U N I V E R S I T Y   O F   S O U T H A M P T O N

T H E   D E T E R M I N A T I O N   O F
I N T E R - S T A G E   S T O C K   L E V E L S
A N D   O U T P U T   P O T E N T I A L S
O F   U N P A C E D   A S S E M B L Y   L I N E S

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C O N T E N T S

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ABSTRACT

FACULTY OF SCIENCE

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THE DETERMINATION OF INTER-STAGE STOCK LEVELS AND OUTPUT POTENTIALS OF UNPACED ASSEMBLY LINES

by William George Wild

The stock level required between two stages of an unpaced assembly line to ensure 100% utilisation of the fed stage in each of a series of fixed time intervals is a function of the time interval and the operation time distributions of the fed and feeding stages. In the case of balanced lines having Erlang operation times, shape parameter = 1,2, 3 or 4 simulation indicates that a negative binomial distribution may be used to describe the variations in the inter-stage stock level requirement. Regression relationships obtained enable the distribution parameters to be estimated for time intervals in the range 20 to 1370 time units.

If inter-stage stock levels are such that there is a high chance of zero idle time then all stages in a line may be considered to operate independently of each other. The short term output potential of such a line is then equivalent to the output potential of the line's final stage. For Erlang operation times the short term output distribution is general Poisson.

If the chance of zero idle time is to remain constant from one interval to another the inter-stage stock must be the same at the end of a given period as it was at the beginning. Therefore the long term output potential is described by the distribution of the minimum value in a sample of size n (equals number of stages) from the output distribution of an individual stage.

If inter-stage stock levels and output potential are considered in the way described the interactions between planning period, inter-stage storage capacity, number of stages, output schedules and chance of zero idle time can be taken into account when a line is being designed. Examples of such interactions are discussed.

CHAPTER 1

GENERAL CONSIDERATIONS

A Definition of an 'assembly line'

This thesis considers the scheduling of assembly lines where:-

a. an assembly line is considered to be a manufacturing process designed to produce a single product on a continuous basis over a significant period of time by means of a predetermined set of operations.

b. operation stages may be in series and/or in parallel.

c. each stage specialises in one particular aspect of the total process necessary for converting raw materials components and/or sub assemblies into finished products.

d. once the line manager has defined, laid down and commissioned a line his prime objective is to meet planned finished goods production schedules for successive planning periods without tying up excessive amounts of stock between stages.

e. no alterations to the mode of operation of the line are foreseen.

Two basic problems associated with the assembly line manager's prime objective

Problem 1: What constitutes 'excessive' stock?

The experienced line manager has an instinctive appreciation of what do and do not constitute excessive inter-stage stocks. However, few scientifically developed yardsticks are available, and to date Operational Research and associated disciplines have provided little practical guidance on this problem.

A common basis by which stock levels are judged is the 'current' financial investment involved versus previous financial investment levels. As a result, by implication, it is often an organisation's financial personnel who have the final say as to what stock levels should be. However, line managers recognise stocks as a 'lubricant' for providing for the smooth running of a line. But in the absence of any absolute measures of what protection (or degree of lubrication) a given inter-stage stock level will provide, the line manager finds it difficult to argue against the financial experts on any basis other than that of precedence.

The identification of the implications of given inter-stage stock levels is an essential first stage in any investigation aimed at helping the line manager to achieve his prime objective. This problem is discussed in some detail in Chapters 4 and 6.

Problem 2: How do variations in operation times affect the probability that a given output schedule will be met?

In practice assembly lines tend to be planned on the basis of output versus time requirements and to be planned and controlled as though they operate in a deterministic manner. In particular the capacity of a line is usually stated in terms of a single figure e.g. line X can produce 40 sets per day. All planning and control is then based on this figure. But the output per week, month, or other time period used for planning and control purposes is a stochastic variable and its statistical characteristics must be taken into account if fully effective planning and control are to be achieved. Findings relating to the implications of given inter-stage stock levels have been taken as a starting point for investigations into the effect on output potential of operation time variation.

The precise form of assembly line investigated

An assembly line contains 4 major elements:-

1. operation stages.
2. operatives and/or machines.
3. stock at operation stages i.e. items being worked on.
4. inter-stage stock i.e. items waiting in front of a stage prior to being worked on by that stage.

A basic assembly line is here considered to be one:-

a. which is made up of any configuration of strings of stages in series and/or in parallel, a string being one or more stages feeding each other in series (See Figures 1A - 1E, pages 5 - 9)

b. in which there is one machine (or operative) per stage and no machine or operative is used in more than one stage.

c. in which a stage can only operate on one item at a time.

d. in which inter-stage storage capacity is, theoretically, infinite.

e. which is balanced i.e. the means of the operation time distributions are identical for all stages.

f. which will be required to produce exactly the same product in exactly the same way for all of the foreseeable future.

g. which is planned and controlled on the basis of fixed, consecutive, time periods of equal length.

h. in which operatives are unpaced but operation times are statistically consistent over time.

i. in which operation times are independent.

FIGURE 1A: A two stage, single string assembly line.

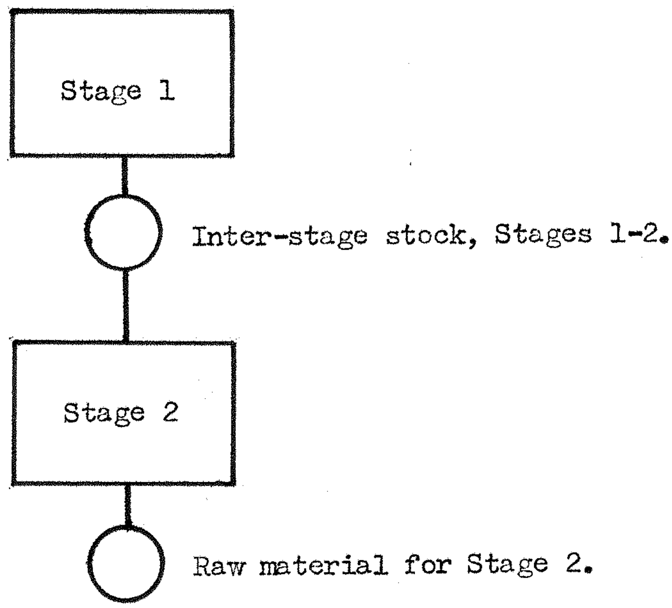


FIGURE 1B: A complex, 9 stage, 3 string assembly line.

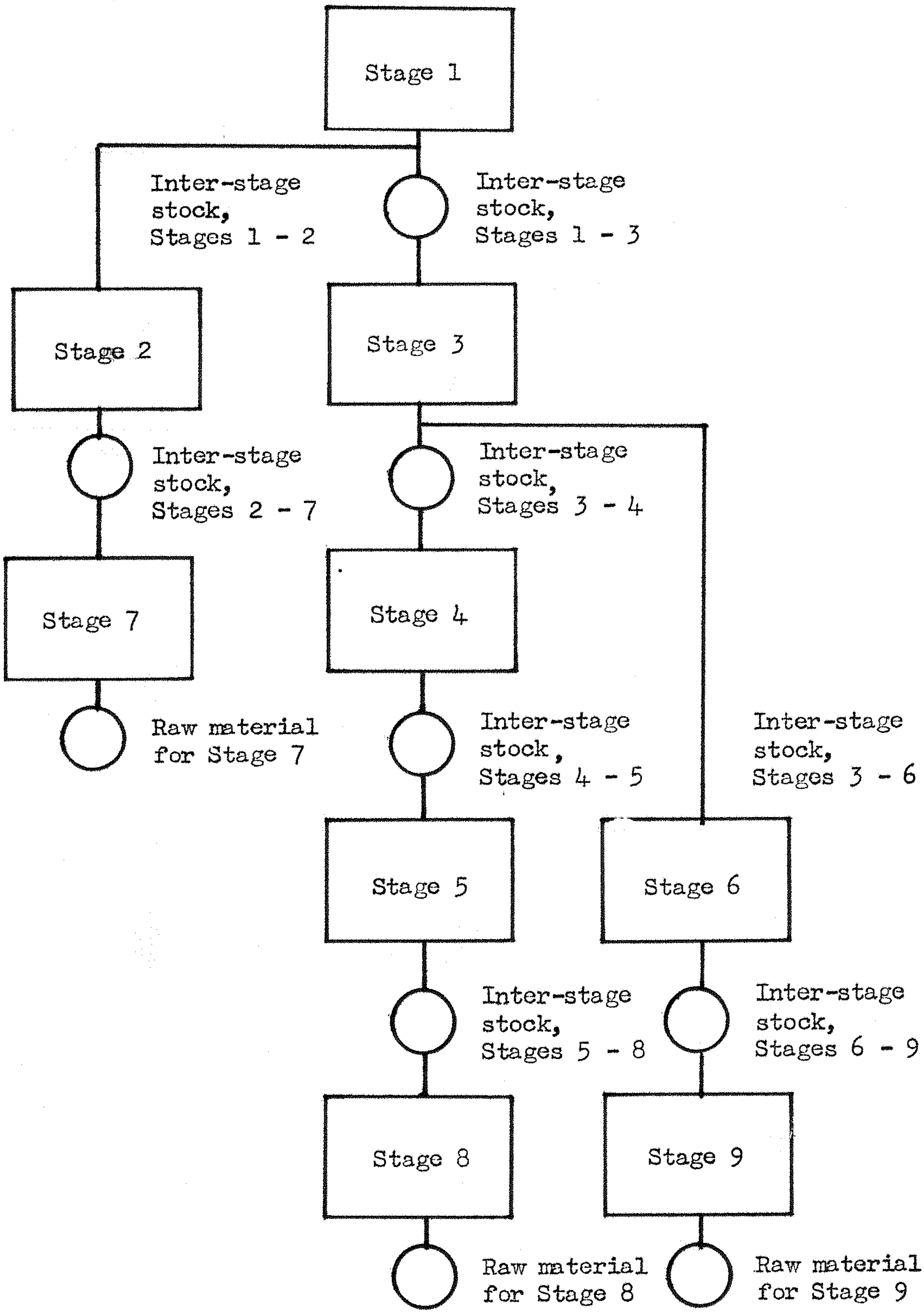


FIGURE 1C: A 3 stage, single string, assembly line.

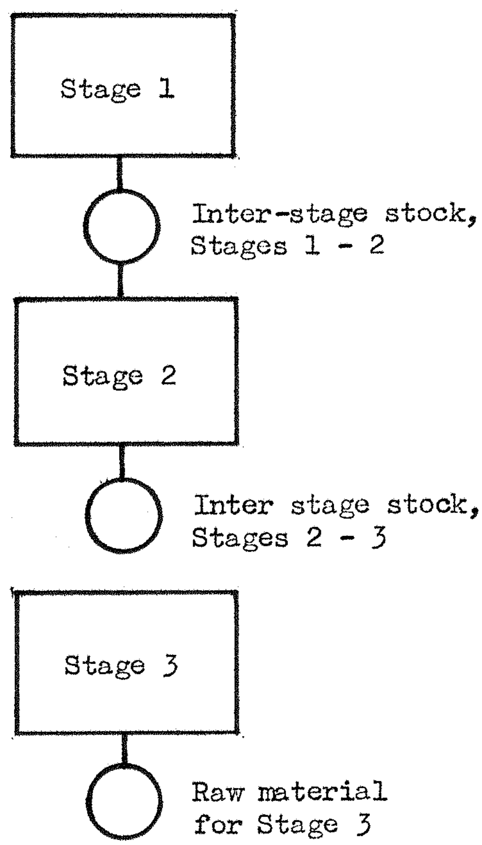


FIGURE 1D: A complex 5 stage, 3 string, assembly line.

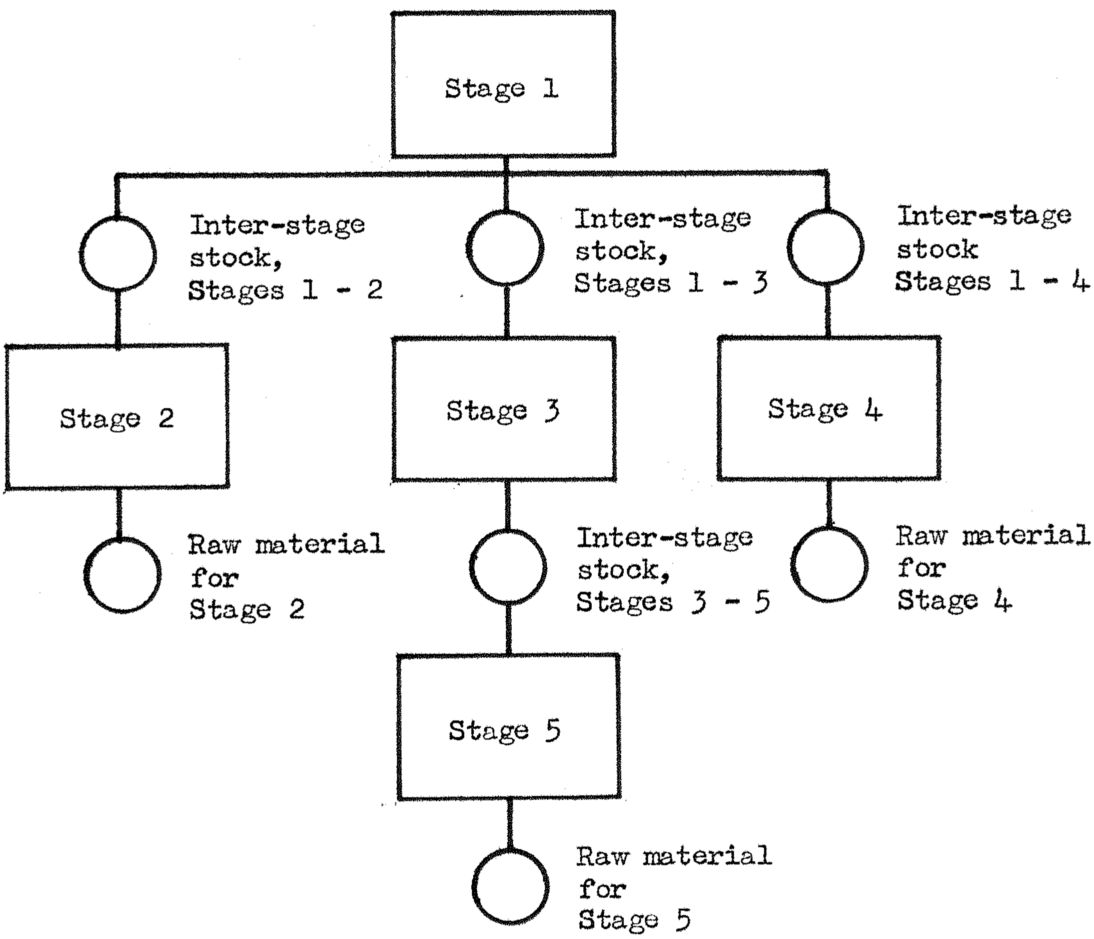
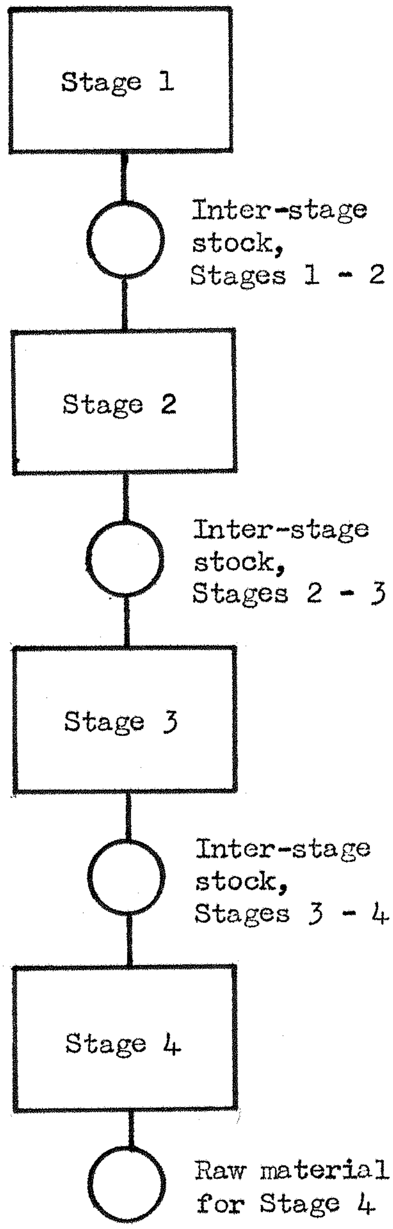


FIGURE 1E: A 4 stage, single string, assembly line.



The results presented in this thesis are based upon investigations into the operations of assembly lines complying with a - d and f - i above. Point e was assumed to apply but in addition operation times were assumed to follow an Erlang distribution. (The factors leading to the assumption of Erlang operation times are discussed in Chapter 2, pages 18-21).

The type of line thus defined will be referred to subsequently as a 'perfectly balanced line'. Literature on the subject of line balancing (Freeman (1964), Ignall (1965)) tends to refer to lines in which all mean stage operation times are the same as being 'balanced'. 'Perfectly' is used here to draw attention to the fact that not only are means identical but so too are all other distribution parameters.

The type of line considered is just one of the very large number of possible abstractions of line that might be studied. However the basic ideas and concepts put forward in this thesis are of general applicability. The form specified was simply a convenient vehicle for the purposes of the investigations carried out.

Accounting of output

It is assumed (g, page 4) that scheduling is based on fixed, consecutive, time periods of equal length. In the short term i.e. the next planning period, the output can reasonably be taken to be that of the final stage. This will be implied when future reference is made to 'short term output'.

In the long term if, as will be shown to be the case, it is possible to specify initial inter-stage stock levels that provide an high, constant, chance of successive planning period output requirements being met then the output figure accredited to a given planning period must be such that the inter-stage stocks are left the same at the end of a period as they were at the beginning. This being so it is the output of the slowest stage in a given period that determines the output for that period and the distribution of the output of the slowest stage in successive periods that governs the overall output potential of the line.

Short and long term output potentials are discussed more fully in Chapters 6 and 7.

CHAPTER 2

OUTLINE OF APPROACH USED

It was stated in Chapter 1 (page 1) that a line manager's prime objective is considered to be to meet planned finished goods production schedules for successive planning periods without tying up excessive amounts of stock between stages. The research described here concentrated upon the first part of this objective i.e. 'a line manager's (prime) objective is to meet planned finished goods production schedules for successive planning periods'.

It was assumed that production schedules are imposed by an external agency e.g. a sales department. Such schedules could be assumed to be reasonable i.e. not in excess of the basic production capacity, but beyond the control of the line manager. It was further assumed that the line had no finished goods storage facilities available to it and that accounting of output was independent from one planning period to the next. Thus a 6 month's schedule for a line with a mean capacity of 100 items per month might be:

Month	Jan.	Feb.	Mar.	Aprl.	May.	June.
Schedule	80	90	100	100	100	90

The line manager's aim would be to meet each of these figures exactly, considering each one independently.

It is considered that the procedure developed and the results obtained provide a sound basis for a subsequent investigation into line economics referred to in the second part of the objective.

Inter-stage stocks and the maximisation of a line's output potential

Different lines are scheduled in different ways, but schedules are normally based on a convenient calendar interval e.g. day, week, month or quarter. Meeting such schedules presents difficulties when there are mechanical failures on the line, breakdowns in the supply of components and/or raw materials to the line, changes of output schedules as a result of changes in marketing policy or demand,etc; but even if the truly unexpected does not occur a fundamental problem is always present i.e. how to set about meeting an output schedule when a foreseeable degree of variability is inevitable. A '(foreseeable) degree of variability' is inevitable because operation times are subject to variation. Therefore the output from a stage per unit time interval will tend to vary too.

If a schedule calls for an output during the planning period from a 1 stage line equal to (planning period/mean operation time) there is a 50% chance (approximately) of meeting the schedule - the exact chance depending upon the relationship between the mean and median of the output distribution. Even this 50% figure can only be approached if the stage can be fed continuously with raw materials and or sub/assemblies and thus be usefully occupied for the whole of the planning period. For a single stage line, or the first stage in each string of a complex line, this is normally assumed to be so. But in general, in multi-stage lines, each 'non-end-of-string' stage is fed by another stage and if two adjacent stages are not in complete synchronization or there is not a significant amount of stock between them then the fed stage will tend to be idle on occasions.

Since operation times are variable and it is assumed here, independent, synchronization is not possible and inter-stage stock must be used if a high degree of utilisation of the fed stage is to be ensured and thus the chance of meeting schedules maximised.

Illustrative operations of a 2 stage line

Table I shows the operations of a 2 stage line over a planning period of 20 time units. It is assumed that the fed and feeding stages each have 1 item available for commencing work on at time 0, but that there is no initial inter-stage stock (IISS).

TABLE I: Operations of a 2 stage line with no initial inter-stage stock.

Item reference	Feeding stage commences work on item at time:	Operation time	Feeding stage finishes with item at time:	Item reference	Fed stage commences work on item at time:	Operation time	Fed stage finishes item at time:
B	0.0	$2.\dot{9}$	$2.\dot{9}$	A	0.0	$0.\dot{9}$	$0.\dot{9}$
C	3.0	$4.\dot{9}$	$7.\dot{9}$	B	3.0	$0.\dot{9}$	$3.\dot{9}$
D	8.0	$2.\dot{9}$	$10.\dot{9}$	C	8.0	$1.\dot{9}$	$9.\dot{9}$
E	11.0	$3.\dot{9}$	$14.\dot{9}$	D	11.0	$3.\dot{9}$	$14.\dot{9}$
F	15.0	$0.\dot{9}$	$15.\dot{9}$	E	15.0	$1.\dot{9}$	$16.\dot{9}$
G	16.0	$0.\dot{9}$	$16.\dot{9}$	F	17.0	$1.\dot{9}$	$18.\dot{9}$
H	17.0	$4.\dot{9}$	$21.\dot{9}$	G	19.0	$3.\dot{9}$	$22.\dot{9}$

The line output during the planning period of 20 time units is seen to be 6 items - item G not being counted since it was completed after the end of the planning period.

The feeding stage had no idle time since it was assumed to be fed from an infinite stock of raw materials. The fed stage was idle for 2 time units from 1 to $2.\dot{9}$, 4 time units from 4 to $7.\dot{9}$ and 1 time unit from 10 to $10.\dot{9}$.

Assuming that the fed and feeding stage operation times follow random independent distributions, output and idle time will vary from planning period to planning period. The line's output is a function of the fed stage's idle time and a reduction of such idle time will tend to lead to an improvement in output. For maximum output no idle time should occur. This can be achieved by having 'sufficient' inter-stage stock available at the beginning of the planning period. This initial inter-stage stock can be used in two ways:

i. output from the feeding stage goes to the end of the 'queue' formed by the initial inter-stage stock and previous items completed by the feeding stage.

ii. the two types of stock - those items produced by the feeding stage during the planning period and those items making up the initial inter-stage stock-are kept separate and when the fed stage requires an item it takes it from the feeding stage's output stock whenever possible. Only when there is no such stock is initial inter-stage stock used.

Examples of movements for each of these models are given in Tables II and III. In both cases an initial inter-stage stock of 3 is sufficient to ensure 100% utilisation of the fed stage. (Due

TABLE II: Operations of a 2 stage line assuming that the three items - reference IISS1, 2 and 3 - making up the initial inter-stage stock are used by the fed stage before any items produced by the feeding stage.

Item reference	Feeding stage commences work on item at time:	Operation time	Feeding stage finishes with item at time:	Item reference	Fed stage commences work on item at time:	Operation time	Fed stage finishes item at time:
B	0.0	$2.\dot{9}$	$2.\dot{9}$	A	0.0	$0.\dot{9}$	$0.\dot{9}$
C	3.0	$4.\dot{9}$	$7.\dot{9}$	IISS1	1.0	$0.\dot{9}$	$1.\dot{9}$
D	8.0	$2.\dot{9}$	$10.\dot{9}$	IISS2	2.0	$1.\dot{9}$	$3.\dot{9}$
E	11.0	$3.\dot{9}$	$14.\dot{9}$	IISS3	4.0	$3.\dot{9}$	$7.\dot{9}$
F	15.0	$0.\dot{9}$	$15.\dot{9}$	B	8.0	$1.\dot{9}$	$9.\dot{9}$
G	16.0	$0.\dot{9}$	$16.\dot{9}$	C	10.0	$1.\dot{9}$	$11.\dot{9}$
H	17.0	$4.\dot{9}$	$21.\dot{9}$	D	12.0	$3.\dot{9}$	$15.\dot{9}$
				E	16.0	$2.\dot{9}$	$18.\dot{9}$
				F	19.0	$1.\dot{9}$	$20.\dot{9}$

to the different rules used for drawing on initial inter-stage stock the movements of individual items differ but the net result is the same - zero idle time and 8 items - compared with 6 when IISS = 0 - produced within the planning period).

During subsequent, independent, planning periods the initial inter-stage stock requirement (IISSR) for 100% utilisation of the fed stage will vary - will in fact follow some statistical distribution.

TABLE III: Operations of a 2 stage line assuming that the three items - reference IISS1, 2 and 3 - making up the initial inter-stage stock are only used when feeding stage output is not available.

Item reference	Feeding stage commences work on item at time:	Operation time	Feeding stage finishes with item at time:	Item reference	Fed stage commences work on item at time:	Operation time	Fed stage finishes item at time:
B	0.0	$2.\dot{9}$	$2.\dot{9}$	A	0.0	$0.\dot{9}$	$0.\dot{9}$
C	3.0	$4.\dot{9}$	$7.\dot{9}$	IISS1	1.0	$0.\dot{9}$	$1.\dot{9}$
D	8.0	$2.\dot{9}$	$10.\dot{9}$	IISS2	2.0	$1.\dot{9}$	$3.\dot{9}$
E	11.0	$3.\dot{9}$	$14.\dot{9}$	B	4.0	$3.\dot{9}$	$7.\dot{9}$
F	15.0	$0.\dot{9}$	$15.\dot{9}$	C	8.0	$1.\dot{9}$	$9.\dot{9}$
G	16.0	$0.\dot{9}$	$16.\dot{9}$	IISS3	10.0	$1.\dot{9}$	$11.\dot{9}$
H	17.0	$4.\dot{9}$	$21.\dot{9}$	D	12.0	$3.\dot{9}$	$15.\dot{9}$
				E	16.0	$2.\dot{9}$	$18.\dot{9}$
				F	19.0	$1.\dot{9}$	$20.\dot{9}$

The parameters of this distribution e.g. the mean, will depend upon the planning period. If an IISS level having only a given chance of running out - and thus of idle time occuring - is to be quoted then this IISSR distribution for the planning period concerned must be known.

Various conceptual models suggest themselves as a basis for seeking an identification of IISSR distributions analytically. However a consideration of such models, even for the most basic of assumptions e.g. operation time distribution for fed and feeding stages negative exponential with identical means, has so far failed to provide an answer. The alternative of a simulation approach was therefore adopted. Simulation has the important advantage that even if the simpler cases do subsequently yield an analytical solution it is unlikely that more complex, more realistic, models would do so. Simulation on the other hand can be used to investigate models of any degree of complexity.

Operation time distributions

Hicks and Young (1962), Murrell (1962), Dudley (1955, 1958 1962, 1963) and Sury (1967) have shown that the distributions of operation (or cycle) times tend to be positively skewed. Since the procedure presented here is based on simulation it can be used for any type of operation time distribution. However in order to develop the procedure it was convenient to assume a standard family of distributions.

In addition to the above quoted evidence that shows that operation time distributions tend to be positively skewed, practical considerations further suggest that such distributions:-

i. will be continous

ii. will have 'completion rates' which increase as time increases, 'completion rates' being analagous to 'failure rates' of Reliability Theory.

Relevant characteristics of standard continuous distributions that might be considered are given in Table IV.

TABLE IV: Skewness and completion rates of common continuous distributions.

