDETERMINANCY AT EACH RELEASE POINT (USING RELATIVE INERTIA)

7.94118E-05	-1.67647E-08	0
0.0143808	1.55615E-08	0
0	0	0

THE R.M. ERROR (USING RELATIVE INERTIA)

-3.04807E-04	-9.01023E-09	0
-1.19231E-04	4.21137E-09	0
6.6346E-05	1.7433E-08	0
6.6346E-05	1.7433E-08	0
6.6346E-05	1.7433E-08	0
6.6346E-05	1.7433E-08	0
6.6346E-05	1.7433E-08	0
6.6346E-05	1.7433E-08	0
6.6346E-05	1.7433E-08	0
-1.19231E-04	4.21137E-09	0
-3.04807E-04	-9.01023E-09	0

THE N.F. ERROR (USING RELATIVE INERTIA)

0	0	0
0	0	0
0	0	0
-2.47435E-06	-1.76288E-10	0
-2.47435E-06	-1.76288E-10	0
-2.47435E-06	-1.76288E-10	0
-2.47435E-06	-1.76288E-10	0
-2.47435E-06	-1.76288E-10	0
0	0	0
0	0	0
0	0	0

ITERATION NO. 4

THE FINAL BENDING MOMENTS RESULTANTS

-268.987	-0.0299473	0
137.432	0.0153008	0
543.852	0.0605488	0
543.852	0.0605488	0
-206.148	-0.0334512	0
-956.148	-0.0644512	0
-206.148	-0.0334512	0
543.852	0.0605488	0
543.852	0.0605488	0
137.432	0.0153008	0
-268.987	-0.0299473	0

THE CORRESPONDING AXIAL FORCES RESULTANTS

-15	-2.5E-03	0
-15	-2.5E-03	0
-15	-2.5E-03	0
-5.41893	-6.03307E-04	0
-5.41893	-6.03307E-04	0
-5.41893	-6.03307E-04	0
-5.41893	-6.03307E-04	0
-5.41893	-6.03307E-04	0
-15	-2.5E-03	0
-15	-2.5E-03	0
-15	-2.5E-03	0

THE LACK OF INDETERMINANCY AT EACH RELEASE POINT (ITERATION NO. 4)

-1.79412E-10	-1.52941E-14	0
-8.35294E-09	8.39118E-12	0
0	0	0

THE FINAL B.M. ERROR (ITERATION NO. 4)

1.16812E-05	-2.24459E-08	0
1.11672E-05	-5.74567E-09	0
1.06533E-05	1.09546E-08	0
1.06533E-05	1.09546E-08	0
1.06533E-05	1.09546E-08	0
1.06533E-05	1.09546E-08	0
1.06533E-05	1.09546E-08	0
1.06533E-05	1.09546E-08	0
1.06533E-05	1.09546E-08	0
1.11672E-05	-5.74567E-09	0
1.16812E-05	-2.24459E-08	0

THE FINAL N.F. ERROR (ITERATION NO. 4)

0	0	0
0	0	0
0	0	0
6.85248E-09	-2.2267E-10	0
6.85248E-09	-2.2267E-10	0
6.85248E-09	-2.2267E-10	0
6.85248E-09	-2.2267E-10	0
6.85248E-09	-2.2267E-10	0
0	0	0
0	0	0
0	0	0

THE HOOGING MOMENT M1 VALUES

137.432
543.852
543.852
0
0
0
543.852
543.852
137.432
0

THE AXIAL FORCE H1 CORRESPONDING TO M1 VALUES

0
-15
-15
-5.41893
0
0
0
-5.41893
-15
-15
0

THE SAGGING MOMENT M2 VALUES

-268.987
0
0
0
-206.148
-956.148
-206.148
0
0
0
-268.987

THE AXIAL FORCE H2 CORRESPONDING TO M2 VALUES

-15
0
0
0
-5.41893
-5.41893
-5.41893
0
0
0
-15

THE DEAD LOAD MOMENT AT EACH STATION

-6.61591
3.38023
13.3764
13.3764
-7.38999
-14.2385
-7.38999
13.3764
13.3764
3.38023
-6.61591

