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Some Non-parametric Methods in Experimental Design

by

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To Mitra, Abtin and Armin

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ABSTRACT

FACULTY OF MATHEMATICAL STUDIES

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Some Non-parametric Procedures

for Experimental Design

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This research consists of two parts involving non-parametric procedures for assessing interaction and main effects . The first part is concerned with the problem of interaction in two-way layouts with one observation per cell. After a survey of the work done so far attention is centered on the work carried out by Wolfe et al. based on orthogonal Latin squares. Analysis is made of the three procedures proposed and it is shown that for one of them the critical values for the test involved and hence the powers are highly dependent on the main effects. Proposals are made to adjust the data for the main effects by aligning the data within the levels of the two factors through row and column means or medians. A power comparison based on a Monte-Carlo simulation study reveals that the modified procedures do stabilize the critical points of the tests and lead to a more consistent power performance.

The second part of the research originates from a blocked factorial experiment at Cambridge Laboratory, Cereals Research Department involving two factors. Measurement of the response variable is not possible and hence a non-parametric procedure is sought for analysis. A Friedman-type procedure is proposed for the analysis which leads to over-estimation of the residual variance and hence reduced power performance when testing one effect in the presence of the others. Two modifications are made to the proposed procedure and through a power study based on simulations it is shown that one of the modifications mitigates the drop in power performance and leads to a procedure which is comparable to the ANOVA under normality and is more efficient when severe deviations from normality occur.

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Chapter 1

Testing Interaction in a Two-Way Layout

1.1 Introduction

In an experimental situation where the effects of two factors are being investigated on a response variable, one is usually interested in the separate individual effect of each factor and also their joint effect. The former are known as main effects and the latter as the interaction and the relationship between the response and the two factors is described through a model. In order to assess the importance of either of the main effects, it is essential that the problem of non-additivity or presence of interaction between the two factors is first dealt with. In classical statistics the usual model for a two-way layout with k observations per cell is as follows:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk} \qquad i = 1, 2, \ldots, I \quad j = 1, 2, \ldots, J$$
$$k = 1, 2, ..., K \tag{1.1}$$

where ϵ_{ijk}'s are mutually independent, identically distributed normal variates with mean equal to 0 and variance σ^2, α_i is the main effect of level i of factor U, β_j is the main effect of level j of factor V and γ_{ij} is the interaction effect arising from a combination of level i of factor U and level j of factor V. Standard analysis of variance procedures can be used to test the main effects and interaction. The degrees of freedom allocation is as follows:

$(I-1)$ for the sum of squares for factor U, $(J-1)$ for the sum of squares for factor V, $((I-1)(J-1))$ for the sum of squares for interactions and $IJ(K-1)$ for the sum of squares for error. The partition of the total sum of squares is:

$$\begin{aligned}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(y_{ijk}-\bar{y}_{...})^2 &= \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(\bar{y}_{i..}-\bar{y}_{...})^2+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(\bar{y}_{.j.}-\bar{y}_{...})^2 \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2 \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(y_{ijk}-\bar{y}_{ij.})^2 \qquad (1.2)\end{aligned}$$

The first term on the right is the sum of squares for factor U, the second is the sum of squares for factor V, the third is the sum of squares for interaction and the last term is the sum of squares for error. Division of each sum of squares by its degree of freedom will give us the relevant mean squares and the division of each mean square by the mean square error will provide us with the F-ratio for testing the effect in question. Here a test for interaction is available because there is an independent estimate for error due to the m replicates in each cell against which all the effects, including the interaction, can be tested.

Now consider the case where we have only one observation per cell. The model is:

$$y_{ij}=\mu+\alpha_i+\beta_j+\gamma_{ij}+\epsilon_{ij} \qquad i=1,2,\ldots,I \quad j=1,2,\ldots,J \qquad (1.3)$$

The partition of the total sum of squares is:

$$\begin{aligned}\sum_{i=1}^{I}\sum_{j=1}^{J}(y_{ij}-\bar{y}_{..})^2 &= \sum_{i=1}^{I}\sum_{j=1}^{J}(\bar{y}_{i.}-\bar{y}_{..})^2+\sum_{i=1}^{I}\sum_{j=1}^{J}(\bar{y}_{.j}-\bar{y}_{..})^2 \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J}(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})^2 \qquad (1.4)\end{aligned}$$

The first component is due to factor U, the second component is due to factor V and the third is due to the interaction between U and V. If we assume there is no interaction, then we can test the main effects because the third component will be the error sum of squares and we can test the main effects against the error. On the other hand, if interaction does exist, then we do not have a standard test for the main effects in the classical setting of normally distributed data because in this case we do not have an estimate for the error. Nor do we have a standard test for interaction. This one-observation per cell situation has always been a difficult problem. Over the years there have been several attempts to address this but all of them have been restrictive in the sense that they are either capable of detecting a particular type of interaction or they are applicable in particular situations because of the conditions imposed upon them. In the next section these methods of assessing interaction are reviewed and in chapter 2 some of the more recent research is considered.

1.2 Previous Work

Tukey (1949) suggested a test for interaction which drew several criticisms and modifications. This test is based on a partition of the error sum of squares into a component with one degree of freedom due to interaction and a remainder component for error and is designed for the assessment of a particular kind of interaction known as product interaction. The null hypothesis is that $\gamma_{ij} = 0$ for all i and j and the alternative hypothesis is that $\gamma_{ij} = \rho\alpha_i\beta_j$ for some constant ρ. Under the assumption of normality the test statistic has an F-distribution with one and $IJ - I - J$ degrees of freedom. Obviously the test is very restrictive for the type of interaction it can detect.

Other approaches have used non-parametric procedures and in such cases the model assumptions are different. The condition of normality is replaced so that the assumptions for ϵ_{ij}'s are that they are mutually independent, identically distributed, continuous random variables with common median θ. We now review the non-parametric tests of interaction for two-way layouts with emphasis on methods applied to cases with one observation per cell.

Rank-based nonparametric approaches to the problem of non-additivity are rare. Most of these approaches require replications in the cells of the two-way layout. Among them are the rank transform approaches of Iman (1974) and Conover and Iman (1976). In this method, observations are replaced by their ranks taken over all observations, and an analysis of variance is carried out on these ranks. Suitable F-ratios are then used to investigate the main effects and the interaction. The first non-parametric procedure available in the literature which can be used in a single observation case is that of DeKroon and Van der Laan (1981) for a special type of interaction which they call rank-interaction. We need to explain this concept of rank interaction and differentiate between it and the more usual kind of interaction.

1.3 Rank Interaction

Intuitively we say that two factors interact if the effect of one factor depends on the levels of the other. Non-existence of an interaction implies that the effect of one factor is the same at the different levels of the other factor. This would indicate that the response curves are parallel in the factor domain. De Kroon and van der Laan (1981) introduced the weaker concept of rank interaction in which the rankings of levels of one factor are compared across levels of another factor. If we have two factors U and V and the ranking of U is the same at all levels of V, then they say there is complete "concordance" in the ranking and hence no rank interaction of U across V, otherwise there is some "discordance" in the rankings and rank interaction is said to exist and the notation $U^*(V)$ is used for this rank interaction. Similarly, if the rankings of factor V is not the same at all levels of factor U then there is rank interaction and it is denoted by

$V^*(U)$. The following points which indicate the limitations of rank interaction should be noted.

Rank interaction is not symmetric with respect to U and V and so rank interaction of type $U^*(V)$ does not imply rank interaction of type $V^*(U)$ and vice versa. In diagram (c) of Figure 1.1 we have a situation where we have only rank interaction of type $V*(U)$. Similarly in diagram (d) only rank interaction $U*(V)$ exists. Consequently the non-existence of $U^*(V)$ does not imply the non-existence of $V^*(U)$. We should note that situations where there is rank interaction form a subset of all those where there is interaction in the usual sense. Thus if we have interaction in the usual sense we may or may not have rank interaction and also if there is no rank interaction we may or may not have interaction in the usual sense. In diagram (b) of Figure 1.1 we have a situation where no rank interaction exists but obviously we do have interaction.

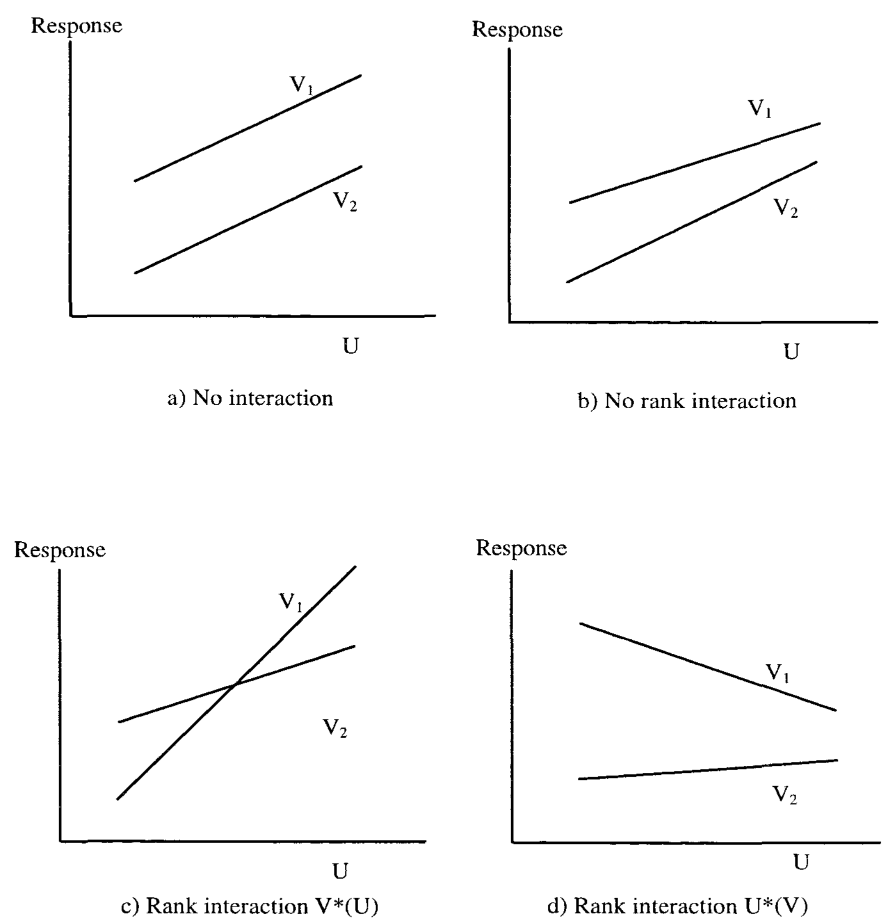


Figure 1.1: Performance curves showing the Response against Factor U at two levels of Factor V in four different situations.

De Kroon and Van Der Laan's method for assessing rank interaction based on model (1.1) uses a Kruscal Wallis type statistic to compare the levels of one factor at all levels of the other factor. The statistic is shown to be made up of two statistics, one showing the overall assessment of one factor(Friedman' Test) and the other assessing the rank interaction. The analysis runs along the following lines:
Consider testing the null hypothesis

$$H_0 : \alpha_1 + \gamma_{1j} = \alpha_2 + \gamma_{2j} = ... = \alpha_I + \gamma_{Ij} \qquad j = 1, 2, \dots, J \tag{1.5}$$

which implies that factor U has no effect at all against the alternative

$$H_1 : H_0 \;\; does \;\; not \;\; hold. \tag{1.6}$$

We are comparing the ranking of levels of factor U across the levels of factor V. For each level of V we rank all the observations at that level. We denote by r_{ijk} the rank of the kth observation at level i of U and level j of V. Thus $\bar{r}_{ij.}$ is the mean rank of the observations in cell (ij) or the mean rank of all the observations at level i of U and level j of V. Similarly $\bar{r}_{i..}$ is the mean rank of all the observations at level i of factor U. Finally $\bar{r}_{...}$ denotes the mean rank of all the observations.

Let $T = \sum_{j=1}^{J}(L_j)$ where L_j is the Kruscal Wallis statistic for testing factor U at level j of factor V and thus T is the sum of J Kruscal Wallis statistics. In that case

$$T = \frac{12}{KI(KI+1)} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (\bar{r}_{ij.} - \bar{r}_{.j.})^2. \tag{1.7}$$

The statistic T can be written as a sum of two statistics T_1 and T_2. T_1 is the Friedman statistic which is:

$$T_1 = \frac{12}{KI(KI+1)} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{K} (\bar{r}_{i..} - \bar{r}_{...})^2 \tag{1.8}$$

T_2 can be obtained by subtraction.

$$
\begin{aligned}
T_2 = T - T_1 &= \frac{12}{KI(KI+1)} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(\bar{r}_{ij.} - \bar{r}_{.j.})^2 \\
&- \frac{12}{KI(KI+1)} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(\bar{r}_{i..} - \bar{r}_{...})^2 \\
&= \frac{12}{KI(KI+1)} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(\bar{r}_{ij.} - \bar{r}_{.j.} - \bar{r}_{i..} + \bar{r}_{...})^2 \qquad (1.9)
\end{aligned}
$$

The above formula for T_2 can be simplified if we note that $\bar{r}_{.j.}$ is equal to $\bar{r}_{...}$. Therefore T_2 is written as :

$$
T_2 = \frac{12}{KI(KI+1)} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m}(\bar{r}_{ij.} - \bar{r}_{i..})^2 \qquad (1.10)
$$

Thus in the rank interaction method, there are three statistics involved when we compare rankings of the I levels of U over the J levels of V. These are:(a) T, a sum of J independent Kruscal-Wallis statistics, one for each level of V;(b) T_1, the overall Friedman statistic for comparing the I levels of U which is sensitive to differences between levels of U; and, finally, (c) $T_2 = T - T_1$, which is sensitive to differences among the rankings of levels of U at different levels of V and is therefore the statistic for testing rank interaction. The statistic T_2 assesses the rank interaction $U^*(V)$. If ranking is carried out within the levels of factor U, and attention is centered on testing the levels of factor V, then following the same reasoning another statistic is obtained for assessing $V^*(U)$. As we notice the two tests do not detect all differences in the γ_{ij}'s as shown in diagram b in Figure 1.1. Instead they are designed to detect discordance either in the ranking vectors of $(\alpha_1 + \gamma_{1j}, \alpha_2 + \gamma_{2j}, ..., \alpha_I + \gamma_{IJ})$ for $j = 1, 2, ..., J$ or in the ranking vectors of $(\beta_1 + \gamma_{i1}, \beta_2 + \gamma_{i2}, ..., \beta_J + \gamma_{IJ})$ for $i = 1, 2, ..., I$. We should note that although the rank interaction procedure has been proposed for situations where there are K observations per cell, it can be used when there is only one observation per cell.

Wolfe, Dean and Hartlaub (1990) proposed a non-parametric ranking procedure for testing interaction in a single replicate two-way layout based on orthogonal Latin squares and later Wolfe, Dean, Wiers, and Hartlaub(1992) used the same ideas to test the main effects in the presence of interaction. In the next chapter we consider their methodology for testing interaction which includes a section on our proposed corrected form for one of the statistics suggested in their paper together with the corrected values for the null distribution percentiles. In chapter 3 an analysis is made of one of the tests and two new procedures for testing interaction are proposed. A power study which assesses the performance of our proposed tests is included.

Chapter 2

Testing Interaction Using Latin Squares

2.1 Introduction

In this chapter we assume that we have a two-way layout with one observation per cell. The model that we entertain is

$$y_{ij} = \alpha_i + \beta_j + \gamma_{ij} + x_{ij} \qquad i = 1, 2, \ldots, I \quad j = 1, 2, \ldots, J \tag{2.1}$$

where α_i is the main effect of level i of factor U, β_j is the main effect of level j of factor V and γ_{ij} is the interaction effect arising from a combination of level i of factor U and level j of factor V and the x_{ij}'s are mutually independent, identically distributed, continuous random variables with common median θ. If we further assume $I = J = n$ and n is a power of a prime, then following Wolfe, Dean, and Hartlaub(1990), the properties of orthogonal Latin squares together with alignment and ranking of data may be used to develop tests for interaction in model (2.1).

2.2 Latin Square Structure

Latin square designs are commonly used when there are three sources of variation, two represented by the rows and columns of the design and the other, treatments for example, are represented by the letters. The following shows an example of a Latin square.

A Latin Square

A	B	C	D	E
B	C	D	E	A
C	D	E	A	B
D	E	A	B	C
E	A	B	C	D

In such a design every treatment appears once and only once in each row and once and only once in each column so that the rows, columns and treatments are orthogonal to each other. This orthogonal property of a Latin square may be used to set up a test for interaction in a two-way layout. In fact, since the interaction is orthogonal to rows and columns in a two-way layout, the 'treatment' component in a Latin square is part of the interaction component in a two-way layout. The properties of a set of orthogonal Latin squares, if one exists, may be used to identify the interaction component. For example, with $n = 5$, a set of four mutually orthogonal Latin squares may be used to partition the sum of squares due to interaction in a two-way layout as a set of 'treatment' components obtained from the set of orthogonal squares. To illustrate the ideas, consider the following set of four orthogonal Latin squares listed in Fisher and Yates Tables :

Four Orthogonal Latin Squares

ABCDE	ABCDE	ABCDE	ABCDE
BCEAD	CEDBA	DABEC	EDACB
CEDBA	DABEC	EDACB	BCEAD
DABEC	EDACB	BCEAD	CEDBA
EDACB	BCEAD	CEDBA	DABEC

For each Latin square design the rows and columns are orthogonal to the treatments represented by the letters. Therefore, if we identify for each square the treatment letters with the corresponding observations in the two-way layout, the treatment sum of squares will form part of the interaction (or error) sum of squares. Use of four Latin squares forming an orthogonal set will enable all sixteen degrees of freedom for error to be associated with four treatment sums of squares each with four degrees of freedom.

For each Latin square five interaction bands may be defined in such a way that each interaction band consists of observations relating to the letters corresponding to one treatment. For example, for the first Latin square the interaction bands are defined as follows:

Interaction Bands for Square 1

$$Band\,A : (X_{11}, X_{24}, X_{35}, X_{42}, X_{53})$$

$$Band\,B : (X_{12}, X_{21}, X_{34}, X_{43}, X_{55})$$

$$Band\,C : (X_{13}, X_{22}, X_{31}, X_{45}, X_{54})$$

$$Band\,D : (X_{14}, X_{25}, X_{33}, X_{41}, X_{52})$$

$$Band\,E : (X_{15}, X_{23}, X_{32}, X_{44}, X_{51})$$

The sum of squares for treatments in this Latin square is part of the interaction sum of squares. In the same way we have similar contributions from each of the other Latin squares. Therefore we have the following partition of the interaction sum of squares in the two-way layout.

$$\sum_{i=1}^{I}\sum_{j=1}^{J}(y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 = I_1 + I_2 + I_3 + I_4 \tag{2.2}$$

where I_l is the sum of squares of treatments relating to Latin square l.

2.3 The Non-parametric Tests

Thus we have a partition of the total sum of squares into six orthogonal sums of squares each with four degrees of freedom one each for rows and columns and four corresponding to the particular Latin squares used to partition the interaction(error)sum of squares. Three different procedures were investigated by Wolfe, Dean and Hartlaub(1990) for testing these interaction components. With the data arranged in a square $n \times n$ table, rows corresponding to the levels of factor U and columns corresponding to the levels of factor V, these procedures are defined as follows:

I- Procedure RC: The data are initially aligned with respect to factor U. This means that for each level of U, the median of the observation at that level is subtracted from each observation at that level. Having aligned the data with respect to U the data are ranked within each level of factor V. The ranks are then summed over the n interaction bands identified by the letters of one of the $n \times n$ Latin squares in the orthogonal set. The ranks are summed in this way using the interaction bands corresponding to each of the squares in the orthogonal set.

A Friedman type statistic can be obtained for each Latin square l as follows :

$$F_l = \sum_{k=1}^{n}(r_{lk} - n(n+1)/2)^2 \quad l = 1, 2, \ldots, (n-1) \tag{2.3}$$

where r_{lk} is the sum of ranks in interaction band k of Latin square l. Thus for each of the $n-1$ orthogonal Latin squares l a statistic F_l is obtained. The test statistic for this procedure is :

$$RC = maximum(F_1, F_2, ..., F_{n-1}) \qquad (2.4)$$

The idea of aligning and ranking is hopefully to remove the nuisance main effects and to provide a ranking method for assessing interaction. Aligning within the levels of U we try to remove the effect due to this factor. Similarly by ranking within the levels of V we try to adjust the data for the nuisance factor V and follow this with a Friedman-type ranking procedure to investigate the differences between the rank sums which provide a test for interaction. How successful these adjustments are in removing the main effects, will be investigated by determining the power of this procedure using a Monte-Carlo Simulation Study as described in section (2.6).

II- Procedure CR: In this procedure we carry out the alignment with respect to factor V and then rank the aligned data within the levels of the U factor. Similar Friedman-type statistics are defined using the rank sums for each interaction band and each Latin square as for procedure I. In this case, we define

$$G_l = \sum_{k=1}^{n} (s_{lk} - n(n+1)/2)^2 \qquad l = 1, 2, \ldots, (n-1) \qquad (2.5)$$

where s_{lk} is the sum of the ranks for interaction band k in Latin square l, and the overall test statistic is defined as

$$CR = maximum(G_1, G_2, ..., G_{n-1}) \qquad (2.6)$$

III- Procedure JR: In this test procedure no adjustments for the main effects are made. The data values are simply ranked from 1 to n^2 and, for each Latin square, a Friedman-type statistic is defined using the rank sums for the associated interaction bands. For the Latin square l define

$$H_l = \sum_{k=1}^{n} (t_{lk} - n(n^2+1)/2)^2 \qquad l = 1, 2, \ldots, (n-1) \qquad (2.7)$$

where t_{lk} is sum of ranks of interaction band k for Latin square l and then define

$$JR = maximum(H_1, H_2, ..., H_{n-1}) \tag{2.8}$$

to provide an alternative test statistic.

Wolfe et al.(1990) define H_l to be

$$H_l = \sum_{k=1}^{n}(t_{lk} - n^2(n^2+1)/2)^2 \tag{2.9}$$

which is different from the definition above and seems to be incorrect. The following reasoning justifies this. If we let r_{ij} denote the rank of the (ij)th entry in the two way table, then

$$E(t_{lk}) = nE(r_{ij}) \tag{2.10}$$

since t_{lk} is the sum of ranks of n observations. If we let $p(r_{ij})$ denote the probability that the rank of the (ij)th entry is r_{ij}, then

$$\begin{aligned} E(r_{ij}) &= \sum_{k=1}^{n}\sum_{l=1}^{n} r_{ij}p(r_{ij}) = 1/n^2\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij} = n^2(n^2+1)/2n^2 \\ &= (n^2+1)/2 \end{aligned} \tag{2.11}$$

therefore,

$$E(t_{lk}) = n(n^2+1)/2 \tag{2.12}$$

We believe that the revised definition of H_l is correct and that the results shown in Wolfe et al.(1990) are adrift by a constant. We shall return to this ambiguity later when discussing the null distribution of JR.

Notice that in applying the JR test there are no adjustments made for the main effects. Obviously the presence of any main effects will affect the performance of the test. This will be confirmed in the power comparisons considered later.

2.4 Null Distributions

The hypotheses of real interest are:

$H_0 : [\gamma_{ij} = 0,\ i = 1, 2, ..., I \text{ and } j = 1, 2, ..., J;\ \alpha_i\text{'s and } \beta_j\text{'s unrestricted}]$

$H_1 : [\gamma_{ij}\text{'s not all zero; } \alpha_i\text{'s and } \beta_j\text{'s unrestricted}]$

Since none of the test statistics RC, CR or JR is exactly distribution free under the above H_0 we consider the critical values for the more restrictive null hypothesis

$$\begin{aligned} H_0^* : [\ \alpha_1 &= \alpha_2 = ... = \alpha_I,\ \beta_1 = \beta_2 = ... = \beta_J \ and \\ \gamma_{11} &= \gamma_{12} = ... = \gamma_{1J} = \gamma_{21} = \gamma_{22} = ... = \gamma_{IJ}] \end{aligned}$$

Using Monte-Carlo simulations null distributions were obtained for the more restrictive case of no main effects. Under these conditions the distributions of RC and CR are identical. Hence, only a single set of critical values is required for the associated tests. To obtain the required critical values for the tests based on RC(CR) and JR, Monte Carlo simulation is employed under H_0^* and normally distributed errors. Independent sets of n^2 mutually independent standard normal variables are generated through a fortran program and the values of the proposed non-parametric statistics RC(CR) and JR are calculated for each of these $n \times n$ tables of data. Combining these values, empirical approximations are obtained to the null (H_0^*) distributions for each of the test statistics. Table 2.1 shows the observed empirical percentiles corresponding to upper tail probabilities immediately below and above (or equal to, as the case may be) the standard nominal significance levels of $.01, .025, .05$ and $.10$ for the new statistics RC(CR) and JR and design sizes $n = 3, 4, 5, 7, 8$ and 9. The values shown in the table were obtained by Wolfe et al. and are given in their paper. Later some corrections required to the table are described and modified values produced by these revised simulations are provided.

Table 2.1: **Simulated Null (H_0^*) Distribution Percentiles for the Statistics RC(CR) and JR Empirical percentiles closest to the Significance Levels $\alpha = .01, .025, .05, .10$ Table Sizes $n = 3, 4, 5, 7, 8, 9$.**

n	*RC(CR)*		*JR*	
	x	$P_0(RC \geq x)$	x	$P_0(JR \geq x)$
3	18	.092	689	.140
			693	.098
			695	.056
			701	.049
			713	.021
			729	.007
4	62	.132	4934	.102
	64	.093	4936	.098
	66	.083	5016	.050
	68	.046	5078	.027
	72	.028	5080	.024
	74	.018	5156	.010
	80	.003		

Table 2.1 continued

N	*RC(CR)*		*JR*	
	x	$P_0(RC \geq x)$	x	$P_0(JR \geq x)$
5	141.5	.102	22746	.100
	142.0	.098	23046	.050
	155.0	.050	23324	.025
	168.0	.025	23652	.010
	182.5	.010		
7	514.5	.100	229561	.100
	562.5	.050	231553	.050
	608.0	.025	233445	.025
	665.0	.010	235763	.010
8	868	.10	575988	.100
	942	.05	580096	.050
	1014	.025	583946	.025
	1104	.010	588904	.010
9	1336.0	.100	1298914	.100
	1445.5	.050	1306568	.050
	1543.5	.025	1314084	.025
	1677.0	.010	1323678	.010

2.5 Null Distributions Corrected

In the previous section it was pointed out that the correct form for the JR statistic for Latin square l is:

$$H_l = \sum_{k=1}^{n}(t_{lk} - n(n^2+1)/2)^2 = \sum_{k=1}^{n} t_{lk}^2 - \frac{n^3(n^2+1)^2}{4} \tag{2.13}$$

The form given by Wolfe et al. (1990) is

$$H_l = \sum_{k=1}^{n}(t_{lk} - n^2(n^2+1)/2)^2 = \sum_{k=1}^{n} t_{lk}^2 - \frac{n^5(n^2+1)^2}{4} \tag{2.14}$$

But on further investigation it seems that the null distribution of JR was obtained by neither of the formulas above. The formula giving rise to the tabulated null distribution seems to have been :

$$H_l = \sum_{k=1}^{n}(t_{lk} - n(n+1)/2)^2 = \sum_{k=1}^{n} t_{lk}^2 - \frac{n^2(n+1)^2}{4} \tag{2.15}$$

This would mean that the values have been inflated by the amount

$$\frac{n^3(n^2+1)^2}{4} - \frac{n^2(n+1)^2}{4} = \frac{n^3(n^4+n^2-2n)}{4} \tag{2.16}$$

Using the above formula, the null distribution of the JR statistic was corrected for each n. The following table shows the corrected form of the JR distribution together with the RC(CR) distribution.

Table 2.1 Corrected

Simulated Null (H_0^*) Distribution Percentiles
for the Statistics RC(CR) and JR
Empirical percentiles closest to the
Significance Levels $\alpha = .01, .025, .05, .10$
Table Sizes $n = 3, 4, 5, 7, 8, 9$.

N	*RC(CR)*			*JR*		
	x	$P_0(RC \geq x)$	$P_0^*(RC \geq x)$	x	$P_0(JR \geq x)$	$P_0^*(RC \geq x)$
3	18	.092	.0968	122	.140	.1438
				126	.098	.1002
				128	.056	.0568
				134	.049	.0495
				146	.021	.0210
				162	.007	.0069

The probabilities P_0 were obtained by Wolfe et al., except that those for JR have been scaled down to allow for the error. The probabilities P_0^* were obtained by us based on $100,000$ sets of n^2 mutually independent standard normal variables. There are some discrepancies. Our simulations show that for $n = 5$ the empirical percentile closest to $\alpha = .05$ for JR is 3032 as shown in table 3.1 on page 31. For some levels closer empirical percentiles can be obtained.

Table 2.1 Corrected continued

Simulated Null (H_0^*) Distribution Percentiles
for the Statistics RC(CR) and JR
Empirical percentiles closest to the
Significance Levels $\alpha = .01, .025, .05, .10$
Table Sizes $n = 3, 4, 5, 7, 8, 9$.

N	*RC(CR)*			*JR*		
	x	$P_0(RC \geq x)$	$P_0^*(RC \geq x)$	X	$P_0(JR \geq x)$	$P_0^*(RC \geq x)$
4	62	.132	.1343	710	.102	.1028
	64	.093	.0952	712	.098	.1001
	66	.083	.0847	792	.050	.0515
	68	.046	.0469	854	.027	.0265
	72	.028	.0293	856	.024	.0256
	74	.018	.0184	932	.010	.0109
	80	.003	.0032			
5	141.5	.102	.1013	2746	.100	.0992
	142.0	.098	.0976	3046	.050	.0480
	155.0	.050	.0495	3324	.025	.0232
	168.0	.025	.0244	3652	.010	.0091
	182.5	.010	.0107			

Table 2.1 Corrected continued

Simulated Null (H_0^*) Distribution Percentiles

for the Statistics RC(CR) and JR

Empirical percentiles closest to the

Significance Levels $\alpha = .01, .025, .05, .10$

Table Sizes $n = 3, 4, 5, 7, 8, 9$.

N	*RC(CR)*			*JR*		
	x	$P_0(RC \geq x)$	$P_0^*(RC \geq x)$	x	$P_0(JR \geq x)$	$P_0^*(RC \geq x)$
7	514.5	.100	.0992	20674	.100	.0972
	562.5	.050	.0496	22666	.050	.0481
	608.0	.025	.0242	24558	.025	.0239
	665.0	.010	.0094	26876	.010	.0094
8	868	.10	.0998	45556	.100	.1037
	942	.05	.0481	49664	.050	.0499
	1014	.025	.0255	53514	.025	.0260
	1104	.010	.0100	58472	.010	.0102
9	1336.0	.100	.0988	91690	.100	.1016
	1445.5	.050	.0507	99344	.050	.0513
	1543.5	.025	.0272	106860	.025	.0253
	1677.0	.010	.0104	116454	.010	.0104

2.6 Monte Carlo Simulation Power Comparisons

In order to compare the power properties of the three test statistics JR, RC, and CR through Monte Carlo Simulation we concentrated on the setting with $n = 5$. The following five patterns were used as the factor U and/or factor V main effects, and are to be regarded as nuisance factors when testing for interaction. They are as follows:

Main Effect 1	-1	$-.5$	0	.5	1
Main Effect 2	-1.5	$-.75$	0	.75	1.5
Main Effect 3	1.5	1	-1	.5	$-.5$
Main Effect 4	3	2	-2	1	-1
Main Effect 5	-1	0	2	0	-1

These main effects represent trends of different types and magnitudes. For example main effect 1 is a linear trend while main effect 5 is quadratic, and main effect 3 is cubic. Main effect 2 is 1.5 times main effect 1 and main effect 4 has twice the magnitude of main effect 3. In the simulations, a number of different interaction matrices γ_{ij} are used. These include the product interaction with γ_{ij} defined to be $\alpha_i\beta_j$ where α_i and β_j are the main effects of U and V respectively, and a class of interactions proposed by Martin(1980).

This class of interaction was suggested following a generalisation (Martin, 1980) of an experimental situation given by Vyvyan(1955). The experiment involved grafting of varieties of apple tree scions and rootstocks in which each apple tree variety was used as both scion and rootstock, resulting in a square design with similar variety rootstock and scion along the diagonal and different combinations in the off diagonal entries. The interaction component in such experiments may be partitioned into three components each assessing a certain attribute of the grafting process. These three features are called the equivalence feature, the union feature and the consistancy feature. The first represents a comparison of the combination of the same varieties versus combinations of different varieties; the second assesses the relative success of the various kinds of grafts within pairs of varieties and the third assesses the relative dominance of rootstock and scion in each pair of varieties. The interactions referred to as Martin interactions are such that certain components in this partition would lead to significant effects.

The following is an example of this type of interaction (Wolfe et al. 1993):

1.84	.92	1.51	2.28	−6.56
−.22	.08	2.08	2.59	−4.55
−.94	−.92	1.21	2.30	−1.64
.09	−1.20	.53	−7.65	8.23
−.77	1.11	−5.33	.48	4.51

To identify different models for the simulation results we shall use the notation $(u_p, v_q, 0)$ or (u_p, v_q, M) where $p, q = 1,2,3,4,5$ represent the case where the main effects present are u_p and v_q and the interactions are either all zero or given by the matrix M.

Table 2.2 shows the power of the JR, RC and CR tests using a 5 percent significance level for $n = 5$ and a selection of alternative models; $(0,0,0)$ represents the null case already included in Table 2.1.

Table 2.2: **Monte Carlo Simulated Rejection Rate Estimates for an Underlying Normal Distribution Significance Level $\alpha = .05$, $n = 5$**

	Tests				*Tests*		
Configurations	*JR*	*RC*	*CR*	*Configurations*	*JR*	*RC*	*CR*
(0,0,0)	.051	.049	.049	$(u_3, v_3, 0)$	.000	.022	.022
$(0, v_1, 0)$	.004	.030	.050	$(u_3, v_4, 0)$	.000	.005	.022
$(0, v_2, 0)$	.000	.019	.049	$(u_3, v_5, 0)$	.000	.023	.022
$(0, v_3, 0)$	.000	.023	.049	$(u_4, v_1, 0)$	.000	.030	.005
$(0, v_4, 0)$	.000	.005	.050	$(u_4, v_2, 0)$	.000	.019	.005
$(0, v_5, 0)$	.000	.024	.050	$(u_4, v_3, 0)$	.000	.022	.005
$(u_1, 0, 0)$	.004	.050	.030	$(u_4, v_4, 0)$	.000	.005	.005
$(u_2, 0, 0)$	.000	.050	.020	$(u_4, v_5, 0)$	.000	.023	.005
$(u_3, 0, 0)$	.001	.049	.022	$(u_5, v_1, 0)$	.000	.030	.023
$(u_4, 0, 0)$	.000	.049	.005	$(u_5, v_2, 0)$	.000	.019	.024
$(u_5, 0, 0)$	.000	.050	.023	$(u_5, v_3, 0)$	.000	.022	.023
$(u_1, v_1, 0)$	.000	.030	.031	$(u_5, v_4, 0)$	.000	.004	.024
$(u_1, v_2, 0)$	.000	.019	.031	$(u_5, v_5, 0)$	.000	.023	.023
$(u_1, v_3, 0)$	.000	.022	.030	$(0, 0, M)$	.429	.168	.136
$(u_1, v_4, 0)$	.000	.005	.030	$(0, v_1, M)$	.312	.131	.137
$(u_1, v_5, 0)$	.000	.024	.030	$(0, v_2, M)$	.223	.098	.135
$(u_2, v_1, 0)$	.000	.030	.018	$(0, v_3, M)$	.233	.115	.136
$(u_2, v_2, 0)$	.000	.019	.018	$(0, v_4, M)$	.016	.073	.137
$(u_2, v_3, 0)$	.000	.022	.019	$(0, v_5, M)$	.155	.152	.136
$(u_2, v_4, 0)$	.000	.005	.020	$(u_1, 0, M)$	.474	.169	.156
$(u_2, v_5, 0)$	.000	.024	.018	$(u_2, 0, M)$	.390	.167	.155
$(u_3, v_1, 0)$	.000	.030	.022	$(u_3, 0, M)$	.152	.168	.126
$(u_3, v_2, 0)$	.000	.019	.022	$(u_4, 0, M)$	.005	.169	.090

Table 2.2 Continued

Monte Carlo Simulated Rejection Rate Estimates for an Underlying Normal Distribution Significance Level $\alpha = .05$, $n = 5$

	Tests				*Tests*		
Configurations	*JR*	*RC*	*CR*	*Configurations*	*JR*	*RC*	*CR*
$(u_5, 0, M)$	.180	.166	.129	(u_3, v_3, M)	.064	.115	.124
(u_1, v_1, M)	.368	.131	.156	(u_3, v_4, M)	.002	.074	.125
(u_1, v_2, M)	.288	.099	.158	(u_3, v_5, M)	.039	.153	.126
(u_1, v_3, M)	.201	.117	.157	(u_4, v_1, M)	.003	.132	.090
(u_1, v_4, M)	.011	.074	.156	(u_4, v_2, M)	.001	.100	.089
(u_1, v_5, M)	.187	.154	.157	(u_4, v_3, M)	.001	.116	.090
(u_2, v_1, M)	.339	.130	.152	(u_4, v_4, M)	.000	.073	.089
(u_2, v_2, M)	.270	.099	.154	(u_4, v_5, M)	.001	.153	.088
(u_2, v_3, M)	.138	.117	.154	(u_5, v_1, M)	.111	.131	.129
(u_2, v_4, M)	.005	.074	.155	(u_5, v_2, M)	.070	.100	.131
(u_2, v_5, M)	.151	.253	.154	(u_5, v_3, M)	.092	.116	.131
(u_3, v_1, M)	.116	.131	.126	(u_5, v_4, M)	.006	.073	.130
(u_3, v_2, M)	.075	.099	.124	(u_5, v_5, M)	.101	.152	.131

When all the three effects are zero, the rejection rates provide an evaluation of the type I error. We can see that the values are close to the significance level of five percent. When the interaction effect is zero and at least one of the main effects is not zero, the rates provide us with information about how well the various nominal levels are maintained over the broader null hypothesis H_0^*. For example when interaction effect and main effect U are zero, the power of the JR test is close to zero and substantially smaller than the nominal type I error. For the RC test the powers are also less than the nominal significance level of five percent. In fact, the values vary from .030 to .005. As may be seen, the larger the main effect V, the smaller the type I error. The explanation for this is that when the main effect V is present the null distributions of RC are shifted to the left. The larger the main effect is the larger the magnitude of the shift. Obviously for RC the ranking is not having the effect of removing the main effect of V. The same feature is evident when U is non-zero and V is zero and it is clear that just ranking the data is not removing the influence of the main effects. For the CR tests, where the model is of the form $(0, v_2, 0)$, the powers are close to the significance level which implies that the effect of the non-zero V main effect is being adequately removed by the aligning which is carried out within the levels of V. In contrast to this, the CR tests when the model is of the form $(u_p, 0, 0)$ behave in the same way as the RC tests for the model$(0, v_2, 0)$ as was to be expected.

When the interaction effect is non-zero, but the main effects are zero the rejection rates are estimates of the powers of the tests in the absence of main effects. In this case we can see that the JR test performs much better than either RC or CR. For example, compare the value of .423 for the JR test with .173 and .138 for the RC and CR tests respectively for the case $(0, 0, M)$. When the interaction effect and also main effects are non-zero, the simulated rejection rates, are, of course, estimates of the powers of the associated tests in the presence of the nuisance main effects. As we can see in situations where the main effects are small, the statistic JR outperforms CR and RC but as soon as any main effect becomes sizable JR's power drops and this conforms with our expectations since JR does not make any adjustments for the main effects. It also appears that aligning is more effective than ranking in adjusting for the main effects. This is apparent since RC performs better than CR when the U effect is large. Similarly we notice that CR performs better than RC when the V effect is large.

Chapter 3

Two Alternative Non-parametric Tests

3.1 Introduction

In chapter 2 the null distributions of the test statistics for testing interaction effects were obtained under the hypothesis of no main effects and no interaction effects. This would mean that when testing for interaction effects we are implicitly assuming that alignment and ranking of the data virtually removes any main effects that are present so that we will be dealing with the null distributions under no main effects and no interaction effects. In this chapter we are going to investigate whether the technique applied is really effective and whether alignment and ranking do adjust the data to account for the possible presence of main effects prior to conducting a rank test for interactions.

3.2 Studying the Critical Values

To investigate whether the null distributions of the test statistics are affected by the presence of main effects, different combinations of the same five patterns in section (2.6) were chosen for the main effects. A series of simulations was carried out to obtain the null distributions of the statistics in the presence of the nuisance main effects.

To find the null distribution under the various combinations of main effects, some modifications were made to the fortran program. For each combination of the two main effects the five percent point of the null distribution was obtained based on a simulation consisting of $100,000$ sets of data. The five percent critical points of the null distribution for each combination of the two main effects are shown in Table 3.1.

Table 3.1: **Five Percent Critical Points for the JR, RC and CR Null Distributions for some combinations of main effects, $N = 5$**

Configurations	*JR Statistic*	*RC Statistic*	*CR Statistic*
(0,0)	3032	154.5	155
$(0, v_1)$	2218	146.5	154.5
$(0, v_2)$	1590	140	155
$(0, v_3)$	1802	141.5	154.5
$(0, v_4)$	770	122	154.5
$(0, v_5)$	1626	142.5	155
$(u_1, 0)$	2206	154	146.5
$(u_2, 0)$	1594	154	139.5
$(u_3, 0)$	1810	154	142
$(u_4, 0)$	766	154.5	122
$(u_5, 0)$	1626	155	143
(u_1, v_1)	1730	146.5	146.5
(u_1, v_2)	1330	140	146.5
(u_1, v_3)	1474	142	146.5
(u_1, v_4)	690	122	146.5
(u_1, v_5)	1346	142.5	146.5

Table 3.1 continued

Five Percent Critical pcnts
for the JR, RC and CR Null Dstributions
for some combinations of main effects, $N = 5$

Configurations	*JR Statistic*	*RC Statisic*	CR Statistic
(u_2, v_1)	1330	146.5	139.5
(u_2, v_2)	1090	139.5	138.5
(u_2, v_3)	1180	142	139.5
(u_2, v_4)	622	122	139.5
(u_2, v_5)	1098	143.5	140
(u_3, v_1)	1472	146.5	142
(u_3, v_2)	1176	139.5	142
(u_3, v_3)	1282	142	142
(u_3, v_4)	644	122	142
(u_3, v_5)	1184	142.5	141.5
(u_4, v_1)	690	146.5	122
(u_4, v_2)	622	139.5	122
(u_4, v_3)	646	141.5	121.5
(u_4, v_4)	474	122	122
(u_4, v_5)	630	142.5	122
(u_5, v_1)	1352	146.5	142.5
(u_5, v_2)	1098	139.5	142.5
(u_5, v_3)	1186	142	143.5
(u_5, v_4)	628	122	142.5
(u_5, v_5)	1118	142.5	142.5

As can be seen the five percent points obtained for the JR and RC statistics under zero main effects and zero interaction were 3032 and 155 respectively. Obviously from the values given in Table 3.1, the two tests suggested are not stable under the presence of main effects. The null distribution changes substantially for different configurations of main effects. The five percent points for the JR statistic under various configurations are much lower than 3032. For the main effect combination (u_4, v_4), the critical point for the JR statistic is 474 which shows a substantial shift. We can see that the larger the size of the main effects, the larger the reduction in the critical point. Thus the power values are affected by this instability and would be greater than those listed in the paper by Wolfe et al. where the approximate critical point values are used. Even for the RC statistic, the five percent point goes down to as low as 122. Obviously the null distributions are shifting downwards due to the presence of main effects. They are certainly highly dependent on the magnitudes of any main effects which might be present. But using 3032 and 155 corresponding to the critical values with no main effects implies that the null distributions remain fixed. The power comparisons are conducted under the assumption that the tests are stable under various configurations of main effects, which is clearly not true. To see how the power values change, when comparisons are made to the specific critical values, simulations based on 100,000 sets of data were carried out using the new critical points. Of course, in practice it would not be possible to know which critical values were the ones to use. The following computations were carried out to illustrate the effect that the presence of main effects can have on the apparent power of the tests involving aligning and ranking.

Power values were obtained for various configurations using the following interaction matrix :

.33	.94	1.2	1.48	−3.94
−.59	.29	1.04	2.62	−3.35
−.90	−.94	.47	2.38	−2.89
1.29	−.94	.91	−6.26	5.00
−.13	−1.22	−3.62	−.21	5.18

The results of the simulation studies with this interaction matrix and various combinations of main effects u and v are shown in Table 3.2, which gives the rejection rates for JR and RC using the fixed critical values 3032 and 155 respectively and using the variable critical values which would be appropriate if the magnitudes of the main effects present were known.