Distribution	Skewness	Completion rate as time from commencement of operation increases
Negative exponential[1]	Positive	Constant
Normal	Symmetrical	Increases
Log normal	Positive	Decreases at extreme values
Weibull	Positive or Negative	Monotonically increasing, constant or decreasing
Gamma	Positive	Asymptotically increasing, constant or decreasing
Erlang[2]	Positive	Asymptotically increasing or constant

Notes 1. The negative exponential is a member of the Weibull, gamma and Erlang distribution families.

2. The Erlang distribution forms a sub-set of the gamma distribution family.

The negative exponential distribution has a constant completion rate and is therefore unlikely to apply to operation times (despite which many investigations e.g. Buzacott (1971) and Hillier and Boling (1966), have assumed such operation times). It does however occur as a special case of both the Weibull and gamma distributions (see below) and has the attraction of being one of the simplest distributions to deal with analytically. Of the other distributions listed in Table IV the normal and log normal distributions appear unlikely candidates for operation time distributions. The Weibull distribution is of considerable importance in Reliability Theory but has not often been associated with operations times or, in the wider sense, Queueing situations. On the other hand gamma distributions have been applied to a wide variety of problems and the Erlang subset has found extensive application in the Queueing field. It was decided therefore to assume Erlang operation times in the simulation investigations. (A small, independent, investigation into the effect of assuming Weibull rather than Erlang operation times was carried out. The results are summarised in Appendix A).

The Erlang probability density function (p.d.f.) is:-

$$\mathrm{erl}(t;K,m') = \frac{\left(\frac{K}{m'}\right)^{K} t^{K-1} e^{-Kt/m'}}{(K-1)!} \qquad \text{for } t \geq 0$$

$$= 0 \qquad \text{elsewhere}$$

where m' = the mean.
K = the shape parameter.

For K = 1 this reduces to the negative exponential distribution's p.d.f. Despite the fact that operation times are unlikely to follow a negative exponential distribution in practice, it was included in the investigation as a particular case of the Erlang distribution because the results could provide a useful first check of any analytical approach that might subsequently be developed.

Secondary advantages to be had by assuming Erlang operation times were:-

a. the family covers a wide range of positively skewed distributions from exponential through to deterministic.

b. Erlang distributions are analytically tractable and convenient to handle in simulation work.

c. research in the more general field of Queues and Queueing has shown that service times are frequently Erlang (Saaty 1957). This has been justified in practice and in theory. The assembly line presents a special form of Queueing problem and operation times are obviously specific examples of service times. It might be that the results presented here have application outside the assembly line field since there are almost certainly analogous problems elsewhere, e.g. dams for water storage.

Implications of the IISSR distribution approach

If (as is the case) it is possible to specify an IISS level which in conjunction with output from the feeding stage has a required chance of keeping the fed stage 100% utilised during a given planning period then in practice - when a high chance of 100% utilisation will tend to be specified - three important consequences follow:

i. each stage may be considered to operate independently of all other stages.

ii. the short term output potential of the line (see Accounting of output, page 10) may be considered to be the output potential of the final stage - which will be 100% utilised.

iii. the long term output potential of the line may be considered to be the output of the slowest stage in a given planning period (since IISS for each stage must always be left at the same level at the end of a planning period as it was at the beginning if the chance of 100% utilisation is to remain constant from planning period to planning period).

Not only will the chance of meeting a given schedule be maximised but both the long and short term scheduling problems will be simplified to the extent that the output distribution of a stage operating independently of all other stages will in general be easier to define than the output distribution of a non-independent stage (which might be the case if IISS is set too low).

Summary of approach used

i. in practice the first priority of a line manager was considered to be the meeting of output schedules.

ii. a major influence on his ability to meet schedules are inter-stage stock levels. For maximum likelihood of success initial inter-stage stocks should be infinite. In practice this is impossible and unnecessary. The distribution of IISSR for 100% utilisation of a fed stage during a given planning period was investigated therefore.

iii. if actual IISS is set at a level that gives a high chance of 100% utilisation of a fed stage then for practical purposes all stages in a line may be considered to be independent. The output potential of a line was determined therefore on the basis of stages 100% utilised.

iv. Erlang operation times were assumed. In practice a variety of operation time distributions might be involved in a given line, ranging from Erlang through to unique distributions defined only by a given set of Work Study observations. In the light of their place in Queueing Theory Erlang operation times could have a particular value, but the general procedure developed could be used for any operation time distributions. Erlang operation times simply provided a convenient and relevant set of operation time distributions with which to develop the basic procedure.

CHAPTER 3

REVIEW OF PUBLISHED PAPERS

Much research has been directed at gaining an understanding of the many problems associated with the design, planning, and control of unpaced assembly lines. Research papers that are directly relevant to the present thesis may be conveniently considered under two headings:

i. those which consider the effects on a line's output potential of given inter-stage storage capacities and, where operation time parameters vary from stage to stage, of the ordering of stages within a line.

ii. those which deal with the characteristics of operation time distributions for unpaced operations.

The more important papers are reviewed under each of the above headings and, where considered appropriate, parallels are drawn with the present author's approach (as outlined in the previous chapter).

Papers which concentrate upon the effect on output potential of inter-stage storage capacity and of the ordering of stages within a line.

Almost without exception inter-stage storage capacity is considered in previously published papers rather than, as is the case here, the actual initial inter-stage stock requirement (IISSR). The difference is important since the capacity approach immediately imposes a restriction on the way the line may operate i.e. that when an inter-stage storage space is full the feeding stage cannot pass any more completed items on. Thus a feeding stage can be idle for want of storage space as well as shortage of items to work on. The assumption of limited storage capacity makes investigation of the upper limit of output potential of a line impossible. In contrast the IISSR approach enables maximum output potential to be determined first and then for inter-stage storage capacity requirements for a given chance of achieving maximum output potential to be specified.

Anderson and Moodie (1969) stated that 'All production lines have a certain amount of inbalance and service time variability at different stages. Thus a problem arises: how to calculate the minimum in-process inventory needed between stages to prevent delays.' They developed models which 'will aid the production line designer to determine the best buffer inventory size for given manufacturing and facility constraints'. They considered the best buffer inventory size ('size' and 'capacity' apparently being synonymous in their paper) to be a function of average delay (idle time), average in-process inventory, storage spaces, delay cost per unit time, inventory cost per unit per unit time, storage space cost per unit per unit time and number of stages. Emphasis was on a 'best' buffer capacity, and empircial formulae, developed by applying analysis of variance and regression analysis to simulation results for lines with normal and exponential operation times, were presented. Both steady state and transient conditions were considered and it was concluded that 'minimum cost operation occurs when buffer inventory limits are maintained at their best steady state level during the entire production run. There is no cost advantage in controlling the inventory during the transient period'. ('Transient period' was considered in the context of a line stopping and starting - perhaps to change production models).

Barten (1962) discussed the effect of limited (0 to 6 items) inter-stage storage capacities on the overall output rate of single strings of up to 10 operations, operation times assumed to be normal with a range of coefficients of variation. Barten commented that 'Infinite storage is the ideal, from the point of view of time, but never exists in reality, and hence is of little interest. Under this circumstance every operation would operate independently of every other one, regardless of the average times required to perform each operation, the distribution and variations of these times, and the total number of operations. The output of such a system would be determined by the production rate of the last operation'. With regard to the development of analytical solutions to line storage and output problems Barten said 'In the sequential storage problem, so long as the capacity of a storage area is extremely large, permitting it to hold a virtually infinite quantity of material, the arrival rate of items into the storage is in fact independent of the rate with which it is removed (the service rate). However, if the storage is limited and can become filled to this limited capacity, the arrival rate becomes dependent upon the service rate and, with the storage becoming and remaining filled, the rate of material arriving can never exceed the rate with which items are removed. It is clear from the foregoing, that the present status of queueing models does not permit the general solution of storage problems in sequential production processes'. His investigations were therefore based on the simulation of (relatively simple) lines having normal operation times, for which numerical results were given.

Buzacott (1971) considered the role of inventory banks in flow line production systems. Whilst the effects of fixed and variable operation times were considered the greater part of the paper related to the effect of machine breakdowns and repairs. In the introduction he stated that 'Many managers feel that the level of their inprocess inventories is too high; however they lack guidance on what the level of such inventories should be; indeed they are often uncertain of the precise function of inventories and storage'. The effect of exponential operation time distributions on production capacity were discussed and, in the case of two identical stages (exponential operation times) it was concluded that 'inventory banks with capacity no greater than 4 or 5 give a significant improvement in the system production capacity'. In the conclusions it was stated that: 'If, with a given inventory capacity, production capacity is inadequate then the manager must investigate the relative merits of increasing inventory capacity.

Davis (1975) described an 'Interactive Simulation' approach to helping a line manager meet his primary objective, i.e. 'The primary objective of a production line manager is to meet production demands, both volume and time, while controlling production costs and inventories.' (Compare with the independently arrived at 'prime objective' given in Chapter 1, page 1, d.) The interactive simulation system presented in the paper enables the line manager to investigate ways of reducing existing inter-stage stocks to 'desired' levels without jeopardizing the attainment of a required output rate. But it was assumed that a line manager can specify the 'desired' levels; no indication was given of how to arrive at such levels.

Hillier and Boling (1966) discussed the effect that the number of work stations, storage capacity and unbalancing of two, three and four station lines with exponentially distributed operation times, has on the production rate. Only small/simple lines were considered e.g. maximum of four stations in series with a maximum buffer storage capacity of 4 units. On the basis of their results they introduced the concept that 'in some cases unbalancing a production line with variable operation times can increase its efficiency.' They suggested that the 'bowl phenomena' they had found indicated that a suitable basis for maximizing a system production rate was the assigning of lower than average operation times to intermediate stations rather than to stations at either end. This idea has been referred to frequently by other authors e.g. Payne, Slack & Wild (1972) and Wild and Slack (1973).

Hillier and Boling (1967) investigated queueing systems with N service channels in series with limited inter-stage storage capacity and Erlang operation times. They were particularly interested in the mean output rate of the final stage - taking into account the effects of blocking (the inability of an item finished with by one stage moving into the queue in front of the next operation, due to that queue - of limited capacity - being full) and the mean number of items in the system. They presented extensive theoretical, approximate and numerical results in support of their approach.

Kala and Hitchings (1973) investigated the operations of a four station, balanced, unpaced assembly line using simulation. Their paper included details of the production rate and build-up of inter-stage stock measured against a wide range of operation time variances. Their results and conclusions were similar to those of the present author but, as with most other researchers they assumed that a significant amount of idle time is inevitable. They then concentrated upon ways of assigning operations within a line with a view to minimising the 'inevitable' idle time and thus maximising output.

Knott (1970) presented an empirical basis for determining the output potential of a series queue system and compared his results with those of Hillier and Boling (1967). Output was considered to be a function of the number of stations, queue capacity and operation time distributions. A range of operation time distributions, including Erlang, were considered and empirically derived formulae, coupled with theoretical rationale, were presented. Inter-stage capacities were emphasised throughout.

Patterson (1964) considered the optimal ordering of stages in a string and the mean output rate of such strings. Attention was concentrated upon the effect of unequal stage mean operation times, the emphasis being upon the line balancing problem.

Payne, Slack and Wild (1972) investigated the operation of a 20 station balanced flow line working with unlimited inventory or queue capacities between stations. The initial objectives of the investigation were to determine the idle time of each station and the maximum queue before each station for various (normal) work time distributions over a specific time period. All simulations started with an empty flow line. It was found that although the lines were balanced as regards the nature of the station work time distributions, stations towards the end of the line incurred more idle time than those at the beginning because each station was dependent upon all predecessors for the supply of work; hence the more predecessors the greater the probability of a station being starved of work.

Payne et al argued that the simulation of a line with zero initial inter-stage stock and operating over a fixed time period (5000 time units, which in conjunction with the constant work time distribution mean of 10 time units meant less than 500 units of production) produced results of practical significance since few practical flow lines operate in a steady state condition. In their final discussion they raised 2 questions:-

1. is it satisfactory to consider line balance to be a valid objective of flow line design?
2. is it valid to attempt to establish methods for determining the single optimum buffer stock capacity for all stations on particular flow lines?

Question 1 was prompted by their results which showed that in order to reduce total station idle time, operations should be allocated to stations so that those towards the end of the line have higher station time means.

The dismissal of the balanced line objective would seem intuitively wrong. But if the balanced line objective is rejected then the answer to question 2 must be 'no' - an optimum buffer stock capacity for each station must then be sought. The resulting complications seem well worth avoiding if at all possible. As has been previously stated the research reported on here concentrated on initial inter-stage stock levels necessary to reduce the probability of any idle time at all to an acceptably low figure. Buffer stock capacity requirements are then obtained as a by-product. The determination of an optimum inter-stage stock level can subsequently be approached via the stock-out-risk level to be used. The intuitive appeal of completely balanced lines is then no longer challenged since idle time can be made to occur at a controlled rate and a single stock level can be sought.

Wild and Slack (1973) considered a related aspect of assembly lines but not one of direct concern here, namely 'single' versus 'double' lines - a single line having one operator per stage and a double line having two operators per stage. Nevertheless it is of interest to note that they used a simulation approach, assumed 'a given maximum capacity' (between each stage) - limited to 1,2, 3 and 4 inter-stage buffer capacity units per operator and assumed the line to be perfectly balanced, all operation times normal with common mean and variance.

Papers relating to operation time distribution characteristics

Hicks and Young (1962) investigated various aspects of elemental time distributions. Their studies suggested that whilst such distributions might tend to be positively skewed, for many purposes an assumption of normality of operation times was reasonable.

Murrell (1962) considered the effects of operator variability and fatigue. The paper is of interest for its discussion of the cycle time distributions for repetitive tasks. The case is argued for positively skewed distributions as opposed to a symmetrical (normal) model.

Much research has been carried out at the University of Birmingham, Department of Engineering Production, into the characteristics of cycle (operation) times of unpaced assembly tasks. In particular Dudley (1955, 1958, 1962, 1963) and Sury (1967) have shown that the distribution of such cycle times is invariably positively skewed.

Mansoor and Ben-Tuvia (1966) concerned themselves with the line balancing aspect of assembly lines but made several relevant points e.g.'............the basic assumption in (the previously mentioned heuristic line balancing solutions) is that work element times are deterministic. In practice this is never so, the work element time is a random variable and experience shows that the distribution closely follows the normal distribution (Hicks and Young (1962) and thus if the average work element time, denoted by u, is used for line balancing then on the average 50 per cent of the items will not be finished within the cycle time, causing serious hold up of the line. Clearly the cycle time must be set at some figure $c > u$. Moodie and Young (1965) consider an assembly line consisting of n stations. They show that if the variability of the work element time is taken into account then it is possible to ensure that a certain high percentage of units are completed without a hold-up of the line. Clearly this percentage is dependent on c.' The paper assumed no inventories were allowed between stages.

Davis (1966) described results obtained in a simulation of an assembly line with and without pacing of operatives. He confirmed Conrad's (1955) conclusion that unpaced operations are superior to paced operations. He gave as a definition of an unpaced operation' (an operation) taking place with an infinite queue at each station so that the operator is insulated from arrivals from prior stations (stages).' It is interesting to note in connection with the comment on page 25 with regard to the concentration of previous investigators on inter-stage capacity rather than inter-stage stock that Davis did not distinguish clearly between the two. At one point he stated that an assembly line cannot perform at maximum efficiency (in terms of operation idle time and units completed) unless queues are provided before each work station. Here 'queues' might reasonably be taken to imply inter-stage stock as considered in this thesis, but at another point (in a table heading) he referred to 'Maximum allowable queue at each station'- clearly a reference to storage capacity. He concluded that infinite queues at each station gave better performance than lesser or no queues.

Slack and Wild (1975) in a comparison of flow lines and collective working as production systems represented operator work time distributions by a Weibull distribution. They justified this on the basis of Slack's (1972) finding that published histograms of manual work may be represented by Weibull distributions.

Muth (1973) in considering the effect of variability on the lower bound of the production rate of work stations discussed the possible failure rate characteristics of operation time distributions. He suggested that in practice it would be more reasonable to assume fixed operation times rather than exponential operation times. He went on to imply that distributions with completion rates that increase as time from commencement of work on an item increases would be more appropriate than distributions with decreasing completion rates.

CHAPTER 4

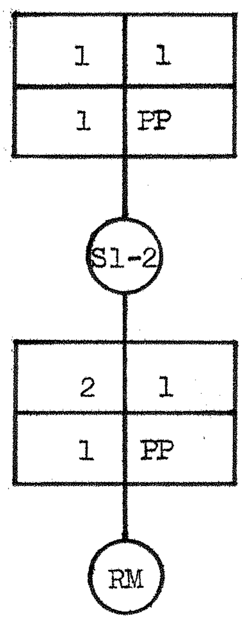
INITIAL INTER-STAGE STOCK REQUIREMENTS OF PERFECTLY BALANCED ERLANG LINES

Figure 2, page 36, represents the simplest form of perfectly balanced line with Erlang operation times. The output of finished goods from Stage 1 during a given planning period is dependent upon the operation time distribution parameters (assumed to be the same for stages 1 and 2) and the inter-stage stock at the beginning of the planning period. It will vary, according to some statistical law, from one planning period to another. The higher the initial inter-stage stock the less will Stage 1's idle time tend to be and the higher will be the probability of a given scheduled output figure being achieved, the highest probability being achieved with infinite initial inter-stage stock.

The problem considered is the determination of the initial inter-stage stock level which, if present at the beginning of a given planning period, will have an (acceptably) high chance of proving sufficient to keep the fed stage 100% utilised. This stock level is a function of the planning period and of both the fed and feeding stage operation time distributions.

The determination of what constitutes an 'acceptably high chance' should be based on economic and managerial considerations. Economic considerations would involve determination of, amongst other things, cost of stage idle time, cost of not meeting production schedules and cost of inter-stage storage. Managerial considerations would take account of the psychological

FIGURE 2: Perfectly balanced 2 stage assembly line, operation times Erlang, mean = 1.0 time unit, K = 1.



<u>Legend</u>

Stage number	Shape parameter K
Mean, m'	Planning Period

S1-2 = inter-stage stock, Stages 1 and 2

RM = raw material

effect of idle time on operators, the effect on supplier/customer relations of non-compliance with delivery schedules, etc. These economic and managerial aspects are not considered here but it seems reasonable to assume that a chance of sufficient stock being available (equals chance of zero idle time) in the range 90 - 99.9% might apply in practice. Examples presented here tend to be based on this preconception. But should figures outside this range be more appropriate no changes of principle would be required: only numerical values would change. Once the initial inter-stage stock requirement (IISSR) figure is established (for a given chance of zero idle time) and made available, Stage 1 will operate, for the majority of planning periods as though it is being fed from an infinite inter-stage stock and therefore as though it is entirely independent of the feeding stage, Stage 2.

A simulation based procedure for investigating initial inter-stage stock requirements

A line may be considered to be made up of a number of stock points. A stage i fed by q_i other stages will 'contribute' q_i independent stock points. Thus in Figure 1D, page 8, Stage 1 is fed by three other stages. Therefore $q_1 = 3$ and Stage 1 contributes 3 inter-stage stock points to the line total of 4 - the fourth being between Stages 3 and 5.

Since IISSR is a function of both fed and feeding stage operation time distributions each inter-stage stock point's requirement will, in principle, have to be investigated separately. Where however two or more inter-stage stock points are identical in terms of their fed and feeding stage operation times the results obtained for one may be used directly for the others. Thus in the case of a perfectly balanced line only one case need be investigated.

For a stock point within a line scheduled on the basis of a specified planning period, the IISSR for a given chance of 100% utilisation of the fed stage can be determined as follows:

i. simulate the operations of the two stages concerned over a number of independent planning periods - independent that is in terms of the operation times that occur within them.

ii. from the simulation results determine the distribution of initial inter-stage stock necessary to ensure that the fed stage did not run short of work during any of the planning periods simulated.

iii. determine, either directly from the simulation results or from a standard distribution fitted to the simulation results, that value of initial inter-stage stock that has the required probability of proving sufficient to keep the fed stage fully occupied.

An investigation based on the above procedure

The computer programme of Appendix B was used to simulate the operations of two stage assembly lines having Erlang operation times, mean = 1.0, K (shape parameter) = 1,2, 3 and 4.

In order to cover a reasonable range of values of planning period (PP) the ratio of successive values of planning period simulated was set equal to $\sqrt{2}$. The first planning period simulated was $8\sqrt{2}$ and thus subsequent values were 16, $16\sqrt{2}$, 32, $32\sqrt{2}$, 64, $64\sqrt{2}$, 128, $128\sqrt{2}$, 256, $256\sqrt{2}$, 512, $512\sqrt{2}$, 1024, and $1024\sqrt{2}$ time units.

The first objective in carrying out the simulations was to gain a general insight into the IISSR distribution. A simulation sample size of 100 for each (PP,K) combination used proved adequate for this purpose and required computer runs of an acceptable duration. The programme of Appendix B simulated the basic operations of two stage lines and identified the level of initial inter-stage stock necessary to keep the fed stage 100% utilised during each of the planning periods simulated for a given (PP,K) combination. The programme summarised the results for each combination in the form of an initial inter-stage stock requirement frequency distribution.

Results

Details of the IISSR frequency distributions are given in Tables VA - VD, pages 40,- 46. Each column in the body of the table gives the results for the (PP,K) combination indicated at the top.

Thus in the 100 simulations for PP = $8\sqrt{2}$, K = 1 there were 13 (independent) planning periods when no initial inter-stage stock was required - it being assumed that the fed and feeding stages each had one item available for commencing work on at the beginning of the planning period; a zero IISSR indicates

TABLE VA: INITIAL INTER-STAGE STOCK REQUIREMENT FIGURES FOR 100% UTILISATION OF A FED STAGE - OBTAINED BY SIMULATING THE OPERATIONS OF TWO STAGE LINES. OPERATION TIMES ERLANG. MEAN = 1.0, K=1,2,3, AND 4. 100 SIMULATIONS CARRIED OUT FOR EACH OF THE PLANNING PERIODS INDICATED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 8/2				16				16/2				32			
K =	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
0	13	12	17	22	7	16	16	17	11	10	16	16	13	8	9	11
1	23	17	25	30	14	15	20	23	15	17	25	21	11	17	15	20
2	13	29	27	21	14	25	20	19	14	16	9	18	9	11	19	19
3	9	20	18	14	16	14	20	13	10	14	16	21	11	14	12	13
4	10	11	9	9	8	6	12	13	9	6	12	9	10	11	7	9
5	11	4	3	3	12	5	8	10	4	12	8	7	9	12	9	12
6	6	3	1	0	11	9	1	3	8	10	4	4	6	10	9	6
7	8	3	0	0	6	8	1	1	4	7	1	2	4	5	5	8
8	1	1	0	0	4	2	2	1	4	3	3	1	4	2	7	1
9	2	0	0	1	3	0	0	0	6	4	4	1	4	2	3	1
10	4	0	0	0	3	0	0	0	4	0	1	0	3	3	1	0
11	0	0	0	0	1	0	0	0	3	0	0	0	5	1	2	0
12	0	0	0	0	1	0	0	0	4	0	1	0	2	2	1	0
13	0	0	0	0	0	0	0	0	1	0	0	0	5	2	1	0
14	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
15	0	0	0	0	0	0	0	0	2	0	0	0	1	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

TABLE VB: INITIAL INTER-STAGE STOCK REQUIREMENT FIGURES FOR 100% UTILISATION OF A FED STAGE - OBTAINED BY SIMULATING THE OPERATIONS OF TWO STAGE LINES, OPERATION TIMES ERLANG, MEAN = 1.0, K=1,2,3, AND 4. 100 SIMULATIONS CARRIED OUT FOR EACH OF THE PLANNING PERIODS INDICATED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 32√2				64				64√2				128			
K =	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
0	8	4	6	7	8	11	8	13	4	4	4	5	7	6	3	7
1	9	12	12	14	5	9	9	14	7	10	10	11	4	1	5	5
2	5	11	11	22	7	9	13	13	4	9	14	14	7	7	8	11
3	7	7	20	14	5	7	9	16	7	11	6	8	1	3	5	13
4	12	16	11	12	8	6	10	12	9	6	7	9	9	6	1	9
5	7	6	11	11	6	12	9	9	9	4	8	9	7	6	5	9
6	5	7	15	6	3	5	11	4	3	4	12	6	3	9	10	6
7	7	12	3	4	4	8	8	8	3	11	6	7	3	6	6	4
8	7	3	7	4	8	4	2	4	9	7	4	7	3	6	11	6
9	5	7	0	3	5	4	9	2	5	5	3	4	2	4	12	7
10	3	3	1	1	8	5	2	1	3	6	2	5	3	8	5	7
11	3	3	3	1	3	4	3	1	5	4	4	5	4	5	7	3
12	4	2	0	1	3	5	3	0	2	3	5	0	4	7	4	2
13	2	1	0	0	2	1	2	1	4	2	3	3	1	4	5	3
14	4	2	0	0	4	3	1	0	2	4	2	1	5	7	3	0
15	5	1	0	0	6	0	1	2	2	4	1	1	4	7	2	1
16	1	0	0	0	2	2	0	0	1	1	2	0	5	1	0	2
17	2	1	0	0	2	0	0	0	3	0	1	3	1	0	3	1
18	0	0	0	0	2	2	0	0	1	2	2	1	4	1	1	1
19	0	1	0	0	2	0	0	0	1	1	0	1	5	0	1	1
20	2	1	0	0	1	1	0	0	4	0	2	0	3	1	0	1
21	0	0	0	0	1	0	0	0	4	0	1	0	2	0	0	0
22	0	0	0	0	0	0	0	0	2	0	0	0	3	1	1	1
23	0	0	0	0	0	0	0	0	1	1	0	0	3	0	1	0
24	1	0	0	0	1	1	0	0	1	0	1	0	0	2	1	0
25	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
26	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0
27	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0
28	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
30	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
40	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0

TABLE VC: INITIAL INTER-STAGE STOCK REQUIREMENT FIGURES FOR 100% UTILISATION OF A FED STAGE - OBTAINED BY SIMULATING THE OPERATIONS OF TWO STAGE LINES, OPERATION TIMES ERLANG, MEAN = 1.0, K=1,2,3, AND 4. 100 SIMULATIONS CARRIED OUT FOR EACH OF THE PLANNING PERIODS INDICATED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 128√2				256				256√2				512			
K =	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
0	2	3	3	5	5	6	0	3	3	5	4	4	3	3	1	3
1	5	3	6	5	3	4	3	8	3	5	9	7	4	1	3	7
2	5	8	9	6	3	8	9	5	6	6	3	4	3	5	6	6
3	2	5	5	8	3	2	6	9	3	4	3	3	2	2	7	5
4	6	4	11	11	2	5	7	6	2	6	3	3	5	3	4	7
5	3	4	7	8	2	4	4	3	3	4	2	8	2	4	8	1
6	3	6	2	10	3	2	6	7	2	3	7	7	5	1	4	4
7	4	3	2	4	5	2	6	12	6	5	4	2	3	2	4	3
8	3	4	7	5	3	4	0	5	3	6	5	3	3	4	7	5
9	9	2	6	5	0	6	2	3	4	2	7	6	7	3	1	4
10	7	9	4	3	6	4	9	3	1	2	6	3	1	3	4	7
11	4	3	4	6	1	5	6	2	0	5	7	4	3	0	2	6
12	1	3	8	4	3	3	5	5	3	3	4	6	3	2	3	4
13	4	4	3	0	5	3	3	4	2	2	1	6	2	4	2	1
14	3	3	5	2	3	1	4	4	2	4	2	1	1	2	3	2
15	4	6	0	6	3	3	2	3	0	2	3	2	3	5	2	2
16	1	1	3	2	3	5	5	4	3	6	1	1	2	4	2	4
17	1	3	1	2	0	2	2	0	0	3	7	3	0	3	3	1
18	2	1	1	1	2	2	2	3	2	1	1	7	7	2	2	6
19	2	1	4	2	1	3	2	3	2	1	4	2	1	2	0	3
20	4	2	0	2	3	4	1	2	1	2	2	3	2	2	5	2
21	2	0	0	0	0	1	1	0	2	4	2	1	2	3	2	1
22	0	5	4	0	4	1	3	0	5	2	0	4	0	1	1	2
23	3	2	2	0	2	4	2	1	2	1	2	1	0	1	2	3
24	1	3	0	3	4	0	1	1	0	2	1	3	2	3	2	0
25	2	2	2	0	1	3	1	1	3	2	1	1	2	5	2	1
26	2	2	0	0	0	1	1	0	2	1	2	2	0	0	0	2
27	3	2	0	0	1	0	0	1	2	0	1	2	0	3	1	0
28	3	2	0	0	2	1	2	0	3	2	2	1	2	1	3	2
29	1	1	0	0	1	3	1	1	2	0	0	0	0	0	0	0
30	0	1	0	0	1	0	1	1	2	1	1	0	1	5	3	0
31	0	0	0	0	1	0	1	0	0	1	0	0	2	2	0	1
32	0	0	0	0	0	3	1	0	0	1	0	0	1	4	1	2
33	0	0	0	0	2	1	0	0	1	0	0	0	1	0	4	0
34	1	0	1	0	2	0	0	0	2	1	0	0	0	0	1	1
35	0	0	0	0	2	0	0	0	0	0	0	0	1	1	0	1
36	1	0	0	0	1	0	0	0	3	0	0	0	2	0	0	0
37	1	0	0	0	0	1	0	0	2	0	1	0	1	0	1	0
38	1	0	0	0	2	2	0	0	0	3	0	0	1	0	1	0
39	1	0	0	0	2	0	1	0	2	0	1	0	1	0	0	0

CONTINUED ON NEXT PAGE.