THE AXIAL FORCE HD DUE TO THE DEAD LOAD AT EACH STATION

-0.552297
-0.552297
-0.552297
-0.133282

-0.133282
-0.133282
-0.133282
-0.552297
-0.552297
-0.552297

THE DEPTH AT EACH STATION

22.0919
22.0919
22.0919
22.0919
22.0919
22.0919
22.0919
22.0919
22.0919
22.0919

THE PRESTRESSING FORCE AT EACH STATION

-65.8009
-65.8009
-65.8009
-126.999
-126.999
-126.999
-126.999
-126.999
-65.8009
-65.8009
-65.8009

THE LOWER LIMIT EA VALUES OF THE LIMITING ZONE AT EACH STATION

3.81343
2.41225
-3.91617
-0.544711
3.74403
3.79796
3.74403
-0.544711
-3.91617
2.41225
3.81343

THE UPPER LIMIT EB VALUES OF THE LIMITING ZONE AT EACH STATION

-0.363788
-3.76425
-3.91617
-3.79117
-2.16153
3.79796
-2.16153
-3.79117
-3.91617
-3.76425
-0.363788

THE PRESTRESSING MOMENT AT EACH STATION

129.998
3.86978
-257.687
-283.495
-7.63171
482.336
-7.63171
-783.495

-257.687
3.86978
129.998

THE LACK OF CONCORDANCY AT EACH RELEASE POINT

4.17647E-10
-2.94118E-09
0

THE ERROR IN THE PRESTRESSING MOMENT

3.04532E-05
-9.5257E-06
-4.95046E-05
-4.95046E-05
-4.95046E-05
-4.95046E-05
-4.95046E-05
-4.95046E-05
-4.95046E-05
-9.5257E-06
3.04532E-05

THE ERROR IN THE PRESTRESSING FORCE

0
0
0
5.33052E-07
5.33052E-07
5.33052E-07
5.33052E-07
5.33052E-07
0
0
0

THE DESIGN OUT PUT DATA AT EACH STATION

STATION NO.	THE DEPTH	THE P. FORCE	THE PROFILE	THE MAX. STRESSES
1	22.0919	-65.8009	1.97563	0.398586
2	22.0919	-65.8009	0.0586104	0.236853
3	22.0919	-65.8009	-3.91617	0.736499
4	22.0919	-126.999	-2.23226	0.69313
5	22.0919	-126.999	-0.0600927	0.277964
6	22.0919	-126.999	3.79796	1.2
7	22.0919	-126.999	-0.0600927	0.277964
8	22.0919	-126.999	-2.23226	0.69313
9	22.0919	-65.8009	-3.91617	0.736499
10	22.0919	-65.8009	0.0586104	0.236853
11	22.0919	-65.8009	1.97563	0.398586

THE WIDTH= 10

STRESSES AND DEFLECTIONS AT THE CRITICAL SECTION OF THE STRUC. UNDER
THE APPLIED LOADS

LOAD.	TOP STRESSES	BOT. STRESSES	DEFLECTION
0	5E-08	-1.15094	-0.0106484
1	-0.04	-1.11258	-8.05989E-03
2	-0.08	-1.07421	-5.47141E-03
3	-0.12	-1.03585	-2.88293E-03
4	-0.16	-0.997483	-2.9445E-04
5	-0.2	-0.959118	2.29403E-03
6	-0.24	-0.920754	4.88251E-03
7	-0.28	-0.882389	7.47099E-03
8	-0.32	-0.844024	0.0100595
9	-0.36	-0.805659	0.012648
10	-0.4	-0.767295	0.0152364
11	-0.44	-0.72893	0.0178249
12	-0.48	-0.690565	0.0204134
13	-0.52	-0.6522	0.0230019
14	-0.56	-0.613836	0.0255904
15	-0.6	-0.575471	0.0281788
16	-0.64	-0.537106	0.0307673
17	-0.68	-0.498741	0.0333558
18	-0.72	-0.460377	0.0359443
19	-0.76	-0.422012	0.0385328
20	-0.8	-0.383647	0.0411212
21	-0.84	-0.345283	0.0437097
22	-0.88	-0.306918	0.0462982
23	-0.92	-0.268553	0.0488867
24	-0.96	-0.230188	0.0514752
25	-1	-0.191824	0.0540636
26	-1.04	-0.153459	0.0566521
27	-1.08	-0.115094	0.0592406
28	-1.12	-0.0767295	0.0618291
29	-1.16	-0.0383647	0.0644175
30	-1.2	0	0.067006

DIMENSIONS IN KN.CM.