Table 3.2: **Monte Carlo Simulated Rejection Rate Estimates for an Underlying Normal Distribution**

Significance Level $\alpha = .05$, $n = 5$

Configurations	*JR*	*JR corrected*	*RC*	*RC corrected*	*CR*	*CR corrected*
$(0, 0)$	.380	.380	.141	.141	.144	.144
$(0, v_1)$	.249	.815	.132	.184	.142	.145
$(0, v_2)$	.154	.992	.123	.218	.144	.144
$(0, v_3)$	.136	.937	.084	.159	.144	.147
$(0, v_4)$	.004	1.000	.055	.270	.145	.147
$(0, v_5)$	.095	.988	.086	.150	.144	.144
$(u_1, 0)$	.312	.861	.140	.148	.152	.207
$(u_2, 0)$	.194	.994	.138	.146	.156	.267
$(u_3, 0)$	.131	.967	.139	.146	.103	.187
$(u_4, 0)$	.003	1.000	.139	.141	.030	.193
$(u_5, 0)$	.073	.982	.139	.139	.036	.077
(u_1, v_1)	.247	.992	.132	.183	.152	.206
(u_1, v_2)	.181	1.000	.124	.217	.152	.207
(u_1, v_3)	.078	.991	.082	.152	.153	.207
(u_1, v_4)	.002	1.000	.056	.272	.153	.208
(u_1, v_5)	.081	.998	.088	.151	.153	.207
(u_2, v_1)	.166	1.000	.131	.183	.153	.265
(u_2, v_2)	.119	1.000	.122	.220	.155	.278
(u_2, v_3)	.036	1.000	.083	.154	.156	.268
(u_2, v_4)	.001	1.000	.056	.274	.156	.269
(u_2, v_5)	.043	1.000	.087	.148	.156	.265

Table 3.2 Continued

Monte Carlo Simulated Rejection Rate Estimates

for an Underlying Normal Distribution

Significance Level $\alpha = .05$, $n = 5$

Configurations	*JR*	*JR corrected*	*RC*	*RC corrected*	*CR*	*CR corrected*
(u_3, v_1)	.068	.997	.133	.185	.106	.189
(u_3, v_2)	.029	1.000	.123	.219	.103	.187
(u_3, v_3)	.045	.999	.082	.153	.104	.187
(u_3, v_4)	.001	1.000	.055	.274	.103	.188
(u_3, v_5)	.023	1.000	.087	.151	.104	.194
(u_4, v_1)	.001	1.000	.134	.186	.029	.193
(u_4, v_2)	.000	1.000	.122	.220	.030	.194
(u_4, v_3)	.001	1.000	.083	.159	.031	.201
(u_4, v_4)	.000	1.000	.055	.271	.030	.193
(u_4, v_5)	.000	1.000	.087	.149	.030	.192
(u_5, v_1)	.049	.998	.131	.182	.036	.079
(u_5, v_2)	.024	1.000	.123	.220	.037	.079
(u_5, v_3)	.012	.999	.083	.152	.036	.076
(u_5, v_4)	.000	1.000	.057	.273	.036	.078
(u_5, v_5)	.013	1.000	.087	.150	.037	.079

As can be seen the power values for the JR test based on the variable critical values are much higher than those using the fixed critical point. This is due to the fact that the null distribution of JR changes substantially with the addition of main effects and using the critical point corresponding to zero main effects for all the different configurations of main effects will result in low values for the power. Thus comparing the JR power values based on fixed critical point with those of the other tests would not provide us with a fair comparison. This should not come to us as a surprise since the JR test does nothing to remove the main effects whatsoever and obviously the main effects do affect the null distribution and as a result the power values for the tests on interaction. In the next chapter we will suggest some changes to the JR test to improve the deficiency noticed.

Now turning our attention to the RC test we see that the corrected critical values are different from the fixed critical value obtained by setting the main effects and interaction to zero but these differences are not as large as they were for the the JR test. We can see improvement on the power values but not as substantial as they were for the JR test. The reason is of course that the RC test does attempt to remove the nuisance factors by aligning and ranking but these adjustments are not completely effective in removing the main effects.

3.3 Conclusions

In order to test the interaction in the presence of the main effects, three different testing procedures were proposed. For two of them alignment and ranking are used to adjust the data for the main effects before obtaining a Friedman-type statistic based on the interaction bands. For the third procedure the whole data are ranked and a Friedman-type statistic is obtained. Analysis of critical points shows that the null distribution is affected by the presence of main effects and the critical points are shifted to the left which leads to a loss of power. The greater the magnitude of the main effects, the more the critical points are shifted, and the greater the power loss. The reduction in power is much more severe for the JR test and this is conceivable since this test does not involve any adjustment to the main effects.

3.4 Alternative Tests for Interaction

In this section we are going to propose two tests for interaction based on the work done so far and then we will follow that with a power comparison. First we show the power results for an interaction effect matrix which acts along the interaction bands. The actual interaction matrix is:

$$\begin{matrix} -3 & 2 & 3 & -1 & -1 \\ 2 & -1 & -3 & 1 & 1 \\ 0 & 1 & 2 & -3 & 0 \\ 1 & 2 & -1 & 0 & -2 \\ 0 & -4 & -1 & 3 & 2 \end{matrix}$$

The values in the above interaction matrix have been chosen in such a way to magnify the differences among the interaction bands. This is done with a view to getting "some feel" for the best power performance that one could expect with the banding technique(Wolfe et al. 1990) and would facilitate the power comparison between the different proposed procedures for testing interaction based on the orthogonal Latin squares methodology.

The following table shows the power calculations for the three tests JR, RC and CR applying the above interaction matrix.

Obviously the three tests are dependent on the main effects though not to the same extent. The power values for the JR test are highly variable and heavily dependent on the main effects. With no main effects the power for JR test is .975 compared with the

Table 3.3: **Monte Carlo Simulated Rejection Rate Estimates for an Underlying Normal Distribution Significance Level $\alpha = .05$, $n = 5$**

	Tests				*Tests*		
Configurations	*JR*	*RC*	*CR*	*Configurations*	*JR*	*RC*	*CR*
(0,0,M)	.975	.634	.686	(u_2, v_3, M)	.498	.599	.583
$(0, v_1, M)$	.871	.523	.687	(u_2, v_4, M)	.026	.483	.583
$(0, v_2, M)$	.664	.470	.688	(u_2, v_5, M)	.373	.558	.584
$(0, v_3, M)$	.818	.601	.689	(u_3, v_1, M)	.635	.528	.710
$(0, v_4, M)$	.081	.481	.687	(u_3, v_2, M)	.382	.470	.710
$(0, v_5, M)$	.752	.560	.686	(u_3, v_3, M)	.513	.601	.709
$(u_1, 0, M)$	.910	.635	.639	(u_3, v_4, M)	.022	.481	.711
$(u_2, 0, M)$	.756	.634	.583	(u_3, v_5, M)	.457	.558	.708
$(u_3, 0, M)$	.848	.631	.710	(u_4, v_1, M)	.038	.525	.571
$(u_4, 0, M)$	.096	.636	.577	(u_4, v_2, M)	.013	.469	.576
$(u_5, 0, M)$	.664	.635	.645	(u_4, v_3, M)	.024	.601	.574
(u_1, v_1, M)	.718	.524	.640	(u_4, v_4, M)	.000	.481	.574
(u_1, v_2, M)	.480	.470	.639	(u_4, v_5, M)	.038	.555	.574
(u_1, v_3, M)	.685	.601	.640	(u_5, v_1, M)	.438	.525	.647
(u_1, v_4, M)	.050	.480	.637	(u_5, v_2, M)	.249	.472	.645
(u_1, v_5, M)	.591	.556	.636	(u_5, v_3, M)	.407	.601	.648
(u_2, v_1, M)	.514	.524	.583	(u_5, v_4, M)	.018	.480	.647
(u_2, v_2, M)	.295	.474	.581	(u_5, v_5, M)	.354	.558	.646

values .634 for RC test and .686 for CR test. Thus in the absence of main effects the JR test is much more powerful than the other two tests. Even when the main effects are small in magnitude the JR test is still more efficient. For main effects (u_1, v_1) the power for the JR test is .718 compared with the values .524 for the RC test and .640 for the CR test. But with increase in the magnitude of the main effects the three tests show loss of power, however the drop in power is much larger for the JR test. For main effects (u_4, v_4) the power for the JR test is only .0003 compared with the values .481 for RC and .574 for the CR test.

3.4.1 Modification of the JR test using medians

Because of the way the JR test is affected by main effects we decided to adjust the data for the main effects before applying the JR test. Adjustments can be made by aligning the rows and columns either through means or through medians. Alignments through medians is done by subtracting from each observation the corresponding row median. Having adjusted the data for row effects we can now make adjustments for column effects. We subtract from each observation the corresponding column median. Having adjusted the data for the main effects we can now rank the adjusted data and apply the JR test. We refer to this modified test using alignment for medians as the JR(med) test.

To see how effective the alignments are in stabilising the critical points, we simulated the null distributions for various combinations of main effects and obtained the critical points for the five percent significance level.

Examination of the critical points shows that the values are much more stable. Although the values are still dependent on the main effects to some extent, the dependency is much less than before. When there are no main effects, the critical point is 3823 and we can see all the other values are smaller than this. Obviously there is a slight shift of the null distribution but it is much less than before.

Table 3.4: **Critical Points for the JR Null Distribution after Alignments with Medians**

Significance Level α=.05, $n = 5$

Configurations	*Critical Points*	*Configuration*	*Critical Points*
(0,0)	3864.5	(u_2, v_3)	3416
$(0, v_1)$	3860	(u_2, v_4)	3423
$(0, v_2)$	3864	(u_2, v_5)	3433.5
$(0, v_3)$	3852	(u_3, v_1)	3485.5
$(0, v_4)$	3854	(u_3, v_2)	3478
$(0, v_5)$	3863	(u_3, v_3)	3474
$(u_1, 0)$	3599.5	(u_3, v_4)	3473
$(u_2, 0)$	3414.5	(u_3, v_5)	3478.5
$(u_3, 0)$	3478.5	(u_4, v_1)	3038.5
$(u_4, 0)$	3025.5	(u_4, v_2)	3044
$(u_5, 0)$	3517.5	(u_4, v_3)	3033.5
(u_1, v_1)	3603.5	(u_4, v_4)	3032
(u_1, v_2)	3605.5	(u_4, v_5)	3038
(u_1, v_3)	3611.5	(u_5, v_1)	3512.5
(u_1, v_4)	3609.5	(u_5, v_2)	3515
(u_1, v_5)	3608	(u_5, v_3)	3524
(u_2, v_1)	3424	(u_5, v_4)	3510
(u_2, v_2)	3413.5	(u_5, v_5)	3509.5

3.4.2 Modification of the JR test using means

A second procedure is proposed for testing interaction by aligning the rows and columns through means before applying a Friedman-type test to the interaction bands. To adjust for the row effect, the corresponding row mean is subtracted from each observation. Having adjusted the data for the row effects, we subtract from each adjusted observation the corresponding column mean. These adjustments are equivalent to obtaining the residuals in a two way analysis of variance which are free from the main effects. These residuals are, then, ranked and the Friedman-type test statistic is obtained for each Latin square. We refer to this modified test using alignment for means as the JR(mean) test.

Again to see the effectiveness of the proposed procedure the null distributions for this alternative statistic JR(mean) were simulated for combinations of main effects and the five percent critical points obtained. The results are shown in Table 3.5.

The new critical values are relatively stable. In fact it can be shown that alignments through means remove the main effects since this is equivalent to estimating the main effects using least squares and then analysing the resulting residuals.

Table 3.5: **Critical Points for the JR Null Distribution after Alignments with Means**

Significance Level α=.05, $n = 5$

Configurations	*Critical Points*	*Configurations*	*Critical Points*
(0,0)	4094	(u_2, v_3)	4088
$(0, v_1)$	4094	(u_2, v_4)	4102
$(0, v_2)$	4108	(u_2, v_5)	4108
$(0, v_3)$	4094	(u_3, v_1)	4094
$(0, v_4)$	4100	(u_3, v_2)	4096
$(0, v_5)$	4096	(u_3, v_3)	4094
$(u_1, 0)$	4086	(u_3, v_4)	4102
$(u_2, 0)$	4096	(u_3, v_5)	4096
$(u_3, 0)$	4092	(u_4, v_1)	4106
$(u_4, 0)$	4094	(u_4, v_2)	4112
$(u_5, 0)$	4102	(u_4, v_3)	4098
(u_1, v_1)	4094	(u_4, v_4)	4098
(u_1, v_2)	4100	(u_4, v_5)	4104
(u_1, v_3)	4094	(u_5, v_1)	4094
(u_1, v_4)	4106	(u_5, v_2)	4094
(u_1, v_5)	4086	(u_5, v_3)	4108
(u_2, v_1)	4096	(u_5, v_4)	4086
(u_2, v_2)	4086	(u_5, v_5)	4102

3.5 A Power Comparison

To see how well our two proposed tests perform, a power study was carried out comparing JR, RC and the two new tests JR(med) and JR(mean). The interaction matrix applied is the one introduced in section (3.4). The results of these simulations for the various combinations of main effects are given in Table 3.6.

Table 3.6: **Monte Carlo Simulated Rejection Rate Estimates for an Underlying Normal Distribution Significance Level $\alpha = .05$, $n = 5$**

	Tests				
Configurations	*JR*	*JR(med)*	*JR(mean)*	*RC*	*CR*
(0,0,M)	.975	.671	.687	.634	.686
$(0, v_1, M)$	.871	.671	.686	.523	.687
$(0, v_2, M)$	.664	.672	.688	.470	.688
$(0, v_3, M)$	.818	.672	.686	.601	.689
$(0, v_4, M)$	.081	.671	.686	.481	.687
$(0, v_5, M)$	.752	.670	.686	.560	.686
$(u_1, 0, M)$	.910	.604	.688	.635	.639
$(u_2, 0, M)$	.756	.534	.687	.634	.583
$(u_3, 0, M)$	.848	.622	.686	.631	.710
$(u_4, 0, M)$	.096	.443	.688	.636	.577
$(u_5, 0, M)$	.664	.566	.687	.635	.645
(u_1, v_1, M)	.718	.605	.689	.524	.640
(u_1, v_2, M)	.480	.606	.688	.470	.639
(u_1, v_3, M)	.685	.606	.690	.601	.640
(u_1, v_4, M)	.050	.603	.687	.480	.637
(u_1, v_5, M)	.591	.600	.686	.556	.636
(u_2, v_1, M)	.514	.534	.687	.524	.583
(u_2, v_2, M)	.295	.535	.689	.474	.581

Table 3.6 Continued

Monte Carlo Simulated Rejection Rate Estimates for an Underlying Normal Distribution Significance Level $\alpha = .05$, $n = 5$

Configurations	*Tests*				
	JR	*JR(med)*	*JR(mean)*	*RC*	*CR*
(u_2, v_3, M)	.498	.536	.688	.599	.583
(u_2, v_4, M)	.026	.537	.687	.483	.583
(u_2, v_5, M)	.373	.535	.689	.558	.584
(u_3, v_1, M)	.635	.621	.688	.528	.710
(u_3, v_2, M)	.382	.623	.687	.470	.710
(u_3, v_3, M)	.513	.622	.687	.601	.709
(u_3, v_4, M)	.022	.623	.688	.481	.711
(u_3, v_5, M)	.457	.622	.688	.558	.708
(u_4, v_1, M)	.038	.441	.686	.525	.571
(u_4, v_2, M)	.013	.442	.688	.469	.576
(u_4, v_3, M)	.024	.444	.690	.601	.574
(u_4, v_4, M)	.000	.442	.688	.481	.574
(u_4, v_5, M)	.038	.442	.687	.555	.574
(u_5, v_1, M)	.438	.568	.687	.525	.647
(u_5, v_2, M)	.249	.567	.690	.472	.645
(u_5, v_3, M)	.407	.567	.689	.601	.648
(u_5, v_4, M)	.018	.569	.688	.480	.647
(u_5, v_5, M)	.354	.568	.691	.558	.646

Comparing the power values we can see that the JR test aligned through means, JR(mean), is the most stable among the five tests and has the best overall power performance. It maintains a power of .69 in the absence of main effects and also in the presence of any combinations of main effects. For most combinations of main effects it shows higher powers than the other tests. In the presence of main effects (u_4, v_4) the power of JR(mean) is .6877 compared with .0003 for JR, .4425 for JR(median), .4813 for RC and .5739 for CR. While the JR(mean) is robust in the presence of main effects the JR(median) is not. We can see that JR(median) is robust in the presence of v effects only. When there is no u effect the power variation is from .670 to .672 for different values of the v effect which is well within the simulation error. However with a u effect and no v effect in the model the power values change from .443 to .622. This feature is evident for all sets of power values with fixed u effect over the variation in the v effect. This behavior of JR(median) is due to the order of alignments applied for removing the main effects. Alignment was used for the u effect first and then applied to adjust the data for the v effects. Obviously this upsets the previous alignment for the u effect. This upset due to the order of alignments does not occur for the JR(mean) test because alignments through means simply reduces the data to the residuals.

Power comparisons were also carried out under multiplicative interaction. The results have not been shown since the powers obtained were all less than 10 percent for all the tests considered here. The low power performance is due to the intrinsic nature of the Latin square procedure. With this type of interaction the model is

$$y_{ij} = \alpha_i + \beta_j + \alpha_i\beta_j + x_{ij} \qquad i = 1, 2, \ldots, I \quad j = 1, 2, \ldots, J \tag{3.1}$$

Here the interaction effect for each cell is the product of the corresponding main effects. The methodology, based on Latin squares picks up interactions along the interaction bands making distinctions between various bands. Because of the way the observations are included in each band, each interaction band carries the effects of all levels of both factors in a product form and thus the rank sums for the bands tend to be similar resulting in a low value for the test statistics. As a result, the procedure fails to detect multiplicative interaction.

We noticed that for the Martin type of interaction the powers were also low and only for the interaction matrix acting along the bands did we have the powers raised to a certain extent. Thus it seems that the proposed methodology is capable of detecting only certain type of interactions i.e. interactions that are compatible with the interaction bands in one of the orthogonal Latin squares and attempts in modifying the tests do not seem to result in more powerful tests. We noticed that even for situations where interactions are along the bands the maximum power observed was .69 which shows that the methodology proposed is not efficient.

3.6 Conclusions

The three non-parametric tests proposed by Wolfe, Dean, Hartlaub were studied and found to be dependent on the main effects. Analysis of the Joint Rank Procedure revealed that the critical values were highly variable and heavily dependent on the main effects. It is shown that alignment through medians prior to ranking stabilizes the critical points, and alignment through the means removes the main effects. Based on this study two non-parametric procedures are proposed for testing the interaction. Our power study revealed that for interaction effect matrices acting close to the interaction bands, the JR mean aligned test has the best overall performance among all the tests studied under the normal distribution. For product interaction all the tests perform poorly showing powers less than 10 percent. We think that the poor performance is due to the structure of the tests. Latin square structure is not capable of detecting this form of interaction because of the way this interaction spreads over the two-way design. In fact the same argument can be used about Martin-type interaction effect matrices with less severity. In brief, all the tests mentioned here based on orthogonal Latin squares properties can be effective for the detection of interaction only if the interaction spreads along the interaction bands. Wolfe et al.(1990) propose "work on the development of ways to tie the selection of a banding scheme (or other clustering mechanism for the data)to the specific type of interaction deemed most likely to be present and/or to be most important in the problem under consideration". But practical applications giving rise to such circumstances are few and far between. For the multiplicative interaction which is common in practice or assessment of interaction in general the methodology has low power of detection.

3.7 Further Work

There are different ways of extending this work:

1. To compare the efficiency of the proposed tests with that of Tukey's and also De Kroon and van der Laan's test on rank interaction, one may embark upon a power study involving all the tests on interaction with only a single replicate discussed so far.

2. One could investigate the efficiency of the proposed procedures when the errors are non-normal.

3. Latin square structure has also been used to test the main effects in the presence of interaction (Wolfe et al., 1992). Similar analysis to the work done so far might be undertaken to see if a more efficient test procedure can be obtained for testing the main effects in a two-way layout with one observation per cell.

Chapter 4

The Use of Ranked Data in Blocked Factorial Experiments

4.1 Introduction

This research originated from an experiment at Cambridge Laboratory, Cereals Research Department, John Innes Centre in which the effects of elevated atmospheric concentration of carbon dioxide and a nitrate fertiliser on the development of disease on plants were investigated.

The aim of the experiment was to see if the amount of disease which developed on plants depended on the atmospheric concentration of carbon dioxide. They were also interested to know if the amount of nitrate fertilizer supplied to the plants affected the development of disease, either separately from carbon dioxide or in an interaction with it.

There were two levels of carbon dioxide concentration and five levels of nitrate fertilizer. Thus we have a factorial experiment with one factor, carbon dioxide, at two levels and the other, nitrate fertilizer, at five levels. They could use up to a maximum of twelve replicates for each combination of treatment levels. It is expected that the design would involve all the ten treatment combinations using twelve replicates. The amount of disease on plants grown under the different treatment combinations would be the response variable.

Consider first how one would analyse such a design if the response could be measured on a continuous scale. A standard factorial analysis would produce an analysis of variance dividing the total sum of squares into the main effects and interaction components as follows:

Source of variation	Degrees of Freedom
Main Effect CO_2	1
Main Effect NO_3	4
Interaction $CO_2 \times NO_3$	4
Residual	110
Total	119

Since the second factor is a quantitative factor it would be possible to divide the effects into components testing the shape of the response curve(or surface) i.e. we could define linear, quadratic, cubic, and quartic components using orthogonal polynomials. This is particularly straight forward if the levels of NO_3 are equally spaced on an interval, since the coefficients of the relevant contrasts may be found in standard tables. These coefficients define contrasts which correspond to the particular shape of the response curve. Thus the full partition of the total sum of squares would then be:

Source of variation	D.F	
Main Effect of CO_2	1	
Main Effect of NO_3	4	
Linear Component of NO_3		1
Quadratic Component of NO_3		1
Cubic Component of NO_3		1
Quartic Component of NO_3		1
Interaction CO_2 X NO_3	4	
CO_2 by Linear NO_3		1
CO_2 by Quadratic NO_3		1
CO_2 by Cubic NO_3		1
CO_2 by Quartic NO_3		1
Residual	110	
Total	119	

The residual sum of squares could be further split up to test other features of the design, such as blocks, if these had been incorporated in the design.

4.2 General Model

Now to generalise the problem, let us assume that we have a factorial experiment with two factors U and V with the number of levels I and J respectively. We further assume that the experiment is run in a randomised block design and the number of blocks used is M. Thus the general model we are considering is:

$$y_{ijm} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_m + \epsilon_{ijm} \tag{4.1}$$

where y_{ijm} is the observation at level i of factor U and level j of factor V in block m, α_i is the effect of level i of factor U, β_j is the effect of level j of factor V, $(\alpha\beta)_{ij}$ is the interaction effect due to the combination of level i of factor U and level j of factor V, γ_m is the effect of block m and ϵ_{ijm} is the error term associated with the observation y_{ijm}. The ϵ_{ijk}'s are assumed to be identically and independently distributed continuous random variables. To be able to use ANOVA we need to add the normality assumption as well. The partition of the total sum of squares is:

$$\begin{aligned}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}(y_{ijk}-\bar{y}_{...})^2 &= \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}(\bar{y}_{i..}-\bar{y}_{...})^2+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}(\bar{y}_{.j.}-\bar{y}_{...})^2 \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}(\bar{y}_{..m}-\bar{y}_{...})^2 \\ &+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}(y_{ijm}-\bar{y}_{ij.}-\bar{y}_{..m}+\bar{y}_{...})^2 \qquad (4.2)\end{aligned}$$

The first sum of squares is for factor U and is denoted by $SS(U)$, the second sum of squares is for factor V and is denoted by $SS(V)$, the third sum of squares is for the interaction between U and V, and is denoted by $SS(U \times V)$, the fourth sum of squares is for the blocks and is denoted by $SS(B)$ and finally the last sum of squares is for the random error and is denoted by $SS(E)$. The total sum of squares is denoted by $SS(T)$. Using the above notation we can now show in the table 4.1 the analysis of variance for the model under study. The main effect and interaction sums of squares are further partitioned into their orthogonal components for quantitative factors to determine the shape of the response as illustrated earlier. The tests on components of main effects and interaction are carried out by contrasts. A typical contrast can be represented as:

$$T = \sum_{i=1}^{K} c_i t_{i.} \qquad (4.3)$$

here K is the number of treatment combinations, $t_{i.}$ is the total on treatment i, and c_i is the contrast coefficient relating to treatment i and there is a constraint on c_i's, $\sum_{i=1}^{K} c_i = 0$. We also assume $\sum_{i=1}^{k} c_i^2 = 1$.

Table 4.1: **Table of Analysis of Variance for a Factorial Experiment with Two Factors Performed in a Randomised Block Design**

Source of Variation	Sum of Squares	D.F	Mean Squares	F-Ratios
Factor U	$SS(U)$	$I-1$	$MS(U) = \frac{SS(U)}{I-1}$	$\frac{MS(U)}{MS(E)}$
Factor V	$SS(V)$	$J-1$	$MS(V) = \frac{SS(V)}{J-1}$	$\frac{MS(V)}{MS(E)}$
Blocks	$SS(B)$	$M-1$	$MS(B) = \frac{SS(B)}{M-1}$	$\frac{MS(B)}{MS(E)}$
Interaction $U \times V$	$SS(U \times V)$	$(I-1)(J-1)$	$MS(U \times V) = \frac{SS(U \times V)}{(I-1)(J-1)}$	$\frac{MS(U \times V)}{MS(E)}$
Error	$SS(E)$	$(IJ-1)(M-1)$	$MS(E) = \frac{SS(E)}{(IJ-1)(M-1)}$	
Total	$SS(T)$	$IJM-1$		

Each contrast is used to test a particular component of one of the main effects or interaction. For instance, one of them would test the linear component of factor U and another would test the linear by linear component of the interaction. The contrasts are orthogonal and with the assumption of normality this would imply that they are independent and thus independent tests can be made by comparing the sum of squares of each contrast with the mean square of the error term in the experiment. Each contrast carries one degree of freedom.

4.3 Problem with the Measurements

The above procedure for the analysis is dependent upon exact measurements for the response variable. Numerical measurements of the amount of disease on a plant leaf, such as the percentage of the leaf area which is covered by the disease, are difficult and time consuming to obtain, and could be unreliable. It is possible in practice, however, to rank the plants according to the level of disease on them, which would provide a comparison between plants. This sounds fine, but ranking all the experimental units as a single group could get extremely tedious and again could be unreliable. Experience shows that it is fairly easy to rank a small number of the most infected and least infected plants but those in the middle are not nearly so easy to rank. Thus if we consider each replicate of the factorial experiment as a block, then ranking the observations in a block would be a relatively simple task.

4.4 A Blocked Factorial Experiment

From a practical point of view then it would be more appropriate to consider the above factorial experiment to be performed in a randomised block design. This would involve setting up blocks each of which would allow a complete replicate of the factorial experiment. Within each block the assignment of treatments to experimental units is randomised. Ranking all the observations in a block would not be difficult. Ranking is done separately in each block. A randomised block experiment in which the responses to a number of different treatments are compared using ranks within blocks may be analysed using Friedman's test. Here we have a similar situation except that the treatments consist of all combinations of the levels of the factors. The question is, is there a test similar to Friedman's test, i.e. one in which responses are ranked within blocks, which would allow a test of the main effects and the interaction component. Such a test would be of more general application since situations like this are quite common in industry, particularly in biological and agricultural experiments where obtaining exact quantitative measurements of the response variable is often difficult. A more general method of analysis not constrained by the requirement of exact measurements is therefore quite desirable.

4.5 A Non-Parametric Approach

Let us assume that we have a factorial experiment with two factors U and V with the number of levels I and J respectively. We are performing the factorial experiment in a randomised block design with M blocks, each block representing a complete replicate of the experiment. Thus we have $K = IJ$ observations in each block, i.e, one observation for every factor combination. As explained before, allocation of treatments to experimental units within each block is carried out at random. Furthermore, we assume that the factors are quantitative and the levels are equally spaced. Here we rank the observations within each block separately to obtain the rankings shown in the table below.

Replication	Treatment				
	1	2	3	...	K
1	R_{11}	R_{12}	R_{13}	...	R_{1K}
2	R_{21}	R_{22}	R_{23}	...	R_{2K}
3	R_{31}	R_{32}	R_{33}	...	R_{3K}
⋮	⋮	⋮	⋮		⋮
M	R_{M1}	R_{M2}	R_{M3}	...	R_{MK}
sum	$R_{.1}$	$R_{.2}$	$R_{.3}$	...	$R_{.K}$

In the table R_{mk} is the rank of treatment k in block m and each row shows the ranks of the observations for each block. In this layout the observations are ranked across the rows from 1 to K so that the ranks in each row will sum to the value $K(K+1)/2$. Let $R_{.k}$ denote the sum of the ranks for treatment combination k, where $k = 1, 2, \ldots, K$. If we simply had K treatments to compare and these were ranked in the rows as described, this would be the situation which might be analysed using Friedman's test statistic. While the Friedman test applies to one factor only, here we have a factorial experiment involving two factors. In the next section we propose a procedure which is an extension of the Friedman test to a factorial experiment with two factors.

4.6 Non-Parametric Test Statistics Based on Ranks

Our proposed non-parametric approach is defined as follows:
To test a particular component of a main effect or interaction we use the relevant parametric contrast and replace the values of the observations involved for each treatment combination with their corresponding ranks which are obtained by ranking the observations in each block separately. Thus the sum of values for each treatment is replaced with the sum of the ranks. The expected value and the variance of each contrast can now be obtained under the null hypothesis. Our non-parametric test statistic for testing the effect under question is defined as the standardised form of the relevant rank-based contrast.

The orthogonal contrasts are defined on treatment rank sums as follows:

$$T = \sum_{k=1}^{K} c_k R_{.k} \tag{4.4}$$

where $R_{.k}$ is the sum of ranks for treatment k, and c_k is the contrast coefficient relating to treatment k and as mentioned before $\sum_{k=1}^{K} c_k = 0$ and $\sum_{k=1}^{K} c_k^2 = 1$. In the next section we shall find the expected value and variance of these rank-based contrasts.

4.7 Expected value and Variance of T

The expected value and variance of T under the null hypothesis H_0 of no treatment effects are obtained as follows:

$$
\begin{aligned}
E(T|H_0) &= E(\sum_{k=1}^{K} c_k R_{.k}) = \sum_{k=1}^{K} c_k E(R_{.k}) && (4.5)\\
E(R_{.k}) &= \sum_{m=1}^{M} E(R_{mk}) \\
E(R_{mk}) &= \sum_{r_{mk}=1}^{K} r_{mk} P(R_{mk} = r_{mk}) = \frac{1}{K}[1 + 2 + \ldots + K] \\
&= \frac{K+1}{2} && (4.6)\\
E(R_{.k}) &= \sum_{m=1}^{M} E(R_{mk}) = M\frac{K+1}{2} && (4.7)\\
E(T|H_0) &= M\frac{K+1}{2}(\sum_{k=1}^{K} c_k) = 0 && (4.8)\\
V(T|H_0) &= V(\sum_{k=1}^{K} c_k R_{.k}) && (4.9)\\
V(\sum_{k=1}^{K} c_k R_{.k}) &= \sum_{k=1}^{K} c_k^2 V(R_{.k}) + \sum_{k=1}^{K}\sum_{t\neq k} c_k c_t Cov(R_{.k}, R_{.t}) && (4.10)\\
&= V(R_{.k})\sum_{k=1}^{K} c_k^2 + Cov(R_{.k}, R_{.t})\sum_{k=1}^{K}\sum_{t\neq k} c_k c_t && (4.11)\\
(\sum_{k=1}^{K} c_k)^2 &= \sum_{k=1}^{K} c_k^2 + \sum_{k=1}^{K}\sum_{t\neq k} c_k c_t && (4.12)
\end{aligned}
$$

The left hand side is the square of the sum of contrast coefficients and is therefore equal to zero. Thus:

$$
\sum_{k=1}^{K} c_k^2 = -\sum_{k=1}^{K}\sum_{t\neq k} c_k c_t \qquad (4.13)
$$

If we use the above result in the formula for the variance then:

$$
V(T|H_0) = (\sum_{k=1}^{K} c_k^2)[V(R_{.k}) - Cov(R_{.k}, R_{.t})] \qquad (4.14)
$$

The terms $V(R_{.k})$ and $Cov(R_{.k}, R_{.t})$ can be derived and then substituted in this equation.

$$V(R_{.k}) = \sum_{m=1}^{M} V(R_{mk}) = MV(R_{mk}) \quad (4.15)$$

because $R_{.k}$ is the sum of M independent ranks.

$$\begin{aligned}
V(R_{mk}) &= E(R_{mk}^2) - (E(R_{mk}))^2 & (4.16)\\
E(R_{mk}^2) &= \sum_{r_{mk}=1}^{K} (r_{mk})^2 P(R_{mk} = r_{mk}) = \frac{1}{K}[1^2 + 2^2 + 3^2 + \ldots + K^2] \\
&= \frac{1}{K}\frac{K((K+1)(2K+1)}{6} \\
&= \frac{(K+1)(2K+1)}{6} & (4.17)\\
V(R_{mk}) &= \frac{(K+1)(2K+1)}{6} - \frac{(K+1)^2}{4} \\
&= \frac{K^2-1}{12} & (4.18)
\end{aligned}$$

If we substitute this result in (4.15), we obtain the variance of $R_{.k}$.

$$V(R_{.k}) = \frac{M(K^2-1)}{12} \quad (4.19)$$

To obtain the covariance of two ranks R_{mk} and R_{mt} in the same block we note that:

$$\begin{aligned}
\sum_{k=1}^{K} R_{mk} &= \frac{K(K+1)}{2} & (4.20)\\
V(\sum_{k=1}^{K} R_{mk}) &= 0 & (4.21)\\
V(\sum_{k=1}^{K} R_{mk}) &= \sum_{k=1}^{K} V(R_{mk}) + \sum_{k=1}^{K}\sum_{t\neq k} Cov(R_{mk}, R_{mt}) & (4.22)\\
&= KV(R_{mk}) + K(K-1)Cov(R_{mk}, R_{mt}) & (4.23)\\
&= 0
\end{aligned}$$

after substitution for the variance of R_{mk}, the covariance of R_{mk} and R_{mt} can be derived.

$$Cov(R_{mk}, R_{mt}) = -\frac{K+1}{12} \tag{4.24}$$

To find the covariance of $R_{.k}$ and $R_{.t}$ we note that:

$$\sum_{k=1}^{K} R_{.k} = \frac{MK(K+1)}{2} \tag{4.25}$$

$$V(\sum_{k=1}^{K} R_{.k}) = \sum_{k=1}^{K} V(R_{.k}) + \sum_{k=1}^{K}\sum_{t\neq k} Cov(R_{.k}, R_{.t}) = 0$$

$$V(\sum_{k=1}^{K} R_{.k}) = KV(R_{.k}) + K(K-1)Cov(R_{.k}, R_{.t})$$

$$V(R_{.k}) = \sum_{m=1}^{M} V(R_{mk}) = \frac{M(K^2-1)}{12} \tag{4.26}$$

After substitution for the variance of $R_{.k}$, the covariance of $R_{.k}$ and $R_{.t}$ can be obtained as:

$$Cov(R_{.k}, R_{.t}) = -\frac{M(K+1)}{12} \tag{4.27}$$

If we substitute for the variance of $R_{.k}$ and covariance of $R_{.k}$ and $R_{.t}$ in the formula (4.14), we obtain a formula for the variance of the contrast:

$$V(T|H_0) = \frac{MK(K+1)}{12} \sum_{i=1}^{K} c_k^2 \tag{4.28}$$

Since the sum of squares of the coefficients is taken to be 1, the formula for the variance of a contrast becomes:

$$V(T|H_0) = \frac{MK(K+1)}{12} \tag{4.29}$$

4.8 Test Statistics

The contrast T can now be standardised and if a normal approximation is used, then one can say that under the null hypothesis the standardised form of the contrast has a standard normal distribution:

$$\left(\frac{T}{\sqrt{V(T)}}|H_0\right) \sim N(0,1) \tag{4.30}$$

With $K = IJ$ treatment combinations, there are $K-1$ orthogonal contrasts leading to $K-1$ test statistics, $I-1$ of them are used to test the components of the main effect due to factor U, $J-1$ of them are used to test the components of the main effect due to factor V, and finally the other $(I-1)(J-1)$ test statistics are used for testing the various components of the interaction.

4.9 Independence of the Contrasts

In the previous section the rank sums were formed into orthogonal contrasts to test the components of the main effects and interaction. Here we verify that such contrasts are, under the null hypothesis of no main effects and interaction, uncorrelated. Suppose we have n random variables $y_1, y_2, \ldots, y_n$ which are identically distributed with mean μ and variance σ^2. ρ is the coefficient of correlation between any two of the variables and is assumed to be fixed. Thus $Cov(y_i, y_j) = \rho\sigma^2 \; for \; all \; i \neq j$. Consider two linear contrasts $T_1 = \mathbf{a}^T\mathbf{y}$ and $T_2 = \mathbf{b}^T\mathbf{y}$ where $\mathbf{y}^T = (y_1, \ldots, y_n)$, $\mathbf{a}^T = (a_1, \ldots, a_n)$ and $\mathbf{b}^T = (b_1, \ldots, b_n)$ such that $\mathbf{a}^T.\mathbf{1} = 0$ and $\mathbf{b}^T.\mathbf{1} = 0$ and here $\mathbf{1}$ denotes a column vector with all its elements equal to one. The covariance of T_1 and T_2 is given by

$$Cov(T_1, T_2) = \mathbf{a}^T\mathbf{V}\mathbf{b} \tag{4.31}$$

where $\mathbf{V} = \{\mathbf{I}(1-\rho) + \mathbf{J}\rho\}\sigma^2$ is the variance-covariance matrix of $\mathbf{y}$, $\mathbf{I}$ is the $(n \times n)$ identity matrix, and $\mathbf{J}$ is an $(n \times n)$ matrix of 1's. This becomes,

$$Cov(T_1, T_2) = \mathbf{a}^T\{\mathbf{I}(1-\rho) + \mathbf{J}\rho\}\mathbf{b}\sigma^2 \tag{4.32}$$

$$= \sigma^2(1-\rho)\mathbf{a}^T\mathbf{b} + \rho\sigma^2\mathbf{a}^T\mathbf{J}\mathbf{b} \tag{4.33}$$

$$= \sigma^2(1-\rho)\mathbf{a}^T\mathbf{b} \tag{4.34}$$

since $\mathbf{a}^T\mathbf{1} = 0$. If T_1 and T_2 are orthogonal contrasts then $\mathbf{a}^T\mathbf{b} = 0$ and T_1 and T_2 are uncorrelated. If the y_i's are normally distributed then T_1 and T_2 will be independent normal variables. Note that independence of T_1 and T_2 does not require independence of the y_i's. Thus the following conclusions can be drawn:

1- If Y_i's denote the treatment totals, then the resulting T_i's will denote the test statistics for the classical procedure. From the previous discussion with the assumption of normality these test statistics are independent, and this would imply that we are dealing with $K-1$ independent tests. We note that the independence assumption is not required here.

2- If Y_i's denote the treatment rank sums, then the resulting T_i's will denote the test statistics for the non-parametric approach proposed. As proved the orthogonality of the contrasts would lead to test statistics which are uncorrelated. If normal approximation is applicable, then again this would lead to mutual independence for the contrasts. As we know the ranks within each block are correlated, but we have shown that this does not affect the independence for the test statistics.

3- If we assume that the two linear combinations are two contrasts based on treatment rank sums and in the formula for the covariance of T_1 and T_2 we change the index 2 to 1, then we obtain a simple formula for the variance of a contrast from the formula for the covariance. Thus $Cov(T_1, T_1) = Var(T_1)$ and therefore we have the following formula for the variance of a contrast based on rank sums.

$$Var(T_1) = \mathbf{a}^T\mathbf{V}\mathbf{a} \tag{4.35}$$

$$= \sigma^2(1-\rho)\mathbf{a}^T\mathbf{a} \tag{4.36}$$

$$= \sigma^2(1-\rho)\sum a_i^2 \tag{4.37}$$

Now if we substitute the covariance of two rank sums $Cov(R_{.k}, R_{.t})$ for $\rho\sigma^2$ and the variance of a rank sum $V(R_{.k})$ for σ^2, then $Var(T_1) = \sum a_i^2[V(R_{.k}) - Cov(R_{.k}, R_{.t})]$ which has the same form as the formula in (4.14) and if we substitute 1 for the sum of squares of the coefficients and substitute for the variance and covariance terms we will arrive at the formula (4.29) for the variance of a contrast.

4.10 Relationship with Friedman's Statistic

The Friedman statistic is defined as

$$F = \frac{12}{MK(K+1)} \sum_{i=1}^{K} \left(R_{i.} - \frac{M(K+1)}{2}\right)^2 \tag{4.38}$$

where M is the number of blocks, K the number of treatments and $R_{i.}$ denotes the sum of ranks on treatment i. Consider the contrasts T_j, $j = 1, 2, \ldots, K$-1 corresponding to the orthogonal components for the main effects and the interaction for a polynomial fit. We can write these contrasts in the following manner.

$$T_j = \sum_{i=1}^{K} c_{ji} R_{i.} \qquad j = 1, 2, \ldots, K-1 \tag{4.39}$$

and here c_{ji} is the coefficient of treatment i in contrast j. Thus the variance of T_j is

$$V(T_j) = \frac{MK(K+1)}{12}. \tag{4.40}$$

We would like to prove the following relationship

$$\frac{12}{MK(K+1)} \sum_{i=1}^{K} \left(R_{i.} - \frac{M(K+1)}{2}\right)^2 = \frac{12}{MK(K+1)} \sum_{j=1}^{k-1} \left(\sum_{i=1}^{K} c_{ji} R_{i.}\right)^2 \tag{4.41}$$

which means the Friedman statistic is equal to the sum of squares of the standardized contrasts discussed earlier. To prove that we should establish the following

$$\sum_{i=1}^{K} \left(R_{i.} - \frac{M(K+1)}{2}\right)^2 = \sum_{j=1}^{K-1} \left(\sum_{i=1}^{K} c_{ji} R_{i.}\right)^2 \tag{4.42}$$

Let

$$\bar{R} = \frac{M(K+1)}{2}.$$

We need to prove that

$$\sum_{i=1}^{K}(R_{i.} - \bar{R})^2 = \sum_{j=1}^{K-1} (\sum_{i=1}^{K} c_{ji} R_{i.})^2. \tag{4.43}$$

Consider the following K transformations

$$\begin{array}{lllllll}
c_{11}R_{1.} & + & c_{12}R_{2.} & + \cdots + & c_{1K}R_{K.} & = & T_1 \\
c_{21}R_{1.} & + & c_{22}R_{2.} & + \cdots + & c_{2K}R_{K.} & = & T_2 \\
\cdots & & & & & & \\
c_{(K-1)1}\, R_{1.} & + & c_{(K-1)2}\, R_{2.} & + \cdots + & c_{(K-1)K}\, R_{K.} & = & T_{K-1} \\
\frac{1}{\sqrt{K}}R_{1.} & + & \frac{1}{\sqrt{K}}R_{2.} & + \cdots + & \frac{1}{\sqrt{K}}R_{k.} & = & T_K
\end{array}$$

We can see that $\sum_{i=1}^{K} \frac{1}{\sqrt{k}} c_{ji} = \frac{1}{\sqrt{k}} \sum_{i=1}^{K} c_{ji} = 0$ for $j = 1, 2, \ldots, K-1$. Hence, $T_1, T_2, \ldots, T_{K-1}$ are orthogonal to T_K and we have a set of K orthogonal transformations.

Let T be the $(K \times 1)$ vector of T_j values, R the $(K \times 1)$ vector of $R_{i.}$ values and A the $(K \times K)$ matrix of the coefficients. Then

$$T = AR \tag{4.44}$$

and $T^T T = (AR)^T AR = R^T A^T AR = R^T R$ since the matrix A is orthogonal. But $\sum_{i=1}^{K} T_j^2 = T^T T$ and $\sum_{i=1}^{K} R_{i.}^2 = R^T R$. Hence

$$\sum_{j=1}^{K} T_j^2 = \sum_{i=1}^{K} R_{i.}^2. \tag{4.45}$$

We note that $T_K^2 = (\frac{1}{\sqrt{K}} \sum_{i=1}^{K} R_{i.})^2 = K\bar{R}^2$ and therefore

$$\sum_{i=1}^{K}(R_{i.} - \bar{R})^2 = \sum_{i=1}^{K} R_{i.}^2 - K\bar{R}^2 = \sum_{i=1}^{K-1} T_j^2 \tag{4.46}$$

But

$$\sum_{i=1}^{K-1} T_j^2 = \sum_{j=1}^{K-1} (\sum_{i=1}^{K} c_{ji} R_{i.})^2$$

and thus the relation (4.41) is established.

4.11 Distribution of Friedman's Statistic

The exact distribution of Friedman's statistic can be obtained using combinatorial analysis and principles of counting. The discussion in the previous section can be used to obtain the asymptotic distribution of Friedman's statistic. By the Central Limit Theorem the asymptotic distribution of each standardized contrast is normal. Thus the asymptotic distribution of the square of each contrast is chi-square with one degree of freedom. We showed that the contrasts are uncorrelated and because of the asymptotic normality they are asymptotically independent. Therefore we have $K - 1$ independent chi-square variables and because of the additivity property of independent chi-square variables their sum has also chi-square distribution. Thus we have shown that the asymptotic distribution of Friedman's statistic is chi-square with $K - 1$ degrees of freedom.

4.12 Null Distributions

The exact null distribution of the test statistics can be obtained using combinatorial analysis. We shall determine the exact null distribution for three small designs: (2 2 2), (2 2 3) and (3 3 2) where the first index indicates the number of levels of one factor, the second shows the number of levels of the other factor, and the last one is the number of blocks used. We then compare the exact distributions with the corresponding simulated distributions and later compare them with the standard normal distribution. One can use the same reasoning to obtain the null distributions for larger designs with the aid of an algorithm to compute the probabilities determined by the involved combinatorics. We shall show that for larger designs the normal distribution provides an adequate approximation for most practical purposes.

4.12.1 Exact Null Distribution for the Design (2 2 2)

For this design we have a factorial experiment involving two factors each with two levels. Here we have three contrasts, one for each of the main effects and one for the interaction. Thus we have four treatments in each block. The number of ways we can rank these four treatments is $4! = 24$. Since we have two blocks the total number of ways we can rank the whole data is $(4!) \times (4!) = 576$ taking into account the fact that ranking is done independently in each block. The contrast coefficients, before being standardised, are -1 and 1. Let us take the contrast for one of the main effects, $(ab) - (b) + (a) - (1)$. For each particular rank configuration we have four ranks contained in $(ab) + (a)$ and four in $-(b) - (1)$. The ranks of treatments (ab) and a have a permutation of 2! in each block assuming there are no ties. Similarly the other two treatments have a permutation of 2! in each block. As far as the value of the statistic is concerned all these permutations give rise to the same value for the test statistic. Thus each particular rank configuration will give rise to a value for the test statistic which is repeated $(2!)^4 = 16$ times and therefore we have just $(576 \div 16) = 36$ rank configurations to consider. The other configurations will just be permutations of the others in the manner explained above and therefore would give rise to the same values for the test statistic.

The rank configuration that gives rise to the maximum value of the test statistic would allocate the ranks $3, 4$ to treatments $(ab), (a)$ and the ranks $1, 2$ to treatments $(b), (1)$ in each block. Thus the maximum value for the contrast is $2(3+4)-2(1+2) = 8$. Similarly the rank configuration that gives rise to the minimum value of the test statistic would allocate the ranks $1, 2$ to treatments $(ab), (a)$ and the ranks $3, 4$ to treatments $(b), (1)$ and this would lead to a value of -8 for the contrast. We can calculate the variance of the test statistic as:
$Var(T|H_0) = \frac{MK(K+1)}{12} \sum_{k=1}^{K} c_k^2 = \frac{2\times4(4+1)}{12} \times 4 = 40/3$. In this way the maximum value

of the statistic is obtained as $8/(\sqrt{40/3}) = 2.191$ and the minimum value is -2.191.