TABLE VC: CONTINUED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 128√2				256				256√2				512			
K =	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
40	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0
41	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
42	0	0	0	0	1	0	0	0	1	0	0	0	0	4	0	0
43	0	0	0	0	1	0	0	0	1	0	0	0	3	2	2	0
44	0	1	0	0	2	0	0	0	0	0	0	0	1	0	0	0
45	0	0	0	0	1	0	0	0	3	1	0	0	1	3	0	1
46	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
47	1	0	0	0	1	0	0	0	5	1	0	0	0	1	0	0
48	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
49	0	0	0	0	3	0	0	0	0	0	0	0	2	1	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
51	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
53	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
55	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
61	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
63	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

TABLE VD: INITIAL INTER-STAGE STOCK REQUIREMENT FIGURES FOR 100% UTILISATION OF A FED STAGE - OBTAINED BY SIMULATING THE OPERATIONS OF TWO STAGE LINES, OPERATION TIMES ERLANG, MEAN = 1.0, K=1,2,3, AND 4. 100 SIMULATIONS CARRIED OUT FOR EACH OF THE PLANNING PERIODS INDICATED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 512/2				1024				1024/2			
K =	1	2	3	4	1	2	3	4	1	2	3	4
0	4	0	2	1	2	2	1	4	3	2	2	1
1	3	1	2	3	4	3	2	5	1	2	0	3
2	2	1	7	0	2	2	5	3	3	4	3	0
3	3	0	3	5	3	5	4	5	2	3	1	3
4	6	4	2	7	2	1	4	2	0	5	2	4
5	1	6	6	1	3	2	3	3	0	3	3	1
6	2	5	3	5	2	1	3	9	1	1	5	2
7	1	4	2	2	2	2	3	3	0	3	4	4
8	4	2	3	3	1	4	1	2	1	2	1	4
9	0	5	2	3	2	2	6	3	2	2	4	3
10	2	4	3	4	1	3	2	6	2	2	6	3
11	4	1	3	3	1	2	6	2	1	0	2	3
12	3	1	2	2	1	0	3	2	1	3	3	4
13	2	3	1	4	2	3	3	3	2	1	2	2
14	0	2	5	4	0	1	1	3	2	4	3	4
15	3	4	3	3	1	4	2	5	1	4	1	0
16	2	8	1	4	1	4	2	1	1	1	1	3
17	1	0	3	4	3	1	3	0	0	0	2	0
18	2	3	2	4	1	2	4	1	1	3	0	3
19	0	2	2	2	1	1	2	2	2	1	4	2
20	2	3	2	2	2	3	0	1	1	0	2	3
21	1	4	1	3	1	3	4	2	0	2	1	0
22	1	0	3	4	0	1	1	2	0	0	5	4
23	4	2	3	0	2	3	3	2	1	2	1	0
24	1	0	1	0	0	1	1	1	1	1	4	3
25	1	1	2	3	0	3	1	2	0	1	3	3
26	3	3	0	3	2	0	1	2	1	1	0	5
27	2	2	2	4	0	1	4	2	4	0	2	0
28	0	0	2	2	1	1	2	1	0	0	2	6
29	0	1	5	1	1	2	1	0	2	0	3	2
30	3	1	3	2	3	1	1	0	1	1	1	2
31	0	0	3	0	5	3	2	1	4	1	1	1
32	1	2	0	2	1	2	1	2	1	2	1	1
33	0	1	0	0	3	1	1	3	0	0	0	2
34	3	2	0	1	0	3	2	2	4	1	1	1
35	1	4	2	1	0	3	0	2	0	1	3	3
36	1	2	1	2	1	2	1	0	1	0	2	0
37	1	1	2	0	2	2	0	0	1	2	1	3
38	1	0	1	2	1	0	3	0	1	2	1	2
39	1	1	0	1	2	3	0	1	1	2	1	1
40	2	1	0	0	1	2	1	3	2	0	0	0
41	0	0	2	0	0	4	1	1	2	0	0	1
42	2	3	1	1	1	1	1	0	1	2	2	0
43	1	0	0	0	2	0	1	1	1	0	1	0
44	1	1	0	1	3	1	0	1	0	3	1	0

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TABLE VD: CONTINUED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 512√2				1024				1024√2			
K =	1	2	3	4	1	2	3	4	1	2	3	4
45	0	0	1	0	0	0	0	0	0	0	2	1
46	1	0	2	0	1	1	0	1	1	1	0	0
47	1	1	0	0	0	0	0	0	1	1	0	1
48	1	0	0	0	0	0	0	0	3	2	1	0
49	0	1	1	0	0	1	1	0	2	1	0	0
50	0	1	0	0	1	1	0	1	2	1	1	1
51	0	1	0	0	0	0	1	1	1	2	1	0
52	1	0	0	0	0	2	1	0	1	0	0	1
53	0	3	0	0	1	0	1	0	1	0	0	0
54	4	0	0	0	1	0	1	0	1	2	1	0
55	1	0	1	0	0	1	0	0	0	0	1	0
56	3	1	1	0	3	0	0	0	0	1	1	0
57	1	0	0	0	1	1	0	0	0	1	0	0
58	2	0	0	0	0	0	0	0	3	3	0	0
59	0	0	0	0	1	0	0	1	1	0	0	0
60	1	0	1	1	1	0	0	0	0	2	0	1
61	0	0	0	0	0	3	0	0	1	0	1	0
62	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	1	0	0	0	3	2	0	0
65	0	0	0	0	1	0	0	0	2	1	0	0
66	2	0	0	0	0	0	1	0	1	1	0	0
67	1	0	0	0	1	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	1	0	1	0
69	0	0	0	0	0	0	0	0	0	1	1	0
70	0	0	0	0	0	0	0	0	2	0	0	0
71	0	0	0	0	0	2	0	0	0	0	0	1
72	0	0	0	0	2	0	0	0	0	0	0	0
73	0	0	0	0	1	0	0	0	0	1	0	0
74	0	0	0	0	1	0	0	0	2	1	0	1
75	0	0	0	0	0	0	1	0	1	2	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	1	0	0	0	0	0	1	0
78	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	1	1	0	0	1	0	0	0
80	0	0	0	0	0	0	0	0	1	0	0	0
81	1	0	0	0	1	0	0	0	0	0	0	0
82	0	0	0	0	1	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0	0	0	0	0	0
84	1	0	0	0	1	0	0	0	0	0	0	0
85	1	1	0	0	1	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	3	1	0	0
88	0	0	0	0	0	0	0	0	1	0	0	0
89	0	0	0	0	0	0	0	0	1	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	0	1	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0
93	0	0	0	0	1	0	0	0	0	0	0	0
94	0	0	0	0	0	0	0	0	0	0	0	0

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TABLE VD: CONTINUED.

INITIAL INTER STAGE STOCK REQUIREMENT	PLANNING PERIOD 512√2				1024				1024√2			
K =	1	2	3	4	1	2	3	4	1	2	3	4
95	0	0	0	0	0	0	0	0	0	0	0	1
96	0	0	0	0	0	0	0	0	0	1	0	0
97	0	0	0	0	1	0	0	0	0	0	0	0
98	0	0	0	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	1	0	0	0	0	0	0	0
101	0	0	0	0	0	0	0	0	2	0	0	0
102	0	0	0	0	0	0	0	0	0	0	0	0
103	0	0	0	0	0	0	0	0	0	0	0	0
104	0	0	0	0	0	0	0	0	0	0	0	0
105	0	0	0	0	0	0	0	0	0	0	0	0
106	0	0	0	0	2	0	0	0	1	0	0	0
107	0	0	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0	0	0	0	0
112	0	0	0	0	0	0	0	0	0	0	0	0
113	0	0	0	0	0	0	0	0	0	0	0	0
114	0	0	0	0	0	0	0	0	0	0	0	0
115	0	0	0	0	0	0	0	0	0	0	0	0
116	0	0	0	0	0	0	0	0	0	0	0	0
117	0	0	0	0	1	0	0	0	0	1	0	0
118	0	0	0	0	0	0	0	0	0	0	0	0
119	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0
121	0	0	0	0	0	0	0	0	0	0	0	0
122	0	0	0	0	0	0	0	0	1	0	0	0
123	0	0	0	0	0	0	0	0	0	0	0	0
124	0	0	0	0	0	0	0	0	0	0	0	0
125	0	0	0	0	0	0	0	0	0	0	0	0
126	0	0	0	0	0	0	0	0	0	0	0	0
127	0	0	0	0	2	0	0	0	0	0	0	0
128	0	0	0	0	0	0	0	0	0	0	0	0
129 TO 149 - ENTRIES ALL ZERO.												
150	0	0	0	0	0	0	0	0	0	0	0	0
151	0	0	0	0	0	0	0	0	1	0	0	0
152	0	0	0	0	0	0	0	0	0	0	0	0
153	0	0	0	0	0	0	0	0	0	0	0	0
154	0	0	0	0	0	0	0	0	0	0	0	0
155	0	0	0	0	0	0	0	0	0	0	0	0
156	0	0	0	0	0	0	0	0	0	0	0	0
157	0	0	0	0	0	0	0	0	0	0	0	0
158	0	0	0	0	0	0	0	0	1	0	0	0
159	0	0	0	0	0	0	0	0	0	0	0	0
160	0	0	0	0	0	0	0	0	0	0	0	0
161	0	0	0	0	0	0	0	0	0	0	0	0
162	0	0	0	0	0	0	0	0	1	0	0	0

that the feeding stage finished at least one item in each time period that it took the fed stage to complete one item. Similarly there were 23 occasions when an initial inter-stage stock of 1 was required, 13 occasions when the requirement was 2, 9 occasions when the requirement was 3, etc. This compares with corresponding figures for K = 1, PP = 16 of 7 occasions when the requirement was 0, 14 occasions when the requirement was 1, 14 occasions when the requirement was 2, 16 occasions when the requirement was 3, etc., the column total equalling 100 (simulations).

The results of Tables VA - VD could be used directly to determine the IISSR for the specific combinations of K and PP simulated. Thus if a two stage line's planning period = 32 time units and the fed and feeding stage operation times are Erlang, mean = 1 time unit, K = 3 then the IISSR could be determined on the basis of the figures of Table VI which is derived from Table VA.

TABLE VI: IISSR for a stock point in a line scheduled on the basis of a planning period of 32 time units, fed and feeding stage operation times Erlang, mean = 1 time unit, K = 3. Figures based on the simulation results of Table VA

Chance of fed stage being 100% utilised. (%)	IISSR
100	13
99	12
98	11
96	10
95	9
92	8
85	7

With regard to the 100% IISSR figure in Table VI theorectically IISSR must be infinite. However the simulation results only indicate that there is a less than approximately 1 in 100 chance of more than 13 items being required.

Using the simulations results directly as indicated above presents certain practical problems such as the difficulty of interpolation over IISSR figures that do not occur in the simulations e.g. for K = 2, PP = 16½ interpreting the difference between an IISSR of 13 compared with an IISSR of 10.

Such problems could be alleviated if a standard distribution could be found to describe the data. Ideally this should be identified by analysis but as has been stated previously (Chapter 2, page 18) this has not proved possible to date. Therefore an empirical approach was used.

The underlying distribution must be discrete and have an infinite range. The Poisson distribution - being one of the simplest and most commonly encountered discrete distributions - was tried first but as the examples in Table VII show it did not provide a uniformally good fit to the data.

TABLE VII: Example chi^2 values for Poisson distributions fitted to the data of Table VA

Planning Period	K = 1		K = 2		K = 3		K = 4	
	Chi^2	df	Chi^2	df	Chi^2	df	Chi^2	df
8½	59.0	5	3.0	4	0.8	3	3.4	3
16	33.0	6	50.6	5	7.3	4	14.8	4
16½	153.7	7	36.7	5	43.4	5	12.2	4

Table VIII gives the chi^2 values obtained by fitting negative binomial distributions to the data of Table V. Only the three values indicated are significant at a 5% level. In the absence of any greater insight into the problem the negative binomial distribution appears to provide a reasonable description over the range of planning periods and K's investigated.

TABLE VIII: Chi^2 values for negative binomial distributions fitted to the data of Table V. The marked values are significant at a 5% level.

Planning Period	K = 1 Chi^2	df	K = 2 Chi^2	df	K = 3 Chi^2	df	K = 4 Chi^2	df
8 √2	6.3	6	3.0	3	0.8	2	0.9	2
16	4.0	6	10.9	4	3.2	4	5.2	4
16 √2	4.9	8	6.5	6	6.5	5	2.7	4
32	3.5	8	5.4	7	5.8	7	4.5	5
32 √2	9.2	10	11.1	8	9.6	5	2.3	5
64	9.4	11	8.9	10	7.3	8	4.0	6
64 √2	16.8	12	11.5	10	10.7	11	3.5	9
128	21.0	12	13.6	12	19.1	11	6.4	10
128 √2	8.1	11	16.7	13	13.1	11	6.2	10
256	14.7	13	12.2	12	14.9	13	13.1	11
256 √2	19.5	14	7.9	13	20.8	13	24.2	13
512	9.1	13	11.3	13	11.6	11	14.2	13
512 √2	18.1	14	18.5	14	13.6	13	7.7	12
1024	23.1	14	11.0	14	6.0	13	12.8	12
1024 √2	15.1	14	18.7	14	6.0	14	10.3	14

The negative binomial distribution may be defined in terms of its mean m'_s and shape parameter p' (See Appendix C). Table IX gives the sample means and maximum likelihood estimates of p' for the data of Table V. (Table VIII gives the chi^2 values obtained when negative binomial distributions with the parameter values of Table IX are fitted to the frequency distributions of Table V).

TABLE IX: Sample means and negative binomial maximum likelihood estimates of p' for the initial inter-stage stock requirement data of Table V.

Planning period	K = 1 Sample mean	K = 1 p	K = 2 Sample mean	K = 2 p	K = 3 Sample mean	K = 3 p	K = 4 Sample mean	K = 4 p
8 √2	3.29	.4148	2.46	1.0000	1.90	1.0000	1.74	.8558
16	4.02	.5052	2.82	.5344	2.37	.7949	2.35	.6995
16 √2	4.73	.2579	3.58	.4745	2.92	.4168	2.46	.6678
32	5.19	.2297	4.05	.4184	3.91	.4124	3.02	.5996
32 √2	7.10	.2010	5.61	.3232	4.03	.6197	3.59	.5513
64	9.04	.1480	6.35	.1924	5.10	.3506	3.75	.3724
64 √2	9.90	.1443	7.10	.2300	6.73	.2267	5.82	.2774
128	11.55	.1153	9.13	.2271	8.32	.2926	6.31	.2577
128 √2	14.70	.0996	12.48	.1276	9.20	.1767	7.94	.2159
256	19.98	.0635	13.27	.0974	11.57	.1620	9.15	.1776
256 √2	21.42	.0586	13.03	.0980	11.90	.1219	11.32	.1458
512	21.08	.0566	19.82	.0825	14.32	.1053	12.05	.1196
512 √2	26.82	.0424	21.44	.0900	19.00	.0761	17.02	.1148
1024	37.83	.0290	24.54	.0604	19.38	.0776	16.86	.0721
1024 √2	44.63	.0303	31.11	.0359	22.91	.0733	22.05	.0795

For K = 2, PP = 16 /2 the sample mean = 3.58 and the maximum likelihood estimate of p' = 0.4745. Table X gives the individual and cumulative probabilities for such a negative binomial distribution.

TABLE X: Negative binomial distribution fitted to K = 2, PP = 16 /2 data of Table V. m'_s = 3.58, p' = 0.4745

Initial inter-stage stock (x)	Probability exactly x used	Probability x is sufficient to keep fed stage 100% utilised	Theoretical frequency for a sample of 100	Observed frequency in a sample of 100 (simulations)
0	0.0898	0.0898	8.98	10
1	0.1526	0.2424	15.26	17
2	0.1697	0.4121	16.97	16
3	0.1555	0.5677	15.55	14
4	0.1274	0.6950	12.74	6
5	0.0968	0.7918	9.68	12
6	0.0698	0.8616	6.98	10
7	0.0484	0.9100	4.84	7
8	0.0325	0.9425	3.25	3
9	0.0213	0.9639	2.13	4
10	0.0137	0.9776	1.37	0
11	0.0087	0.9862	0.87	0
12	0.0054	0.9916	0.54	0
13	0.0033	0.9950	0.33	0
14	0.0020	0.9970	0.20	1

On the basis of these figures it can be said that if initial inter-stage stock equals 10 there is a 97.76% chance of 100% utilisation of the fed stage whereas if initial inter-stage stock equals 13 there is a 99.5% chance of 100% utilisation. Thus the fitting of a standard distribution enables sensible interpolation within simulation results to be carried out.

An extension of the negative binomial results.

The procedure used to develop the negative binomial stock requirement model involved simulating two stage lines with planning periods set at specific convenient values. It will be shown later that irrespective of the complexity of a line or the particular operation time distribution involved, if assembly line scheduling is carried out on the basis of the concepts advanced in this thesis then such simulations as need to be carried out have only to be for appropriate 2 stage lines. On the other hand the results obtained for a given planning period cannot be used directly to determine the IISSR distribution for another planning period.

If (regression) relationships could be established for the m_s and p estimates of Table IX i.e. eight relationships altogether - four for sample means (one for each value of K) and four for p estimates, then it would be possible, by interpolation, to specify the negative binomial IISSR parameters for planning periods other than those simulated.

Figures 3A - 3D and 4A - 4D, pages 53 - 60, show that the relationships between the planning period and m_s and planning period and p are non-linear. The data of Table IX was used to identify transformations that would yield linear relationships - linear relationships making subsequent interpolation or extrapolation simpler.

TABLE IX, PAGE 50, DATA.

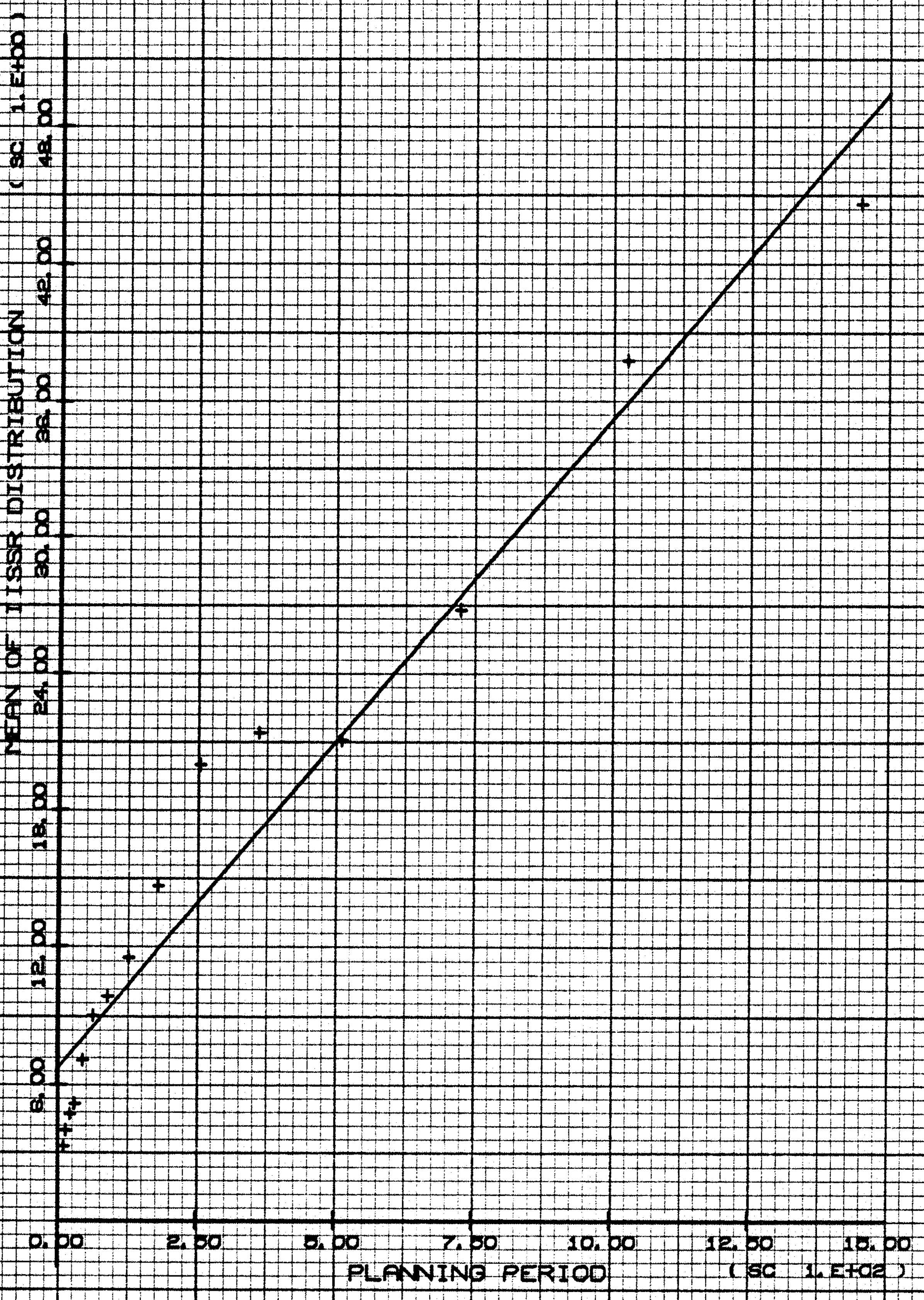

FIGURE 3A: K=1 SAMPLE MEANS AGAINST LEAST SQUARES LINE.

TABLE IX, PAGE 50, DATA.

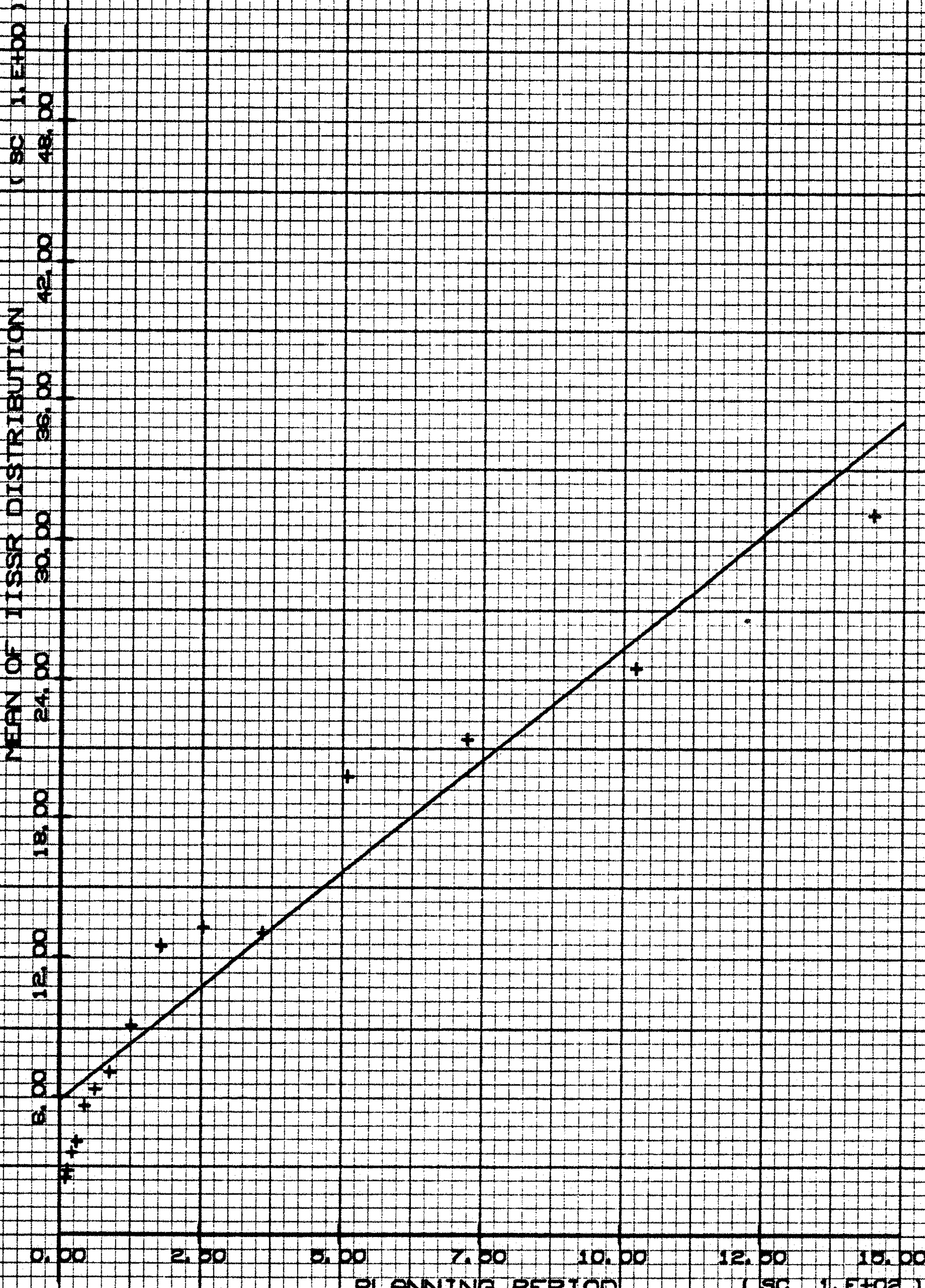

FIGURE 3B: K=2 SAMPLE MEANS AGAINST LEAST SQUARES LINE.

TABLE IX, PAGE 50, DATA.