9170 END

:CLOSE

APPENDIX C

Program output data for the test arch

THE BUCKLING MOMENT M1 VALUES

0
58.3662
104.231
0
0
0
104.231
58.3662
0

THE AXIAL FORCE N1 CORRESPONDING TO M1 VALUES

0
-34.5489
-34.5878
0
0
0
-34.5878
-34.5489
0

THE SAGGING MOMENT M2 VALUES

-83.8439
0
0
-36.2508
-333.078
-36.2508
0
0
-83.8439

THE AXIAL FORCE N2 CORRESPONDING TO M2 VALUES

-33.7912
0
0
-33.615
-31.2692
-33.615
0
0
-33.7912

THE DEAD LOAD MOMENT AT EACH STATION

1.41509
0.477516
-0.169748
-0.569525
-0.707542
-0.569525
-0.169748
0.477516
1.41509

THE AXIAL FORCE N3 DUE TO THE DEAD LOAD AT EACH STATION

-0.58301
-0.530926
-0.475791
-0.441794
-0.435801
-0.441794
-0.475791
-0.530926
-0.58301

THE LOWER LIMIT EA VALUES OF THE LIMITING ZONE AT EACH STATION

2.37839
2.27895
1.9961
2.40896
2.41125
2.40896
1.9961
2.27895
2.37839

THE UPPER LIMIT EB VALUES OF THE LIMITING ZONE AT EACH STATION

-2.36676
-2.41118
-2.39675
-3.19837
2.41125
-3.19837
-2.39675
-2.41118
-2.36676

THE PRESTRESSING MOMENT AT EACH STATION

69.2085
100.607
-16.8236
-142.01
130.053
-142.01
-16.8236
100.607
69.2085

THE LACK OF CONCORDANCY AT EACH RELEASE POINT

3.38235E-10
-6.47059E-09
0

THE DESIGN OUT PUT DATA AT EACH STATION

STATION NO.	THE DEPTH	THE P. FORCE	THE PROFILE	THE MAX. STRESSES
1	14.2735	-53.9359	1.28316	0.483665
2	14.2735	-53.9359	1.8653	0.502292
3	14.2735	-53.9359	-0.311918	0.549285
4	14.2735	-53.9359	-2.63295	0.342267
5	14.2735	-53.9359	2.41125	1.2
6	14.2735	-53.9359	-2.63295	0.342267
7	14.2735	-53.9359	-0.311918	0.549285
8	14.2735	-53.9359	1.8653	0.502292
9	14.2735	-53.9359	1.28316	0.483665

THE WIDTH= 10

STRESSES AND DEFLECTIONS AT THE CRITICAL SECTION OF THE STRUC. UNDER
THE APPLIED LOADS

LOAD.	TOP STRESSES	BOT. STRESSES	DEFLECTION
0	-1E-08	-0.761857	0.0231211
1	-0.04	-0.736461	0.0291344
2	-0.08	-0.711066	0.0301477
3	-0.12	-0.685671	0.031161
4	-0.16	-0.660276	0.0321743
5	-0.2	-0.634881	0.0331876
6	-0.24	-0.609485	0.0342009
7	-0.28	-0.58409	0.0352141
8	-0.32	-0.558695	0.0362274
9	-0.36	-0.5333	0.0372407
10	-0.4	-0.507904	0.038254
11	-0.44	-0.482509	0.0392673
12	-0.48	-0.457114	0.0402806
13	-0.52	-0.431719	0.0412939
14	-0.56	-0.406324	0.0423072
15	-0.6	-0.380928	0.0433205
16	-0.64	-0.355533	0.0443337
17	-0.68	-0.330138	0.045347
18	-0.72	-0.304743	0.0463603
19	-0.76	-0.279347	0.0473736
20	-0.8	-0.253952	0.0483869
21	-0.84	-0.228557	0.0494002
22	-0.88	-0.203162	0.0504135
23	-0.92	-0.177767	0.0514268
24	-0.96	-0.152371	0.0524401
25	-1	-0.126976	0.0534534
26	-1.04	-0.101581	0.0544666
27	-1.08	-0.0761857	0.0554799
28	-1.12	-0.0507905	0.0564932
29	-1.16	-0.0253953	0.0575065
30	-1.2	-2E-08	0.0585198

DIMENSIONS IN MM. CM.

9170 END

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