To find the exact probability for each value of the test statistic we count the number of the corresponding possible rank configurations. For the maximum value, we must allocate the ranks 1, 2 to treatments $(b), (1)$ and the ranks 3, 4 to treatments $(ab), (a)$ in each block. If we now disregard the common number of repetitions due to the permutations explained before, we can have only one rank configuration out of the possible 36. Thus the probability is 1/36.

To be able to explain the probabilities for the other values in an easier manner we just concentrate on the rank allocation to treatments $(b), (1)$ in each block since this allocation would automatically determine the rank allocation to the other two treatments in the block as well and thus we can find the values of the test statistic.

To find the probability for the second largest value of the statistic we notice that we must allocate the ranks 1, 2 to treatments $(b), (1)$ in one block and the ranks 1, 3 in the other block. Thus the number of possible configurations is 2 which is the number of permutations of the two blocks with different allocations. The value of the test statistic is obtained by finding the contrast value first and then dividing it by the standard deviation. Thus the contrast value is $(3+4+2+4)-(1+2+1+3)) = 6$ and the value of the statistic is $6/(\sqrt{40/3}) = 1.643$ The probability for this value is, then, 2/36 assuming there are no ties. For the next largest value we must allocate the ranks 1, 2 to treatments $(b), (1)$ in one block and 1, 4 in the other block or 1, 2 in one block and 2, 3 in the other block or 1, 3 in both blocks. The first two would give rise to two rank configurations each since the pattern in each block is different, while the third case would give rise to just one rank configuration since the pattern in the blocks is the same. The corresponding contrast value is $(12) - (8) = 4$ and the value of the test statistic is $4/(\sqrt{40/3}) = 1.095$. We have a total of 5 possibilities for this value which gives us the probability 5/36. Other values are obtained in a similar fashion until the contrast value becomes 0. These values and their associated probabilities, are shown

Table 4.2: **The Exact Null Distribution of the Test Statistic**

Design (2 2 2)

Values	−2.191	−1.643	−1.095	−.548	0	.548	1.095	1.643	2.191
Exact Probability	1/36	2/36	5/36	6/36	8/36	6/36	5/36	2/36	1/36

in Table 4.2. Because of the symmetry of the distribution the probabilities associated with negative test scores can be derived in a similar fashion.

4.12.2 Exact Null Distribution for the Design (2 2 3)

The number of rank configurations for this design is $(4!)^3 = 13824$. Here again we can reason that for each particular rank configuration we have $(2!)^6 = 64$ permutations giving rise to the same value for the test statistic. Thus here we have $13824/64 = 216$ rank configurations to consider. In a similar manner we can find the values of the test statistic and the corresponding probabilities counting the combinations that give rise to the same values of the statistic. The table 4.3 gives the full exact distribution of the linear component for this design.

4.12.3 Exact Null Distribution for the Design (2 3 2)

The number of treatments per block is $2 \times 3 = 6$ and so the number of possible rankings in each block is $(6!) = 720$ and thus the total number of rank configurations for all the data is $720^2 = 518400$. We now concentrate on the null distribution of the linear component of the first factor. As before the contrast coefficients are -1 and 1. Here for each rank configuration we have $(3!)^4 = 1296$ permutations giving rise to the same value for the statistic. Thus we have $518400/1296 = 400$ configurations to consider. The maximum value of the statistic is obtained when the ranks $1, 2, 3$ are

Table 4.3: **The Exact Null Distribution of the Test Statistic**

Design (2 2 3)

Values	Probability	Values	Probability
-2.683	1/216	0.447	33/216
-2.236	3/216	0.894	27/216
-1.789	9/216	1.342	16/216
-1.342	16/216	1.789	9/216
-0.894	27/216	2.236	3/216
-0.447	33/216	2.683	1/216
0	38/216		

allocated to one level of the factor and the ranks $4, 5, 6$ to the other in each block. The contrast value is $2(4 + 5 + 6) - 2(1 + 2 + 3) = 18$. The variance of the test statistic is $2(6)(7)(6)/12 = 42$ and so the maximum value of the statistic is $18/\sqrt{42} = 2.777$. The probability for this value is $1/400$. The second largest value is obtained when the ranks $1, 2, 3$ are allocated to one level of the factor in one block while in the other block the ranks $1, 2, 4$ are allocated to the same level. the contrast value is $(4 + 5 + 6 + 3 + 5 + 6) - (1 + 2 + 3 + 1 + 2 + 4) = 29 - 13 = 16$ and the value of the statistic is $16/\sqrt{42} = 2.4689$ with the corresponding probability of $2/400$. For the next largest value we have three configurations to consider, namely, allocating the ranks $1, 2, 4$ to one level of the factor in each block, or allocating $1, 2, 3$ in one block and $1, 2, 5$ in the other or allocating $1, 2, 3$ in one block and $1, 3, 4$ in the other. The first one would count as one rank configuration and the others as two to allow for the block permutation. Thus the probability is $5/400$ relating to the value of $14/\sqrt{42} = 2.160$ for the test statistic. Continuing in the same manner we can obtain the other values of the statistic and keeping track of the combinations leading to the same values we can

Table 4.4: **The Exact Null Distribution of the Test Statistic Design** (2 3 2)

Values	Probability	Values	Probability	Values	Probability
-2.777	1/400	-.617	40/400	1.543	16/400
-2.469	2/400	-.309	45/400	1.852	10/400
-2.160	5/400	0	48/400	2.160	5/400
-1.852	10/400	.309	45/400	2.469	2/400
-1.543	16/400	.617	40/400	2.777	1/400
-1.234	24/400	.926	33/400		
-.926	33/400	1.234	24/400		

obtain the corresponding probabilities.

For larger designs the exact null distribution can be obtained in a similar fashion. As the number of levels of the factors and also the number of blocks increase it would be very difficult and impractical to follow the various combinations and do the counting by hand. However one can write a computer program and the exact distributions can be obtained, if necessary, fairly easily in this way.

4.12.4 Comparison with the Simulated Distributions

To be able to assess the accuracy of the null distributions obtained by simulation we compare the exact distribution with the corresponding simulated distribution for the three designs considered above. The tables 4.5, 4.6, and 4.7 compare the exact distributions with their simulated counterparts. Each simulated distribution is based on $100,000$ independent repetitions of the experiment. For each run of the experiment K random numbers are taken from the standard normal distribution for each block

Table 4.5: **Comparison of the Exact Null Distribution and the Simulated Distribution Design** (2 2 2)

Values	−2.191	−1.643	−1.095	−.548	0	.548	1.095	1.643	2.191
Exact Probability	.0278	.0555	.1389	.1667	.2222	.1667	.1389	.0555	.0278
Simulated Prob.	.0271	.0555	.1397	.1666	.2228	.1666	.1389	.0549	.0279

Table 4.6: **Comparison of the Exact Null Distribution and the Simulated Distribution Design** (2 2 3)

Values	Exact Probability	Simulated	Values	Exact Probability	Simulated
-2.683	.00463	.00434	.447	.15278	.15248
-2.236	.01389	.01372	.894	.12500	.12768
-1.789	.04167	.04204	1.342	.07407	.07522
-1.342	.07407	.07420	1.789	.04167	.04210
-0.894	.12500	.12680	2.236	.01389	.01418
-0.447	.15278	.15024	2.683	.00463	.00434
0	.17592	.17266			

through a fortran program which is included in the appendix. The random observations thus obtained are ranked separately within each block and then for each treatment combination the corresponding rank sum is obtained. Thus the value of each contrast is calculated. In this way for each contrast a vector of calculated values is obtained from which the relevant simulated null distribution is derived.

Table 4.7: **Comparison of the Exact Null Distribution and the Simulated Distribution Design** (2 3 2)

Values	Exact Probability	Simulated	Values	Exact Probability	Simulated
-2.777	.00250	.00246	0.309	.11250	.11134
-2.469	.00500	.00470	0.617	.10000	.10098
-2.160	.01250	.01326	0.926	.08250	.08176
-1.852	.02500	.02424	1.234	.06000	.06320
-1.543	.04000	.03988	1.543	.04000	.04000
-1.234	.06000	.06000	1.852	.02500	.02376
-0.926	.08250	.08340	2.160	.01250	.01282
-0.617	.10000	.09878	2.469	.00500	.00526
-0.309	.11250	.11216	2.777	.00250	.00264
0	.12000	.11936			

4.13 Simulation Error

The simulated null distributions presented throughout this work are all based on 100,000 simulations. A 95% confidence interval for a simulated probability would be given by

$$p \pm 1.96(\sqrt{p(1-p)/100000})$$

where p is the exact probability. Table 4.8 shows the margin of error, given by

$$1.96(\sqrt{p(1-p)/100000})$$

for a range of values of p together with corresponding percentage errors. It can be seen that for the exact probability of .025 or more we should expect the simulated probabilities to be in error by at most 4 percent. With p larger than .3 we expect the the simulated probabilities to be in error by less than 1% and for probabilities above 90% the margin of error is .21% at most.

Comparison of the exact probabilities with the simulated values shown in Tables 4.5, 4.6 and 4.7 assures us of the accuracy of the simulated probabilities. We can see that the errors are within the expected accuracies set in Table 4.8. Thus the simulated null distributions based on 100,000 independent runs are good approximations for the null distributions of the test statistics and can be used in testing hypotheses and in power calculations where exact distributions are not available or when the accuracy of normal approximation is in ¡doubt.

Table 4.8: **Expected Accuracy of the Simulated Null Distributions Based on 100,000 Simulations**

Probability	Simulation Error	% Error	Probability	Simulation Error	% Error
.01	.0006	6.2	.60	.0030	.51
.025	.0010	3.9	.70	.0028	.41
.05	.0013	2.7	.80	.0025	.31
.10	.0019	1.9	.90	.0019	.21
.20	.0025	1.2	.95	.0013	.14
.30	.0028	.95	.975	.0010	.10
.40	.0030	.76	.99	.0006	.06
.50	.0031	.62			

4.14 Comparison with Normal Distribution

To see how close the normal approximation is to the exact distribution, the cumulative distribution functions were evaluated at each point of the distributions using the normal approximation and the exact distributions. Tables 4.9, 4.10, and 4.11 give a comparison of the exact and approximate distribution functions. We can see that with the increase in block size and also in the number of levels of the factors, the distribution of the test statistic tends to get closer to normality.

To be able to see the closeness graphically, we have compared the probability distributions with the normal density and also the cumulative distributions with the cumulative normal. To compare the probability distributions, we first adjust the exact distributions to draw the probability polygons. Each exact probability would denote the area of a rectangle the height of which can be obtained by dividing the probability

Table 4.9: **Comparison of the Cumulative Distribution Functions of the Statistic and the Normal Distribution Design** (2 2 2)

Values	−2.191	−1.643	−1.095	−.548	0	.548	1.095	1.643	2.191
Exact	.0278	.0833	.2222	.3889	.6111	.7778	.9167	.9722	1.0000
Normal	.0142	.0502	.1368	.2918	.5000	.7081	.8632	.9498	.9858

Table 4.10: **Comparison of the Cumulative Distribution Functions of the Statistic and the Normal Distribution Design** (2 2 3)

Values	Exact	Normal	Values	Exact	Normal
-2.683	.0046	.0036	.447	.7407	.6726
-2.236	.0185	.0127	.894	.8657	.8144
-1.789	.0602	.0368	1.342	.9398	.9101
-1.342	.1342	.0898	1.789	.9815	.9632
-0.894	.2593	.1857	2.236	.9954	.9873
-0.447	.4120	.3274	2.683	1.0000	.9964
0	.5880	.5000			

Table 4.11: **Comparison of the Cumulative Distribution Functions of the Statistic and the Normal Distribution Design** (2 3 2)

Values	Exact	Normal	Values	Exact	Normal
-2.777	.0025	.0027	0.309	.6725	.6212
-2.469	.0075	.0068	0.617	.7725	.7315
-2.160	.0200	.0154	0.926	.8550	.8227
-1.852	.0450	.0320	1.234	.9150	.8915
-1.543	.0850	.0614	1.543	.9550	.9386
-1.234	.1450	.1086	1.852	.9800	.9680
-0.926	.2275	.1772	2.160	.9925	.9846
-0.617	.3275	.2686	2.469	.9975	.9932
-0.309	.4400	.3787	2.777	1.0000	.9973
0	.5600	.5000			

by the difference between two consecutive values of the test statistic. Having obtained the adjusted values we can draw the probability polygons for each distribution. To be able to assess the normal approximation we superimpose the normal density graph. Fig 4.1 shows the comparisons. For the cumulative distributions we join the points for each design and compare the approximated cumulatives with the line for the standard normal cumulative. Fig 4.2 shows the comparison for the cumulative distributions.

Comparison of the tabulated values and also the graphical presentations demonstrate that the distribution of the test statistic approaches normality when the number of blocks increases and also when the number of levels of the factors increases. For larger designs the normal distribution would provide sufficient accuracy, in particular null percentage points obtained from the normal approximation would be sufficiently accurate for practical purposes as we shall see in the next section.

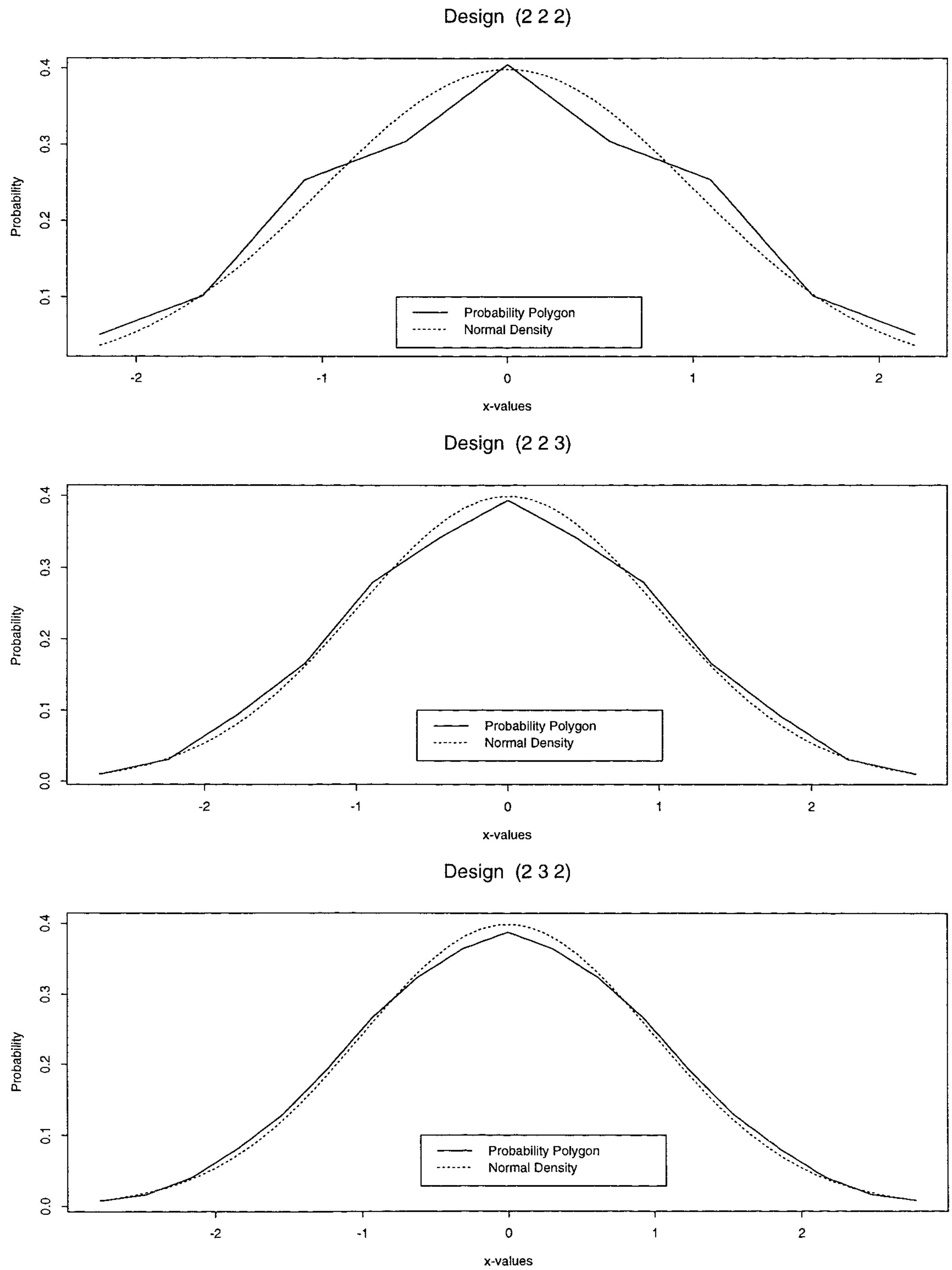


Figure 4.1: The Probability Polygon and Normal Density presentation for the three designs (2 2 2), (2 2 3) and (2 3 2).

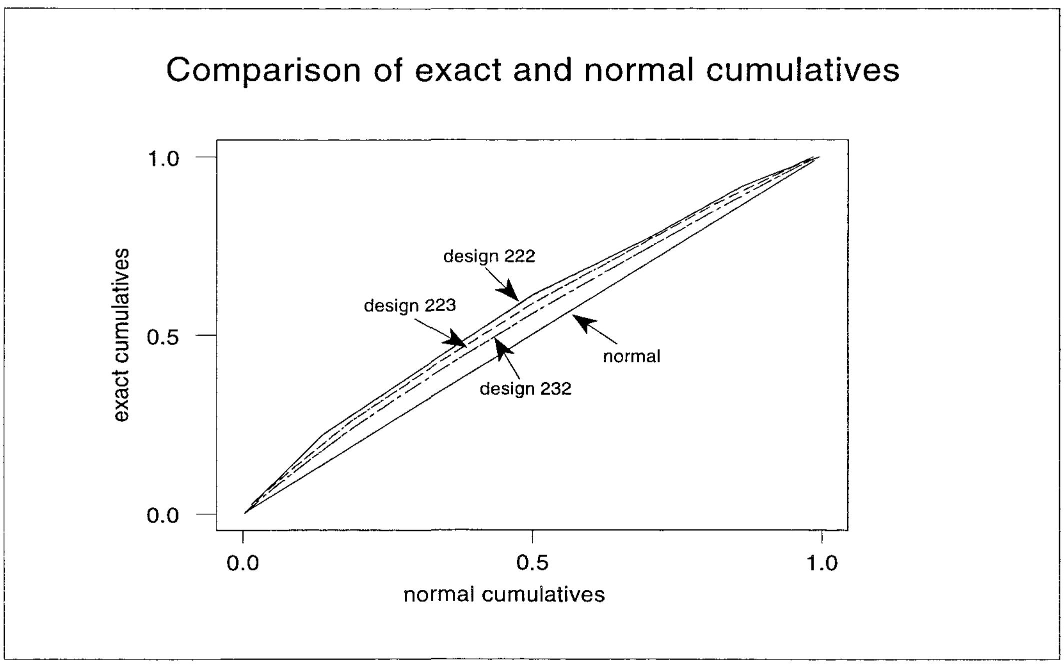


Figure 4.2: Comparison of the Exact and Normal Cumulatives for the three Designs (2 2 2), (2 2 3) and (2 3 2). The solid line is for the cumulative normal.

4.15 Simulated Null Distributions

Simulation methods were used to investigate the null distributions of these rank-based statistics. A computer program was written in fortran for this purpose. For each design considered, a simulation of 100,000 experiments was used and various percentage points obtained. These percentage points were subsequently compared with those of the normal distribution to investigate the adequacy of the asymptotic results. This was done for a wide range of design sizes starting with two levels for each factor gradually increasing the number of levels. For each specific design, the distribution was obtained for different numbers of blocks. These results are given in Tables 4.12 to 4.18. For each case, the first line shows the percentage points for the normal distribution and then the corresponding percentage points for each test statistic are given. Each line shows the results for a main effect or an interaction component as indicated. We can see that tests for designs with 2 factors at 2 levels are not adequately represented by the normal approximation. This is due to the discrete nature of the statistics and as a result some of the percentage points coincide. For these cases the exact distributions obtained, either theoretically by permutation methods or by simulation, can be used in our testing procedures. But we can see that as we increase the number of levels of the factors and blocks, the discreteness reduces gradually and the distributions tend to converge to the asymptotic results so that for a design as small as 3 3 2 the approximation seems reasonable. For example the 95 percent point for the linear component of factor U is 1.687 and for the linear by linear interaction it is 1.678, fairly close to the asymptotic value of 1.645. Similarly, for the design 2 5 5 the values are 1.635, 1.651 and 1.618 for the main effects U, V and the linear interaction respectively. Thus, apart from very small designs, normal approximation provides an accurate and practical test procedure based on ranks for all the other designs. Simulations for various other designs have been performed and are available with the author.

Table 4.12: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 2, \quad J = 2, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.095	1.643	2.191	2.191	2.191
Linear V	1.095	1.643	2.191	2.191	2.191
Lin$U$ × Lin$V$	1.095	1.643	2.191	2.191	2.191

Table 4.13: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 2, \quad J = 2, \quad M = 3$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.342	1.789	1.789	2.236	2.236
Linear V	1.342	1.789	1.789	2.236	2.236
Lin$U$ × Lin$V$	1.342	1.789	1.789	2.236	2.236

Table 4.14: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 2, \quad J = 3, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.234	1.543	1.852	2.160	2.469
Linear V	1.323	1.701	1.890	2.268	2.457
Quadratic V	1.309	1.637	1.964	2.291	2.291
Lin$U$ x Lin$V$	1.323	1.701	1.890	2.268	2.457
Lin$U$ x Quad$V$	1.309	1.637	1.964	2.182	2.400

Table 4.15: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 2, \quad J = 3, \quad M = 3$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.386	1.638	1.890	2.394	2.646
Linear V	1.234	1.697	2.006	2.315	2.469
Quadratic$V$	1.336	1.604	1.871	2.405	2.405
Lin$U$ x Lin$V$	1.234	1.697	2.006	2.315	2.469
Lin$U$ x Quad$V$	1.336	1.604	1.960	2.227	2.405

Table 4.16: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 3, \quad J = 3, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.265	1.687	1.897	2.214	2.424
Quadratic U	1.278	1.643	2.008	2.191	2.556
Linear V	1.265	1.687	1.897	2.214	2.424
Quadratic V	1.278	1.643	2.008	2.191	2.556
Lin$U$ x Lin$V$	1.291	1.678	1.936	2.195	2.453
Lin$U$ x Quad$V$	1.267	1.640	1.938	2.236	2.460
Quad$U$ x Lin$V$	1.267	1.640	1.938	2.236	2.460
Quad$U$ x Quad$V$	1.291	1.678	1.936	2.195	2.453

Table 4.17: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 3, \quad J = 3, \quad M = 3$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.291	1.635	1.980	2.238	2.496
Quadratic U	1.342	1.640	1.938	2.236	2.534
Linear V	1.291	1.635	1.980	2.238	2.496
Quadratic V	1.342	1.640	1.938	2.236	2.534
Lin$U$ x Lin$V$	1.265	1.687	1.897	2.214	2.530
Lin$U$ x Quad$V$	1.278	1.643	1.947	2.252	2.495
Quad$U$ x Lin$V$	1.278	1.643	1.947	2.252	2.495
Quad$U$ x Quad$V$	1.265	1.687	1.897	2.319	2.530

Table 4.18: **Simulated Percentage Points of the Distributions of the Test Statistics under the Corresponding Null Hypotheses Design Parameters** $I = 2, \quad J = 5, \quad M = 5$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	1.282	1.645	1.960	2.326	2.576
Linear U	1.261	1.635	1.915	2.289	2.569
Linear V	1.288	1.651	1.949	2.312	2.510
Quadratic V	1.284	1.647	1.954	2.345	2.596
Cubic V	1.288	1.651	1.949	2.312	2.543
Quartic V	1.311	1.685	1.997	2.309	2.559
Lin$U$ x Lin$V$	1.288	1.618	1.949	2.312	2.510
Lin$U$ x Quad$V$	1.284	1.647	1.954	2.289	2.540
Lin$U$ x Cubic$V$	1.288	1.651	1.949	2.312	2.510
Linear$U$ x Quartic$V$	1.286	1.648	1.960	2.309	2.534

4.16 Power Study

In the power studies that are described here the analysis of variance procedure will be used as the optimal method and the alternative rank-based procedures will be compared with this. This assumes that the actual responses are available, which favours the ANOVA procedure. If, as in the practical situation which initiated these investigations, the actual responses are not available then it will not be possible to consider the analysis of variance and it will be necessary to employ a suitable rank-based method.

In order to assess the efficiency of the proposed non-parametric procedure, a power study was undertaken. Simulations, based on 30,000 experiments, were carried out to compare the powers of the non-parametric method and the standard analysis of variance procedure based on F-tests. We assume that the two factors are quantitative and levels of each factor are equidistant. Thus we can use the coefficients of orthogonal polynomials to obtain the contrast for the effect under study. In this work we shall concentrate on examining the linear component of the main effects and the linear by linear component of the interaction for a variety of design sizes and for a range of models which include linear main effects and interaction components of varying magnitudes. The power study included designs IJM, where I and J are the numbers of levels of the two factors U and V respectively and M is the number of blocks, taking the following values:

1) $I = 2,\ J = 5,\ M = 2$
2) $I = 4,\ J = 4,\ M = 2$
3) $I = 5,\ J = 5,\ M = 2$
4) $I = 4,\ J = 6,\ M = 2$

The alternative models used in the power study include terms to represent linear components of the main effects and linear by linear interaction. Without loss of generality the constant term μ and the block effects γ_m in model (4.1) are set to zero. A series of models is considered of the form

$$Model\ 1: \quad (U,0,0) \quad y_{ijm} = g_1(i-\bar{u}_i) + \epsilon_{ijm} \tag{4.47}$$

$$Model\ 2: \quad (0,V,0) \quad y_{ijm} = g_2(j-\bar{v}_j) + \epsilon_{ijm} \tag{4.48}$$

$$Model\ 3: \quad (0,0,UV) \quad y_{ijm} = g_3(i-\bar{u}_i)(j-\bar{v}_j) + \epsilon_{ijm} \tag{4.49}$$

$$Model\ 4: \quad (U,V,0) \quad y_{ijm} = g_1(i-\bar{u}_i) + g_2(j-\bar{v}_j) + \epsilon_{ijm} \tag{4.50}$$

$$Model\ 5: \quad (U,0,UV) \quad y_{ijm} = g_1(i-\bar{u}_i) + g_3(i-\bar{u}_i)(j-\bar{v}_j) + \epsilon_{ijm} \tag{4.51}$$

$$Model\ 6: (U,V,UV) \quad y_{ijm} = g_1(i-\bar{u}_i) + g_2(j-\bar{v}_j) + g_3(i-\bar{u}_i)(j-\bar{v}_j) + \epsilon_{ijm} \tag{4.52}$$

where $\bar{u}_i$ and $\bar{v}_j$ depend on the number of factor levels and are $1.5, 2, 2.5, 3$ and 3.5 for $2, 3, 4, 5$ and 6 factor levels respectively. The coefficients g_1, g_2 and g_3 ranged over different values to increase the magnitude of the relevant components. Comparisons of powers were arranged to show the influence of additional components on the power of the tests to detect specific components. For example, a comparison of the results for model 1 $(U,0,0)$, model 4 $(U,V,0)$ and model 6 (U,V,UV) illustrates the changes in the powers to detect the linear component of U where a linear component of V and a linear $\times$ linear component of the interaction are present. Similarly, a comparison of the results for model 3 $(0,0,UV)$, model 5 $(U,0,UV)$ and model 6 (U,V,UV) illustrates the changes in the powers to detect the linear $\times$ linear component of interaction when a linear component of U and a linear component of V are present.

The errors ϵ_{ijm} are standard normal variates and were generated through a fortran subroutine written for this purpose. To obtain the power performance for each effect various values were chosen for each coefficient. For each set of values for the coefficients, 30,000 standard normal variates were generated and the y_{ijm}'s were obtained through the corresponding model. For each data set thus obtained the values of the corresponding statistics were computed and compared with the appropriate critical values for both the ANOVA procedure and the Ranking Method for the same data sets. In this way powers were calculated for different values of the coefficients under study and tabulated. Tabulated values of power were then processed through an S-Plus program to provide us with the relevant power performance curves.

4.17 Accuracy of Simulated Powers

The power calculations presented in the next section are based on $30,000$ simulated experiments. A 95% confidence interval for the true power P would be given approximately by

$$p \pm 1.96(\sqrt{p(1-p)/30000})$$

where p is the estimated power. The following table shows the margin of error, given by

$$1.96(\sqrt{p(1-p)/30000})$$

for a range of values of p. It can be seen that for powers above 95 percent we should expect the simulated powers to be in error by at most 0.25 percent. Even for values of the power in the middle of the range near 0.5, the error is likely to be only about 1%. Therefore $30,000$ simulated experiments should be sufficient for the required comparisons.

Table 4.19 shows the error margin for various values of p.

Table 4.19: **Error Margin for Various Values of P**

Simulated Power	Error Margin
.01 or .99	.00112
.02 or .97	.00158
.05 or .95	.00247
.10 or .90	.00339
.20 or .80	.00452
.30 or .70	.00518
.40 or .60	.00554
.50	.00565

The tabulated results are discussed and illustrated in the next section.

4.18 Power Comparison

Tables 4.20 to 4.29 show the power comparisons for the designs under study and the Figures 4.3, 4.4, 4.5, and 4.6 are the corresponding graphical presentations.

Tables 4.20, 4.21, 4.22 and Fig 4.3 illustrate the power comparisons for ANOVA and the Ranking Method for the linear main effects and the linear by linear interaction for design 2 5 2. For each effect we can see the influence of the the relevant extraneous components in the model. Concentrating on factor U, we can see the following features. The power curve for ANOVA is not affected by the presence of other effects in the model and thus we have the same power curve for factor U for all models incorporating different extraneous effects. For the Ranking Method the power performance depends highly on the other effects present. Rank1 shows the powers when only the effect under study, namely, factor U is included in the model (Model 1) while Rank2 shows the power for U when the main effect V is included in the model as well(Model 4) and finally Rank3 is the power curve for U in the presence of both the main effect V and the interaction effect UV(Model 6).

As we can see the power curve for the Ranking Method is almost the same as that for the ANOVA when there is no nuisance effect present. When the main effect V is added the powers for the Ranking Method decrease as seen by Rank2. With the main effect V and interaction UV both included there is a further drop in power for U as indicated by Rank3. Thus we can see that with more extraneous effects present the power drops substantially.

The same trend is observed when we investigate the power performance for factor V. Again here, as with factor U, when we have only the main effect V in the model(Model 2), we can see that the powers for ANOVA and Rank1 are almost identical and with the addition of the extraneous effects U and interaction UV there is substantial loss of power. Rank2 shows the powers for V in the presence of U effect(Model 4) and Rank3 is the power curve for V in the presence of main effect U and interaction UVModel 6). With the main effect U and interaction UV both in the model, the power drops from .985 to .441 for $g_2 = \pm.750$, a drop of more than 50 percent in power due to presence of U effect and interaction.

As we can see the power curves for the interaction are the same as those for the V effect within the error accuracy due to the simulation. This is due to the fact that we have only two levels for factor U. Here Rank1 is the power curve for the interaction UV for the Ranking Method in the absence of any significant main effects(Model 3). Rank2 shows the powers for UV in the presence of U effect only(Model 5) and finally Rank3 is the power curve for interaction UV in the presence of both main effects U and V(Model 6).

Tables 4.23, 4.24 and Fig 4.4 show the power comparisons for design 442. Again we can see that the powers for the Ranking Method and ANOVA are quite close when there are no extraneous effects present and with V effect and interaction effect UV added we can see the drop in power though not as much as the drop for the design 252. For $g_1 = -.8$ the power for U is .995 under Model 1 compared with the power of .785 under the full model and .997 for the ANOVA. We can see the same trend for the interaction. Tables 4.25, 4.26, 4.27 and Figure 4.5 show the power curves for the main effect and interaction for design 462. The same trend is observed though with less intensity. As we can see for this design the effect of extraneous effects on power performance for both the main effects and the interaction is less severe than that for the smaller designs. And finally Tables 4.28, 4.29 and Figure 4.6 illustrate the power curves for the main effects U, V and the interaction UV for design 5 5 2. With increase in design size the decrease in powers due to the presence of extraneous effects becomes less. For the U effect in design 5 5 2 when $g_1 = -.510$ the power under Model 1 is .998 compared with the value of .952 under the full model, a drop of less than 5 percent in power only.

Table 4.20: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear Component of Factor U**

Design Parameters $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	Rank1	Rank2	Rank3	g_1	ANOVA	Rank1	Rank2	Rank3
-2.250	0.993	0.992	0.901	0.605	0.150	0.061	0.055	0.053	0.052
-2.100	0.986	0.983	0.875	0.585	0.300	0.092	0.082	0.081	0.077
-1.950	0.972	0.969	0.842	0.573	0.450	0.153	0.139	0.132	0.121
-1.800	0.945	0.939	0.788	0.533	0.600	0.231	0.214	0.193	0.170
-1.650	0.907	0.897	0.737	0.512	0.750	0.324	0.308	0.263	0.227
-1.500	0.848	0.836	0.676	0.474	0.900	0.435	0.416	0.345	0.282
-1.350	0.767	0.749	0.601	0.434	1.050	0.554	0.535	0.432	0.335
-1.200	0.665	0.650	0.520	0.390	1.200	0.663	0.650	0.519	0.389
-1.050	0.550	0.530	0.428	0.332	1.350	0.766	0.751	0.599	0.434
-0.900	0.435	0.417	0.349	0.284	1.500	0.844	0.832	0.675	0.474
-0.750	0.324	0.304	0.261	0.223	1.650	0.907	0.898	0.741	0.513
-0.600	0.225	0.211	0.186	0.165	1.800	0.944	0.941	0.792	0.539
-0.450	0.149	0.140	0.129	0.120	1.950	0.972	0.968	0.839	0.567
-0.300	0.092	0.083	0.080	0.076	2.100	0.985	0.984	0.874	0.589
-0.150	0.061	0.052	0.053	0.052	2.250	0.994	0.994	0.904	0.608
0.000	0.050	0.045	0.045	0.045					

Table 4.21: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear Component of Factor** V

Design Parameters $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_2	ANOVA	Rank1	Rank2	Rank3	g_2	ANOVA	Rank1	Rank2	Rank3
-0.750	0.987	0.985	0.823	0.441	0.050	0.060	0.057	0.056	0.055
-0.700	0.974	0.969	0.790	0.433	0.100	0.088	0.084	0.082	0.079
-0.650	0.957	0.953	0.752	0.415	0.150	0.139	0.134	0.123	0.112
-0.600	0.919	0.913	0.708	0.399	0.200	0.205	0.198	0.177	0.150
-0.550	0.871	0.864	0.651	0.381	0.250	0.291	0.282	0.239	0.190
-0.500	0.802	0.797	0.593	0.360	0.300	0.395	0.385	0.308	0.235
-0.450	0.719	0.709	0.526	0.330	0.350	0.510	0.500	0.387	0.274
-0.400	0.618	0.606	0.459	0.305	0.400	0.610	0.606	0.459	0.308
-0.350	0.511	0.500	0.388	0.274	0.450	0.718	0.708	0.527	0.336
-0.300	0.399	0.390	0.312	0.234	0.500	0.802	0.792	0.595	0.357
-0.250	0.294	0.286	0.238	0.192	0.550	0.872	0.865	0.651	0.382
-0.200	0.204	0.199	0.176	0.150	0.600	0.920	0.915	0.704	0.398
-0.150	0.136	0.131	0.120	0.109	0.650	0.953	0.949	0.751	0.412
-0.100	0.089	0.085	0.083	0.079	0.700	0.974	0.970	0.785	0.426
-0.050	0.059	0.056	0.056	0.056	0.750	0.988	0.986	0.823	0.440
0.000	0.050	0.049	0.049	0.049					

Table 4.22: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-1.500	0.987	0.985	0.824	0.441	0.100	0.061	0.058	0.057	0.056
-1.400	0.974	0.970	0.785	0.424	0.200	0.085	0.085	0.081	0.078
-1.300	0.953	0.951	0.747	0.412	0.300	0.138	0.133	0.122	0.110
-1.200	0.919	0.914	0.707	0.404	0.400	0.209	0.204	0.175	0.153
-1.100	0.872	0.863	0.651	0.382	0.500	0.291	0.284	0.238	0.196
-1.000	0.807	0.798	0.601	0.361	0.600	0.398	0.390	0.312	0.237
-0.900	0.723	0.708	0.530	0.338	0.700	0.510	0.505	0.387	0.276
-0.800	0.618	0.610	0.459	0.306	0.800	0.611	0.606	0.456	0.305
-0.700	0.509	0.499	0.387	0.276	0.900	0.716	0.708	0.527	0.334
-0.600	0.395	0.393	0.313	0.235	1.000	0.803	0.795	0.595	0.363
-0.500	0.297	0.287	0.239	0.192	1.100	0.868	0.861	0.649	0.377
-0.400	0.203	0.198	0.176	0.148	1.200	0.921	0.915	0.703	0.398
-0.300	0.136	0.133	0.122	0.112	1.300	0.954	0.950	0.745	0.414
-0.200	0.089	0.087	0.084	0.079	1.400	0.974	0.971	0.790	0.430
-0.100	0.061	0.059	0.058	0.058	1.500	0.987	0.986	0.825	0.437
0.000	0.050	0.048	0.048	0.048					

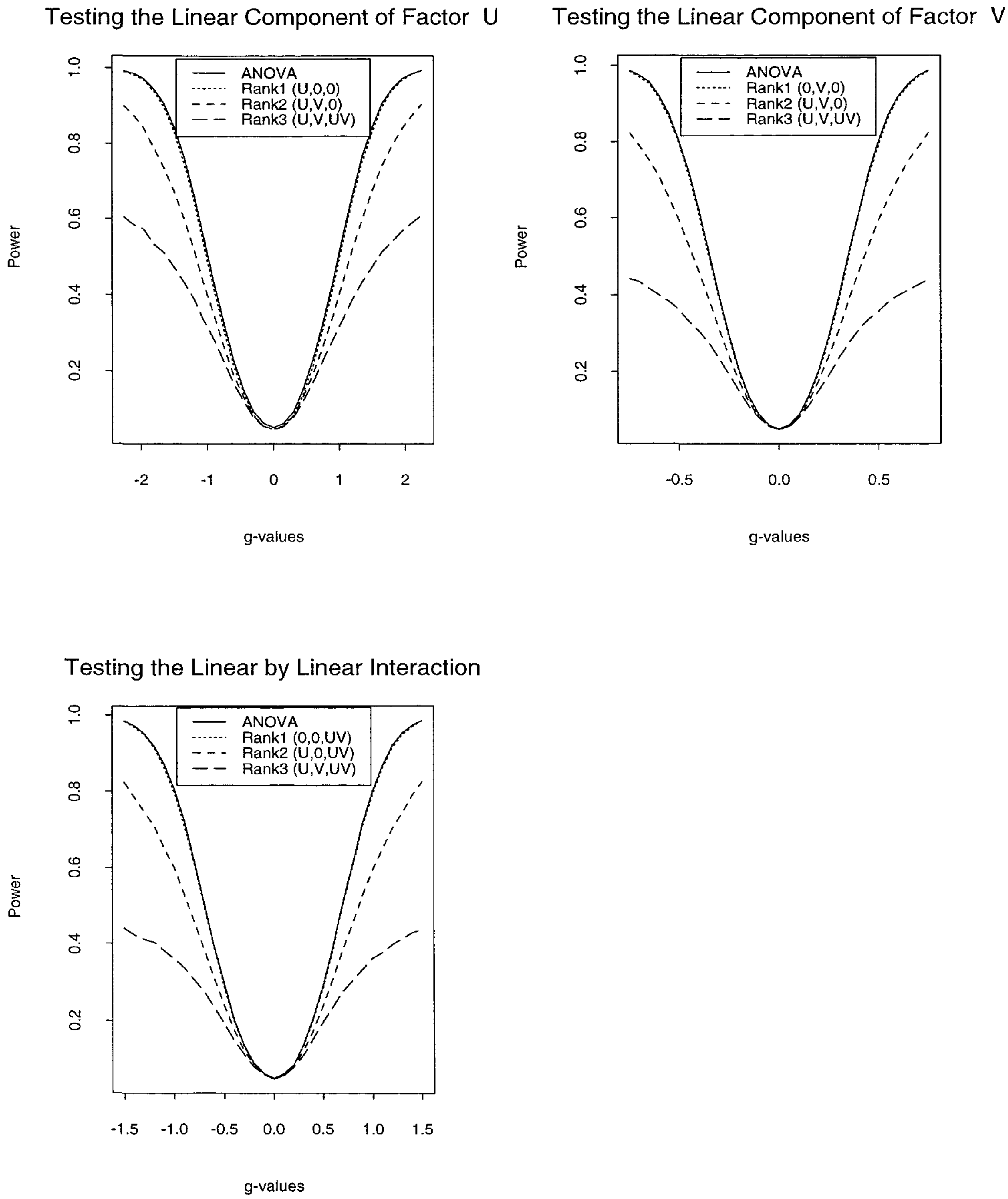


Figure 4.3: Power Comparisons for Design 2 5 2 showing the effect of extraneous components on main effects and interaction under the Ranking Procedure and Normal Distribution.

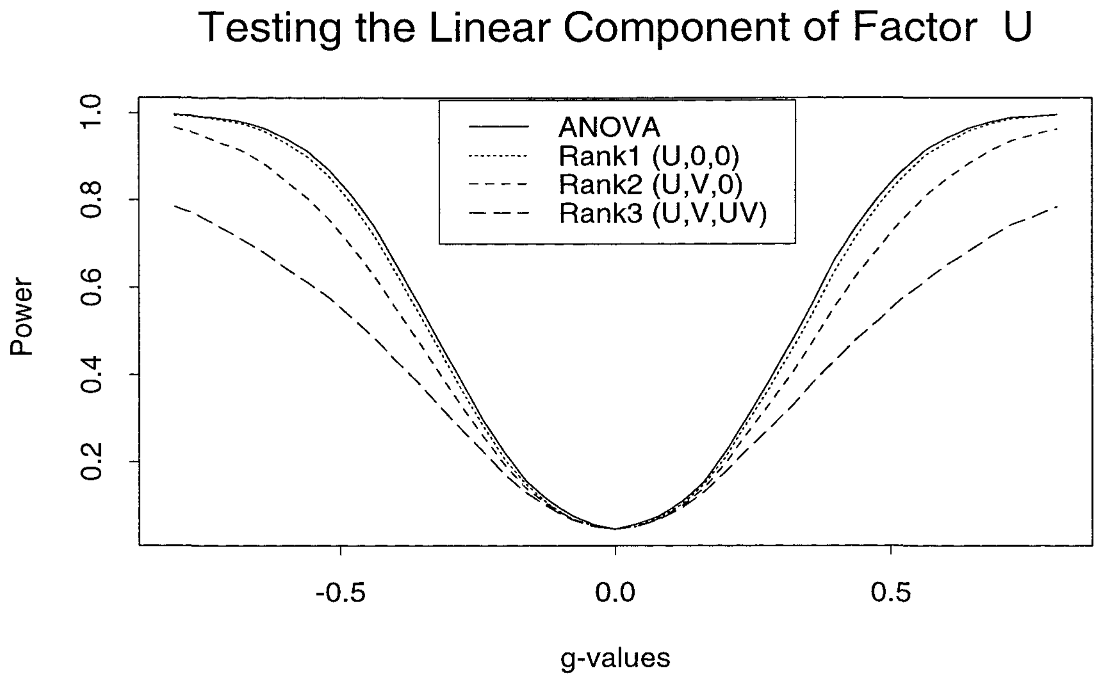
Table 4.23: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear Component of Factor U**

Design Parameters $I = 4, \quad J = 4, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	Rank1	Rank2	Rank3	g_1	ANOVA	Rank1	Rank2	Rank3
-0.800	0.997	0.995	0.967	0.785	0.040	0.058	0.053	0.053	0.053
-0.760	0.994	0.993	0.954	0.767	0.080	0.077	0.073	0.071	0.069
-0.720	0.988	0.985	0.933	0.740	0.120	0.109	0.103	0.099	0.095
-0.680	0.981	0.976	0.915	0.712	0.160	0.153	0.147	0.139	0.130
-0.640	0.967	0.960	0.886	0.682	0.200	0.223	0.211	0.197	0.178
-0.600	0.943	0.932	0.846	0.645	0.240	0.302	0.290	0.265	0.228
-0.560	0.913	0.900	0.805	0.612	0.280	0.386	0.368	0.332	0.276
-0.520	0.869	0.854	0.755	0.576	0.320	0.474	0.453	0.400	0.326
-0.480	0.811	0.792	0.694	0.531	0.360	0.566	0.542	0.477	0.382
-0.440	0.743	0.721	0.629	0.485	0.400	0.666	0.640	0.558	0.433
-0.400	0.657	0.636	0.554	0.433	0.440	0.742	0.719	0.629	0.482
-0.360	0.568	0.548	0.480	0.383	0.480	0.810	0.792	0.692	0.526
-0.320	0.475	0.457	0.404	0.329	0.520	0.866	0.850	0.753	0.573
-0.280	0.386	0.366	0.329	0.275	0.560	0.912	0.900	0.806	0.612
-0.240	0.296	0.283	0.259	0.224	0.600	0.942	0.931	0.848	0.650
-0.200	0.219	0.204	0.192	0.170	0.640	0.965	0.958	0.883	0.682
-0.160	0.154	0.146	0.138	0.128	0.680	0.980	0.975	0.914	0.716
-0.120	0.111	0.103	0.100	0.096	0.720	0.990	0.987	0.938	0.745
-0.080	0.079	0.071	0.071	0.069	0.760	0.993	0.992	0.952	0.763
-0.040	0.057	0.054	0.054	0.053	0.800	0.997	0.996	0.964	0.784
0.000	0.047	0.047	0.047	0.047					

Table 4.24: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 4, \quad J = 4, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-0.720	0.997	0.996	0.956	0.849	0.040	0.059	0.057	0.057	0.057
-0.680	0.995	0.992	0.942	0.821	0.080	0.083	0.082	0.080	0.079
-0.640	0.988	0.985	0.923	0.798	0.120	0.126	0.120	0.116	0.111
-0.600	0.977	0.970	0.892	0.764	0.160	0.185	0.179	0.169	0.161
-0.560	0.959	0.952	0.864	0.733	0.200	0.265	0.259	0.236	0.216
-0.520	0.929	0.917	0.820	0.688	0.240	0.357	0.344	0.308	0.276
-0.480	0.886	0.872	0.767	0.644	0.280	0.455	0.441	0.389	0.341
-0.440	0.828	0.813	0.707	0.588	0.320	0.564	0.545	0.476	0.412
-0.400	0.753	0.734	0.635	0.538	0.360	0.660	0.639	0.556	0.472
-0.360	0.663	0.643	0.555	0.476	0.400	0.757	0.737	0.639	0.542
-0.320	0.561	0.542	0.473	0.409	0.440	0.825	0.811	0.707	0.592
-0.280	0.458	0.439	0.390	0.344	0.480	0.885	0.871	0.768	0.645
-0.240	0.357	0.342	0.312	0.280	0.520	0.929	0.916	0.816	0.688
-0.200	0.264	0.256	0.234	0.214	0.560	0.959	0.951	0.862	0.728
-0.160	0.185	0.177	0.167	0.159	0.600	0.978	0.973	0.897	0.765
-0.120	0.126	0.121	0.117	0.113	0.640	0.988	0.985	0.921	0.798
-0.080	0.082	0.081	0.079	0.077	0.680	0.994	0.991	0.940	0.822
-0.040	0.058	0.058	0.057	0.057	0.720	0.998	0.996	0.956	0.844
0.000	0.051	0.051	0.051	0.051					



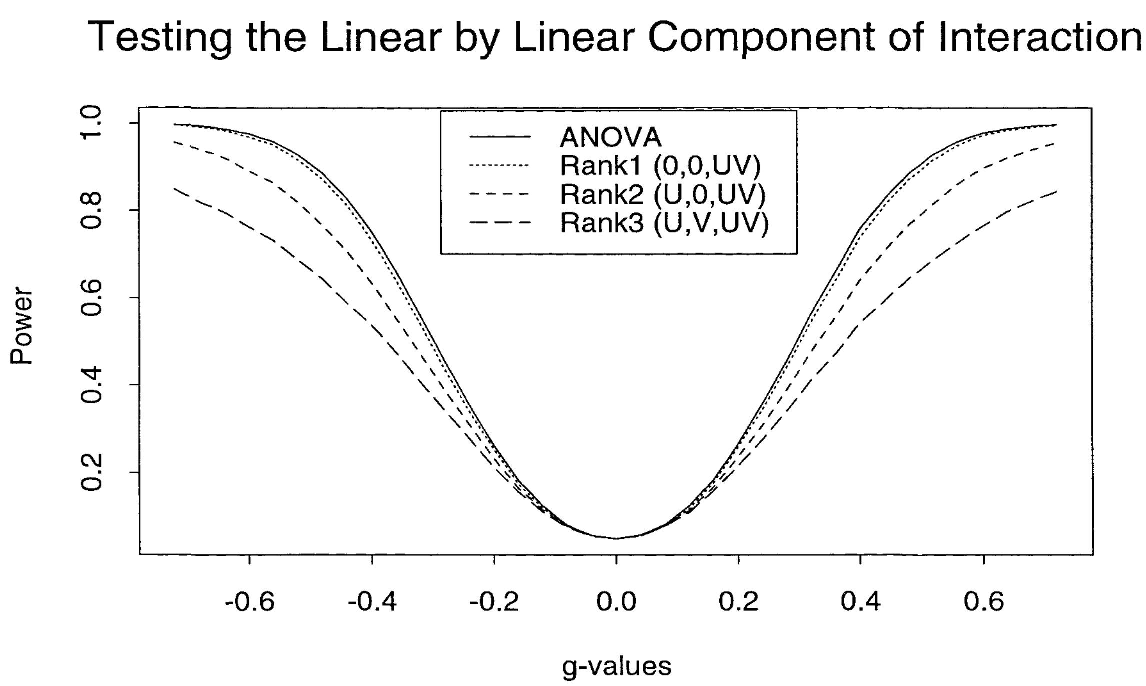


Figure 4.4: Power Comparisons for Design 4 4 2 showing the effect of extraneous components on main effects and interaction under the Ranking Procedure and Normal Distribution.