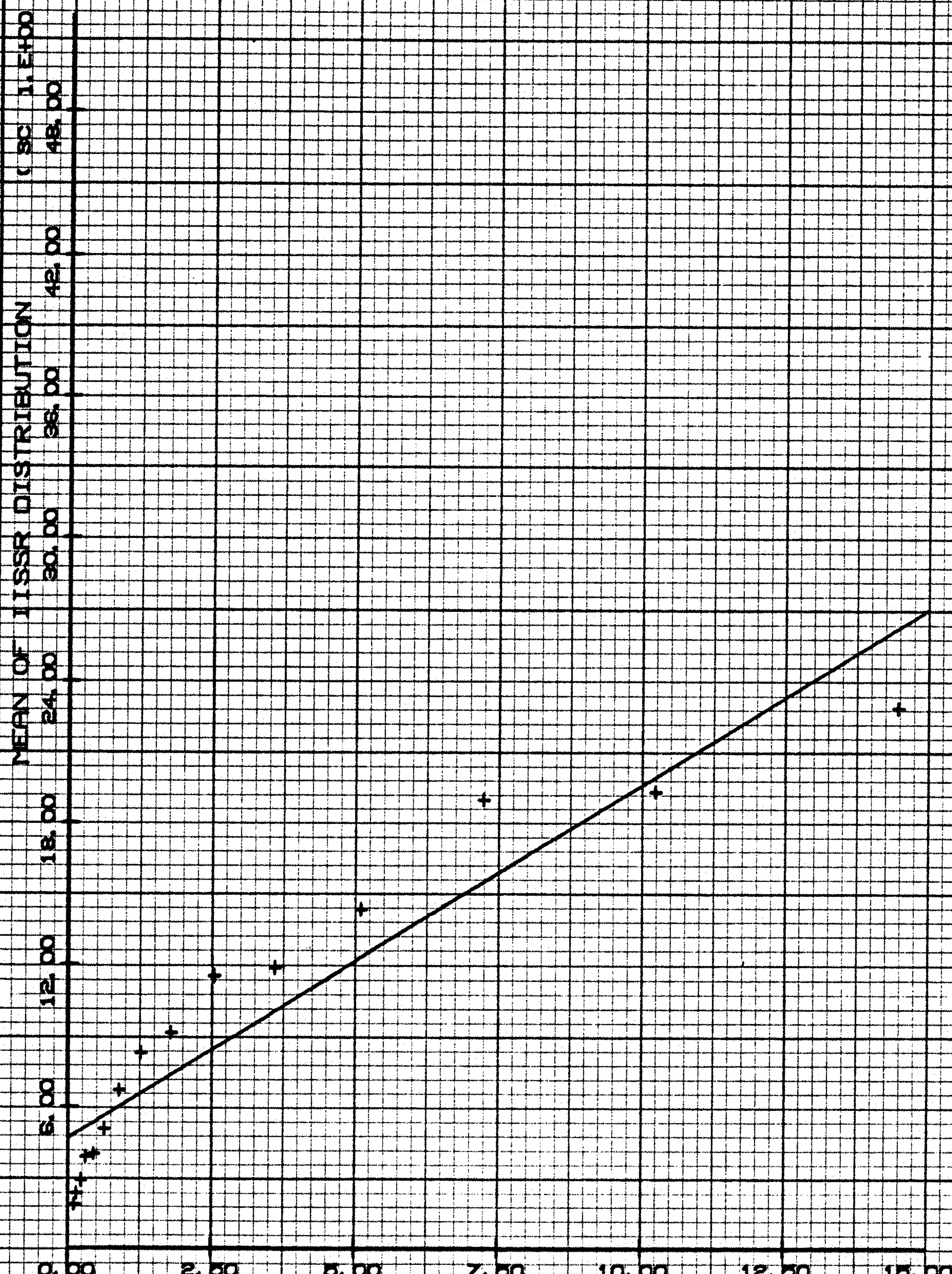

FIGURE 3C. K=3 SAMPLE MEANS AGAINST LEAST SQUARES LINE.

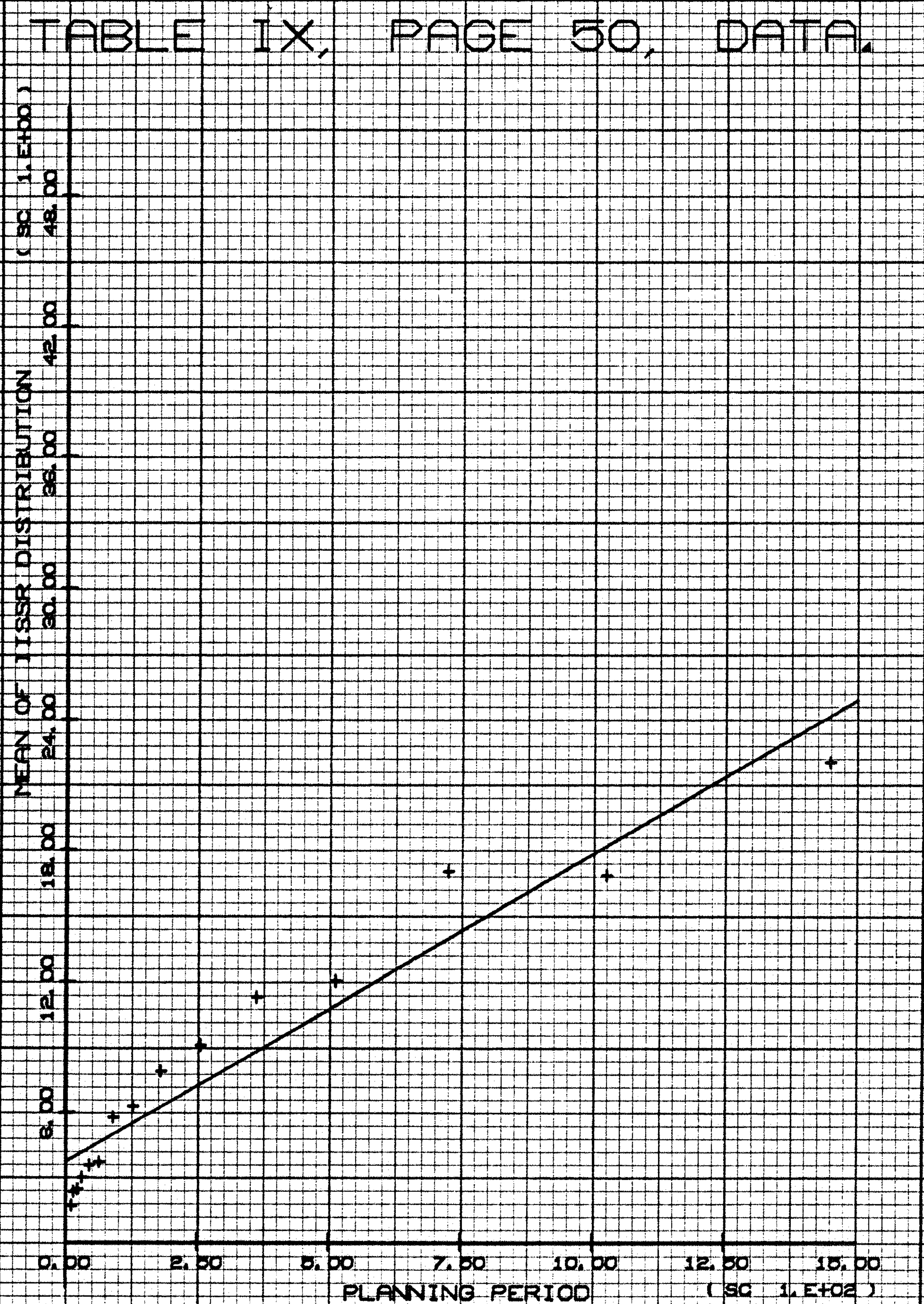

FIGURE 30: K=4 SAMPLE MEANS AGAINST LEAST SQUARES LINE.

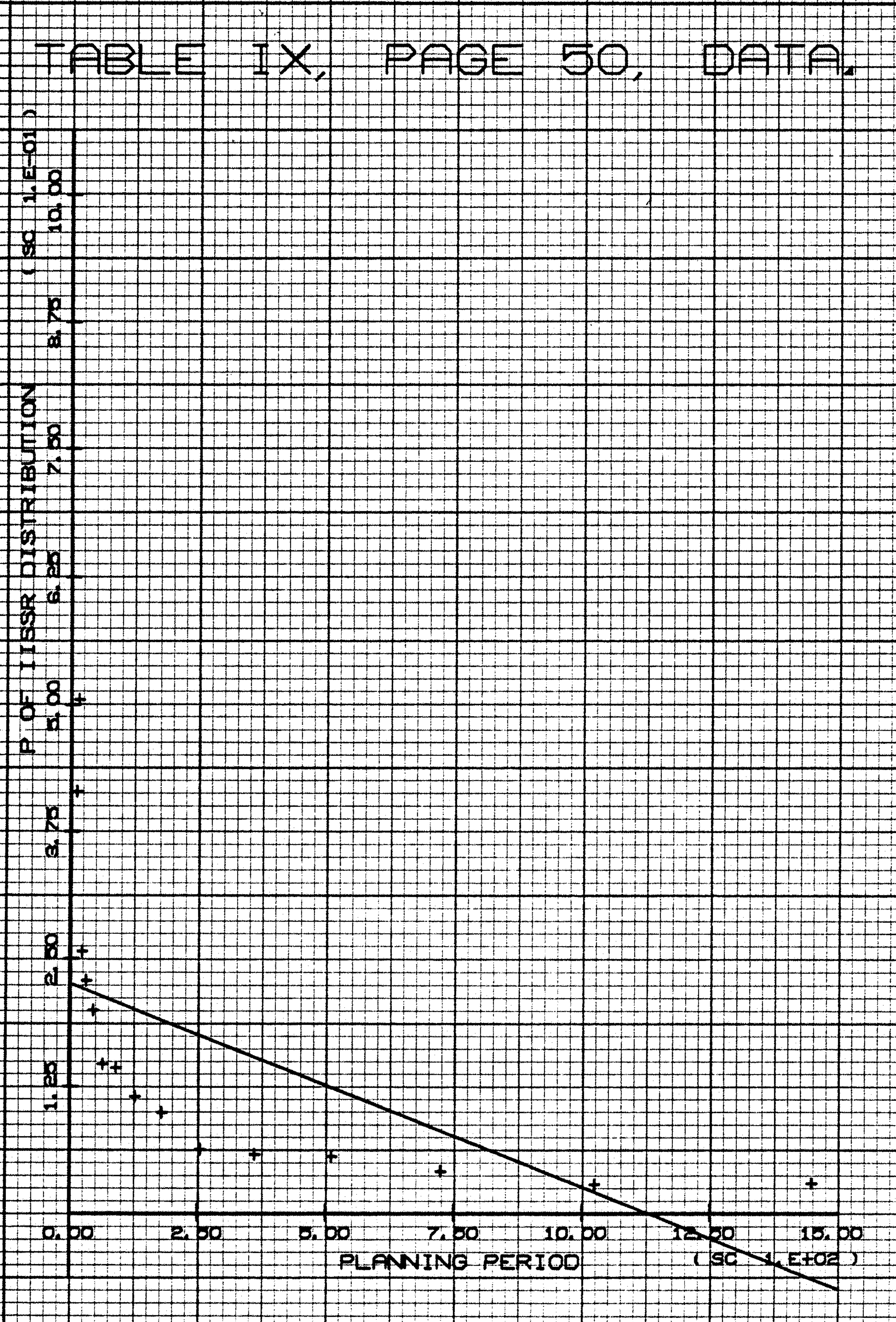

FIGURE 4A: K=1 P'S AGAINST LEAST SQUARES LINE.

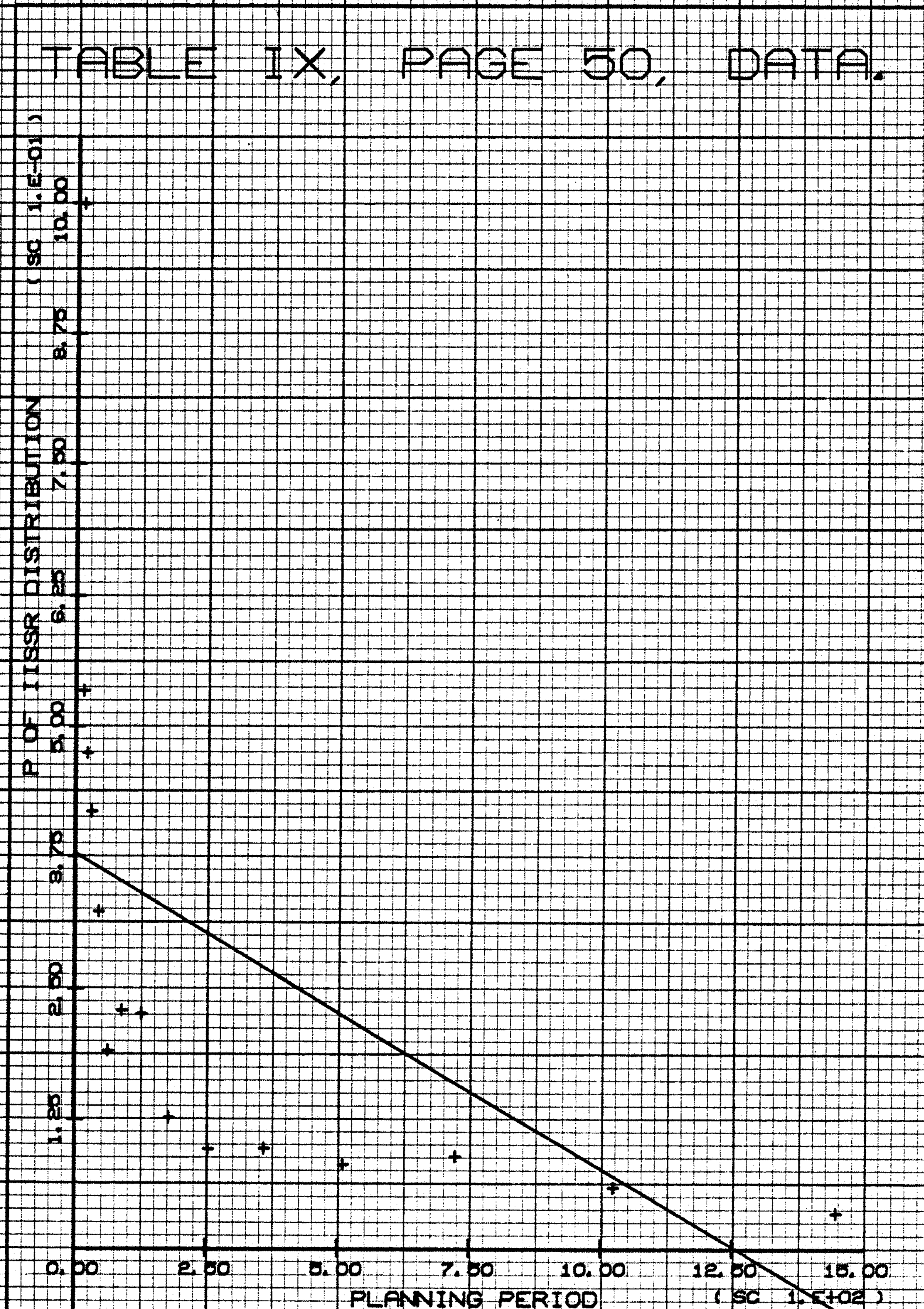

FIGURE 4B: K=2 P'S AGAINST LEAST SQUARES LINE.

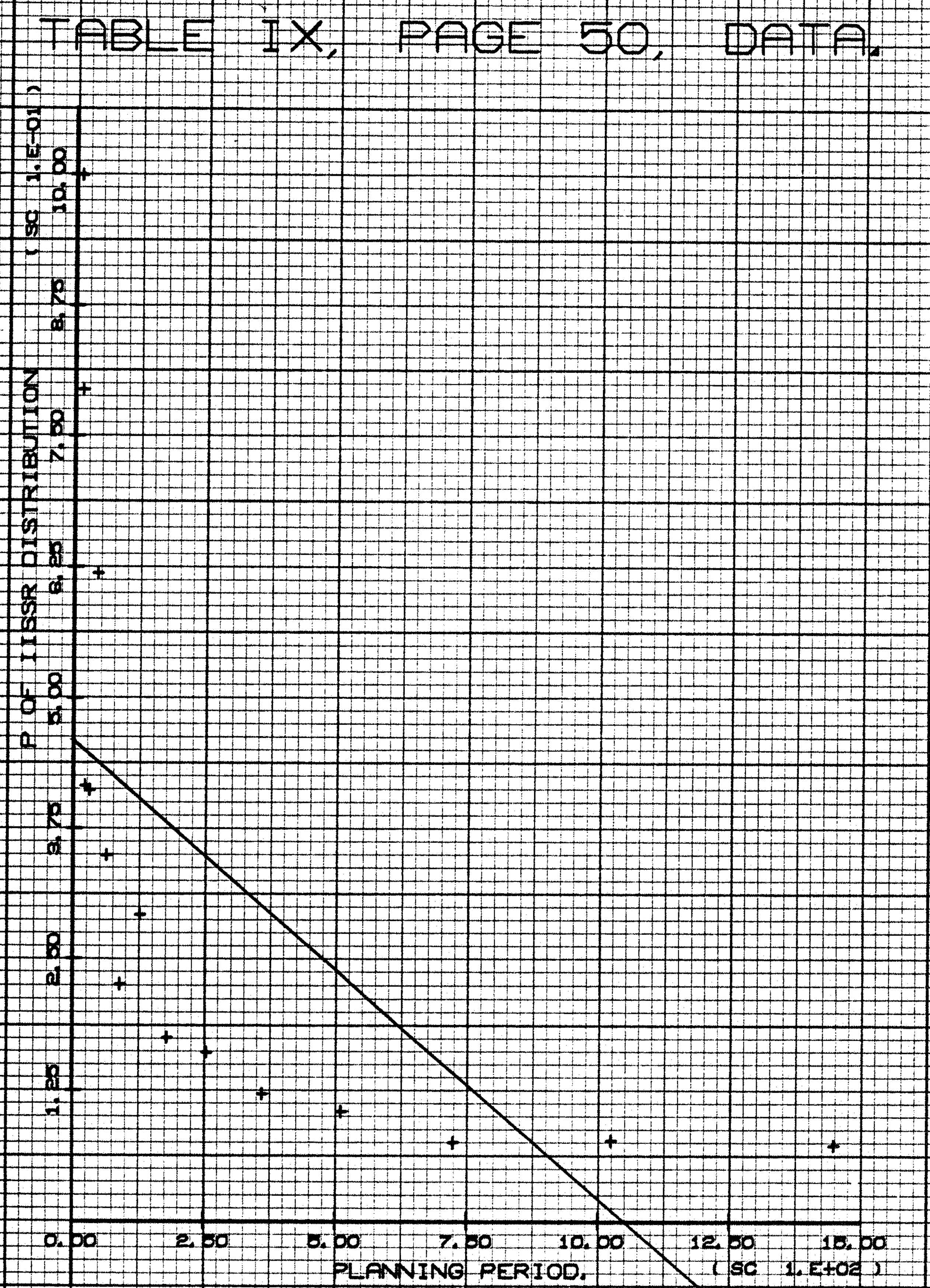

FIGURE 4C. K=3 P'S AGAINST LEAST SQUARES LINE.

FIGURE 4D: K=4 P'S AGAINST LEAST SQUARES LINE.

The following transformations were tried:

$$m_s \text{ v } \log_{10} PP$$

$$\log_{10} m_s \text{ v } \log_{10} PP$$

$$\log_{10} m_s \text{ v } PP$$

$$m_s \text{ v } \sqrt{PP}$$

$$p \text{ v } \log_{10} PP$$

$$\log_{10} p \text{ v } \log_{10} PP$$

$$\log_{10} p \text{ v } PP$$

$$p \text{ v } \sqrt{PP}$$

where m_s = sample mean used as an estimate of the negative binomial distribution mean.

p = maximum likelihood estimate of negative binomial distribution parameter p'

It was found that using:

$$m_s \text{ v } \sqrt{PP}$$

and $\log_{10} p$ v $\log_{10} PP$

relationships that could reasonably be assumed to be linear over the range of planning periods investigated were obtained. (See Figures 5A - 5D and 6A - 6D, pages 62 - 69).

The data of Table IX was used to identify the above relationships. Further, independent, simulations were carried out with the specific aim of determining the regression coefficients for these relationships. The most efficient experimental method for determining a slope parameter requires all experimentation (simulation here) to be carried out at two extreme values of the independent variable since the variance of least squares slope

TABLE IX, PAGE 50, DATA.

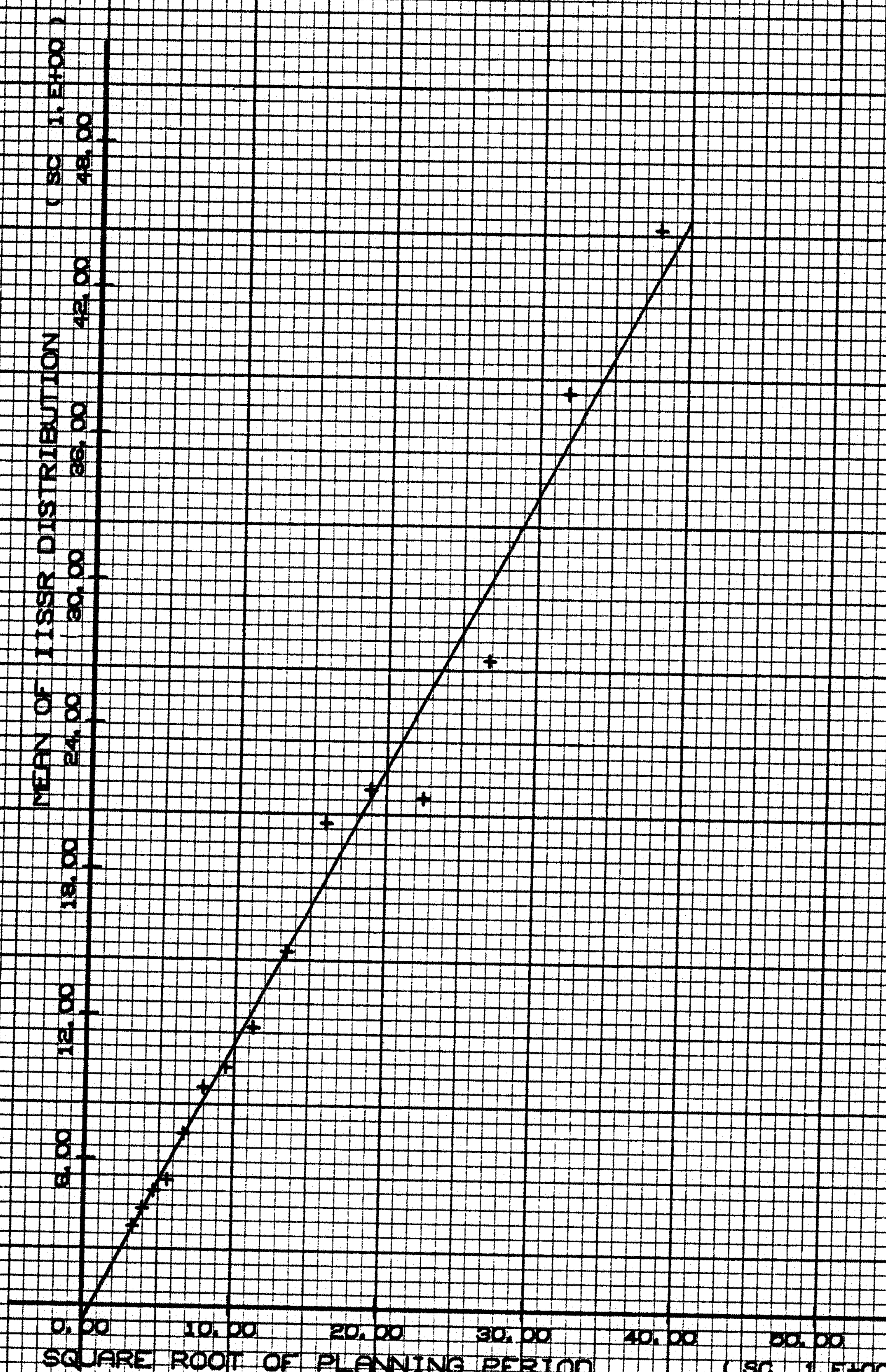

FIGURE 5A: K=1 SAMPLE MEANS V √PP V LEAST SQUARES LINE.

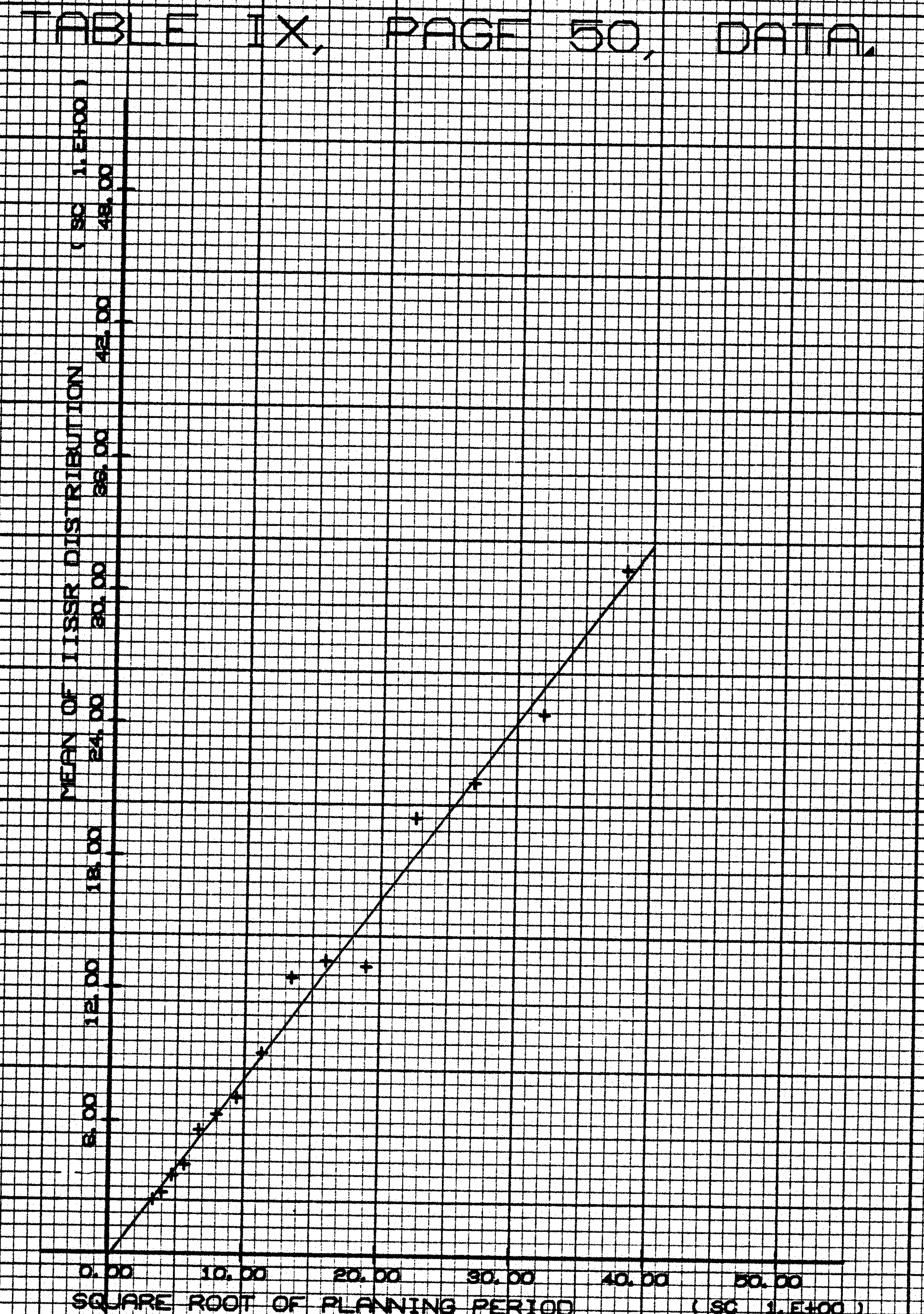

FIGURE 5B. K-2 SAMPLE MEANS V √PP V LEAST SQUARES LINE.

TABLE IX, PAGE 50, DATA.

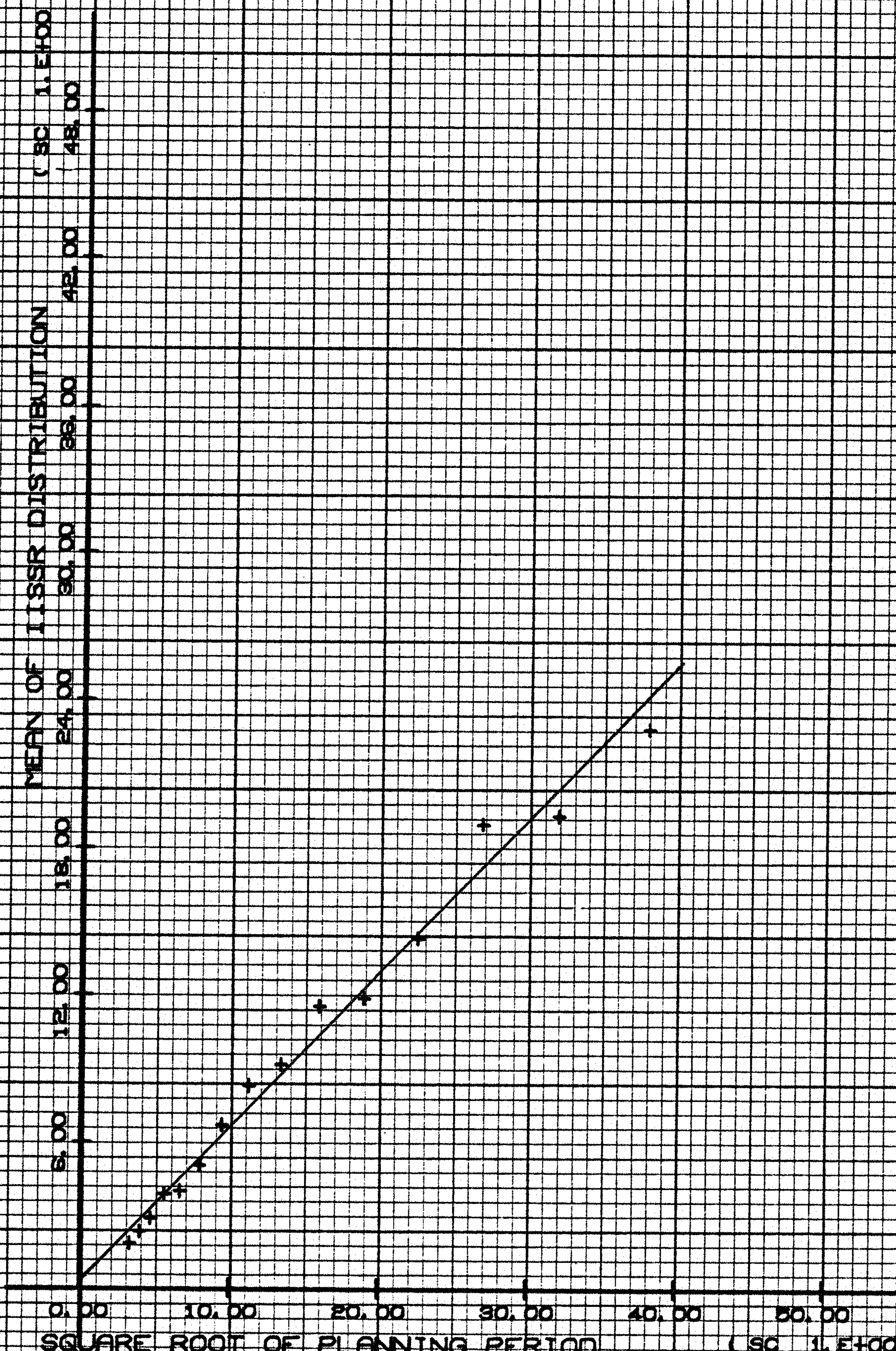

FIGURE 5C: K=3 SAMPLE MEANS V √PP V LEAST SQUARES LINE.

TABLE IX, PAGE 50, DATA.

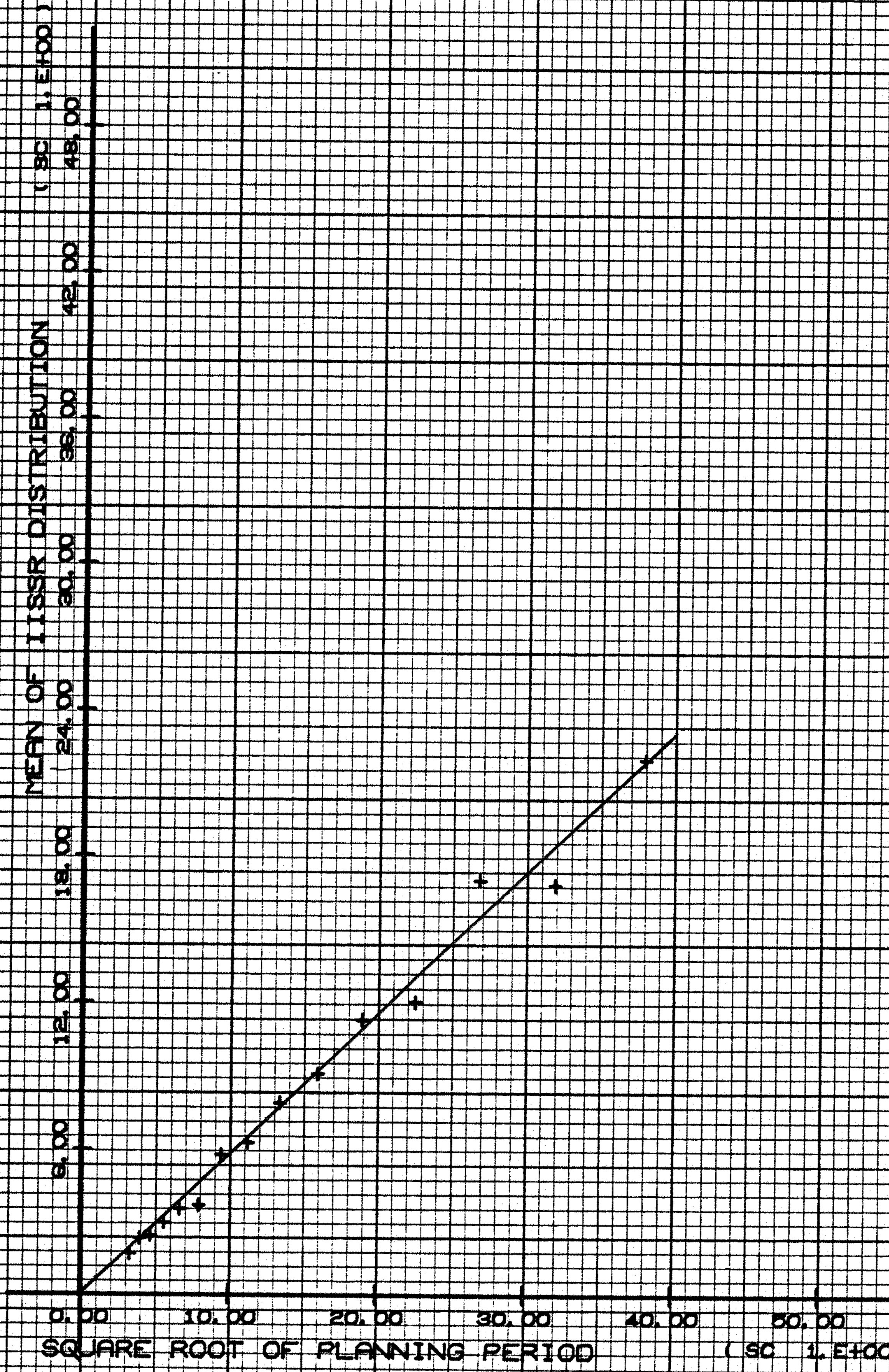

FIGURE 5D: K=4 SAMPLE MEANS V √PP V LEAST SQUARES LINE.

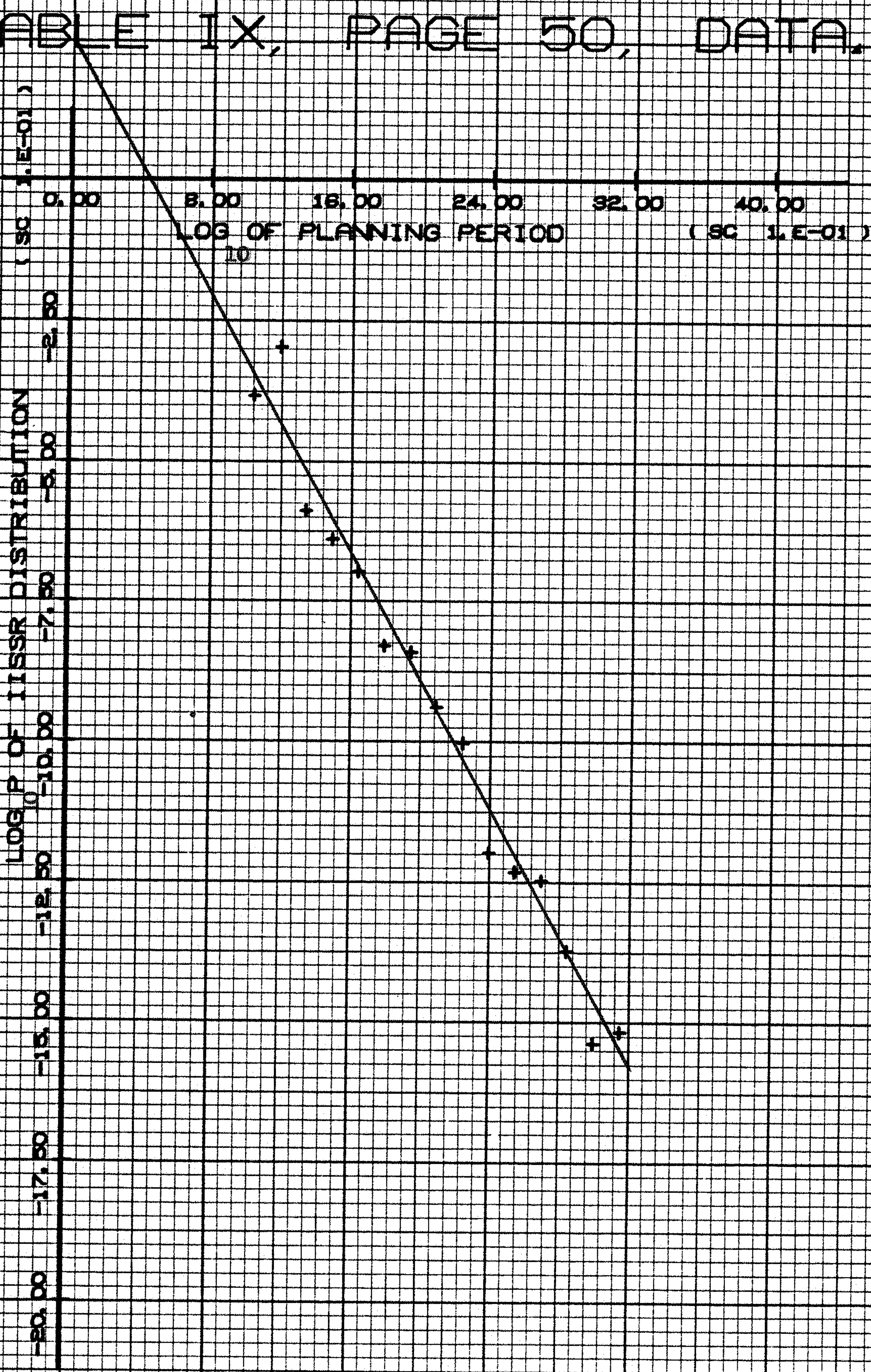

FIGURE 6A: LOG OF K=1 SAMPLE P'S V LOG PP
10

TABLE IX, PAGE 50, DATA.

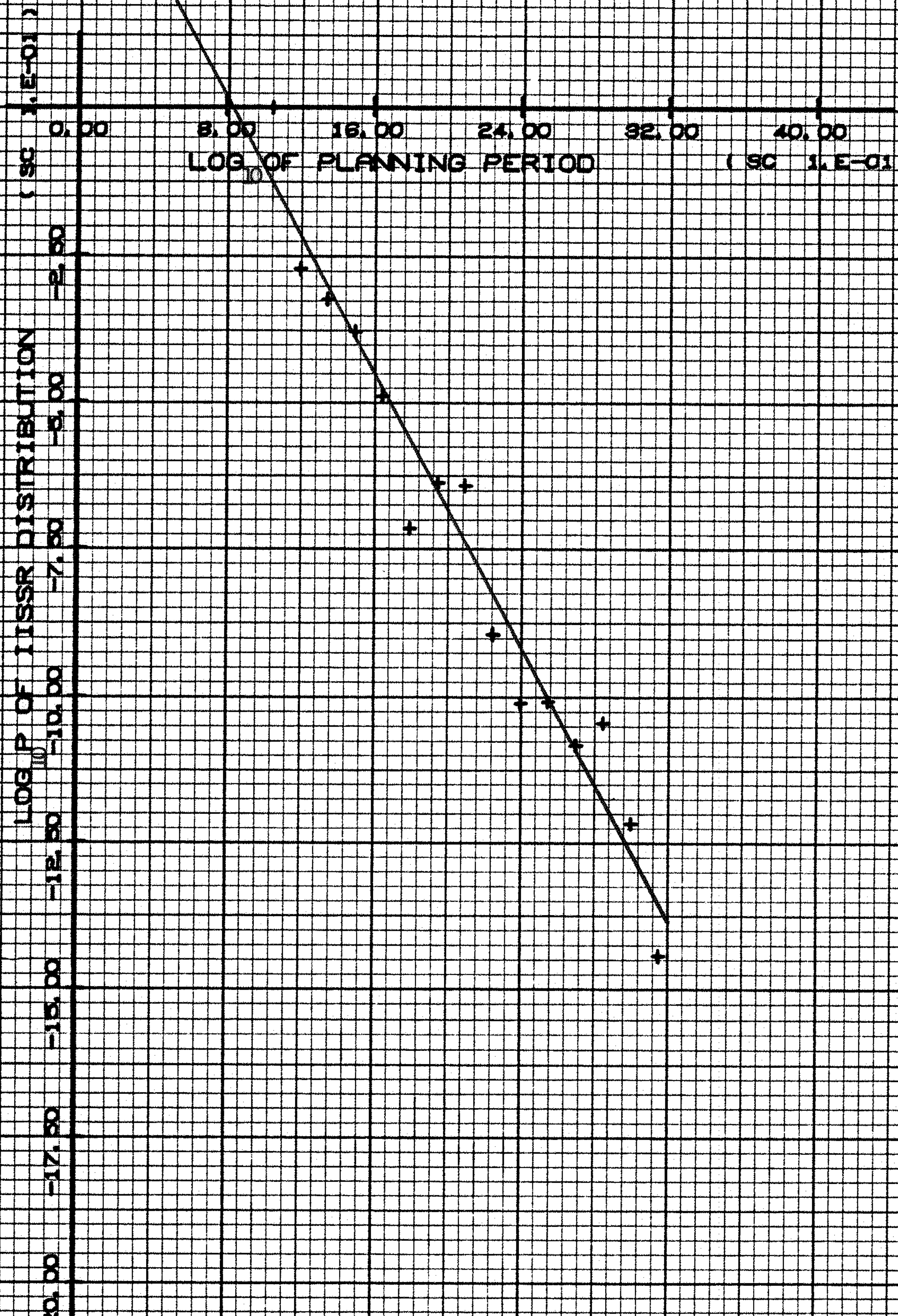

FIGURE 6B: LOG_{10} OF K=2 SAMPLE P'S V LOG PP

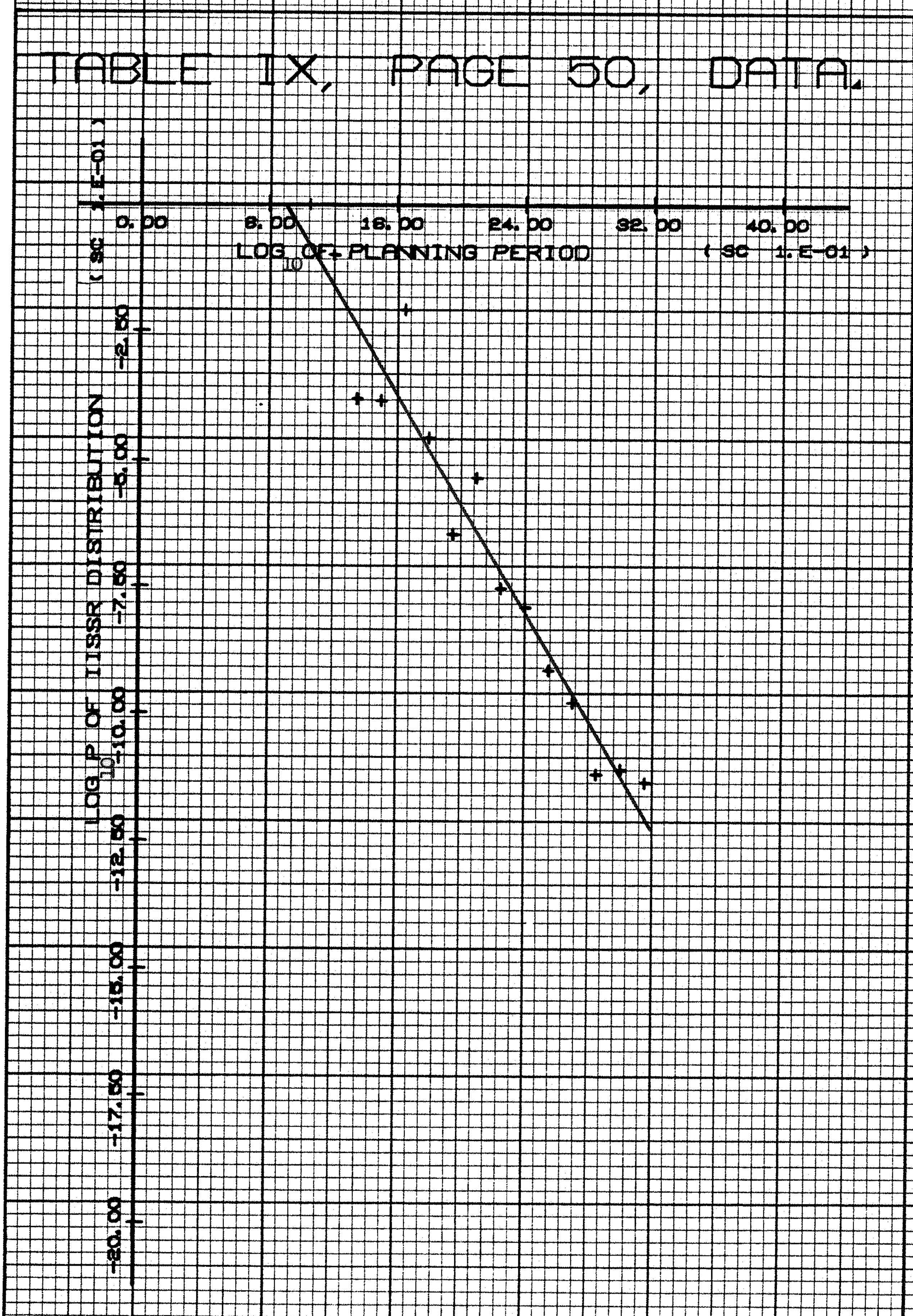

FIGURE 6C. LOG_{10} OF K=3 SAMPLE P'S V LOG PP

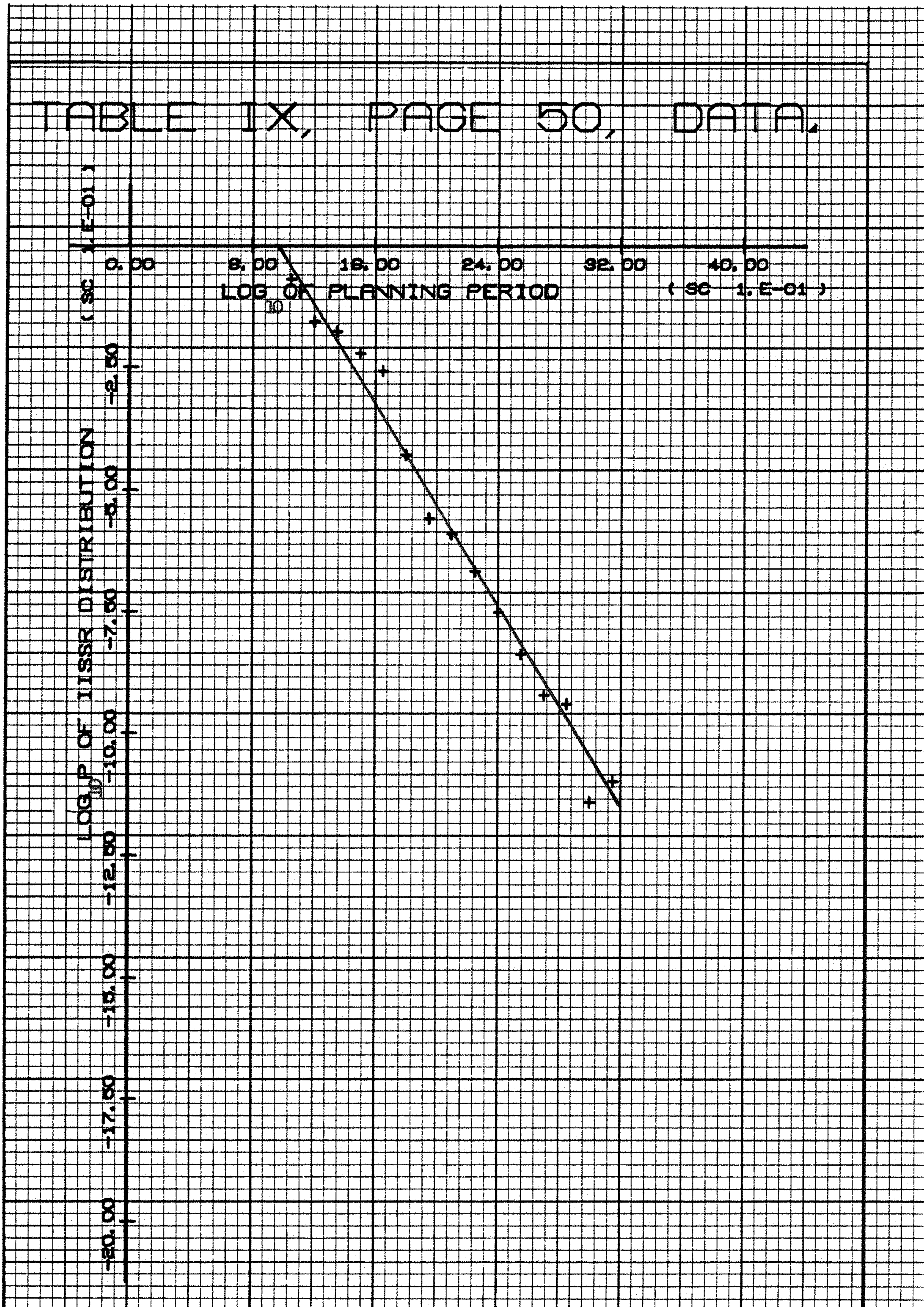

FIGURE 60. LOG10 OF K-4 SAMPLE P'S V LOG PP

parameter estimates is:

$$\frac{s^2_{y/x}}{(x_i - \bar{x})^2}$$

Where $s^2_{y/x}$ = the variance of the scatter, in the vertical (y) direction, of the observed points about the regression line.

x_i = the i^{th} independent observation value.

$\bar{x}$ = the mean of the independent observation values.

Concentrating the observations at the two extreme points maximises the denominator of the above expression thus minimising slope parameter estimation variance. Such additional simulations were carried out at 'extreme' planning periods of 20 and 1370 time units.

The programme of Appendix B was used to generate 15, independent, IISSR samples for each of the eight K:PP combinations i.e. 1:20, 2:20, 3:20, 4:20, 1:1370, 2:1370, 3:1370, and 4:1370. For PP = 20 combinations a sample size of 100 was used for each of the 15 samples and for PP = 1370 combinations a sample size of 10 was used - these sample sizes requiring computer runs of an acceptable duration.

TABLE XIA: Sample means of 15 samples generated for each (planning period = 20 time units: K = 1, 2, 3 and 4) combination. Sample size = 100.

Sample means, planning period = 20 time units, K = 1	2	3	4
4.72	3.27	2.82	2.29
4.51	3.30	2.66	2.46
4.20	3.28	3.02	2.34
4.59	3.76	2.73	2.23
4.10	3.37	2.68	2.28
4.52	2.83	2.90	2.84
4.36	3.74	2.77	2.30
4.23	3.20	2.53	2.33
5.02	3.23	2.97	2.63
4.47	4.27	2.79	2.31
4.61	3.29	2.70	2.35
4.36	3.70	3.03	2.40
4.48	2.90	2.70	2.38
4.55	3.16	2.61	2.55
4.38	3.37	2.80	2.51

TABLE XIB: Sample means of 15 samples generated for each (planning period = 1370 time units: K = 1, 2, 3 and 4) combination. Sample size = 10.

Sample means, planning period = 1370 time units, K = 1	2	3	4
29.7	24.3	22.3	19.6
31.3	45.9	18.6	22.1
23.6	22.0	18.4	19.2
37.8	22.6	24.1	19.7
48.2	17.3	22.7	27.7
31.7	18.7	9.2	19.2
29.0	37.1	21.4	13.3
47.4	31.1	26.3	16.0
41.5	49.2	20.4	17.0
47.6	29.8	24.1	13.5
57.5	22.3	27.6	23.6
40.2	33.8	24.6	27.2
53.3	23.7	26.4	20.2
38.4	16.7	26.9	21.4
48.7	22.0	27.5	23.1

TABLE XIC: Maximum likelihood estimate of p' of 15 samples generated for each (planning period = 20 time units: K = 1, 2, 3 and 4) combination. Sample size = 100.

Maximum likelihood estimate of p', planning period = 20 time units K =			
1	2	3	4
.2996	.3695	.6207	.6705
.2799	.6065	.5689	.5683
.2528	.5013	.6930	.5776
.2936	.4950	.5233	.5872
.3275	.5077	.5485	.6599
.2327	.5672	.6311	.7741
.3741	.4730	.6392	.7054
.2814	.4465	.6733	.6359
.2410	.5086	.4851	.5664
.2401	.4512	.5334	.6264
.2857	.4878	.5442	.5759
.2521	.3647	.4676	.6388
.2904	.5290	.6060	.6731
.3158	.4120	.5776	.6614
.3379	.4316	.4891	.8528

TABLE XID: Maximum likelihood estimate of p' of 15 samples generated for each (planning period = 1370 time units: K = 1, 2, 3 and 4) combination. Sample size = 10.

Maximum likelihood estimate of p', planning period = 1370 time units K =			
1	2	3	4
.0801	.0438	.0521	.0682
.0339	.1218	.1154	.1246
.0244	.0629	.1007	.0999
.0330	.0407	.0887	.0790
.0415	.0739	.0842	.0614
.0703	.0821	.0811	.0444
.0552	.0432	.1021	.0587
.0278	.1263	.1105	.1561
.0310	.0634	.0528	.1215
.0307	.0350	.2208	.0964
.0436	.0607	.0796	.1093
.0464	.0415	.0191	.1001
.0211	.1342	.0276	.1212
.0430	.0781	.0887	.0685
.0236	.0546	.0639	.1057

Tables XIA - D show the sample mean and maximum likelihood estimate of p' values obtained. Thus for PP = 20, K = 1, Table XIA shows that the first of the 15 samples had a sample mean of 4.72 and Table XIC shows that it had a maximum likelihood estimate of p' of 0.2996; the second sample for PP = 20, K = 1, had a sample mean of 4.51 and a maximum likelihood estimate of p' of 0.2799. Table XIB shows that for PP = 1370, K = 2 the first of the 15 samples had a sample mean of 24.3 and Table XID shows that it had a maximum likelihood estimate of p' of 0.0438; the second sample for PP = 1370, K = 2 had a sample mean of 45.9 and a maximum likelihood estimate of p' of 0.1218.

The regression parameters obtained from the results of Tables XIA - D are given in Table XII.

TABLE XII: Regression parameters estimated from the data of Tables XIA - D for the relationships:

$$m'_s = a_K \sqrt{PP} + b_K$$

$$\text{and} \quad \log_{10} p' = c_K \log_{10} PP + d_K$$

Operation time shape parameter K (all means = 1.0 time units)	a_K	b_K	Correlation coefficient for mean	c_K	d_K	Correlation coefficient for p
1	1.1038	-0.4631	0.9352	-0.4794	0.0777	-0.9631
2	0.7495	0.0263	0.8728	-0.4704	0.2862	-0.9542
3	0.6121	0.0432	0.9497	-0.4820	0.3826	-0.9267
4	0.5462	-0.0292	0.9504	-0.4682	0.4202	-0.9697

In the light of the good visual fit shown in Figures 5A - 5D and 6A - 6D and the highly significant correlation coefficients given in Table XII it would seem reasonable to use the relationships defined in Table XII to determine the parameters of the negative binomial IISSR distribution for a perfectly balanced Erlang line, K = 1 to 4, scheduled according to a planning period in the range investigated.