Table 4.25: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear Component of Factor U**

Design Parameters $I = 4,\quad J = 6,\quad M = 2,\quad \alpha = .05$

g_1	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-0.600	0.993	0.992	0.971	0.911	0.035	0.058	0.057	0.056	0.057
-0.565	0.986	0.983	0.956	0.893	0.071	0.083	0.080	0.079	0.079
-0.529	0.976	0.970	0.935	0.864	0.106	0.125	0.123	0.119	0.116
-0.494	0.955	0.948	0.901	0.820	0.141	0.185	0.181	0.173	0.166
-0.459	0.924	0.914	0.862	0.778	0.176	0.262	0.254	0.239	0.226
-0.424	0.882	0.870	0.810	0.724	0.212	0.347	0.335	0.314	0.290
-0.388	0.822	0.803	0.742	0.664	0.247	0.446	0.431	0.400	0.364
-0.353	0.743	0.725	0.665	0.595	0.282	0.554	0.539	0.496	0.448
-0.318	0.657	0.637	0.585	0.525	0.318	0.659	0.635	0.586	0.528
-0.282	0.563	0.544	0.503	0.456	0.353	0.747	0.727	0.670	0.600
-0.247	0.453	0.436	0.403	0.370	0.388	0.825	0.808	0.748	0.670
-0.212	0.348	0.339	0.317	0.293	0.424	0.882	0.869	0.810	0.724
-0.176	0.256	0.249	0.236	0.222	0.459	0.924	0.915	0.861	0.776
-0.141	0.179	0.172	0.167	0.159	0.494	0.956	0.948	0.903	0.821
-0.106	0.123	0.118	0.115	0.112	0.529	0.976	0.971	0.934	0.860
-0.071	0.078	0.077	0.077	0.076	0.565	0.986	0.984	0.956	0.887
-0.035	0.057	0.056	0.055	0.055	0.600	0.994	0.992	0.971	0.915
0.000	0.051	0.050	0.050	0.050					

Table 4.26: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear Component of Factor V**

Design Parameters $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_2	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-0.400	0.995	0.993	0.977	0.927	0.024	0.060	0.059	0.059	0.059
-0.376	0.990	0.986	0.962	0.901	0.047	0.082	0.080	0.079	0.078
-0.353	0.978	0.976	0.945	0.876	0.071	0.130	0.127	0.124	0.119
-0.329	0.962	0.956	0.916	0.841	0.094	0.188	0.183	0.175	0.169
-0.306	0.938	0.929	0.882	0.801	0.118	0.261	0.256	0.244	0.229
-0.282	0.895	0.885	0.836	0.754	0.141	0.359	0.349	0.326	0.301
-0.259	0.837	0.825	0.768	0.688	0.165	0.460	0.452	0.422	0.384
-0.235	0.762	0.750	0.692	0.619	0.188	0.572	0.558	0.521	0.472
-0.212	0.670	0.656	0.609	0.544	0.212	0.671	0.657	0.610	0.547
-0.188	0.566	0.553	0.513	0.462	0.235	0.763	0.746	0.694	0.616
-0.165	0.466	0.450	0.421	0.384	0.259	0.837	0.823	0.768	0.686
-0.141	0.360	0.350	0.329	0.304	0.282	0.894	0.882	0.832	0.747
-0.118	0.270	0.264	0.250	0.235	0.306	0.934	0.926	0.880	0.799
-0.094	0.188	0.184	0.178	0.170	0.329	0.960	0.954	0.914	0.835
-0.071	0.130	0.124	0.122	0.120	0.353	0.980	0.977	0.946	0.874
-0.047	0.083	0.082	0.082	0.079	0.376	0.989	0.987	0.965	0.901
-0.024	0.060	0.060	0.060	0.059	0.400	0.995	0.994	0.977	0.926
0.000	0.049	0.049	0.049	0.049					

Table 4.27: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 4,\ J = 6,\ M = 2,\ \alpha = .05$

g_3	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-0.340	0.990	0.986	0.942	0.831	0.020	0.056	0.056	0.056	0.056
-0.320	0.983	0.977	0.922	0.806	0.040	0.080	0.077	0.075	0.074
-0.300	0.966	0.960	0.893	0.771	0.060	0.118	0.114	0.111	0.109
-0.280	0.945	0.933	0.860	0.737	0.080	0.173	0.163	0.156	0.148
-0.260	0.909	0.895	0.809	0.689	0.100	0.243	0.235	0.218	0.203
-0.240	0.861	0.843	0.757	0.645	0.120	0.331	0.320	0.292	0.268
-0.220	0.795	0.778	0.693	0.590	0.140	0.429	0.410	0.374	0.334
-0.200	0.716	0.696	0.618	0.531	0.160	0.525	0.504	0.452	0.397
-0.180	0.628	0.605	0.538	0.467	0.180	0.626	0.605	0.539	0.468
-0.160	0.529	0.509	0.456	0.403	0.200	0.720	0.697	0.618	0.531
-0.140	0.427	0.406	0.369	0.331	0.220	0.794	0.775	0.688	0.586
-0.120	0.331	0.320	0.295	0.268	0.240	0.861	0.843	0.756	0.643
-0.100	0.244	0.236	0.222	0.205	0.260	0.913	0.899	0.813	0.694
-0.080	0.173	0.165	0.158	0.151	0.280	0.944	0.934	0.856	0.734
-0.060	0.120	0.114	0.111	0.107	0.300	0.966	0.960	0.893	0.774
-0.040	0.080	0.081	0.079	0.077	0.320	0.983	0.978	0.922	0.804
-0.020	0.056	0.056	0.055	0.056	0.340	0.990	0.986	0.943	0.828
0.000	0.051	0.049	0.049	0.049					

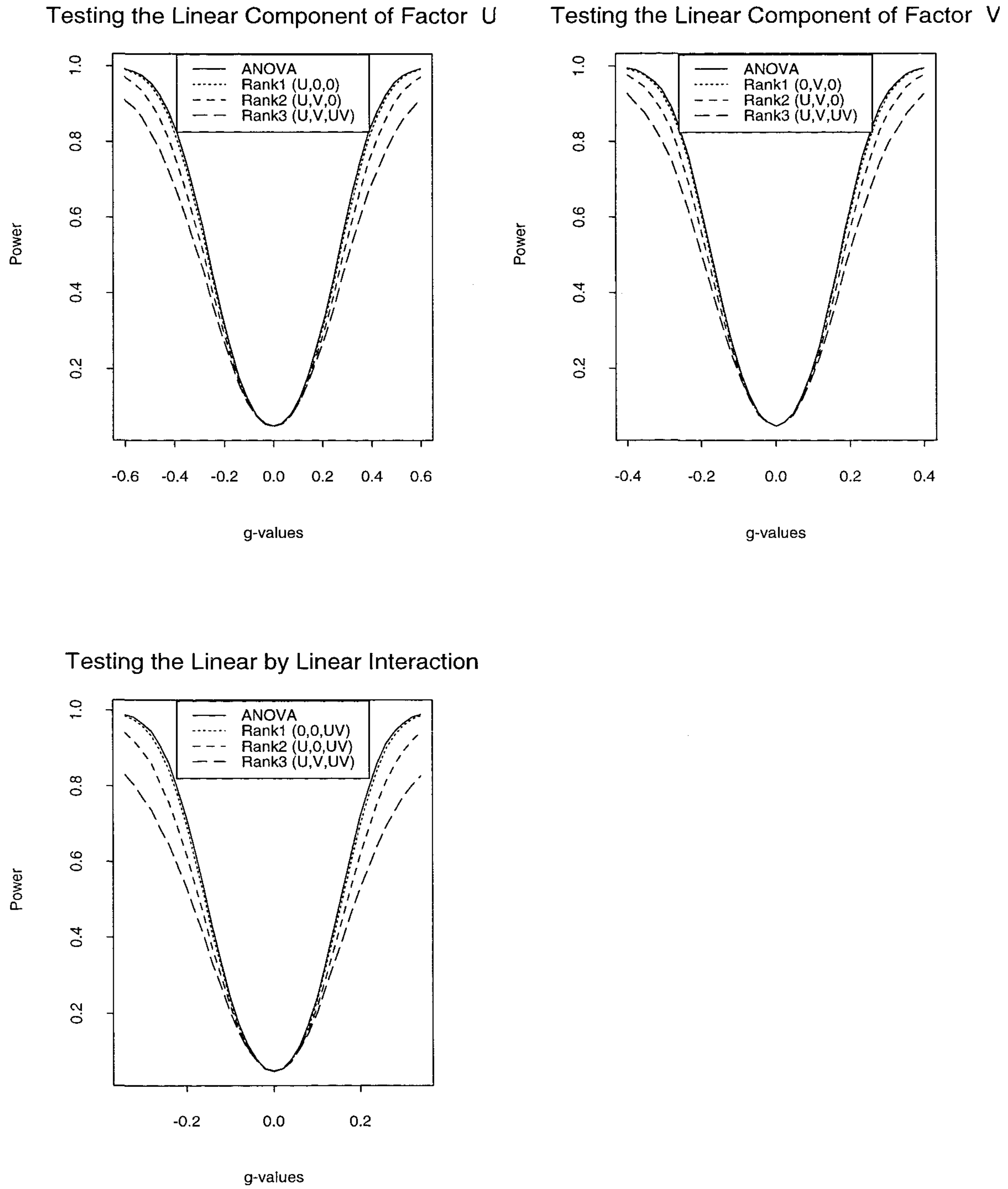


Figure 4.5: Power Comparisons for Design 4 6 2 showing the effect of extraneous components on main effects and interaction under the Ranking Procedure and Normal Distribution.

Table 4.28: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear Component of Factor U**

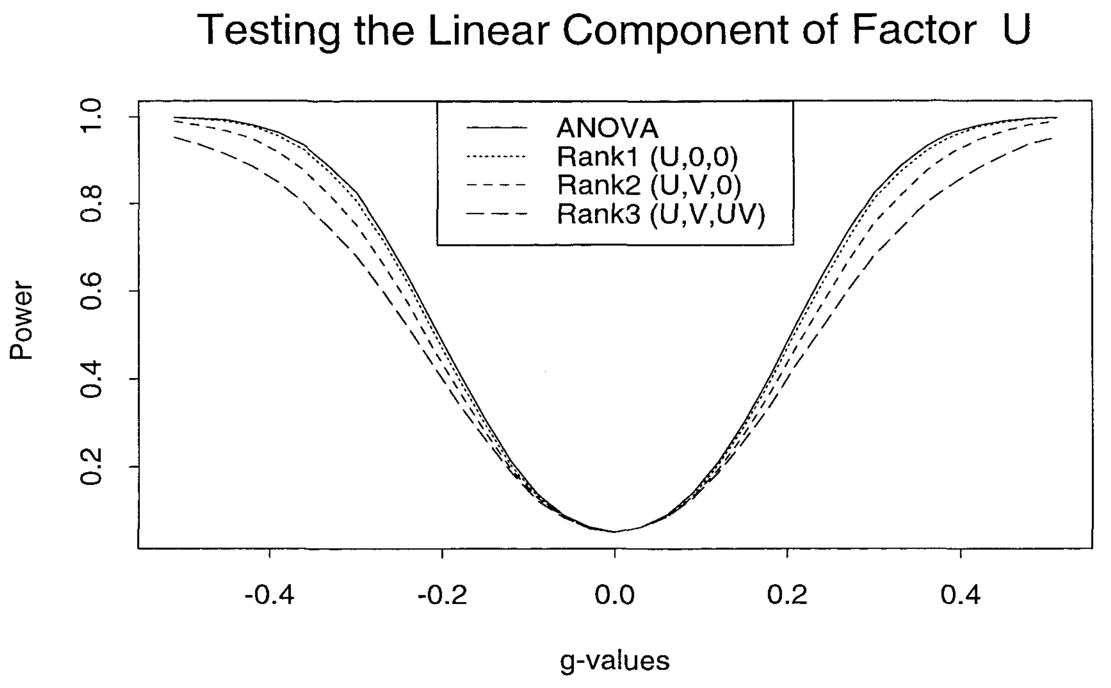
Design Parameters $I = 5,\ \ J = 5,\ \ M = 2,\ \ \alpha = .05$

g_1	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-0.510	0.998	0.998	0.989	0.952	0.030	0.060	0.060	0.060	0.060
-0.480	0.996	0.995	0.980	0.935	0.060	0.088	0.086	0.084	0.084
-0.450	0.992	0.989	0.967	0.912	0.090	0.140	0.132	0.130	0.127
-0.420	0.981	0.978	0.949	0.884	0.120	0.213	0.206	0.195	0.188
-0.390	0.963	0.955	0.917	0.848	0.150	0.301	0.292	0.275	0.260
-0.360	0.934	0.923	0.875	0.800	0.180	0.407	0.395	0.368	0.339
-0.330	0.882	0.870	0.817	0.743	0.210	0.524	0.509	0.472	0.432
-0.300	0.825	0.806	0.752	0.681	0.240	0.636	0.617	0.571	0.517
-0.270	0.735	0.720	0.666	0.602	0.270	0.736	0.718	0.668	0.602
-0.240	0.633	0.617	0.572	0.518	0.300	0.824	0.811	0.755	0.681
-0.210	0.521	0.505	0.466	0.429	0.330	0.885	0.872	0.818	0.741
-0.180	0.411	0.396	0.369	0.340	0.360	0.931	0.921	0.875	0.802
-0.150	0.306	0.294	0.277	0.261	0.390	0.963	0.955	0.917	0.845
-0.120	0.210	0.201	0.192	0.186	0.420	0.980	0.976	0.946	0.881
-0.090	0.137	0.131	0.128	0.122	0.450	0.990	0.987	0.967	0.911
-0.060	0.088	0.084	0.084	0.083	0.480	0.996	0.995	0.982	0.938
-0.030	0.061	0.057	0.057	0.058	0.510	0.998	0.998	0.989	0.952
0.000	0.049	0.049	0.049	0.049					

Table 4.29: **Simulated Powers for ANOVA and the Ranking Method Testing the Linear by Linear Component of Interaction**

Design Parameters $I = 5, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	Rank1	Rank2	Rank3	g_3	ANOVA	Rank1	Rank2	Rank3
-0.340	0.996	0.995	0.961	0.860	0.020	0.059	0.058	0.058	0.057
-0.320	0.993	0.989	0.946	0.840	0.040	0.083	0.083	0.082	0.079
-0.300	0.983	0.980	0.925	0.807	0.060	0.126	0.123	0.120	0.115
-0.280	0.968	0.961	0.891	0.775	0.080	0.187	0.184	0.175	0.163
-0.260	0.940	0.931	0.852	0.732	0.100	0.273	0.260	0.241	0.223
-0.240	0.899	0.885	0.798	0.686	0.120	0.367	0.353	0.321	0.292
-0.220	0.845	0.830	0.739	0.630	0.140	0.478	0.460	0.413	0.369
-0.200	0.770	0.753	0.666	0.573	0.160	0.586	0.566	0.503	0.441
-0.180	0.686	0.662	0.589	0.511	0.180	0.684	0.662	0.587	0.511
-0.160	0.586	0.566	0.502	0.441	0.200	0.775	0.757	0.673	0.579
-0.140	0.474	0.456	0.412	0.367	0.220	0.850	0.833	0.741	0.638
-0.120	0.368	0.358	0.323	0.292	0.240	0.903	0.889	0.801	0.691
-0.100	0.275	0.266	0.247	0.230	0.260	0.942	0.931	0.851	0.733
-0.080	0.193	0.187	0.177	0.166	0.280	0.968	0.961	0.897	0.782
-0.060	0.127	0.125	0.121	0.116	0.300	0.982	0.979	0.920	0.806
-0.040	0.085	0.083	0.080	0.080	0.320	0.991	0.989	0.944	0.839
-0.020	0.056	0.054	0.054	0.054	0.340	0.996	0.994	0.960	0.858
0.000	0.050	0.047	0.047	0.047					



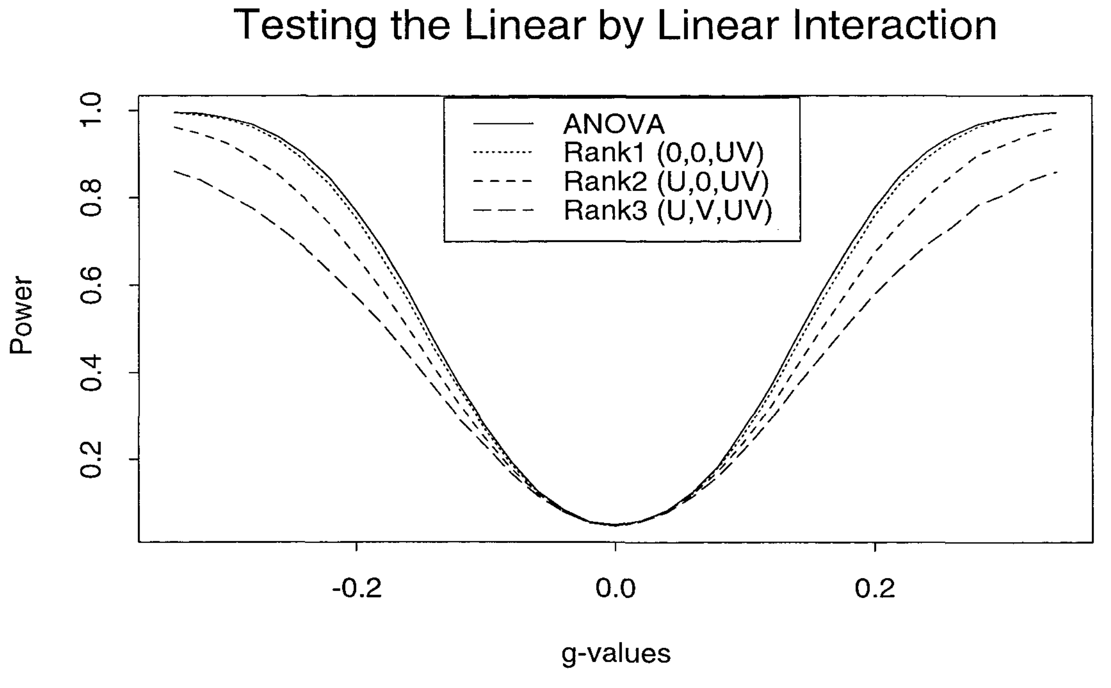


Figure 4.6: Power Comparisons for Design 5 5 2 showing the effect of extraneous components on main effects and interaction under the Ranking Procedure and Normal Distribution.

4.19 Problem with the Variance

In classical analysis of variance procedures the tests of significance are based on the residual mean square. The residual mean square error is an unbiased estimator for the residual variation. This quantity remains valid as an estimate for the variance when non-zero effects are present. Thus the estimate for the variance of each observation is the same whether the null hypothesis is true or not and furthermore when we are using orthogonal contrasts for testing various effects it does not make any difference whether we test each effect in the presence of other effects or alone. That is the reason why the power curve for any effect in the model using the ANOVA method is exactly the same whether the test is conducted alone or in the presence of other non-zero effects.

However, when we are using non-parametric methods the situation is different. If we have K observations, then under the null hypothesis of no treatment effect, the variance of each observation is $\frac{(K-1)(K+1)}{12}$. This is the situation when each observation is equally likely to have any available rank. The important point to observe is that under the alternative hypothesis the above quantity is no longer an unbiased estimator for the residual variation. In fact it will provide an over-estimate for the variance because in this case the variability in some subsets of the data is "constrained"(Shirley, 1986). The problem of over-estimation of variance has been addressed repeatedly in multiple comparison procedures, see for example Steel(1960), Shorack(1967), Shirley(1977), and Williams(1986).

This over-estimation of the variance leads to the loss of power. Furthermore, if we are conducting the test in the presence of other non-zero effects there will be more constraint on the variability and thus the over-estimation of variance will be more severe. Consequently there will be further power loss. In fact one can say that the more non-zero effects present, the more serious is the problem of over-estimation of variance, and the greater the drop in power. In the next chapter we shall use the available data to estimate the variance and use it to test the main effects and the interaction as a means of reducing the variance over-estimation and the consequent power loss. In the next section we show that the variance of any contrast is maximum under the null hypothesis of no treatment effect.

4.20 Variance Reduction Under H_1

We now illustrate that under the alternative hypothesis the variance of a contrast is reduced. To establish this we first apply a heuristic approach to arrive at two conjectures and then prove a lemma. The problem can be put down in the following basic form.

The initial assumption is that we have K identically distributed independent random variables which we denote as $X_1, X_2, \ldots, X_K$. The ranks $R_1, R_2, \ldots, R_K$ are allocated to these sample values corresponding to the increasing order of the $X_i, i = 1, 2, \ldots, K$. The ranks are such that $\sum_{i=1}^{K} R_i = \frac{K(K+1)}{2}$.

Under this assumption the probability function of R_i is

$$P_i(r) = \frac{1}{K} \quad for\ r = 1, 2, \ldots, K. \tag{4.53}$$

for each $i = 1, 2, \ldots, K$. The joint probability function of R_i, R_j is

$$P_{i,j}(r,s) = \frac{1}{K(K-1)} \quad for\ \ r,s = 1,2,\ldots,K, \quad r \neq s \tag{4.54}$$

for $i, j = 1, 2, \ldots, K$ with $i \neq j$. In this case, the variance-covariance matrix of $\mathbf{R} = (R_1, R_2, \ldots, R_K)^T$ is given by $\mathbf{V} = a\mathbf{I} + b\mathbf{J}$ where

$$a + b = Var(R_i) = \frac{K^2 - 1}{12}$$

and

$$b = Cov(R_i, R_j) = -\frac{K+1}{12}.$$

The variance of any contrast $T = \mathbf{c^T R}$, where $\mathbf{c}$ is a vector of coefficients such that $\mathbf{1}^T\mathbf{c} = 0$ (and $\mathbf{c}^T\mathbf{c} = 1$), is given by

$$\begin{aligned} Var(T) = \mathbf{c}^T\mathbf{V}\mathbf{c} &= \mathbf{c}^T(a\mathbf{I} + b\mathbf{J})\mathbf{c} && (4.55) \\ &= a\mathbf{c}^T\mathbf{I}\mathbf{c} + b\mathbf{c}^T\mathbf{J}\mathbf{c} && (4.56) \\ &= a + b.0 && (4.57) \\ &= a && (4.58) \\ i.e. \quad Var(T) &= \frac{K^2-1}{12} + \frac{K+1}{12} = \frac{K(K+1)}{12} && (4.59) \end{aligned}$$

Now suppose that the initial assumption is not true and that the $X_1, X_2, \ldots, X_k$ are K independent observations from K populations with different means. One or more of the populations have means which are different from the others. With no loss of generality suppose that population 1 has a mean greater than populations 2 to K so that X_1 will tend to be higher than the other observations, then its rank will tend to be larger than the ranks of the other observations. The probability function of R_1 will be of the form $p_1(r)$, for $r = 1, 2, \ldots, K$, with $p_1(r) < \frac{1}{K}$ for small values of r and $p_1(r) > \frac{1}{K}$ for larger values of r. In other words, $p_1(r)$ will not be uniform over $r = 1, 2, \ldots, K$. As a consequence of this, $p_i(r)$ will not be uniform for $i = 2, 3. \ldots, K$ either.

We can use similar reasoning for $p_{1,j}(r, s)$ the joint probability function of R_1, R_j. Since the mean of population 1 is greater than the mean of population j, then the observation X_1 will tend to be greater than the observation X_j, then R_1 (the rank of the observation from population 1) will tend to be greater than R_j(the rank of the observation from population j). Thus $p_{1,j}(r, s) > p_{1,j}(s, r)$ for $r > s$ and therefore we do not have the uniformity along the non-diagonal points attained under the assumption of X_i's coming from the same distribution. As a result of this we will not have the uniformity for any other joint probability function of the ranks either.

Having illustrated the properties of the distribution of each random variable R_i and the joint distribution of each two variables R_i and R_j under the alternative assumption we can now compare these distributions with their counterparts under the the initial assumption and arrive at the following two conjectures:

Conjecture 1

$$(Var(R_i) \mid H_1) \leq (Var(R_i) \mid H_0) \qquad (i = 1, 2, \ldots, K)$$

Here H_0 refers to the assumption of identical means and H_1 refers to situations where not all the means are equal.

Conjecture 2

$$(Cov(R_i, R_j) \mid H_1) \geq (Cov(R_i, R_j) \mid H_0) \qquad (i = 1, 2, \ldots, K,\ j = 1, 2, \ldots, K,\ j \neq i)$$

Since the covariances are all negative we can say that the absolute value of each covariance is at its maximum under the assumption of identical means.

The following Lemma establishes the relation between each variance and the covariances.

Lemma 1

$$-Var(R_j) = \sum_{i \neq j} Cov(R_j, R_i) \qquad (j = 1, 2, \ldots, K). \tag{4.60}$$

Proof of Lemma 1:

Since

$$\sum_{i=1}^{K} R_i = K(K+1)/2 \tag{4.61}$$

$$\sum_{i \neq j} R_i = \frac{K(K+1)}{2} - R_j \qquad (j = 1, 2, \ldots, K) \tag{4.62}$$

On taking variance of both sides we obtain:

$$Var(\sum_{i \neq j} R_i) = Var(R_j) \qquad (j = 1, 2, \ldots, K) \tag{4.63}$$

However, since

$$\sum_{i=1}^{K} R_i = \sum_{i \neq j} R_i + R_j \tag{4.64}$$

$$Var(\sum_{i=1}^{K} R_i) = Var(\sum_{i \neq j} R_i) + Var(R_j) + 2Cov(R_j, \sum_{i \neq j} R_i) = 0 \tag{4.65}$$

If we substitute $Var(R_j)$ for $Var(\sum_{i \neq j} R_i)$ we obtain

$$-Var(R_j) = Cov(R_j, \sum_{i \neq j} R_i) = \sum_{i \neq j} Cov(R_j, R_i) \qquad (j = 1, 2, \ldots, K). \tag{4.66}$$

Now we can use Conjectures 1,2, and Lemma 1 to establish the following theorem about the variance of a linear combination of the random variables R_i's.

Theorem:

The variance of a linear combination of the ranks R_i's is maximum under the assumption of K identical distributions.

Proof of the Theorem:

Let $T = \sum_{i=1}^{K} c_i R_i$ be a linear combination of ranks where the c_i's are constants. Then

$$Var(\sum_{i=1}^{K} c_i R_i) = \sum_{i=1}^{K} c_i^2 Var(R_i) + \sum_{i=1}^{K} \sum_{j \neq i} c_i c_j Cov(R_i, R_j). \tag{4.67}$$

The first sum consists of terms which are all positive. The second sum consists of both positive and negative terms depending upon the signs of c_i and c_j. If c_i and c_j have opposite signs, then, with the covariance being, negative the corresponding term will be positive. On the other hand, if the coefficients have the same sign, then the corresponding term will be negative. If all the terms in the second sum are positive then, as a result of Conjecture 1 and Conjecture 2, it follows that the variance of the linear combination would be maximum when all the variances are maximum and all the covariances are minimum which would mean that the underlying distributions are all identical. Now let us assume that there is a negative term in the second summation.

It follows that the product of the corresponding coefficients c_m and c_n is positive. Now suppose that the maximum variance for the linear combination is attained where the corresponding covariance is not at its minimum. Let a denote the increase in the value of the covariance. Then the total increase in the second summation due to this increase in the value of the covariance is $2c_m c_n a$. Now by lemma 1 the increase a in $Cov(R_m, R_n)$ will induce a corresponding decrease a in the variances of R_m and R_n. Thus the corresponding total decrease in the first summation is $(c_m^2 + c_n^2)a$. We note that

$$(c_m^2 + c_n^2)a \geq 2c_m c_n a$$

We have shown that the increase in the second summation due to an increase in one of the covariances is less than the resulting decrease in the first summation. We conclude that the maximum cannot be attained at a point other than when all the covariances are at their minimum. Thus the maximum variance is obtained only when the variances are maximum and the covariances at their minimum, that is under the null hypothesis of K identical distributions. We notice that the result of the theorem applies to any linear combination of the ranks and therefore to any contrast based on the ranks.

To get an idea of the magnitude of the variance reduction involved we give an example for $K = 2$ and $M = 1$. Under the initial assumption, $P_1(r) = 1/2$ and also $P_2(r) = 1/2$ for $r = 1, 2$. $Var(R_1) = Var(R_2) = 1/4$ and $Cov(R_1, R_2) = -1/4$. Thus the variance of the only contrast $R_1 - R_2$ is 1. Now if we change the probabilities we can calculate the variances and the covariance involved and thus work out the variance of the contrast for each set of probabilities under the alternative assumption. Let $P_1(r = 1) = p$, then $Var(R_1) = Var(R_2) = p(1-p)$ and $Cov(R_1, R_2) = -p(1-p)$. Thus $Var(R_1 - R_2) = 4p(1-p)$ and the contrast variance can be calculated in terms of the probability p.

Contrast Variance under H_1 $K = 2,\ M = 1$

Probability	1/2	1/3	1/4	1/5	1/6	1/7
$Var(T)$	1	.89	.75	.65	.55	.49

We can see that as p changes from 1/2 to 1/7 the reduction in variance is more than 50 percent which is fairly large.

4.21 Summary and Conclusions

The origin of our research was a problem in biology that required the design of a blocked factorial experiment involving two factors. Exact measurements were not available and thus we needed a non-parametric method for the analysis. A ranking procedure was proposed for the analysis which was an extension of Friedman's test. The relationship between the two procedures was established. Orthogonal contrasts were used to test the linear components of the main effects and the interaction. Null distributions for normal error were investigated. Exact distributions were obtained for three small designs and simulation methods were employed to find the null distributions for larger designs. We found that unless the design under study is very small we can use the normal distribution as a good approximation to the null distributions. The power performance of the procedure was obtained for a number of designs with varying sizes and a comparison was made with analysis of variance. We have shown that when the errors in the underlying model are normally distributed, the powers are quite comparable if there are no extraneous effects in the model. In fact in this case our procedure reduces to Friedman's test when applied to the linear component of the effect in question and thus the high powers are just another justification of the efficiency of Friedman's test. But if there are other effects present in the model the proposed procedure loses power and with more nuisance effects in the model the power loss becomes substantial for small designs. With the help of two conjectures and a lemma a theorem is proved showing that the variance of a contrast is minimum under the null hypothesis of no treatment effect and thus the power loss observed is shown to be due to over-estimation of the residual variance under the alternative hypothesis. In the next chapter we shall apply a modification to the non-parametric procedure with a view to overcoming this problem.

Chapter 5

Modification of the Proposed Procedure

5.1 Introduction

In chapter 4 we saw that the proposed ranking procedure had shortcomings in that the power of the test on one effect depended on the extraneous factors present in the model. In this chapter, in order to tackle the problem of over-estimation of the contrast variance, we shall estimate the error variance by pooling all the components of interaction except the linear by linear component. The test statistic for each effect is obtained by squaring the relevant contrast and dividing by the error variance. In the next section we shall show the simulated null distributions for a number of designs and this is followed by a section on power comparisons with the analysis of variance procedure.

5.2 Null Distributions

The smallest design for which our modified method could be used is the design 2 3 2 since we cannot apply this method to the design 2 2 2 because here we have just the linear by linear component for the interaction. The largest design considered is the design 5 5 2. The null distributions of the test statistics for testing the main effects and the linear by linear component of the interaction have been obtained for a number of designs in this range. To be able to assess the accuracy of the asymptotic F-distribution in approximating the null distributions, a comparison is made between the simulated distributions and the F-distribution. Samples were generated through a random normal generator using a fortran program. For each design 100,000 sets of data were generated and the value of our proposed non-parametric statistic calculated. The approximate null distributions for a number of designs are shown in Tables 5.1 to 5.7. For design 2 3 2 Table 5.1 shows that the percentage points beyond 95 percent are at infinity. This is due to the fact that the probability of getting a zero in the denominator of the test statistic is about .045. For design 2 4 2 this probability reduces to 0.0027 and we can see that simulated percentage points are much closer to the asymptotic points. For the 95 percent point the simulated value for the linear components is 18.000 compared with the asymptotic value of 18.510. For higher percentage points the differences are more. It can be seen that for larger designs the simulated values are close to the asymptotic values. For design 2 5 2 the simulated 95 percent points range from 10.075 to 10.428 compared with the asymptotic value of 10.130. With increase in design size the approximation improves substantially so that for design 5 5 2 the simulated 95 percent points range from 4.507 to 4.576 which are fairly close to the asymptotic value of 4.543.

Thus apart from designs 232 and 242 the F-distribution provides a fairly good approximation for the null distributions of the test statistics. For designs 232 and 242 one can use the simulated distributions as approximations to the exact distributions.

Table 5.1: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 2, \quad J = 3, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	39.860	161.400	647.800	4052.000	16211.000
Linear U	32.000	392.000	∞	∞	∞
Linear V	40.333	363.000	∞	∞	∞
Quadratic V	49.000	225.000	∞	∞	∞
Lin$U\times$Lin$V$	48.000	363.000	∞	∞	∞

Table 5.2: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 2, \quad J = 4, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	8.526	18.510	38.510	98.500	198.500
Linear U	8.621	18.000	40.000	106.667	250.000
Linear V	8.522	18.000	38.400	98.000	242.000
Quadratic V	8.780	18.000	40.000	111.111	250.000
Cubic V	8.914	19.600	40.500	108.000	280.333
Lin$U\times$Lin$V$	8.244	18.000	40.500	98.000	288.000

Table 5.3: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 3, \quad J = 3, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	5.538	10.130	17.440	34.120	55.550
Linear U	5.505	10.138	17.357	33.750	54.188
Quadratic U	5.586	10.376	17.909	34.615	56.250
Linear V	5.503	10.138	17.455	33.346	54.000
Quadratic V	5.678	10.500	18.000	34.615	57.836
Lin$U\times$Lin$V$	5.491	9.991	17.640	33.923	54.730

Table 5.4: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 2, \quad J = 5, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	5.538	10.130	17.440	34.120	55.550
Linear U	5.580	10.213	17.552	34.714	56.250
Linear V	5.449	10.075	17.455	34.679	55.418
Quadratic V	5.637	10.313	17.857	35.336	56.317
Cubic V	5.616	10.347	17.827	36.000	60.750
Quartic V	5.717	10.428	18.263	35.424	57.677
Lin$U$ x Lin$V$	5.491	10.075	17.361	34.306	54.000

Table 5.5: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 4, \quad J = 4, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.458	5.318	7.571	11.260	14.690
Linear U	3.488	5.360	7.620	11.605	15.037
Quadratic U	3.508	5.461	7.834	11.529	15.367
Cubic U	3.483	5.396	7.701	11.561	15.149
Linear V	3.546	5.427	7.800	11.272	14.352
Quadratic V	3.517	5.470	7.789	11.604	14.936
Cubic V	3.515	5.462	7.779	11.654	15.319
Lin$U\times$Lin$V$	3.553	5.405	7.676	11.612	15.226

Table 5.6: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 4, \quad J = 6, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.102	4.600	6.298	8.862	11.060
Linear U	3.138	4.651	6.416	9.035	11.361
Quadratic U	3.099	4.605	6.427	9.073	11.362
Cubic U	3.136	4.678	6.460	8.985	11.283
Quartic U	3.095	4.603	6.427	9.221	11.608
Linear V	3.164	4.706	6.523	9.097	11.408
Quadratic V	3.112	4.601	6.342	8.892	11.130
Cubic V	3.143	4.757	6.586	9.449	11.954
Quartic V	3.127	4.667	6.470	9.204	11.568
Lin$U\times$Lin$V$	3.113	4.599	6.310	8.760	10.899

Table 5.7: **Simulated Percentage Points of the Distributions of the Test Statistics under the Modified Method**

Design Parameters $I = 5,\ \ J = 5,\ \ M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.073	4.543	6.200	8.683	10.800
Linear U	3.076	4.538	6.266	8.899	11.291
Quadratic U	3.076	4.576	6.247	8.653	10.677
Cubic U	3.074	4.539	6.198	8.820	10.964
Quartic U	3.078	4.533	6.209	8.748	10.922
Linear V	3.058	4.507	6.178	8.691	10.655
Quadratic V	3.073	4.542	6.230	8.912	11.052
Cubic V	3.068	4.551	6.228	8.744	10.972
Quartic V	3.056	4.541	6.219	8.857	11.192
Lin$U\times$Lin$V$	3.100	4.556	6.204	8.768	11.179

5.3 Power Comparisons

A power study was undertaken to analyse the performance of the modified method and to see if the shortcomings of our Ranking Method as described in chapter 4 have been overcome. The power comparisons were performed on the same four designs discussed in the previous chapter and for each design the same six models as in the previous chapter were incorporated to enable us to assess the efficiency of our modification in removing the effect of the nuisance factors. In the Figures used for power comparisons Mod1 refers to the simulated powers under the modified procedure when the only effect included in the model is the effect under study. For testing main effects Mod2 refers to the model when only the two main effects are included in the model and for the test on interaction, Mod2 refers to the model when only the interaction effect and the U effect are included in the model. Similarly Mod3 refers to the simulated powers under the modified procedure when the three effects(linear main effects and the linear by linear interaction) are all included in the model. The power curves Mod1, Mod2, and Mod3 refer to power performance under the modified procedure while ANOVA refers to the power performance for the analysis of variance procedure which is the same under all the models specified above. The power comparisons were performed under the assumption of normal errors and thus standard normal variates were generated and added to the model. As before the number of simulation runs for each calculation was $30,000$.

Tables 5.8 to 5.17 show the power comparisons and Figures 5.1 to 5.4 illustrate the corresponding power curves for the various model comparisons for the four designs indicated.

Table 5.8: **Simulated Powers for ANOVA and the Modified Method Testing the Linear Component of Factor U**

Design Parameters $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	Mod1	Mod2	Mod3	g_1	ANOVA	Mod1	Mod2	Mod3
-2.250	0.993	0.894	0.852	0.584	0.150	0.061	0.058	0.057	0.057
-2.100	0.986	0.856	0.812	0.568	0.300	0.092	0.078	0.076	0.075
-1.950	0.972	0.812	0.762	0.551	0.450	0.153	0.114	0.114	0.111
-1.800	0.945	0.753	0.702	0.517	0.600	0.231	0.156	0.154	0.149
-1.650	0.907	0.686	0.639	0.491	0.750	0.324	0.217	0.213	0.202
-1.500	0.848	0.608	0.570	0.448	0.900	0.435	0.287	0.272	0.252
-1.350	0.767	0.528	0.496	0.408	1.050	0.554	0.361	0.345	0.310
-1.200	0.665	0.442	0.414	0.355	1.200	0.663	0.444	0.415	0.361
-1.050	0.550	0.362	0.339	0.305	1.350	0.766	0.533	0.494	0.410
-0.900	0.435	0.285	0.273	0.254	1.500	0.844	0.609	0.570	0.452
-0.750	0.324	0.212	0.207	0.199	1.650	0.907	0.686	0.642	0.493
-0.600	0.225	0.156	0.152	0.146	1.800	0.944	0.748	0.702	0.519
-0.450	0.149	0.114	0.111	0.109	1.950	0.972	0.811	0.762	0.551
-0.300	0.092	0.078	0.077	0.078	2.100	0.985	0.856	0.812	0.567
-0.150	0.061	0.056	0.056	0.058	2.250	0.994	0.896	0.852	0.586
0.000	0.050	0.051	0.051	0.051					

Table 5.9: **Simulated Powers for ANOVA and the Modified Method Testing the Linear Component of Factor V**

Design Parameters $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_2	ANOVA	Mod1	Mod2	Mod3	g_2	ANOVA	Mod1	Mod2	Mod3
-0.750	0.987	0.830	0.800	0.470	0.050	0.060	0.057	0.057	0.057
-0.700	0.974	0.783	0.757	0.459	0.100	0.088	0.069	0.068	0.068
-0.650	0.957	0.736	0.708	0.438	0.150	0.139	0.099	0.100	0.098
-0.600	0.919	0.670	0.648	0.417	0.200	0.205	0.138	0.138	0.129
-0.550	0.871	0.603	0.580	0.389	0.250	0.291	0.186	0.180	0.166
-0.500	0.802	0.530	0.507	0.360	0.300	0.395	0.243	0.240	0.208
-0.450	0.719	0.455	0.436	0.329	0.350	0.510	0.314	0.305	0.254
-0.400	0.618	0.377	0.368	0.295	0.400	0.610	0.380	0.367	0.293
-0.350	0.511	0.310	0.303	0.253	0.450	0.718	0.457	0.441	0.329
-0.300	0.399	0.244	0.241	0.211	0.500	0.802	0.529	0.513	0.362
-0.250	0.294	0.183	0.181	0.169	0.550	0.872	0.605	0.580	0.392
-0.200	0.204	0.139	0.136	0.129	0.600	0.920	0.670	0.643	0.414
-0.150	0.136	0.100	0.101	0.096	0.650	0.953	0.729	0.703	0.439
-0.100	0.089	0.071	0.072	0.072	0.700	0.974	0.783	0.756	0.453
-0.050	0.059	0.054	0.055	0.054	0.750	0.988	0.831	0.801	0.468
0.000	0.050	0.048	0.048	0.048					

Table 5.10: **Simulated Powers for ANOVA and the Modified Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 2,\ J = 5,\ M = 2,\ \alpha = .05$

g_3	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-1.500	0.987	0.830	0.801	0.472	0.100	0.061	0.056	0.056	0.056
-1.400	0.974	0.782	0.755	0.451	0.200	0.085	0.071	0.070	0.069
-1.300	0.953	0.731	0.702	0.439	0.300	0.138	0.100	0.099	0.097
-1.200	0.919	0.675	0.645	0.420	0.400	0.209	0.139	0.137	0.129
-1.100	0.872	0.606	0.580	0.390	0.500	0.291	0.187	0.184	0.167
-1.000	0.807	0.532	0.513	0.363	0.600	0.398	0.246	0.241	0.209
-0.900	0.723	0.461	0.443	0.335	0.700	0.510	0.314	0.307	0.256
-0.800	0.618	0.379	0.368	0.293	0.800	0.611	0.382	0.366	0.291
-0.700	0.509	0.312	0.305	0.254	0.900	0.716	0.460	0.438	0.329
-0.600	0.395	0.245	0.241	0.211	1.000	0.803	0.533	0.514	0.364
-0.500	0.297	0.185	0.184	0.167	1.100	0.868	0.604	0.579	0.388
-0.400	0.203	0.138	0.138	0.127	1.200	0.921	0.668	0.643	0.414
-0.300	0.136	0.097	0.100	0.096	1.300	0.954	0.728	0.700	0.437
-0.200	0.089	0.073	0.074	0.072	1.400	0.974	0.781	0.752	0.453
-0.100	0.061	0.056	0.056	0.056	1.500	0.987	0.833	0.803	0.467
0.000	0.050	0.049	0.049	0.049					

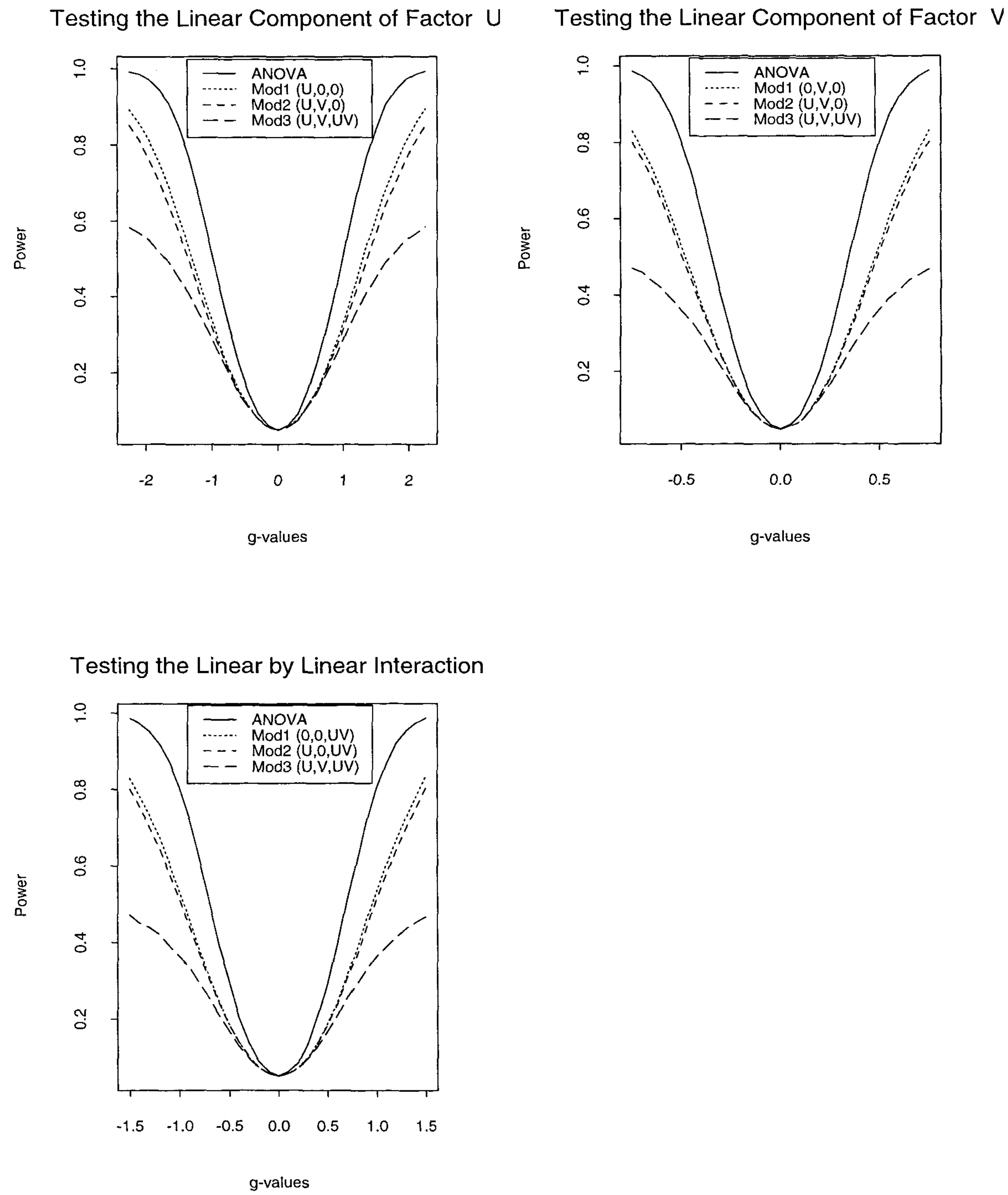


Figure 5.1: Power Comparisons for Design 2 5 2 showing the effect of extraneous components on main effects and interaction under the Modified Procedure and Normal Distribution.