CHAPTER 5

TESTING OF HYPOTHESES

A composite test of the following hypotheses was carried out:

Hypothesis 1: The distribution of initial inter-stage stock required to keep the fed stage of a two stage Erlang line 100% utilised over a planning period in the range 20 to 1370 time units is negative binomial.

Hypothesis 2: The relationships of Table XII, page 75, may be used to estimate the mean and shape parameter of the negative binomial initial inter-stage stock requirement distribution of Hypothesis 1.

The design of the test

A factorial experiment was set up involving the simulation of the operations of two stage, perfectly balanced Erlang lines, mean operation time = 1.0 time units.

Three factors were varied:

Factor 1	Operation time shape parameter K
Factor 2	Planning period
Factor 3	Stock-out-risk

Factor 1 was applied at four levels i.e. K = 1,2, 3 and 4.

Factor 2 was applied at three levels i.e. 20, 695 and 1370 time units.

Factor 3 was applied at three levels i.e. 0.1, 0.3 and 0.5 but differing marginally from these values because of the discrete nature of the negative binomial IISSR distribution.

Changing factors 1 and 2 (K and planning period) was straight forward. Factor 3 was varied by fixing IISS at levels predicted as necessary, by the negative binomial and regression hypotheses, to achieve stock-out-risks as near 0.1, 0.3 and 0.5 as possible. For example for K = 1 the regression relationships of Table XII are:

$$m'_s = 1.1038\sqrt{PP} - 0.4631$$

$$\log_{10}p' = -0.4794\log_{10}PP + 0.0777$$

Thus for PP = 20 time units:

$$m'_s = 4.47$$

$$p' = 0.2844$$ (0.284410 when calculated on a computer using unrounded regression parameters).

Table XIII gives details of the negative binomial distribution

$m'_s = 4.47$, $p' = 0.284410$

TABLE XIII: Negative binomial IISSR distribution - $m'_s = 4.47$, $p' = 0.284410$ - for perfectly balanced Erlang line K = 1, PP = 20 time units

Initial Inter Stage Stock X	Probability Exactly X used	Probability X is sufficient to keep fed stage 100% utilised
0	0.1069	0.1069
1	0.1361	0.2430
2	0.1352	0.3782
3	0.1219	0.5001
4	0.1042	0.6043
5	0.0861	0.6904
6	0.0696	0.7600
7	0.0554	0.8154
8	0.0435	0.8589
9	0.0338	0.8927
10	0.0261	0.9187

This shows that if hypotheses 1 and 2 are correct then for a 0.1073 stock-out-risk IISS should be 9, for a 0.3096 stock-out-risk 5 and for a 0.4999 stock-out-risk 3.

IISS levels were established by this procedure for each of the combinations of factors. The IISS levels arrived at are given in Tables XVA-D, column 4, pages 81-4. Details of the m'_s and p' values used are given in Table XIV

TABLE XIV: Negative binomial m'_s and p' values for (K:PP) combinations on the basis of the regression relationships of Table XII, page 75. (Values calculated on a computer using unrounded regression parameter estimates).

K	Planning period	m'_s	p'
1	20	4.47	0.284410
1	695	28.64	0.051910
1	1370	40.39	0.037482
2	20	3.38	0.472324
2	695	19.79	0.088988
2	1370	27.77	0.064675
3	20	2.78	0.569507
3	695	16.18	0.102982
3	1370	22.70	0.074237
4	20	2.41	0.647288
4	695	14.37	0.122909
4	1370	20.19	0.089473

Having determined the factor values to be used the operations of each of the 36 (two stage) lines thus specified were simulated using the simulation programme of Appendix D. Two replications were carried out for each of the 36 lines. An individual simulation was terminated immediately the fed stage became idle because of a stock-out. If the hypotheses being tested were correct then the proportion of stock-outs for each of the 36 factor combinations should, allowing for random variations, be in line with the stock-out-risks built in by the IISS levels specified. An analysis of variance was applied to the difference between the observed number of stock-outs in each simulation set i.e. simulations for given combination of factors, and that predicted by the hypotheses for the IISS level specified.

For an analysis of variance to be strictly applicable it is necessary, amongst other things, that the variables concerned come from normal distributions with equal variances. In order to meet these requirements as far as possible the sample (simulation) size was fixed according to the predicted stock-out-risk. The distribution of the number of stock-outs occurring in a sample of size N must be binomial, parameters N and S, the stock-out-risk value. Values of N were chosen so that each 'number of stock-outs' distribution tended towards a normal distribution with a variance of approximately 9.5

The resulting sample sizes and variances are given in Tables XVA-D pages 81 - 84.

Analysis of variance is generally considered to be a robust technique and it is undoubtedly used in situations where far less is known about the underlying structure of the data. It was considered reasonable therefore to use it to analyse the simulation results thus obtained.

TABLE XVA: DATA RELATING TO THE TEST OF THE NEGATIVE BINOMIAL/ REGRESSION HYPOTHESES AS A BASIS FOR PREDICTING THE PROPORTION OF STOCK-OUTS TO BE EXPECTED IN A 2 STAGE LINE WITH A GIVEN INITIAL INTER-STAGE STOCK. OPERATION TIMES ERLANG, K = 1

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXACT STOCK-OUT-RISK	IISSR	SAMPLE SIZE	EXPECTED SAMPLE VARIANCE	EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	0.1073	9	99.	9.48	10.62
20.	0.30	0.3096	5	44.	9.40	13.62
20.	0.50	0.4999	3	38.	9.50	19.00
695.	0.10	0.1016	59	104.	9.49	10.57
695.	0.30	0.2957	35	46.	9.58	13.60
695.	0.50	0.5033	22	38.	9.50	19.13
1370.	0.10	0.0985	84	107.	9.50	10.54
1370.	0.30	0.3003	49	45.	9.46	13.51
1370.	0.50	0.4940	32	38.	9.50	18.77

N.B. FOR VARIANCE OF 9.5 SAMPLE SIZE = 9.5/(EXACT STOCK-OUT RISK X (1-EXACT STOCK-OUT-RISK))

TABLE XVB: DATA RELATING TO THE TEST OF THE NEGATIVE BINOMIAL/ REGRESSION HYPOTHESES AS A BASIS FOR PREDICTING THE PROPORTION OF STOCK-OUTS TO BE EXPECTED IN A 2 STAGE LINE WITH A GIVEN INITIAL INTER-STAGE STOCK. OPERATION TIMES ERLANG. K = 2

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXACT STOCK-OUT-RISK	IISSR	SAMPLE SIZE	EXPECTED SAMPLE VARIANCE	EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	0.0785	7	131.	9.48	10.28
20.	0.30	0.2779	4	47.	9.43	13.06
20.	0.50	0.5560	2	38.	9.38	21.13
695.	0.10	0.1015	39	104.	9.48	10.56
695.	0.30	0.2970	24	46.	9.60	13.66
695.	0.50	0.4952	16	38.	9.50	18.82
1370.	0.10	0.0996	55	106.	9.51	10.56
1370.	0.30	0.2941	34	46.	9.55	13.53
1370.	0.50	0.5100	22	38.	9.50	19.38

N.B. FOR VARIANCE OF 9.5 SAMPLE SIZE =
9.5/(EXACT STOCK-OUT RISK X (1-EXACT STOCK-OUT-RISK))

TABLE XVC: DATA RELATING TO THE TEST OF THE NEGATIVE BINOMIAL/ REGRESSION HYPOTHESES AS A BASIS FOR PREDICTING THE PROPORTION OF STOCK-OUTS TO BE EXPECTED IN A 2 STAGE LINE WITH A GIVEN INITIAL INTER-STAGE STOCK. OPERATION TIMES ERLANG, K = 3

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXACT STOCK-OUT-RISK	IISSR	SAMPLE SIZE	EXPECTED SAMPLE VARIANCE	EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	0.1139	5	94.	9.49	10.71
20.	0.30	0.3095	3	44.	9.40	13.62
20.	0.50	0.4733	2	38.	9.47	17.99
695.	0.10	0.1037	32	102.	9.48	10.58
695.	0.30	0.3109	19	44.	9.43	13.68
695.	0.50	0.4902	13	38.	9.50	18.63
1370.	0.10	0.0970	46	108.	9.46	10.48
1370.	0.30	0.3072	27	45.	9.58	13.82
1370.	0.50	0.5011	18	38.	9.50	19.04

N.B. FOR VARIANCE OF 9.5 SAMPLE SIZE = 9.5/(EXACT STOCK-OUT RISK X (1-EXACT STOCK-OUT-RISK))

TABLE XVD: DATA RELATING TO THE TEST OF THE NEGATIVE BINOMIAL/ REGRESSION HYPOTHESES AS A BASIS FOR PREDICTING THE PROPORTION OF STOCK-OUTS TO BE EXPECTED IN A 2 STAGE LINE WITH A GIVEN INITIAL INTER-STAGE STOCK. OPERATION TIMES ERLANG, K = 4

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXACT STOCK-OUT-RISK	IISSR	SAMPLE SIZE	EXPECTED SAMPLE VARIANCE	EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	0.0722	5	142.	9.51	10.25
20.	0.30	0.2442	3	51.	9.41	12.45
20.	0.50	0.4089	2	39.	9.43	15.95
695.	0.10	0.1034	28	102.	9.46	10.55
695.	0.30	0.3065	17	45.	9.57	13.79
695.	0.50	0.5170	11	38.	9.49	19.65
1370.	0.10	0.0982	40	107.	9.48	10.51
1370.	0.30	0.3067	24	45.	9.57	13.80
1370.	0.50	0.5075	16	38.	9.50	19.29

N.B. FOR VARIANCE OF 9.5 SAMPLE SIZE = 9.5/(EXACT STOCK-OUT RISK X (1-EXACT STOCK-OUT-RISK))

TABLE XVIA: EXPECTED AND OBSERVED NUMBER OF STOCK-OUTS AND THEIR DIFFERENCE FOR THE SIMULATION RUNS CARRIED OUT TO TEST THE NEGATIVE BINOMIAL/REGRESSION HYPOTHESES FOR ERLANG OPERATION TIMES, K = 1

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 1:- OBSERVED NUMBER OF STOCK-OUTS	OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 2:- OBSERVED NUMBER OF STOCK-OUTS	OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	10.62	15.	4.38	7.	-3.62
20.	0.30	13.62	13.	-0.62	10.	-3.62
20.	0.50	19.00	17.	-2.00	26.	7.00
695.	0.10	10.57	13.	2.43	11.	0.43
695.	0.30	13.60	12.	-1.60	20.	6.40
695.	0.50	19.13	20.	0.87	17.	-2.13
1370.	0.10	10.54	10.	-0.54	8.	-2.54
1370.	0.30	13.51	18.	4.49	11.	-2.51
1370.	0.50	18.77	20.	1.23	20.	1.23

TABLE XVIB: EXPECTED AND OBSERVED NUMBER OF STOCK-OUTS AND THEIR DIFFERENCE FOR THE SIMULATION RUNS CARRIED OUT TO TEST THE NEGATIVE BINOMIAL/REGRESSION HYPOTHESES FOR ERLANG OPERATION TIMES, K = 2

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 1:- OBSERVED NUMBER OF STOCK-OUTS	REPLICATE 1:- OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 2:- OBSERVED NUMBER OF STOCK-OUTS	REPLICATE 2:- OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	10.28	11.	0.72	14.	3.72
20.	0.30	13.06	12.	-1.06	16.	2.94
20.	0.50	21.13	18.	-3.13	19.	-2.13
695.	0.10	10.56	16.	5.44	11.	0.44
695.	0.30	13.66	15.	1.34	18.	4.34
695.	0.50	18.82	18.	-0.82	19.	0.18
1370.	0.10	10.56	10.	-0.56	12.	1.44
1370.	0.30	13.53	16.	2.47	15.	1.47
1370.	0.50	19.38	16.	-3.38	18.	-1.38

TABLE XVIC: EXPECTED AND OBSERVED NUMBER OF STOCK-OUTS AND THEIR DIFFERENCE FOR THE SIMULATION RUNS CARRIED OUT TO TEST THE NEGATIVE BINOMIAL/REGRESSION HYPOTHESES FOR ERLANG OPERATION TIMES, K = 3

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 1:- OBSERVED NUMBER OF STOCK-OUTS	OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 2:- OBSERVED NUMBER OF STOCK-OUTS	OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	10.71	7.	-3.71	9.	-1.71
20.	0.30	13.62	11.	-2.62	13.	-0.62
20.	0.50	17.99	17.	-0.99	19.	1.01
695.	0.10	10.58	12.	1.42	15.	4.42
695.	0.30	13.68	10.	-3.68	13.	-0.68
695.	0.50	18.63	19.	0.37	22.	3.37
1370.	0.10	10.48	13.	2.52	10.	-0.48
1370.	0.30	13.82	12.	-1.82	18.	4.18
1370.	0.50	19.04	27.	7.96	22.	2.96

TABLE XVID: EXPECTED AND OBSERVED NUMBER OF STOCK-OUTS AND THEIR DIFFERENCE FOR THE SIMULATION RUNS CARRIED OUT TO TEST THE NEGATIVE BINOMIAL/REGRESSION HYPOTHESES FOR ERLANG OPERATION TIMES, K = 4

PLANNING PERIOD	TARGET STOCK-OUT-RISK	EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 1:- OBSERVED NUMBER OF STOCK-OUTS	REPLICATE 1:- OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS	REPLICATE 2:- OBSERVED NUMBER OF STOCK-OUTS	REPLICATE 2:- OBSERVED MINUS EXPECTED NUMBER OF STOCK-OUTS
20.	0.10	10.25	13.	2.75	3.	-7.25
20.	0.30	12.45	17.	4.55	23.	10.55
20.	0.50	15.95	17.	1.05	13.	-2.95
695.	0.10	10.55	18.	7.45	8.	-2.55
695.	0.30	13.79	13.	-0.79	9.	-4.79
695.	0.50	19.65	25.	5.35	20.	0.35
1370.	0.10	10.51	18.	7.49	19.	8.49
1370.	0.30	13.80	12.	-1.80	19.	5.20
1370.	0.50	19.29	21.	1.72	17.	-2.29

TABLE XVII Analysis of variance applied to the 'observed minus expected number of stock-outs' data of Table XVIA-XVID, pages 85-88.

Source	Degrees of freedom	Sum of Squares	Mean Square	F	5% Significance value of F greater than:
Erlang shape parameter K	3	19.45	6.48	0.57	2.84
Planning period	2	24.55	12.28	1.08	3.23
Stock-out risk	2	6.12	3.06	0.27	3.23
Interaction between K and planning period	6	55.42	9.24	0.81	2.34
Interaction between K and stock-out-risk	6	99.59	16.60	1.45	2.34
Interaction between planning period and stock-out-risk	4	34.94	8.73	0.76	2.61
Interaction between K planning period and stock-out-risk	12	247.71	20.64	1.81	2.00
Within replicates	36	411.04	11.42		
Total	71	898.82			

Results and Analyses

Tables XVIA-D, pages 85-88, give the observed number of stock-outs, expected number of stock-outs and their difference for each of the 72 simulation samples. Table XVII, page 89, gives the results of an analysis of variance applied to the 72 differences. No F value in Table XVII is significant at a 5% level.

Test of the overall sample mean

A test was applied to the difference between the overall mean of the 72 differences and the predicted mean difference of zero. Details are given in Table XVIII. (The use of the predicted variance of 9.5 instead of the sample variance in calculating t provided the more stringent test). The t value of 2.51 is significant at a 1% level thus casting doubt on the negative binomial/regression hypotheses.

TABLE XVIII: Sample statistics and t and F values for the differences of Tables XVIA-D, pages 85-88. t values based on sample means, predicted variance of 9.5 and predicted mean of zero. F values based on sample variances and predicted variance of 9.5

K	Sample size	Sample mean	Sample variance	t	F
1,2,3 plus 4	72	0.9132	12.66	2.51	1.33
1 plus 2	36	0.5922	8.31	1.15	0.87
3 plus 4	36	1.2342	17.16	2.40	1.81

Test of normality of the differences

If the experimental design is valid then the differences should be normally distributed - with a variance of 9.5 and a mean of zero. A normal distribution with these parameters fitted to the 72 differences gives a chi^2 value that would be significant only at a 20% level (see Appendix G). Thus considered collectively the differences might reasonably be assumed to follow a normal distribution.

F tests applied to the differences

An F test of the sample variance against the predicted variance of 9.5 gives an F value of 1.33. The 95% point for the F distribution with 71/infinity degrees of freedom is less than 1.33. Thus there is reason to doubt that the true variance is 9.5

Considering the 72 differences in two groups gives an F of 0.87 (see Table XVIII, page 90) for the K=1 plus K=2 differences and an F of 1.81 for the K=3 plus K=4 differences. This second value is above the 99.5% point of the 35/infinity F distribution. Thus it appears that the deviation from the planned variance of 9.5 may be largely accounted for by the K=3 and K=4 samples. Further the t value for the K=3 plus K=4 differences (see Table XVIII) is significant at the 1% level whilst the K=1 plus K=2 value is unexceptional.

Conclusions

The most likely explanation of the above results is that the regression parameters used for estimating m'_s and p' for K=3 and 4 are inaccurate. However the possibility that the negative binomial distribution is only a reasonable description for K=1 and 2 and becomes less adequate for higher values of K cannot be discounted.

Despite the test results the general degree of correspondence between the observed and expected number of stock-outs in Table XVIA-D, pages 85-88, provides some encouragement and the hypotheses might, in the absence of any alternative, be considered to provide a reasonable first approximation to the solution of the problem of defining inter-stage stock levels.

C H A P T E R 6

USING INITIAL INTER-STAGE STOCK REQUIREMENT DISTRIBUTIONS

In Chapter 4 a general basis for determining the initial inter-stage stock requirement (IISSR) distribution for any given line was discussed and results for perfectly balanced Erlang lines were presented.

If the IISSR distribution relative to a given line can be identified then for each fed stage/feeding stage pair in the line it is possible to state an initial stock level that has a required probability of keeping each fed stage 100% utilised during a planning period. In general the higher the required probability, the higher the IISSR figure.

If initial inter-stage stocks are set at a level such that the probability of one or more of the fed stages running short of work is small then for the majority of planning periods each stage, being 100% utilised, will operate as though entirely independent of all other stages. It is then possible to consider the output potential of each stage as being governed only by its own operation time distribution.

An example of the use of an initial inter-stage stock requirement distribution.

Determining the IISSR for a given line

In Chapter 5 it was shown that for perfectly balanced Erlang lines, K = 1 to 4, the IISSR distribution for 100% utilisation of a fed stage might reasonably be assumed to be negative binomial. Further, it was shown that relationships of the form:

$$m'_s = a_K \sqrt{PP} + b_K$$

and $$\text{Log}_{10} p' = c_K \log_{10} PP + d_K$$

with parameter values as per Table XII, page 75, could be used to obtain estimates of this negative binomial distribution's parameters.

Figure 7, page 94 , represents a perfectly balanced four stage Erlang line, planning period = 124 time units, operation time mean = 1.0 time units, K = 4.

The p' and m'_s relationships for K = 4 are, from Table XII:-

$$m'_s = 0.5462 \sqrt{PP} - 0.0292$$
$$p' = -0.4682 \log_{10} PP + 0.4202$$

Thus for PP = 124 time units:

$$m'_s = 6.05$$
$$p' = 0.2755$$

Details of the negative binomial distribution $m'_s = 6.05$, p' = 0.2755 are given in Table XIX, page 95. Table XIX enables the probability of a given level of initial inter-stage stock (IISS) keeping a fed stage 100% utilised during a planning period of 124 time units to be determined e.g. if IISS = 6, probability of 100% utilisation = 0.6249. Conversely Table XIX can be used to determine IISS for a required chance of 100% utilisation of a fed stage e.g. if

FIGURE 7: Perfectly balanced 4 stage line, operation times Erlang, mean = 1.0 time unit, K = 4.

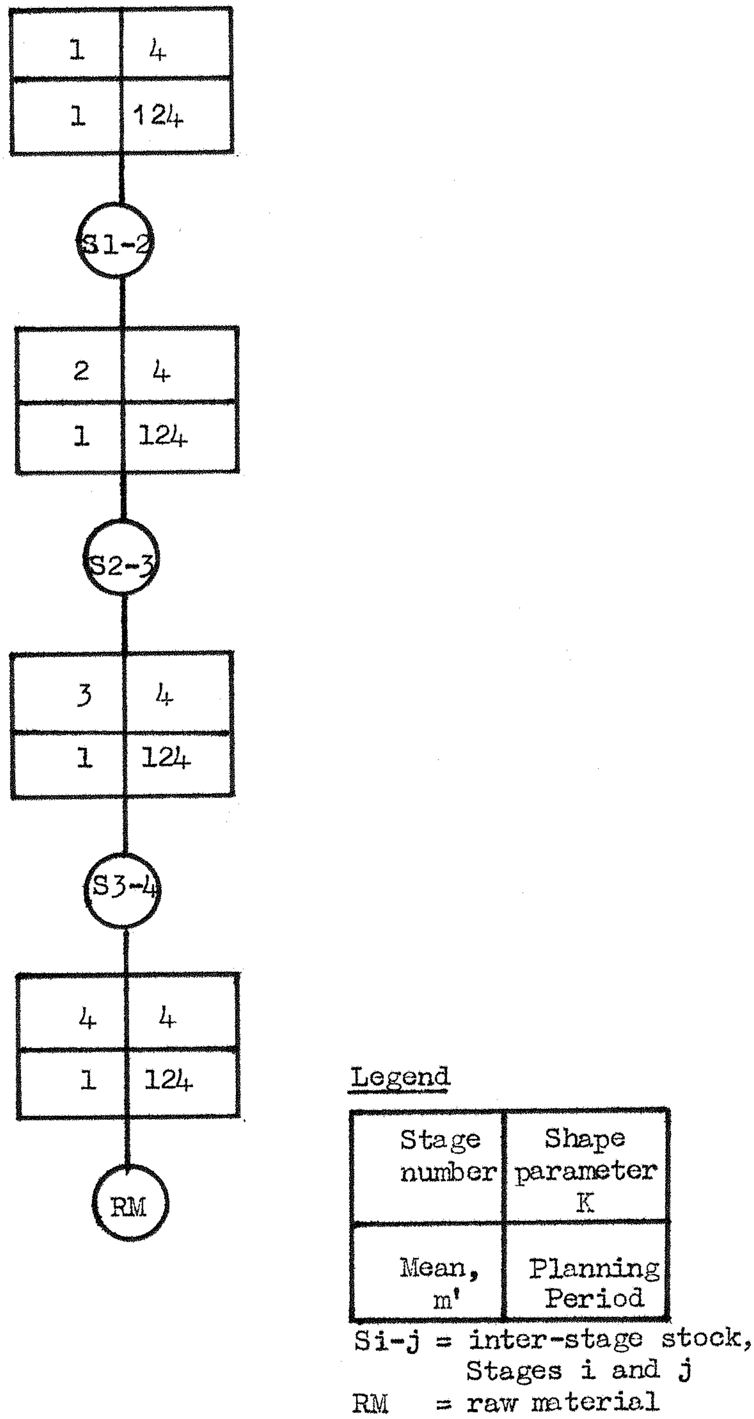

TABLE XIX: Negative binomial IISSR distribution for the line of Figure 7, page 94. $m'_s = 6.05$, $p' = 0.2755$

Initial inter stage stock X	Probability exactly X used	Probability X is sufficient to keep fed stage 100% utilised
0	0.0515	0.0515
1	0.0859	0.1374
2	0.1027	0.2401
3	0.1066	0.3467
4	0.1024	0.4490
5	0.0935	0.5425
6	0.0824	0.6249
7	0.0708	0.6957
8	0.0596	0.7553
9	0.0494	0.8047
10	0.0405	0.8452
11	0.0328	0.8780
12	0.0263	0.9043
13	0.0210	0.9253
14	0.0166	0.9419
15	0.0131	0.9550
16	0.0102	0.9653
17	0.0080	0.9733
18	0.0062	0.9795
19	0.0048	0.9843
20	0.0037	0.9880
21	0.0029	0.9908
22	0.0022	0.9930
23	0.0017	0.9947
24	0.0013	0.9960
25	0.0010	0.9970
26	0.0007	0.9977
27	0.0006	0.9983
28	0.0004	0.9987
29	0.0003	0.9990

a 90% chance is required then an IISS of 12 is necessary. Because of the discrete nature of the distribution it is generally only possible to approximate to a required probability figure. Thus in this case 90.43% is possible as opposed to the 90% specified.

Table XIX relates to the stock between any two adjacent (fed and feeding) stages in the line. The implications of a given common probability of 100% utilisation for the line as a whole would need to be considered in practice. Thus for the line of Figure 7 there are 3 inter-stage stocks. If each of the three initial inter-stage stocks is set at 12 then, since the probability of an individual IISS proving sufficient is 0.9043, the probability of all three proving sufficient in a given planning period is $0.9043^3 = 0.7395$ i.e. there is a 73.95% chance that no stage will become idle.

The stock used to feed the initial stage (Stage 4) has been ignored in the foregoing. Because it is not fed from an earlier operation stage the distribution of stock required will not (necessarily) be negative binomial. If, as is often the case in practice, this stock comes from a store then provided the store is run efficiently it is reasonable to assume that the probability that this initial stage runs short of stock will be small. If however it is considered desirable to specify a stock level in front of the initial stage that gives the same level of protection as the initial inter-stage stocks then a different model must be used.

For 100% utilisation of the initial stage as many items must be available in the feeding store as are completed by the stage during the planning period. Thus the distribution of stock requirement is the same as the distribution of output from a 100% utilised stage having an Erlang operation time distribution.

For exponential service times mean = m' a standard result is that the output per fixed (planning) period PP is Poisson, mean = PP/m' (Burke, 1956). Further it can be shown (Cox, 1962) that if the operation time is Erlang, parameters K and m', then the distribution of output will be general Poisson, parameters K and PP/m'. (See Appendix F for further details of the general Poisson distribution).

Table XX, page 98, gives the output distribution for a 100% utilised stage, planning period = 124 time units, operation time Erlang, mean = 1 time unit, K = 4. The main use of this table in this thesis is to determine the short term output potential of the line of Figure 7 but from the foregoing it can be seen that the table might be used to determine the stock level necessary to give a 90% chance of stage 4 being 100% utilised i.e. it shows that a stock level of 131 would be required.

Determining the output potential of the line of Figure 7

Two methods of reckoning output during a given period were outlined in Chapter 1, page 5. The output potential of a line (in terms of an output per planning period distribution) will be different for the short term and long term accounting methods.

Short term output potential

If all initial inter-stage stocks are maintained at a level that will provide a high probability of keeping fed stages fully occupied, Stage 1 will effectively be decoupled from the rest of the line. The short term output distribution is then that of a 100% utilised Stage 1.

As has been stated above in connection with determining an initial stock level for Stage 4 the output distribution of a 100% utilised stage

TABLE XX: Short term output distribution (general Poisson, K = 4, PP/m' = 124) for the line of Figure 7, page 94.

Output X	Probability output = X	Probability output X
110	0.0029	0.0070
111	0.0045	0.0115
112	0.0069	0.0184
113	0.0101	0.0285
114	0.0143	0.0429
115	0.0196	0.0624
116	0.0259	0.0883
117	0.0330	0.1213
118	0.0407	0.1620
119	0.0486	0.2106
120	0.0560	0.2666
121	0.0625	0.3291
122	0.0675	0.3966
123	0.0706	0.4672
124	0.0714	0.5387
125	0.0700	0.6087
126	0.0665	0.6752
127	0.0612	0.7364
128	0.0546	0.7909
129	0.0472	0.8381
130	0.0395	0.8776
131	0.0321	0.9098
132	0.0253	0.9351
133	0.0194	0.9545

having an Erlang operation time distribution is general Poisson, parameters K and PP/m'.

The general Poisson probability mass function is:-

$$gp(x;K,u) = \sum_{i=1}^{K} \frac{e^{-u} \quad u^{Kx+i-1}}{(Kx+i-1)!}$$

where K = shape parameter

u = PP/m' here = location parameter.

With K = 1 the Poisson distribution is obtained. u then equals the mean.

Thus the short term output potential of the line of Figure 7 is governed by the general Poisson distribution gp(x;4,124). The probability mass function and cumulative distribution function are evaluated in Table XX. This table shows that there is a 9.02% chance of an output of 132 items or more in a planning period of 124 time units since the probability of 131 or less is 0.9098. Similarly there is:-

a 20.91% chance of 129 or more items,

a 39.13% chance of 126 or more items,

a 46.13% chance of 125 or more items,

a 53.28% chance of 124 or more items,

a 91.17% chance of 117 or more items,

and a 99.30% chance of 111 or more items.

The difference between the expected output figure (expected by a production planner using a deterministic approach) of 124 items and a figure that has a reasonable chance of being achieved e.g. 99.3% chance of producing 111, could well be significant in practice.

Long term output potential

It has been pointed out (Chapter 1 page 10) that if it is assumed that from a long term point of view it is desirable to maintain the probability of meeting any given output figure at a constant level over all foreseeable planning periods then it is necessary to keep initial inter-stage stock constant. If this is to be so then only the output of the slowest stage i.e. the stage producing the smallest number of items during a given planning period can be 'counted'.

Thus in the case of perfectly balanced Erlang lines the long term output potential is governed by the distribution of the minimum value in successive samples of n (n = number of stages) from the general Poisson output distribution that applies to each of the n stages.

In general an extreme (minimum) value distribution for a sample of n may be derived as follows:

Let $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$

be a random sample of discrete variables independently distributed according to the same probability density function p(x).

Let $z_n = \min(x_1, x_2, x_3, \ldots\ldots x_n)$

Let probability x_i greater than $x = S(x)$

$\therefore$ p(all x_i greater than z) $= (S(z))^n$

and p (all x_i greater than (z-1)) $= (S(z-1))^n$

$$\therefore p(z_n = z) = (S(z-1))^n - (S(z))^n$$
$$= (S(z) + p(z))^n - (S(z))^n$$

$$\therefore p(z_n = z) = \sum_{i=1}^{n} \binom{n}{i} (p(z))^i (S(z))^{n-i}$$

where $\binom{n}{i} = n!/i!\,(n-i)!$

In the case of the long term output distribution for perfectly balanced Erlang lines:-

$p(z_n = z)$ = probability minimum output of n stages = z

This will be written as gpmin(z; K, u, n). Thus:

z	=	minimum output figure
K	=	operation time shape parameter
u	=	ratio of planning period to mean operation time i.e. PP/m'
n	=	number of stages
p(z)	=	output distribution of individual stages i.e. general Poisson distribution
S(z)	=	probability output of an individual stage is greater than z.

Thus $$S(z) = 1 - \sum_{x=0}^{z} \sum_{i=1}^{K} \frac{e^{-u} u^{Kx+i-1}}{(Kx + i-1)!}$$

gpmin(z; 4, 124, 4) for the line of Figure 7 is evaluated in Table XXI page 102. This table shows that there is an 8.06% chance of an output of 124 items or more. Similarly there is (corresponding short term figures in brackets):

	a	20.26%	chance of 122 or more items (20.91%:129),
	a	38.84%	chance of 120 or more items (39.13%:126),
	a	49.31%	chance of 119 or more items (46.13%:125, 53.28%;124),
	a	89.07%	chance of 114 or more items (91.17%:117),
and	a	99.08%	chance of 109 or more items (99.30%:111).

Reference has already been made to the difference between the 'expected' output figure of 124 items per planning period and any short term output figure that has a good chance of being achieved. The difference between the expected figure and any reasonably certain long

TABLE XXI: Long term output distribution gpmin($z;4,124,4$) for the line of Figure 7, page 94.

Output z	Probability output $= z$	Probability output $\leq z$
104	0.0004	0.0007
105	0.0007	0.0014
106	0.0013	0.0027
107	0.0024	0.0051
108	0.0042	0.0092
109	0.0070	0.0162
110	0.0114	0.0276
111	0.0177	0.0452
112	0.0264	0.0716
113	0.0377	0.1093
114	0.0514	0.1607
115	0.0666	0.2273
116	0.0818	0.3091
117	0.0947	0.4039
118	0.1030	0.5069
119	0.1047	0.6116
120	0.0990	0.7107
121	0.0867	0.7974
122	0.0700	0.8675
123	0.0520	0.9194
124	0.0353	0.9547
125	0.0218	0.9766
126	0.0123	0.9889
127	0.0063	0.9952

term output figure is even greater. The contrast is perhaps even more marked when it is considered that Table XXI shows that since the probability of 123 or less is 0.9194, there is only an 8.06% chance of achieving the long term output figure of 124 items.

Interest in the short term output potential of a line is likely to arise in emergency situations. At such times the main concern will probably be centered around getting as much output as possible or even simply getting any output. Whilst a knowledge of the probability of a given output figure being achieved would almost certainly be useful on such occasions the precise magnitude of a probability would tend not to be of great importance.

From the long term point of view however effective planning and control of a line require that schedules be based on output figures that do have a high chance of being achieved. The approach to long term output potential presented above provides a basis for determining such figures. In addition it enables the vital role that inter-stage stocks play in maximising a line's output potential to be quantified.

CHAPTER 7

SOME DESIGN IMPLICATIONS OF THE IISSR AND OUTPUT POTENTIAL CONCEPTS

The approach to inter-stage stock and output potential put forward has important implications at the line design stage. In particular it provides a basis for considering the effects of:

i. desired or implied chance of zero line idle time i.e. chance of no stage being idle.

ii. desired or implied chance of zero stage idle time i.e. chance of an individual stage having zero idle time.

iii. planning period.

iv. initial inter-stage stock

v. inter-stage storage capacity.

vi. desired chance of meeting an output schedule.

vii. number of stages.

viii. financial limits on stock investment.

Some examples of the way in which such factors act follow. The discussions tend to be based on the line of Figure 7, page 94 but whereas in Chapter 6 a planning period of 124 time units was assumed here it will be assumed that the planning period must be a multiple of 31 time units i.e. 31, 62, 93, 124, etc.

Chance of zero line idle time versus chance of zero stage idle time.

There are three inter-stage stock points in the line of Figure 7. The negative binomial/regression analysis results of Chapter 4 enable the chance of zero idle time at an individual stage to be controlled by the setting of an appropriate initial inter-stage stock level. The chance of zero stage idle time could be the same for all stages or could vary from stage to stage. This latter approach might be desirable because perhaps certain stages need higher utilisation levels than others for their efficient operation. Here however it will be assumed that a common figure is to be adopted. Given such a common figure the necessary initial inter-stage stock level can be determined from the appropriate IISSR distribution defined by Table XII, page 75.

If however the chance of zero line idle time is given then:

if the chance of zero line idle time = C,

chance of zero stage idle time = S

and number of inter-stage stock points = D

then $C = S^D$

and thus $S = C^{1/D}$

Tables XXII, page 106, and XXIII, page 107, illustrate the effect of this relationship for a range of 'chance of zero line idle time' and 'number of inter-stage stock points'. These tables are based on the assumption that the chance of zero line idle time is specified and that the above relationship is then used to determine the 'implied' chance of zero stage idle time.

TABLE XXII: Initial inter-stage stock versus possible planning periods for given chance of zero line idle time for the line of Figure 7, page 94. Figures in brackets based on negative binomial regression hypotheses using the K = 4 relationships of Table XII, page 75.

Required chance of zero line idle time during planning period	Implied chance of zero stage idle time during planning period	Initial inter-stage stock for a planning period of:			
		31	62	93	124
.5	.7937	4	6	8	9
		(.7742)	(.7875)	(.8170)	(.8047)
.6	.8434	5	7	9	10
		(.8583)	(.8467)	(.8604)	(.8452)
.7	.8879	5	8	10	11
		(.8583)	(.8909)	(.8943)	(.8780)
.8	.9283	6	9	11	13
		(.9136)	(.9232)	(.9205)	(.9253)
.9	.9655	8	11	14	16
		(.9701)	(.9629)	(.9672)	(.9653)
.95	.9830	9	13	16	19
		(.9828)	(.9826)	(.9822)	(.9843)
.98	.9933	11	15	19	22
		(.9946)	(.9920)	(.9931)	(.9930)
.99	.9967	12	17	21	25
		(.9970)	(.9964)	(.9964)	(.9970)
.995	.9983	13	19	23	27
		(.9984)	(.9984)	(.9981)	(.9983)

TABLE XXIII: Effect of number of stages on inter-stage stock requirements. Stock requirements based on negative binomial regression hypotheses using the K = 4 relationships of Table XII, page 75.

Number of inter - stage stocks	Required chance of zero line idle time during planning period	Implied chance of zero stage idle time during planning period	Initial inter-stage stock for a planning period of:			
			31	62	93	124
2	.90	.9487	7	10	13	15
2	.95	.9747	8	12	15	17
2	.99	.9950	11	16	20	23
3	.90	.9655	8	11	14	16
3	.95	.9830	9	13	16	19
3	.99	.9967	12	17	21	25
4	.90	.9740	8	12	15	17
4	.95	.9873	10	14	17	20
4	.99	.9975	12	18	22	26
5	.90	.9791	9	13	16	18
5	.95	.9898	10	14	18	21
5	.99	.9980	13	18	23	26
6	.90	.9826	9	13	16	19
6	.95	.9915	10	15	18	21
6	.99	.9983	13	19	23	27
8	.90	.9869	10	14	17	20
8	.95	.9936	11	16	19	22
8	.99	.9987	13	20	24	28
10	.90	.9895	10	14	18	21
10	.95	.9949	11	16	20	23
10	.99	.9990	14	20	25	29

Interaction of chance of zero line/stage idle time, planning period and initial inter-stage stock

It is assumed that the line of Figure 7 can be planned on the basis of a time interval that is a multiple of 31 time units. Columns 1 and 2, Table XXII cover a range of chance of zero line idle time and corresponding chance of zero stage idle time given that there are 3 inter-stage stock points. Columns 3-6 give the initial inter-stage stock necessary to achieve the required chance of zero line/stage idle time over the planning period indicated at the top of each column. These IISSR figures are based on the negative binomial distribution having parameters defined by the K = 4 figures of Table XII, page 75 i.e.

$$m'_s = 0.5462/PP - 0.0292$$

$$\log_{10} p' = -0.4682 \log_{10} PP + 0.4202$$

Thus for PP = 31 $\quad m'_s = 3.01$ and $p' = 0.5272$,

for PP = 62 $\quad m'_s = 4.27$ and $p' = 0.3811$,

for PP = 93 $\quad m'_s = 5.24$ and $p' = 0.3152$

and for PP = 124 $\quad m'_s = 6.05$ and $p' = 0.2755$

Table XXIV, page 109, gives details of the IISSR distribution for PP = 31 and Table XIX, page 95, for PP = 124. Table XXIV shows that if a planning period of 31 time units is used then if IISS equals 4 there is a 0.7742 chance of zero stage idle time. With three inter-stage stock points the implied chance of zero stage idle time for 0.5 chance of zero

TABLE XXIV: Negative binomial initial inter-stage stock requirement distribution for K = 4, planning period = 31 time units.

$m'_s = 0.5462\sqrt{PP} - 0.0292 = 3.01$

$\log_{10}p' = -0.4682\log_{10} PP + 0.4202 = 0.5272$

Initial inter stage stock X	Probability exactly X used	Probability X is sufficient to keep fed stage 100% utilised
0	0.1166	0.1166
1	0.1851	0.3017
2	0.1906	0.4924
3	0.1609	0.6533
4	0.1209	0.7742
5	0.0841	0.8583
6	0.0554	0.9136
7	0.0350	0.9486
8	0.0214	0.9701
9	0.0128	0.9828
10	0.0075	0.9903
11	0.0043	0.9946
12	0.0024	0.9970
13	0.0014	0.9984
14	0.0007	0.9991

line idle time is $0.5^{1/3} = 0.7937$. Because the IISSR distribution is discrete the nearest figure to this is the above 0.7742. Thus 4 is the figure that appears in line 1, column 3 of Table XXII with the achievable figure of 0.7742 in brackets beneath it. Similarly the table shows that for a 0.5 chance of zero line idle time over a planning period of 62 time units an IISS of 6 is necessary, the exact chance of zero stage idle time being 0.7875.

Table XXII shows that for a given required chance of zero line idle time IISSR increases as the planning period increases and that for a given planning period as required chance of zero line idle time increases so IISSR increases. Such information could be used to good effect at the line design stage.

If there was an upper limit to the IISS figure - perhaps because of a physical limitation on inter-stage stock because of financial constraints -Table XXII shows the alternatives available. If for instance the upper limit was 11 items then the maximum possible chance of zero stage idle time is 0.98 in conjunction with a planning period of 31 time units. If in addition the planning period must be 124 time units then the maximum possible chance of zero stage idle time is 0.7

Inter-stage storage capacity

The stock figures given in the body of Table XXII refer to the stock level that must exist at the beginning of each planning period if the chance of zero line idle time indicated is to be achieved. Over a given planning period physical stock would tend to vary about this value. Thus inter-stage storage capacity would need to be some figure greater than this initial figure. The research carried out did not consider this aspect of the problem but the procedure used could easily be adapted to give the necessary information. The table does however provide an indication of the minimum inter-stage capacity that must be provided for a given planning period/chance of zero line idle time combination and this alone is a significant improvement on the kind of information that is currently available to the line designer.

Financial limits on stock investment

Table XXII gives an insight into the financial investment requirements of the line. In general the higher the probability of zero line idle time and the longer the planning period the greater IISSR and thus the higher the financial investment involved. As was stated in the previous paragraph actual inter-stage stock will fluctuate about a stock figure quoted in the table; but it could provide the basis for a good first approximation to the financial investment that would have to be made in stock and thus an indication as to whether financial constraints are likely to be violated. Any attempt to find a solution within such constraints could proceed on the basis of the various references to inter-stage stock levels before and after this paragraph.

Number of stages versus initial inter-stage stock requirement versus planning period

A key decision at the line design stage concerns how many stages to have. This tends to be influenced by technological and ergonomic considerations and the required average output rate. Such factors are likely to be paramount but inter-stage stock and risk of idle time implications warrant consideration.

Columns 2 and 3 of Table XXIII, page 107, show how, for a given chance of zero line idle time, the chance of zero stage idle time - being equal to $C^{1/D}$, C = chance of zero line idle time, D = number of inter-stage stock points - increases as D increases. Therefore for a given planning period IISS must also increase. This effect is shown in columns 4, 5, 6 and 7.

Thus for a 0.99 chance of zero line idle time and using a planning period of 124 time units, IISSR = 23 for a three stage line (giving two inter-stage stock points if assumed to be a single string). If however an eleven stage line is used - giving ten inter - stage stock points - then the individual inter-stage stock requirement increases by 6 to 29. The total amount of stock between stages increases from 2 X 23 = 46 units to 10 X 29 = 290 units - an increase that could have significant financial and physical storage implications.

Effect of the choice of planning period on output potential

Table XXII shows that if an upper limit is placed on initial inter-stage stock then for a given risk of idle time the choice of planning period may be restricted.

As has been stated previously the short term output potential of a line would normally be of interest in emergency situations when the emphasis would tend to be on producing as many items as possible in the immediate period. If this period equals the planning period the 'short term' output potential as previously defined on page 10 applies. If the period is greater than the planning period then there is an increased risk of stock-outs between stages and the reckonable output potential will tend to be reduced.

In the case of the long term output potential if the choice of planning period is restricted then the long term output potential may be adversely affected - to an extent that might be significant.

In the case of the line of Figure 7 for instance if the preferred planning period is 124 time units and the probability of zero line idle time is required to be 0.90 but there is an upper limit to initial inter-stage stock of 8 items then from Table XXII it can be seen that a planning period of 124 cannot be accommodated and one of 31 time units must be used. If output schedules are to be based on a figure that is achievable 90% of the time then Table XXI, page 102 shows that for a planning period of 124 time units the output figure that has only a 4.52% chance of not being achieved is 112.

If however a planning period of 31 time units is used then the distribution of output per planning period of 31 time units is as shown in Table XX V, page 114, and the distribution of output per four consecutive, independent, planning periods of 31 time units each, obtained by combining four 31 time units output distributions is as shown in Table XXVI, page 115. This shows that with a 31 time units planning period the output figure corresponding to the 4.52% figure of the previous paragraph is 107/108.

TABLE XXV: Long term output distribution per 31 time units for the line of Figure 7, page 94. Planning period = 31 time units.

Output X	Probability output = X	Probability output ≤ X
19	.0001	.0001
20	.0001	.0002
21	.0006	.0008
22	.0023	.0030
23	.0078	.0108
24	.0222	.0330
25	.0529	.0859
26	.1041	.1900
27	.1656	.3556
28	.2071	.5627
29	.1970	.7597
30	.1383	.8980
31	.0698	.9678
32	.0249	.9927
33	.0061	.9988
34	.0010	.9999
35	.0001	1.0000

TABLE XXVI: Long term output distribution per 124 time units for the line of Figure 7, page 94. Planning period = 31 time units. Distribution based on the combination of the long term output for four consecutive, independent, planning periods of 31 time units. (See Table XXIV, page 114).

Output X	Probability output = X	Probability output ≤ X
98	.0001	.0001
99	.0002	.0003
100	.0003	.0006
101	.0007	.0013
102	.0015	.0028
103	.0029	.0057
104	.0053	.0109
105	.0092	.0201
106	.0153	.0354
107	.0240	.0594
108	.0355	.0949
109	.0497	.1446
110	.0654	.2101
111	.0809	.2910
112	.0940	.3850
113	.1022	.4872
114	.1040	.5912
115	.0987	.6898
116	.0873	.7771
117	.0718	.8489
118	.0549	.9038
119	.0388	.9426
120	.0254	.9681
121	.0154	.9835
122	.0086	.9921
123	.0044	.9965
124	.0021	.9986
125	.0009	.9995
126	.0004	.9998
127	.0001	.9999

Conclusion

The factors and interactions considered in this chapter are by no means exhaustive but they help to demonstrate the way in which the concepts developed can throw new light on some of the problems that must be considered at the line design stage.

C H A P T E R 8

SUMMARY

The research carried out had as its main aims:

1. the development of a procedure for determining, for any assembly line, the initial (to a planning period) inter-stage stock necessary to provide a given chance of zero idle time occuring during a planning period.
2. assuming that an initial inter-stage stock level giving a high chance of zero idle time could be stated - definition of the output potential of a line in terms of its short and long term output distributions.

The results obtained have prompted consideration of the effect of the interaction of such factors as planning period, initial inter-stage stock, risk of idle time and number of stages on the performance of a line.

With regard to aim 1 above, for a two stage line a method based on simulation has been developed. This enables the required stock level to be estimated directly; the larger the sample size (number of simulations) the more reliable will the estimate be. For two stage lines using planning periods in the range $8\sqrt{2}$ to $1024\sqrt{2}$ time units, and having operation times Erlang, mean 1.0 time unit, shape parameter = 1, 2, 3 or 4, it has been shown that the distribution of initial inter-stage stock requirement might reasonably be assumed to be negative binomial. For lines with more than two stages and thus more than one inter-stage stock point the two stage results could be used directly. However caution must be exercised since the two stage results are themselves approximate and in addition the effect of applying them to a line with more than two stages is sensitive to the number of inter-stage stock points.

Further research

There is a need for further development of the concepts put forward. In particular the following aspects require further investigation:

i. analytical identification of the true initial inter-stage stock requirement distribution.

ii. investigation of the sensitivity of initial inter-stage stock requirements to the number of inter-stage stock points, D, and determination of the range of values of D for which the procedures developed may be applied in practical situations.

iii. if no alternative to the negative binomial hypothesis is found - refinement of the regression relationships of Table XII, page 75.

iv. extension/testing of the results obtained for perfectly balanced Erlang lines over a wider range of K and planning period.

v. consideration of the economic aspect of the scheduling problem e.g. determination of 'economic' initial inter-stage stock levels and 'optimum' stock-out-risks.

vi. consideration of lines having more complex characteristics than perfectly balanced Erlang lines. For example:

a. lines with Erlang operation times having identical means but different values of K.

b. balanced lines with non-Erlang operation times.

c. balanced lines with unique operation time distributions - perhaps assumed to exactly follow the results of work study investigations.

d. unbalanced lines.

e. lines exhibiting the common characteristics of assembly lines e.g. pacing, incentive schemes, operative grouping schemes, mechanical (conveyor belt) movement of items along a line, etc.

vii. investigation of the problems involved in applying the concepts developed in practice. e.g. the effects of limiting output at stages other than the slowest during a planning period - as implied in the long term output model presented.

Conclusion

The results and concepts presented provide an insight into factors crucial for effective design, planning and control of a line. The significance of any one factor or interaction of factors will depend upon the particular line being designed, planned or controlled but an awareness of the implications of decisions relating to such factors and interactions can only improvd the chance of a line manager achieving his prime objective:

to meet planned finished goods production schedules for successive planning periods without tying up excessive amounts of stock between stages.

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APPENDIX A

Pilot investigation into initial inter-stage stock requirements when operation times are Weibull

For reasons stated in Chapter 2, pages 18-21, only Erlang operation times were considered in the main investigations. However a small independent investigation was carried out into the possible effect on results if Weibull operation times are assumed.

A Weibull distribution may be defined by two parameters a' and b' such that the cumulative distribution function is given by:

$$W(0 \leq t \leq T; a', b') = 1 - e^{-(b'T)^{a'}}$$

and:

$$\text{mean} = \frac{\Gamma(1 + a'^{-1})}{b'}$$

$$\text{variance} = \frac{\Gamma(1 + 2a'^{-1}) - (\Gamma(1 + a'^{-1}))^2}{b'^2}$$

The operation time distributions used in generating the results of Tables VA - D, pages 40-46, were Erlang, mean m' = 1 time unit, K = 1,2,3 and 4, variance = 1/K.

The simulations carried out with Erlang operations times, K = 2, 3 and 4, planning period = 16, 16 $\sqrt{2}$, 32, 32 $\sqrt{2}$ and 64 time units were repeated using Weibull operation times with corresponding means and variances. The required values of a' and b', identified by means of the above Weibull mean and variance expressions are given in Table AI, page 128.

An example of comparable distributionsis given in Table AIII, page 129.

TABLE A I: Weibull distribution parameters to give operation time distributions equivalent to those used in the simulations that produced the results of Tables VA-VD, pages 40-46

Erlang mean	Erlang shape parameter K	Equivalent Weibull distribution defined by: a'	b'
1.0	2	1.44	0.9075
1.0	3	1.79	0.8896
1.0	4	2.10	0.8857

The IISSR frequency distributions obtained using Weibull distribution operation times were compared with the corresponding Erlang results using the Kolmogorov-Smirnov two sample test. The maximum differences in cumulative frequencies-focused upon by the test - are given in Table A II. The 10% significance level for the test is in general given by $1.22\sqrt{(n_1 + n_2) / n_1 n_2}$ where n_1 and n_2 are the sample sizes (Seigal (1956)). Here n_1 and n_2 = 100 giving a 10% value of 0.1725. Thus the figures of Table A II indicate no significant difference between the distributions - which cover the range of planning periods 16 to 64 time units.

TABLE A II: Maximum observed differences between the cumulative IISSR frequency distributions for Erlang and Weibull operation times.

Planning period	K = 2	3	4
16	.05	.12	.12
16/2	.06	.12	.11
32	.09	.09	.09
32/2	.08	.10	.06
64	.12	.13	.13

This is not to say however that for longer planning periods, when the increasing completion rate of the Weibull distribution might have more effect, the distributions might not diverge significantly.

TABLE A III: Comparable initial inter-stage stock requirement frequency distributions obtained by simulating the operations of a two stage line first with Erlang and then with Weibull operation times, mean = 1.0 time units, variance = 0.5 i.e. K=2, a' = 1.44, b' = 0.9075. Planning period = 16 time units.