Table 5.11: **Simulated Powers for ANOVA and the Modified Method**

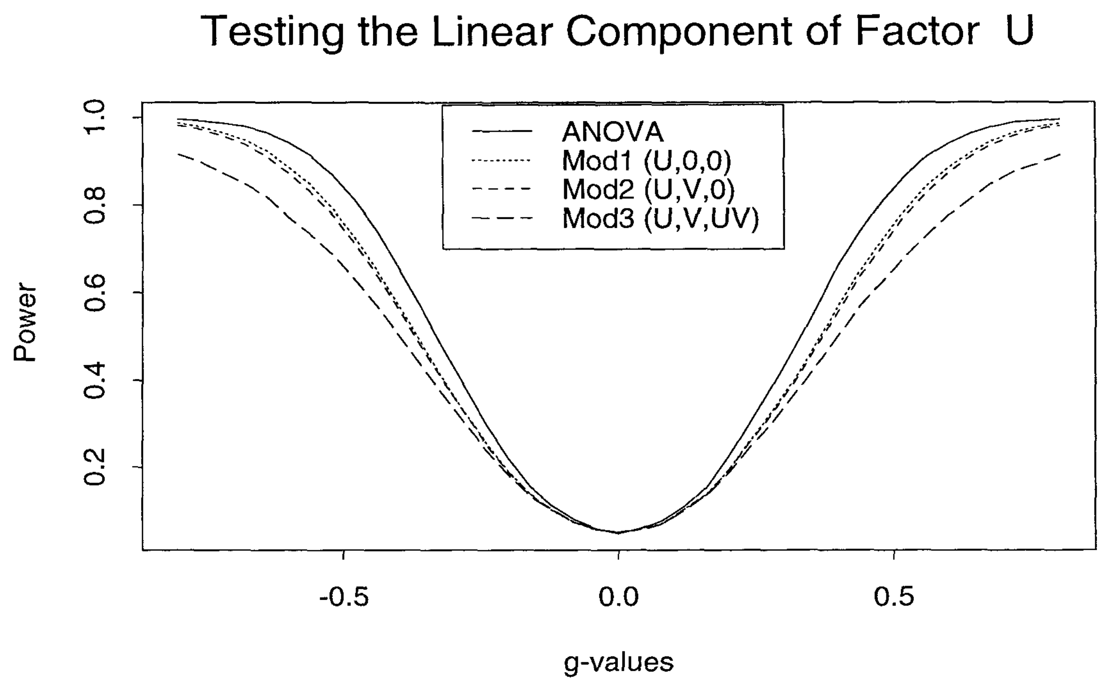
Testing the Linear Component of Factor U

Design Parameters $I = 4, \quad J = 4, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	Mod1	Mod2	Mod3	g_1	ANOVA	Mod1	Mod2	Mod3
-0.800	0.997	0.988	0.983	0.916	0.040	0.058	0.057	0.056	0.055
-0.760	0.994	0.980	0.974	0.900	0.080	0.077	0.070	0.069	0.069
-0.720	0.988	0.967	0.959	0.874	0.120	0.109	0.102	0.102	0.099
-0.680	0.981	0.950	0.940	0.850	0.160	0.153	0.138	0.137	0.135
-0.640	0.967	0.925	0.912	0.820	0.200	0.223	0.192	0.192	0.185
-0.600	0.943	0.887	0.873	0.773	0.240	0.302	0.260	0.257	0.245
-0.560	0.913	0.847	0.830	0.733	0.280	0.386	0.328	0.322	0.302
-0.520	0.869	0.792	0.778	0.687	0.320	0.474	0.401	0.395	0.366
-0.480	0.811	0.726	0.711	0.630	0.360	0.566	0.484	0.477	0.434
-0.440	0.743	0.652	0.641	0.570	0.400	0.666	0.572	0.559	0.502
-0.400	0.657	0.571	0.563	0.503	0.440	0.742	0.651	0.639	0.571
-0.360	0.568	0.488	0.479	0.435	0.480	0.810	0.722	0.707	0.625
-0.320	0.475	0.404	0.401	0.368	0.520	0.866	0.789	0.776	0.683
-0.280	0.386	0.325	0.324	0.301	0.560	0.912	0.845	0.830	0.731
-0.240	0.296	0.255	0.250	0.238	0.600	0.942	0.886	0.875	0.778
-0.200	0.219	0.189	0.187	0.181	0.640	0.965	0.921	0.912	0.816
-0.160	0.154	0.137	0.137	0.133	0.680	0.980	0.948	0.940	0.852
-0.120	0.111	0.100	0.102	0.101	0.720	0.990	0.967	0.960	0.878
-0.080	0.079	0.071	0.072	0.072	0.760	0.993	0.979	0.973	0.894
-0.040	0.057	0.055	0.055	0.056	0.800	0.997	0.987	0.982	0.915
0.000	0.047	0.050	0.050	0.050					

Table 5.12: **Simulated Powers for ANOVA and the Modified Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 4, \quad J = 4, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-0.720	0.997	0.986	0.970	0.933	0.040	0.059	0.057	0.056	0.057
-0.680	0.995	0.975	0.955	0.909	0.080	0.083	0.077	0.076	0.077
-0.640	0.988	0.960	0.936	0.885	0.120	0.126	0.109	0.108	0.106
-0.600	0.977	0.933	0.905	0.849	0.160	0.185	0.156	0.154	0.151
-0.560	0.959	0.903	0.870	0.811	0.200	0.265	0.222	0.216	0.211
-0.520	0.929	0.858	0.821	0.767	0.240	0.357	0.294	0.287	0.278
-0.480	0.886	0.798	0.763	0.708	0.280	0.455	0.379	0.363	0.348
-0.440	0.828	0.734	0.697	0.646	0.320	0.564	0.472	0.451	0.428
-0.400	0.753	0.652	0.616	0.581	0.360	0.660	0.558	0.533	0.502
-0.360	0.663	0.559	0.535	0.503	0.400	0.757	0.653	0.621	0.581
-0.320	0.561	0.469	0.450	0.425	0.440	0.825	0.732	0.696	0.650
-0.280	0.458	0.380	0.365	0.350	0.480	0.885	0.801	0.761	0.712
-0.240	0.357	0.298	0.288	0.279	0.520	0.929	0.853	0.819	0.765
-0.200	0.264	0.219	0.215	0.208	0.560	0.959	0.901	0.869	0.812
-0.160	0.185	0.157	0.153	0.154	0.600	0.978	0.938	0.907	0.855
-0.120	0.126	0.109	0.110	0.108	0.640	0.988	0.960	0.934	0.886
-0.080	0.082	0.078	0.078	0.076	0.680	0.994	0.975	0.954	0.910
-0.040	0.058	0.057	0.057	0.056	0.720	0.998	0.986	0.970	0.930
0.000	0.051	0.052	0.052	0.052					



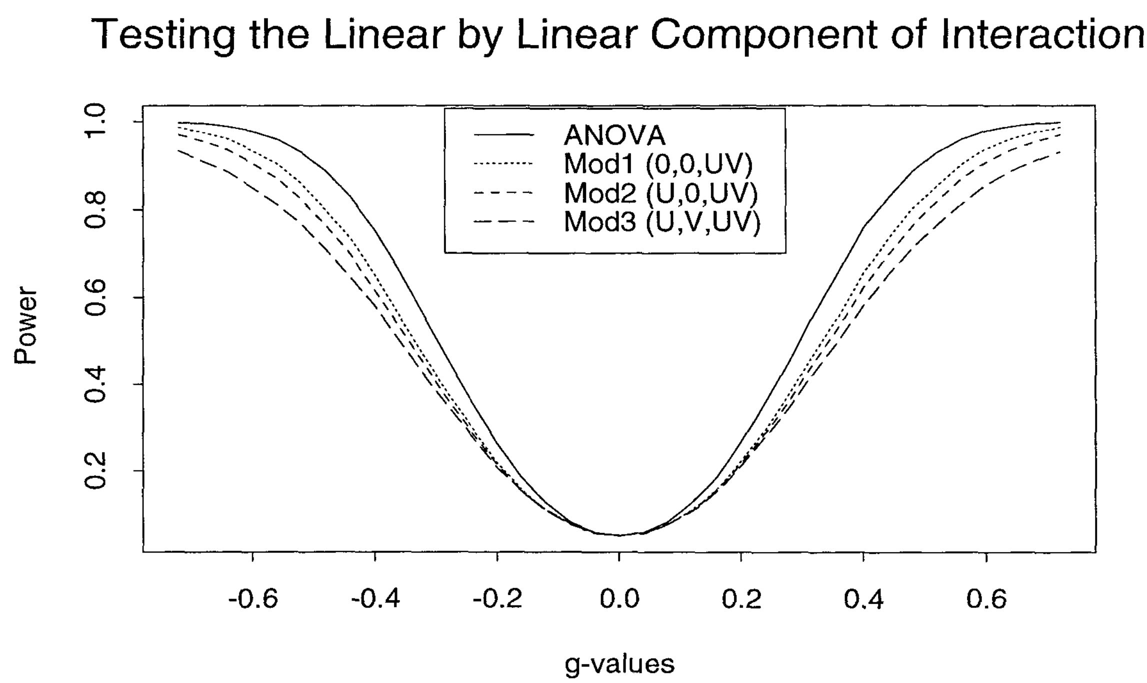


Figure 5.2: Power Comparisons for Design 4 4 2 showing the effect of extraneous components on main effects and interaction under the Modified Procedure and Normal Distribution.

Table 5.13: **Simulated Powers for ANOVA and the Modified Method Testing the Linear Component of Factor U**

Design Parameters $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-0.600	0.993	0.986	0.983	0.962	0.035	0.058	0.059	0.059	0.059
-0.565	0.986	0.975	0.972	0.946	0.071	0.083	0.079	0.079	0.080
-0.529	0.976	0.955	0.950	0.920	0.106	0.125	0.117	0.117	0.114
-0.494	0.955	0.927	0.921	0.886	0.141	0.185	0.169	0.170	0.168
-0.459	0.924	0.888	0.882	0.847	0.176	0.262	0.237	0.237	0.232
-0.424	0.882	0.838	0.830	0.790	0.212	0.347	0.316	0.314	0.307
-0.388	0.822	0.767	0.760	0.726	0.247	0.446	0.401	0.400	0.386
-0.353	0.743	0.687	0.681	0.650	0.282	0.554	0.507	0.502	0.482
-0.318	0.657	0.604	0.597	0.570	0.318	0.659	0.600	0.597	0.571
-0.282	0.563	0.510	0.506	0.488	0.353	0.747	0.687	0.683	0.654
-0.247	0.453	0.408	0.405	0.392	0.388	0.825	0.774	0.765	0.731
-0.212	0.348	0.319	0.316	0.309	0.424	0.882	0.837	0.831	0.791
-0.176	0.256	0.234	0.232	0.229	0.459	0.924	0.888	0.883	0.845
-0.141	0.179	0.164	0.165	0.164	0.494	0.956	0.928	0.924	0.886
-0.106	0.123	0.114	0.115	0.114	0.529	0.976	0.958	0.952	0.922
-0.071	0.078	0.077	0.077	0.076	0.565	0.986	0.974	0.970	0.945
-0.035	0.057	0.057	0.056	0.056	0.600	0.994	0.985	0.983	0.964
0.000	0.051	0.051	0.051	0.051					

Table 5.14: **Simulated Powers for ANOVA and the Modified Method Testing the Linear Component of Factor V**

Design Parameters $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_2	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-0.400	0.995	0.988	0.986	0.968	0.024	0.060	0.057	0.057	0.058
-0.376	0.990	0.978	0.974	0.950	0.047	0.082	0.077	0.077	0.076
-0.353	0.978	0.961	0.958	0.930	0.071	0.130	0.120	0.120	0.118
-0.329	0.962	0.937	0.930	0.897	0.094	0.188	0.171	0.172	0.170
-0.306	0.938	0.903	0.898	0.859	0.118	0.261	0.239	0.239	0.234
-0.282	0.895	0.854	0.848	0.810	0.141	0.359	0.320	0.318	0.309
-0.259	0.837	0.787	0.780	0.743	0.165	0.460	0.418	0.417	0.401
-0.235	0.762	0.708	0.699	0.666	0.188	0.572	0.518	0.517	0.495
-0.212	0.670	0.615	0.611	0.584	0.212	0.671	0.617	0.611	0.585
-0.188	0.566	0.513	0.510	0.487	0.235	0.763	0.704	0.699	0.665
-0.165	0.466	0.416	0.415	0.401	0.259	0.837	0.785	0.778	0.743
-0.141	0.360	0.324	0.321	0.313	0.282	0.894	0.852	0.847	0.807
-0.118	0.270	0.245	0.244	0.240	0.306	0.934	0.901	0.895	0.859
-0.094	0.188	0.173	0.172	0.170	0.329	0.960	0.935	0.930	0.897
-0.071	0.130	0.118	0.118	0.118	0.353	0.980	0.961	0.957	0.928
-0.047	0.083	0.079	0.078	0.078	0.376	0.989	0.978	0.975	0.949
-0.024	0.060	0.058	0.058	0.058	0.400	0.995	0.989	0.986	0.967
0.000	0.049	0.051	0.051	0.051					

Table 5.15: **Simulated Powers for ANOVA and the Modified Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-0.340	0.990	0.978	0.963	0.932	0.020	0.056	0.057	0.057	0.057
-0.320	0.983	0.963	0.946	0.908	0.040	0.080	0.077	0.076	0.075
-0.300	0.966	0.939	0.918	0.877	0.060	0.118	0.110	0.110	0.108
-0.280	0.945	0.909	0.885	0.840	0.080	0.173	0.155	0.154	0.152
-0.260	0.909	0.865	0.837	0.793	0.100	0.243	0.217	0.212	0.207
-0.240	0.861	0.807	0.778	0.735	0.120	0.331	0.296	0.289	0.281
-0.220	0.795	0.735	0.709	0.671	0.140	0.429	0.381	0.370	0.355
-0.200	0.716	0.656	0.632	0.597	0.160	0.525	0.468	0.451	0.433
-0.180	0.628	0.566	0.548	0.518	0.180	0.626	0.565	0.547	0.519
-0.160	0.529	0.473	0.459	0.439	0.200	0.720	0.655	0.632	0.597
-0.140	0.427	0.377	0.367	0.354	0.220	0.794	0.735	0.706	0.668
-0.120	0.331	0.298	0.290	0.282	0.240	0.861	0.807	0.778	0.732
-0.100	0.244	0.221	0.218	0.211	0.260	0.913	0.868	0.839	0.794
-0.080	0.173	0.158	0.154	0.153	0.280	0.944	0.908	0.882	0.841
-0.060	0.120	0.112	0.110	0.109	0.300	0.966	0.940	0.918	0.878
-0.040	0.080	0.079	0.078	0.079	0.320	0.983	0.965	0.947	0.910
-0.020	0.056	0.055	0.055	0.056	0.340	0.990	0.978	0.965	0.931
0.000	0.051	0.050	0.050	0.050					

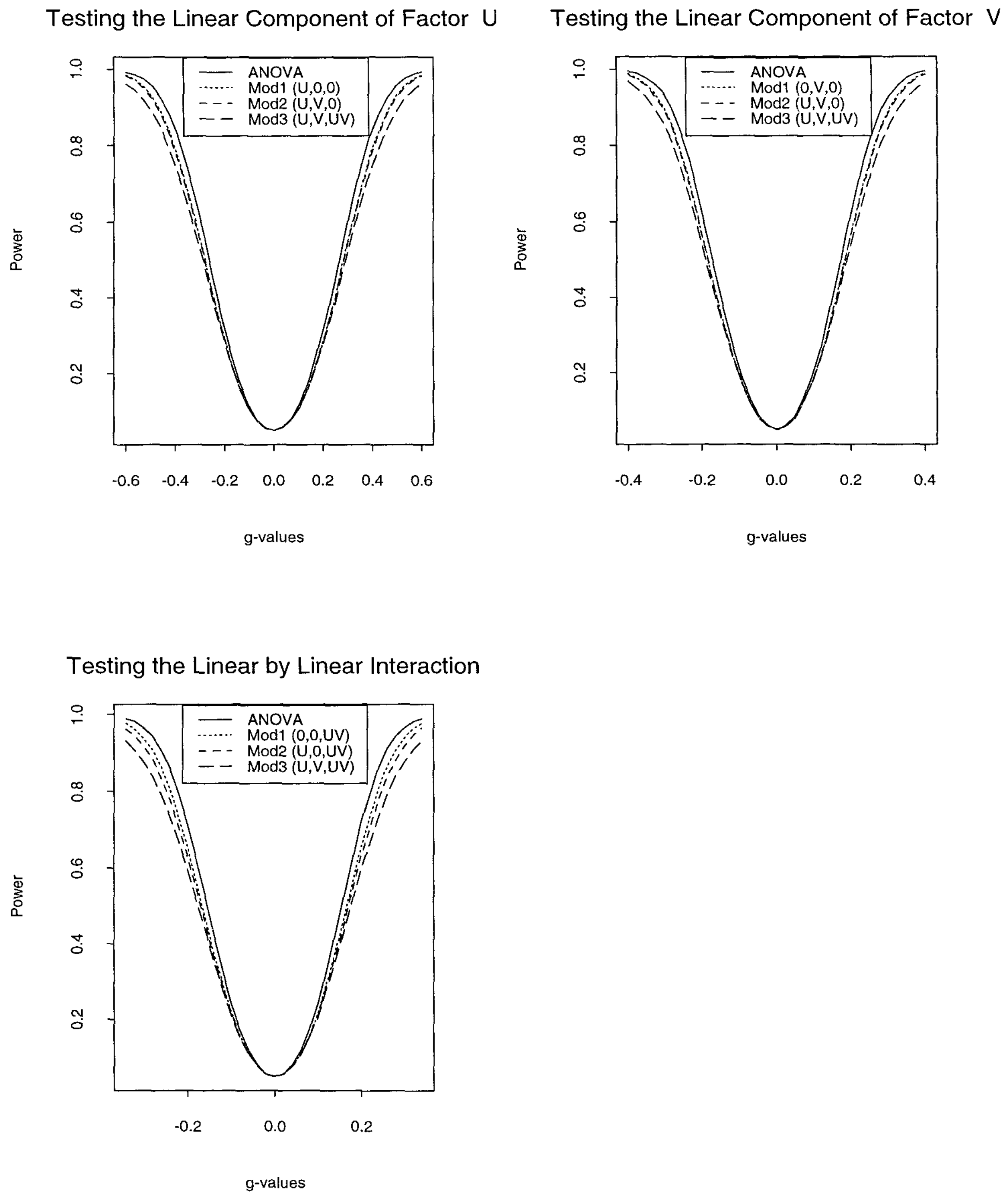


Figure 5.3: Power Comparisons for Design 4 6 2 showing the effect of extraneous components on main effects and interaction under the Modified Procedure and Normal Distribution.

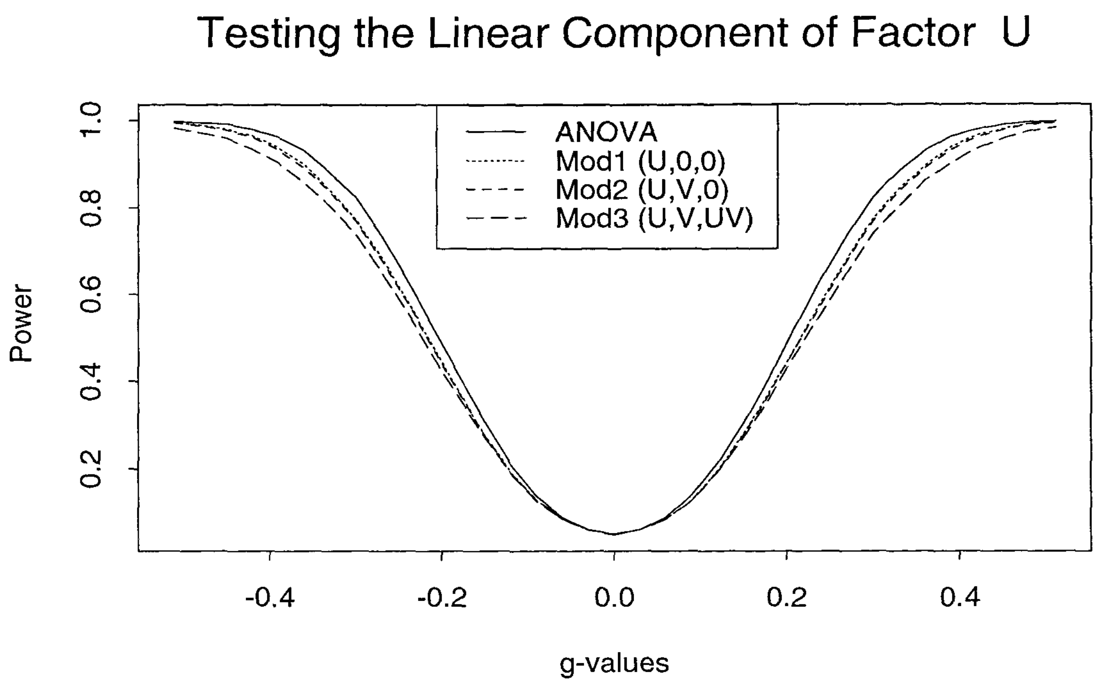
Table 5.16: **Simulated Powers for ANOVA and the Modified Method Testing the Linear Component of Factor** U

Design Parameters $I = 5, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-0.510	0.998	0.996	0.995	0.983	0.030	0.060	0.061	0.061	0.060
-0.480	0.996	0.990	0.988	0.973	0.060	0.088	0.084	0.084	0.084
-0.450	0.992	0.981	0.979	0.960	0.090	0.140	0.127	0.127	0.127
-0.420	0.981	0.967	0.963	0.936	0.120	0.213	0.195	0.193	0.192
-0.390	0.963	0.940	0.935	0.905	0.150	0.301	0.278	0.276	0.269
-0.360	0.934	0.901	0.893	0.860	0.180	0.407	0.374	0.370	0.359
-0.330	0.882	0.842	0.838	0.805	0.210	0.524	0.476	0.475	0.461
-0.300	0.825	0.773	0.770	0.739	0.240	0.636	0.583	0.577	0.555
-0.270	0.735	0.685	0.679	0.651	0.270	0.736	0.683	0.679	0.651
-0.240	0.633	0.584	0.581	0.557	0.300	0.824	0.776	0.771	0.741
-0.210	0.521	0.477	0.472	0.454	0.330	0.885	0.844	0.838	0.804
-0.180	0.411	0.374	0.371	0.360	0.360	0.931	0.897	0.892	0.863
-0.150	0.306	0.275	0.274	0.269	0.390	0.963	0.940	0.934	0.904
-0.120	0.210	0.193	0.191	0.190	0.420	0.980	0.966	0.961	0.936
-0.090	0.137	0.127	0.128	0.125	0.450	0.990	0.981	0.978	0.958
-0.060	0.088	0.085	0.085	0.085	0.480	0.996	0.991	0.990	0.975
-0.030	0.061	0.059	0.059	0.059	0.510	0.998	0.996	0.995	0.984
0.000	0.049	0.051	0.051	0.051					

Table 5.17: **Simulated Powers for ANOVA and the Modified Method Testing the Linear by Linear Component of Interaction Design Parameters** $I = 5, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	Mod1	Mod2	Mod3	g_3	ANOVA	Mod1	Mod2	Mod3
-0.340	0.996	0.990	0.979	0.954	0.020	0.059	0.056	0.056	0.056
-0.320	0.993	0.981	0.968	0.938	0.040	0.083	0.080	0.080	0.079
-0.300	0.983	0.968	0.950	0.915	0.060	0.126	0.116	0.116	0.117
-0.280	0.968	0.944	0.921	0.881	0.080	0.187	0.173	0.170	0.166
-0.260	0.940	0.906	0.880	0.841	0.100	0.273	0.243	0.241	0.233
-0.240	0.899	0.855	0.827	0.784	0.120	0.367	0.326	0.318	0.309
-0.220	0.845	0.794	0.764	0.724	0.140	0.478	0.430	0.415	0.400
-0.200	0.770	0.712	0.686	0.652	0.160	0.586	0.529	0.509	0.487
-0.180	0.686	0.625	0.601	0.571	0.180	0.684	0.624	0.599	0.574
-0.160	0.586	0.529	0.508	0.486	0.200	0.775	0.718	0.694	0.655
-0.140	0.474	0.425	0.414	0.397	0.220	0.850	0.795	0.766	0.727
-0.120	0.368	0.332	0.325	0.313	0.240	0.903	0.860	0.829	0.789
-0.100	0.275	0.250	0.244	0.238	0.260	0.942	0.909	0.881	0.839
-0.080	0.193	0.175	0.173	0.170	0.280	0.968	0.944	0.923	0.885
-0.060	0.127	0.119	0.117	0.116	0.300	0.982	0.966	0.946	0.910
-0.040	0.085	0.080	0.079	0.079	0.320	0.991	0.982	0.968	0.936
-0.020	0.056	0.054	0.054	0.054	0.340	0.996	0.989	0.980	0.951
0.000	0.050	0.048	0.048	0.048					



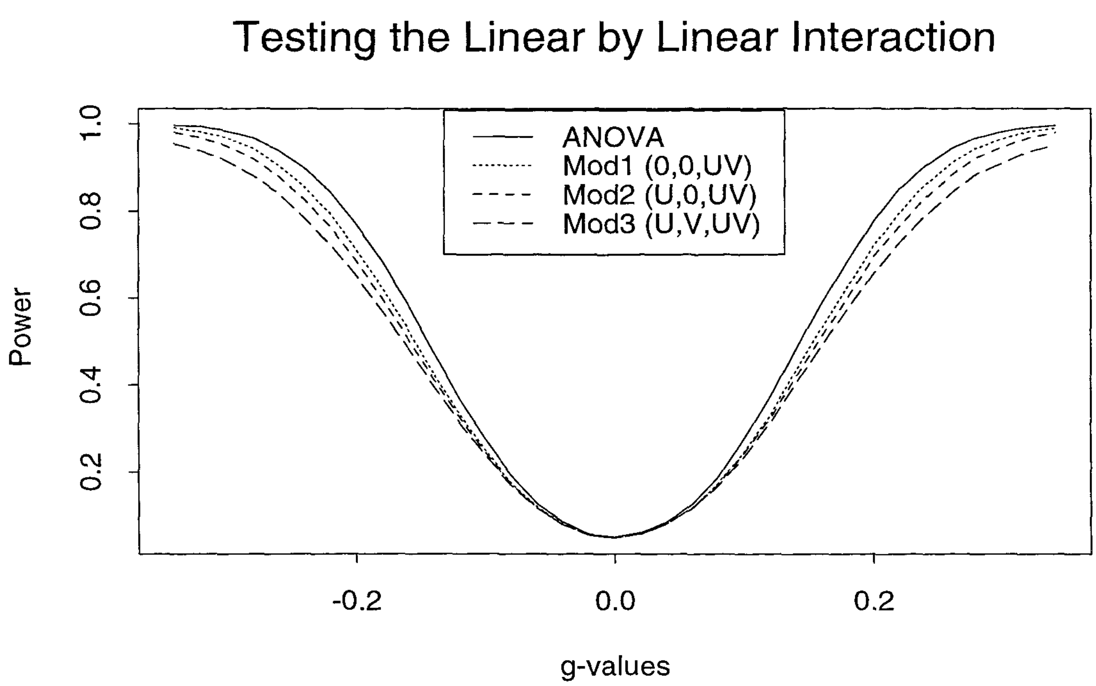


Figure 5.4: Power Comparisons for Design 5 5 2 showing the effect of extraneous components on main effects and interaction under the Modified Procedure and Normal Distribution.

Tables 5.8, 5.9, 5.10 and Fig 5.1 show the power comparisons for ANOVA and the Modified Method for testing the linear main effects and the linear by linear component of interaction under models incorporating different extraneous effects when the underlying design is 252. As before the power curves for the analysis of variance procedures remain the same under different models whether extraneous effects are present or not, while the power curves for the Modified Method depend heavily on the presence of extraneous effects. For the factor U, Mod1 shows the powers when only the effect under study, namely, factor U is included in the model while mod2 shows the power for U when the main effect V is included in the model as well and finally Mod3 is the power curve for U in the presence of both the main effect V and the interaction effect UV.

A comparison of the power performance of the Ranking Method and the Modified Method for testing the U effect in design 252 can be made by referring to figures 4.3 and 5.1 which indicate that the Ranking Method performs better than the modified method both in the presence and absence of extraneous effects. For $g_1 = -2.250$ the power under Rank1 is .992 compared with the value of .894 under Mod1. For the full model the power is .605 under Rank3 compared with the value of .584 under Mod3.

The difference between Mod1 and Mod2 in Fig 4.3 is less than the corresponding difference between Rank1 and Rank2 in Fig 5.1. It seems that with the modified procedure the test for factor U is less affected by the presence of factor V compared with the corresponding test under the Ranking Method. For $g_1 = -2.25$ the reduction in power due to the presence of factor V is 9 percent for the Ranking Method compared with a 4 percent reduction for the Modified Method. However the addition of interaction causes the gap to widen and we notice that we have not been able to remove the effect of the extraneous factors in the model. The same distinctions are noted when we compare the power performances for testing the V effect and also the interaction effect UV. Thus we note that as far as the design 252 is concerned the Modified Method has had an adverse effect on power performance in that the modification has not only resulted in power loss but also our main purpose for removing the effect of the extraneous factors has not been fulfilled either.

Tables 5.11, 5.12 and Fig 5.2 show the power curves under the Modified Method for the design 442. Comparing them with those in Fig 4.4 for the Ranking Method the following observations can be noted:

1- The powers for Rank1 are greater than those for Mod1 which indicates that the Ranking Method is still performing better than the Modified Method when there are no extraneous effects present.

2- The powers for Mod2 are greater than those for Rank2 and also the powers for Mod3 are greater than those for Rank3 which would indicate that in the presence of extraneous effects the Modified Method performs better than the Ranking Method.

3- The differences between the power curves for the Modified Method are less than the corresponding differences for the Ranking Method and this would indicate that relative to the Ranking Method the Modified Method is less affected by the presence of extraneous effects. In fact, the difference between Mod1 and Mod2 is almost negligible which shows that the test on one main effect is not much affected by the presence of the other factor in the model.

Similar distinctions are made if we compare the powers for the interaction effect. The Ranking Method performs better than the Modified Method when there are no main effects but when main effects are present the Modified Method performs better. In the same way as noticed for the main effects, the test for interaction under the Modified Method is also less affected by the presence of main effects. Thus we can see that although the Ranking Method shows superiority in the absence of other effects in the model, nevertheless, with this increase in design size the Modified Method not only gains power but also is less affected by the nuisance effects.

Tables 5.13, 5.14, 5.15 and Figure 5.3 illustrate the power performance curves for the main effects and interaction for design 462 under the Modified Method. Again, comparison of the performance curves with their counterparts for the Ranking Method, in Figure 4.5 demonstrate that apart from the situation where there are no nuisance effects there is some gain in power for the Modified Method and, for the test on main effects, the effect of the extraneous main effect almost vanishes and it is only the presence of interaction that lowers the powers. Finally Tables 5.16, 5.17 and Figure 5.4 show the performance of the tests on main effect U and interaction UV respectively for the design 552 under the Modified Method. Again comparison with the corresponding Figure 4.6 shows that, in the absence of nuisance effects, the Ranking Method still demonstrates superiority but the difference between the two methods becomes almost negligible with the increasing values of g. For this increased design size there is further gain in power and the effects of nuisance factors become less. For the U effect, when $g_1 = -.510$ the power under the ANOVA is .998 compared with the value of .996 under Mod1 and .983 under the full model, a reduction of 1.2 percent in power only.

5.4 Conclusions

A modified form of our proposed Ranking Method was introduced. The modification was obtained by estimating the error variance by pooling the nonlinear components of the interaction with a view to removing the over-estimation of the contrast variance when testing one effect in the presence of other effects. Our study of the power curves for a number of designs of varying sizes showed that for small designs such as 252 the modification has adverse effects. Not only have we not been able to remove the effect of the nuisance factors but there is a drop in power as well when we compare the power performance of our modified procedure with that of our original proposed Ranking Method based on a partition of Friedman's test statistic.

However with increase in design size the performance of the modified test improves.

Although in the absence of nuisance effects the Ranking Method still has higher powers, there is a gain in power under the modified method when nuisance effects exist. Furthermore, for the tests on main effects the reduction in power due to the extraneous effect of the other factor becomes negligible. In general the effect of the nuisance factors diminishes so that for a design such as 552 the Modified Method can be a good substitute for ANOVA. However our modified procedure has shortcomings in that it is not suitable for small designs and the problem of nuisance effects still exists, although the drop in performance is rather small with increase in design size. In the next section we shall introduce yet another modification to overcome the shortcomings of our modified procedure.

Chapter 6

The Analogue

6.1 Introduction

In this chapter we attempt to overcome the difficulties encountered with both the extension to Friedman's test(ranking) and the modified version of this test(modified) by estimating the contrast error variance using the blocking structure of the experiment. Here we estimate the error variance for each treatment using the ranks of the corresponding observations in the blocks and averaging over all treatments, i.e.

$$\hat{\sigma}^2 = \sum_{k=1}^{K} \sum_{m=1}^{M} (R_{km} - \bar{R}_{k\cdot})^2 / (K-1)(M-1).$$

The test statistic for each effect is obtained by squaring the relevant contrast and dividing by the estimated error variance. The resulting statistics are shown to have F distributions under the null hypothesis. One advantage of using this method is that it is easy to implement since it is equivalent to applying the usual ANOVA to the ranks within the blocked design. Since it is related to ANOVA in this way we shall refer to this method as the "Analogue" method. We confirm that except for the small designs considered, the resulting test statistics are distributed as F variables under the null hypothesis.

6.2 Null Distribution

A computer program was written to simulate the null distributions. The simulation run applied was 100,000 experiments. For each simulation the value of each statistic was calculated under the null hypothesis of no treatment effect and thus a vector of null values was obtained and subsequently ordered to yield the null distribution of each statistic. Tables 6.1 to 6.8 show the percentage points of these simulated null distributions for a number of designs with different sizes.

For design 2 2 2 the percentage points beyond 95 percent are indicated as infinity. For this design the probability of the sum of squares error becoming zero is $(1/4!) = .0417$ and thus the simulated percentage points are quite different from the points for the F distribution due to the discrete nature of the distributions. For design 2 3 2 the probability of getting a zero in the denominator is $(1/6!) = .00139$ and we can see that the simulated percentage points are much closer to the F values compared with the 2 2 2 design. For the 95 percent point the simulated values range from 8.000 to 8.352 compared with the value 6.608 for the F distribution. With increase in design size there is substantial improvement in the approximation. For design 2 5 2 the probability of zero is $(1/10!) = 2.75 \times 10^{-7}$. For the 95 percent point the simulated values range from 5.326 to 5.409 compared with the asymptotic value of 5.117. For larger designs the differences are quite negligible. For design 5 5 2 the simulated values range from 4.246 to 4.310 compared with the value 4.260 for the F distribution.

We conclude that for the small designs 2 2 2, 2 3 2, the asymptotic results are not accurate due to the high probability of zero in the denominator of the test statistics. For these designs one can use the simulated distributions as approximations to the exact distributions. For larger designs the asymptotic values provide us with good approximations to the distributions of the test statistics.

Table 6.1: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue Design Parameters** $I = 2, \quad J = 2, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	5.538	10.130	17.440	34.120	55.550
Linear U	12.000	24.000	∞	∞	∞
Linear V	12.000	24.000	∞	∞	∞
Lin$U\times$Lin$V$	12.000	24.000	∞	∞	∞

Table 6.2: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue Design Parameters** $I = 2, \quad J = 3, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	4.060	6.608	10.010	16.260	22.780
Linear U	4.615	8.167	13.333	26.667	41.667
Linear V	4.444	8.182	15.000	25.312	40.833
Quadratic V	4.286	8.352	15.000	23.438	45.938
Lin$U\times$Lin$V$	4.375	8.000	13.333	25.312	40.833

Table 6.3: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue**

Design Parameters $I = 2, \quad J = 4, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.589	5.591	8.073	12.250	16.240
Linear U	3.730	6.034	8.823	14.175	19.717
Linear V	3.746	6.050	9.113	14.787	19.886
Quadratic V	3.728	6.034	9.000	14.787	21.000
Cubic V	3.765	6.050	9.100	14.450	19.785
Lin$U \times$Lin$V$	3.733	5.973	8.960	14.400	19.886

Table 6.4: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue**

Design Parameters $I = 3, \quad J = 3, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.458	5.318	7.571	11.260	14.690
Linear U	3.568	5.633	8.170	12.800	17.333
Quadratic U	3.507	5.556	8.167	12.522	17.043
Linear V	3.539	5.597	8.202	12.789	17.361
Quadratic V	3.571	5.628	8.167	13.000	17.818
Lin$U \times$Lin$V$	3.596	5.633	8.258	12.600	16.941

Table 6.5: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue Design Parameters** $I = 2,\ \ J = 5,\ \ M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.360	5.117	7.209	10.560	13.610
Linear U	3.456	5.358	7.855	12.166	16.200
Linear V	3.422	5.326	7.766	11.719	15.532
Quadratic V	3.496	5.403	7.727	11.716	15.572
Cubic V	3.457	5.409	7.850	11.912	16.200
Quartic V	3.426	5.369	7.653	11.716	15.366
Lin$U$ x Lin$V$	3.451	5.358	7.766	11.683	15.817

Table 6.6: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue Design Parameters** $I = 4, \; J = 4, \; M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	3.073	4.543	6.200	8.683	10.800
Linear U	3.062	4.576	6.295	8.883	11.235
Quadratic U	3.097	4.601	6.368	9.007	11.188
Cubic U	3.091	4.562	6.178	8.834	11.129
Linear V	3.052	4.474	6.125	8.606	10.756
Quadratic V	3.066	4.559	6.207	8.744	10.920
Cubic V	3.115	4.641	6.445	9.143	11.341
Lin$U\times$Lin$V$	3.044	4.510	6.123	8.670	10.898

Table 6.7: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue Design Parameters** $I = 4, \quad J = 6, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	2.937	4.279	5.750	7.881	9.635
Linear U	2.960	4.360	5.889	8.084	9.946
Quadratic U	2.943	4.327	5.854	8.186	10.232
Cubic U	2.969	4.361	5.873	8.167	10.089
Quartic U	2.927	4.292	5.804	8.082	9.825
Linear V	2.996	4.366	5.858	8.286	10.228
Quadratic V	2.934	4.261	5.787	7.965	9.794
Cubic V	2.933	4.351	5.962	8.192	10.223
Quartic V	2.936	4.288	5.827	8.062	9.753
Lin$U\times$Lin$V$	2.938	4.334	5.943	8.155	10.084

Table 6.8: **Simulated Percentage Points of the Distributions of the Test Statistics under the Analogue Design Parameters** $I = 5, \quad J = 5, \quad M = 2$

Points	90%	95%	97.5%	99%	99.5%
Asymptotic	2.927	4.260	5.717	7.823	9.551
Linear U	2.940	4.284	5.772	7.948	9.707
Quadratic U	2.928	4.277	5.764	7.963	9.791
Cubic U	2.950	4.293	5.824	7.918	9.644
Quartic U	2.954	4.283	5.782	7.972	9.823
Linear V	2.923	4.282	5.805	7.919	9.765
Quadratic V	2.938	4.288	5.803	8.139	10.057
Cubic V	2.922	4.271	5.745	7.939	9.827
Quartic V	2.917	4.246	5.792	7.975	9.876
Lin$U\times$Lin$V$	2.958	4.310	5.801	7.937	9.794

6.3 Power Comparisons

To assess the efficiency of the Analogue, power comparisons were made for the same four designs studied in chapters 4 and 5. As before for each effect under study four power comparisons are made. ANOVA refers to the powers under analysis of variance procedure, Anal1 refers to the powers under the analogue in the absence of any extraneous effects(Models 1, 2, 3), Anal3 refers to the powers in the presence of both extraneous effects(Model 6). For the power comparisons involving main effects Anal2 refers to the powers for the linear main effect under study in the presence of only the other main effect(Model 4) while for the interaction effect it refers to the powers for the linear by linear component of interaction in the presence of only the linear component of factor U(Model 5). Tables 6.9 to 6.18 show the power values for the above comparisons and Figures 6.1, 6.2, 6.3 and 6.4 illustrate the results.

Comparison of the power curves for the Analogue and those for the Ranking Method and the Modified Method reveals the following points.

1- In the absence of extraneous effects the Ranking Method performs better than the Modified and the Analogue for all the effects studied for the four designs and this superiority shows itself for all values of the coefficients g_1, g_2, g_3. Furthermore, we can see that the powers for the Ranking Method and the Analogue are quite close and comparable to those of ANOVA. For design 2 5 2, in testing the U effect with no other component in the model when $g_1 = 1.65$, the Ranking Method shows the power value of .898, compared with .686 for the Modified, .883 for the Analogue and .907 for the ANOVA. For the test on interaction when $g_3 = -1.5$, the power for the ANOVA is .987 compared with .985 for the Ranking Method, .976 for the Analogue, and .830 for the Modified Method. We can see that the difference between the Ranking Method and the Analogue is negligible. In fact for testing the main effects in the absence of extraneous effects the Ranking procedure reduces to ordinary Friedman test for a one-way layout where instead of having one observation at each level we have replications and the number of replications is the number of levels of the other factor while for the test on interaction the procedure is the same as the test on the linear component of Friedman's test. Thus here we are comparing the linear component of the Friedman test with that of ANOVA.

2- The differences between the power curves for the Analogue are less than the corresponding differences for the Modified and the Ranking Method which illustrate the fact that the Analogue is less affected by the presence of extraneous effects than the other two methods. For design 2 5 2 for the U effect when $g_1 = 2.250$ the reduction in power due to the presence of the nuisance components is 8 percent for the Analogue compared with 34.6 percent for the Modified Method and 38.8 percent for the Ranking Method. For larger designs the differences between these power losses are reduced. For design 4 4 2 for the U effect, when $g_1 = -.8$ the power loss is 2.8 percent for the Analogue compared with 7.3 percent for the Modified Method and 21.1 percent for the Ranking Method.

3- In the absence of interaction effect under the Analogue the power curves for the main effects indicate that the power loss due to the presence of the other main effect is quite small in small designs and becomes negligible in large designs. This same feature can also be seen for the Modified Method but it is more pronounced for the Analogue. For design 2 5 2 for $g_1 = 2.250$, the reduction in power due to presence of the factor V is .6 percent for the Analogue compared with 4.9 percent for the Modified Method and 9 percent reduction for the Ranking Method.

4- For the test on interaction under the Analogue the power loss due to the presence of only one main effect is quite small. For the 2 5 2 design when $g_3 = -1.5$, the power loss due to the presence of the U effect is .6 percent for the Analogue compared with 3.5 percent for the Modified Method and 16.3 percent for the Ranking Method.

In the next section we shall compare the power performance of the four procedures studied under the full model.

Table 6.9: **Simulated Powers for ANOVA and the Analogue Testing the Linear Component of Factor U**

Design Parameters $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	anal1	anal2	anal3	g_1	ANOVA	anal1	anal2	anal3
-2.250	0.993	0.989	0.983	0.907	0.150	0.061	0.065	0.065	0.065
-2.100	0.986	0.978	0.969	0.886	0.300	0.092	0.095	0.094	0.095
-1.950	0.972	0.960	0.950	0.858	0.450	0.153	0.151	0.153	0.153
-1.800	0.945	0.927	0.912	0.815	0.600	0.231	0.221	0.221	0.220
-1.650	0.907	0.883	0.869	0.779	0.750	0.324	0.311	0.308	0.302
-1.500	0.848	0.820	0.803	0.717	0.900	0.435	0.414	0.410	0.393
-1.350	0.767	0.734	0.723	0.650	1.050	0.554	0.529	0.521	0.486
-1.200	0.665	0.638	0.626	0.570	1.200	0.663	0.635	0.624	0.572
-1.050	0.550	0.524	0.516	0.478	1.350	0.766	0.733	0.723	0.651
-0.900	0.435	0.415	0.412	0.392	1.500	0.844	0.816	0.802	0.715
-0.750	0.324	0.311	0.311	0.300	1.650	0.907	0.883	0.870	0.775
-0.600	0.225	0.219	0.218	0.215	1.800	0.944	0.928	0.915	0.817
-0.450	0.149	0.148	0.147	0.147	1.950	0.972	0.959	0.949	0.859
-0.300	0.092	0.093	0.093	0.093	2.100	0.985	0.978	0.969	0.889
-0.150	0.061	0.062	0.062	0.063	2.250	0.994	0.989	0.983	0.910
0.000	0.050	0.054	0.054	0.054					

Table 6.10: **Simulated Powers for ANOVA and the Analogue Testing the Linear Component of Factor V**

Design Parameters $I = 2, \quad J = 5, \quad M = 2, \quad \alpha = .05$

g_2	ANOVA	anal1	anal2	anal3	g_2	ANOVA	anal1	anal2	anal3
-0.750	0.987	0.976	0.970	0.792	0.050	0.060	0.063	0.063	0.062
-0.700	0.974	0.958	0.949	0.764	0.100	0.088	0.089	0.090	0.090
-0.650	0.957	0.934	0.925	0.730	0.150	0.139	0.137	0.139	0.132
-0.600	0.919	0.891	0.879	0.691	0.200	0.205	0.196	0.201	0.187
-0.550	0.871	0.836	0.821	0.648	0.250	0.291	0.278	0.272	0.251
-0.500	0.802	0.766	0.751	0.600	0.300	0.395	0.370	0.366	0.324
-0.450	0.719	0.677	0.664	0.538	0.350	0.510	0.480	0.471	0.404
-0.400	0.618	0.580	0.568	0.474	0.400	0.610	0.578	0.568	0.472
-0.350	0.511	0.476	0.471	0.404	0.450	0.718	0.676	0.661	0.540
-0.300	0.399	0.375	0.370	0.327	0.500	0.802	0.762	0.751	0.597
-0.250	0.294	0.279	0.279	0.253	0.550	0.872	0.836	0.826	0.650
-0.200	0.204	0.198	0.198	0.186	0.600	0.920	0.893	0.879	0.693
-0.150	0.136	0.132	0.132	0.128	0.650	0.953	0.931	0.919	0.729
-0.100	0.089	0.091	0.090	0.088	0.700	0.974	0.956	0.949	0.761
-0.050	0.059	0.061	0.061	0.061	0.750	0.988	0.978	0.971	0.787
0.000	0.050	0.054	0.054	0.054					

Table 6.11: **Simulated Powers for ANOVA and the Analogue Testing the Linear by Linear Component of Interaction Design Parameters** $I = 2,\ J = 5,\ M = 2,\ \alpha = .05$

g_3	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-1.500	0.987	0.976	0.970	0.792	0.100	0.061	0.063	0.064	0.064
-1.400	0.974	0.957	0.949	0.760	0.200	0.085	0.088	0.090	0.087
-1.300	0.953	0.932	0.922	0.730	0.300	0.138	0.136	0.135	0.130
-1.200	0.919	0.890	0.878	0.692	0.400	0.209	0.202	0.199	0.189
-1.100	0.872	0.836	0.823	0.649	0.500	0.291	0.278	0.276	0.255
-1.000	0.807	0.765	0.754	0.606	0.600	0.398	0.371	0.368	0.327
-0.900	0.723	0.678	0.669	0.543	0.700	0.510	0.480	0.470	0.405
-0.800	0.618	0.583	0.572	0.476	0.800	0.611	0.575	0.566	0.472
-0.700	0.509	0.478	0.470	0.410	0.900	0.716	0.674	0.664	0.539
-0.600	0.395	0.375	0.370	0.330	1.000	0.803	0.765	0.751	0.600
-0.500	0.297	0.281	0.277	0.253	1.100	0.868	0.834	0.818	0.648
-0.400	0.203	0.195	0.196	0.183	1.200	0.921	0.892	0.879	0.693
-0.300	0.136	0.134	0.132	0.131	1.300	0.954	0.933	0.919	0.730
-0.200	0.089	0.090	0.089	0.088	1.400	0.974	0.959	0.949	0.766
-0.100	0.061	0.065	0.064	0.065	1.500	0.987	0.977	0.971	0.787
0.000	0.050	0.054	0.054	0.054					

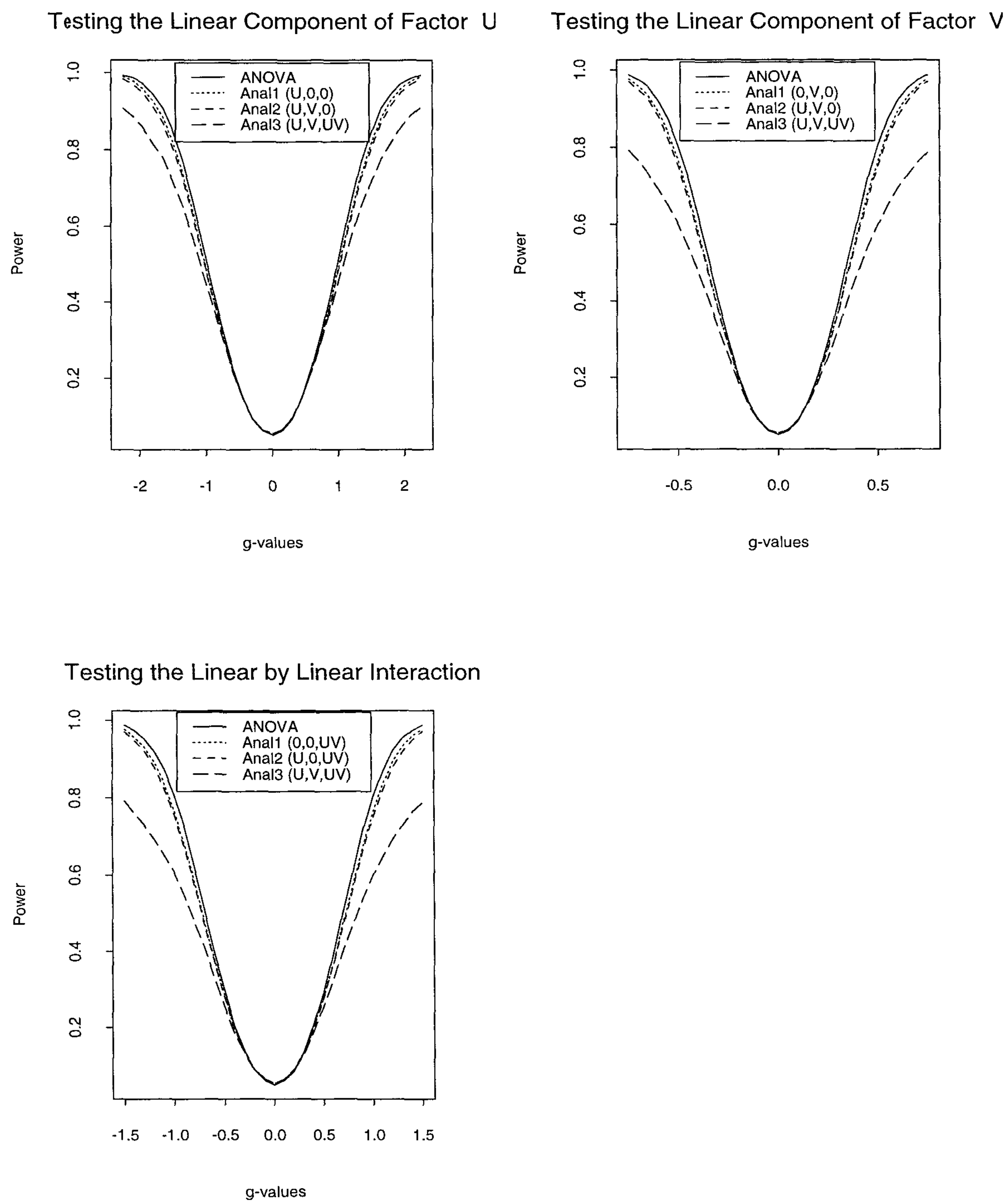


Figure 6.1: Power Comparisons for Design 2 5 2 showing the effect of extraneous components on main effects and interaction under the Analogue Procedure and Normal Distribution.

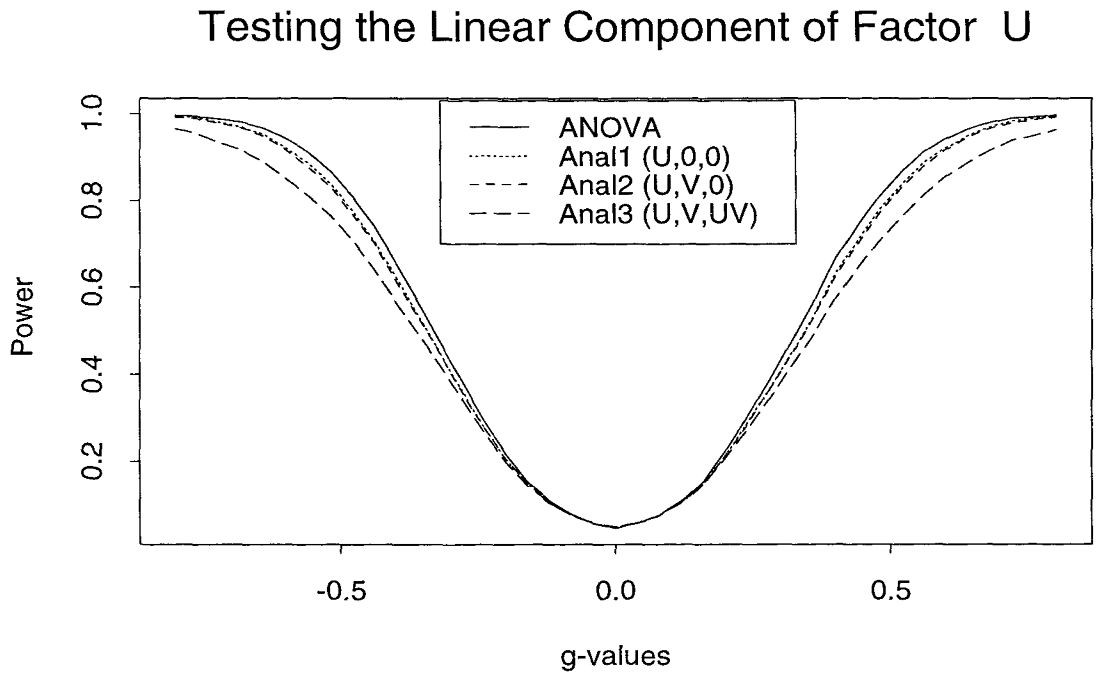
Table 6.12: **Simulated Powers for ANOVA and the Analogue Testing the Linear Component of Factor** U

Design Parameters $I = 4, \quad J = 4, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	anal1	anal2	anal3	g_1	ANOVA	anal1	anal2	anal3
-0.800	0.997	0.994	0.992	0.966	0.040	0.058	0.057	0.057	0.058
-0.760	0.994	0.990	0.988	0.954	0.080	0.077	0.077	0.077	0.076
-0.720	0.988	0.981	0.978	0.934	0.120	0.109	0.107	0.106	0.105
-0.680	0.981	0.969	0.967	0.916	0.160	0.153	0.148	0.148	0.146
-0.640	0.967	0.953	0.948	0.889	0.200	0.223	0.212	0.212	0.206
-0.600	0.943	0.921	0.917	0.851	0.240	0.302	0.288	0.284	0.273
-0.560	0.913	0.888	0.879	0.812	0.280	0.386	0.365	0.364	0.343
-0.520	0.869	0.840	0.833	0.767	0.320	0.474	0.446	0.442	0.417
-0.480	0.811	0.779	0.771	0.710	0.360	0.566	0.534	0.530	0.492
-0.440	0.743	0.706	0.703	0.642	0.400	0.666	0.632	0.626	0.575
-0.400	0.657	0.625	0.615	0.567	0.440	0.742	0.707	0.699	0.643
-0.360	0.568	0.538	0.534	0.495	0.480	0.810	0.779	0.771	0.707
-0.320	0.475	0.450	0.449	0.421	0.520	0.866	0.837	0.831	0.762
-0.280	0.386	0.363	0.360	0.345	0.560	0.912	0.887	0.879	0.813
-0.240	0.296	0.280	0.279	0.269	0.600	0.942	0.921	0.916	0.855
-0.200	0.219	0.208	0.208	0.200	0.640	0.965	0.950	0.946	0.887
-0.160	0.154	0.150	0.150	0.144	0.680	0.980	0.969	0.966	0.915
-0.120	0.111	0.107	0.106	0.106	0.720	0.990	0.984	0.980	0.940
-0.080	0.079	0.077	0.078	0.077	0.760	0.993	0.989	0.986	0.950
-0.040	0.057	0.057	0.058	0.057	0.800	0.997	0.994	0.992	0.964
0.000	0.047	0.050	0.050	0.050					

Table 6.13: **Simulated Powers for ANOVA and the Analogue Testing the Linear by Linear Component of Interaction Design Parameters** $I = 4, \quad J = 4, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-0.720	0.997	0.994	0.986	0.967	0.040	0.059	0.060	0.060	0.060
-0.680	0.995	0.988	0.979	0.951	0.080	0.083	0.082	0.082	0.082
-0.640	0.988	0.980	0.964	0.933	0.120	0.126	0.122	0.121	0.120
-0.600	0.977	0.961	0.941	0.905	0.160	0.185	0.175	0.173	0.171
-0.560	0.959	0.940	0.915	0.879	0.200	0.265	0.248	0.242	0.238
-0.520	0.929	0.901	0.874	0.833	0.240	0.357	0.331	0.321	0.311
-0.480	0.886	0.852	0.821	0.778	0.280	0.455	0.423	0.407	0.392
-0.440	0.828	0.793	0.759	0.716	0.320	0.564	0.528	0.509	0.486
-0.400	0.753	0.711	0.678	0.647	0.360	0.660	0.615	0.591	0.565
-0.360	0.663	0.622	0.596	0.567	0.400	0.757	0.714	0.686	0.653
-0.320	0.561	0.523	0.503	0.484	0.440	0.825	0.789	0.760	0.717
-0.280	0.458	0.425	0.410	0.397	0.480	0.885	0.852	0.822	0.779
-0.240	0.357	0.332	0.323	0.310	0.520	0.929	0.900	0.872	0.831
-0.200	0.264	0.249	0.243	0.236	0.560	0.959	0.937	0.912	0.874
-0.160	0.185	0.172	0.169	0.168	0.600	0.978	0.964	0.945	0.907
-0.120	0.126	0.120	0.120	0.118	0.640	0.988	0.978	0.963	0.934
-0.080	0.082	0.083	0.082	0.081	0.680	0.994	0.988	0.977	0.951
-0.040	0.058	0.059	0.059	0.059	0.720	0.998	0.994	0.986	0.964
0.000	0.051	0.052	0.052	0.052					



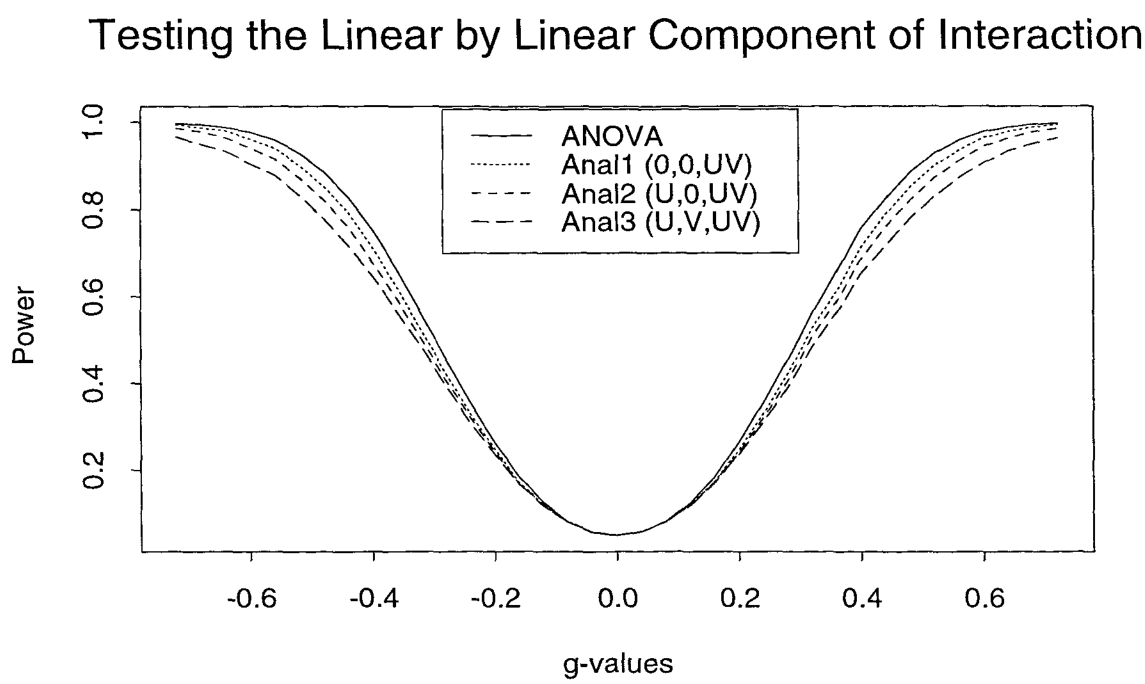


Figure 6.2: Power Comparisons for Design 4 4 2 showing the effect of extraneous components on main effects and interaction under the Analogue Procedure and Normal Distribution.

Table 6.14: **Simulated Powers for ANOVA and the Analogue Testing the Linear Component of Factor U**

Design Parameters $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_1	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-0.635	0.997	0.994	0.994	0.984	0.035	0.058	0.058	0.058	0.059
-0.600	0.993	0.989	0.988	0.974	0.071	0.083	0.081	0.081	0.081
-0.565	0.986	0.980	0.978	0.961	0.106	0.125	0.122	0.122	0.122
-0.529	0.976	0.965	0.963	0.941	0.141	0.185	0.177	0.176	0.175
-0.494	0.955	0.940	0.937	0.911	0.176	0.262	0.248	0.247	0.243
-0.459	0.924	0.906	0.903	0.872	0.212	0.347	0.329	0.325	0.319
-0.424	0.882	0.859	0.856	0.820	0.247	0.446	0.422	0.424	0.409
-0.388	0.822	0.793	0.790	0.762	0.282	0.554	0.527	0.525	0.506
-0.353	0.743	0.713	0.706	0.679	0.318	0.659	0.625	0.624	0.602
-0.318	0.657	0.623	0.622	0.598	0.353	0.747	0.713	0.710	0.687
-0.282	0.563	0.533	0.533	0.514	0.388	0.825	0.795	0.790	0.762
-0.247	0.453	0.428	0.427	0.417	0.424	0.882	0.858	0.853	0.823
-0.212	0.348	0.332	0.331	0.324	0.459	0.924	0.906	0.902	0.873
-0.176	0.256	0.244	0.243	0.241	0.494	0.956	0.941	0.938	0.910
-0.141	0.179	0.171	0.172	0.169	0.529	0.976	0.965	0.964	0.941
-0.106	0.123	0.119	0.120	0.118	0.565	0.986	0.980	0.978	0.960
-0.071	0.078	0.078	0.078	0.078	0.600	0.994	0.990	0.988	0.975
-0.035	0.057	0.057	0.057	0.057	0.635	0.997	0.994	0.994	0.984
0.000	0.051	0.052	0.052	0.052					

Table 6.15: **Simulated Powers for ANOVA and the Analogue Testing the Linear Component of Factor** V

Design Parameters $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_2	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-0.424	0.998	0.996	0.995	0.986	0.024	0.060	0.060	0.061	0.061
-0.400	0.995	0.991	0.990	0.979	0.047	0.082	0.081	0.080	0.079
-0.376	0.990	0.983	0.981	0.964	0.071	0.130	0.124	0.125	0.123
-0.353	0.978	0.970	0.968	0.946	0.094	0.188	0.180	0.179	0.177
-0.329	0.962	0.948	0.946	0.920	0.118	0.261	0.249	0.250	0.245
-0.306	0.938	0.920	0.917	0.887	0.141	0.359	0.339	0.335	0.330
-0.282	0.895	0.873	0.870	0.838	0.165	0.460	0.437	0.436	0.424
-0.259	0.837	0.809	0.807	0.775	0.188	0.572	0.543	0.543	0.524
-0.235	0.762	0.734	0.728	0.699	0.212	0.671	0.642	0.637	0.618
-0.212	0.670	0.637	0.636	0.613	0.235	0.763	0.733	0.729	0.700
-0.188	0.566	0.537	0.536	0.516	0.259	0.837	0.810	0.804	0.772
-0.165	0.466	0.439	0.439	0.425	0.282	0.894	0.868	0.866	0.835
-0.141	0.360	0.340	0.339	0.331	0.306	0.934	0.916	0.914	0.884
-0.118	0.270	0.256	0.255	0.251	0.329	0.960	0.946	0.945	0.916
-0.094	0.188	0.180	0.179	0.178	0.353	0.980	0.972	0.969	0.947
-0.071	0.130	0.124	0.124	0.122	0.376	0.989	0.983	0.981	0.964
-0.047	0.083	0.082	0.081	0.081	0.400	0.995	0.991	0.990	0.979
-0.024	0.060	0.060	0.060	0.059	0.424	0.998	0.995	0.995	0.985
0.000	0.049	0.050	0.050	0.050					

Table 6.16: **Simulated Powers for ANOVA and the Analogue Testing the Linear by Linear Component of Interaction Design Parameters** $I = 4, \quad J = 6, \quad M = 2, \quad \alpha = .05$

g_3	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-0.360	0.996	0.992	0.984	0.965	0.020	0.056	0.056	0.056	0.057
-0.340	0.990	0.984	0.974	0.950	0.040	0.080	0.078	0.077	0.077
-0.320	0.983	0.972	0.961	0.930	0.060	0.118	0.113	0.112	0.112
-0.300	0.966	0.952	0.935	0.903	0.080	0.173	0.161	0.161	0.158
-0.280	0.945	0.926	0.905	0.870	0.100	0.243	0.230	0.226	0.221
-0.260	0.909	0.882	0.858	0.822	0.120	0.331	0.315	0.306	0.297
-0.240	0.861	0.829	0.807	0.768	0.140	0.429	0.401	0.391	0.380
-0.220	0.795	0.765	0.740	0.705	0.160	0.525	0.494	0.478	0.461
-0.200	0.716	0.682	0.659	0.628	0.180	0.626	0.591	0.576	0.549
-0.180	0.628	0.592	0.573	0.547	0.200	0.720	0.684	0.660	0.630
-0.160	0.529	0.498	0.484	0.465	0.220	0.794	0.760	0.738	0.703
-0.140	0.427	0.399	0.391	0.376	0.240	0.861	0.833	0.806	0.768
-0.120	0.331	0.312	0.307	0.298	0.260	0.913	0.886	0.864	0.825
-0.100	0.244	0.231	0.228	0.222	0.280	0.944	0.927	0.906	0.870
-0.080	0.173	0.163	0.162	0.160	0.300	0.966	0.954	0.935	0.902
-0.060	0.120	0.114	0.114	0.111	0.320	0.983	0.973	0.961	0.930
-0.040	0.080	0.079	0.079	0.078	0.340	0.990	0.982	0.974	0.949
-0.020	0.056	0.058	0.057	0.056	0.360	0.996	0.992	0.985	0.964
0.000	0.051	0.052	0.052	0.052					

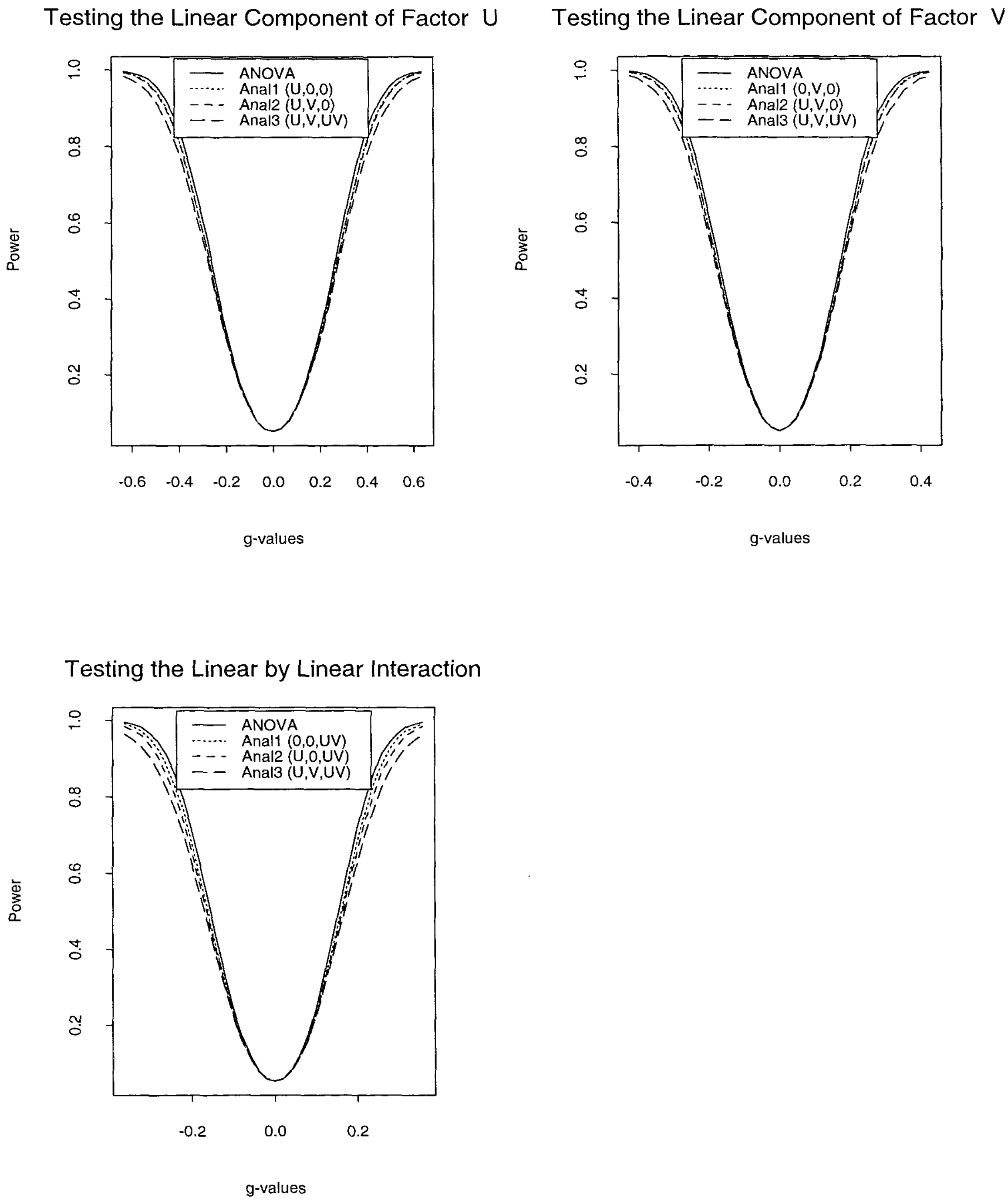


Figure 6.3: Power Comparisons for Design 4 6 2 showing the effect of extraneous components on main effects and interaction under the Analogue Procedure and Normal Distribution.

Table 6.17: **Simulated Powers for ANOVA and the Analogue Testing the Linear Component of Factor U Design Parameters $I = 5, \quad J = 5, \quad M = 2, \quad \alpha = .05$**

g_1	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-0.510	0.998	0.997	0.996	0.989	0.030	0.060	0.061	0.061	0.061
-0.480	0.996	0.994	0.993	0.982	0.060	0.088	0.086	0.087	0.086
-0.450	0.992	0.986	0.985	0.971	0.090	0.140	0.134	0.135	0.134
-0.420	0.981	0.973	0.971	0.952	0.120	0.213	0.203	0.203	0.200
-0.390	0.963	0.950	0.947	0.924	0.150	0.301	0.285	0.283	0.279
-0.360	0.934	0.916	0.912	0.884	0.180	0.407	0.387	0.387	0.373
-0.330	0.882	0.862	0.858	0.828	0.210	0.524	0.499	0.497	0.482
-0.300	0.825	0.797	0.793	0.767	0.240	0.636	0.607	0.602	0.584
-0.270	0.735	0.709	0.700	0.680	0.270	0.736	0.705	0.705	0.679
-0.240	0.633	0.605	0.604	0.581	0.300	0.824	0.798	0.793	0.768
-0.210	0.521	0.496	0.492	0.479	0.330	0.885	0.862	0.859	0.829
-0.180	0.411	0.387	0.384	0.375	0.360	0.931	0.914	0.908	0.884
-0.150	0.306	0.292	0.290	0.285	0.390	0.963	0.950	0.947	0.924
-0.120	0.210	0.200	0.199	0.197	0.420	0.980	0.973	0.970	0.951
-0.090	0.137	0.134	0.132	0.131	0.450	0.990	0.985	0.984	0.970
-0.060	0.088	0.086	0.086	0.086	0.480	0.996	0.994	0.993	0.982
-0.030	0.061	0.060	0.060	0.061	0.510	0.998	0.997	0.997	0.990
0.000	0.049	0.050	0.050	0.050					

Table 6.18: **Simulated Powers for ANOVA and the Analogue Testing the Linear by Linear Component of Interaction Design Parameters** $I = 5,\ J = 5,\ M = 2,\ \alpha = .05$

g_3	ANOVA	anal1	anal2	anal3	g_3	ANOVA	anal1	anal2	anal3
-0.340	0.996	0.993	0.985	0.967	0.020	0.059	0.060	0.059	0.059
-0.320	0.993	0.986	0.978	0.954	0.040	0.083	0.081	0.081	0.080
-0.300	0.983	0.975	0.961	0.931	0.060	0.126	0.121	0.120	0.117
-0.280	0.968	0.954	0.937	0.903	0.080	0.187	0.180	0.178	0.175
-0.260	0.940	0.920	0.900	0.863	0.100	0.273	0.255	0.252	0.247
-0.240	0.899	0.874	0.848	0.812	0.120	0.367	0.343	0.336	0.325
-0.220	0.845	0.817	0.788	0.750	0.140	0.478	0.449	0.437	0.420
-0.200	0.770	0.736	0.711	0.682	0.160	0.586	0.552	0.534	0.513
-0.180	0.686	0.647	0.626	0.600	0.180	0.684	0.647	0.624	0.599
-0.160	0.586	0.554	0.532	0.511	0.200	0.775	0.741	0.716	0.685
-0.140	0.474	0.446	0.431	0.418	0.220	0.850	0.818	0.792	0.757
-0.120	0.368	0.347	0.339	0.327	0.240	0.903	0.876	0.852	0.815
-0.100	0.275	0.260	0.254	0.250	0.260	0.942	0.922	0.900	0.864
-0.080	0.193	0.182	0.180	0.176	0.280	0.968	0.955	0.937	0.905
-0.060	0.127	0.123	0.123	0.120	0.300	0.982	0.973	0.959	0.932
-0.040	0.085	0.084	0.082	0.082	0.320	0.991	0.986	0.976	0.953
-0.020	0.056	0.056	0.056	0.055	0.340	0.996	0.993	0.986	0.966
0.000	0.050	0.049	0.049	0.049					

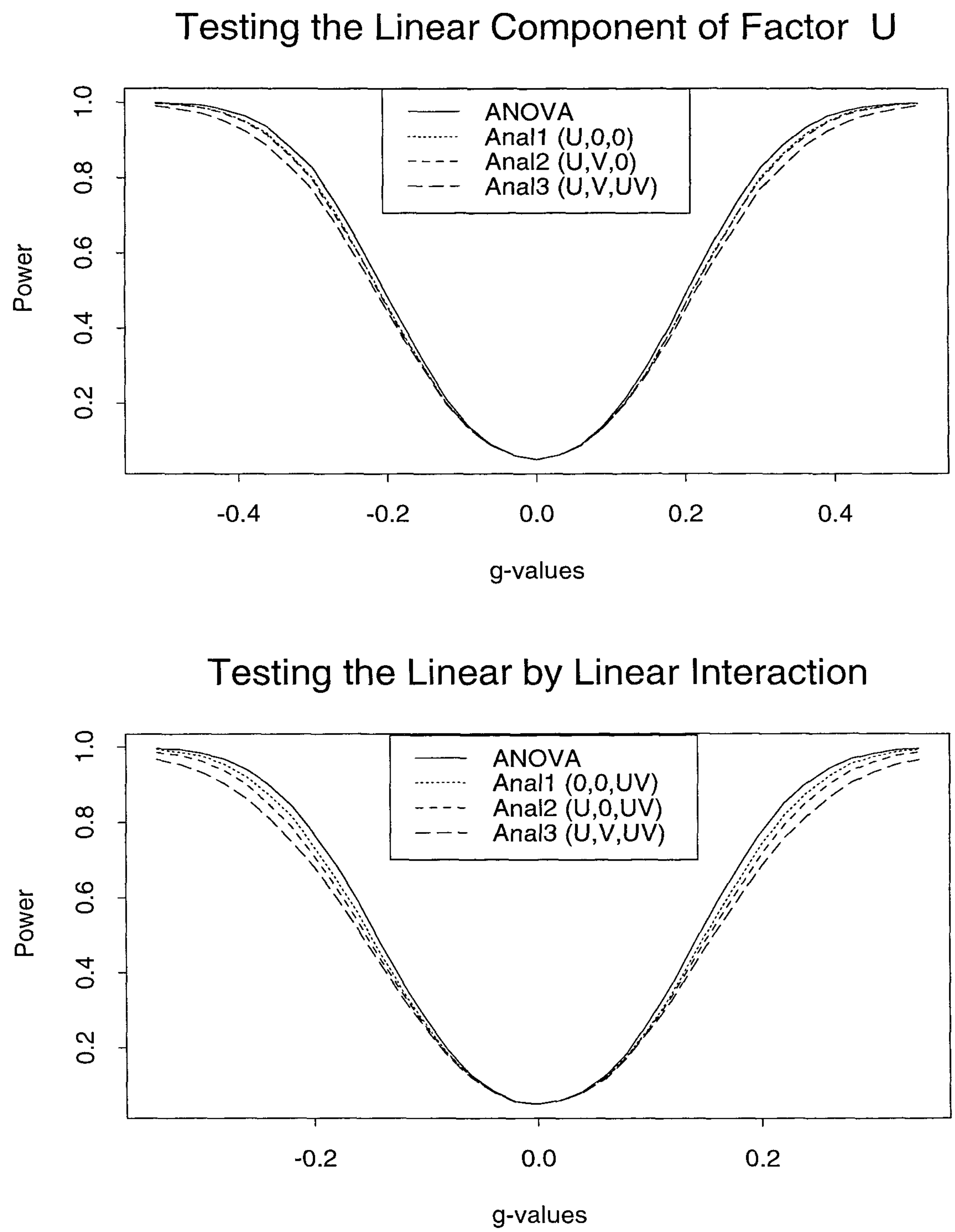


Figure 6.4: Power Comparisons for Design 5 5 2 showing the effect of extraneous components on main effects and interaction under the Analogue Procedure and Normal Distribution.

6.4 A Comparison of the four Methods

In order to have an overall assessment of the four methods applied, namely, ANOVA, the Ranking Method, the Modified and the Analogue we make a simultaneous power comparison for each effect in the presence of the other two. Figure 6.5 shows the comparisons for factor U for the four designs studied in this work. Similarly, Figures 6.6 and 6.7 show the power comparisons for factor V and the interaction respectively.

In the presence of the extraneous effects the difference in powers is quite considerable for design 252. For the U effect when $g_1 = -2.550$, the power for the ANOVA is .999 compared with .940 for the Analogue, .608 for the Modified, and .636 for the Ranking Method. For the interaction when $g_3 = -1.7$, the power for the ANOVA is .997 compared with .835 for the Analogue, .493 for the Modified Method, and .462 for the Ranking Method. Thus we can see that the Analogue performs much better than the other two methods. Thus the weakness that was noticed for the Ranking Method and the Modified Method with regard to the nuisance effects has been mitigated substantially by the Analogue.

With increase in design size the difference in powers is reduced. For design 4 6 2 for the U effect when $g_1 = -.6$ the power for the ANOVA is .994 compared with .974 for the Analogue, .962 for the Modified, and .915 for the Ranking Method and for the interaction when $g_3 = -.38$, the power for the ANOVA is .998 compared with .976 for the Analogue, .963 for the Modified and .875 for the Ranking Method. We can see that, regardless of the design and the effect under study, the Analogue has higher powers than the Modified and the Ranking Method.

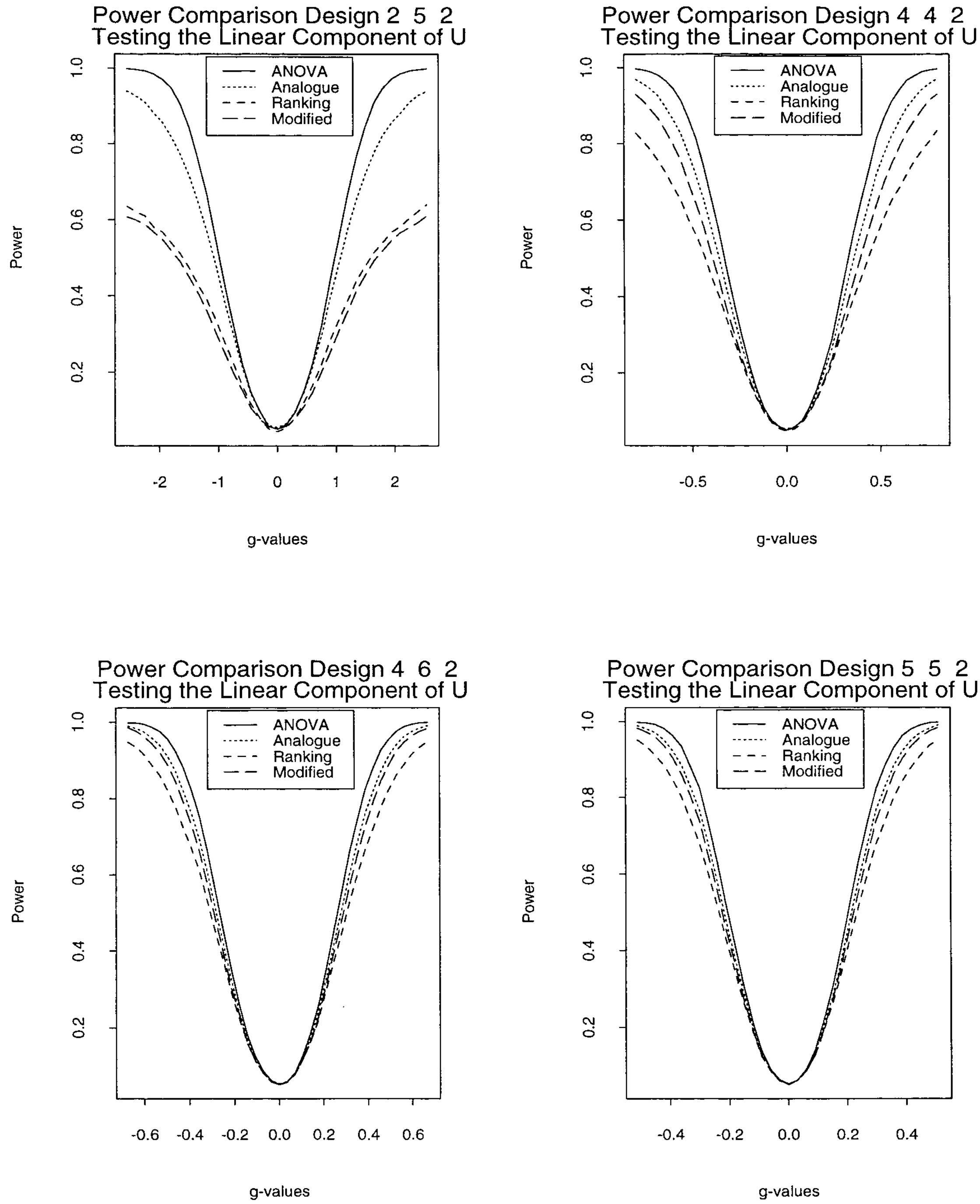


Figure 6.5: Power Comparison for Testing the Linear Component of Factor U in the Presence of the Linear Component of Factor V and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Normal Distribution.

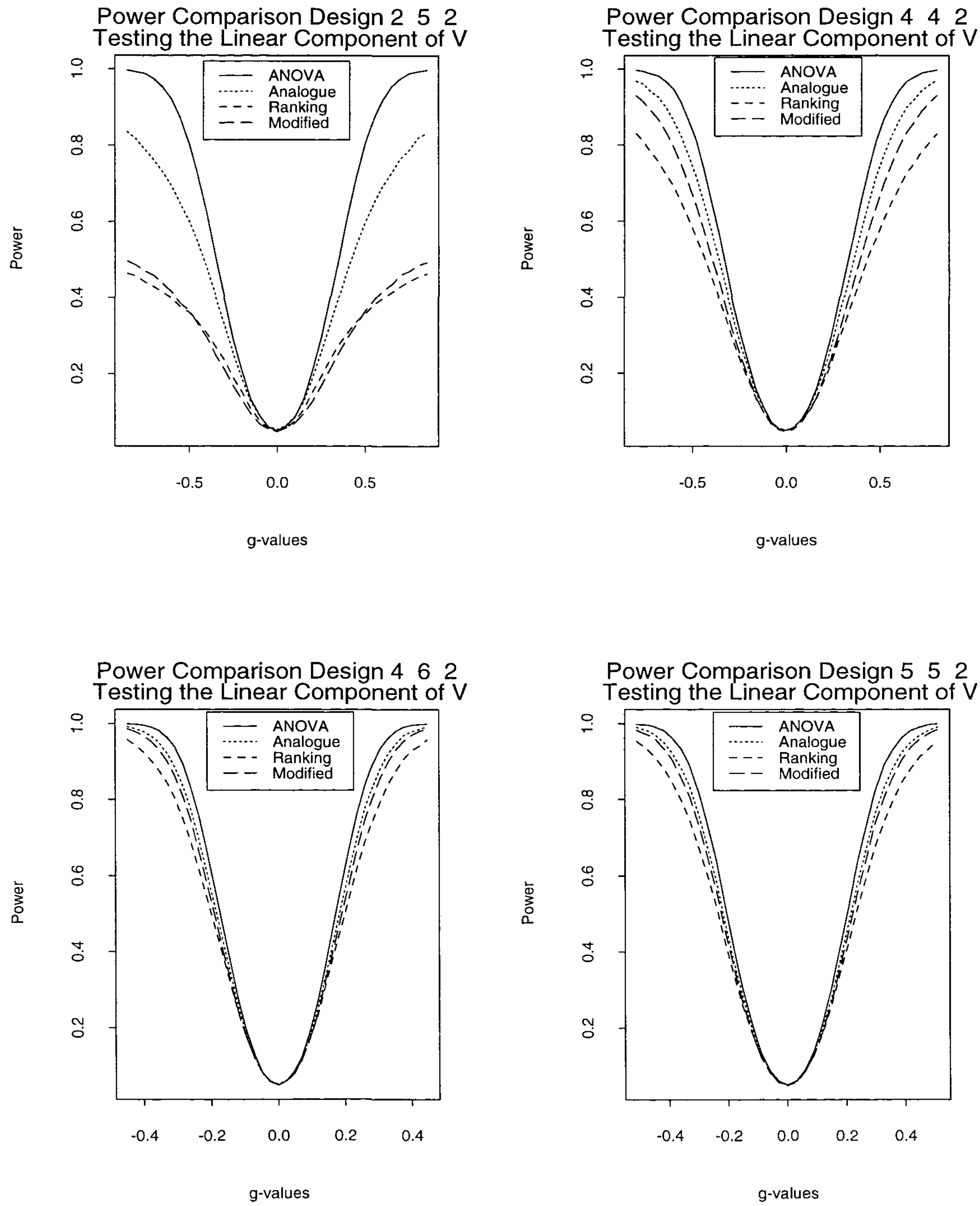


Figure 6.6: Power Comparison for Testing the Linear Component of Factor V in the Presence of the Linear Component of Factor U and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Normal Distribution.