IISSR	Erlang line frequency	Weibull line frequency
0	16	16
1	15	18
2	25	17
3	14	14
4	6	14
5	5	3
6	9	10
7	8	3
8	2	2
9		2
10		1

APPENDIX B

Listing of Fortran programme used to simulate the operations of two stage assembly lines having Erlang operation times, mean = 1.0, K = 1,2,3 and 4

```
C-----PROGRAMME CODE NUMBER 740628 1155.
C-----BUFFER STOCK REQUIREMENT SIMULATOR, ERLANG OPERATION TIMES.
C-----STATEMENT NUMBERS IN RANGES 20-32, 100-128, 998-1004 AND 2000-2026
C-----USED.
      REAL TIME1(2000),TIME2(2000),MEAN
     1,PP(4),ALPHA(10)
      INTEGER DIST(2000)
      SQRT2=SQRT(2.0)
      READ (7,1004) DATE
      READ (7,1004)          ALPHA
 1004 FORMAT (10A8)
      ALPHG1=8H********
      READ (7,1001) LASTNO,NSIM,K1,P
C-----LASTNO=RANDOM NUMBER GENERATOR SEED.
C-----NSIM=SAMPLE (SIMULATION) SIZE.
C-----K1=FIRST ERLANG SHAPE PARAMETER.   PROGRAMME CYCLES FROM K=K1 TO 4
C-----P=PLANNING PERIOD. PROGRAMME AUTOMATICALLV PRODUCES PLANNING
C-----PERIOD THAT IS AN ODD MULTIPLE OF /2.
C-----IF INTEGER PP REQUIRED PUNCH PP*-1 ON DATA CARD.
C-----FOR /2PP PUNCH INTEGER PP, 1 BELOW PP REQUIRED.
 1001 FORMAT (3I0,F0.0)
      K=K1
      IF (P.LT.0.0) GO TO 128
      P=P*SQRT2
  128 CONTINUE
      IF (P.LT.0.0) P=P*(-1.0)
      MEAN=1.0
      NDONE=0
      NI30=0
  126 CONTINUE
      DO 30 I30=1,15
C-----PROGRAMME WILL AUTOMATICALLY REPEAT SIMULATIONS 15 TIMES.
      WRITE (2,999)
  999 FORMAT (1H1)
      DO 33 K=K1,4
```

```
      WRITE (2,2020)
      DO 31 I31=1,2000
      TIME1(I31)=-2.0
      TIME2(I31)=-2.0
   31 CONTINUE
      NI30=NI30+1
      IF (NI30.EQ.3) WRITE (2,2020)
 2020 FORMAT (///20X,62HOUTPUT FROM IISSR SIMULATOR, FRLANG OPERATION TI
     1MES.
     2/20X,'RANDOM NUMBER MULTIPLIER = 2893.'                      /)
      IF (NI30.EQ.3) NI30=0
      WRITE (2,2015) (ALPHG1,I=1,8)
 2015 FORMAT (20X,8A8)
      IF (NDONE.EQ.0) WRITE (2,2011) LASTNO
 2011 FORMAT (19X,22H RANDOM NUMBER SEED =        ,I10)
      NDONE=1
      NPR=100
      WRITE (2,2026) DATE
 2026 FORMAT (30X,'THIS OUTPUT PRODUCED ',A8)
      WRITE (2,2025) ALPHA
 2025 FORMAT (20X,10A8)
      WRITE (2,2008) K,MEAN,P,NSIM
 2008 FORMAT (20X,4HK = ,I2,3X,7HMEAN = ,F4.1,3X,11HTIME UNITS,/20X,
     1 18HPLANNING PERIOD = ,F14.2,3X,11HTIME UNITS,      ,
     2 /20X,24HNUMBER OF SIMULATIONS =      ,I4//)
      WRITE (2,2012) LASTNO
      DO 25 I25=1,2000
      DIST(I25)=0
   25 CONTINUE
      NPR=400
      NSIMS=1
  105 CONTINUE
      NWIPPR=0
      TIME1(1)=0.0
      TIME2(1)=0.0
```

```
      DO 20 I20=2,2000
C-----DO 20 LOOP GENERATES RANDOM TIMES FOR FED STAGE.
      TIME2(I20)=-0.0
      N1INP=I20
      CALL ERLANG(K,MEAN,LASTNO,RTIME)
      TIME1(I20)=TIME1(I20-1)+RTIME
      IF (TIME1(I20).GE.P) GO TO 100
   20 CONTINUE
      WRITE (2,2000) TIME1(2000)
 2000 FORMAT (7H 1 2000  ,F10.4)
      STOP
  100 N1INP=N1INP-1
      TIME1(N1INP+1)=-1.0
  121 CONTINUE
C-----N1INP NOW GIVES NO OF ITEMS COMPLETED DURING PLANNING PERIOD.
      N1INP=N1INP-1
      DO 21 I21=2,3000
C-----DO 21 LOOP GENERATES RANDOM TIMES FOR FEEDING STAGE.
      IF (I21.GT.(N1INP+3)) TIME1(I21)=-1.0
      N2INP=I21
      CALL ERLANG(K,MEAN,LASTNO,RTIME)
      TIME2(I21)=TIME2(I21-1)+RTIME
      IF (TIME2(I21).GE.P) GO TO 101
   21 CONTINUE
      WRITE (2,2001) TIME2(I21)
 2001 FORMAT (7H 2 2000    ,F10.4)
      STOP
  101 N2INP=N2INP-1
      IF (N1INP.LE.N2INP) NMIN=N1INP
      IF (N1INP.GT.N2INP) NMIN=N2INP
C-----COMPLETED ITEMS ARE PAIRED OFF BELOW IN DO 22 LOOP. NO POINT IN
C-----CONSIDERING MORE THAN THE SMALLER OF N1INP AND N2INP=NMIN.
      IF (N1INP.EQ.0) NWIPPR=0
      IF (N1INP.EQ.0) GO TO 120
  122 CONTINUE
```

```
C-----N1INP REFERS TO FED STAGE.
C-----N2INP REFERS TO FEEDING STAGE.
      NWITH=1
      DO 22 I22=1,NMIN
C-----NI22 USED AS COUNT OF ITEMS FROM FEEDING STAGE USEFUL TO FED STAGE
C-----DURING PLANNING PERIOD.
      NI22=I22
  111 IF (TIME2(I22).LE.TIME1(NWITH)) GO TO 113
C-----TIME1(1,2,3,....N1INP) COMPARED WITH TIME2(I22) UNTIL ONE GREATER
C-----FOUND INDICATING FEEDING STAGE ITEM FINISHED AT TIME2(I22) CAN
C-----BECOME FED STAGE ITEM STARTED AT TIME1(NWITH).
C-----DISCARDED TIME1 S MUST BE SUPPLIED FROM ISS.
      NWITH=NWITH+1
C----IF FED STAGE ITEMS EXHAUSTED GOES TO 114.
      IF (NWITH.GT.N1INP) GO TO 114
      GO TO 111
  113 NWITH=NWITH+1
C-----COMES THRU HERE WHEN A MATCH HAS BEEN MADE.
      IF (NWITH.GT.N1INP) GO TO 112
   22 CONTINUE
C-----COMES THRU HERE WHEN FEEDING STAGE ITEMS ARE EXHAUSTED.
      GO TO 112
  114 NI22=NI22-1
C-----NI22 ADJUSTED IF LAST FEEDING STAGE ITEM CONSIDERED COULD NOT BE
C-----USED.
C-----NWIPPR=IISSR THIS SIMULATION.
  112 NWIPPR=N1INP-NI22
C-----IISSR=NWIPPR
  120 CONTINUE
  118 CONTINUE
  115 CONTINUE
C-----IISSR DISTRIBUTION BUILT UP IN ARRAY DIST (ALLOWS FOR IISSR OF
C-----UP TO 1999)
      IF ((NWIPPR+1).GT.2000) WRITE (2,2019)
 2019 FORMAT (12X,24H WIP TOO GREAT FOR DIST.      )
```

```
      IF ((NWIPPR+1).GT.2000) GO TO 123
      DIST(NWIPPR+1)=DIST(NWIPPR+1)+1
  123 CONTINUE
C-----IF MORE SIMULATIONS TO BE DONE GOES TO 108.
      IF (NSIMS.LT.NSIM) GO TO 108
C-----PRINTS OUT IISSR DISTRIBUTION AND STATISTICS.
      NSIZE=2000
      CALL POUTSZ(DIST,NSIZE,J)
      WRITE (2,998)
      WRITE (2,2016) (I,I=0,9)
 2016 FORMAT (20X,45H DISTRIBUTION OF IISS REQUIRED BY FED STAGE
     1 //23X,10I4/)
      WRITE (2,2010) (DIST(I),I=1,J)
  127 CONTINUE
      NINAR1=2000
      CALL MUVAR(DIST,J,NINAR1 )
 2010 FORMAT (20X,3H  0,10I4/20X,3H 10,10I4/20X,3H 20,10I4/20X,3H 30,10I
     1 4/20X,3H 40,10I4/20X,3H 50,10I4/20X,3H 60,10I4/20X,3H 70,10I4/
     2 20X,3H 80,10I4/20X,3H 90,10I4/
     3 20X,3H100,10I4/20X,3H110,10I4/20X,3H120,10I4/20X,3H130,10I4/
     420X,3H140,10I4/20X,3H150,10I4/20X,3H160,10I4/20X,3H170,10I4/
     5 20X,3H180,10I4/20X,3H190,10I4/20X,3H200,10I4/      )
  998 FORMAT (1H )
      GO TO 117
  108 CONTINUE
      NSIMS=NSIMS+1
      GO TO 105
  117 CONTINUE
      WRITE (2,2012) LASTNO
 2012 FORMAT (20X,21HLAST RANDOM NUMBER =    ,I10)
      WRITE (2,2015) (ALPHG1,I=1,8)
      WRITE (2,999)
   33 CONTINUE
      K1=1
   30 CONTINUE
```

```
      P=P*SQRT2
      IF (P.GT.(1024.0*SQRT2))  WRITE (2,2023) LASTNO
 2023 FORMAT (/10X,15HSTOPPED AT 2023  /10X,22HFINAL RANDOM NUMBER = ,
     1 I10)
      IF (P.GT.(1024.0*SQRT2)) STOP
   29 CONTINUE
      GO TO 128
      STOP
      END
C-----DETERMINES SIZE OF DISTRIBUTION PRINT OUT.
      SUBROUTINE POUTSZ(DIST,NSIZE,NUPTO)
      INTEGER DIST(NSIZE)
      DO 20 I20=1,NSIZE
      NEGI20=NSIZE-I20+1
      IF (DIST(NEGI20).NE.0) GO TO 100
   20 CONTINUE
  100 NUPTO=NEGI20
      RETURN
      END
C-----RANDOM ERLANG OPERATION TIME GENERATOR.
      SUBROUTINE ERLANG(KERL,MEAN,LASTNO,RTIME)
      REAL MEAN
      FMEAN=MEAN/FLOAT(KERL)
      K=4194304
      FK=4194304.
      MULTI=2893
      RTIME=0
      DO 20 I21=1,KERL
C-----RANDOM NUMBER GENERATOR BASED ON A. R. EDMONDS PAPER IN THE
C-----COMPUTER JOURNAL, VOLUME 2, 1959-60.
      LASTNO=LASTNO*MULTI
      IF (LASTNO) 50,51,51
   50 LASTNO=LASTNO+K+K
   51 FLASTN=LASTNO
      RANDNO=FLASTN/(2*FK)
```

```
      REXPN=(ALOG(RANDNO))*FMEAN*(-1.0)
      RTIME=RTIME+REXPN
   20 CONTINUE
      RETURN
      END

C-----CALCULATES DISTRIBUTION PARAMETERS.
      SUBROUTINE MUVAR(DIST,J,NINAR1)
      INTEGER DIST(NINAR1)
      SUMX=0.0
      SUMX2=0.0
      SUMF=0.0
      DO 20 I20=1,J
      FREQ=FLOAT(DIST(I20))
      SUMF=SUMF+FREQ
      X=FLOAT(I20-1)
      SUMX=SUMX+(X*FREQ)
      SUMX2=SUMX2+(X*X*FREQ)
   20 CONTINUE
      FMEAN=SUMX/SUMF
      VAR=(SUMX2-((SUMX*SUMX)/SUMF))/(SUMF-1.0)
      SU=SQRT(VAR)
      WRITE (2,2000) SUMF,FMEAN,VAR,SU
 2000 FORMAT (/20X,14HSAMPLE SIZE = ,F6.0,2X,
     1 7HMEAN = ,F10.4/20X,11HVARIANCE = ,F12.4,2X,
     2 21HSTANDARD DEVIATION = ,F12.4/)
      WRITE (2,2001) SUMX,SUMX2
 2001 FORMAT (20X,15HSUM OF THE X =     ,F8.0,3X,
     1  23HSUM OF THE X SQUARED =      ,F9.0/)
      RETURN
      END
C-----RANDOM NUMBER GENERATOR.
      SUBROUTINE RANDN(RANDNO,LASTNO)
      K=4194304
      FK=FLOAT(K)
```

```
      NA=2893
      LASTNO=LASTNO*NA
      IF (LASTNO.LT.0) LASTNO=LASTNO+K+K
      FLASTN=FLOAT(LASTNO)
      RANDNO=FLASTN/(2.0*FK)
      RETURN
      END
C-----RANDOM WEIBULL OPERATION TIME GENERATOR.  NOT USED IN THIS VERSION
      SUBROUTINE WEIBLL(A,B,LASTNO,WBLV)
      FMEAN=1.0/A
      CALL ERLANG(1,FMEAN,LASTNO,EXPV)
      X=ALOG(EXPV*A)/A
      WBLV=EXP(X)/B
      RETURN
      END
```

Example of output from the programme of Appendix B

```
OUTPUT FROM IISSR SIMULATOR, ERLANG OPERATION TIMES.
RANDOM NUMBER MULTIPLIER = 2893.

****************************************************************************
RANDOM NUMBER SEED =      2191034
          THIS OUTPUT PRODUCED 5/5/76
FIRST OF THE 15 PP=20 SAMPLES USED TO ESTIMATE REGRESSION PARAMS
K =  1    MEAN = 1.0   TIME UNITS.
PLANNING PERIOD =           20.00   TIME UNITS.
NUMBER OF SIMULATIONS = 100

LAST RANDOM NUMBER =      2191034

 DISTRIBUTION OF IISS REQUIRED BY FED STAGE

       0   1   2   3   4   5   6   7   8   9

  0  10  12  13  14   4   9   9   6   8   3
 10   2   5   2   0   2   0   0   1

SAMPLE SIZE =    100.  MEAN =       4.7200
VARIANCE =         14.1228  STANDARD DEVIATION =          3.7580

SUM OF THE X =       472.   SUM OF THE X SQUARED =        3626.

LAST RANDOM NUMBER =       740858
****************************************************************************
```

A P P E N D I X C

Negative Binomial or Pascal Distribution

A Negative Binomial distribution may be defined by two parameters p' and k such that the probability mass function is given by:

$$nb(0;p',k) = p^k$$

and:

$$nb(x;p',k) = p^k \left[\frac{k(k+1)(k+2) \ldots\ldots (k+x-1)(1-p)^x}{x!} \right]$$

for $x \geq 1$.

The population mean $= k(1-p')/p'$

and population

variance $= k(1-p')^2/p'^2$

(mean $<$ variance)

Least squares estimates for p' and k may be obtained from the sample mean $\bar{x}$ and variance s^2 as:-

$$p' = \bar{x}/s^2$$

$$k = \bar{x}^2/(s^2 - \bar{x})$$

Maximum likelihood estimates of p' and k may be obtained from:-

$$L \log_e p' + \sum_{i=a+1}^{L} \left(\frac{1}{k} + \frac{1}{(k+1)} + \frac{1}{(k+2)} \ldots\ldots \frac{1}{(k+x_i-1)}\right) = 0$$

where:-

L = sample size

a = number of zeros in sample

$k = \bar{x}\, p'/(1-p')$

An iterative procedure may be used to solve the above maximum likelihood relationship.

Reference:- Bissell (1970)

APPENDIX D

Listing of Fortran simulation programme used to test the negative binomial/regression hypotheses.

```
C-----PROGRAMME CODE NUMBER 760402 1640.
C-----SIMULATOR OF A 2 STAGE LINE FOR IDENTIFYING STOCK-OUTS.
C-----STATEMENTS NUMBERS IN THE RANGES 100-119, 903-1000 AND 2000-2011
C-----USED.
      REAL MC(2),ALPHA(10)
      READ (7,1000) DATE
 1000 FORMAT (10A8)
      READ (7,1000) ALPHA
C-----ALPHA=LABEL OF UP TO 80 CHARACTERS.
      READ (7,1002) LASTNO
C-----LASTNO=RANDOM NUMBER GENERATOR SEED.
      WRITE (2,2006) DATE
      WRITE (2,2007) ALPHA
      WRITE (2,2009)
 2009 FORMAT (/10X,'LASTNO INPUT ON A CARD'/)
      GO TO 118
  108 CONTINUE
      WRITE (2,2006) DATE
 2006 FORMAT (30X,'THIS OUTPUT PRODUCED',1X,A8,'ON 760402 1640.'/)
      WRITE (2,2011)
 2011 FORMAT (61X,'2899 VERSION.'/)
C-----SEE RANDOM NUMBER GENERATOR SUBROUTINE 'RANDN' FOR EXPLANATION OF
C----- '2899 AND 2893' VERSIONS.
      WRITE (2,2007) ALPHA
 2007 FORMAT (10X,10A8/)
      WRITE (2,2010)
 2010 FORMAT (/10X,'LASTNO CARRIED FORWARD FROM PREVIOUS SAMPLE'/)
  118 CONTINUE
      READ (7,1003) PSO,NSIMTL,PPRIOD,IISS,KERL
C-----PSO=STOCK-OUT-RISK IMPLIED BY INITIAL INTER-STAGE STOCK - IISS.
C-----NSIMTL=SAMPLE (SIMULATION) SIZE.
C-----PPRIOD=PLANNING PERIOD.
C-----KERL=ERLANG OPERATION TIME SHAPE PARAMETER K.
      WRITE (2,2000) KERL,PPRIOD,PSO,IISS,NSIMTL
 1002 FORMAT (I0)
```

```
 1003 FORMAT (F0.0,I0,F0.0,2I0)
 2000 FORMAT (10X,'K= ',I3,1X,'PLANNING PERIOD = ',F8.1,1X,
     1'STOCK-OUT-RISK = ',F7.4/
     1 10X,
     2 'INITIAL INTER-STAGE STOCK = ',I3,1X,
     2'SAMPLE SIZE = ',I4/)
      WRITE (2,2005) LASTNO
 2005 FORMAT (10X,'RANDOM NUMBER SEED = ',I10/)
      ERLMN=1.0/FLOAT(KERL)
      NROUTS=0
      NSIM=0
      N1=0
      N2=0
  110 CONTINUE
      MC(1)=0.0
      MC(2)=0.0
C-----1 ADDED TO IISS TO ALLOW FOR START UP ITEM.
      ISS=IISS+1
C-----CLOCK GIVES AN ITEM'S START TIME.
      CLOCK=0.0
      NSIM=NSIM+1
  106 CONTINUE
C-----IF THERE IS A STOCK-OUT RTIME IS USED TO IDENTIFY WHEN THE ITEM
C-----AFFECTED WOULD HAVE FINISHED.   IF AFTER END OF PLANNING PERIOD
C-----IT IS NOT COUNTED AS A STOCK-OUT.
      LLSTNO=LASTNO
      CALL RANERL(KERL,LASTNO,RTIME,ERLMN)
      IF (ISS.LE.0) GO TO 101
      ISS=ISS-1
      NSTAGE=1
      MC(1)=CLOCK+RTIME
      N1=N1+1
  111 CONTINUE
      IF (MC(2).GT.0.0) GO TO 102
  100 CONTINUE
```

```
      CALL RANERL(KERL,LASTNO,RTIME,ERLMN)
      NSTAGE=2
      MC(2)=CLOCK+RTIME
      N2=N2+1
  102 CONTINUE
      IF (MC(1)-MC(2)) 103,104,105
  103 CLOCK=MC(1)
      IF (CLOCK.GE.PPRIOD) GO TO 107
      MC(1)=0.0
      GO TO 106
  104 CLOCK=MC(1)
      IF (CLOCK.GE.PPRIOD) GO TO 107
      ISS=ISS+1
      MC(1)=0.0
      MC(2)=0.0
      GO TO 106
  105 CLOCK=MC(2)
      IF (CLOCK.GE.PPRIOD) GO TO 107
      MC(2)=0.0
      ISS=ISS+1
      GO TO 100
  101 CONTINUE
      XTCLK=CLOCK+RTIME
      IF (XTCLK.GE.PPRIOD) GO TO 119
      LASTNO=LLSTNO
      NROUTS=NROUTS+1
      WRITE (2,2001) NROUTS,NSIM
     1 ,CLOCK,XTCLK            ,LASTNO
 2001 FORMAT (10X,'SO ',I3,1X,'AT =',I4,1X,'PREV FIN TIME = ',F7.2,1X,
     1 'NEXT = ',F7.2,1X,'LASTNO = ',I8)
  119 CONTINUE
  107 IF (NSIM.LT.NSIMTL) GO TO 110
      WRITE (2,903)
  903 FORMAT (///)
      WRITE (2,2008)          LASTNO
```

```
 2008 FORMAT (//10X,'LAST NO = ',I8/)
      WRITE (2,999)
  999 FORMAT (1H1)
      GO TO 108
  109 STOP
      END
C-----RANDOM ERLANG OPERATION TIME GENERATOR.
      SUBROUTINE RANERL(KERL,LASTNO,RERLNG,FMEAN)
      RERLNG=0.0
      DO 20 I20=1,KERL
      CALL RANDN(RN,LASTNO)
      REXPN=(ALOG(RN))*FMEAN*(-1.0)
      RERLNG=RERLNG+REXPN
   20 CONTINUE
      RETURN
      END
      SUBROUTINE RANDN(RN,LASTNO)
C-----RANDOM NUMBER GENERATOR BASED ON A. R. EDMONDS PAPER IN THE
C-----COMPUTER JOURNAL, VOLUME 2, 1959-60.
      K=4194304
      FK=4194304.0
      MULTI=2893
      MULTI=2899
C-----2899 USED IN REPLICATE 1.
C-----2893 USED IN REPLICATE 2.
      LASTNO=LASTNO*MULTI
      IF (LASTNO) 50, 51, 51
   50 LASTNO=LASTNO+K+K
   51 FLASTN=FLOAT(LASTNO)
      RN=FLASTN/(2*FK)
   52 CONTINUE
   53 CONTINUE
      RETURN
      END
```

Example of output from the programme of Appendix D

```
                         THIS OUTPUT PRODUCED 27/5/76 ON 760402 1640.

RESULTS FOR K=1, PP=20, STOCK-OUT-RISK=0.1073, REPLICATE 1, TABLE XVIA.

LASTNO INPUT ON A CARD

K=    1 PLANNING PERIOD =      20.0 STOCK-OUT-RISK =  0.1073
INITIAL INTER-STAGE STOCK =   9 SAMPLE SIZE =   99

RANDOM NUMBER SEED =    8025740

SO   1 AT =   2 PREV FIN TIME =   16.94 NEXT =   17.74 LASTNO =  2374060
SO   2 AT =   6 PREV FIN TIME =   18.60 NEXT =   19.22 LASTNO =  7524964
SO   3 AT =  10 PREV FIN TIME =   15.48 NEXT =   16.66 LASTNO =  7828132
SO   4 AT =  15 PREV FIN TIME =    8.25 NEXT =    8.35 LASTNO =  3000420
SO   5 AT =  18 PREV FIN TIME =    5.51 NEXT =    5.55 LASTNO =  3420140
SO   6 AT =  25 PREV FIN TIME =   11.51 NEXT =   13.46 LASTNO =  7937612
SO   7 AT =  27 PREV FIN TIME =   15.19 NEXT =   15.36 LASTNO =  3011812
SO   8 AT =  28 PREV FIN TIME =   14.31 NEXT =   15.55 LASTNO =  7174124
SO   9 AT =  34 PREV FIN TIME =   15.75 NEXT =   15.80 LASTNO =  8368204
SO  10 AT =  42 PREV FIN TIME =   14.98 NEXT =   15.19 LASTNO =  6275716
SO  11 AT =  55 PREV FIN TIME =    5.05 NEXT =    5.69 LASTNO =  8054476
SO  12 AT =  60 PREV FIN TIME =    7.99 NEXT =    8.32 LASTNO =  4990692
SO  13 AT =  70 PREV FIN TIME =   19.07 NEXT =   19.79 LASTNO =  1980652
SO  14 AT =  83 PREV FIN TIME =   14.71 NEXT =   14.71 LASTNO =  5188260
SO  15 AT =  93 PREV FIN TIME =   12.99 NEXT =   14.01 LASTNO =  7055684

LAST NO =  3397028
```

APPENDIX E

Erlang Distributions

The Erlang family of distributions has the probability density function:

$$erl(t;K,m') = \frac{(K/m')^K t^{K-1} e^{-Kt/m'}}{(K-1)!}$$

where:-

K, an integer, = a shape parameter

m' = the mean

$e = 2.7183$

The mode is at $t = (K-1)/(K/m')$

and the variance $= m'^2/K$

For $K = 1$ $erl(t;K,m') = e^{-t/m'}/m'$, the probability density function of the negative exponential distribution.

For $K = \infty$ the mode is at m' and the variance is zero.

Erlang distributions form a subset of the gamma distribution family:

$$gamma(t;a',b') = \frac{(a'/b')^{a'} t^{a'-1} e^{-a't/b'}}{(a'-1)!}$$

$a' > 0$

b' positive

In the Erlang subset a' is a positive integer.

The Erlang distribution is identified as a special case of the gamma distribution family because of its importance in Queueing Theory where it is frequently encountered as a service time distribution (Saaty 1957).

In general the gamma cumulative distribution:

$$\text{GAMMA}\ (0 \leq t \leq T\ ; a',\ b') = \int_0^T \frac{(a'/b')^{a'} t^{a'-1} e^{-a't/b'}}{(a'-1)!}\, dt$$

must be evaluated by numerical methods but if a' is a positive integer, as in the case of the Erlang family then:

$$\text{GAMMA}\ (0 \leq t \leq T; a',\ b') = \text{ERL}(0 \leq t \leq T; K, m')$$

$$= 1 - \left[1 + \frac{m't}{K} + \frac{1}{2!} \frac{(m't)^2}{(K)^2} + \frac{1}{3!} \frac{(m't)^3}{(K)^3} \cdots \frac{1}{(K-1)!} \frac{(m't)^{K-1}}{(K)^{K-1}} \right] e^{-m't/K}$$

References:-

Sasieni, Yaspan & Friedman (1959)

Mood & Graybill (1963)

APPENDIX F

The General Poisson Distribution

A standard result (Haight (1967), Cox (1962), Morlat (1952)) is that if the intervals between arrivals at a point follow a negative exponential distribution then the output per unit time is Poisson.

A generalisation of this result can be obtained by allowing the interval between arrivals to be Erlang(negative exponential intervals then being a particular case).Haight (1967) refers to two possibilities:-

i. synchronous counting - when counting commences immediately after an arrival.

ii. asynchronous counting - when counting commences at a time independent of any arrival time.

The synchronous counting model is relevant to the fixed planning period situation. Goodman (1952), Morlat (1952), Haight (1959) and Jewell (1960) give the probability mass function for the output distribution for this model as:

$$gp(x;K,u) = \sum_{i=1}^{K} \frac{e^{-u} \quad u^{Kx+i-1}}{(Kx+i-1)!}$$

where K = shape parameter,

and u = location parameter.

This distribution has been referred to as the general, or generalized, Poisson distribution.

To fit a general Poisson distribution to data Haight (1959) suggested trying different values of K until a minimum chi^2 value is obtained. To do so u should be set equal to $K\bar{x} + 0.5K - 0.5$ where $\bar{x}$ = the sample mean. This is a procedure which Haight found to work in practice and experience during the course of the present research confirms this.

APPENDIX G

Tests of the normality of the differences of Tables XVIA-D, pages 85-88

In order to test the assumption that the 72 differences are normally distributed the three 'versions' of normal distribution indicated in Table GI were considered. 0.9132 and 3.5580 are the sample mean and standard deviation respectively. 0 and $\sqrt{9.5}$ are the predicted mean and standard deviation under the negative binomial/regression hypotheses of page 77.

TABLE GI: Goodness-of-fit results for normal distributions fitted to the 72 differences of Tables XVIA-D, pages 85-88.

Mean	Standard deviation	Chi^2	df	Significant at (% level)
0.9132	3.5580	11.11	7	20
0	3.5580	9.59	7	25
0	$\sqrt{9.5}$	8.85	5	20

Table GI shows that the observations could reasonably be assumed to come from any of the three normal distributions tested.

G L O S S A R Y

Key abbreviations and symbols

a_K — slope parameter of regression relationship for negative binomial initial inter-stage stock requirement distribution mean. (See Table XII, page 75).

b_K — intercept parameter of regression relationship for negative binomial initial inter-stage stock requirement distribution mean. (See Table XII, page 75).

$\binom{n}{i}$ — number of combinations of n items taken i at a time.

C — chance of zero line idle time i.e. of all stages in a line being 100% utilised.

c_K — slope parameter of regression relationship for negative binomial initial inter-stage stock requirement distribution parameter p'. (See Table XII, page 75).

d_K — intercept parameter of regression relationship for negative binomial initial inter-stage stock requirement distribution parameter p'. (See Table XII, page 75).

D — number of inter-stage stock points in a line.

IISS — initial inter-stage stock. Equals stock present between two stages at the beginning of a planning period.

IISSR	initial inter-stage stock requirement. Equals the stock that must be present between two stages at the beginning of a planning period if a given chance of the fed stage being 100% utilised is to be achieved.
K	Erlang operation time shape parameter.
m'	mean of an operation time distribution.
m_s	IISSR sample mean.
m'_s	true or assumed (on basis of regression relationships of Table XII, page 75) mean of an IISSR distribution.
n	number of stages in a line.
N	sample size used in factorial experiment of Chapter 5.
p	IISSR sample shape parameter. (Distribution assumed to be negative binomial).
p'	true or assumed (on basis of regression relationships of Table XII, page 75) IISSR distribution shape parameter. (Distribution assumed to be negative binomial).
PP	planning period. A line is assumed to be planned, scheduled and controlled on the basis of consecutive time intervals of constant and predetermined duration i.e. on the basis of the planning period.
q_i	number of independent stock points in front of stage i.
S	chance of zero stage idle time i.e. of an individual stage being 100% utilised.

u location parameter of general Poisson distribution. Equals planning period/mean operation time in the output distribution context.

Distribution notation used

Probability density functions (p.d.f.) and probability mass functions (p.m.f.) are indicated by one or more descriptive letters - in lower case - followed in brackets by a symbol representing a random variable, a semicolon and then parameter symbols separated by commas:

$erl(t;K,m')$ = Erlang p.d.f.

$gamma\ (t;a',b')$ = gamma p.d.f.

$gp(x;K,u)$ = general Poisson p.m.f.

$gpmin\ (x;K,u,n)$ = p.m.f. of minimum value in a sample of size n from a general Poisson distribution.

$nb(x;p',k)$ = negative binomial p.m.f.

Cumulative distribution functions (c.d.f.) are indicated in a related fashion, capital letters being used instead of lower case for the descriptive letters and the sense of the accumulation appearing before the semicolon:

$ERL(0 \leq t \leq T;K,m')$ = Erlang c.d.f.

$GAMMA(0 \leq t \leq T;a',b')$ = gamma c.d.f.

$W(0 \leq t \leq T;a',b')$ = Weibull c.d.f.