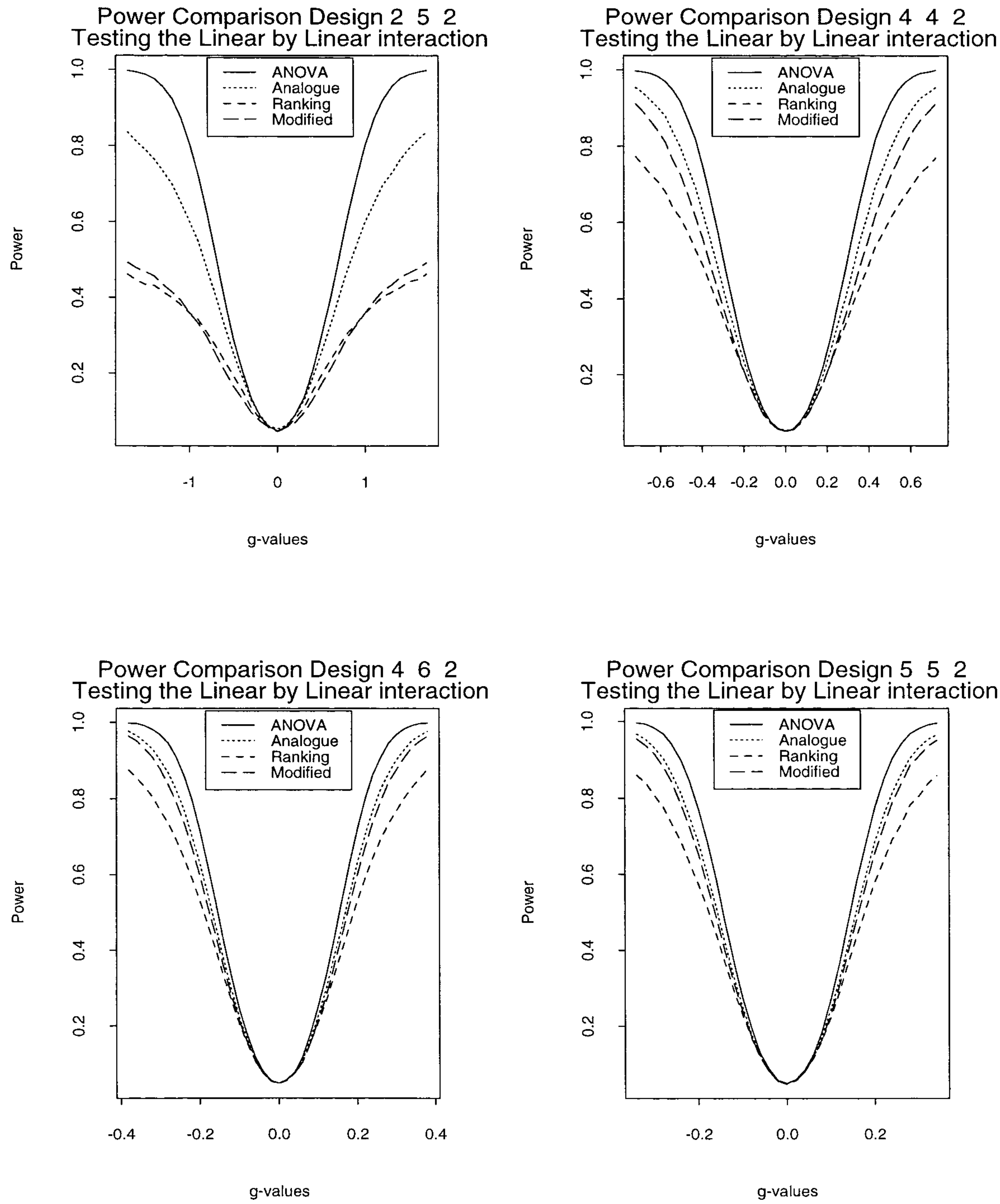


Figure 6.7: Power Comparison for Testing the Linear by Linear Component of Interaction in the Presence of the Linear Component of Factor U and the Linear Component of Factor V for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Normal Distribution.

6.5 Summary and Conclusions

Making use of the rank variation of each treatment in different blocks the error variance is estimated which leads to an estimate for the contrast variance and an alternative procedure for analysis. The procedure obtained called the "Analogue" mitigates the problem of power loss due to the presence of nuisance components encountered before. It is shown that in the absence of extraneous effects the powers are quite close to those for the Ranking Method and in the presence of them the powers are higher than those for the two procedures studied before. The superiority in power performance is significant for small designs. The substantial loss of power which was observed for the Ranking Method and the Modified Method under the full model is now greatly reduced for the Analogue. Furthermore, it is shown that that for the tests on main effects the power loss is mainly due to the presence of the interaction rather than the other main effect while for the test on interaction the loss in power is due to the joint presence of the two main effects in the model.

The powers of the Analogue are, in general comparable to those of the Analysis of Variance procedure and, therefore, the procedure is a good substitute for the ANOVA whenever the application of the latter is not possible due to unavailability of the observations for the response variable or when the accuracy of the ANOVA is in doubt due to the violation of the normality assumption.

In the next chapter we shall study the efficiency of our proposed procedure when non-normality prevails.

Chapter 7

Performance under Non-normal Distributions

7.1 Introduction

In the previous chapters we saw that when the errors are normally distributed ANOVA is more powerful than the non-parametric methods proposed. In cases where data are non-normal, standard procedures can be applied to find a suitable transformation so that the transformed data follow a normal distribution making it possible to apply ANOVA. The alternative would be to apply a non-parametric procedure.

The analysis of variance procedure is based on the assumption of normality while the non-parametric methods are not, thus we expect the ANOVA to lose efficiency when deviations from normality occur. On the other hand, the non-parametric methods are distribution free under the null hypothesis, however, under the alternative hypothesis, they are dependent on the underlying distribution too. As a result the power performance of ANOVA and the non-parametric methods all depend upon the underlying distribution. It would be interesting to know how they compare. In this chapter we shall investigate the powers under non-normal conditions. To be able to assess the efficiency of the non-parametric methods against ANOVA under various non-normal conditions, we compared the powers when the errors have one of the following four distributions:

1- Exponential Distribution

2- Chi-Square One Distribution

3- Chi-Square Four Distribution

4- Cauchy Distribution.

The choice of the exponential distribution was its application in practical problems. Chi-square one was chosen to see how the powers compare under a severely skewed distribution. With the increase in degrees of freedom for the chi-square distribution, skewness is reduced and with the approach to normality the relative performance of ANOVA is expected to improve. Finally the choice of the Cauchy distribution was to see how the powers compare when the data are exposed to severe outliers on both sides.

Again, as for the normal distribution, the powers for each effect were calculated in the presence of the other effects. In the following sections we shall examine the power study for each distribution separately. In each case results were produced for the same four designs as investigated in previous sections. In section 7.7 a summary is given of the work done so far and the conclusions reached and finally in section 7.8 we make some recommendations about possible future research on the topic.

7.2 Power Comparisons under Exponential Distribution

Random variables were generated from an exponential distribution with parameter $\alpha = 1$ and were included in the model as described in section 4.14. Powers were calculated based on 30,000 independent simulations. Figures 7.1, 7.2, 7.3 show the power comparisons for the main effect U, V and interaction UV respectively. Comparison of power curves for the main effect U under exponential distribution and the corresponding ones in Figure 6.5 under normal distribution reveal the following points.

1- For designs 2 5 2 and 4 4 2 the Analogue has a slightly better performance for small magnitudes of g but after a certain point the powers for ANOVA tend to be higher, though the difference between the two is less than the corresponding difference under normality. For design 2 5 2 the ANOVA procedure is not as powerful as the Analogue for values of g around zero corresponding to powers less than 50% for negative g and less than 40% for positive g. We define the points where the power curves cross as "change over points", where the relative power performance of the ANOVA procedure becomes less than the power for the Analogue Method. For design 4 4 2 the change over points are around the power value of 80%. There is a slight lack of symmetry for the power curves of the non-parametric methods and this lack of symmetry is reduced with increase in design size.

2- For designs 4 6 2 and 5 5 2 the Analogue has higher powers for powers up to 98% after which the powers are practically the same as for the ANOVA. The Ranking Method and the Modified have higher powers than ANOVA for small g values but after a certain point ANOVA shows superiority.

For the V effect the power curves are the same for the designs 4 4 2 and 5 5 2 as those for the U effect within the random error fluctuation due to the symmetry of the design. For the 2 5 2 design the difference between the Analogue and the ANOVA is less for the U effect than the V effect but for the 4 6 2 design the power curves for the two effects are similar. But for the interaction the pattern is slightly different for the designs 4 6 2 and 5 5 2. For these two designs the change over point for the negative values of g is different from the one for the positive g values. For the negative values for the powers up to 95% the Analogue has higher powers while for the positive values the change over point is around 80%. Thus, under exponential distribution, except for the design 2 5 2 where ANOVA still maintains its superiority for the larger magnitudes of the g value, for the other three designs the Analogue has a better overall performance. The superiority in performance for the Analogue is evident with increase in design size.

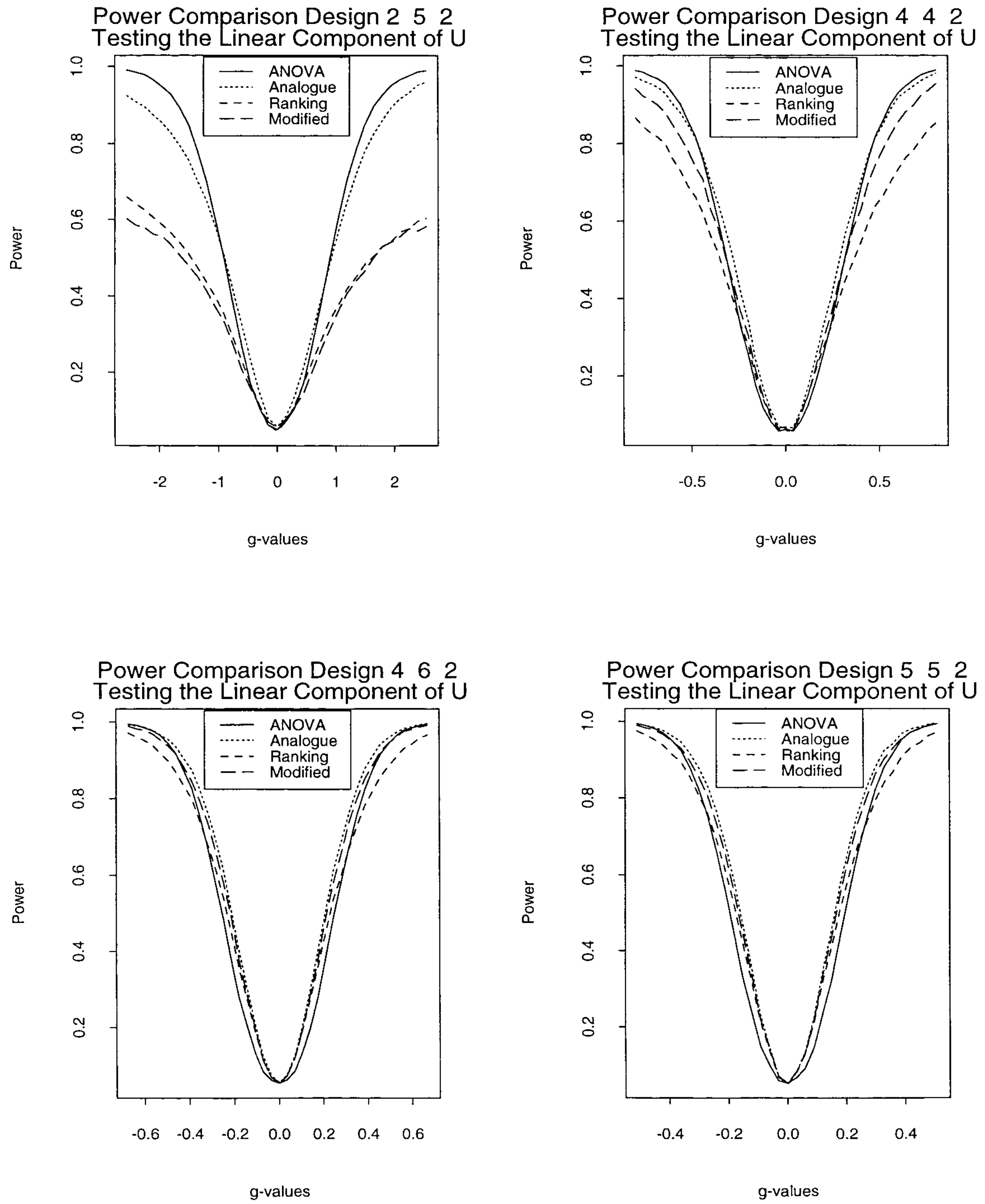


Figure 7.1: Power Comparison for Testing the Linear Component of Factor U in the Presence of the Linear Component of Factor V and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Exponential Distribution.

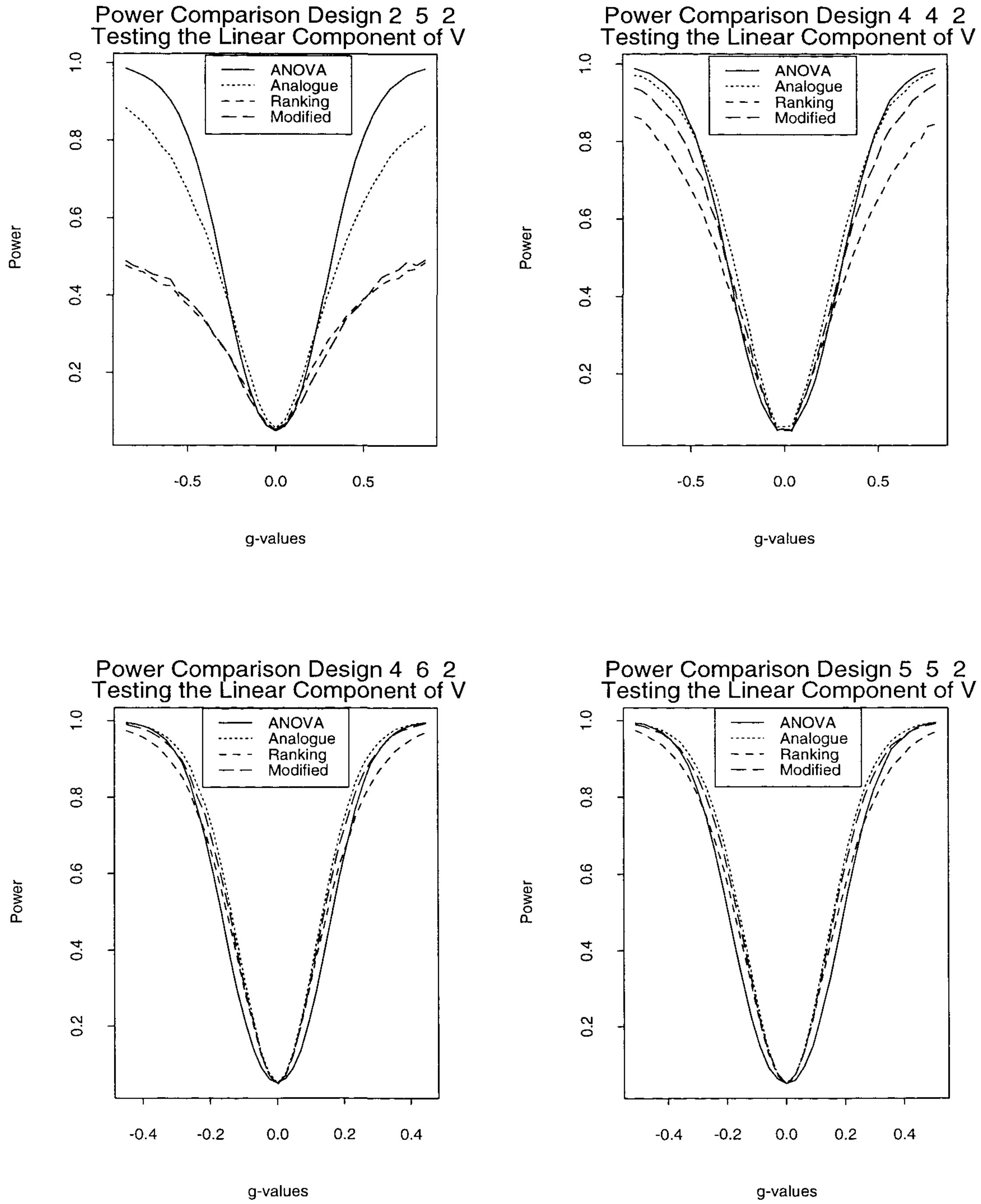


Figure 7.2: Power Comparison for Testing the Linear Component of Factor V in the Presence of the Linear Component of Factor U and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Exponential Distribution.

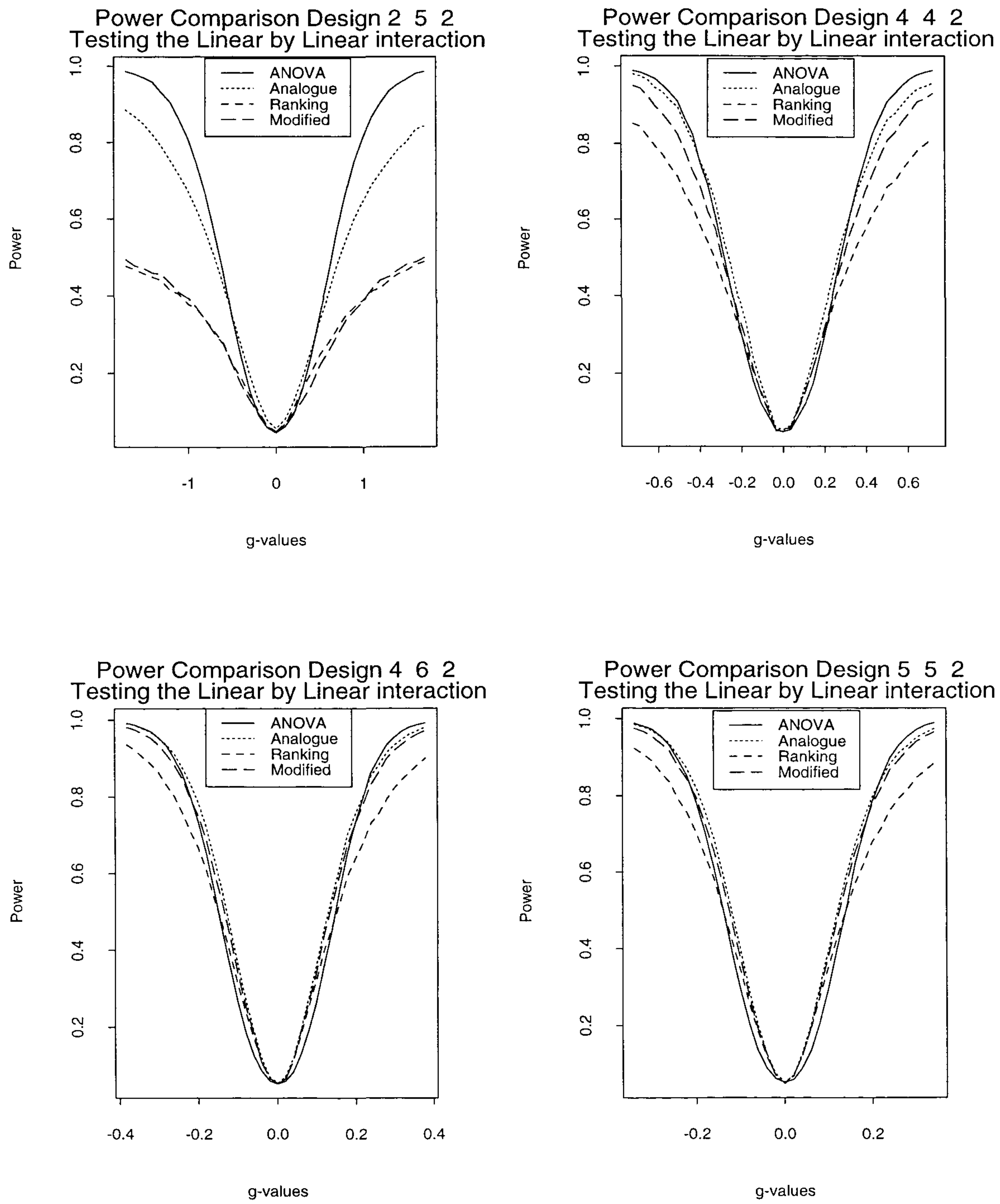


Figure 7.3: Power Comparison for Testing the Linear by Linear Component of Interaction in the Presence of the Linear Component of Factor U and the Linear Component of Factor V for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Exponential Distribution.

7.3 Power Comparison under Chi-Square One Distribution

Figures 7.4, 7.5 and 7.6 show the powers for testing the effects U, V and interaction for the four designs under Chi-square distribution with one degree of freedom respectively. For the U effect the following features can be noted.

1- For design 2 5 2 the analogue has better performance up to the power value around 60% after which the ANOVA is more powerful.

2- For design 4 4 2 for the negative values of g the analogue has better performance up to the power value around 94% after which the ANOVA tends to be slightly more powerful though for the positive values the Analogue has higher powers for practically all the g values.

3- For designs 4 6 2 and 5 5 2 the analogue and the modified method are more powerful than the ANOVA.

For the V effect the pattern is similar. For design 2 5 2 the change over point for the negative g values is around power value of 50% while for the positive g values the change over point is around power value of 40%. For designs 4 4 2 and 5 5 2 the pattern is the same as for the U effect due to the symmetry of the design. Again there is a slight lack of symmetry for the power curves of the non-parametric methods for the designs 2 5 2 and 4 4 2 and again with increase in design size the asymmetry is reduced for the designs 4 6 2 and 5 5 2. There is very little trace of asymmetry for the Analogue. For design 4 6 2 the analogue and the modified are more powerful than the ANOVA.

For the interaction effect lack of symmetry is noticeable for all the four designs under study. For design 2 5 2 the power curves for interaction are the same as those for the V effect due to the intrinsic structure of the design. Thus the asymmetry observed for the V effect shows itself for the interaction as well. For design 4 4 2 for the negative values of g the Analogue has higher powers. For the positive g values for powers less the 87% the Analogue has higher powers but for higher powers the ANOVA is more powerful. For the designs 4 6 2 and 5 5 2 for the negative g values the Analogue is superior to the ANOVA. For the positive g values there is a slight decline for the Analogue but the power performance of Analogue is still superior.

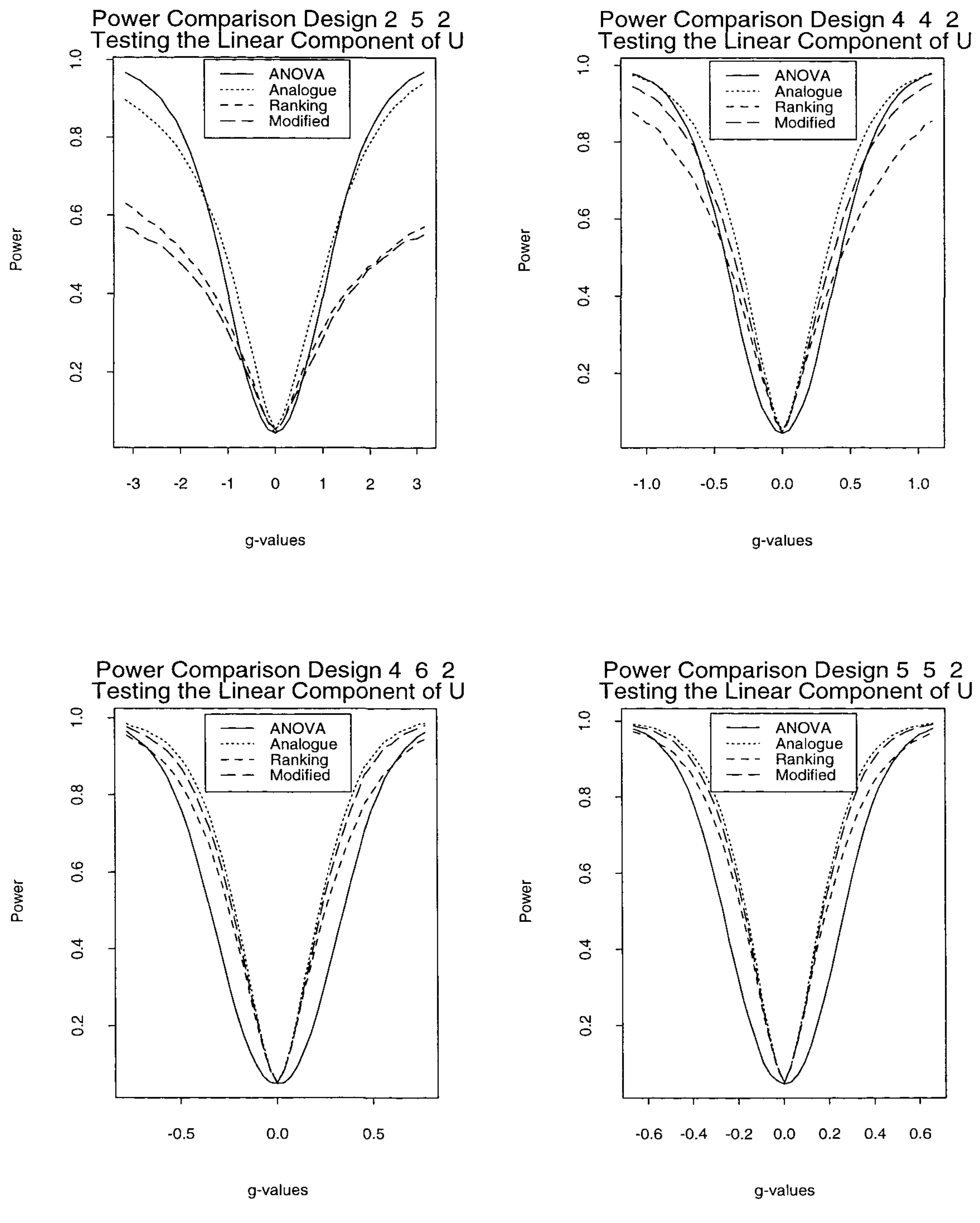


Figure 7.4: Power Comparison for Testing the Linear Component of Factor U in the Presence of the Linear Component of Factor V and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Chi-Square One Distribution.

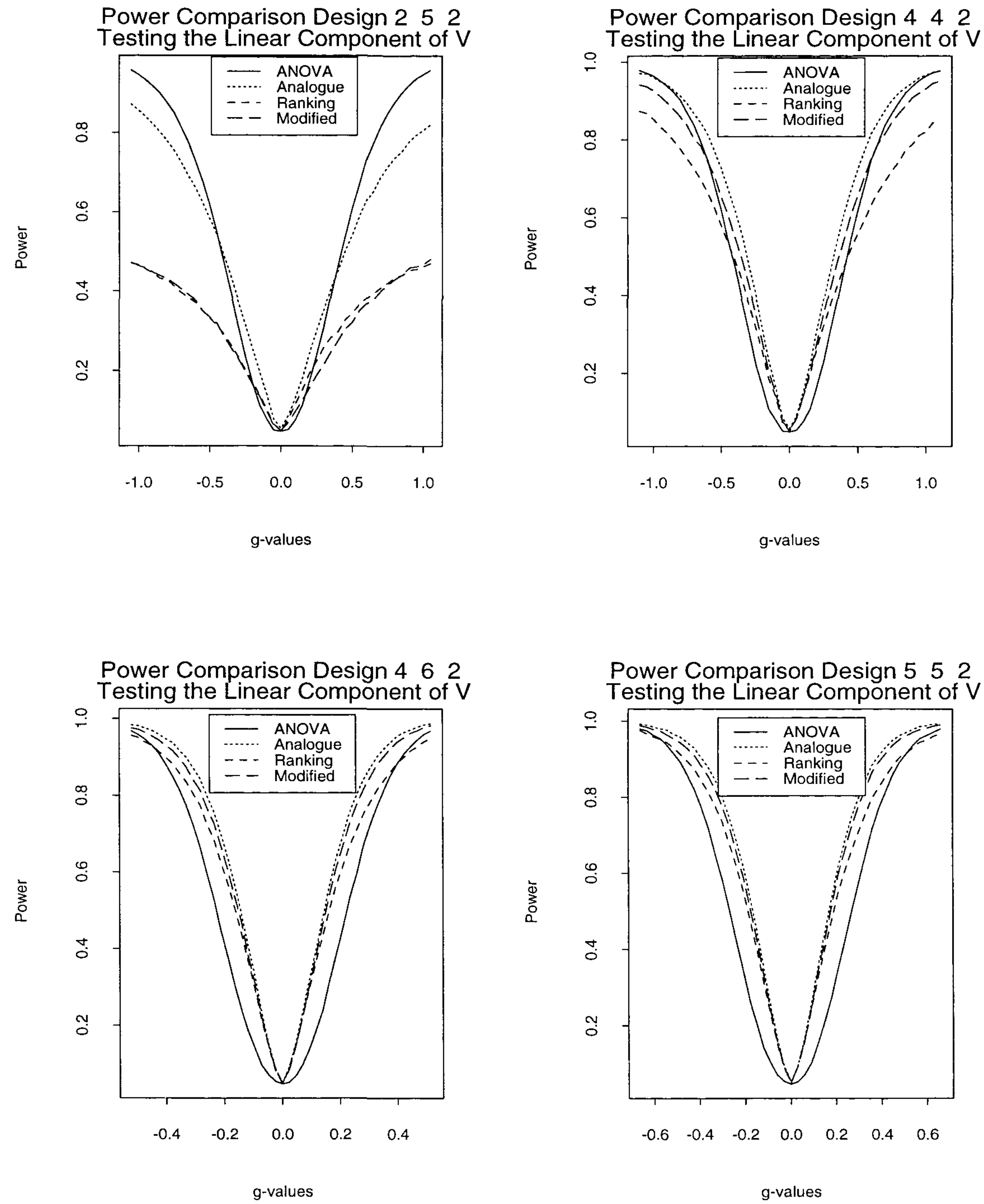


Figure 7.5: Power Comparison for Testing the Linear Component of Factor V in the Presence of the Linear Component of Factor U and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Chi-Square One Distribution.

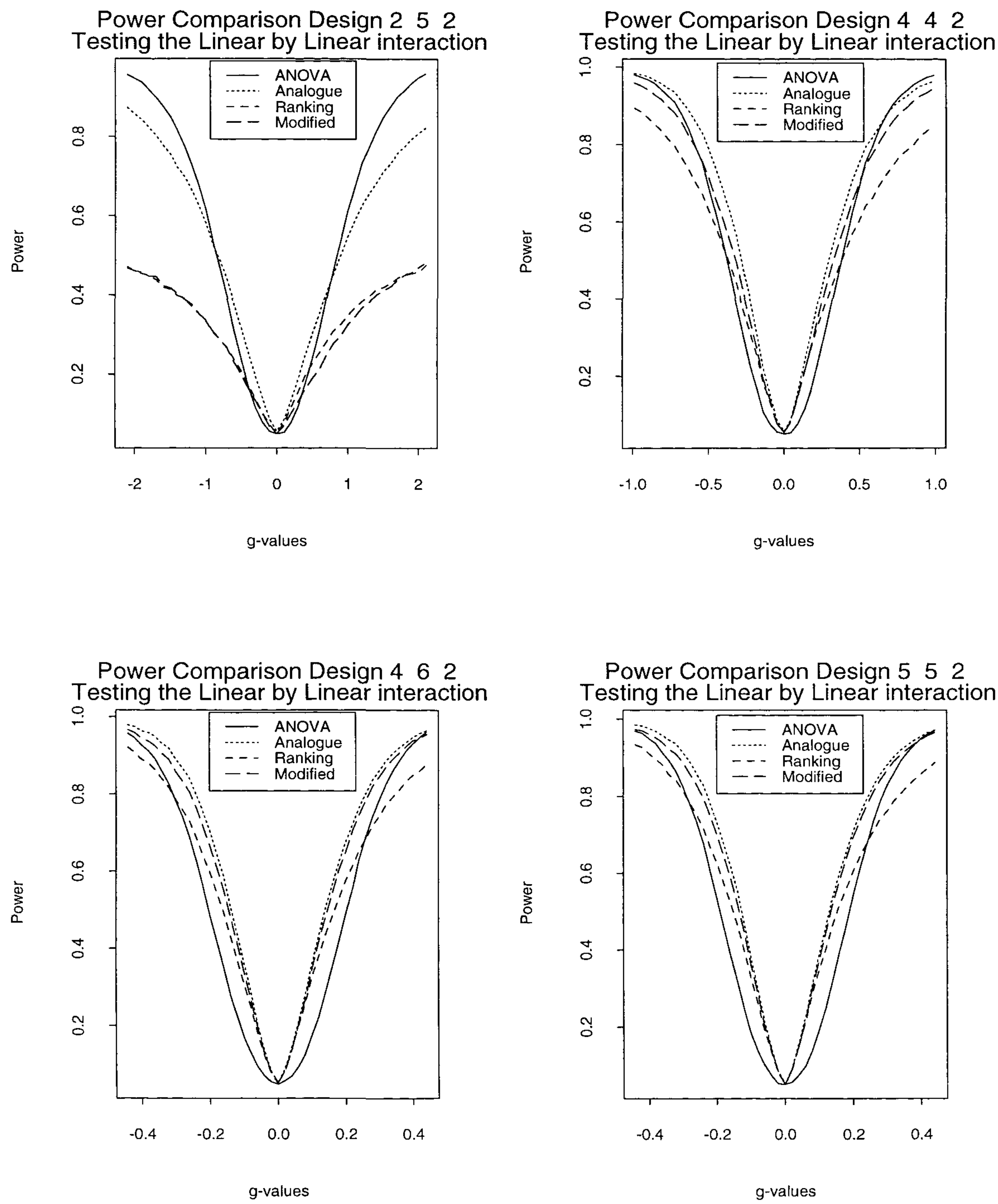


Figure 7.6: Power Comparison for Testing the Linear by Linear Component of Interaction in the Presence of the Linear Component of Factor U and the Linear Component of Factor V for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Chi-Square One Distribution.

7.4 Power Comparison under Chi-Square Four Distribution

Figures 7.7, 7.8 and 7.9 show the power comparisons for the effects U, V and interaction respectively under chi-square distribution with four degrees of freedom. For the U effect the following features are noted.

1- For designs 2 5 2 and 4 4 2 the Analogue has higher powers for g values with small magnitudes but for large magnitudes the ANOVA is more powerful. For design 2 5 2 the approximate change over points are at power values 40% and 27% and at power values of 57% and 48% for the 4 4 2 design.

2- For design 4 6 2 the change over points are at powers 80% and 86% and finally for design 5 5 2 they are at powers 86% and 91% but for large magnitudes the differences between the ANOVA and the Analogue are quite small.

For the V effect the pattern is quite similar except that for the 2 5 2 design the difference between the ANOVA and the Analogue is more than that for the U effect when large magnitudes of g are being considered. For the interaction effect the patterns for the designs 4 4 2, 4 6 2 and 5 5 2 are the same except that the gaps between the ANOVA ana the Analogue have widened. For the 2 5 2 design the power curves are the same as those for the V effect as mentioned before.

Chi-square one distribution is highly skewed and we saw that, in general, the Analogue was more powerful than the ANOVA procedure. Chi-square four distribution is less skewed and indeed with the relative power performance of ANOVA.. Thus whereas for Chi-square one the Analogue was the more powerful procedure, under Chi-square four the ANOVA has gained efficiency so that for large magnitudes of g values the powers for the ANOVA procedure are now higher than those for the Analogue. This superiority in power is more distinct in small designs and with increase in design size tends to lose intensity.

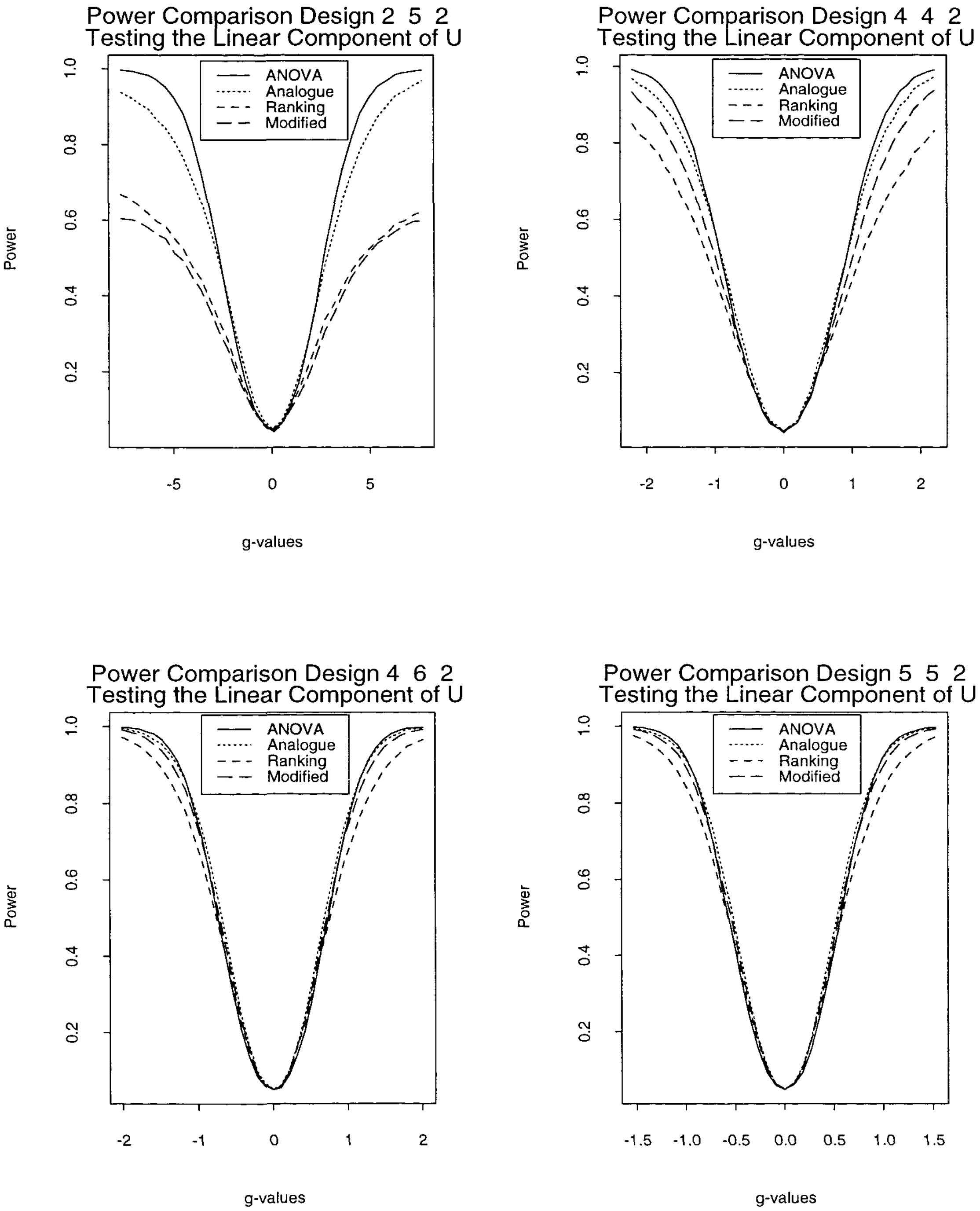


Figure 7.7: Power Comparison for Testing the Linear Component of Factor U in the Presence of the Linear Component of Factor V and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Chi-Square Four Distribution.

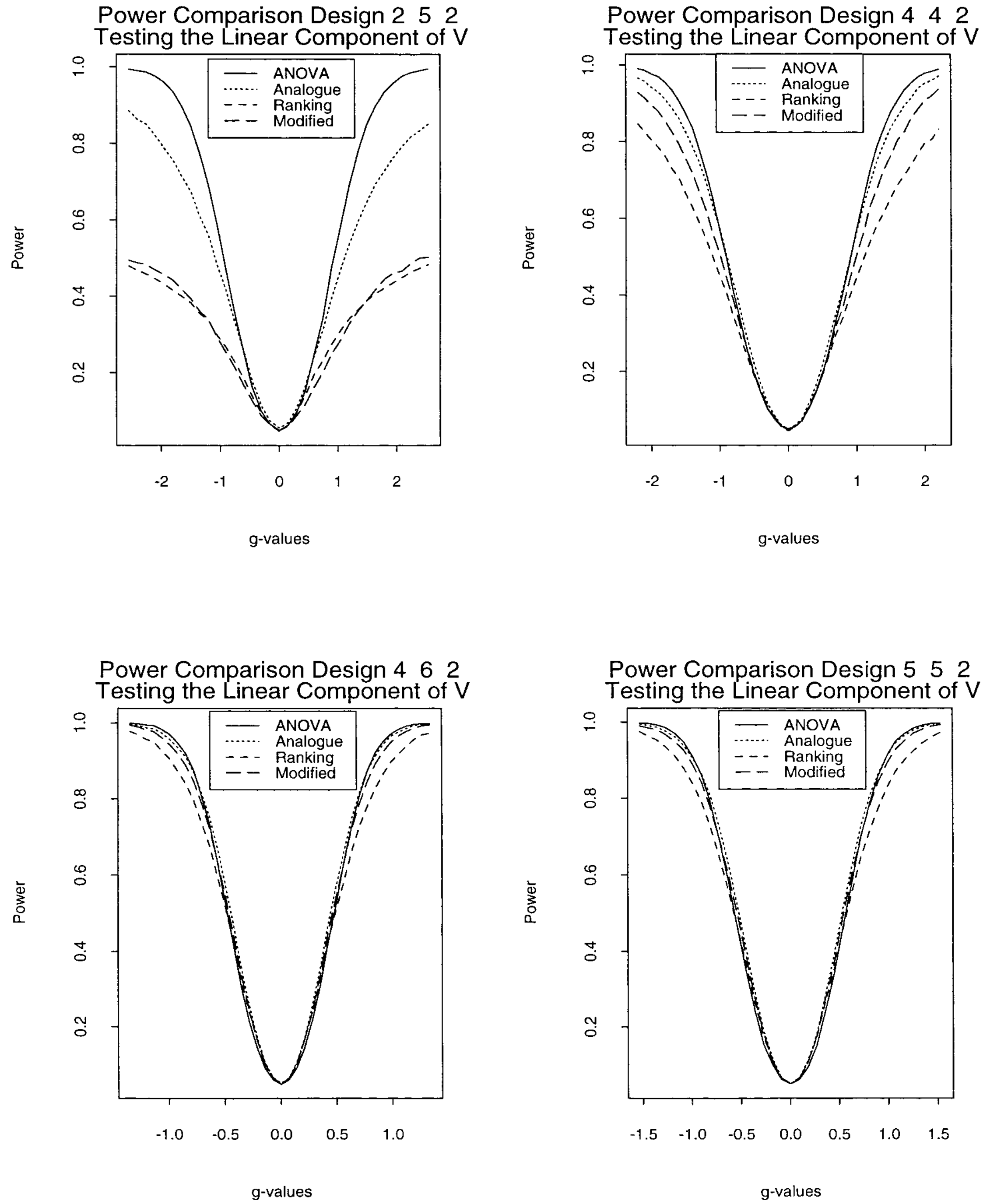


Figure 7.8: Power Comparison for Testing the Linear Component of Factor V in the Presence of the Linear Component of Factor U and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Chi-Square Four Distribution.

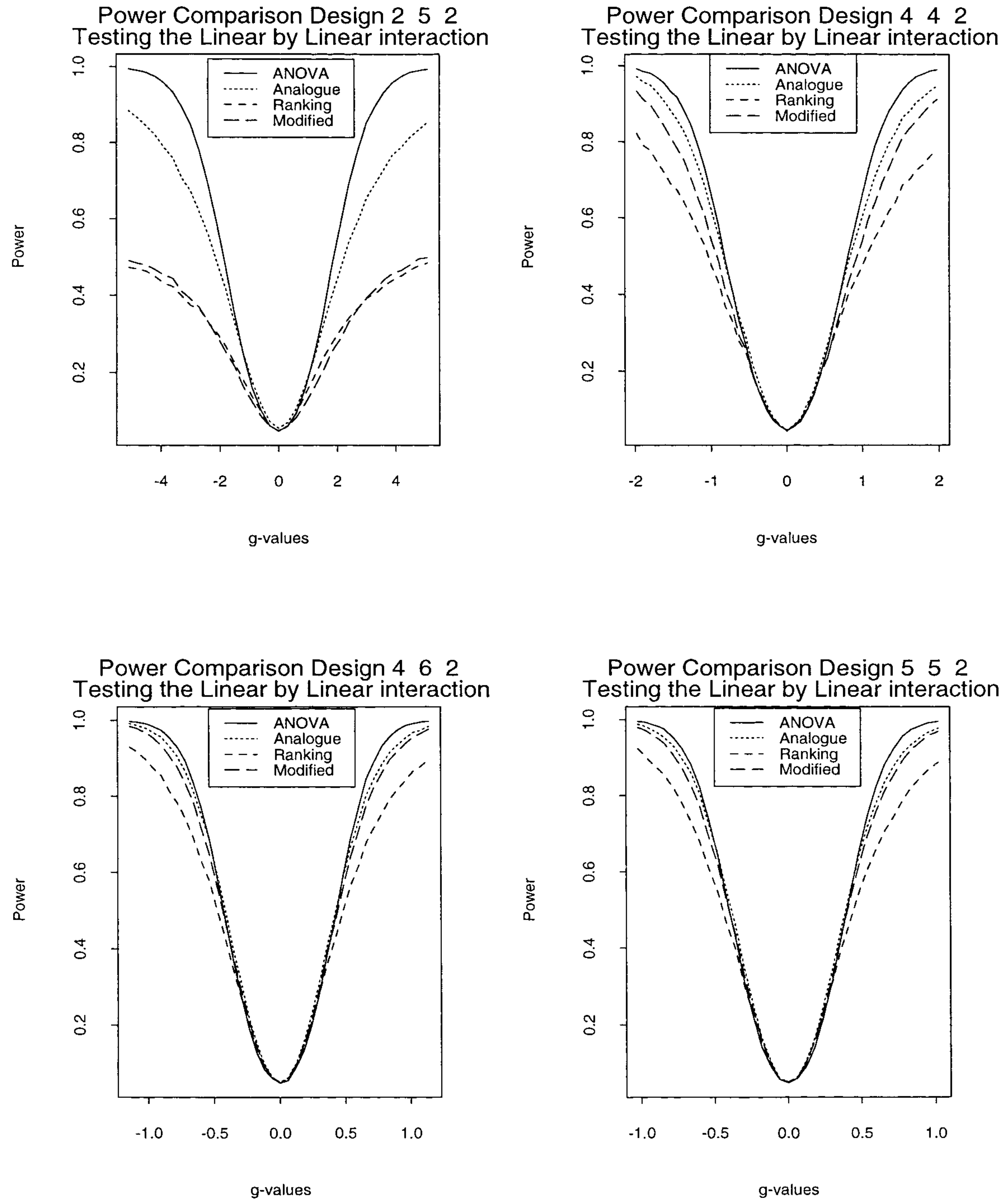


Figure 7.9: Power Comparison for Testing the Linear by Linear Component of Interaction in the Presence of the Linear Component of Factor U and the Linear Component of Factor V for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Chi-Square Four Distribution.

7.5 Power Comparisons under Cauchy Distribution

In previous sections we showed that when the error distribution is skewed then the Analogue performs better than the Analysis of Variance procedure, the more skewed the underlying distribution, the better the relative power performance of the Analogue. In this section we shall investigate the powers under Cauchy distribution. Here although we have an error distribution which is symmetric around zero, yet the errors are exposed to extreme outliers much larger than normal and thus we expect power loss for the ANOVA due to deviation from normality. The non-parametric procedures, however, being concerned only with the ranks of the observations are not so much affected.

Figures 7.10, 7.11 and 7.12 show the power comparisons for the U effect, V effect and interaction respectively. We notice that ANOVA does not hold the significance level $\alpha = 0.05$ because here the outliers inflate the error which leads to the test not even retaining the nominal level. We can also see that for the three designs 4 4 2, 4 6 2 and 5 5 2 the three non-parametric methods proposed perform much better than the ANOVA. For the 2 5 2 design only the Analogue has higher powers than ANOVA. For this design for the U effect when $g_1 = -35.7$, the power for the Analogue is .957 compared with .917 for the ANOVA, .834 for the Ranking Method and .522 for the Modified Method. With increase in design size the differences become dramatic. For design 5 5 2 for U effect when $g_1 = -2.550$, the power for the Analogue is .994 compared with .989 for the Modified Method, .976 for the Ranking Method and .650 for the ANOVA. The same pattern is seen for the V effect and the interaction. Thus we notice there is a huge power loss on the ANOVA procedure due to the existence of outliers.

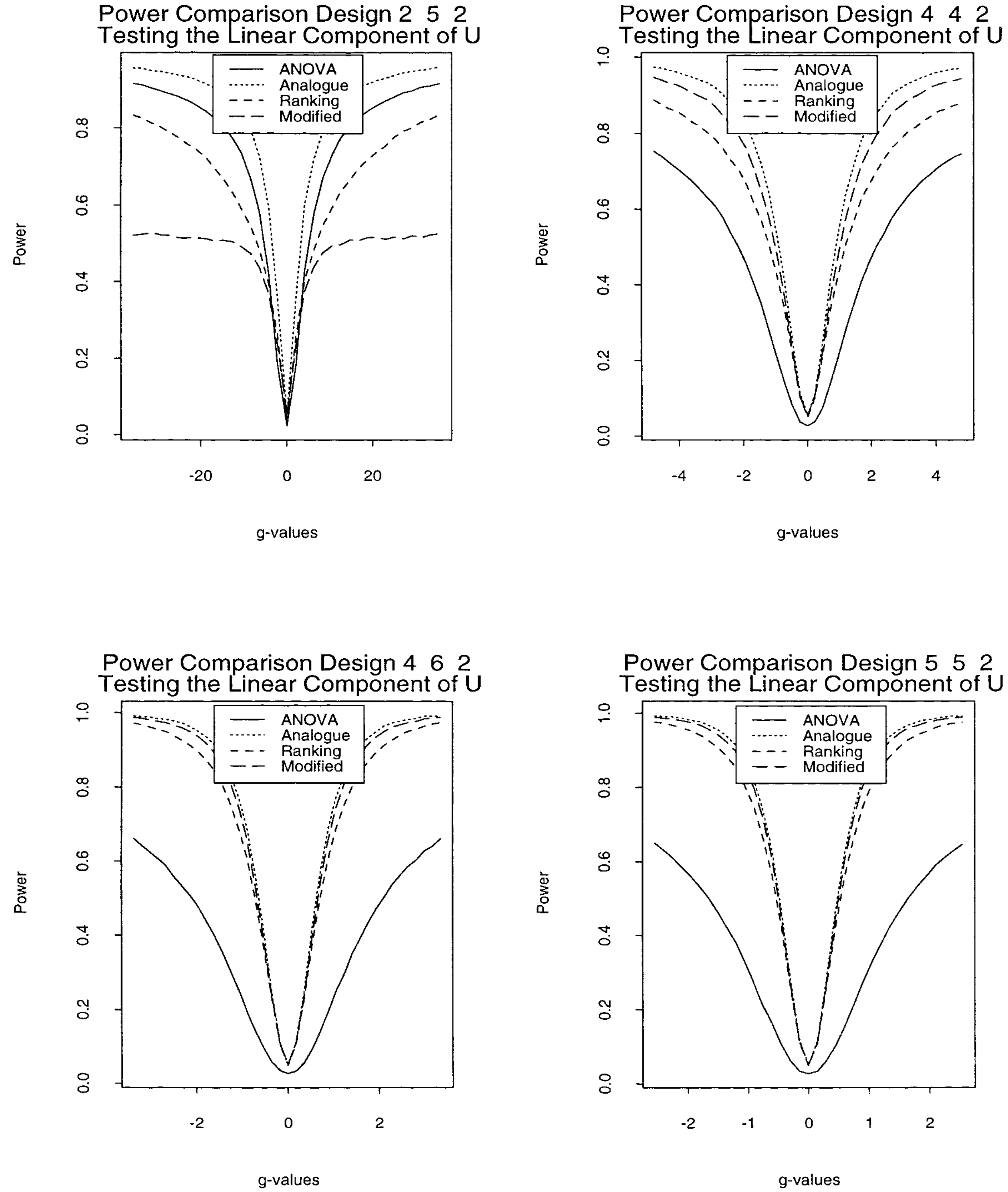


Figure 7.10: Power Comparison for Testing the Linear Component of Factor U in the Presence of the Linear Component of Factor V and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Cauchy Distribution.

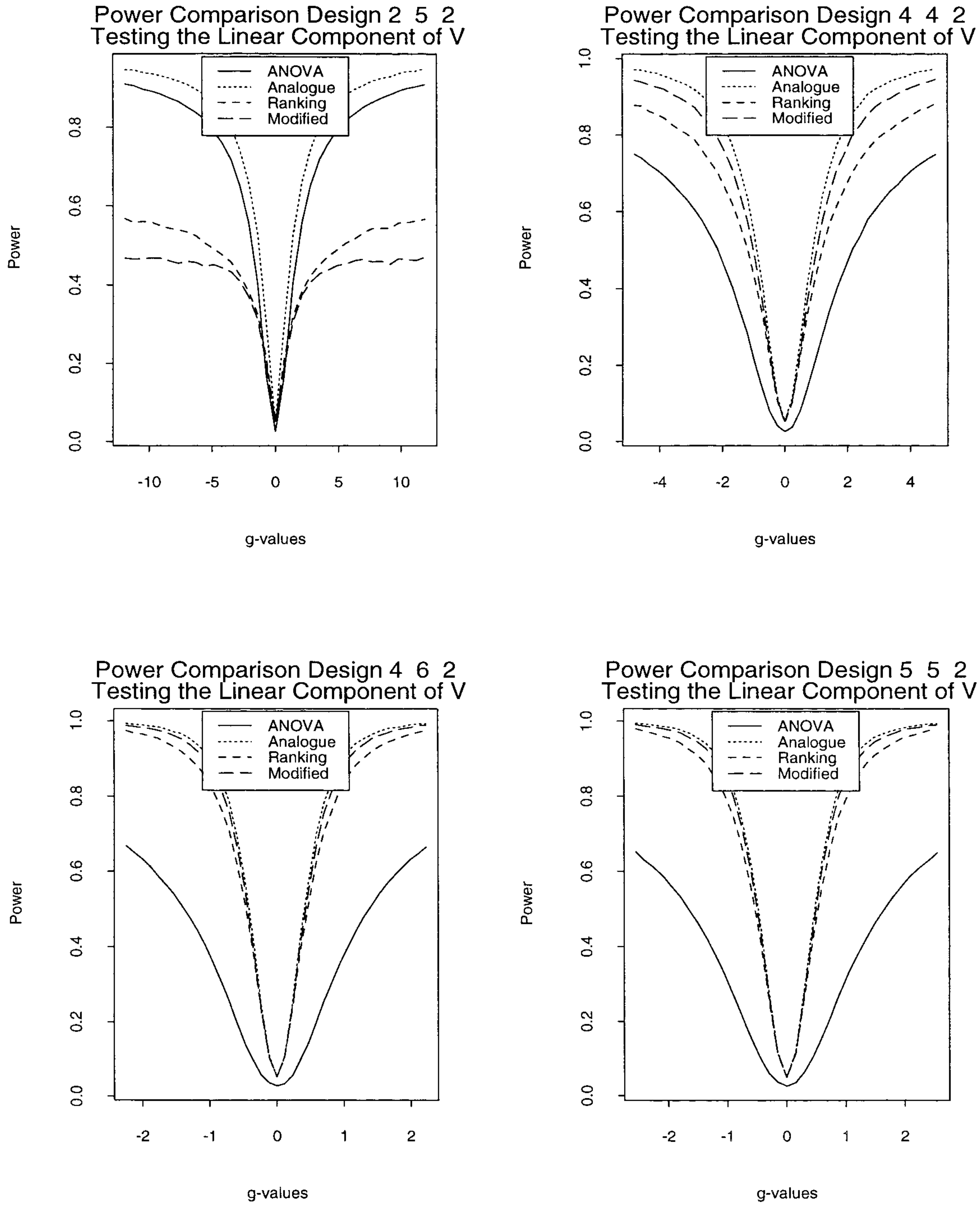


Figure 7.11: Power Comparison for Testing the Linear Component of Factor V in the Presence of the Linear Component of Factor U and the Linear by Linear Component of Interaction for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Cauchy Distribution.

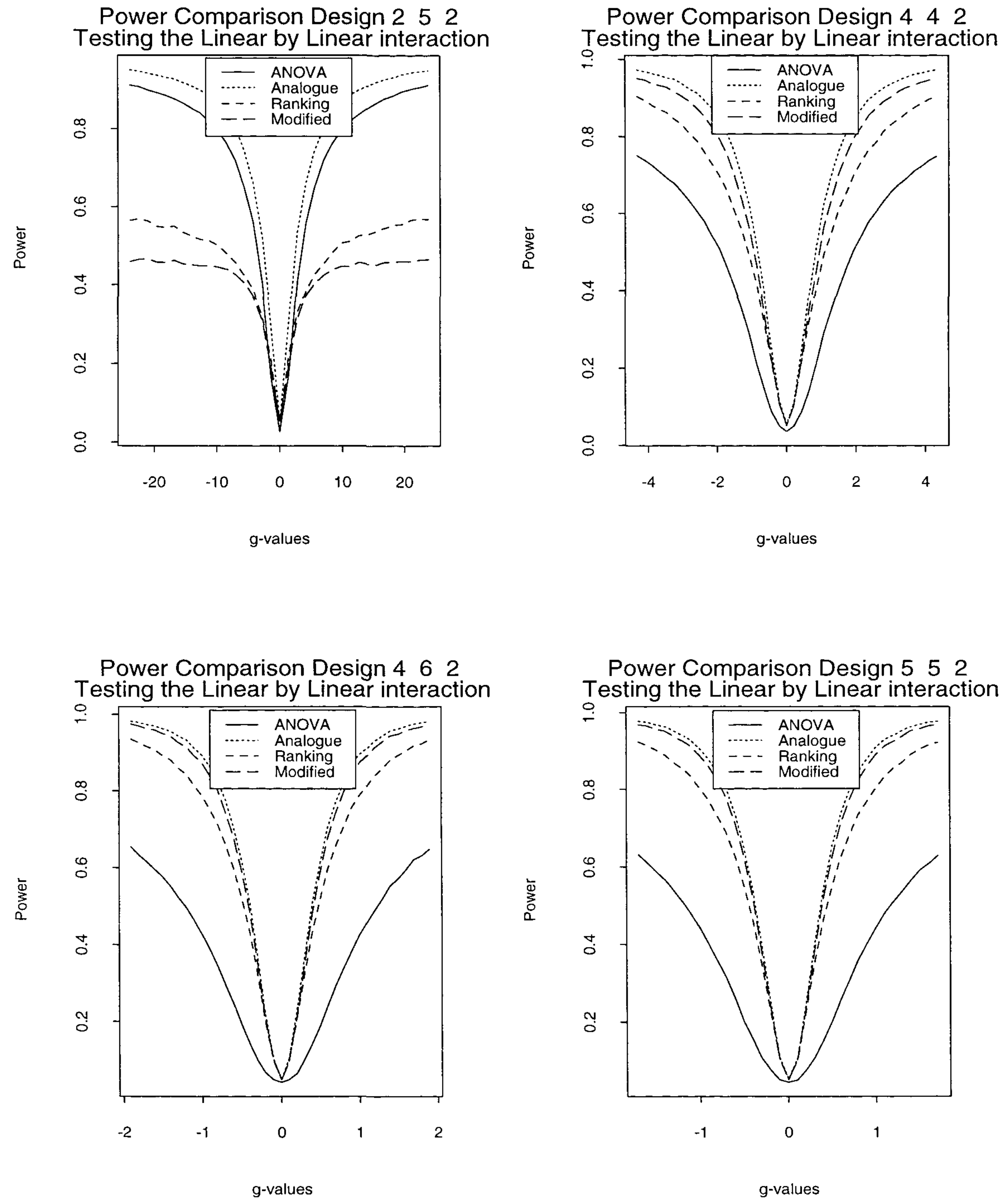


Figure 7.12: Power Comparison for Testing the Linear by Linear Component of Interaction in the Presence of the Linear Component of Factor U and the Linear Component of Factor V for the Designs 2 5 2, 4 4 2, 4 6 2, and 5 5 2 under Cauchy Distribution.

7.6 Conclusions

Power comparisons under exponential distribution, chi-square one, chi-square four and Cauchy distribution reveals the fact that deviation from normality affects the ANOVA performance severely and that the relative performance of the Analogue against the ANOVA improves under the four distributions studied. Under Cauchy distribution not only does the Analogue perform much better but the Modified and the Ranking method also have higher powers when the design size is not small. Under exponential and chi-square one distributions the Analogue has the better overall performance when the design size is not small. Under chi-square four distribution the overall performance of the Analogue is slightly better than the ANOVA for the larger designs 4 6 2 and 5 5 2 but for the other two designs the ANOVA has the better overall performance. Thus with decrease in skewness and approach to normality the ANOVA gains power.

This research has revealed the following:

1- Under conditions of normality the analysis of variance procedure has higher powers than the non-parametric procedures proposed for the analysis of a factorial experiment with two factors run in a completely randomised block structure. However the powers under ANOVA are only slightly higher than those for the Analogue.

2- When the normality assumption is violated, the ANOVA is no longer the optimum method for the analysis and the non-parametric procedure proposed in this work is more efficient if the design size is not small.

3- If the underlying population has a Cauchy distribution, the Analogue has much better power performance for all the designs studied.

7.7 Summary of the Work

The origin of our research was a problem in biology that required the design and analysis of a blocked factorial experiment. The nature of the problem was such that accurate measurements of the response variable were not possible and thus the analysis of variance procedure was not applicable. An attempt was made to extend the Friedman test to a factorial experiment. Ranking was applied independently within each block and orthogonal polynomial analysis used to test the components of main effects and interaction. Exact null distributions were obtained for small designs and the results tabulated. It was shown that, with a slight increase in design size, application of a normal approximation was feasible. Power comparisons revealed that the procedure was powerful and comparable to ANOVA if the effect under study was isolated, but with other components being present the test showed a substantial power loss. It was shown that the loss in power was due to over-estimation of the variance under the alternative hypothesis and that the variance of the contrast was maximum under the null hypothesis of no treatment effects.

A modification was proposed by estimating the variance through pooling the non-linear components of the interaction. Null distributions were simulated and shown to have approximate normal distributions with increase in design size. Power comparisons indicated that although there was some improvement in power performance yet the effect of extraneous components was still evident. Thus a further modification was adopted by estimating the contrast variance through the error variance between replications in different blocks. This was found to be effective in reducing the power loss due to the presence of the nuisance components. Power analysis showed that the procedure was efficient and comparable to ANOVA.

All the comparisons carried out so far were based upon the assumption of normality which is essential to the analysis of variance procedure. Finally power comparisons were carried out under various non-normal conditions. The analysis revealed that when the error distribution under study is skewed the non-parametric procedure developed in this work is in general more powerful than the analysis of variance procedure if the design size is not too small. The more skewed the underlying distribution the more relative gain in power for the non-parametric method. Power comparisons under Cauchy distribution revealed that the proposed non-parametric procedure was superior to the ANOVA procedure for all the parameter values for the four designs under study.

7.8 Recommendations for Future Work

The work done so far can be extended in the following ways.

1. The power comparisons were used only for the linear components of the main effects and interactions. The analysis can be extended by including higher order components in the model.
2. The method was used for a factorial experiment with two factors only, the procedure may be applied to larger factorial experiments.
3. In some cicumstances where the normal distribution is not applicable a contaminated normal distribution might be a good approximation for the population under study when outliers occur or when there is skewness. Power comparisons can be made under various contaminated normal distributions.
4. There are instances where exact measurements are available but it may be assumed that the underlying continuous variable does not follow a normal distribution. In such cicumstances one might apply the ANOVA procedure based on normal scores. A power comparison can be made between this method and the proposed procedures based on ranks.

5. Throughout this work, assessment of the main effects and interactions has been made based on tests of hypothesis. Attempts can be made to find confidence intervals for the effects using ranks.

   Theoretical work can also be undertaken in the following directions.

6. The two conjectures made to prove the theorem that the variance of a contrast attains its maximum under the assumption of no treatment effects can be proved.

7. The distribution of the rank of each treatment under the null hypothesis is a uniform discrete distribution. We could study the distribution of the ranks under a range of alternative treatment means.

8. Asymptotic null distributions have been dealt with heuristically throughout the work based on simulations. One could establish them theoretically.

9. One could study the asymptotic relative efficiency of the Analogue.

Appendix

```
C      This program is for a power comparison between the ANOVA procedure
C    and three non-parametric methods for a factorial experiment with two
C    factors in a randomised block structure.  The analysis is  based  on
C    orthogonal contrasts corresponding to orthogonal polynomials.
C    For the non-parametric methods the data  are  ranked  separately  in
C    each block and the three methods are constructed as  following.  For
C    the first method the contrasts are constructed using the ranks.  The
C    contrasts are then divided by their corresponding standard deviation
c    under the null hypothesis of no effect at all. For the second method
c    all the interaction components except the linear by linear component
c    are pooled together to give us an estimate for the variance  of  the
c    contrasts. F distribution provides a good approximation for the null
c    distributions.  For the third method the analogue of ANOVA is  carried
c    out on the ranks.  Data is generated from a normal distribution  and
c    for any desired model the power function is  obtained  by  simulation
c    for every method and the results are then compared for all the  com-
c    ponents of the main effects and interaction considered in the model.
C************************************************************************
      implicit double precision (A-H,O-Z)
      dimension DATAO(10,10,11)
      dimension g1(50),g2(50),g3(50)
      dimension FR(10),FC(10),p(50,25),p1(50,25),p2(50,25),p3(50,25)
      double precision tmp(30000)
      dimension F1(25,30000),F(25,30000),F2(25,30000),F3(25,30000)
      dimension FI(10,10),CR(10,10),CC(10,10),CMR(10)
      dimension CMC(10),CMI(10,10)
      dimension CMCN(10),CMRN(10),CMIN
      dimension A(10,10,11),B(10,10,11)
      common // ic, ir, in, i1
      common /test/ g1,g2,g3,i2
      OPEN( UNIT=11, FILE='n123252i')
      OPEN( UNIT=12, FILE='n123252o')
      READ(11,*)  ISEED1,ISEED2
      DO 10 I=1,10
      CALL RAND(R1,R2,ISEED1,ISEED2)
   10 CONTINUE
      READ(11,*)IR,IC,IN,id,if
```

```
      DO   25    I=1,IR-1
 25   READ(11,*)(CR(I,J), J=1,IR)
      DO   30    I=1,IC-1
 30   READ(11,*)(CC(I,J), J=1,IC)
      read(11,*)c,c1,c2
      do   99  i2=1,if
      g3(i2)=-1.7+.1*float(i2-1)
      g2(i2)=g3(i2)/2.
      g1(i2)=1.5*g3(i2)
      DO   90    I1=1,ID
      it=i1
111   DO   21    I=1,IR
      DO   21    K=1,IN
      do   21    j=1,ic
      call normal(z1,z2, iseed1,iseed2)
      z=z1
      DATAO(I,J,K)=g1(i2)*float(i-1.5)+g2(i2)*float(j-3)+
     +g3(i2)*float((i-1.5)*(j-3))+z
 21   A(I,J,K)=datao(i,j,k)
      CALL FACT2 (CR,CC,DATAO,FR,FC,FI,F)
      if(it.eq.i1)then
      go to 112
      else
      i1=i1+1
      go to  111
      endif
112   do  31  i=1,ir*ic-1
 31   F3(i,I1)=F(i,I1)
      call rankc(ir,ic,in,a,b)
      DO   20    k=1,IN
      DO   20    J=1,IC
      DO   20    I=1,IR
 20   DATAO(I,J,K)=b(i,j,k)
      CALL FACT2 (CR,CC,DATAO,FR,FC,FI,F)
      if(it.eq.i1)then
      go  to  113
      else
      i1=i1+1
      go  to  111
      endif
113   CALL RANK2 (CR,CC,DATAO,CMR,CMC,CMI,CMRN,CMCN,CMIN,F1,F2)
      if(it.eq.i1)then
      go to 90
      else
      i1=i1+1
      go  to  111
      endif
 90   continue
      call stat (ID,F,TMP)
      call stat (ID,F3,TMP)
```

```
      call stat (ID,F1,TMP)
      call stat (ID,F2,TMP)
      call  PVAL(F,C,ID,P,i2)
      call  PVAL(F3,C,ID,P3,i2)
      call  PVAL(F1,C1,ID,P1,i2)
      call  PVAL(F2,C2,ID,P2,i2)
      write(12,14)
   14 format(9x,'Power Comparison for the Non-parametric Methods And
     + the ANOVA'/,30x,'Normal  Distribution')
      write(12,16) ir,ic,in,id
  16  format(5x,'  Model  ',5x, 'IR=',I2,5x,'IC='I2,5x,'IN='I2,
     +10x,'ID='I6)
      write(12,9) d1,d2,d3
   9  format(2x ,'g = ',f8.3,'   g1 = ',f8.3,'   g2 = ',f8.3/)
      write (12,7)
   7  format(13x,'Simulated Powers for the ANOVA and Ranking methods'/
     +'************************************************************')
      write(12,11)
   11 format(5x,'         p = 0.050                        p= 0.025
     +    '/4x,'anova   analog  ranks  modified    anova   analog  ranks
     +   modified')
         do   97  i=1,ir+ic-1
      write (12,8) P3(i2,i),P(i2,i),P1(i2,i),P2(i2,i)
   8  FORMAT(2x,F7.3,1x,F7.3,1x,F7.3,2x,F7.3,3x,F7.3,1x,F7.3,1x,
     +F7.3,2x,F7.3)
  97  continue
   99 continue
      write(12,60)
   60 format(9x,'Power Comparison for the Non-parametric Methods And
     + the ANOVA'/,29x,'Normal  Distribution')
      write(12,61) ir,ic,in,id
  61  format(5x,'  Model  ',5x, 'IR=',I2,5x,'IC='I2,5x,'IN='I2,
     +10x,'ID='I6)
       write(12,*)'            y=g1(i-1.5)+g2(j-3)+g3(i-1.5)(j-3)+z'
c      write(12,62) d1,d2,d3
c 62   format(2x ,'g = ',f8.3,'   g1 = ',f8.3,'   g2 = ',f8.3/)
      write (12,63)
  63  format(13x,'Simulated Powers for the ANOVA and Ranking methods'/
     +'**********************Linear Main Effect U*****************')
      write(12,64)
   64 format(5x,'                   p = 0.050
     +    '/14x,'g-value    anova    analog   ranks   modified')
      do   66  i=1,if
      write (12,65) g1(i),P3(i,1),P(i,1),P1(i,1),P2(i,1)
   65  FORMAT(13x,F7.3,4x,F7.3,3x,F7.3,1x,F7.3,3x,F7.3)
  66  continue
      write(12,*)'****************************************************'

      write (12,73)
  73  format(13x,'Simulated Powers for the ANOVA and Ranking methods'/
```

```
     +'**********************Linear Main Effect V*****************')
      write(12,64)
      do   77  i=1,if
              j=ir
      write (12,65) g2(i),P3(i,j),P(i,j),P1(i,j),P2(i,j)
  77  continue
      write(12,*)'*****************************************************'
      write (12,15)
  15   format(13x,'Simulated Powers for the ANOVA and Ranking methods'/
     +'********************   Linear by Linear Interaction *********')
      write(12,64)
      do   98  i=1,if
              j=ir+ic-1
      write (12,65) g3(i),P3(i,j),P(i,j),P1(i,j),P2(i,j)
  98  continue
      write(12,*)'*****************************************************'
      stop
      end
C     *************************************************************
      SUBROUTINE RAND(R1,R2,IN,IM)
      implicit double precision (A-H,O-Z)
      DOUBLE PRECISION U1, U2, C
      DATA G, A, B/1.0, 129.0, 4097.0/
      DATA C, D/3435973836.8D1, 0.788/
c*      print *, 'Entered RAND'
      IF (G .EQ. 0.0) GOTO 10
      U1=IN
      U2=IM
      G=0.0
   10 U1= U1*B + 1.0
      U1 = DMOD(U1,C)
      U2=(U2*A + D*C)*A+D*C
      U2=DMOD(U2,C)
      R1=U1/C
      R2=U2/C
c*        *, 'Exit RAND'
      RETURN
      END
c     ******************************
      SUBROUTINE NORMAL(Z1,Z2,IN,IM)
c*      print *, 'Entered NORMAL'
      implicit double precision (A-H,O-Z)
      CALL RAND(R1,R2,IN,IM)
      P1=SQRT(-2.0*LOG(R1))
      P2=6.284318*R2
      Z1=P1*COS(P2)
      Z2=P1*SIN(P2)
c*      print *, 'Exit NORMAL'
      RETURN
      END
```

```
c     ********************************
      SUBROUTINE FACT2 (CR,CC,DATAO,FR,FC,FI,F)
      implicit double precision (A-H,O-Z)
      double precision DATAO(10,10,11),DATA(10,10),RSUM(10)
      double precision CSUM(10),SSR(10),SSC(10),SSI(10,10)
      double precision FR(10),FC(10)
      double precision F(25,30000)
      double precision FI(10,10),CR(10,10),CC(10,10),CMR(10)
      double precision CMC(10),CMI(10,10),DR(10),DC(10),D(10,10)
      double precision msrow, mscol, msint,mse,msbl
      common // ic, ir, in, i1
      common /test/ g1(50),g2(50),g3(50),i2

C         *********CALCULATION OF TOTAL SUM OF SQUARES**********
c*      print *, 'Entered FACT2'
      T=0
      SS=0
      DO   100    I=1,IR
      DO   101    J=1,IC
      DATA(I,J)=0
      DO   102    K=1,IN
      T=T+DATAO(I,J,K)
      SS=SS+ (DATAO(I,J,K))**2
      DATA(I,J)=DATA(I,J)+DATAO(I,J,K)
 102  CONTINUE
 101  CONTINUE
 100  CONTINUE
      C=(T**2)/(IR*IC*IN)
      SST=SS-C
C     ******************ROW AND COLUMN TOTALS********************
      DO   103    I=1,IR
      RSUM(I)=0
 103  CONTINUE
      DO   104    I=1,IR
      DO   105    J=1,IC
      RSUM(I)=RSUM(I)+DATA(I,J)
 105  CONTINUE
 104  CONTINUE
      DO   106    J=1,IC
      CSUM(J)=0
 106  CONTINUE
      DO   107    J=1,IC
      DO   108    I=1,IR
      CSUM(J)=CSUM(J)+DATA(I,J)
 108  CONTINUE
 107  CONTINUE
C     *************ORTHOGONAL COMPONENTS FOR ROW EFFECTS*************
      SSROW=0
      DO   201   I=1,IR-1
      CMR(I)=0
```

```
      DR(I)=0
      DO   200   J=1,IR
      CMR(I)=CMR(I)+RSUM(J)*CR(I,J)
      DR(I)=DR(I)+CR(I,J)**2
 200  CONTINUE
      SSR(I)=CMR(I)**2/(IC*IN*DR(I))
      SSROW=SSROW+SSR(I)
      MSROW=SSROW/(IR-1)
 201  CONTINUE
C     **********ORTHOGONAL COMPONENTS FOR COLUMN EFFECTS*********
      SSCOL=0
      DO   301   I=1,IC-1
      CMC(I)=0
      DC(I)=0
      DO   300   J=1,IC
      CMC(I)=CMC(I)+CSUM(J)*CC(I,J)
      DC(I)=DC(I)+CC(I,J)**2
 300  CONTINUE
      SSC(I)=CMC(I)**2/(IR*IN*DC(I))
      SSCOL=SSCOL+SSC(I)
      MSCOL=SSCOL/(IC-1)
 301  CONTINUE
C       ******************SUM OF SQUARES BLOCKS*********************
      TBL=0
      DO  5  K=1,IN
      TK=0
      DO  6  J=1,IC
      DO  7  I=1,IR
      TK=TK+DATAO(I,J,K)
   7  CONTINUE
   6  CONTINUE
      TBL=TBL+TK**2
   5  CONTINUE
      SSBL=(TBL/(IR*IC))-C
      MSBL=SSBL/(IN-1)
C     ************ORTHOGONAL COMPONENTS FOR INTERACTION EFFECTS*****
      SSINT=0
      DO   400    I=1,IR-1
      DO   401    J=1,IC-1
      D(I,J)=0
      CMI(I,J)=0
      DO   402    K=1,IR
      DO   403    L=1,IC
      CMI(I,J)=CMI(I,J)+CR(I,K)*CC(J,L)*DATA(K,L)
      D(I,J)=D(I,J)+(CR(I,K)*CC(J,L))**2
 403  CONTINUE
 402  CONTINUE
      SSI(I,J)=CMI(I,J)**2/(IN*D(I,J))
      SSINT=SSINT+SSI(I,J)
      MSINT=SSINT/((IR-1)*(IC-1))
```

```
 401  CONTINUE
 400  CONTINUE
      SSE=SST-SSROW-SSCOL-SSINT-SSBL
      MSE=SSE/((IR*IC-1)*(IN-1))
      if (mse .eq. 0.0 ) then
      write(12,444), g1(i2),g2(i2),g3(i2), sst,ssrow,sscol,ssint,ssbl
 444  format(5x, 'g1(i2)=',f6.2,5x, 'g2(i2)=',f6.2,5x, 'g3(i2)=',f6.2,/
     +2x,'sst=',f8.2,2x,'ssrow=',f8.2,2x,'sscol=',f8.2,'ssint=',f8.2,
     +2x,'ssbl='f8.2/)
      write(12,445), i1, i2
 445  format(5x,'i1=',I6, 5x,'i2=',I2/)
        do 999, n=1,in
        do 998, l=1,ir
          write(12, '(10F7.1)'), (datao(l,m,n), m=1,ic)
 998    continue
 999    continue
       i1=i1-1
       else
C     **************************************************************
      DO   454   I=1,IR-1
      FR(I)=SSR(I)/MSE
 454  CONTINUE
C     ****************************************************************
      DO   455   I= 1,IC-1
      FC(I)=SSC(I)/MSE
 455  CONTINUE
C     ******************************************************************
      DO   456   I=1,IR-1
      DO   457   J=1,IC-1
      FI(I,J)=SSI(I,J)/MSE
 457  CONTINUE
 456  CONTINUE
C     ******************************************************************
      J=0
      DO  410  I=1,IR-1
      J=J+1
      F(J,I1)=FR(I)
  410 CONTINUE
      DO  411  I=1,IC-1
      J=J+1
      F(J,I1)=FC(I)
  411 CONTINUE
      J=J+1
      F(J,I1)=FI(1,1)
      endif
c*      print *, 'Exit FACT2'
      RETURN
      END
c     **********************************
      SUBROUTINE RANKC(M1, M2, m3, A, B)
```

```
C     This routine ranks the blocks of the matrix A and stores
C     the results in the matrix B.
      implicit double precision (A-H,O-Z)
      INTEGER M1, M2, M3, ORD(100)
      double precision A(10,10,10), B(10,10,10), TEMP(100), TEMP2(100)
c*     print *, 'Entered RANKC'
C     Loop through the blocks of A.
      do 300 k=1,m3
C     Create a temporary vector for a block of A.
      DO 100 J = 1, M2
         do 100 i = 1, m1
         l = i + (j-1)*m1
         TEMP(l) = A(i,j,k)
  100    CONTINUE
      mr=m1*m2
C     Sort the vector and place the ranks in the matrix B.
         CALL JQSORT (TEMP, mr, ORD)
         CALL TIES (TEMP, Mr, ORD, TEMP2)
         DO 200 J = 1, M2
         DO 200 I = 1 , M1
         L = I + (J-1)*m1
            B(I,J,K) = TEMP2(L)
  200    CONTINUE
  300 CONTINUE
c*      print *, 'Exit RANKC'
      RETURN
      END
c*********************************************
      SUBROUTINE JQSORT (A,N,ORD)
C     ACM QUICKSORT - ALGORITHM #402 - IMPLEMENTED IN FORTRAN BY
C                                WILLIAM H. VERITY
C                                COMPUTATION CENTER
C                                PENNSYLVANIA STATE UNIVERSITY
C                                UNIVERSITY PARK, PA.  16802
C
      implicit double precision (A-H,O-Z)
      INTEGER ORD,IPPLST,P,Q,U,U1,YP
      double precision A(N), X, XX, Z, ZZ, Y
      DIMENSION ORD(N),IPPLST(2,20)
C
C     TO SORT DIFFERENT INPUT LISTS, CHANGE THE FOLLOWING
C     SPECIFICATION STATEMENTS.
C
C
C     FOR ALL BUT CHARACTER SORTS, THE ARRAY "A" AND THE SIX SCALARS
C     WILL BE THE SAME TYPE.  THE SIX SCALARS HOLD VARIOUS VALUES OF THE
C     "A" VECTOR DURING EXECUTION.
*      print *, 'Entered JQSORT'
      NDEEP = 0
      U1 = N
```

```
      L1 = 1
C                                          THE FOLLOWING TWO LINES ARE
C                                          NEEDED IF NO PREVIOUS ORDER
C                                          IS GIVEN.
      DO 100  I = 1,N
  100 ORD(I) = I
  105 IF (U1.GT.L1) GO TO 110
*      print *, 'Exit JQSORT'
      RETURN
C
  110 L = L1
      U = U1
C
C PART
C
  115 P = L
      Q = U
C     FOR CHARACTER SORTS, THE FOLLOWING 3 STATEMENTS WOULD BECOME
C     X = ORD(P)
C     Z = ORD(Q)
C     IF (CLE(A(X),A(Z),LEN))GO TO 2
C
C     WHERE "CLE" IS A LOGICAL FUNCTION WHICH RETURNS "TRUE" IF THE
C     FIRST ARGUMENT IS LESS THAN OR EQUAL TO THE SECOND, BASED ON "LEN"
C     CHARACTERS.
C
      KDUM = ORD(P)
      X = A(KDUM)
      KDUM = ORD(Q)
      Z = A(KDUM)
      IF (X.LE.Z) GO TO 120
      Y = X
      X = Z
      Z = Y
      YP = ORD(P)
      ORD(P) = ORD(Q)
      ORD(Q) = YP
  120 IF (U-L.LE.1) GO TO 170
      XX = X
      IX = P
      ZZ = Z
      IZ = Q
C
C LEFT
C
  125 P = P+1
      IF (P.GE.Q) GO TO 130
      KDUM = ORD(P)
      X = A(KDUM)
      IF (X.GE.XX) GO TO 135
```

```
      GO TO 125
  130 P = Q-1
      GO TO 160
C
C RIGHT
C
  135 Q = Q-1
      IF (Q.LE.P) GO TO 140
      KDUM = ORD(Q)
      Z = A(KDUM)
      IF (Z.LE.ZZ) GO TO 145
      GO TO 135
  140 Q = P
      P = P-1
      Z = X
      KDUM = ORD(P)
      X = A(KDUM)
C
C DIST
C
  145 IF (X.LE.Z) GO TO 150
      Y = X
      X = Z
      Z = Y
      IP = ORD(P)
      ORD(P) = ORD(Q)
      ORD(Q) = IP
  150 IF (X.LE.XX) GO TO 155
      XX = X
      IX = P
  155 IF (Z.GE.ZZ) GO TO 125
      ZZ = Z
      IZ = Q
      GO TO 125
C
C OUT
C
  160 CONTINUE
      IF (.NOT.(P.NE.IX.AND.X.NE.XX)) GO TO 165
      IP = ORD(P)
      ORD(P) = ORD(IX)
      ORD(IX) = IP
  165 CONTINUE
      IF (.NOT.(Q.NE.IZ.AND.Z.NE.ZZ)) GO TO 170
      IQ = ORD(Q)
      ORD(Q) = ORD(IZ)
      ORD(IZ) = IQ
  170 CONTINUE
      IF (U-Q.LE.P-L) GO TO 175
      L1 = L
```

```
      U1 = P-1
      L = Q+1
      GO TO 180
  175 U1 = U
      L1 = Q+1
      U = P-1
  180 CONTINUE
      IF (U1.LE.L1) GO TO 185
C
C START RECURSIVE CALL
C
      NDEEP = NDEEP+1
      IPPLST(1,NDEEP) = U
      IPPLST(2,NDEEP) = L
      GO TO 110
  185 IF (U.GT.L) GO TO 115
C
C POP BACK UP IN THE RECURSION LIST
C
      IF (NDEEP.EQ.0) GO TO 105
      U = IPPLST(1,NDEEP)
      L = IPPLST(2,NDEEP)
      NDEEP = NDEEP-1
      GO TO 185
C     END QSORT
      END
c*********************************************************************
      SUBROUTINE TIES (E, N, ORD, F)
      implicit double precision (A-H,O-Z)
      INTEGER N, ORD(N), IPOS, K, ISUM, COUNT
      double precision E(N), F(N), G(81), H(81)
c*      print *, 'Entered TIES'
      DO 100 I = 1, N
         G(I) = E(ORD(I))
  100 CONTINUE
      H(N) = FLOAT(N)
      IPOS = 0
      K = 1
  125 IF (K.LT.N) THEN
         ISUM = K
         COUNT = 1
  135    IF (G(K+1).EQ.G(K)) THEN
            ISUM = ISUM + (K+1)
            K = K + 1
            COUNT = COUNT + 1
            IF (K.EQ.N) GO TO 145
            GO TO 135
         ELSE
  145       DO 150 I = 1, COUNT
      if (count .eq. 0) stop 'zero count'
```

```
               H(IPOS+I) = (FLOAT(ISUM))/COUNT
  150       CONTINUE
            IPOS = IPOS + COUNT
            K = K + 1
            GO TO 125
         END IF
      ELSE
         DO 200 I = 1, N
            F(ORD(I)) = H(I)
  200    CONTINUE
      END IF
c*      print *, 'Exit TIES'
      RETURN
      END
C     ******************************
      SUBROUTINE RANK2 (CR,CC,DATAO,CMR,CMC,CMI,CMRN,CMCN,CMIN,F1,F2)
      implicit double precision (A-H,O-Z)
      double precision DATAO(10,10,11),DATA(10,10),RSUM(10)
      double precision CSUM(10)
      double precision F1(25,30000),F2(25,30000)
      double precision CR(10,10),CC(10,10),CMR(10)
      double precision CMC(10),CMI(10,10),DR(10),DC(10),D(10,10)
      double precision CMRN(10), CMCN(10), CMIN
          common // ic, ir, in, i1
      common /test/ g1(50),g2(50),g3(50),i2
c*      print *, 'Entered RANK2'
      DO   100    I=1,IR
      DO   101    J=1,IC
      DATA(I,J)=0
      DO   102    K=1,IN
      DATA(I,J)=DATA(I,J)+DATAO(I,J,K)
 102  CONTINUE
 101  CONTINUE
 100  CONTINUE
C     *******************ROW AND COLUMN TOTALS*******************
      DO   103    I=1,IR
      RSUM(I)=0
 103  CONTINUE
      DO   104    I=1,IR
      DO   105    J=1,IC
      RSUM(I)=RSUM(I)+DATA(I,J)
 105  CONTINUE
 104  CONTINUE
      DO   106    J=1,IC
      CSUM(J)=0
 106  CONTINUE
      DO   107    J=1,IC
      DO   108    I=1,IR
      CSUM(J)=CSUM(J)+DATA(I,J)
 108  CONTINUE
```

```
 107  CONTINUE
C     ************ORTHOGONAL COMPONENTS FOR ROW EFFECTS*********
      DO   201   I=1,IR-1
      CMR(I)=0
      DR(I)=0
      DO   200   J=1,IR
      CMR(I)=CMR(I)+RSUM(J)*CR(I,J)
      DR(I)=DR(I)+CR(I,J)**2
 200  CONTINUE
      G=((IR*IC*float((IR*IC+1))/12.)*IC*IN*DR(I))**.5
      if (g .eq. 0.0 ) stop 'its G'
      CMR(I)=CMR(I)/G
      cmr(I)=(cmr(I))**2
 201  CONTINUE
C     **************ORTHOGONAL COMPONENTS FOR COLUMN EFFECTS********
      DO   301   I=1,IC-1
      CMC(I)=0
      DC(I)=0
      DO   300   J=1,IC
      CMC(I)=CMC(I)+CSUM(J)*CC(I,J)
      DC(I)=DC(I)+CC(I,J)**2
 300  CONTINUE
      G=((IR*IC*float((IR*IC+1))/12.)*IR*IN*DC(I))**.5
      if (g .eq. 0.0 ) stop 'its 2nd G'
      CMC(I)=CMC(I)/G
      cmc(I)=(cmc(I))**2
 301  CONTINUE
C     ***********ORTHOGONAL COMPONENTS FOR INTERACTION EFFECTS*******
       SCMI=0
      DO   400    I=1,IR-1
      DO   401    J=1,IC-1
      D(I,J)=0
      CMI(I,J)=0
      DO   402    K=1,IR
      DO   403    L=1,IC
      CMI(I,J)=CMI(I,J)+CR(I,K)*CC(J,L)*DATA(K,L)
      D(I,J)=D(I,J)+(CR(I,K)*CC(J,L))**2
 403  CONTINUE
 402  CONTINUE
      G=((IR*IC*float((IR*IC+1))/12.)*IN*D(I,J))**.5
      if (g .eq. 0.0 ) stop 'its 3rd G'
      CMI(I,J)=CMI(I,J)/G
      cmi(I,J)=(cmi(I,J))**2
c      SCMI  =SCMI+CMI(I,J)**2
      SCMI  =SCMI+CMI(I,J)
 401  CONTINUE
 400  CONTINUE
C      *****************************************************
      SCMI=SCMI-CMI(1,1)
      if (scmi .eq. 0.0 ) then
```

```
       write(12,555),  g1(i2),g2(i2),g3(i2)
 555  format(5x, 'g1(i2)=',f6.2,5x, 'g2(i2)=',f6.2,5x, 'g3(i2)=',f6.2)
       write(12,445), i1,i2
 445  format(5x,'i1=',I6, 5x,'i2=',I2)
       do 999, n=1,2
       do 998, l=1,2
        write(12, '(5F7.1)'), (datao(l,m,n), m=1,5)
 998    continue
 999    continue
        write(12,*), 'cmi:'
        write(12,*), (cmi(1,m),m=1,4)
      i1=i1-1
      else
C     *****************The New F-Ratios***************************
      DO   501   I=1,IR-1
      CMRN(I)=(FLOAT((IC-1)*(IR-1)-1))*CMR(I)/SCMI
  501 CONTINUE
      DO   502   I=1, IC-1
      CMCN(I)=(FLOAT((IC-1)*(IR-1)-1))*CMC(I)/SCMI
  502 CONTINUE
      CMIN   =(FLOAT((IC-1)*(IR-1)-1))*CMI(1,1)/SCMI
cc      write(12,*)'***************************************** '
      J=0
      DO  410  I=1,IR-1
      J=J+1
      F2(J,I1)=CMRN(I)
      F1(J,I1)=CMR(I)
  410 CONTINUE
      DO  411  I=1,IC-1
      J=J+1
      F2(J,I1)=CMCN(I)
      F1(J,I1)=CMC(I)
  411 CONTINUE
      J=J+1
      F2(J,I1)=CMIN
      F1(J,I1)=CMI(1,1)
      endif
c*      print *, 'Exit RANK2'
      RETURN
      END
C     **************************************
      SUBROUTINE STAT(ID,F,TMP)
      implicit double precision (A-H,O-Z)
      double precision tmp(50000)
      double precision f(25,30000)
      common // ic, ir
c*      print *, 'Entered STAT'
      DO  91   J=1,ir+ic-1
      DO  92   I=1,ID
   92 TMP(I)=F(J,I)
```

```
      CALL m01caf(TMP,1,ID,'A',ifail)
      K1=18.0*ID/20.0
      K2=19.0*ID/20.0
      K3=19.5*ID/20.0
      K4=19.8*ID/20.0
      K5=19.9*ID/20.0
c      WRITE(12,93) TMP(K1),TMP(K2),TMP(K3),TMP(K4),TMP(K5)
c  93  FORMAT(2X,5(3X,F11.3))
      do 94 i=1,id
  94  f(j,i)=tmp(i)
  91  CONTINUE
c*      print *, 'Exit STAT'
      RETURN
      END
c     **********************************************************
      SUBROUTINE PVAL(F,C,ID,P,k)
      implicit double precision (A-H,O-Z)
      double precision F(25,30000),P(50,25)
      common // ic, ir
c      print *, 'Entered PVAL'
      DO  10  I=1,ir+ic-1
      j=id
  15  z=abs(f(i,j))
      IF(z.Ge.C) GO TO 25
      p(k,i)=float((ID-J))/float(ID)
      GO TO 20
  25  J=J-1
      if(j.eq.0) then
      go to 26
      else
      GO TO 15
      endif
  20  j=1
  35  z=abs(f(i,j))
      if(z.ge.c) go to 45
      P(k,i)=p(k,i)+float(j-1)/float(id)
      go to 10
  45  j=j+1
      if(j-id.gt.0) go to 26
      go to 35
  26  p(k,i)=1
  10  CONTINUE
c      print *, 'Exit PVAL'
      RETURN
      END
c     *************************************************************
      integer function myhandler( sig, code, context )
      call abort()
      end
```

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